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BINOMIAL THEOREM

Revision Module

CBSE-XI

MATHEMATICS

Conceptual Review: Immerse yourself in a focused review of the Binomial Theorem, revisiting fundamental concepts such as binomial coefficients, expansion formulae, and the general term. Strengthen your understanding of the unique patterns exhibited in binomial expansions.

Formulae Mastery: Reinforce your knowledge of essential formulae related to the Binomial Theorem. Navigate through expansions like (a+b)n and understand the relationships between coefficients.

Problem-Solving Techniques: Benefit from a plethora of solved examples and problem-solving techniques, empowering you to tackle a diverse range of problems related to coordinate geometry and straight lines.

Pattern Recognition: Develop a keen eye for recognizing patterns in binomial expansions. Practice identifying coefficients and terms efficiently, enhancing your problem-solving skills.

Applications in Algebraic Problem-Solving: Connect theoretical knowledge to practical applications in algebra. Explore how the Binomial Theorem is employed in problem-solving, providing you with a versatile tool for tackling mathematical challenges.

Practice Exercises: Engage in targeted practice exercises covering different difficulty levels. The module ensures you are well-prepared for assessments and class examinations.

Online Accessibility: The revision module is accessible online, allowing you to study anytime anywhere. This flexibility enables you to integrate revision seamlessly into your schedule.

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Binomial Theorem

Binomial Theorem for any Index

If n is negative integer, then n! is not defined. We state bionomial theorem in another form

$$(a+b)^n = a^n + \frac{n}{1!}a^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + --- + \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}a^{n-r}b^r + \cdots\cdots + b^n$$

Here, $T_{r+1} = \frac{n(n-1)(n-2)\cdots\cdots(n-r+1)}{r!}a^{n-r}b^r$

General & Middle Terms in Expansion of $(a+b)^n$

The general term of an expansion $(a+b)^n$ is

$T_{r+1} = {}^nC_r\, a^{n-r} b^r,\ 0 \le r \le n, r \in N$

Middle Terms:

1. In $(a+b)^n$, if n is even, then the no. of terms in the expansion is odd. Therefore, there is only one middle term and it is $\left(\frac{n+2}{2}\right)^{th}$ term.
2. In $(a+b)^n$, if n is odd then the no. of terms in the expansion is even. Therefore, there are two middle terms and those are $\left(\frac{n+1}{2}\right)^{th}$ *and* $\left(\frac{n+3}{2}\right)^{th}$ terms.

Some Special Cases $(a+b)^n$

In the expansion of (a+b)",

(i) Taking a=x and b=–y, we obtain

$(x-y)^n = {}^nC_0x^n - {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + --- + (-1)^n\, {}^nC_ny^n$

(ii) Taking a=1, b=x, we obtain

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \text{-------} + {}^nC_nx^n$

(iii) Taking a=1, b=–x, we obtain

$(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 + \text{----} + (-1)^n\, {}^nC_nx^n$

Definition

If a, b$\in R$ and n$\in N$, then

$$(a+b)^n = {}^nC_0\, a^nb^0 + {}^nC_1\, a^{n-1}b^1 + {}^nC_2\, a^{n-2}b^2 + ------ + {}^nC_n\, a^0b^n$$

- **Remarks:** If the index of the binomial is n then the expansion contains n+1 terms.
- In each term, the sum of indices of a and b is always n.
- Coefficients of the terms in binomial expansion equidistant from both the ends are equal.

$$(a-b)^n = {}^nC_0\, a^nb^0 - {}^nC_1a^{n-1}b^1 + {}^nC_2\, a^{n-2}b^2 + ------ + (-1)^n\, {}^nC_na^0b^n$$

Binomial Coefficients

The coefficient ${}^nC_0, {}^nC_1, {}^nC_2, \text{------}\, {}^nC_n$, in the expansion of $(a+b)^n$ are called binomial coefficients and denoted by $C_0, C_1, C_2 \text{----} C_n$ respectively

Properties of binomial coefficients:

(i) $C_0 + C_1 + C_2 + ----- + C_n = 2^n$

(ii) $C_0 - C_1 + C_2 ------ + (-1)^n\, C_n = 0$

(iii) $C_0 + C_2 + C_4 + ---- = C_1 + C_3 + C_5 + ---- = 2^{n-1}$

(iv) ${}^nC_{r_1} = {}^nC_{r_2} \Rightarrow r_1 = r_2$ or $r_1 + r_2 = n$

(v) ${}^nC_r + {}^nC_{r-1} = {}^{1+n}C_r$

(vi) ${}^nC_r = \frac{n}{r}\, {}^{n-1}C_{r-1}$

BINOMIAL THEOREM

Revision Module

CBSE-XI

MATHEMATICS

SECTION A NCERT EXERCISES

EXERCISE 8.1

Expand each of the expressions in Exercises 1 – 5.

1. $(1-2x)^5$

Sol. $(1-2x)^5$

$= {}^5C_0.1^5 + {}^5C_1.1^4.(-2x) + {}^5C_2.1^3(-2x)^2 + {}^5C_3.1^2(-2x)^3 + {}^5C_4.1^1(-2x)^4 + {}^5C_5 1^0(-2x)^5$

$= 1.1 + 5.1.(-2x) + \frac{5.4}{1.2}.1.\,4x^2 + \frac{5.4}{1.2}.1(-8x^3) + \frac{5}{1}.1.16x^4 + (-32x^5)$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5.$

2. $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Sol. $\left[\frac{2}{x}+\left(-\frac{x}{2}\right)\right]^5$

$= C(5,0)\left(\frac{2}{x}\right)^5 + C(5,1)\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + C(5,2)\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + C(5,3)\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + C(5,4)\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + C(5,5)\left(-\frac{x}{2}\right)^5$

$= 1\left(\frac{2}{x}\right)^5 + 5\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + 10\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + 10\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + 5\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + \left(-\frac{x}{2}\right)^5$

$= 32x^{-5} - 40x^{-3} + 20x^{-1} - 5x + \frac{5}{8}x^3 - \frac{1}{32}x^5$

3. $(2x-3)^6$

Sol. $(2x-3)^6$

$= {}^6C_0(2x)^6 + {}^6C_1(2x)^5(-3) + {}^6C_2(2x)^4(-3)^2 + {}^6C_3(2x)^3(-3)^3 + {}^6C_4(2x)^2(-3)^4 + {}^6C_5(2x)(-3)^5 + {}^6C_6(2x)^0(-3)^6$

$= 64x^6 + \frac{6}{1}(32x^5)(-3) + \frac{6\,.\,5}{1\,.\,2}(16x^4)9 + \frac{6\,.\,5\,.\,4}{1\,.\,2\,.\,3}(8x^3)(-27) + \frac{6\,.\,5}{1\,.\,2}(4x^2)81 + \frac{6}{1}(2x)(-243) + 729$

$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

4. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Sol. $\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3$

$$+{}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4+{}^5C_5\left(\frac{x}{3}\right)^0\left(\frac{1}{x}\right)^5$$

$$=\frac{x^5}{243}+\frac{5}{1}\cdot\frac{x^4}{81}\cdot\frac{1}{x}+\frac{5.4}{1.2}\cdot\frac{x^3}{27}\cdot\frac{1}{x^2}$$

$$+\frac{5.4}{1.2}\cdot\frac{x^2}{9}\cdot\frac{1}{x^3}+\frac{5}{1}\cdot\frac{x}{3}\cdot\frac{1}{x^4}+\frac{1}{x^5}$$

$$=\frac{x^5}{243}+\frac{5}{81}x^3+\frac{10}{27}x+\frac{10}{9}\cdot\frac{1}{x}+\frac{5}{3}\cdot\frac{1}{x^3}+\frac{1}{x^5}$$

5. $\left(x+\frac{1}{x}\right)^6$

Sol. $\left(x+\frac{1}{x}\right)^6={}^6C_0x^6\left(\frac{1}{x}\right)^0+{}^6C_1x^5\left(\frac{1}{x}\right)$

$$+{}^6C_2x^4\left(\frac{1}{x}\right)^2+{}^6C_3x^3\left(\frac{1}{x}\right)^3+{}^6C_4x^2\left(\frac{1}{x}\right)^4$$

$$+{}^6C_5x\left(\frac{1}{x}\right)^5+{}^6C_6x^0\left(\frac{1}{x}\right)^6$$

$$=x^6+\frac{6}{1}x^4+\frac{6.5}{1.2}x^2+\frac{6.5.4}{1.2.3}$$

$$+\frac{6.5}{1.2}\left(\frac{1}{x^2}\right)+\frac{6}{1}\left(\frac{1}{x^4}\right)+\left(\frac{1}{x^6}\right)$$

$$=x^6+6x^4+15x^2+20+15\frac{1}{x^2}+\frac{6}{x^4}+\frac{1}{x^6}$$

6. Using Binomial Theorem, evaluate the following:
$(96)^3$

Sol. We express 96 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem.

Write $96 = 100 - 4$

Therefore

$(96)^3 = (100-4)^3$

$= {}^3C_0\ (100)^3 - {}^3C_1\ (100)^2\ (4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3$

$= 1000000 - 3(10000)(4) + 3(100)(16) - (64)$

$= 1000000 - 120000 + 4800 - 64 = 884736$

7. Using binomial theorem, evaluate the value of $(102)^5$

Sol. $(102)^5=(100+2)^5$

$=100^5+{}^5C_1(100)^4 2+{}^5C_2(100)^3 2^2$

$+{}^5C_3(100)^2 2^3+{}^5C_4(100)2^4+{}^5C_5(100)^0 2^5$

$=10000000000+5\times(100000000)\times 2$

$+\frac{5\ .\ 4}{1\ .\ 2}(1000000)\times 4+\frac{5\ .\ 4}{1\ .\ 2}(10000)\times 8$

$+\frac{5}{1}(100)\times 16+32$

$=10000000000+1000000000+40000000$

$+800000+8000+32$

$=11040808032$

8. Using Binomial Theorem, evaluate the following :
$(101)^4$

Sol. $(101)^4=(100+1)^4$

$={}^4C_0(100)^4+{}^4C_1(100)^3(1)+{}^4C_2(100)^2(1)^2$

$+{}^4C_3(100)^1(1)^3+{}^4C_4(1)^4$

$=100000000+4(1000000)+6(10000)+4(100)+1$

$=100000000+4\,000000+60000+4\,00+1$

$=104060401$

9. Using Binomial theorem evaluate $(99)^5$

Sol. $(99)^5=(100-1)^5$

$={}^5C_0(100)^5+{}^5C_1(100)^4(-1)$

$+{}^5C_2(100)^3(-1)^2+{}^5C_3(100)^2(-1)^3$

$+{}^5C_4(100)(-1)^4+(-1)^5$

$=10000000000-500000000+10000000$

$-100000+500-1=9509900499$

10. Using Binomial theorem indicate which number is larger $(1.1)^{10000}$ or 1000.

Sol. $(1.1)^{10000}=[1+(0.1)]^{10000}$

Expanding by binomial theorem

$=C(10000,0)(1)^{10000}$

$+C(10000,1)(1)^{10000-1}(0.1)+$ other terms

$=1+10000\times 0.1+$ other terms

$=1001+$ other terms

Hence, $(1.1)^{10000}>1000$.

11. Find $(a+b)^4-(a-b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$

Sol. (i) $(a+b)^4 = a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4a^0b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \quad \ldots(i)$

$(a-b)^4 = a^4 + {}^4C_1a^3(-b) + {}^4C_2a^2(-b)^2 + {}^4C_3a(-b)^3 + {}^4C_4(-b)^4$

$= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \quad \ldots(ii)$

Subtracting (ii) from (i)

$(a+b)^4 - (a-b)^4 = 2[4a^3b + 4ab^3]$

$= 8ab(a^2+b^2) \quad \ldots(iii)$

Putting $a=\sqrt{3}, b=\sqrt{2}$ in equ. (iii)

$\therefore \quad (\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

$= 8.\sqrt{3}.\sqrt{2}\left[(\sqrt{3})^2 + (\sqrt{2})^2\right]$

$= 8\sqrt{6}(3+2) = 40\sqrt{6}$

12. Find $(x+1)^6+(x-1)^6$, Hence or otherwise evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$

Sol. $(x+1)^6 = x^6 + {}^6C_1.x^5.1 + {}^6C_2.x^4.1^2 + {}^6C_3.x^3.1^3 + {}^6C_4.x^2.1^4 + {}^6C_5.x.1^5 + {}^6C_6.x^0.1^6$

$= x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1 \quad \ldots(i)$

$(x-1)^6 = x^6 + {}^6C_1.x^5.(-1) + {}^6C_2.x^4.(-1)^2 + {}^6C_3.x^3.(-1)^3 + {}^6C_4.x^2.(-1)^4 + {}^6C_5.x.(-1)^5 + {}^6C_6.x^0.(-1)^6$

$= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1 \quad \ldots(ii)$

Adding (i) and (ii)

$(x+1)^6 + (x-1)^6 = 2[x^6 + 15x^4 + 15x^2 + 1]$

Putting $x=\sqrt{2}$

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$= 2[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$

$= 2[8 + 60 + 30 + 1] = 2 \times 99 = 198$

13. Show that $9^{n+1} - 8n - 9$ is divisible by 64 whenever n is a positive integer.

Sol. We have

$(1+x)^{n+1} = C(n+1,0) + C(n+1,1)x + C(n+1,2)x^2 + C(n+1,3)x^3 + \ldots\ldots\ldots + C(n+1,n+1)x^{n+1}$

Putting x = 8, we get

$(1+8)^{n+1} = C(n+1,0) + C(n+1,1)8 + C(n+1,2)8^2 + C(n+1,3)8^3 + \ldots\ldots\ldots + C(n+1,n+1)8^{n+1}$

$9^{n+1} = C(n+1,0) + C(n+1,1)8 + C(n+1,2)8^2 + C(n+1,3)8^3 + \ldots\ldots\ldots + C(n+1,n+1)8^{n+1}$

$[\because C(n+1,0) = 1 \text{ and } C(n+1,1) = n+1]$

or $\quad 9^{n+1} = 1 + 8n + 8 + C(n+1,2)8^2 + C(n+1,3)8^3 + \ldots\ldots\ldots + C(n+1,n+1)8^{n+1}$

or $\quad 9^{n+1} - 8n - 9$

$= C(n+1,2)8^2 + C(n+1,3)8^3 + \ldots\ldots\ldots + C(n+1,n+1)8^{n+1}$

$= 8^2[C(n+1,2) + C(n+1,3)8 + C(n+1,4)8^2 + \ldots\ldots\ldots + C(n+1,n+1)8^{n-1}]$

$9^{n+1} - 8n - 9 = 64 \times$ some constant quantity.

Hence, $9^{n+1} - 8n - 9$ is divisible by 64 whenever n is a positive integer.

14. Prove that $\sum_{r=0}^{n} 3^r\, {}^nC_r = 4^n$

Sol. $\sum_{r=0}^{n} 3^r\, {}^nC_r = {}^nC_0 + {}^nC_1.3^1 + {}^nC_2 3^2 + \ldots\ldots\ldots + {}^nC_r.3^r + \ldots\ldots + {}^nC_n.3^n$

$= (1+3)^n = 4^n$

EXERCISE 8.2

1. Find the coefficient of x^5 in $(x+3)^8$.

Sol. General term in $(x+3)^8 = {}^8C_r x^{8-r}.3^r$

We have to find the coefficient of x^5

$8 - r = 5, r = 8 - 5 = 3$

$\therefore$ Coefficient of x^5 (putting $r=3$)

$= {}^8C_3 . 3^3 = \frac{8.7.6}{1.2.3} . 27 = 56.27 = 1512$

2. Find the coefficient of a^5b^7 in $(a-2b)^{12}$.

Sol. $(a-2b)^{12} = [a + (-2b)]^{12}$

General term $T_{r+1} = C(12,r)\, a^{12-r}(-2b)^r$.

Putting $12-r=5$ or $12-5=r \Rightarrow r=7$

$T_{7+1} = C(12,7)\, a^{12-7}(-2b)^7$

$= C(12,7)\, a^5(-2b)^7 = C(12,7)(-2)^7 a^5 b^7$

Hence required coefficient is $C(12,7)(-2)^7$

$= -\frac{12!}{7!\,5!}.2^7$

$-\frac{-12\times 11\times 10\times 9\times 8\times 7!}{7!\times 5\times 4\times 3\times 2\times 1}\times 2^7$

$= 8\times -11\times 9\times 2^7$

$= -99\times 8\times 128 = -101376$

3. Write the general term in the expansion of $(x^2-y)^6$.

Sol. General term $= T_{r+1} = {}^6C_r(x^2)^{6-r}(-y)^r$

$= (-1)^r\frac{6!}{r!(6-r)!}.x^{12-2r}.y^r$

4. Write the general term in the expansion of $(x^2-yx)^{12}, x\neq 0$.

Sol. Binomial expansion is $(x^2-yx)^{12}$

General term $T_{r+1} = {}^{12}C_r(x^2)^{12-r}.(-yx)^r$

$= \frac{12!}{r!(12-r)!}.x^{24-2r}.(-1)^r.y^r x^r$

$= \frac{(-1)^r 12!}{r!(12-r)!}.x^{24-r}.y^r$

5. Find the 4th term in the expansion of $(x-2y)^{12}$

Sol. 4th term $= T_{3+1}$ in the expansion of $(x+(-2y))^{12}$

$= {}^{12}C_3 x^{12-3}[-2y]^3$

$= \frac{12.11.10}{1.2.3}x^9(-1)^3.2^3.y^3$

$= -220\times 8.x^9.y^3 = -1760\,x^9 y^3.$

6. Find the 13th term in the expansion of

$$\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x\neq 0$$

Sol. 13th term, $T_{13} = T_{12+1}$

$= {}^{18}C_{12}(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$= {}^{18}C_6 9^6.x^6(-1)^{12}.\frac{1}{3^{12}}\times\frac{1}{x^6}$

$= 18564\times(3^2)^6.\frac{1}{3^{12}}\times\frac{x^6}{x^6}$

$= 18564\times\frac{3^{12}}{3^{12}} = 18564$

7. Find the middle term in the expansion of

$$\left(3-\frac{x^3}{6}\right)^7.$$

Sol. Number of terms in the expansion is $7+1=8$

There are two middle terms which are

$\left(\frac{8}{2}\right)^{th}$ & $\left(\frac{7+3}{2}\right)^{th}$ i.e., 4th & 5th

Hence, we have to find T_4 and T_5 in the given expansion $\left(3-\frac{x^3}{6}\right)^7 = \left[3+\left(-\frac{x^3}{6}\right)\right]^7$

$T_{r+1} = C(7,r)3^{7-r}\left(-\frac{x^3}{6}\right)^r$ ...(i)

Now $T_{r+1} = T_4$ or $r+1=4$ $\therefore$ $r=3$

Putting $r=3$, we have

$T_{3+1} = C(7,3)3^{7-3}\left(-\frac{x^3}{6}\right)^3$

$= C(7,3)3^4(-1)^3\frac{x^9}{6^3} = \frac{-7!}{3!\,4!}\frac{3}{2^3}x^9$

$= \frac{-7\times 6\times 5\times 4!}{3\times 2\times 1\times 4!}.\frac{3}{2^3}.x^9 = \frac{-105}{8}x^9$

Again $T_{r+1} = T_5$ or $r+1=5$ or $r=4$

Putting $r=4$ in (i), we have

$T_{4+1} = T_5 = C(7,4)\,3^{7-4}(-1)^4\frac{x^{12}}{6^4}$

$= \frac{7!}{4!\,3!}\frac{3^3x^{12}}{3^4 2^4} = \frac{7\times 6\times 5\times 4!}{3\times 2\times 1\times 4!}\times\frac{x^{12}}{3\times 2^4}$

$= \frac{35}{48}x^{12}$

8. Find the middle term in the expansion of

$$\left(\frac{x}{3}+9y\right)^{10}$$

Sol. Number of terms in the expansion is

$10+1=11$ (odd)

Middle term of the expansion is $\left(\frac{n}{2}+1\right)^{th}$ term

$= (5+1)^{\text{th}}$ term $= 6^{\text{th}}$ term

$$T_6 = T_{5+1} = C(10,5)\left(\frac{x}{3}\right)^{10-5}(9y)^5$$

$$= C(10,5)\frac{x^5}{3^5}9^5y^5 = C(10,5)3^5x^5y^5$$

$$= \frac{10!}{5!(10-5)!}3^5x^5y^5 = \frac{10!}{5!\,5!}3^5x^5y^5$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}3^5x^5y^5$$

$$= 61236x^5y^5$$

9. In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Sol. General term in the expansion of $(1+a)^{m+n}$ is

$T_{r+1} = {}^{m+n}C_r a^r$

Putting $r = m$

$$T_{m+1} = {}^{m+n}C_m a^m \quad \ldots (i)$$

$\therefore$ Coefficient of $a^m = {}^{m+n}C_m$

Again putting $r = n$

$$T_{n+1} = {}^{m+n}C_n a^n$$

Coefficient of $a^n = {}^{m+n}C_n = {}^{m+n}C_m \quad \ldots (ii)$

$[\because {}^nC_r = {}^nC_{n-r}]$

From (i) and (ii) coefficient of a^m is equal to coefficient of a^n.

10. The coefficients of the $(r-1)^{\text{th}}$, r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(x+1)^n$ are in the ratio of 1 : 3 : 5. Find n and r.

Sol. General term in the expansion of $(x+1)^n$ is

$$T_{r-1} = T_{(r-2)} = {}^nC_{r-2}.x^{r-2}$$

$$T_r = T_{(r-1)} = {}^nC_{r-1}.x^{r-1}$$

$$T_{r+1} = {}^nC_r.x^r$$

$C(n,r-2) : C(n,r-1) : C(n,r) = 1 : 3 : 5$

or $\dfrac{C(n,r-2)}{1} = \dfrac{C(n,r-1)}{3} = \dfrac{C(n,r)}{5}$

If $\dfrac{C(n,r-2)}{1} = \dfrac{C(n,r-1)}{3}$

or $3C(n,r-2) = C(n,r-1)$

or $3\dfrac{n!}{(r-2)!(n+2-r)!} = \dfrac{n!}{(r-1)!(n+1-r)!}$

or $\dfrac{3}{(r-2)!(n+2-r)(n+1-r)!}$

$= \dfrac{1}{(r-1)(r-2)!(n+1-r)!}$

or $\dfrac{3}{n+2-r} = \dfrac{1}{r-1}$

or $3r-3 = n+2-r$

or $4r = n+5 \quad \ldots (i)$

Again if $\dfrac{C(n,r-1)}{3} = \dfrac{C(n,r)}{5}$

or $5C(n,r-1) = 3C(n,r)$

or $5\dfrac{n!}{(r-1)!(n+1-r)!} = 3\dfrac{n!}{r!(n-r)!}$

or $\dfrac{5}{(r-1)!(n+1-r)(n-r)!} = \dfrac{3}{r(r-1)!(n-r)!}$

or $5r = 3(n+1-r)$ or $8r = 3n+3 \quad \ldots (ii)$

From (i) & (ii) $2n+10 = 3n+3$

or $3n - 2n = 10 - 3 \Rightarrow n = 7$

From (ii) $8r = 21 + 3 = 24$

$\therefore \quad r = 3$

$\therefore \quad n = 7, r = 3$

11. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

Sol. General term in the expansion of $(1+x)^{2n}$ is

$T_{r+1} = C(2n, r)x^r$

Putting r = n, we have

$T_{n+1} = C(2n, n)x^n$

Coefficient of $x^n = C(2n, n)$

Again general term in the expansion of

$(1+x)^{2n-1}$ is $T_{r+1} = C(2n-1, r)x^r$

Putting $r = n$, we have

$T_{n+1} = C(2n-1, n)x^n$

Coefficient of x^n in the expansion of $(1+x)^{2n-1}$ is $C(2n-1, n)$

According to the problem, we have to prove that

$$C(2n, n) = 2\times C(2n-1, n)$$

or $\dfrac{2n!}{n!(2n-n)!} = 2.\dfrac{(2n-1)!}{n!(2n-1-n)!}$

or $\dfrac{2n!}{n!n!} = 2.\dfrac{(2n-1)!}{n!(n-1)!}$

Multiplying N^r and D^r by n on RHS, we have

$$\frac{2n!}{n!n!} = \frac{2n(2n-1)!}{n!n(n-1)!}$$

i.e $\dfrac{2n!}{n!n!} = \dfrac{2n!}{n!n!}$, Which is true.

Hence proved.

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

Sol. Given expansion is $(1+x)^m$. Now,

General term $= T_{r+1} = {}^mC_r\, x^r$

Put r = 2, we have

$T_3 = {}^mC_2 . x^2$

According to the question $C(m, 2) = 6$

or $\dfrac{m(m-1)}{2!} = 6$

$\Rightarrow$ $m^2 - m = 12$

or $m^2 - m - 12 = 0$

$\Rightarrow$ $m^2 - 4m + 3m - 12 = 0$

or $(m-4)(m+3) = 0$

$\therefore$ $m = 4$, since $m \neq -3$

MISCELLANEOUS EXERCISE

1. Find a, b and n in the expansion of $(a+b)^n$. If the first three terms of the expansion are 729, 7290 and 30375, respectively.

Sol. T_1 of $(a+b)^n = a^n = 729$ ...(i)

T_2 of $(a+b)^n = {}^nC_1 a^{n-1} b = 7290$ ...(ii)

T_3 of $(a+b)^n = {}^nC_2 a^{n-2} b^2 = 30375$ ...(iii)

Dividing (i) by (ii),

$$\frac{a^n}{{}^nC_1 a^{n-1} b} = \frac{729}{7290} = \frac{1}{10} \text{ or } \frac{a}{nb} = \frac{1}{10} \quad \text{...(iv)}$$

Dividing (ii) by (iii)

$$\frac{{}^nC_1 a^{n-1} b}{{}^nC_2 a^{n-2} b^2} = \frac{7290}{30375}$$

or $\dfrac{n a^{n-1} b}{\dfrac{n(n-1)}{2} a^{n-2} b^2} = \dfrac{7290}{30375} = \dfrac{6}{25}$

or $\dfrac{2}{n-1}\times\dfrac{a}{b} = \dfrac{6}{25}$ ...(v)

Dividing (iv) by (v)

$$\frac{a}{nb}\times\frac{(n-1)b}{2a} = \frac{1}{10}\times\frac{25}{6} = \frac{5}{12}$$

or $\dfrac{n-1}{2n} = \dfrac{5}{12}$

or $12n - 12 = 10n$

or $2n = 12$ or $n = 6$

Also, putting $n = 6$ in (i) $a^6 = 729 \therefore a = 3$

Putting $n = 6, a = 3$ in eqn (iv)

$$\frac{3}{6b} = \frac{1}{10} \quad \therefore b = \frac{3\times 10}{6} = 5$$

Thus $a = 3, b = 5, n = 6$

2. Find a if the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal.

Sol. General term $= T_{r+1} = {}^9C_r . 3^{9-r} a^r x^r$

Putting $r = 2$

Coefficient of $x^2 = {}^9C_2 . 3^{9-2} a^2$

$$= \frac{9\times 8}{2} . 3^7 a^2 = 4.\ 3^9\ a^2 \quad \text{...(i)}$$

Putting $r = 3$

Coefficient of $x^3 = {}^9C_3 . 3^{9-3}$

$a^3 = \dfrac{9\times 8\times 7}{6}\times 3^6\times a^3$

$= 4\times 7\times 3^7 . a^3$ ...(ii)

Equating (i) & (ii)

$4.\ 3^9 . a^2 = 4\times 7\times 3^7\times a^3$

or $3^2 = 7a \Rightarrow a = \dfrac{9}{7}$

3. Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

Sol. $(1+2x)^6$

$$= {}^6C_0 . 1 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$

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$=1+12x+60x^2+20\times 8x^3+15\times 16x^4$

$+6\times 32x^5+64x^6$

$=1+12x+60x^2+160x^3+240x^4$

$+192x^5+64x^6$ ... (i)

$(1-x)^7 = 1-{}^7C_1x+{}^7C_2x^2-{}^7C_3x^3+{}^7C_4x^4$

$-{}^7C_5x^5+{}^7C_6x^6-{}^7C_7x^7$

$= 1-7x+21x^2-35x^3+35x^4$

$-21x^5+7x^6-x^7$ ...(ii)

Multiplying (i) and (ii) and collecting the coefficient of x^5

$\therefore$ Coefficient of x^5 in the product $(1+2x)^6(1-x)^7$

$= 1\times(-21)+12\times 35+60\times(-35)+160\times 21$

$+240\times(-7)+192\times 1$

$=-21+420-2100+3360-1680+192=171$

4. If a and b are distinct integers, prove that $a-b$ is a factor of a^n-b^n, whenever n is a positive integer.

Sol. Now, $a=a+b-b=b+(a-b)$

$a^n=\{b+(a-b)\}^n$

$= b^n+{}^nC_1b^{n-1}(a-b)+{}^nC_2b^{n-2}(a-b)^2$

$+\ldots\ldots\ldots\ldots+(a-b)^n$

or $a^n-b^n={}^nC_1b^{n-1}(a-b)+{}^nC_2(a-b)^2b^{n-2}$

$+\ldots\ldots\ldots\ldots+(a-b)^n$

$=(a-b)[{}^nC_1b^{n-1}+{}^nC_2b^{n-2}(a-b)$

$+\ldots\ldots+(a-b)^{n-1}]$

Thus, $(a-b)$ is a factor of (a^n-b^n).

5. Evaluate $(\sqrt{3}+\sqrt{2})^6-(\sqrt{3}-\sqrt{2})^6$

Sol. $(\sqrt{3}+\sqrt{2})^6$

$=(\sqrt{3})^6+{}^6C_1(\sqrt{3})^5(\sqrt{2})+{}^6C_2(\sqrt{3})^4(\sqrt{2})^2$

$+{}^6C_3(\sqrt{3})^3(\sqrt{2})^3+{}^6C_4(\sqrt{3})^2(\sqrt{2})^4$

$+{}^6C_5(\sqrt{3})(\sqrt{2})^5+(\sqrt{2})^6$ ... (i)

$(\sqrt{3}-\sqrt{2})^6$

$=(\sqrt{3})^6-{}^6C_1(\sqrt{3})^5(\sqrt{2})+{}^6C_2(\sqrt{3})^4(\sqrt{2})^2$

$-{}^6C_3(\sqrt{3})^3(\sqrt{2})^3+{}^6C_4(\sqrt{3})^2(\sqrt{2})^4$

$-{}^6C_5(\sqrt{3})(\sqrt{2})^5+(\sqrt{2})^6$ ... (ii)

Subtracting (ii) from (i)

$(\sqrt{3}+\sqrt{2})^6-(\sqrt{3}-\sqrt{2})^6$

$=2[{}^6C_1(\sqrt{3})^5(\sqrt{2})+{}^6C_3(\sqrt{3})^3(\sqrt{2})^3$

$+{}^6C_5(\sqrt{3})(\sqrt{2})^5]$

$=2\left[6.3^{5/2}.2^{1/2}+20.3^{3/2}.2^{3/2}+6.3^{1/2}.2^{5/2}\right]$

$=2.3^{1/2}.2^{1/2}[6.3^2+20.3.2+6.2^2]$

$=2\sqrt{6}[54+120+24]$

$=2\sqrt{6}\times 198=396\sqrt{6}$

6. Find the value of $(a^2+\sqrt{a^2-1})^4+(a^2-\sqrt{a^2-1})^4$.

Sol. Put $a^2=x$, $\sqrt{a^2-1}=y$

$\therefore\quad (x+y)^4 = x^4+{}^4C_1x^3y+{}^4C_2x^2y^2$

$+{}^4C_3xy^3+{}^4C_4y^4$ ... (i)

$(x-y)^4=x^4-{}^4C_1x^3y+{}^4C_2x^2y^2$

$-{}^4C_3xy^3+{}^4C_4y^4$ ... (ii)

Adding (i) & (ii)

$(x+y)^4+(x-y)^4=2[x^4+{}^4C_2x^2y^2+{}^4C_4y^4]$

$=2[x^4+6x^2y^2+y^4]$

$\therefore\ (a^2+\sqrt{a^2-1})^4+(a^2-\sqrt{a^2-1})^4$

$=2[(a^2)^4+6(a^2)^2(\sqrt{a^2-1})^2+(\sqrt{a^2-1})^4]$

$=2[a^8+6a^4(a^2-1)+(a^2-1)^2]$

$=2[a^8+6a^4(a^2-1)+a^4-2a^2+1]$

$=2[a^8+6a^6-5a^4-2a^2+1]$

7. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Sol. $(0.99)^5=(1-0.01)^5$

$=1-{}^5C_1\times(0.01)+{}^5C_2\times(0.01)^2\ldots\ldots\ldots$

$=1-0.05+10\times 0.0001\ldots\ldots\ldots\ldots$

$=1.001-0.05=0.951$

8. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\dfrac{1}{\sqrt[4]{3}}\right)$ is $\sqrt{6}:1$.

Sol. T_5 in $\left(2^{1/4}+\dfrac{1}{3^{1/4}}\right)^n={}^nC_4.(2^{1/4})^{n-4}.\left(\dfrac{1}{3^{1/4}}\right)^4$

$$\left(\because T_{n-3} = T_{(n-4)+1}\right)$$

$$= {}^nC_4 . 2^{(n-4)/4} . \frac{1}{3} \quad \ldots\text{(i)}$$

Total number of terms $= n+1$
Fifth term from the end
$= [(n+1)-5+1]^{\text{th}}$ term from the beginning
$= (n-3)^{\text{th}}$ term

$$= {}^nC_{n-4} . (2^{1/4})^{n-(n-4)} . \left(\frac{1}{3^{1/4}}\right)^{n-4}$$

$$= {}^nC_{n-4} . 2 . \left(\frac{1}{3}\right)^{n-4/4} = {}^nC_4 . 2 . \left(\frac{1}{3}\right)^{n-4/4} \quad \ldots\text{(ii)}$$

Dividing (i) by (ii)

$$\frac{{}^nC_4 2^{(n-4)/4} . \frac{1}{3}}{{}^nC_4 . 2\left(\frac{1}{3}\right)^{(n-4)/4}} = \frac{\sqrt{6}}{1}$$

or $$\frac{2^{\frac{n}{4}-2}}{\left(\frac{1}{3}\right)^{\frac{n}{4}-2}} = \frac{\sqrt{6}}{1}$$

or $$2^{\frac{n}{4}-2} . 3^{\frac{n}{4}-2} = 6^{\frac{1}{2}}$$

or $$6^{\frac{n}{4}-2} = 6^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{n}{4} - 2 = \frac{1}{2} \quad \text{or} \quad \frac{n}{4} = \frac{5}{2}$$

or $$n = \frac{5}{2} \times 4 = 10$$

9. Expand using Binomial theorem

$$\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0$$

Sol. $$\left[\left(1+\frac{x}{2}\right) - \frac{2}{x}\right]^4$$

$$= \left(1+\frac{x}{2}\right)^4 + {}^4C_1\left(1+\frac{x}{2}\right)^3\left(-\frac{2}{x}\right)$$

$$+ {}^4C_2\left(1+\frac{x}{2}\right)^2\left(-\frac{2}{x}\right)^2$$

$$+ {}^4C_3\left(1+\frac{x}{2}\right)\left(-\frac{2}{x}\right)^3 + {}^4C_4\left(-\frac{2}{x}\right)^4$$

$$= \left(1+\frac{x}{2}\right)^4 - 8.\frac{1}{x}\left(1+\frac{x}{2}\right)^3 + 24\frac{1}{x^2}\left(1+\frac{x}{2}\right)^2$$

$$-32\frac{1}{x^3}\left(1+\frac{x}{2}\right) + \frac{16}{x^4}$$

Expanding, $\left(1+\frac{x}{2}\right)^4, \left(1+\frac{x}{2}\right)^3, \left(1+\frac{x}{2}\right)^2$

$$\left[\left(1+\frac{n}{2}\right) - \frac{2}{x}\right]^4 = \left(1 + 4\frac{x}{2} + 6.\frac{x^2}{4} + 4\frac{x^3}{8} + \frac{x^4}{16}\right)$$

$$-8.\frac{1}{x}\left(1 + 3.\frac{x}{2} + 3.\frac{x^2}{4} + \frac{x^3}{8}\right)$$

$$+24.\frac{1}{x}\left(1 + x + \frac{x^2}{4}\right) - 32 \times \frac{1}{x^3}\left(1+\frac{x}{2}\right) + \frac{-16}{x^4}$$

$$= \left(1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{x^4}{16}\right)$$

$$-\left(\frac{8}{x} + 12 + 6x + x^2\right) + \left(\frac{24}{x^2} + \frac{24}{x} + 6\right)$$

$$-\left(\frac{32}{x^3} + \frac{16}{x^2}\right) + \frac{16}{x^4}$$

$$= \frac{x^4}{16} + \frac{x^3}{2} + \frac{x^2}{2} - 4x - 5 + \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4}$$

10. Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.

Sol. $$\left[3x^2 - a(2x - 3a)\right]^3$$

$$= (3x^2)^3 - {}^3C_1(3x^2)^2.a(2x-3a)$$

$$+ {}^3C_2(3x^2).a^2(2x-3a) - a^3(2x-3a)^3$$

$$= 27x^6 - 27x^4a(2x-3a)$$

$$+9x^2a^2(4x^2 - 12ax + 9a^2)$$

$$-a^3(3.4x^2.3a - 3.2x.9a^2 - 27a^3)$$

$$= 27x^6 - 57x^5a + 81a^2x^4 + 36a^2x^4$$

$$-108a^3x^3 + 81a^4x^2 - 8a^3x^3 + 36a^4x^2 - 54a^5x + 27a^6$$

$$= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3$$

$$+117a^4x^2 - 54a^5x + 27a^6$$

SECTION B PRACTICE QUESTIONS

Short Answer Questions

1. If the coefficient of x in $(x^2 + k/x)^5$ is 270 then find the value of k.
2. Find the term independent of x in the expansion $\left(x^2 - \frac{1}{x}\right)^9$.
3. Evaluate the number of terms in the expansion $(x+y+z)^{10}$.
4. If the coefficient of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal , then find the value of n.
5. In the expansion of $(1+x)^{50}$, Evaluate the sum of the coefficient of odd powers of x.
6. In the expansion of $(1+x)^m.(1-x)^n$, the coefficient of x and x^2 are 3 and –6 respectively, then find the value of m.
7. Find the approximate value of $(7.995)^{1/3}$ correct to four decimal places.
8. Evaluate $3\cdot {}^nC_0 - 8.{}^nC_1 + 13.{}^nC_2 - 18.{}^nC_3 + \ldots\ldots\ldots$ to $(n+1)$ term.
9. Find the coefficient of x^{-12} in the expansion of $\left(x+\frac{y}{x^3}\right)^{20}$.
10. Find the middle term in the expansion of $\left(\frac{2}{3}x-\frac{3}{2}y\right)^{20}$.
11. Find the 7th term from the end in the expansion of $\left(x-\frac{2}{x^2}\right)^{10}$.
12. Show that $2^{4n} - 15n - 1$ is divisible by 225.
13. If the coefficients of $(r-5)^{th}$ and $(2r - 1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal, find r.
14. Find the coefficient of x^{50} after simplifying and collecting the like terms in the expansion of $(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + x^{1000}$.

Long Answer Questions

1. If the r^{th} term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4, then determine the value of r.
2. Evaluate the coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m}.2^m$
3. Find the coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$.
4. Find the number of zero terms in the expansion of $(1+3\sqrt{2}x)^9 + (1-3\sqrt{2}x)^9$.
5. If the third term in the expansion of $[x + x^{\log_{10} x}]^5$ is equal to 10,00,000, then find the value of x.
6. Given positive integers $r > 1$, $n > 2$ and the coefficients of $(3r)^{th}$ and $(r+2)^{th}$ terms in the Binomial expansion of $(1+x)^{2n}$ are equal. Find the relation between n and r.
7. Find the greatest coefficient in the expansion of $(1+x)^{10}$.
8. Determine the greatest coefficient in the expansion of $(1+x)^{2n+2}$
9. If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{2n}x^{2n}$ then find the value of $a_0 + a_2 + a_4 + \ldots\ldots + a_{2n}$.
10. Evaluate $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \ldots\ldots$

11. Find the number of integral terms in the expansion of $\left(5^{1/2}+7^{1/8}\right)^{1024}$.

12. Find the term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$.

13. Determine the value of

$${}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+.....+(-1)^n\,{}^nC_n.$$

14. Evaluate

$$C_0+3C_1+5C_2+7C_3+..........+(2n+1)\,C_n.$$

15. Evaluate $1+\frac{1}{3}x+\frac{1}{3}.\frac{4}{6}x^2+\frac{1.4.7}{3.6.9}x^3+........$

16. Find the value of

$$1-\frac{1}{2}.\frac{1}{2}+\frac{1.3}{2.4}.\frac{1}{2^2}-\frac{1.3.5}{2.4.6}\frac{1}{2^3}+.......$$

17. Determine the coefficient of x^r $(0\le r\le n-1)$ in the expansion of

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+...............+(x+2)^{n-1}$$

18. Find numerically greatest term in the expansion of $(2+3x)^2$ when $x=\frac{5}{6}$

19. Find the sum of series

$$\frac{2(n/2)!(n/2)!}{n!}\{C_0{}^2-2C_1{}^2+3C_2{}^2-4C_3{}^2+..........+(-1)^n(n+1)C_n{}^2\}$$

where n is an even positive integer.

20. Find the value of

$$1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5\,8}{3.6.9}\left(\frac{1}{2}\right)^3+.........$$

21. Find the value of

$$C_0C_r+C_1C_{r+1}+........+C_{n-r}C_n$$

22. If $(3\sqrt{3}+5)^7=P+F$, where P is an integer and F is a proper fraction, then find the value of $F.(P+F)$.

23. If the fourth term in the expansion of $[\sqrt{\{x^{1/(1+\log x)}\}}+x^{1/2}]^6$ is equal to 200 and $x>1$, then find the value of x.

24. Find the value of x for which the 6th term in the expansion of

$$\left\{2^{\log_2\sqrt{(9^{x-1}+7)}}+\frac{1}{2^{1/5\log 2(3^{x-1}+1)}}\right\}^7$$ is 84.

25. Find the ratio of 11th term from the beginning and 11th term from the end in the expansion of

$$\left(2x-\frac{1}{x^2}\right)^{25}$$

26. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.......(2n-1)}{n!}2^n x^n$.

27. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$

28. If P be the sum of odd terms and Q that of even terms in the expansion of $(x+a)^n$, prove that :

(i) $(x^2-a^2)^n=\mathrm{P}^2-\mathrm{Q}^2$

(ii) $(x+a)^{2n}-(x-a)^{2n}=4\mathrm{PQ}$

(iii) $(x+a)^{2n}+(x-a)^{2n}=2\,(\mathrm{P}^2+\mathrm{Q}^2)$

29. If the coefficients of a^{r-1}, a^r and a^{r+1} in the expansion of $(1+a)^n$ are in arithmetic progression prove that

$$n^2-n(4r+1)+4r^2-2=0.$$

30. Show that $2^{4n+4}-15n-16$, where $n\in N$ is divisible by 225.

PRACTICE QUESTION'S SOLUTIONS

Short Answer Questions

1. ***Ans*** : 3

Hint - $T_{r+1} = {}^5C_r (x^2)^{5-r} (k/x)^r = {}^5C_r k^r x^{10-3r}$

For coefficient of x, $10 - 3r = 1 \Rightarrow r = 3$

coefficient $x = {}^5C_3\, k^3 = 270$

$$\Rightarrow \quad k^3 = \frac{270}{10} = 27 \quad \therefore k = 3$$

2. ***Ans*** : 84

Hint -

$$T_{r+1} = {}^9C_r (x^2)^{9-r} \left(-\frac{1}{x}\right)^r = {}^9C_r (-1)^r x^{18-3r}$$

for term, independent of x, $18 - 3r = 0 \Rightarrow r = 6$

$\therefore$ term independent of $x = {}^9C_6 (-1)^6 = 84$

3. ***Ans*** : 66

Hint - Number of term in the expansion

$$= \frac{1}{2}(n+1)(n+2) = \frac{1}{2}(10+1)(10+2) = 66$$

4. ***Ans*** : 55

Hint.- Coefficient of x^7 = Coefficient of x^8

$$\Rightarrow {}^nC_7 . 2^{n-7} \left(\frac{1}{3}\right)^7 = {}^nC_8 . 2^{n-8} \left(\frac{1}{3}\right)^8$$

$$\Rightarrow 7! \frac{n!}{(n-7)!} . 2 = \frac{n!}{8!(n-8)!} . \frac{1}{3}$$

$$\Rightarrow \frac{2}{n-7} = \frac{1}{8.3} \Rightarrow n = 55$$

5. ***Ans*** : 2^{49}

Hint - $(1+x)^{50} = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots\ldots\ldots + C_{50} x^{50}$ ... (i)

Replacing $x = 1$ and $x = -1$ Successively in (i)

$2^{50} = C_0 + C_1 + C_2 + C_3 + \ldots\ldots\ldots + C_{50}$ ... (ii)

$0 = C_0 - C_1 + C_2 - C_3 + \ldots\ldots\ldots + C_{50}$ ... (iii)

Subtracting (iii) from (ii)

$$2^{50} = 2[C_1 + C_3 + C_5 + \ldots\ldots + C_{49}]$$

i.e. $C_1 + C_3 + C_5 + \ldots\ldots\ldots + C_{49} = 2^{49}$

6. ***Ans*** : 12

Hint -

$$(1+x)^m (1-x)^n = \left[1 + mx + \frac{m(m-1)}{2!}x^2 + \ldots\ldots\right]$$

$$\times \left[1 - nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots\right]$$

$\therefore$ Coeff. of $x = m - n = 3$ ... (i)

and coeff. of $x^2 = \frac{m}{2}(m-1) + \frac{n}{2}(n-1) - mn$

$= -6$ ... (ii)

Solving (i) & (ii), we get m = 12

7. **Hint** - $(7.995)^{1/3} = (8 - .005)^{1/3}$

$$= (8)^{1/3}(1 - .005/8)^{1/3}$$

$$= 2\left[1 - \frac{1}{3} \times \frac{0.005}{8} + \frac{(1/3)(1/3-1)}{2!}\left(\frac{.005}{8}\right)^2\right]$$

$= 2[1 - 0.000208] = 1.999584 = 1.9996$

8. $3 \cdot {}^nC_0 - 8 . {}^nC_1 + 13 . {}^nC_2 - 18 . {}^nC_3 + \ldots\ldots\ldots$ to $(n+1)$ term

$= 3[C_0 - C_1 + C_2 + \ldots\ldots\ldots] - 5[C_1 - 2C_2 + 3C_3]$

$= 3 \times 0 - 5 \times 0 = 0$

9. Suppose x^{-12} occurs in $(r+1)$th term. We have

$$T_{r+1} = {}^{20}C_r\, x^{20-r} \left(\frac{y}{x^3}\right)^r = {}^{20}C_r\, x^{20-4r}\; y^r$$

This term contains x^{-12} if $20 - 4r = -12$

or $r = 8$.

$\therefore$ The coefficient of x^{-12} is ${}^{20}C_8\, y^8$.

10. The binomial expansion of $\left(\frac{2}{3}x - \frac{3}{2}y\right)^{20}$ consists of 21 terms. Therefore $\left(\frac{20}{2}+1\right)$ th term,

i.e., 11th term is the middle term.

Hence the middle term

$= T_{11} = {}^{20}C_{10} \left(\frac{2}{3}x\right)^{20-10} \left(-\frac{3}{2}y\right)^{10}$

$= {}^{20}C_{10}\, x^{10}\, y^{10}.$

11. The 7th term from the end = 5th term from beginning

$T_5 = {}^{10}C_4 x^6 \left(-\frac{2}{x^2}\right)^4 = {}^{10}C_4 \cdot 2^4 \left(\frac{1}{x^2}\right)$

12. $2^{4n} = (16)^n = (1+15)^n$

$\therefore\ 2^{4n} = 1 + {}^nC_1 15 + {}^nC_2 15^2 + ... + {}^nC_n 15^n$

$\therefore\ 2^{4n} - 1 - 15n = 15^2\,[{}^nC_2 + {}^nC_3 . 15 + ... + {}^nC_n 15^{n-2}\,] = 225\,k.$

Where k is an integer.

Hence $2^{4n} - 15n - 1$ is divisible by 225.

13. The coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms of the expansion $(1+x)^{34}$ are ${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2,}$ respectively. Since they are equal so ${}^{34}C_{r-6,} = {}^{34}C_{2r-2.}$

Therefore, either $r - 6 = 2r - 2$

or $\quad r \quad 6 = 34 \quad (2r \quad 2)$

[Using the fact that if ${}^nC_r = {}^nC_p$, then either $r = p$ or $r = n - p$]

So, we get $r = -4$ or $r = 14$. r being a natural number, $r = -4$ is not possible. So, $r = 14$.

14. Since the above series is a geometric series with the common ratio $\frac{x}{1+x}$, its sum is

$$\frac{(1+x)^{1000}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{\left[1 \quad \left(\frac{x}{1+x}\right)\right]} = \frac{(1+x)^{1000} - \frac{x^{1001}}{1+x}}{\frac{1+x-x}{1+x}}$$

$= (1+x)^{1001} - x^{1001}$

Hence, coefficient of x^{50} is given by

$${}^{1001}C_{50} = \frac{(1001)!}{(50)!\ (951)!}$$

Long Answer Questions

1. ***Ans*** **:** 3

Hint - $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-(r-1)}\left(-\frac{2}{x^2}\right)^{r-1}$

$= {}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r} .(-2)^{r-1} x^{13-3r}$

for coefficient of x^4, $13 - 3r = 4 \Rightarrow r = 3$

2. ***Ans*** **:** $-{}^{100}C_{53}$

Hint - Given series

$= (x-3)^{100} + {}^{100}C_1 (x-3)^{99}.2^1 + {}^{100}C_2 (x-3)^{98}.2^2 + ... + {}^{100}C_{100}\, 2^{100}$

$= [(x-3)+2]^{100} = (x-1)^{100} = (1-x)^{100}$

$\therefore$ Coefficient of $x^{54} = {}^{100}C_{53}(-1)^{53} = -{}^{100}C_{53}$

3. ***Ans*** **:** ${}^nC_4 + {}^nC_{2} + {}^nC_1 . {}^nC_2$

Hint - $(1 + x + x^2 + x^3)^n = (1+x)^n (1+x^2)^n$

$= (1 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + ... + x^n)$

$\times [1 + {}^nC_1 x^2 + {}^nC_2 x^4 ...... + (x^2)^n\,]$

$\therefore$ Coefficient of $x^4 = {}^nC_4 + {}^nC_2 . {}^nC_1 + {}^nC_2$

4. ***Ans*** **:** 5

Hint - Given expression

$= 2[1 + {}^9C_2 (3\sqrt{2}x)^2 + {}^9C_4 (3\sqrt{2}x)^4 + {}^9C_6 (3\sqrt{2}x)^6 + {}^9C_8 (3\sqrt{2}x)^8\,]$

$\therefore$ then number of non-zero terms is 5

5. ***Ans*** **:** 10

Hint - $T_3 = T_{2+1} = {}^5C_2 x^3 (x \log_{10} x)^2 = 10^6$

$\therefore 3\log_{10} x + 2(\log_{10} x)^2 = 5$

$\therefore 2(\log_{10} x)^2 + 5\log_{10} x - 2\log_{10} x - 5 = 0$

$\therefore\ (\log_{10} x - 1)(2\log_{10} x + 5) = 0$

or $x = 10$ or $x = 10^{-5/2}$ $\therefore\ x = 10$

6. ***Ans*** **:** $n = 2r$

Hint - Coefficient of $(3r)^{th}$ term = Coefficient of $(r+2)^{th}$ term

$\Rightarrow {}^{2n}C_{3r-1} = {}^{2n}C_{r+1} \Rightarrow (3r-1) + (r+1) = 2n$

$\Rightarrow n = 2r$

7. ***Ans :*** $\frac{10!}{(5!)^2}$

Hint - Coefficient of x^r is ${}^{10}C_r$ which is max. if

$r=\frac{10}{2}=5$ (as 10 is even)

Hence greatest coefficient is ${}^{10}C_5=\frac{10!}{(5!)^2}$

8. ***Ans :*** $\frac{(2n+2)!}{\{(n+1)!\}^2}$

Hint - In the expansion of $(1+x)^{2n+2}$, the coefficient of x^r, i.e ${}^{2n+2}C_r$ term is the greatest coefficient

if $r=\frac{2n+2}{2}\Rightarrow r=n+1$

$\therefore$ Greatest Coefficient $={}^{2n+2}C_{n+1}$

$$=\frac{(2n+2)!}{(n+1)!(n+1)!}=\frac{(2n+1)!}{\{(n+1)!\}^2}$$

9. ***Ans :*** $\frac{1}{2}(1-3^n)$

Hint - we have

$(1-x+x^2)^n=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+\text{.........}+a_{2n}x^{2n}$ ... (i)

Replacing $x=1$ and $x=-1$ successively in (i)

$1=a_0+a_1+a_2+a_3+a_4+\text{..........}+a_{2n}$ ...(ii)

$3^n=a_0-a_1+a_2-a_3+a_4-\text{..........}+a_{2n}$ ...(iii)

Adding (ii) & (iii)

$3^n+1=2[a_0+a_2+a_4+\text{........}+a_{2n}]$

$\Rightarrow a_0+a_2+a_4+\text{.........}+a_{2n}=\frac{1}{2}(3^n+1)$

10. ***Ans :*** $\frac{1}{n+1}$

Hint - $(1-x)^n=C_0-C_1x+C_2x^2\text{.......}+(-1)^n.C_nx^n$

Integration,

$-\frac{1}{n+1}(1-x)^{n+1}+A$

$=C_0x-\frac{C_1x^2}{2}+\frac{C_2x^3}{3}+\text{........}+(-1)^n\frac{C_nx^n}{n+1}$

putting $x=0$, $A=\frac{1}{n+1}$

Now, putting $x=1$, we get

$C_0-\frac{C_1}{2}+\frac{C_2}{3}-\frac{C_3}{4}+\text{.....}+(-1)^n\frac{C_n}{n+1}=\frac{1}{n+1}$

11. ***Ans :*** 129

Hint - $[5^{1/2}+7^{1/8}]^{1024}=[(5^{1/2}+7^{1/8})^8]^{128}$

$=[5^4+{}^8C_1 5^{7/2}7^{1/8}+\text{.......}$

$+{}^8C_7 5^{1/2}7^{7/8}+{}^8C_8.7]^{128}$

$=[5^4+7]^{128}$ + (non - integral term)

= 129 no. of integral terms

12. ***Ans :*** $C_0^2+C_1^2+\text{.......}+C_n^2$

Hint - $(1+x)^n\left(1+\frac{1}{x}\right)^n$

$=[C_0+C_1x+C_2x^2+\text{........}+C_nx^n]$

$\times\left[C_0+C_1\left(\frac{1}{x}\right)+C_2\left(\frac{1}{x}\right)^2+\text{.....}+C_n\left(\frac{1}{x}\right)^n\right]$

$\therefore$ Term independent of x in the expansion

$=C_0^2+C_1^2+C_2^2+\text{.........}+C_n^2$

13. ***Ans :*** 0

Hint - $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\text{................}+{}^nC_nx^n$

Putting $x=-1$, we get

${}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\text{.........}+(-1)^n\,{}^nC_n=0$

14. ***Ans :*** $2^n(n+1)$

Hint - We have $C_0+3C_1+5C_2+7C_3+\text{.........}+(2n+1)C_n$

$=(C_0+C_1+C_2+C_3+\text{...........}+C_n)$

$+2(C+2C_2+3C_3+\text{...............}+{}^nC_n)$

$=2^n+2.n\,2^{n-1}=2^n(n+1)$

15. ***Ans*** **:** $(1-x)^{-1/3}$

Hint - Let

$$1+\frac{1}{3}x+\frac{1}{3}.\frac{4}{6}x^2+\frac{1.4.7}{3.6.9}x^3+..........$$

$$=(1+y)^n=1+ny+\frac{n(n-1)}{2!}y^2+............ \quad (i)$$

$$\therefore\ ny=\frac{1}{3}x,\ \frac{n(n-1)}{2!}y^2=\frac{1.4}{4.6}x^2$$

Solving, $n=-\frac{1}{3},\ y=-x$

Given series $=(1-x)^{-1/3}$

16. ***Ans*** **:** $\sqrt{2}/3$

Hint - The given series

$$1-\frac{1}{2}.\frac{1}{2}+\frac{\frac{1}{2}.\frac{3}{2}}{1.2}\left(\frac{1}{2}\right)^2-\frac{1.3.5}{1.2.3}\left(\frac{1}{2}\right)^3+......$$

$$=\left(1+\frac{1}{2}\right)^{-1/2}=\left(\frac{3}{2}\right)^{-1/2}=\sqrt{\frac{2}{3}}$$

17. ***Ans*** **:** ${}^nC_r(3^{n-r}-2^{n-r})$

[Hint

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+....+(x+2)^{n-1}$$

$$=\frac{(x+3)^n-(x+2)^n}{(x+3)-(x+2)}=(x+3)^n-(x+2)^n$$

$\therefore$ Coefficient of $x^r={}^nC_r3^{n-r}-{}^nC_r2^{n-r}$

18. ***Ans*** **:** ${}^{12}C_7\left(\frac{5}{2}\right)^7.2^5$

Hint - $(2+3x)^{12}$ at $x=\frac{5}{6}=\left(\frac{5}{2}\right)^{12}\left[1+\frac{4}{5}\right]^{12}$;

$|T_r|<=>|T_{r+1}|$ as $\left|\frac{r}{(n-r+1)x}\right|<=>1$

i.e. $\left|\frac{r.5}{(13-r)4}\right|<=>1$ i.e $r<=>\frac{52}{9}$;

$r=5<\frac{52}{9}\Rightarrow T_5<T_6$; $r=6>\frac{52}{9}\Rightarrow T_6>T_7$

$\therefore T_6$ is the greatest term in the expansion of $\left[1+\frac{4}{5}\right]^{12}$.

Hence the greatest term in the expansion of $(2+3x)^{12}$ at $x=\frac{5}{6}$ is

$$\left(\frac{5}{2}\right)^{12}.{}^{12}C_5\left(\frac{4}{5}\right)^5 \text{ i.e., } {}^{12}C_7\left(\frac{5}{2}\right)^7.2^5$$

19. ***Ans*** **:** $(-1)^{n/2}(n+2)$

Hint - $C_0+C_1x+C_2x^2+C_3x^3+..........+C_nx^n$
$=(1+x)^n \quad ...(i)$

Diff. (i) after multiplying by x, we get
$C_0+2C_1x+3C_2x^2+..........+(n+1)C_nx^n$
$=(1+x)^n+nx(1+x)^{n-1} \quad ...(ii)$

Replacing x by $\left(-\frac{1}{x}\right)$ in (i), we get

$$C_0-\frac{C_1}{x}+\frac{C_2}{x^2}-\frac{C_3}{x^3}+.........$$

$$=\left(1-\frac{1}{x}\right)^n=\frac{(1-x)^n}{x^n} \quad ...(iii)$$

From (ii) & (iii)

$C_0{}^2-2C_1{}^2+3C_2{}^2-........=$ Coefficient of the term independent of x in the expansion of

$(1-x)^n[(1+x)^n+nx(1+x)^{n-1}]/x^n$

$=$ Coefficient of x^n in
$[(1-x^2)^n+nx(1-x)(1-x^2)^{n-1}]$
$=(-1)^{n/2}\,{}^nC_{n/2}-n.{}^{n-1}C_{(n-2)/2}(-1)^{(n-2)/2}$

$$=(-1)^{n/2}\left[\frac{n!}{(n/2)!(n/2)!}+\frac{n(n-1)!(n/2)}{\{(n-2)/2\}!(n/2)!n/2}\right]$$

$$= (-1)^{n/2} \cdot \frac{n+2}{2} \frac{n!}{\left(\frac{n}{2}\right)! \left(\frac{n}{2}\right)!}$$

$$\therefore \frac{2(n/2)!(n/2)!}{n!} \{C_0^2 - 2C_1^2 + 3C_2^2 - 4C_3^2$$

$$+ \ldots\ldots (-1)^n (n+1)$$

$$= (-1)^{n/2} (n+2)$$

20. ***Ans*** **:** $4^{1/3}$

Hint

$$S = 1 + \frac{2}{3} \cdot \frac{1}{2} + \frac{2.5}{3.6}\left(\frac{1}{2}\right)^2 + \frac{2.5\,8}{3.6.9}\left(\frac{1}{2}\right)^3 + \ldots\ldots$$

$$= 1 + \frac{2/3}{1} \cdot \left(\frac{1}{2}\right) + \frac{(2/3).(5/3)}{2!}\left(\frac{1}{2}\right)^2 .$$

$$+ \frac{(2/3).(5/3)(8/3)}{3!}\left(\frac{1}{2}\right)^3 + \ldots\ldots$$

$$= \left(1 - \frac{1}{2}\right)^{-2/3} = \left(\frac{1}{2}\right)^{-2/3} = 2^{2/3} = 4^{1/3}$$

21. ***Ans*** **:** $\frac{2n!}{\{(n-r)!(n+r)!\}}$

Hint

$$C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n = (1+x)^n$$

$$C_0 + \frac{C_1}{x} + \ldots\ldots + \frac{C_r}{x^r} + \frac{C_{r+1}}{x^{r+1}} + \ldots\ldots + \frac{C_n}{x^n}$$

$$= \left(1 + \frac{1}{x}\right)^n = \left(\frac{1+x}{x^n}\right)^n$$

Multiplying , we get

$$C_0 C_r + C_1 C_{r+1} + C_2 C_{r+2} + \ldots\ldots$$

$\therefore$ Coefficient of $\frac{1}{x^r}$ in $\frac{(1+x)^{2n}}{x^n}$

= coefficient of x^{n-r} in $x\,(1+x)^{2n}$

$$= {}^{2n}C_{n-r} = \frac{(2n)!}{(n-r)!(n+r)!}$$

22. ***Ans*** **:** 2^7

Hint - Given that

$$(3\sqrt{3}+5)^7 = P + F, \ 0 \le F < 1 \qquad \ldots (i)$$

Let $(3\sqrt{3}+5)^7 = S$

as $0 < 3\sqrt{3} - 5 = \frac{2}{(3\sqrt{3}+5)} < 1$

So, $0 < S < 1 \qquad \ldots$ (ii)

$\therefore \quad (P+F) - S = (3\sqrt{3}+5)^7 - (3\sqrt{3}-5)^7$

$= 2[{}^7C_1(3\sqrt{3})^6.5 + {}^7C_3(3\sqrt{3})^4.5^3 + \ldots..]$

= even integer

Since P is an integer F–S must be integer.

$\because \quad -1 < F - S < 1$ so $F - S = 0$ i.e. $F = S$

$\therefore \quad F(P+F) = S(P+F)$

$= (3\sqrt{3}-5)^7 (3\sqrt{3}+5)^7 = 2^7$

23. ***Ans*** **:** 10

Hint $= T_4 = {}^6C_3\left[x^{\frac{3}{2(1+\log_{10} x)}}\right] x^{1/4} = 200$

$$\Rightarrow x^{\left[\frac{6+\log x+1}{4(1+\log x)}\right]} = 10$$

Putting $\log_{10} x = y$, we have

$$\frac{6+y+1}{4(y+1)} = \frac{1}{y} \Rightarrow y^2 + 3y - 4 = 0$$

$\Rightarrow \quad y = -4, y = 1 \Rightarrow x = 10^{-4}$ or 10

Since $x > 1, \ \therefore x = 10$

24. $\left\{2^{\log_2 \sqrt{(9^{x-1}+7)}} + \frac{1}{2^{1/5 \log 2(3^{x-1}+1)}}\right\}^7$

$$= \left\{(9^{x-1}+7)^{1/2} + \frac{1}{(3^{x-1}+1)^{1/5}}\right\}^7$$

$$\therefore T_6 = T_{5+1} = {}^7C_5\left[9^{x-1}+7\right]\frac{1}{3^{x-1}+1} = 84$$

$$\therefore 9^{x-1} + 7 = 4(3^{x-1}+1)$$

Putting $3^x = y$, we get.

$$y^2/9+7=4\left(\frac{y}{3}+1\right) \Rightarrow y^2+63=12y+36$$

$\Rightarrow y^2-12y+27=0 \Rightarrow y=3,9$
$y=3, \Rightarrow 3^x=3 \Rightarrow x=1$
$y=9 \Rightarrow 3^x=3^2 \Rightarrow x=2 \quad \therefore x=1,2$

25. The $(r+1)$th term in the binomial expansion of $(x+a)^n$ is $T_{r+1} = {}^nC_r\, x^{n-r} a^r$

$\therefore$ 11th term from the beginning $= T_{11} = T_{10+1}$

$$= {}^{25}C_{10}(2x)^{25-10}\left(-\frac{1}{x^2}\right)^{10} = {}^{25}C_{10}\, 2^{15} \cdot \frac{1}{x^5}$$

And 11th term from the end $= (26-10)$th $=$ 16th term from the beginning $= {}^{25}C_{15}(2x)^{10}$

$$\left(-\frac{1}{x^2}\right)^{15}$$

$$= -{}^{25}C_{15}\, 2^{10}\, \frac{1}{x^{20}}. \text{ Hence,}$$

$$\text{the desired ratio} = \frac{{}^{25}C_{10}\, 2^{15}\, \frac{1}{x^5}}{-{}^{25}C_{15}\, 2^{10}\, \frac{1}{x^{20}}} = -2^5 x^{15}$$

26. The given expression is $(1+x)^{2n}$. Here the index $2n$ is even.

So $\left(\frac{2n}{2}+1\right)$ i.e. $(n+1)$th term is the middle term.

$$\therefore T_{n+1} = {}^{2n}C_n (1)^{2n-n}. x^n = {}^{2n}C_n x^n$$

$$= \frac{(2n)!}{n!(2n-n)!}x^n = \frac{1.2.3.4....(2n-3)(2n-2)(2n-1)(2n)}{n!n!}x^n$$

$$= x^n\frac{\{1.3.5......(2n-3)(2n-1)\}\{2.4.6......(2n-2)(2n)\}}{n!n!}$$

$$= \frac{1.3.5......(2n-1)2^n.x^n}{n!}$$

27. $a_1 = {}^nC_r, a_2 = {}^nC_{r+1}, a_3 = {}^nC_{r+2}, a_4 = {}^nC_{r+3}$

$a_1 + a_2 = {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

[using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

$a_2 + a_3 = {}^nC_{r+1} + {}^nC_{r+2} = {}^{n+1}C_{r+2}$

$a_3 + a_4 = {}^nC_{r+2} + {}^nC_{r+3} = {}^{n+1}C_{r+3}$

L.H.S.

$$= \frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}$$

$$= \frac{\frac{n!}{r!(n-r)!}}{\frac{(n+1)!}{(r+1)!(n-r)!}} + \frac{\frac{n!}{(r+2)!(n-r-2)!}}{\frac{(n+1)!}{(r+3)!(n-r-2)!}}$$

$$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1}$$

$$\text{R.H.S.} = \frac{2a_2}{a_2+a_3} = \frac{2\frac{n!}{(r+1)!(n-r-1)!}}{\frac{(n+1)!}{(r+2)!(n-r-1)!}} = \frac{2(r+2)}{n+1}$$

$= $ L.H.S

28. (i) Since,

$(x+a)^n$
$= {}^nC_0\, x^n + {}^nC_1\, x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \ldots {}^nC_n\, a^n$
$= T_1 + T_2 + T_3 + T_4 + \ldots.. T_{n+1}$
$= (T_1 + T_3 + T_5 \ldots..) + (T_2 + T_4 + T_6 + \ldots..)$
$= P + Q$

Also $(x-a)^n = {}^nC_0 x^n - {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 - {}^nC_3 x^{n-3} a^3 + \ldots.(-1)^n\, {}^nC_n a^n$

$= T_1 - T_2 + T_3 - T_4 + T_5 \ldots..$
$= (T_1 + T_3 + T_5 \ldots..) - (T_2 + T_4 + T_6 + \ldots..)$
$= P - Q$

Now, $(x^2-a^2)^n = [(x+a)(x-a)]^n$
$= (x+a)^n (x-a)^n$
$= (P+Q)(P-Q) = P^2 - Q^2$

(ii) From above

$(x+a)^{2n}-(x-a)^{2n}-(P+Q)^2-(P-Q)^2$

$=(P^2+Q^2+2PQ)-(P^2+Q^2-2PQ)$

$=4PQ$

(iii) From above (ii)

$(x+a)^{2n}+(x-a)^{2n}=(P+Q)^2+(P-Q)^2$

$=(P^2+Q^2+2PQ+P^2+Q^2-2PQ)$

$=2(P^2+Q^2)$

29. The $(r+1)^{th}$ term in the expansion is ${}^nC_r a^r$. Thus it can be seen that a^r occurs in the $(r+1)^{th}$ term, and its coefficient is ${}^nC_r$. Hence the coefficients of a^{r-1}, a^r and a^{r+1} are ${}^nC_{r-1}$, ${}^nC_r$ and ${}^nC_{r+1}$, respectively. Since these coefficients are in arithmetic progression, so we have,

${}^nC_{r-1}+{}^nC_{r+1}=2.\,{}^nC_r$. This gives

$$\frac{n!}{(r-1)!(n-r+1)!}+\frac{n!}{(r+1)!(n-r-1)!}=2\times\frac{n!}{r!(n-r)!}$$

i.e., $$\frac{1}{(r-1)!(n-r+1)(n-r)(n-r-1)!}+\frac{1}{(r+1)(r)(r-1)!(n-r-1)!}$$

$$=2\times\frac{1}{r(r-1)!(n-r)(n-r-1)!}$$

or $$\frac{1}{(r-1)!(n-r-1)!}\left[\frac{1}{(n-r)(n-r+1)}+\frac{1}{(r+1)(r)}\right]$$

$$=2\times\frac{1}{(r-1)!(n-r-1)![r(n-r)]}$$

i.e., $$\frac{1}{(n-r+1)(n-r)}+\frac{1}{r(r+1)}=\frac{2}{r(n-r)},$$

or $$\frac{r(r+1)+(n-r)(n-r+1)}{(n-r)(n-r+1)r(r+1)}=\frac{2}{r(n-r)}$$

or $r(r+1)+(n-r)(n-r+1)$

$=2(r+1)(n-r+1)$

or $r^2+r+n^2-nr+n-nr+r^2-r$

$=2(nr-r^2+r+n-r+1)$

or $n^2-4nr-n+4r^2-2=0$

i.e., $n^2-n(4r+1)+4r^2-2=0$

30. We have

$2^{4n+4}-15n-16=2^{4(n+1)}-15n-16$

$=16^{n+1}-15n-16$

$=(1+15)^{n+1}-15n-16$

$={}^{n+1}C_0\,15^0+{}^{n+1}C_1\,15^1+{}^{n+1}C_2\,15^2$
$+{}^{n+1}C_3\,15^3+\ldots+{}^{n+1}C_{n+1}(15)^{n+1}-15n-16$

$=1+(n+1)\,15+{}^{n+1}C_2\,15^2+{}^{n+1}C_3\,15^3$
$+\ldots+{}^{n+1}C_{n+1}\,(15)^{n+1}-15n-16$

$=1+15n+15+{}^{n+1}C_2\,15^2+{}^{n+1}C_3\,15^3$
$+\ldots+{}^{n+1}C_{n+1}\,(15)^{n+1}-15n-16$

$=15^2\,[{}^{n+1}C_2+{}^{n+1}C_3\,15+\ldots$ so on$]$

Thus, $2^{4n+4}-15n-16$ is divisible by 225.

SECTION C NCERT EXEMPLAR QUESTIONS

Fill in the Blanks

1. The largest coefficient in the expansion of $(1+x)^{30}$ is ________.
2. The number of terms in the expansion of $(x+y+z)^n$ ________.
3. In the expansion of $\left(x^2-\frac{1}{x^2}\right)^{16}$, the value of constant term is ________.
4. If the seventh terms from the beginning and the end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ are equal, then n equals ________.
5. The coefficient of $a^{-6}b^4$ in the expansion of $\left(\frac{1}{a}-\frac{2b}{3}\right)^{10}$ is ________.
6. Middle term in the expansion of $(a^3+ba)^{28}$ is ________.
7. The ratio of the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ is ________.
8. The position of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is ________.
9. If 25^{15} is divided by 13, the remainder is ________.

True or False

1. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$ is $2^{19}+\frac{{}^{20}C_{10}}{2}$.
2. The expression 7^9+9^7 is divisible by 64.
3. The number of terms in the expansion of $[(2x+y^3)^4]^7$ is 8.
4. The sum of coefficients of the two middle terms in the expansion of $(1+x)^{2n-1}$ is equal to ${}^{2n-1}C_n$.
5. The last two digits of the numbers 3^{400} are 01.
6. If the expansion of $\left(x-\frac{1}{x^2}\right)^{2n}$ contains a term independent of x, then n is a multiple of 2.
7. Number of terms in the expansion of $(a+b)^n$ where $n \in \mathbf{N}$, is one less than the power n.

Short Answer Questions

1. Find the term independent of x, where $x \neq 0$, in the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^{15}$.
2. If the term free from x in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then find the value of k.
3. Find the coefficient of x in the expansion of $(1-3x+7x^2)(1-x)^{16}$.
4. Find the term independent of x in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$.
5. Find the middle term (terms) in the expansion of (i) $\left(\frac{x}{a}-\frac{a}{x}\right)^{10}$ (ii) $\left(3x-\frac{x^3}{6}\right)^9$
6. Find the coefficient of x^{15} in the expansion of $(x-x^2)^{10}$.
7. Find the coefficient of $\frac{1}{x^{17}}$ in the expoansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$.
8. Find the sixth term of the expansion $(y^{1/2}+x^{1/3})^n$, if the Binomial coefficient of the third term from the end is 45.
9. Find the value of r, if the coefficient of $(2r+4)$th and $(r-2)$th terms in the expansion of $(1+x)^{18}$ are equal.
10. If the coefficient of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in AP, then show that $2n^2-9n+7=0$.
11. Find the coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$.

LONG ANSWER QUESTIONS

1. If p is a real number and the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, then find the value of p.

2. Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1\times3\times5\times...\times(2n-1)}{n!}\times(-2)^n$.

3. Find n in the Binomial $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7th term from the beginning to the 7th term from the end is $\frac{1}{6}$.

4. In the expansion of $(x + a)^n$, if the sum of odd terms is denoted by O and the sum of even term by E. Then, prove that
   (i) $O^2 - E^2 = (x^2 - a^2)^n$.
   (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$.

5. If x^p occurs in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$, then prove that its coefficient is $\frac{2n!}{\frac{(4n-p)!}{3!}\frac{(2n+p)!}{3!}}$.

6. Find the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

NCERT EXEMPLAR SOLUTIONS

Fill in the Blanks

1. $^{30}C_{15}$ **2.** $\frac{(n+1)(n+2)}{2}$ **3.** $^{16}C_8$

4. $n=12$ **5.** $\frac{1120}{27}$

6. $^{28}C_{14}\ a^{56}\ b^{14}$ **7.** 1 : 1

8. Third term **9.** 12

True or False

1. F **2.** T **3.** F **4.** F

5. T **6.** F **7.** F

Short Answer Questions

1. Given expression is $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^{15}$.

Let T_{r+1} term is the general term.

Then, $T_{r+1}={}^{15}C_r\left(\frac{3x^2}{2}\right)^{15-r}\left(-\frac{1}{3x}\right)^r$

$= {}^{15}C_r\, 3^{15\quad r} x^{30\quad 2r}\, 2^{r\quad 15}\,(-1)^r\cdot 3^{\quad r}\cdot x^{\quad r}$

$= {}^{15}C_r\,(-1)^r\, 3^{15-2r}\, 2^{r-15}\, x^{30-3r}$

For the term independent of x,

$30 - 3r = 0$

$3r = 30 \Rightarrow r = 10$

$\because \quad T_{r+1} = T_{10+1}$

$= 11$th term is independent of x.

$\therefore \quad T_{10+1} = {}^{15}C_{10}\,(-1)^{10}\, 3^{15\quad 20}\, 2^{10\quad 15}$

$= {}^{15}C_{10}\, 3^{-5}\, 2^{-5}$

$= {}^{15}C_{10}\,(6)^{-5}$

$= {}^{15}C_{10}\left(\frac{1}{6}\right)^5$

2. Given expression is $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$.

Let T_{r+1} is the general term.

Then, $T_{r+1} = {}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{-k}{x^2}\right)^r$

$= {}^{10}C_r(x)^{\frac{1}{2}(10-r)}(-k)^r\cdot x^{-2r}$

$= {}^{10}C_r\, x^{5-\frac{r}{2}}(-k)^r\cdot x^{-2r}$

$= {}^{10}C_r\, x^{5-\frac{r}{2}-2r}(-k)^r$

$= {}^{10}C_r\, x^{\frac{10\quad 5r}{2}}(-k)^r$

For the term independent of x, $\frac{10-5r}{2}=0$

$\Rightarrow \quad 10-5r=0 \Rightarrow r=2$

Since $T_{2+1}=T_3$ is free from x.

$\therefore \quad T_{2+1}={}^{10}C_2(-k)^2=405$

$\Rightarrow \quad \dfrac{10\times 9\times 8!}{2!\times 8!}(-k)^2=405$

$\Rightarrow \quad 45k^2=405 \Rightarrow k^2=\dfrac{405}{45}=9$

$\therefore \quad k=\pm 3$

3. Given expression $=(1-3x+7x^2)(1-x)^{16}$.

$=(1-3x+7x^2)\,({}^{16}C_0\,1^{16}-{}^{16}C_1\,1^{15}x^1 + {}^{16}C_2\,1^{14}x^2+\ldots+{}^{16}C_{16}x^{16})$

$=(1-3x+7x^2)(1-16x+120x^2+\ldots)$

$\therefore$ Coefficient of $x=-3-16=-19$

4. Given expression is $\left(3x-\dfrac{2}{x^2}\right)^{15}$.

Let T_{r+1} is the general term.

$\therefore \quad T_{r+1}={}^{15}C_r(3x)^{15-r}\left(\dfrac{-2}{x^2}\right)^r$

$={}^{15}C_r(3x)^{15-r}(-2)^r x^{-2r}$

$={}^{15}C_r\,3^{15-r}x^{15-3r}(-2)^r$

For the term independent of x,

$15-3r=0 \Rightarrow r=5$

$\therefore T_{5+1}$ or T_6 is the term independent of x.

$T_{5+1}={}^{15}C_5\,3^{15-5}(-2)^5$

$=-\dfrac{15\times14\times13\times12\times11\times10!}{5\times4\times3\times2\times1\times10!}\cdot 3^{10}\cdot 2^5$

$=-3003\cdot 3^{10}\cdot 2^5$

5. (i) Given expression is $\left(\dfrac{x}{a}-\dfrac{a}{x}\right)^{10}$.

Here, the power of Binomial $(n)=10$ (even)

$\therefore$ it has one middle term i.e., $\left(\dfrac{10}{2}+1\right)$th term or 6th term.

$\therefore \quad T_6=T_{5+1}={}^{10}C_5\left(\dfrac{x}{a}\right)^{10-5}\left(\dfrac{-a}{x}\right)^5$

$=-{}^{10}C_5\left(\dfrac{x}{a}\right)^5\left(\dfrac{a}{x}\right)^5$

$=-\dfrac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2\times1}\left(\dfrac{x}{a}\right)^5\left(\dfrac{x}{a}\right)^{-5}$

$=-9\times4\times7=-252$

(ii) Given expression is $\left(3x-\dfrac{x^3}{6}\right)^9$.

Here, $n=9$ [odd]

Since, the Binomial expansion has two middle terms i.e., $\left(\dfrac{9+1}{2}\right)$th and $\left(\dfrac{9+1}{2}+1\right)$th

or, 5th term and 6th term

$\therefore \quad T_5=T_{(4+1)}={}^9C_4(3x)^{9-4}\left(-\dfrac{x^3}{6}\right)^4$

$=\dfrac{9\times8\times7\times6\times5!}{4\times3\times2\times1\times5!}\,3^5x^5x^{12}6^{-4}$

$=\dfrac{7\times6\times3\times3^1}{2^4}x^{17}=\dfrac{189}{8}x^{17}$

and $T_6=T_{5+1}={}^9C_5(3x)^{9-5}\left(-\dfrac{x^3}{6}\right)^5$

$=-\dfrac{9\times8\times7\times6\times5!}{5!\times4\times3\times2\times1}\cdot3^4\cdot x^4\cdot x^{15}\cdot6^{-5}$

$=\dfrac{-21\times6}{3\times2^5}x^{19}=\dfrac{-21}{16}x^{19}$

6. Here the given expression is $(x-x^2)^{10}$.

Let the term T_{r+1} is the general term.

$\therefore \quad T_{r+1}={}^{10}C_r\,x^{10-r}(-x^2)^r$

$=(-1)^r\cdot{}^{10}C_r\cdot x^{10-r}\cdot x^{2r}$

$=(-1)^r\,{}^{10}C_r\,x^{10+r}$

For the coefficient of x^{15},

$10+r=15 \Rightarrow r=5$

$\therefore \quad T_{5+1}=(-1)^5\,{}^{10}C_5\,x^{15}$

$\therefore$ Coefficient of x^{15}

$=-1\dfrac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}$

$=-3\times2\times7\times6=-252$

7. Here the given expression is $\left(x^4-\dfrac{1}{x^3}\right)^{15}$.

Let the term T_{r+1} contains the coefficient of $\dfrac{1}{x^{17}}$ i.e., x^{-17}.

$\therefore \quad T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$= {}^{15}C_r\, x^{60-4r}\,(-1)^r\, x^{-3r}$

$= {}^{15}C_r\, x^{60-7r}\,(-1)^r$

For the coefficient of x^{-17},

$60 - 7r = -17$

$\Rightarrow \quad 7r = 77 \quad \Rightarrow \quad r = 11$

$\therefore \quad T_{11+1} = {}^{15}C_{11}\, x^{60-77}\,(-1)^{11}$

$\therefore$ Coefficient of x^{-17}

$$= \frac{-15\times14\times13\times12\times11!}{11!\times4\times3\times2\times1}$$

$= -15\times7\times13 = -1365$

8. Here the given expression is $(y^{1/2} + x^{1/3})^n$.

$\therefore$ The sixth term

$= T_6 = T_{5+1} = {}^nC_5 (y^{1/2})^{n-5}\,(x^{1/3})^5$ ...(i)

Now, given that the Binomial coefficient of the third term from the end is 45.

We know that, Binomial coefficient of third term from the end = Binomial coefficient of third term from the beginning of $(x^{1/3} + y^{1/2})^n = {}^nC_2$

$\because \quad {}^nC_2 = 45$

$\Rightarrow \quad \frac{n(n-1)(n-2)!}{2!(n-2)!} = 45$

$\Rightarrow \quad n(n-1) = 90$

$\Rightarrow \quad n^2 - n - 90 = 0$

$\Rightarrow \quad n^2 - 10n + 9n - 90 = 0$

$\Rightarrow n(n-10) + 9(n-10) = 0$

$\Rightarrow (n-10)(n+9) = 0$

$\Rightarrow (n+9) = 0$ or $(n-10) = 0$

$\therefore n = 10 \quad [\because n \neq -9]$

Put the value of $n = 10$ in eq. (i),

$T_6 = {}^{10}C_5\, y^{5/2}\, x^{5/3} = 252\, y^{5/2}\cdot x^{5/3}$

9. Given expression is $(1+x)^{18}$

Now, $(2r+4)$th term or T_{2r+3+1}

$= {}^{18}C_{2r+3}\,(1)^{18-2r-3}\,(x)^{2r+3}$

$= {}^{18}C_{2r+3}\, x^{2r+3}$

Now, $(r-2)$th term or $T_{r-3+1} = {}^{18}C_{r-3}\, x^{r-3}$

Since $\quad {}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow \quad 2r + 3 + r - 3 = 18$

$[\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$

$\Rightarrow \quad 3r = 18$

$\therefore \quad r = 6$

10. Here the given expression is $(1+x)^{2n}$.

Now, coefficient of 2nd term $= {}^{2n}C_1$

Coefficient of 3rd term $= {}^{2n}C_2$

Coefficient of 4th term $= {}^{2n}C_3$

Since, ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are in AP.

$\therefore \quad 2\cdot{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow \quad 2\left[\frac{2n(2n-1)(2n-2)!}{2\times1\times(2n-2)!}\right]$

$$= \frac{2n(2n-1)!}{(2n-1)!} + \frac{2n(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!}$$

$\Rightarrow \quad n(2n-1) = n + \frac{n(2n-1)(2n-2)}{6}$

$\Rightarrow \quad n(12n-6) = n(6 + 4n^2 - 4n - 2n + 2)$

$\Rightarrow \quad 12n - 6 = (4n^2 - 6n + 8)$

$\Rightarrow \quad 6(2n-1) = 2(2n^2 - 3n + 4)$

$\Rightarrow \quad 3(2n-1) = 2n^2 - 3n + 4$

$\Rightarrow \quad 2n^2 - 3n + 4 - 6n + 3 = 0$

$\Rightarrow \quad 2n^2 - 9n + 7 = 0$

11. Given expression $= (1 + x + x^2 + x^3)^{11}$

$= [(1+x) + x^2(1+x)]^{11}$

$= [(1+x)(1+x^2)]^{11} = (1+x)^{11}\cdot(1+x^2)^{11}$

$= ({}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + {}^{11}C_3x^3 + {}^{11}C_4x^4 + \ldots)({}^{11}C_0 + 11C_1x^2 + {}^{11}C_2x^4 + \ldots)$

$= (1 + 11x + 55x^2 + 165x^3 + 330x^4 + \ldots)(1 + 11x^2 + 55x^4 + \ldots)$

$\therefore$ Coefficient of $x^4 = 55 + 605 + 330 = 990$

Long Answer Questions

1. Here the given expression is $\left(\frac{p}{2} + 2\right)^8$.

$\therefore n = 8$ [even]

$\therefore$ the Binomial expansion of the given expression has only one middle term i.e.,

$\left(\frac{8}{2} + 1\right)$th $=$ 5th term

$\therefore T_5 = T_{4+1} = {}^8C_4\left(\frac{p}{2}\right)^{8-4}\cdot 2^4$

$\Rightarrow 1120 = {}^8C_4\, p^4\cdot 2^{-4}\, 2^4$

$\Rightarrow 1120 = \frac{8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1}p^4$

$\Rightarrow 1120 = 7 \times 2 \times 5 \times p^4$

$\Rightarrow p^4 = \frac{1120}{70} = 16 \Rightarrow p^4 = 2^4$

$\Rightarrow p^2 = 4 \Rightarrow p = \pm 2$

2. Given expression is $\left(x - \frac{1}{x}\right)^{2n}$. This Binomial expansion has even power. So, it has one middle term.

i.e., $\left(\frac{2n}{2}+1\right)$ th term or $(n+1)$th term

$\therefore T_{n+1} = {}^{2n}C_n (x)^{2n-n}\left(-\frac{1}{x}\right)^n$

$= {}^{2n}C_n x^n(-1)^n x^{-n}$

$= {}^{2n}C_n (-1)^n = (-1)^n \frac{(2n)!}{n!\,n!}$

$= \frac{1\cdot 2\cdot 3\cdot 4\cdot 5 \ldots (2n-1)(2n)}{n!\,n!}(-1)^n$

$= \frac{1\cdot 3\cdot 5 \ldots (2n-1)\cdot 2\cdot 4\cdot 6 \ldots (2n)}{1\cdot 2\cdot 3 \cdot \ldots n(n!)}(-1)^n$

$= \frac{1\cdot 3\cdot 5 \ldots (2n-1)\cdot 2^n (1\cdot 2\cdot 3 \ldots n)(-1)^n}{(1\cdot 2\cdot 3\cdot \ldots n)(n!)}$

$= \frac{[1\cdot 3\cdot 5 \ldots (2n-1)]}{n!}(-2)^n$

3. Here, the Binomial expression is $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$.

Now, 7th term from beginning $= T_7 = T_{6+1}$

$= {}^nC_6 (\sqrt[3]{2})^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6$ ...(1)

and 7th term from end i.e., T_7 from the beginning of $\left(\frac{1}{\sqrt[3]{3}} + \sqrt[3]{2}\right)^n$

i.e. $T_7 = {}^nC_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}(\sqrt[3]{2})^6$ ...(ii)

Given that, $\frac{{}^nC_6(\sqrt[3]{2})^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6}{{}^nC_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}(\sqrt[3]{2})^6} = \frac{1}{6}$

$\Rightarrow \frac{2^{\frac{n-6}{3}}\cdot 3^{-6/3}}{3^{-\left(\frac{n-6}{3}\right)}\cdot 2^{6/3}} = \frac{1}{6}$

$\Rightarrow \left(2^{\frac{n-6}{3}}\cdot 2^{\frac{-6}{3}}\right)\left(3^{\frac{-6}{3}}\cdot 3^{\frac{(n-6)}{3}}\right) = 6^{-1}$

$\Rightarrow \left(2^{\frac{n-6}{3}-\frac{6}{3}}\right)\cdot\left(3^{\frac{n-6}{3}-\frac{6}{3}}\right) = 6^{-1}$

$\Rightarrow (2\cdot 3)^{\frac{n}{3}-4} = 6^{-1}$

$\Rightarrow \frac{n}{3} - 4 = -1 \Rightarrow \frac{n}{3} = 3 \Rightarrow n = 9$

4. (i) Given expression is $(x+a)^n$.

$\therefore (x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1}a^1 + {}^nC_2 x^{n-2}a^2 + {}^nC_3 x^{n-3}a^3 + \ldots + {}^nC_n a^n$

Now, sum of odd terms

$O = {}^nC_0 x^n + {}^nC_2 x^{n-2}a^2 + \ldots$

and sum of even terms $E = {}^nC_1 x^{n-1}a + {}^nC_3 x^{n-3}a^3 + \ldots$

$\because \quad (x+a)^n = O + E$ ...(i)

Similarly, $(x-a)^n = O - E$ ...(ii)

multiply eqs. (i) and (ii)

we have $(O+E)(O-E) = (x+a)^n (x-a)^n$

$\Rightarrow O^2 - E^2 = (x^2 - a^2)^n$

(ii) $4OE = (O+E)^2 - (O-E)^2$

$= [(x+a)^n]^2 - [(x-a)^n]^2$ [using eqs. (i) and (ii)]

$= (x+a)^{2n} - (x-a)^{2n}$

5. Here the given expression is $\left(x^2 + \frac{1}{x}\right)^{2n}$.

$T_{r+1} = {}^{2n}C_r (x^2)^{2n-r}\left(\frac{1}{x}\right)^r$

$= {}^{2n}C_r x^{4n-2r} x^{-r} = {}^{2n}C_r x^{4n-3r}$

Let $\quad 4n - 3r = p$

$\left[\text{as } x^p \text{ occurs in expansion of } \left(x^2 + \frac{1}{x}\right)^{2n}\right]$

$\Rightarrow 3r = 4n - p \Rightarrow r = \frac{4n-p}{3}$

$\therefore$ Coefficient of $x^p = {}^{2n}C_r = \dfrac{(2n)!}{r!\,(2n-r)!}$

$$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(2n-\frac{4n-p}{3}\right)!}$$

$$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{6n-4n+p}{3}\right)!}$$

$$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$$

6. Given expression is $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

Now, for $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$, the general term is:

$$T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

$$= {}^9C_r\left(\frac{3}{2}\right)^{9-r} x^{18-2r}\left(-\frac{1}{3}\right)^r x^{-r}$$

$$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

Therefore, the general term in the expansion of

$(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

$$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r} + {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{19-3r} + 2\cdot{}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{21-3r}$$

For term independent of x

$18-3r=0$, $19-3r=0$ and $21-3r=0$

$\Rightarrow r=6, r=19/3, r=7$

Thus, the possible value of r are 6 and 7.

$\therefore$ The term independent of

$$x = {}^9C_6\frac{3^{9-6}}{2}\left(-\frac{1}{3}\right)^6 + 2\cdot{}^9C_7\frac{3^{9-7}}{2}\left(-\frac{1}{3}\right)^7$$

$$= \frac{9\times8\times7\times6!}{6!\times3\times2}\cdot\frac{3^3}{2^3}\cdot\frac{1}{3^6} - 2\cdot\frac{9\times8\times7!}{7!\times2\times1}\cdot\frac{3^2}{2^2}\cdot\frac{1}{3^7}$$

$$= \frac{21}{2}\cdot\frac{1}{3^3} - \frac{9}{1}\cdot\frac{2}{3^5} = \frac{7}{18} - \frac{2}{27} = \frac{21-4}{54} = \frac{17}{54}$$