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MATHEMATICS COMPLEX NUMBER QUESTIONS AND ANSWERS

QUICK REVISION

Imaginary Numbers

The square root of a negative real number is called imaginary number, e.g. $\sqrt{-2}, \sqrt{-5}$ etc.

The quantity $\sqrt{-1}$ is an imaginary unit and it is denoted by 'i' called **iota**.

Integral Power of IOTA (i)

$$i = \sqrt{-1}, i^2 = -1, i^3 = -i, i^4 = 1$$

So, $i^{4n+1} = i, i^{4n+2} = -1, i^{4n+3} = -i, i^{4n} = 1$

- For any two real numbers a and b, the result $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is true only, when atleast one of the given numbers is either zero or positive.
$$\sqrt{-a} \times \sqrt{-b} \neq \sqrt{ab} \text{ So, } i^2 = \sqrt{-1} \times \sqrt{-1} \neq 1$$
- 'i' is neither positive, zero nor negative.
- $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$

Complex Numbers

A number of the form $x + iy$, where x and y are real numbers, is called a complex number. Here, x is called real part and y is called imaginary part of the complex number, i.e. $\operatorname{Re}(z) = x$ and $\operatorname{Im}(z) = y$.

Purely Real and Purely Imaginary Complex Numbers

A complex number $z = x + iy$ is a purely real if its imaginary part is 0, i.e. $\operatorname{Im}(z) = 0$ and purely imaginary if its real part is 0 i.e. $\operatorname{Re}(z) = 0$.

Equality of Complex Numbers

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal, iff $x_1 = x_2$ and $y_1 = y_2$

i.e. $\operatorname{Re}(z_1) = \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) = \operatorname{Im}(z_2)$

Other relation 'greater than' and 'less than' are not defined for complex number.

Algebra of Complex Numbers

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be any two complex numbers.

(i) **Addition of Complex Numbers**

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2)$$

Properties of addition

- **Closure** $z_1 + z_2$ is also a complex number.
- **Commutative** $z_1 + z_2 = z_2 + z_1$
- **Associative** $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$
- **Existence of additive identity**
$$z + 0 = z = 0 + z$$
Here, 0 is additive identity.
- **Existence of Additive inverse**
$$z + (-z) = 0 = (-z) + z$$
Here, $-z$ is additive inverse.

(ii) **Subtraction of Complex Numbers**

$$z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) = (x_1 - x_2) + i(y_1 - y_2)$$

(iii) **Multiplication of Complex Numbers**

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

Properties of multiplication

- **Closure** $z_1 z_2$ is also a complex number.
- **Commutative** $z_1 z_2 = z_2 z_1$
- **Associative** $z_1 (z_2 z_3) = (z_1 z_2) z_3$
- **Existence of multiplicative identity**

$$z \cdot 1 = z = 1 \cdot z$$

Here, 1 is multiplicative identity.

- **Existence of multiplicative inverse** For every non-zero complex number z, there exists a complex number z_1 such that $z \cdot z_1 = 1 = z_1 \cdot z$
- **Distributive law** $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$

(iv) **Division of Complex Numbers**

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2}$$

where, $z_2 \neq 0$

Conjugate of a Complex Number

Let $z = x + iy$, if 'i' is replaced by $(-i)$, then it is said to be conjugate of the complex number z and denoted by $\bar{z}$, i.e. $\bar{z} = x - iy$.

Properties of Conjugate

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2\,\text{Re}(z)$, $z - \bar{z} = 2i\,\text{Im}(z)$

(iii) $z = \bar{z}$, if z is purely real

(iv) $z + \bar{z} = 0 \Leftrightarrow z$ is purely imaginary

(v) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(vi) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(vii) $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

(viii) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}$, $\bar{z}_2 \neq 0$

(ix) $z \cdot \bar{z} = \{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2$

(x) $z_1 \bar{z}_2 + \bar{z}_1 z_2 = 2\,\text{Re}(\bar{z}_1 z_2) = 2\,\text{Re}(z_1 \bar{z}_2)$

(xi) If $z = f(z_1)$, then $\bar{z} = f(\bar{z}_1)$

(xii) $(\bar{z})^n = \overline{(z^n)}$

Modulus (Absolute Value) of a Complex Number

Let $z = x + iy$ be a complex number. Then, the positive square root of the sum of square of real part and square of imaginary part is called modulus (absolute values) of z and it is denoted by $|z|$ i.e. $|z| = \sqrt{x^2 + y^2}$.

Properties of Modulus

(i) $|z| \geq 0$

(ii) If $|z| = 0$, then $z = 0$ i.e. $\text{Re}(z) = 0 = \text{Im}(z)$

(iii) $-|z| \leq \text{Re}(z) \leq |z|$ and $-|z| \leq |\text{Im} z) \leq |z|$

(iv) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

(v) $z \cdot \bar{z} = |z|^2$

(vi) $|z_1 z_2| = |z_1||z_2|$

(vii) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, $z_2 \neq 0$

(viii) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\text{Re}(z_1 \bar{z}_2)$

(ix) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\text{Re}(z_1 \bar{z}_2)$

(x) $|z_1 + z_2| \leq |z_1| + |z_2|$

(xi) $|z_1 - z_2| \geq |z_1| - |z_2|$

Argand Plane

A complex number $z = a + ib$ can be represented by a unique point $P(a, b)$ in the cartesian plane referred to a pair of rectangular axes.

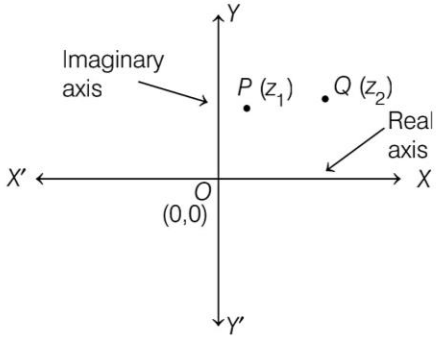
A purely real number a, i.e. $(a + 0i)$ is represented by the point $(a, 0)$ on X-axis. Therefore, X-axis is called **real axis**.

A purely imaginary number ib i.e. $(0 + i\,b)$ is represented by the point $(0, b)$ on Y-axis. Therefore, Y-axis is called **imaginary axis.** The intersection (common) of two axes is called zero complex number i.e. $z = 0 + 0i$.

Similarly, the representation of complex numbers as points in the plane is known as **argand diagram**. The plane representing complex numbers as points, is called **complex plane** or **argand plane** or **gaussian plane.**

If two complex numbers z_1 and z_2 are represented by the points P and Q in the complex plane, then

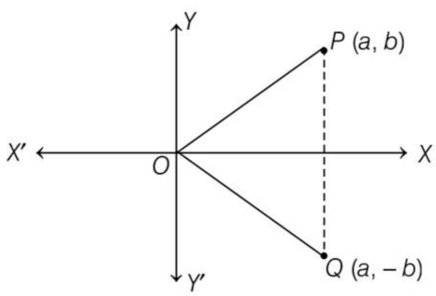
$|z_1 - z_2| = PQ$ = Distance between P and Q



Representation of Conjugate of z on Argand Plane

Geometrically, the mirror image of the complex number $z = a + ib$ (represented by the ordered pair (a, b) about the X-axis is called **conjugate of z** which is represented by the ordered pair $(a, -b)$.

If $z = a + ib$, then $\bar{z} = a - ib$.

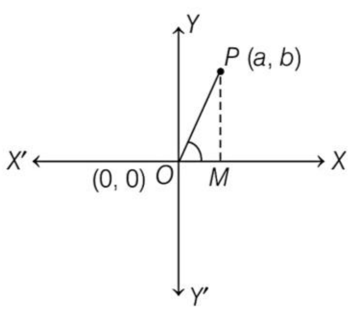


Representation of Modulus of z on Argand Plane

Geometrically, the distance of the complex number $z = a + ib$ [represented by the ordered pair (a, b)] from origin, is called the modulus of z.

$$\therefore \quad OP = \sqrt{(a-0)^2 + (b-0)^2}$$

$$= \sqrt{a^2 + b^2} = \sqrt{\{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2} = |a + ib|$$



Quadratic Equation

An equation of the form $ax^2 + bx + c, a \neq 0$ is called quadratic equation in variable x, where a, b and c are numbers (real or complex).

Nature of Roots of Quadratic Equation

The roots of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Now, if we look at these roots of quadratic equation $ax^2 + bx + c = 0$; $a \neq 0$, we observe that the roots depend upon the value of the quantity $b^2 - 4ac$. This quantity is known as the **discriminant** of the quadratic equation and denoted by D.

There are following four cases arise :

***Case* I** If $b^2 - 4ac = 0$ i.e. $D = 0$,

then $\qquad \alpha = \beta = -\dfrac{b}{2a}$.

Thus, if $b^2 - 4ac = 0$, then the quadratic equation has real and equal roots and each equal to $-b / 2a$.

***Case* II** If a, b and c are rational numbers and $b^2 - 4ac > 0$ and it is a perfect square, then $D = \sqrt{b^2 - 4ac}$ is a rational number and hence α and β are rational and unequal.

***Case* III** If $b^2 - 4ac > 0$ and it is not a perfect square, then roots are irrational and unequal.

***Case* IV** If $b^2 - 4ac < 0$, then the roots are complex conjugate of each other.

Quadratic Equations with Real Coefficients

Let us consider the following quadratic equation $ax^2 + bx + c = 0$ with real coefficients a, b, c and $a \neq 0$. Also, let us assume that $b^2 - 4ac < 0$. Now, we can find the square root of negative real numbers in the set of complex numbers. Therefore, the solutions of the above equation are available in the set of complex numbers which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{-(4ac - b^2)}}{2a}$$

$$= \frac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$$

OBJECTIVE QUESTION

Multiple Choice Questions

1. If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then the values of x and y are

(a) $x = 3, y = 4$ (b) $x = \frac{3}{4}, y = \frac{33}{4}$
(c) $x = 4, y = 3$ (d) $x = 33, y = 4$

2. If $(1 - i)x + (1 + i)y = 1 - 3i$, then (x, y) is equal to

(a) $(2, -1)$ (b) $(-2, 1)$
(c) $(-2, -1)$ (d) $(2, 1)$

3. If $i^{103} = a + ib$, then $a + b$ is equal to

(a) 1 (b) -1
(c) 0 (d) 2

4. $1 + i^{10} + i^{20} + i^{30}$ is a

(a) Real number (b) Complex number
(c) Natural number (d) None of these

5. The value of $\frac{i^{4x+1} - i^{4x-1}}{2}$ is equal to

(a) i (b) -1
(c) $-i$ (d) 0

6. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $z_1 + z_2$ equals to

(a) $5 + 5i$ (b) $5 + 10i$
(c) $4 + 6i$ (d) $6 + 4i$

7. If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$, then $z_1 - z_2$ is equal to

(a) $-1 + 5i$ (b) $5 - i$
(c) $i + 5$ (d) None of these

8. If $(-i)(3i)\left(\frac{-1}{6}i\right)^3 = a + ib$ then a is equal to

(a) 1 (b) 0
(c) -1 (d) 2

9. If $Z_1 = 2 + 3i$ and $Z_2 = 1 - 4i$ then $Z_1 Z_2$ is equal to

(a) $14 - 5i$ (b) $14 + 5i$
(c) $-14 + 5i$ (d) $-14 - 5i$

10. If $3(7 + 7i) + i(7 + 7i) = a + ib$ then $\frac{b}{a}$ is equal to

(a) 2 (b) 1
(c) 3 (d) -1

11. If $\frac{(1 + i)^2}{2 - i} = x + iy$ then the value of $x + y$ is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

12. If $(1 - i)^4 = a + ib$, then the value of a and b are respectively

(a) $-4, 0$ (b) $0, -4$
(c) $4, 0$ (d) $0, 4$

13. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\frac{z_1}{z_2}$ is equal to

(a) $\frac{1}{5}(9 + 12i)$ (b) $9 + 12i$
(c) $3 + 2i$ (d) $\frac{1}{5}(12 + 9i)$

14. The multiplicative inverse of $\frac{3 + 5i}{4 - 3i}$ is equal to

(a) $\frac{-3}{34} - \frac{29i}{34}$ (b) $\frac{3}{34} + \frac{29i}{34}$
(c) $\frac{3}{34} - \frac{29i}{34}$ (d) $\frac{-1}{34} - \frac{29i}{34}$

15. If $Z_1 = \sqrt{3} + \sqrt{3i}$ and $Z_2 = \sqrt{3} + i$ then the quadrant in which $\left(\frac{Z_1}{Z_2}\right)$ lies is

(a) First (b) Second
(c) Third (d) Fourth

16. If $Z_1 = 1 + 2i$ and $Z_2 = 2 + 3i$, then sum of Z_1 and additive inverse of Z_2 is equal to

(a) $1+2i$ (b) $3+i$
(c) $3+5i$ (d) $-1-i$

17. $a + ib$ form of complex number $\overline{9-i} + \overline{6+i^3} - \overline{9+i^2}$ is given by

(a) $7-2i$ (b) $7+2i$
(c) $-7-2i$ (d) $-7+2i$

18. If $Z_1 = 3 + 2i$ and $Z_2 = 2 - i$ then $\overline{Z_1 + Z_2}$ is given by

(a) $5-i$ (b) $5+i$
(c) $-5+i$ (d) $-5-i$

19. If $Z_1 = 1 + i$, $Z_2 = 2 - i$ and $\overline{Z_1 Z_2} = a + ib$, then $a + b$ is equal to

(a) 2 (b) 1
(c) 3 (d) 4

20. The conjugate of $\dfrac{2-i}{(1-2i)^2}$ is equal to

(a) $\frac{-2}{25}+\frac{11}{25}i$ (b) $\frac{-2}{25}-\frac{11}{25}i$
(c) $\frac{2}{25}+\frac{11}{25}i$ (d) $\frac{2}{25}-\frac{11}{25}i$

21. If $Z_1 = 3 + 5i$ and $Z_2 = 2 - 3i$, then $\overline{\left(\dfrac{Z_1}{Z_2}\right)}$ is equal to

(a) $\frac{9}{13}+\frac{19}{13}i$ (b) $\frac{9}{13}-\frac{19}{13}i$
(c) $\frac{-9}{13}-\frac{19}{13}i$ (d) $\frac{-9}{13}+\frac{19}{13}i$

22. Let $Z_1 = 2 - i$, $Z_2 = -2 + i$ and $\dfrac{Z_1 Z_2}{\overline{Z}_1} = a + ib$, then a is equal to

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$
(c) $\frac{11}{5}$ (d) $\frac{-2}{5}$

23. If $Z = -5i^{-15} - 6i^{-8}$ then $\overline{Z}$ is equal to

(a) $-6-5i$ (b) $-6+5i$
(c) $6-5i$ (d) $6+5i$

24. The modulus of the complex number $4 + 3i^7$ is equal to

(a) 5 (b) -5
(c) 2 (d) 3

25. If $Z_1 = 1 + 3i$ and $Z_2 = 2 + 4i$ then $|Z_2 - Z_1|^2$ is equal to

(a) 1 (b) 2
(c) 3 (d) 4

26. If $Z = \dfrac{(1+i)(2+i)}{(3+i)}$, then $|Z|$ is equal to

(a) 1 (b) 0
(c) 2 (d) 3

27. The modulus of the complex number $(1-i)^{-2} + (1+i)^{-2}$ is equal to

(a) 1 (b) 2
(c) 3 (d) 0

28. If $Z_1 = 3 + 2i$ and $Z_2 = 2 - 4i$ then the value of $|z_1 + z_2|^2 + |z_1 - z_2|^2$ is equal to

(a) 11 (b) 22
(c) 66 (d) 55

29. Roots of $9x^2 + 16 = 0$ is given by

(a) $\pm\left(\frac{4}{3}i\right)$ (b) $\pm\left(\frac{3}{4}\right)i$
(c) $\pm\left(\frac{3}{2}\right)i$ (d) $\pm\left(\frac{2}{3}\right)i$

30. Roots of $x^2 + 2 = 0$ are

(a) $\pm\sqrt{2}i$ (b) 2
(c) $2i$ (d) None of these

31. Roots of $x^2 + 3x + 9 = 0$ are

(a) $\frac{-3\pm 3\sqrt{3}i}{2}$ (b) $\frac{3\pm 3\sqrt{3}i}{2}$
(c) $\frac{3\pm\sqrt{3}i}{2}$ (d) $\frac{-3\pm\sqrt{3}i}{2}$

32. Roots of $x^2 + x + 1 = 0$ are

(a) $\frac{-1\pm\sqrt{3}i}{2}$ (b) $\frac{1\pm\sqrt{3}i}{2}$
(c) $\frac{2\pm\sqrt{3}i}{2}$ (d) $\frac{-2\pm\sqrt{3}i}{2}$

33. Roots of $\sqrt{2}x^2 + x + \sqrt{2} = 0$ is given by

(a) $\frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{-1 \pm i\sqrt{7}}{\sqrt{2}}$

(c) $\frac{1 \pm \sqrt{7}i}{2}$ (d) None of these

34. Roots of $(y+1)(y-3)+7=0$ is given by

(a) $-1 \pm \sqrt{3}i$ (b) $1 \pm \sqrt{3}i$

(c) $1 \pm \sqrt{2}i$ (d) $-1 \pm \sqrt{2}i$

35. If difference in roots of the equation $x^2 - px + 8 = 0$ is 2, then p is equal to

(a) ± 6 (b) ± 2

(c) ± 1 (d) ± 5

Assertion-Reasoning MCQs

Directions (Q. Nos. 36-50) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.

(b) A is true, R is true; R is not a correct explanation of A.

(c) A is true; R is false

(d) A is false; R is true.

36. Assertion (A) If $i = \sqrt{-1}$, then $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$ and $i^{4k+3} = -i$.

Reason (R) $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3} = 1$.

37. Assertion (A) Simplest form of i^{-35} is $-i$.

Reason (R) Additive inverse of $(1-i)$ is equal to $-1+i$.

38. Assertion (A) Simplest form of $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ is $1-2\sqrt{2}i$.

Reason (R) The value of $(1+i)^5(1-i)^5$ is 32.

39. Assertion (A) If $Z_1 = 2+3i$ and $Z_2 = 3-2i$, then $Z_1 - Z_2 = -1+5i$.

Reason (R) If $Z_1 = a+ib$ and $Z_2 = c+id$, then $Z_1 - Z_2 = (a-c)+i(b-d)$

40. Assertion (A) If $(1+i)^6 = a+ib$, then $b = -8$.

Reason(R) If $(1-i)^3 = a+ib$, then $\frac{a}{b} = 1$.

41. Assertion (A) If $(1+i)(x+iy) = 2-5i$, then $x = \frac{-3}{2}$ and $y = \frac{-7}{2}$.

Reason (R) If $a+ib = c+id$, then $a = c$ and $b = d$.

42. Assertion (A) Multiplicative inverse of $2-3i$ is $2+3i$.

Reason (R) If $z = 3+4i$, then $\bar{Z} = 3-4i$.

43. Assertion (A) $(2+3i)[(3+2i)+(2+i)] = 1+21i$.

Reason (R) $z_1(z_2+z_3) = z_1z_2 + z_1z_3$.

44. Assertion (A) If $z = 5i\left(\frac{-3}{5}i\right)$, then z is equal to $3+0i$.

Reason (R) If $z_1 = a+ib$ and $z_2 = c+id$, then $z_1 + z_2 = (a+c)+i(b+d)$.

45. Assertion (A) If $z = \frac{1+2i}{1-3i}$, then $|z| = \frac{1}{\sqrt{2}}$.

Reason (R) If $z = a+ib$, then $|z| = \sqrt{a^2+b^2}$.

46. Assertion (A) If $x+4iy = ix+y+3$, then $x = 1$ and $y = 4$.

Reason (R) The reciprocal of $3+\sqrt{7}i$ is equal to $\frac{3}{16} - \frac{\sqrt{7}}{16}i$.

47. **Assertion (A)** If $z = i^9 + i^{19}$, then z is equal to $0 + 0i$.

Reason (R) The value of $1 + i^2 + i^4 + i^6 + ... + i^{20}$ is equal to -1.

48. **Assertion (A)** If $x^2 + 1 = 0$, then solution is $\pm i$.

Reason (R) The value of i^{-1097} is equal to i.

49. **Assertion (A)** If $3x^2 + 4x + 2 = 0$, then equation has imaginary roots.

Reason (R) In a quadratic equation, $ax^2 + bx + c = 0$, if $D = b^2 - 4ac$ is less than zero, then the equation will have imaginary roots.

50. **Assertion (A)** Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \dfrac{-3 \pm i\sqrt{11}}{2}$.

Reason (R) If $x^2 - x + 2 = 0$ is a quadratic equation, then its roots are $\dfrac{1 \pm i\sqrt{7}}{2}$.

Case Based MCQs

51. Two complex numbers $Z_1 = a + ib$ and $Z_2 = c + id$ are said to be equal, if $a = c$ and $b = d$.

On the basis of above information, answer the following questions.

(i) If $(3a - 6) + 2ib = -6b + (6 + a)i$, then the real values of a and b are respectively

(a) $-2, 2$ (b) $2, -2$
(c) $3, -3$ (d) $4, 2$

(ii) If $(2a + 2b) + i(b - a) = -4i$, then the real values of a and b are respectively

(a) 2, 3 (b) $2, -2$
(c) 3, 1 (d) $-2, 2$

(iii) If $\left(\dfrac{1-i}{1+i}\right)^{100} = a + ib$, then the values of a and b are respectively

(a) 1, 0 (b) 0, 1
(c) 1, 2 (d) 2, 1

(iv) If $\dfrac{(1+i)^2}{2-i} = x + iy$, then the value of $x + y$ is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{2}{5}$

(v) If $(x + y) + i(x - y) = 4 + 6i$, then xy is equal to

(a) 5 (b) -5
(c) 4 (d) -4

52. A complex number z is pure real if and only if $\bar{z} = z$ and is pure imaginary if and only if $\bar{z} = -z$.

Based on the above information, answer the following questions.

(i) If $(1 + i)z = (1 - i)\bar{z}$, then $-i\bar{z}$ is

(a) $-\bar{z}$ (b) z
(c) $\bar{z}$ (d) z^{-1}

(ii) $\overline{z_1 z_2}$ is

(a) $\bar{z}_1 \bar{z}_2$ (b) $\bar{z}_1 + \bar{z}_2$
(c) $\dfrac{\bar{z}_1}{\bar{z}_2}$ (d) $\dfrac{1}{\bar{z}_1 \bar{z}_2}$

(iii) If x and y are real numbers and the complex number $\dfrac{(2+i)x - i}{4+i} + \dfrac{(1-i)y + 2i}{4i}$ is pure real, the relation between x and y is

(a) $8x - 17y = 16$ (b) $8x + 17y = 16$
(c) $17x - 8y = 16$ (d) $17x - 8y = -16$

(iv) If $z = \dfrac{3 + 2i \sin\theta}{1 - 2i \sin\theta}\left(0 < \theta \le \dfrac{\pi}{2}\right)$ is pure imaginary, then θ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{12}$

(v) If z_1 and z_2 are complex numbers such that $\left|\dfrac{z_1 - z_2}{z_1 + z_2}\right| = 1$

(a) $\dfrac{z_1}{z_2}$ is pure real

(b) $\dfrac{z_1}{z_2}$ is pure imaginary

(c) z_1 is pure real

(d) z_1 and z_2 are pure imaginary

53. We have, $i = \sqrt{-1}$. So, we can write the higher powers of i as follows

(i) $i^2 = -1$

(ii) $i^3 = i^2 \cdot i = (-1)\cdot i = -i$

(iii) $i^4 = (i^2)^2 = (-1)^2 = 1$

(iv) $i^5 = i^{4+1} = i^4 \cdot i = 1 \cdot i = i$

(v) $i^6 = i^{4+2} = i^4 \cdot i^2 = 1 \cdot i^2 = -1$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

In order to compute i^n for $n > 4$, write $i^n = i^{4q+r}$ for some $q, r \in N$ and $0 \le r \le 3$. Then, $i^n = i^{4q} \cdot i^r = (i^4)^q \cdot i^r = (1)^q \cdot i^r = i^r$.

In general for any integer k, $i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1$ and $i^{4k+3} = -i$.

On the basis of above information, answer the following questions.

(i) The value of i^{37} is equal to

(a) i (b) $-i$
(c) 1 (d) -1

(ii) The value of i^{-30} is equal to

(a) i (b) 1
(c) -1 (d) $-i$

(iii) If $z = i^9 + i^{19}$, then z is equal to

(a) $0 + 0i$
(b) $1 + 0i$
(c) $0 + i$
(d) $1 + 2i$

(iv) The value of $\left[i^{19} + \left(\dfrac{1}{i}\right)^{25}\right]^2$ is equal to

(a) -4 (b) 4 (c) i (d) 1

(v) If $z = i^{-39}$, then simplest form of z is equal to

(a) $1 + 0i$ (b) $0 + i$
(c) $0 + 0i$ (d) $1 + i$

54. The conjugate of a complex number z, is the complex number, obtained by changing the sign of imaginary part of z. It is denoted by $\bar{z}$.

The **modulus** (or absolute value) of a complex number, $z = a + ib$ is defined as the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by $|z|$. i.e. $|z| = \sqrt{a^2 + b^2}$

Multiplicative inverse of z is $\dfrac{\bar{z}}{|z|^2}$. It is also called reciprocal of z.

$z\bar{z} = |z|^2$.

On the basis of above information, answer the following questions.

(i) If $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$, then the value of $x + y$ is equal to

(a) 0 (b) 1
(c) 2 (d) 3

(ii) The value of $(z + 3)(\bar{z} + 3)$ is equivalent to

(a) $|z + 3|^2$ (b) $|z - 3|$
(c) $z^2 + 3$ (d) None of these

(iii) If $f(z) = \dfrac{7 - z}{1 - z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is equal to

(a) $\dfrac{|z|}{2}$ (b) $|z|$
(c) $2|z|$ (d) None of these

(iv) If $z_1 = 1 - 3i$ and $z_2 = -2 + 4i$, then $|z_1 + z_2|$ is equal to

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{3}$ (d) 1

(v) If $z = 3 + 4i$, then $\frac{z + \bar{z}}{2}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

55. An equation of the form $ax^2 + bx + c$, $a \neq 0$ is called quadratic equation in variable x, where a, b and c are numbers (real or complex).

The roots of quadratic equation $ax^2 + bx + c = 0, a \neq 0$ are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Now, if we look at these roots of quadratic equation $ax^2 + bx + c = 0$; $a \neq 0$, we observe that the roots depend upon the value of the quantity $b^2 - 4ac$. This quantity is known as the discriminant of the quadratic equation and denoted by D.

There are following cases :

***Case* I** If $b^2 - 4ac = 0$ i.e. $D = 0$, then $\alpha = \beta = -\frac{b}{2a}$.

Thus, if $b^2 - 4ac = 0$, then the quadratic equation has real and equal roots and each equal to $-b / 2a$.

***Case* II** If a, b and c are rational numbers and $b^2 - 4ac > 0$ and it is a perfect square, then $D = \sqrt{b^2 - 4ac}$ is a rational number and hence α and β are rational and unequal.

***Case* III** If $b^2 - 4ac > 0$ and it is not a perfect square, then roots are irrational and unequal.

***Case* IV** If $b^2 - 4ac < 0$, then the roots are complex conjugate of each other.

Based on above information, answer the following questions

(i) Roots of quadratic equation $2x^2 - 2\sqrt{3}x + \frac{21}{8} = 0$ are given by

(a) $\frac{\sqrt{3}}{2} \pm \frac{3}{4}i$ (b) $-\frac{\sqrt{3}}{2} \pm \frac{3}{4}i$

(c) $\frac{3}{4} \pm \frac{\sqrt{3}}{2}i$ (d) $-\frac{3}{4} \pm \frac{\sqrt{3}}{2}i$

(ii) Roots of quadratic equation $25x^2 - 30x + 11 = 0$ are given by

(a) $\frac{\sqrt{2}}{5} \pm \frac{3}{5}i$ (b) $\frac{3}{5} \pm \frac{\sqrt{2}}{5}i$

(c) $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ (d) $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

(iii) Roots of quadratic equation $2x^2 + x + 1 = 0$ are given by

(a) $\frac{-1 \pm \sqrt{7}i}{4}$ (b) $\frac{\sqrt{7} \pm i}{4}$

(c) $\frac{3 \pm \sqrt{3}i}{4}$ (d) $\frac{3 \pm \sqrt{7}i}{4}$

(iv) Roots of quadratic equation $-x^2 + x - 2 = 0$ are given by

(a) $\frac{1 \pm \sqrt{7}i}{2}$ (b) $\frac{1 \pm \sqrt{5}i}{2}$

(c) $\frac{-1 \pm \sqrt{7}i}{-2}$ (d) $\frac{3 \pm 2i}{4}$

(v) Roots of quadratic equation $3x^2 - 4x + \frac{20}{3} = 0$ are given by

(a) $\frac{2}{3} \pm \frac{4}{3}i$ (b) $\frac{4}{3} \pm \frac{2}{3}i$

(c) $\frac{3}{4} \pm \frac{5}{4}i$ (d) $-\frac{3}{4} \pm \frac{5}{4}i$

ANSWERS

Multiple Choice Questions

1. (b)	2. (a)	3. (b)	4. (a)	5. (a)	6. (a)	7. (a)	8. (b)	9. (a)	10. (a)
11. (b)	12. (a)	13. (a)	14. (a)	15. (a)	16. (d)	17. (b)	18. (a)	19. (a)	20. (b)
21. (c)	22. (d)	23. (b)	24. (a)	25. (b)	26. (a)	27. (d)	28. (c)	29. (a)	30. (a)
31. (a)	32. (a)	33. (a)	34. (b)	35. (a)					

Assertion-Reasoning MCQs

36. (c)	37. (d)	38. (d)	39. (a)	40. (b)	41. (a)	42. (d)	43. (a)	44. (b)	45. (a)
46. (d)	47. (c)	48. (c)	49. (a)	50. (b)					

Case Based MCQs

51. (i) - (a); (ii) - (b); (iii) - (a); (iv) - (d); (v) - (b)
52. (i) - (b); (ii) - (a); (iii) - (a); (iv) - (c); (v) - (b)
53. (i) - (a); (ii) - (c); (iii) - (a); (iv) - (a); (v) - (b)
54. (i) - (a); (ii) - (a); (iii) - (a); (iv) - (a); (v) - (c)
55. (i) - (a); (ii) - (b); (iii) - (a); (iv) - (c); (v) - (a)

SOLUTIONS

1. We have, $4x + i(3x - y) = 3 + i(-6)$ ...(i)

Equating the real and the imaginary parts of Eq. (i), we get

$$4x = 3,\ 3x - y = -6$$

which on solving simultaneously, give

$x = \frac{3}{4}$ and $y = \frac{33}{4}$.

2. $(1 - i)x + (1 + i)y = 1 - 3i$

$(x + y) + i(y - x) = 1 - 3i$

Two complex numbers are equal, if their real and imaginary parts are equal.

$\therefore\ x + y = 1$ and $y - x = -3$

By simplification $x = 2,\ y = -1$

Here, (x, y) is $(2, -1)$.

3. $i^{103} = i^{25\times 4 + 3} = (i^4)^{25}.\, i^3$

$= (1)^{25}.(-i) = -i = 0 - i$

$0 - i = a + ib \Rightarrow a = 0, b = -1$

$\therefore a + b = 0 - 1 = -1$

4. $1 + i^{10} + i^{20} + i^{30} = 1 + (i^4)^2 i^2 + (i^4)^5 + (i^4)^7 i^2$

$= 1 - 1 + 1 - 1 = 0$

5. Consider, $\frac{i^{4x+1} - i^{4x-1}}{2}$

$$= \frac{i^{4x}\cdot i - i^{4x}\cdot i^{-1}}{2} = \frac{i - \frac{1}{i}}{2} \quad [\because i^{4x} = 1]$$

$$= \frac{i^2 - 1}{2i} = \frac{-2}{2i} \quad [\because i^2 = -1]$$

$$= \frac{-1}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i \quad [\because i^2 = -1]$$

6. $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$,

$z_1 + z_2 = (2 + 3i) + (3 + 2i)$

$= (2 + 3) + i(3 + 2) = 5 + 5i$

7. Here, $z_1 = 2 + 3i,\ z_2 = 3 - 2i$, then

$z_1 - z_2 = 2 + 3i - (3 - 2i)$

$= 2 + 3i - 3 + 2i = -1 + 5i$

8. $(-i)(3i)\left(-\frac{1}{6}i\right)^3 = (-3i^2)\left(-\frac{1}{216}i^3\right)$

$= (-3\times(-1))\left(-\frac{1}{216}(-i)\right)$

$[\because i^2 = -1 \text{ and } i^3 = -i]$

$= 3\times\frac{1}{216}\times i$

$= \frac{i}{72} = 0 + \frac{1}{72}i$

$\Rightarrow\ a = 0$

9. $Z_1 = 2 + 3i,\ Z_2 = 1 - 4i$

$\therefore\ Z_1Z_2 = (2 + 3i)(1 - 4i)$

$= 2 - 8i + 3i + 12$

$= 14 - 5i$

10. We have, $3(7+7i)+i(7+7i)$

$$=21+21i+7i-7$$
$$=14+28i=a+ib$$

$\Rightarrow a=14$ and $b=28$

$$\therefore \frac{b}{a}=\frac{28}{14}=2$$

11. $$\frac{(1+i)^2}{2-i}=\frac{1+i^2+2i}{2-i}$$
$$=\frac{2i}{2-i}\times\frac{2+i}{2+i}=\frac{4i-2}{4+1}=\frac{2}{5}$$

12. $(1-i)^4=((1-i)^2)^2=((1)^2-2(1)(i)+(i)^2)^2$

$$[\because (z_1-z_2)^2=z_1^2-2z_1z_2+z_2^2]$$
$$=(1-2i-1)^2 \qquad [\because i^2=-1]$$
$$=(-2i)^2=4i^2=-4=-4+0i$$

which is in the form of $a+ib$.

$\therefore a=-4$ and $b=0$

13. We have, $z_1=6+3i$ and $z_2=2-i$

$$\therefore \quad \frac{z_1}{z_2}=(6+3i)\frac{1}{2-i}=\frac{(6+3i)(2+i)}{(2-i)(2+i)}$$
$$=(6+3i)\left(\frac{2}{5}+i\frac{1}{5}\right)$$
$$=(6+3i)\frac{(2+i)}{5}$$
$$=\frac{1}{5}(9+12i)$$

14. The multiplicative inverse of complex quantities $\frac{3+5i}{4-3i}=\frac{4-3i}{3+5i}$

$$\left[\because \text{multiplicative inverse of } z=\frac{1}{z}\right]$$
$$=\frac{4-3i}{3+5i}\times\frac{3-5i}{3-5i}$$

[Multiply numerator and denominator by the conjugate of denominator i.e. $(3-5i)$]

$$=\frac{(12-15)+i(-9-20)}{9+25}$$
$$=\frac{-3+i(-29)}{34}=-\frac{3}{34}-\frac{29i}{34}$$

15. We have, $z_1=\sqrt{3}+i\sqrt{3}$ and $z_2=\sqrt{3}+i$.

$$\therefore \quad \frac{z_1}{z_2}=\frac{\sqrt{3}(1+i)}{\sqrt{3}+i}$$
$$=\frac{\sqrt{3}(1+i)}{(\sqrt{3}+i)}\times\frac{\sqrt{3}-i}{\sqrt{3}-i}$$

[by rationalising the denominator]

$$=\frac{\sqrt{3}(1+i)(\sqrt{3}-i)}{(\sqrt{3})^2-(i)^2}$$
$$[\because (z_1+z_2)(z_1-z_2)=z_1^2-z_2^2]$$
$$=\frac{\sqrt{3}(\sqrt{3}-i+i\sqrt{3}-i^2)}{3-i^2}$$
$$=\frac{\sqrt{3}(\sqrt{3}+i(\sqrt{3}-1)+1)}{3+1} \qquad [\because i^2=-1]$$
$$=\frac{\sqrt{3}}{4}((\sqrt{3}+1)+i(\sqrt{3}-1))$$
$$=\frac{\sqrt{3}(\sqrt{3}+1)}{4}+\frac{i\sqrt{3}(\sqrt{3}-1)}{4}$$

which is represented by a point in first quadrant.

16. $z_1=1+2i$; $z_2=2+3i$

Additive inverse of $z_2=-2-3i$

$$z_1+(-z_2)=1+2i-2-3i$$
$$=-1-i$$

17. Firstly, write each complex number in standard form and then find its conjugate.

$$\overline{9-i}+\overline{6+i^3}-\overline{9+i^2}$$
$$=(9+i)+\overline{6-i}-\overline{9-1}$$
$$[\because i^3=-i \text{ and } i^2=-1]$$
$$=(9+i)+(6+i)-\overline{8}$$
$$=15+2i-8$$
$$=7+2i$$

18. Given, $Z_1=3+2i$, $Z_2=2-i$

Now, $Z_1+Z_2=(3+2i)+(2-i)=5+i$

$$\overline{Z_1+Z_2}=\overline{5+i}=5-i$$

19. Given, $Z_1=1+i \Rightarrow \overline{Z_1}=1-i$

$$Z_2=2-i \Rightarrow \overline{Z_2}=2+i$$

Now, $\overline{Z_1}\,\overline{Z_2}=(1-i)(2+i)$

$$=2+i-2i-i^2$$
$$=2-i+1=3-i$$

Now, $3-i=a+ib$

$\Rightarrow \quad a=3,\ b=-1$

$\therefore \quad a+b=3-1=2$

20. Given that, $z = \frac{2-i}{(1-2i)^2} = \frac{2-i}{1+4i^2-4i}$

$= \frac{2-i}{1-4-4i} = \frac{2-i}{-3-4i}$

$= \frac{(2-i)}{-(3+4i)} = -\left[\frac{(2-i)(3-4i)}{(3+4i)(3-4i)}\right]$

$= -\left(\frac{6-8i-3i+4i^2}{9+16}\right)$

$= -\frac{(-11i+2)}{25} = \frac{-1}{25}(2-11i)$

$\Rightarrow \quad z = \frac{1}{25}(-2+11i)$

$\therefore \quad \bar{z} = \frac{1}{25}(-2-11i) = \frac{-2}{25} - \frac{11}{25}i$

21. Given, $z_1 = 3+5i$ and $z_2 = 2-3i$

Now, $\frac{z_1}{z_2} = \frac{3+5i}{2-3i} = \frac{3+5i}{2-3i} \times \frac{2+3i}{2+3i}$

[by rationalising the denominator]

$= \frac{6+9i+10i+15i^2}{4-9i^2} = \frac{6+19i-15}{4+9}$ $[\because i^2 = -1]$

$= \frac{-9+19i}{13} = \frac{-9}{13} + \frac{19}{13}i$ ...(i)

$\therefore \quad \overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{-9}{13} + \frac{19}{13}i\right)} = \frac{-9}{13} - \frac{19}{13}i$

22. We have, $z_1 = 2-i$ and $z_2 = -2+i$

Now,

$\frac{z_1 z_2}{\bar{z}_1} = \frac{(2-i)(-2+i)}{\overline{(2-i)}} = \frac{-(2-i)(2-i)}{2+i}$

$= -\frac{(4+i^2-4i)}{2+i} = -\frac{(4-1-4i)}{2+i}$

$= -\frac{(3-4i)}{2+i} \times \frac{2-i}{2-i}$

[by rationalising the denominator]

$= -\frac{(6-3i-8i+4i^2)}{4-i^2}$

$[\because (z_1+z_2)(z_1-z_2) = z_1^2 - z_2^2]$

$= -\frac{(6-11i-4)}{5}$ $[\because i^2 = -1]$

$= -\frac{2-11i}{5} = \frac{-2}{5} + \frac{11}{5}i = a + ib$

$\Rightarrow \quad a = -\frac{2}{5}$

23. Let $z = -5i^{-15} - 6i^{-8}$

$= \frac{-5}{i^{15}} - \frac{6}{i^8} = \frac{-5}{(i^4)^3 \cdot i^3} - \frac{6}{(i^4)^2}$

$[\because i^{15} = i^{4\times3+3}]$

$= \frac{-5}{(1)^3 \cdot (-i)} - \frac{6}{(1)^2}$ $[\because i^4 = 1 \text{ and } i^3 = -i]$

$= \frac{-5}{-i} - 6 = \frac{5}{i} - 6$

$= \frac{5-6i}{i} = \frac{(5-6i)i}{i \cdot i}$

[by rationalising the denominator]

$= \frac{5i-6i^2}{i^2} = \frac{5i+6}{-1}$

$= -6-5i$ $[\because i^2 = -1]$

$\therefore \quad \bar{z} = -6+5i$

24. We have, $4+3i^7 = 4+3(i^4)(i^2)i$

$= 4+3(1)(-1)i$

$[\because i^4 = 1, \ i^2 = -1]$

$= 4-3i$

$\therefore$ Modulus $= |4+3i^7| = |4-3i|$

$= \sqrt{4^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

25. Given, $Z_1 = 1+3i, \ Z_2 = 2+4i$

$\therefore \quad Z_2 - Z_1 = (2+4i)-(1+3i) = 1+i$

$\Rightarrow |Z_2 - Z_1|^2 = (\sqrt{1^2+1^2})^2$

$= 1+1 = 2$

26. Given that,

$\left|(1+i)\frac{(2+i)}{(3+i)}\right| = \left|\frac{(2+i+2i+i^2)}{(3+i)}\right|$

$= \left|\frac{2+3i-1}{3+i}\right|$

$= \left|\frac{1+3i}{3+i}\right| = \left|\frac{(1+3i)(3-i)}{(3+i)(3-i)}\right|$

$= \left|\frac{3+9i-i-3i^2}{9-i^2}\right|$

$= \left|\frac{3+8i+3}{9+1}\right| = \left|\frac{6+8i}{10}\right|$

$= \sqrt{\frac{6^2}{100} + \frac{8^2}{100}}$

$= \sqrt{\frac{36+64}{100}} = \sqrt{\frac{100}{100}} = 1$

27. Let $z = (1-i)^{-2} + (1+i)^{-2}$

$$= \frac{1}{(1-i)^2} + \frac{1}{(1+i)^2} = \frac{(1+i)^2 + (1-i)^2}{(1-i)^2 (1+i)^2}$$

$$= \frac{1 + i^2 + 2i + 1 + i^2 - 2i}{(1-i^2)^2}$$

$$= \frac{1-1+1-1}{(1+1)^2} = \frac{0}{4} \qquad [\because i^2 = -1]$$

$$= 0 = 0 + 0\,i$$

$\therefore\ |z| = \sqrt{0+0} = 0$

28. We have, $z_1 = 3 + 2i$ and $z_2 = 2 - 4i$

$$|z_1 + z_2|^2 + |z_1 - z_2|^2$$

On substituting the values of z_1 and z_2, we get

$$|3 + 2i + 2 - 4i|^2 + |3 + 2i - 2 + 4i|$$

$$= |5 - 2i|^2 + |1 + 6i|^2$$

$$= (5)^2 + (-2)^2 + (1)^2 + (6)^2$$

[if $z = a + ib$, then $|z|^2 = a^2 + b^2$]

$$= 25 + 4 + 1 + 36$$

$\therefore\quad |z_1 + z_2|^2 + |z_1 - z_2|^2 = 66$

29. We have, $9x^2 + 16 = 0$

$\Rightarrow\quad 9x^2 = -16$

$\Rightarrow\quad x^2 = -\dfrac{16}{9}$

$\Rightarrow\quad x = \pm\sqrt{-\dfrac{16}{9}}$

[taking square root both sides]

$\Rightarrow\quad x = \pm\left(\sqrt{\dfrac{16}{9}} \times \sqrt{-1}\right)$

$\therefore\quad x = \pm\left(\dfrac{4}{3}i\right) \qquad [\because \sqrt{-1} = i]$

Hence, the roots are $\dfrac{4}{3}i$ and $-\dfrac{4}{3}i$.

30. $x^2 + 2 = 0$

$\Rightarrow\ x^2 = -2 \Rightarrow x = \pm\sqrt{-2} = \pm\sqrt{2}i$

$\therefore\quad x = \pm\sqrt{2}i$

31. Given, $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = 1,\ b = 3,\ c = 9$$

Now, $D = b^2 - 4ac = (3)^2 - 4 \times 1 \times 9$

$$= 9 - 36 = -27 < 0$$

$\Rightarrow\quad x = \dfrac{-3 \pm \sqrt{-27}}{2 \times 1},\ x = \dfrac{-3 \pm i\sqrt{27}}{2}$

$[\because \sqrt{-1} = i]$

$$= \frac{-3 \pm i\sqrt{9 \times 3}}{2} = \frac{-3 \pm i\,3\sqrt{3}}{2}$$

32. $x^2 + x + 1 = 0$

$\because\quad x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$\Rightarrow\quad x = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow\quad x = \dfrac{-1 \pm i\sqrt{3}}{2}$

$\therefore\quad x = \dfrac{-1 + i\sqrt{3}}{2},\ \dfrac{-1 - i\sqrt{3}}{2}$

33. Given, $\sqrt{2}\,x^2 + x + \sqrt{2} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = \sqrt{2},\ b = 1,\ c = \sqrt{2}$$

$\therefore\ D = b^2 - 4ac = (1)^2 - 4 \times \sqrt{2} \times \sqrt{2}$

$$= 1 - 4 \times 2 = 1 - 8 = -7 < 0$$

$\Rightarrow\ x = \dfrac{-1 \pm \sqrt{-7}}{2 \times \sqrt{2}}$

$\Rightarrow\ x = \dfrac{-1 \pm i\sqrt{7}}{2\sqrt{2}} \qquad [\because \sqrt{-1} = i]$

34. We have, $(y+1)(y-3) + 7 = 0$

$\Rightarrow\quad y^2 - 2y - 3 + 7 = 0$

$\Rightarrow\quad y^2 - 2y + 4 = 0$

On comparing with $ay^2 + by + c = 0$, we get

$$a = 1,\ b = -2 \text{ and } c = 4$$

$\because\quad y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore\quad y = \dfrac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times 4}}{2 \times 1} = \dfrac{2 \pm \sqrt{4 - 16}}{2}$

$$= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2\sqrt{3}\,i}{2} = 1 \pm \sqrt{3}\,i$$

$[\because \sqrt{-1} = i]$

$\therefore\quad y = 1 + \sqrt{3}\,i$

or $\quad y = 1 - \sqrt{3}\,i$

Hence, the roots of the given equation are $1 + \sqrt{3}\,i$ and $1 - i\sqrt{3}$.

35. Let α, β be the roots of the equation $x^2 - px + 8 = 0$.

Therefore, $\alpha + \beta = p$

and $\alpha \cdot \beta = 8$

Now, $\alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

Therefore, $2 = \pm\sqrt{P^2 - 32}$

$\Rightarrow$ $P^2 - 32 = 4$, i.e. $P = \pm 6$

36. **Assertion** We know that, $i = \sqrt{-1}$

$\because$ $i^{4k} = (i^4)^k = 1^k = 1$

$\Rightarrow$ $i^{4k+1} = i^{4k} \cdot i = 1 \times i = i$

$\Rightarrow$ $i^{4k+2} = i^{4k} \cdot i^2 = 1 \times -1 = -1$

$\Rightarrow$ $i^{4k+3} = i^{4k} \cdot i^3 = 1 \times -i = -i$

Reason $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$

$= i^{4k}(1 + i + i^2 + i^3)$

$= i^{4k}(1 + i - 1 - i) = i^{4k}\ 0 = 0$

Hence, Assertion is true and Reason is false.

37. **Assertion**

$i^{-35} = \frac{1}{i^{35}} = \frac{1}{(i^2)^{17} i} = \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = i$

Reason Additive inverse of z is $-z$.

$\therefore$ Additive inverse of $(1-i)$ is

$-(1-i) = -1 + i$

Hence, Assertion is false and Reason is true.

38. **Assertion** We have,

$\frac{5+\sqrt{2}i}{1-\sqrt{2}i} = \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i}$

$= \frac{5 + 5\sqrt{2}i + \sqrt{2}i - 2}{1 - (\sqrt{2}i)^2}$

$= \frac{3 + 6\sqrt{2}i}{1+2} = \frac{3(1 + 2\sqrt{2}i)}{3}$

$= 1 + 2\sqrt{2}i$

Reason $(1+i)^5(1-i)^5 = (1-i^2)^5$

$= 2^5 = 32$

Hence Assertion is false and Reason is true.

39. **Assertion** Given, $Z_1 = 2 + 3i$, $Z_2 = 3 - 2i$

$\therefore$ $Z_1 - Z_2 = (2+3i) - (3-2i)$

$= (2-3) + i(3-(-2)) = -1 + 5i$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

40. **Assertion** We have,

$(1+i)^6 = ((1+i)^2)^3$

$= (1 + i^2 + 2i)^3$ $[\because (z_1+z_2)^2 = z_1^2 + z_2^2 + 2z_1z_2]$

$= (1 - 1 + 2i)^3$ $[\because i^2 = -1]$

$\Rightarrow (1+i)^6 = (2i)^3 = 8i^3 = -8i$ $[\because i^3 = -1]$

$= a + ib$

$\therefore$ $b = -8$

Reason $(1-i)^3 = 1^3 - i^3 - 3(1)^2 i + 3(1)(i)^2$

$[\because (z_1 - z_2)^3 = z_1^3 - 3z_1^2 z_2 + 3z_1 z_2^2 - z_2^3]$

$= 1 - (-i) - 3i - 3$

$[\because i^3 = -i$ and $i^2 = -1]$

$\Rightarrow$ $(1-i)^3 = -2 - 2i$

$= a + ib$

$\Rightarrow$ $a = -2$ and $b = -2$

$\therefore$ $\frac{a}{b} = \frac{-2}{-2} = 1$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

41. **Assertion** We have,

$(1+i)(x+iy) = 2 - 5i$

$\Rightarrow$ $x + iy + ix + i^2 y = 2 - 5i$

$\Rightarrow$ $x + i(y+x) - y = 2 - 5i$ $[\because i^2 = -1]$

$\Rightarrow$ $(x-y) + i(x+y) = 2 - 5i$

On equating real and imaginary parts from both sides, we get

$x - y = 2$ ...(i)

and $x + y = -5$ ...(ii)

On adding Eqs. (i) and (ii), we get

$x - y + x + y = 2 - 5$

$\Rightarrow$ $2x = -3$

$\Rightarrow$ $x = \frac{-3}{2}$

On substituting $x = \frac{-3}{2}$ in Eq. (ii), we get

$\frac{-3}{2} + y = -5$

$\Rightarrow$ $y = -5 + \frac{3}{2} = \frac{-10+3}{2} = \frac{-7}{2}$

$\therefore$ $x = \frac{-3}{2}$ and $y = \frac{-7}{2}$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

42. **Assertion** Let $z = 2 - 3i$

Then, $\bar{z} = 2 + 3i$ and $|z|^2 = 2^2 + (-3)^2 = 13$

Therefore, the multiplicative inverse of $2 - 3i$ is

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{2+3i}{13} = \frac{2}{13} + \frac{3}{13}i$$

The above working can be reproduced in the following manner also,

$$z^{-1} = \frac{1}{2-3i} = \frac{2+3i}{(2-3i)(2+3i)}$$
$$= \frac{2+3i}{2^2-(3i)^2} = \frac{2+3i}{13}$$
$$= \frac{2}{13} + \frac{3}{13}i$$

Reason If $Z = a + ib$, then conjugate of Z

i.e. $\bar{z} = a - ib$

$\therefore$ $z = 3 + 4i$

$\Rightarrow$ $\bar{z} = 3 - 4i$

Hence, Assertion is false and Reason is true.

43. **Assertion** For any three complex numbers z_1, z_2 and z_3, distributive law is

$z_1(z_2 + z_3) = z_1z_2 + z_1z_3$ and $(z_1 + z_2)z_3 = z_1z_3 + z_2z_3$.

$\therefore (2+3i)[(3+2i)+(2+i)]$
$$= (2+3i)(3+2i) + (2+3i)(2+i)$$
$[\because z_1(z_2 + z_{3)} = z_1 z_2 + z_1 z_3]$
$$= (6-6) + 13i + (4-3) + 8i$$
$$= 1 + 21i$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

44. **Assertion** $5i\left(\frac{-3}{5}i\right) = 5 \times \frac{-3}{5}i^2$

$$= -3(-1) = 3 = 3 + 0i$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

45. **Assertion** Let $z = \frac{1+2i}{1-3i}$

$$\therefore \quad z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2-(3i)^2}$$

$[\because (a+b)(a-b) = a^2 - b^2]$

$$= \frac{1+5i+6(-1)}{1-9i^2} \qquad [\because i^2 = -1]$$

$$= \frac{1+5i-6}{1+9} = \frac{-5+5i}{10} = \frac{-1+i}{2}$$

$$\Rightarrow \quad z = -\frac{1}{2} + \frac{1}{2}i$$

$$\therefore \quad |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$[\because |a+ib| = \sqrt{a^2+b^2}]$

$$= \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

46. **Assertion** $x + 4iy = ix + y + 3$

$\Rightarrow$ $x = y + 3$ ...(i)

$\Rightarrow$ $4y = x$ ...(ii)

From Eqs. (i) and (ii), we get

$4y = y + 3$

$\Rightarrow$ $3y = 3$

$\Rightarrow$ $y = 1$

From Eq. (i), we get

$x = 1 + 3 = 4$

Reason Let $z = 3 + \sqrt{7}i$

$$\therefore \quad \frac{1}{z} = \frac{1}{3+\sqrt{7}i} \times \frac{3-\sqrt{7}i}{3-\sqrt{7}i}$$
$$= \frac{3-\sqrt{7}i}{9+7} = \frac{3}{16} - \frac{\sqrt{7}}{16}\hat{i}$$

Hence, Assertion is false and Reason is true.

47. **Assertion** $i^9 + i^{19} = i^9(1 + i^{10}) = i^9[1 + (i^2)^5]$

$$= i^9[1 + (-1)^5] = i^9(1-1) = 0 = 0 + 0i$$

Reason $1 + i^2 + i^4 + \dots i^{20}$

$$= \frac{1[(i^2)^{11} - 1]}{(i)^2 - 1} = \frac{1(-1-1)}{-1-1} = 1$$

Hence, Assertion is true and Reason is false.

48. **Assertion** $x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1} \Rightarrow x = \pm i$

Reason $i^{-1097} = \frac{1}{i^{4 \times 274 + 1}}$

$$= \frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

Hence, Assertion is true and Reason is false.

49. Assertion

$3x^2 + 4x + 2 = 0,\ a = 3, b = 4, c = 2$

$$D = b^2 - 4ac = 16 - 4(3)(2) = 16 - 24 = -8$$

$\Rightarrow \quad D < 0$

$\because b^2 - 4ac < 0$, so above equation has imaginary roots.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

50. Assertion Given, $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 3, c = 5$$

Now, $D = b^2 - 4ac = (3)^2 - 4 \times 1 \times 5$

$$= 9 - 20 = -11 < 0$$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{-11}}{2 \times 1}$$

$$\therefore \quad x = \frac{-3 \pm i\sqrt{11}}{2} \qquad [\because \sqrt{-1} = i]$$

Reason Given, $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -1, c = 2$$

Now, $D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2$

$$= 1 - 8 = -7 < 0$$

$$\Rightarrow \quad x = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1}$$

$$= \frac{1 \pm i\sqrt{7}}{2} \qquad [\because \sqrt{-1} = i]$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

51. (i) We have, $(3a - 6) + 2ib = -6b + (6 + a)i$

On equating real and imaginary parts, we get

$$3a - 6 = -6b \qquad \ldots(i)$$

and $$2b = 6 + a \qquad \ldots(ii)$$

Above equations can be rewritten as

$$3a + 6b = 6 \qquad \ldots(iii)$$

and $$a - 2b = -6 \qquad \ldots(iv)$$

On multiplying Eq. (iv) by 3 and then adding with Eq. (iii), we get

$$3a + 6b + 3a - 6b = 6 - 18$$

$$\Rightarrow \quad 6a = -12 \Rightarrow a = -2$$

On substituting $a = -2$ in Eq. (iv), we get

$$-2 - 2b = -6$$

$$\Rightarrow \quad -2b = -6 + 2 \Rightarrow b = \frac{-4}{-2} = 2$$

$\therefore \quad a = -2$ and $b = 2$.

(ii) We have, $(2a + 2b) + i(b - a) = -4i$, which can be rewritten as

$$(2a + 2b) + i(b - a) = 0 - 4i$$

On equating real and imaginary parts, we get

$$2a + 2b = 0$$

$$\Rightarrow \quad a + b = 0 \qquad [\because 2 \neq 0] \ldots(i)$$

and $$b - a = -4 \qquad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$a + b + b - a = 0 - 4$$

$$\Rightarrow \quad 2b = -4 \Rightarrow b = -2$$

On substituting $b = -2$ in Eq. (i), we get

$$a - 2 = 0 \Rightarrow a = 2$$

$$\therefore \quad a = 2 \text{ and } b = -2$$

(iii) Given that, $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$

$$\Rightarrow \quad \left[\frac{(1-i)}{(1+i)} \cdot \frac{(1-i)}{(1-i)}\right]^{100} = a + ib$$

$$\Rightarrow \quad \left(\frac{1 + i^2 - 2i}{1 - i^2}\right)^{100} = a + ib$$

$$\Rightarrow \quad \left(\frac{-2i}{2}\right)^{100} = a + ib \qquad [\because i^2 = -1]$$

$$\Rightarrow \quad (i^4)^{25} = a + ib$$

$$\Rightarrow \quad 1 = a + ib \qquad [\because i^4 = 1]$$

On comparing real and imaginary parts both sides, we get

$$a = 1 \text{ and } b = 0$$

$$\therefore \quad (a, b) = (1, 0)$$

(iv) Given that, $\frac{(1+i)^2}{2-i} = x + iy$

$$\Rightarrow \quad \frac{(1 + i^2 + 2i)}{2 - i} = x + iy$$

$$\Rightarrow \quad \frac{2i}{2 - i} = x + iy$$

$$\Rightarrow \quad \frac{2i(2+i)}{(2-i)\ (2+i)} = x + iy$$

$$\Rightarrow \quad \frac{4i + 2i^2}{4 - i^2} = x + iy$$

$$\Rightarrow \quad \frac{4i - 2}{4 + 1} = x + iy$$

$$\Rightarrow \quad \frac{-2}{5} + \frac{4i}{5} = x + iy$$

On comparing both sides, we get

$$x = -\frac{2}{5} \text{ and } y = \frac{4}{5}$$

$$\therefore \quad x + y = \frac{-2}{5} + \frac{4}{5} = \frac{2}{5}$$

(v) Given that, $(x + y) + i(x - y) = 4 + 6i$

On comparing both sides, we get

$$x + y = 4 \quad \ldots(i)$$

and $x - y = 6 \quad \ldots(ii)$

Adding Eqs. (i) and (ii), we get

$$2x = 10$$

$$\Rightarrow \quad x = 5$$

∴ From Eq. (i), we get

$$5 + y = 4$$

$$\Rightarrow \quad y = 4 - 5 = -1$$

$$\therefore \quad xy = 5(-1) = -5$$

52. (i) Since, $(1 + i)z = (1 - i)\bar{z}$

$$\Rightarrow \quad \frac{z}{\bar{z}} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{1-i^2}$$

$$= \frac{1 + i^2 - 2i}{1 + 1} = -i$$

$$\Rightarrow \quad z = -i\,\bar{z}$$

(ii) $\because \overline{z_1 z_2} = \bar{z}_1\ \bar{z}_2$

(iii) Let $z = \dfrac{(2+i)x - i}{4+i} + \dfrac{(1-i)y + 2i}{4i}$

$$= \frac{2x + (x-1)i}{4+i} + \frac{y + (2-y)i}{4i} \times \frac{i}{i}$$

$$= \frac{(2x + (x-1)i)\ (4-i)}{(4+i)(4-i)} + \frac{-iy + (2-y)}{4}$$

$$= \frac{8x + x - 1 + i(4x - 4 - 2x)}{16 + 1} + \frac{(2-y) - iy}{4}$$

$$= \frac{9x - 1 + i(2x - 4)}{17} + \frac{2 - y - iy}{4}$$

Since, z is real $\Rightarrow \bar{z} = z$

$$\Rightarrow \quad \text{Im}\, z = 0$$

$$\Rightarrow \quad \frac{2x - 4}{17} - \frac{y}{4} = 0$$

$$\Rightarrow \quad 8x - 16 = 17y \Rightarrow 8x - 17y = 16$$

(iv) $z = \dfrac{3 + 2i \sin\theta}{1 - 2i \sin\theta} \times \dfrac{(1 + 2i\sin\theta)}{(1 + 2i \sin\theta)}$

$$= \frac{(3 + 2i\sin\theta)(1 + 2i\sin\theta)}{1 + 4\sin^2\theta}$$

$$= \frac{(3 - 4\sin^2\theta) + i(8\sin\theta)}{1 + 4\sin^2\theta}$$

Since, z is pure imaginary.

$$\Rightarrow \quad \text{Re}(z) = 0$$

$$\Rightarrow \quad \frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} = 0$$

$$\Rightarrow \quad \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3} \left(\text{since, } 0 < \theta \le \frac{\pi}{2}\right)$$

(v) We have, $\left|\dfrac{z_1 - z_2}{z_1 + z_2}\right| = 1$

$$\Rightarrow \quad |z_1 - z_2| = |z_1 + z_2|$$

$$\Rightarrow \quad |z_1 - z_2|^2 = |z_1 + z_2|^2$$

$$\Rightarrow (z_1 - z_2)(\bar{z}_1 - \bar{z}_2) = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$$

$$[\because |z|^2 = z\bar{z}]$$

$$\Rightarrow \quad 2 z_1 \bar{z}_2 = -2\bar{z}_1 z_2$$

$$\Rightarrow \quad \frac{z_1}{z_2} = -\frac{\bar{z}_1}{\bar{z}_2} = -\overline{\left(\frac{z_1}{z_2}\right)}$$

$\Rightarrow \dfrac{z_1}{z_2}$ is pure imaginary.

53. (i) We have, $i^{37} = (i)^{36+1} = (i)^{4\times 9} i$

$$= (i^4)^9 \cdot i = (1)^9 \cdot i = i \quad [\because i^4 = 1]$$

(ii) We have, $i^{-30} = \dfrac{1}{i^{30}}$

Now, $i^{30} = (i)^{4\times 7 + 2}$

$$= (i^{4\times 7})\, i^2 = (i^4)^7 (-1) \quad [\because i^2 = -1]$$

$$= (1)^7 (-1) = -1 \quad [\because i^4 = 1]$$

$$\Rightarrow \quad i^{-30} = \frac{1}{(-1)} = -1$$

(iii) $i^9 + i^{19} = i^9(1 + i^{10}) = i^9[1 + (i^2)^5]$

[taking i^9 common]

$= i^9[(1 + (-1)^5] = i^9(1-1) = 0$ $[\because i^2 = -1]$

$= 0 + 0i$

(iv) $\left[i^{19} + \left(\frac{1}{i}\right)^{25}\right]^2 = \left[i^{4\times4+3} + \frac{1}{i^{4\times6+1}}\right]^2$

$= \left[(i^4)^4(i)^3 + \frac{1}{(i^4)^6 i}\right]^2 = \left[(1)^4(i)^3 + \frac{1}{(1)^6 i}\right]^2$ $[\because i^4 = 1]$

$= \left(-i + \frac{1}{i}\right)^2 = \left(-i + \frac{i}{i^2}\right)^2$ $[\because i^3 = -i]$

$= \left(-i + \frac{i}{i\times i}\right)^2 = \left(-i + \frac{i}{-1}\right)^2$ $[\because i^2 = -1]$

$= (-i-i)^2 = (-2i)^2 = 4i^2$

$= -4$ $[\because i^2 = -1]$

(v) $i^{-39} = \frac{1}{i^{39}}$

Multiplying and dividing by i, we get

$= \frac{i}{i^{40}} = \frac{i}{(i^4)^{10}} = \frac{i}{(1)^{10}} = \frac{i}{1} = i$ $[\because i^4 = 1]$

$= 0 + i$

54. (i) We have, $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

$\Rightarrow (x - iy)(3 + 5i) = -6 + 24i$

[$\because$ conjugate of $-6 - 24i = -6 + 24i$]

$\Rightarrow \quad 3x - 3iy + 5ix - 5i^2y = -6 + 24i$

$\Rightarrow \quad (3x + 5y) + i(5x - 3y) = -6 + 24i$

$[\because \ i^2 = -1]$ ...(i)

On equating real and imaginary parts both sides of Eq. (i), we get

$3x + 5y = -6$ ...(ii)

and $5x - 3y = 24$ ...(iii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 5, then adding the result, we get

$9x + 15y + 25x - 15y = -18 + 120$

$\Rightarrow \quad 34x = 102 \Rightarrow x = 3$

On substituting $x = 3$ in Eq. (ii), we get

$9 + 5y = -6 \Rightarrow 5y = -15 \Rightarrow y = -3$

Now, $x + y = 3 + (-3) = 0$

(ii) Given that, $(z + 3)(\bar{z} + 3)$

Let $z = x + iy$

$\Rightarrow (z + 3)(\bar{z} + 3) = (x + iy + 3)(x + 3 - iy)$

$= (x + 3)^2 - (iy)^2 = (x + 3)^2 + y^2$

$= |x + 3 + iy|^2 = |z + 3|^2$

(iii) Let $z = 1 + 2i$

$\Rightarrow \quad |z| = \sqrt{1 + 4} = \sqrt{5}$

Now, $f(z) = \frac{7 - z}{1 - z^2} = \frac{7 - 1 - 2i}{1 - (1 + 2i)^2}$

$= \frac{6 - 2i}{1 - 1 - 4i^2 - 4i} = \frac{6 - 2i}{4 - 4i}$

$= \frac{3 - i}{2 - 2i} = \frac{(3 - i)(2 + 2i)}{(2 - 2i)(2 + 2i)}$

$= \frac{6 - 2i + 6i - 2i^2}{4 - 4i^2}$

$= \frac{6 + 4i + 2}{4 + 4}$

$= \frac{8 + 4i}{8} = 1 + \frac{1}{2}i$

$f(z) = 1 + \frac{1}{2}i$

$\therefore |f(z)| = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{4 + 1}{4}}$

$= \frac{\sqrt{5}}{2} = \frac{|z|}{2}$

(iv) Given, $z_1 = 1 - 3i$ and $z_2 = -2 + 4i$

$\therefore z_1 + z_2 = (1 - 3i) + (-2 + 4i) = -1 + i$

$|z_1 + z_2| = \sqrt{(-1)^2 + (1)^2} = \sqrt{1 + 1} = \sqrt{2}$

(v) Given, $z = 3 + 4i$

$\therefore \quad \bar{z} = 3 - 4i$

$\Rightarrow z + \bar{z} = (3 + 4i) + (3 - 4i) = 6$

Now, $\frac{z + \bar{z}}{2} = \frac{6}{2} = 3$

55. (i) We have, $2x^2 - 2\sqrt{3}\,x + \frac{21}{8} = 0$ ...(i)

On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

$a = 2, b = -2\sqrt{3}$ and $c = \frac{21}{8}$

$\because \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore x = \frac{-(-2\sqrt{3}) \pm \sqrt{(-2\sqrt{3})^2 - 4 \times 2 \times \frac{21}{8}}}{2 \times 2}$$

$$= \frac{2\sqrt{3} \pm \sqrt{12-21}}{4} = \frac{2\sqrt{3} \pm \sqrt{-9}}{4}$$

$$= \frac{2\sqrt{3} \pm 3i}{4} = \frac{\sqrt{3}}{2} \pm \frac{3}{4} i \qquad [\because \sqrt{-1} = i]$$

Hence, the roots are $\frac{\sqrt{3}}{2} + \frac{3}{4} i$ and $\frac{\sqrt{3}}{2} - \frac{3}{4} i$.

(ii) Given, $25x^2 - 30x + 11 = 0$ ...(i)

On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

$$a = 25,\ b = -30 \text{ and } c = 11$$

$$\because \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{30 \pm \sqrt{(-30)^2 - 4 \times 25 \times 11}}{2 \times 25}$$

$$\Rightarrow \quad x = \frac{30 \pm \sqrt{900 - 1100}}{50}$$

$$\Rightarrow \quad x = \frac{30 \pm \sqrt{-200}}{50}$$

$$\Rightarrow \quad x = \frac{30 \pm 10 i\sqrt{2}}{50} \qquad [\because \sqrt{-1} = i]$$

$$\Rightarrow \quad x = \frac{3}{5} \pm \frac{\sqrt{2}}{5} i$$

Hence, the roots are $\frac{3}{5} + \frac{\sqrt{2}}{5} i$ and $\frac{3}{5} - \frac{\sqrt{2}}{5} i$.

(iii) Given, $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = 2,\ b = 1,\ c = 1$$

Now, $D = b^2 - 4ac = (1)^2 - 4 \times 2 \times 1$

$$= 1 - 8 = -7 < 0$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{-7}}{2 \times 2} = \frac{-1 \pm i\sqrt{7}}{4}$$

$$[\because \sqrt{-1} = i]$$

(iv) Given, $-x^2 + x - 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = -1,\ b = 1,\ c = -2$$

Now, $D = b^2 - 4ac = (1)^2 - 4(-1)(-2)$

$$= 1 - 4 \times 1 \times 2$$

$$= 1 - 8 = -7 < 0$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm i\sqrt{7}}{-2}$$

$$[\because \sqrt{-1} = i]$$

(v) Given, $3x^2 - 4x + \frac{20}{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = 3,\ b = -4,\ c = \frac{20}{3}$$

$$\therefore \quad D = b^2 - 4ac = (-4)^2 - 4 \times 3 \times \frac{20}{3}$$

$$= 16 - 80 = -64 < 0$$

$$\therefore \quad x = \frac{-(-4) \pm \sqrt{-64}}{2 \times 3}$$

$$= \frac{4 \pm 8i}{2 \times 3}$$

$$= \frac{2(2 \pm 4i)}{2 \times 3}$$

$$= \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3} i$$