AEP STUDY CIRCLE

SINCE 2001...

IIT-NEET-CBSE

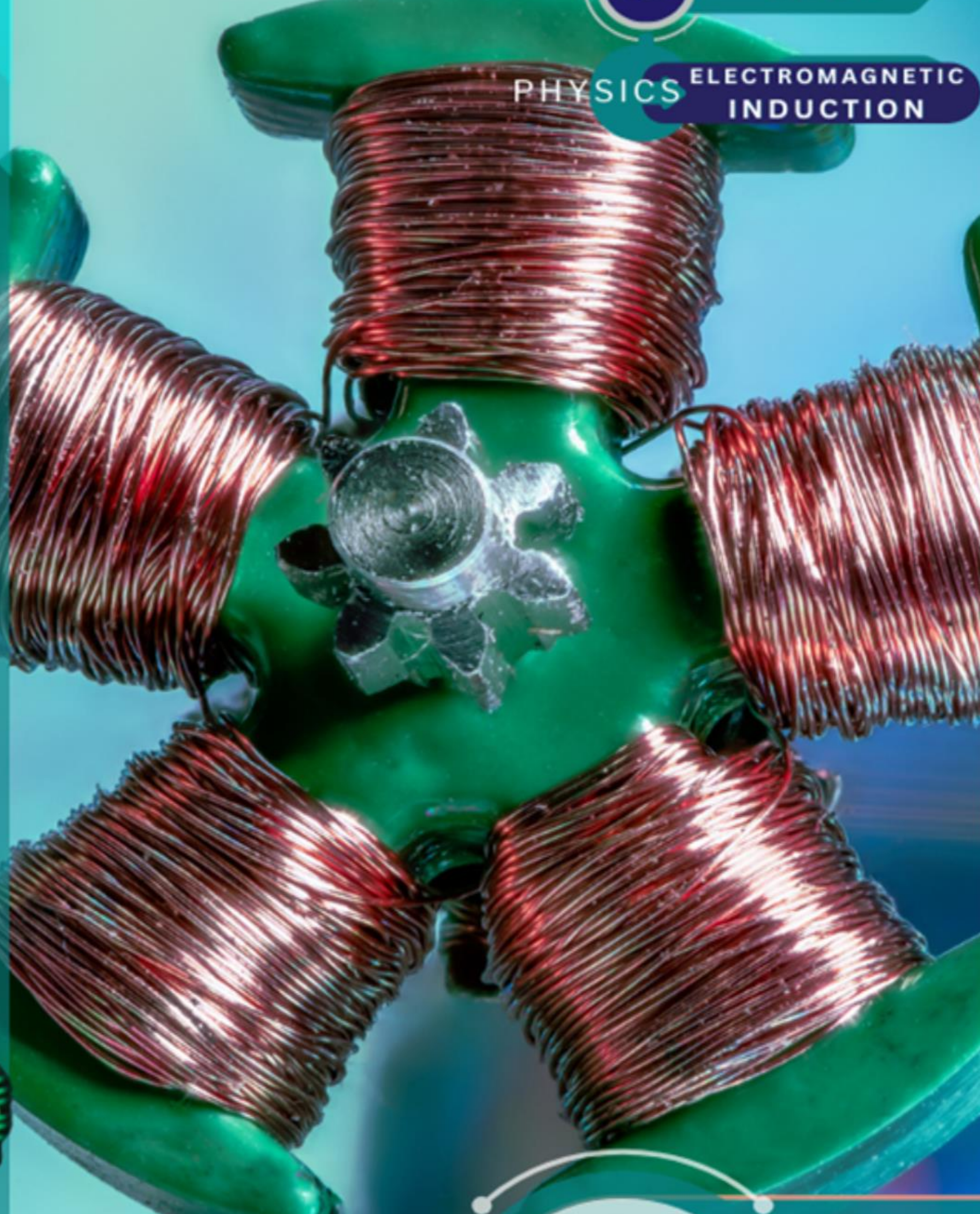
ACCENTS EDUCATIONAL PROMOTERS

24 YEARS OF LEGACY

XII CBSE

PHYSICS

ELECTROMAGNETIC INDUCTION



YOUR GATEWAY TO EXCELLENCE IN

IIT-JEE, NEET AND CBSE EXAMS

ELECTROMAGNETIC INDUCTION

CHAPTER : 01

UNIT: IV

IIT-JEE

NEET

CBSE

CONTACT US:

+91-9939586130

+91-9955930311

www.aepstudycircle.com

aepstudycircle@gmail.com

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

ELECTRO-MAGNETIC INDUCTION

UNIT : 04 CHAPTER : 01

It was discovered by orested that an electric current produces a magnetic field. In 1931 Michel faraday demonstrated the converse effect i.e., the production of electric current, with the help of magnetic fields, such a phenomenon is known as electromagnetic induction.

► When a coil made of copper wire is placed inside a magnetic field, magnetic flux is linked with the coil. **Faraday** found that when the magnetic flux linked with the coil is changed, the change is magnetic flux linked with the coil may be caused by ***varying the strength*** of the **magnetic field** or by the **relative motion between the source of magnetic flux and coil,** an electric current starts flowing in the coil, provided the coil is closed.

► In case the ***coil is open*** an ***emf*** is set up across the ***two end*** of the ***coil.***

Thus

"The phenomenon of generating current/ emf by changing the no. of magnetic lines of force associated with a conductor is called electromagnetic induction."

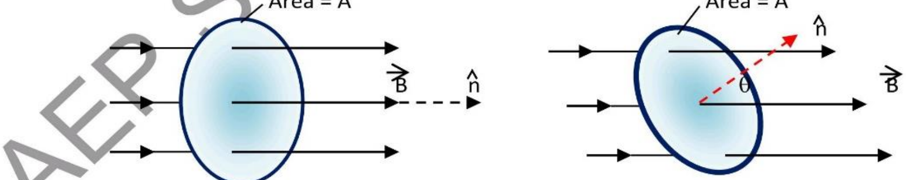
► The current of emf so produced are called ***included current*** and ***included emf*** respectively.

MAGNETIC FLUX

Magnetic flux: The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally. It is measured as the product of the component of the magnetic field normal to the surface and the surface area.

► **Magnetic flux is a scalar quantity**, denoted by ϕ or ϕ_B.

If a uniform magnetic field $\vec{B}$ passes normally through a plane surface area A, as shown in Fig. (a), then the magnetic flux through this area is $\phi = BA$



[Magnetic flux through an area depends on its orientation w.r.t. the magnetic field]

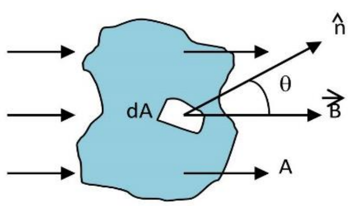
If the field B makes angle θ with the normal drawn to the area A, as shown in Fig. (b), then the component of the field normal to this area will be B cos θ, so that

$$\phi = B\cos\theta \times A = BA\cos\theta$$

or $\quad \phi = \vec{B}.\vec{A}$

Here the direction of vector $\vec{A}$ is the direction of the outward drawn normal to the surface.

In general, the field $\vec{B}$ over an area A may not be uniform. However, over a small area element $\vec{dA}$, the field $\vec{B}$ may be assumed to be uniform. As shown in Fig. if θ is the angle between B and the normal drawn to area element $\vec{dA}$, then the component of $\vec{B}$ normal to $\vec{dA}$ will be B cos θ.



[Surface A in a magnetic field]

∴ Flux through area element $\vec{dA}$ is

$$d\phi = B_{\perp}\,dA = B\cos\theta\,dA$$

$$= B\,dA\cos\theta = \vec{B}.\vec{dA}$$

Then the flux of $\vec{B}$ through the whole area A is

$$\phi = \int_A \vec{B}.\vec{dA}$$

Dimensions of magnetic flux: As we know that

$$\phi = BA$$

But $\quad B = \dfrac{F}{qv\sin\theta}$

∴ $\quad \phi = \dfrac{F}{qv\sin\theta}.A$

Dimensions of flux, $\phi = \dfrac{MLT^{-2}}{C.\,LT^{-1}.\,1}.\,L^2 = \dfrac{ML^2T^{-2}}{A}$ $\quad$ [∵ $1\ CT^{-1} = 1\ A$] $\quad$ or $\quad$ **$[\phi] = [ML^2A^{-1}T^{-2}]$**

►**SI unit of magnetic flux:** The SI unit of magnetic flux is weber (Wb). ***One weber is the flux produced when a uniform magnetic field of one tesla acts normally over an area of 1 m².***

1 weber = 1 tesla × 1 metre^2

or 1 Wb = 1 Tm^2

►**CGS unit of magnetic flux:** The CGS unit of magnetic flux is Maxwell (Mx). ***One Maxwell is the flux produced when a uniform magnetic field of one gauss acts normally over an area of 1 cm².***

1 maxwell = 1 gauss × 1 cm^2

or 1 Mx = 1 G cm^2

Relation between weber and maxwell.

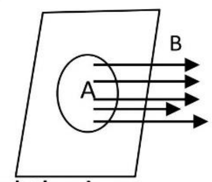
$1\text{ Wb} = 1\text{ T} \times 1\text{ m}^2 = 10^4\text{ G} \times 10^4\text{ cm}^2$ or $1\text{ Wb} = 10^8$ maxwell.

Maximum and zero Magnetic flux: --

● **Case :- I** : When $\vartheta = 0^o$ i.e., uniform magnetic field is acting $\perp^{ar}$ to the plane of the surface then,

$\Phi = BA \cos 0^o$

$\phi = BA$ **(maximum)**

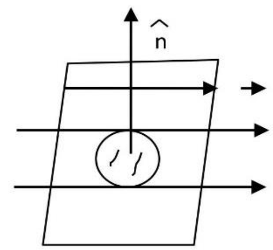


► **Magnetic flux through a given surface is maximum no. of magnetic lines of force passes through the given surface, When θ = 0°**

● **Case:-II**: When $\vartheta = 90^o$ i.e. when the magnetic field is touching the surface tangentially then

$\phi = BA \cos 90^o = 0$ (minimum)

► **Magnetic flux through a given surface is zero when θ = 90°**



● Definition of **magnetic flux density** (B) (or magnetic induction):

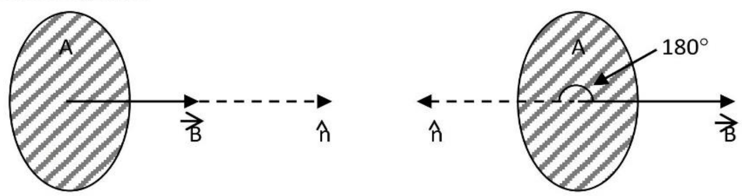
Since, $\phi_{max} = BA$, $\therefore B = \phi_{max} / A$

"Magnetic flux density (B) is defined as the magnetic flux associated normally per unit area".

⟹ If the coil has N turns, total amount of magnetic flux linked with the coil is $\phi = N(\vec{B} \cdot \vec{A})$

$$\Phi = NBA \cos \theta$$

►**Positive and negative flux**: A normal to a plane can be drawn from either side. If the normal drawn to a plane points out in the direction of the field, then θ = 0°. and the flux is taken as positive. If the normal points in the opposite direction of the field, then θ = 180° and the flux is taken as negative.



θ = 0°, $\phi = +BA$ θ = 180°, $\phi = -BA$

[(a) Positive flux and (b) negative flux]

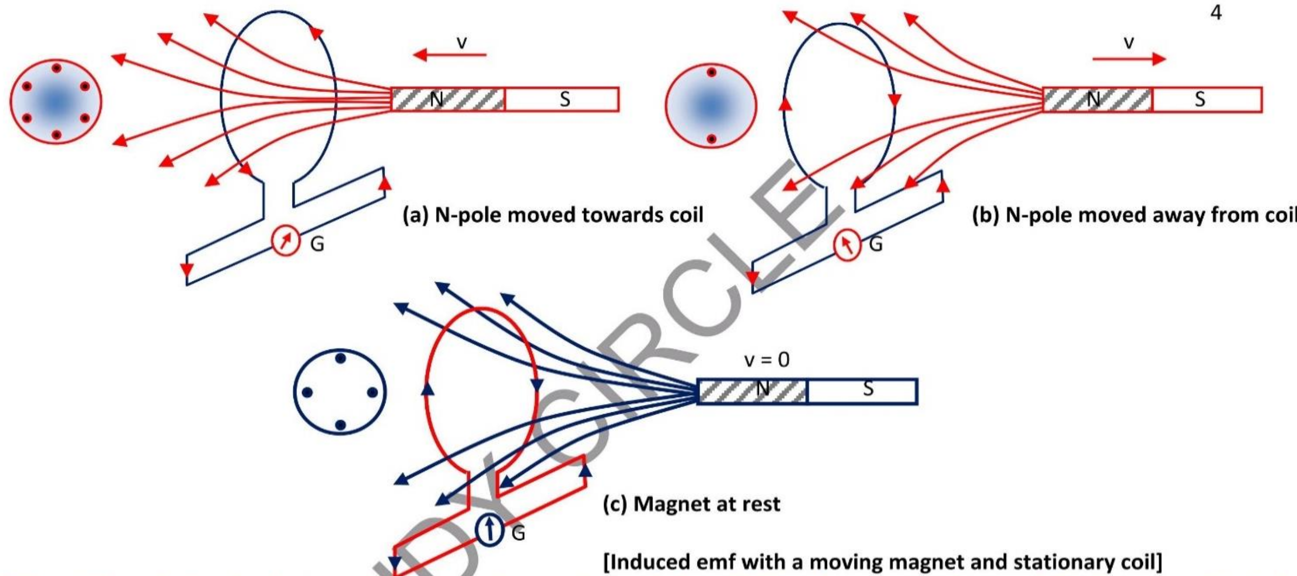
ELECTROMAGNETIC INDUCTION:

Electromagnetic induction: Electricity and magnetism are intimately connected. currents are produced in a loop of wire if a magnet is suddenly moved towards the loop or away from the loop such that the magnetic flux across the loop changes. The current in the loop lasts so long as the flux is changes. The current in the loop lasts so long as the flux is changing. This phenomenon is called electromagnetic induction which means inducing electricity by magnetism.

FARADAY'S EXPERIMENTS

The phenomenon of electromagnetic induction was discovered and understood on the basis of the following experiments performed by Faradays and Henry.

❑**Experiment 1: Induced emf with a stationary coil and moving magnet.** As shown in Fig. take a circular coil of thick insulated copper wire connected to a sensitive galvanometer.

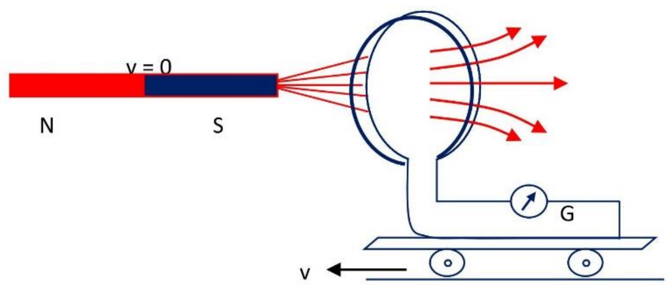


[Induced emf with a moving magnet and stationary coil]

- **(i) When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection, say to the right of the zero mark [Fig. (a)].**
- **(ii) When the N-pole of the bar magnet is moved away from the coil, the galvanometer shows a deflection in the opposite direction [Fig. (b)].**
- **(iii) If the above experiments are repeated by bringing the S-pole of the magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.**
- **(iv) When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection [Fig. (c)].**

Explanation: When a bar magnet is placed near a coil, a number of lines of force pass through it. As the magnet is moved closer to the coil, the magnetic flux (the total number of magnetic lines of force) linked with the coil increases, an induced emf and hence an induced current is set up in the coil in one direction. As the magnet is moved away from the coil, the magnetic flux linked with the coil decreases, an induced emf and hence an induced current is set up in the coil in the opposite direction. As soon as the relative motion between the magnet and the coil ceases, the magnetic flux linked with the coil stops changing and so the induced current through the coil becomes zero.

Experiment 2: Induced emf with a stationary magnet and moving coil. Similar results as in experiment 1 are obtained if the magnet is held stationary and the coil is moved, as shown in fig. When the relative motion between the coil and the magnet is fast, the deflection in the galvanometer is large and when the relative motion is slow, the galvanometer deflection is small.

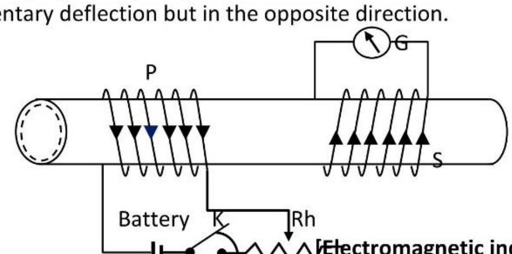


[Electromagnetic induction with a stationary magnet and moving coil]

Faster the relative motion between the magnet and the coil, greater is the rate of change of magnetic flux linked with the coil and larger is the induced current set up in the coil.

Experiment 3: Induced emf by varying current in the neighbouring coil. Fig. shown two coils P and S wound independently on a cylindrical support. The coil P, called primary coil, is connected to a battery and a rheostat through a tapping key K. The coil S, called secondary coil, is connected to a sensitive galvanometer.

(i) When the tapping key is pressed, the galvanometer shows a momentary deflection in one direction. When the key is released, it again shows a momentary deflection but in the opposite direction.



[Electromagnetic induction by varying current in the neighbouring coil]

- (ii) If the tapping key is kept pressed and steady current flows through the primary coil, the galvanometer does not show any deflection.
- (iii) As the current in the primary coil is increased with the help of the rheostat, the induced current flows in the secondary in the same direction as that at the make of the primary circuit.
- (iv) As the current in the primary coil is decreased, the induced current flows in the same direction as that at the break of the primary circuit.
- (v) The deflections in the galvanometer become larger if we use a cylindrical support made of iron.

Explanation: When a current flows through a coil, a magnetic field gets associated with it. As the primary circuit is closed, the current through it increases from zero to a certain steady value. The magnetic flux linked with the primary and hence with the secondary also increases. This sets up an induced current in the secondary coil in one direction. As the primary circuit is broken, the current decreases from the steady value of zero, the magnetic flux through the secondary coil decreases. An induced current is set up in the secondary coil but in the opposite direction. When steady current flows in the primary coil, the magnetic flux linked with the primary coil does not change and no current is induced in the secondary coil.

Conclusion:

1. Whenever the magnetic flux linked with a closed circuit changes, an induced emf and hence an induced current is set up in it.
2. The higher the rate of change of magnetic flux linked with the closed circuit, the greater is the induced emf or current.

LAWS OF ELECTROMAGNETIC INDUCTION

Laws of electromagnetic induction: There are two types of laws which govern the phenomenon of electromagnetic induction:

¶ A. Faraday's laws which give us the magnitude of induced emf.
¶ B. Lenz's law which gives us the direction of induced emf.

A. Faraday's laws of electromagnetic induction: These can be stated as follows:

- **FIRST LAW:** Whenever the magnetic flux linked with a closed circuit changes, an emf (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomenon is called electromagnetic induction.
- **SECOND LAW:** This magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit. Mathematically,

$$|e| = \frac{d\phi}{dt}$$

B. Lenz's law: This law states that the direction of induced current is such that it opposes the cause which produces it, i.e., it opposes the change in magnetic flux.

Mathematical form of the laws of electromagnetic induction: Expression for induced emf.

According to the Faraday's flux rule,

Magnitude of induced emf = Rate of change of magnetic flux

or $$|e| = \frac{d\phi}{dt}$$

Taking into account Lenz's rule for the direction of induced emf, Faraday's law takes the form:

$$e = -\frac{d\phi}{dt}$$

The negative sign indicates that the direction of induced emf is such that it opposes the change in magnetic flux.

If the coil consists of N tightly wound turns, then the emfs developed in all these turns will be equal and in the same direction and hence get added up. Total induced emf will be

$$e = -N\frac{d\phi}{dt}$$

If the flux changes from ϕ_1 to ϕ_2 in time t, then the average induced emf will be

$$e = -N\frac{\phi_2 - \phi_1}{t}$$

If ϕ is in webers and t in seconds, then 'e' will be in volts.

also $e = \vec{E}\cdot\vec{dl}$ from unit –1st , $E = \frac{\text{Work done}}{\text{Unit charge}}$

$$\therefore \oint \vec{E}\cdot\vec{dl} = -d\phi / dt$$

(integral form of faraday's law)

→ If the coil has 'N'turns, than, **$e = -Nd\phi/dt$**

Included current, $I = \frac{\text{Induced emf}}{\text{Resistance in the circuit}}$

$$I = -\frac{d\phi}{R\,dt}$$

EXPLANATION OF LENZ'S LAW

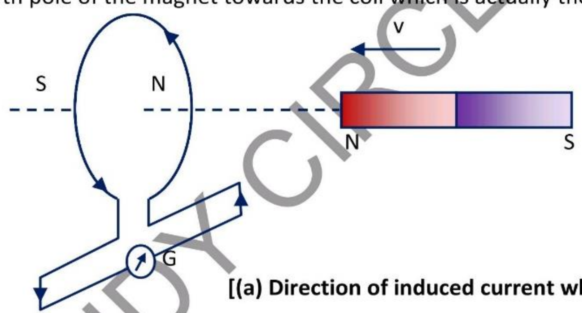
Lenz's law: In 1833, German physicist Heinrich Lens gave a general law for determining **the direction of induced emf and hence that of induced current in a circuit.**

Lenz's law states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it.

Thus, if the magnetic flux linked with a closed circuit increases, the induced current flows in such a direction so as to create a magnetic flux in the opposite direction of the original magnetic flux. If the magnetic flux linked with the closed circuit decreases, the induced current flows in such a direction so as to create a magnetic flux in the direction of the original flux.

☑ Illustrations of Lenz's law:

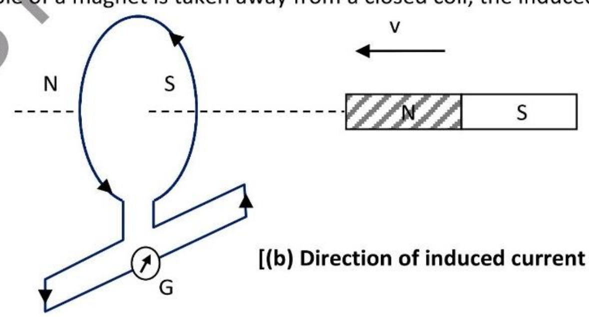
(i) When the north pole of a bar magnet is moved towards a closed coil, the induced current in the coil flows in the anticlockwise direction, as seen from the magnet side closely. The face of the coil towards the magnet develops north polarity and thus, it opposes the motion of the north pole of the magnet towards the coil which is actually the cause of the induced current in the coil.



[(a) Direction of induced current when a magnet moves towards a coil]

In other words, the motion of the magnet increases the flux through the coil. The induced current generates flux in opposite direction, and hence opposes and reduces this flux.

(ii) When the north pole of a magnet is taken away from a closed coil, the induced current in the coil flows clockwise, as seen



[(b) Direction of induced current when a magnet moves away from a coil]

from the magnet side. The face of the coil towards the magnet develops south polarity and attracts the north pole of the magnet i.e., the motion of the magnet away from the coil is opposed which is really the cause of the induced current.

In other words, the motion of the magnet decreases the flux through the coil. The induced current generates flux in the same direction and hence opposes and increases this flux.

◘ *Lenz's law is a consequence of the law of conservation of energy.*:

Lenz's law and law of conservation of energy: Whether a magnet is moved towards or away from a closed coil, the induced current always opposes the motion of the magnet, as predicted by Lenz's law. For example, when the north pole of a magnet is brought closer to a coil [Fig (a)], its face towards the magnet develops north polarity and thus repels north pole of the magnet. Work has to be done in moving the magnet closer to the coil against this force of repulsion. Similarly, when the north pole of the magnet is moved away from the coil [Fig. (b)], its face towards the magnet develops south polarity and thus attracts the north pole of the magnet. Here work has to be done in moving the magnet away from the coil against this force of attraction. It is this work done against the force of repulsion or attraction that appears as electric energy in the form of induced current.

Suppose that the Lenz's law is not valid. Then the induced current flows through the coil in a direction opposite to one dictated by Lenz's law. The resulting force on the magnet makes it move faster and faster, i.e., the magnet gains speed and hence kinetic energy without expanding an equivalent amount of energy. This sets up a perpetual motion machine, violating the law of conservation of energy. Thus Lenz's law is valid and is a consequence of the law of conservation of energy.

Examples based on (i) Magnetic flux (ii) Laws of Electromagnetic Induction

◆ **Formula Used**

1. Magnetic flux, $\phi = BA\cos\theta = \vec{B}\cdot\vec{A}$
2. Induced emf, $\mathcal{E} = -N\frac{d\phi}{dt}$
3. Average induced emf, $\mathcal{E} = -N\frac{\phi_1 - \phi_2}{t}$
4. Induced current, $I = \frac{|\mathcal{E}|}{R}$

◆ **UNIT USED**

Magnetic field B is in tesla, flux ϕ in Wb, area a in m^2, induced emf $\mathcal{E}$ in volt, induced current I in ampere.

Q. 1. A rectangular loop of area 20 cm × 30 cm is placed in a magnetic field of 0.3 T with its plane (i) normal to the field (ii) inclined 30° to the field and (iii) parallel to the field. Find the flux linked with the coil in each case.

Sol. $A = 20\text{ cm} \times 30\text{ cm} = 6 \times 10^{-2}\text{ m}^2$, B = 0.3 T

Let θ be the angle made by the field B with the normal to the plane of the coil.

(i) Here $\theta = 90° - 90° = 0°$

$\therefore \quad \phi = BA\cos\theta = 0.3 \times 6 \times 10^{-2} \times \cos 0°$

$= 1.8 \times 10^{-2}$ Wb.

(iii) Here $\theta = 90°$

$\therefore \quad \phi = 0.3 \times 6 \times 10^{-2} \times \cos 90°$ = zero.

(ii) Here $\theta = 90° - 30° = 60°$

$\therefore \quad \phi = 0.3 \times 6 \times 10^{-2} \times \cos 60°$

$= 0.9 \times 10^{-2}$ Wb.

Q. 2. A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 s. The magnetic flux between the pole pieces is known to be 8×10^{-4} Wb. Estimate the emf induced in the wire.

Sol. Here dt = 0.5s, $d\phi = 8 \times 10^{-4} - 0 = 8 \times 10^{-4}$ Wb

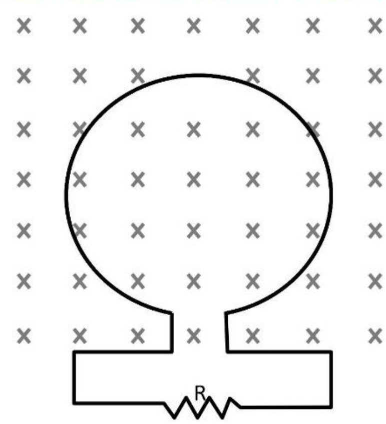
The emf induced in the wire,

$$|e| = \frac{d\phi}{dt} = \frac{8 \times 10^{-4}}{0.5} = 1.6 \times 10^{-3}\text{ V}$$

Q. 3. The magnet flux passing perpendicular to the plane of a coil and directed into the paper is given by $\phi = 5t^2 + 6t + 2$, where ϕ is in milliwabers and t in seconds.

(a) What is the magnitude of the induced emf set up in the loop at t = 1 s?

(b) What is the direction of current through the resistor R?



Sol. Given $\phi = (5t^2 + 6t + 2)$ mWb

$\therefore$ Induced emf, $\mathcal{E} = \frac{d\phi}{dt} = (10t + 6)$ mV

At t = 1s, e = 16 mV

(b) By Lenz's law, the direction of the induced current is such as to oppose the change in flux, For this the induced current should generate outward flux by flowing in the anticlockwise sense. So the induced current flows through the resistor R from left to right.

Q. 4. The magnitude flux through a coil perpendicular to the plane is varying according to the relation:

$\phi = (5t^3 + 4t^2 + 2t - 5)$ Wb

Calculate the induced current through the coil at t = 2s, if the resistance of the coil is 5 Ω.

Sol. the magnitude of induced emf set up at any instant t will be

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^3 + 4t^2 + 2t - 5) = 15t^2 + 8t + 2$$

At t = 2 s,

$|e| = 15(2)^2 + 8(2) + 2 = 60 + 16 + 2 = 78$ V

Resistance of the coil, R = 5 Ω

Induced current, $I = \frac{|e|}{R} = \frac{78}{5} = 15.6$ A

Q. 5. A square loop of side 10 cm and resistance 0.70 Ω is placed vertically in the earth-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time-interval.

Sol. Clearly, the angle made by the normal to the plane (east-west plane) of the coil with the magnetic field in north-east direction is

$\theta = 45°$

Maximum flux through the coil,

$\phi_{max} = BA\cos\theta = 0.1 \times (0.10 \times 0.10)\cos 45°$

$= \frac{10^{-3}}{\sqrt{2}} = 0.7 \times 10^{-3}$ Wb

This flux is set up in 0.7 s. So the magnitude of induced emf,

$$|e| = \frac{d\phi}{dt} = \frac{\phi_{max} - 0}{dt} = \frac{0.7 \times 10^{-3}\text{ Wb}}{0.7\text{ s}}$$

The magnitude of induced current,

$$I = \frac{|e|}{R} = \frac{1\text{ mV}}{0.7\,\Omega} = 0.4\text{ mA}$$

Q. 6. A 10 Ω resistance coil has 1000 turns and at a time 5.5×10^{-4} Wb of flux passes through it. If the flux falls to 0.5×10^{-4} Wb in 0.1 second, find the emf generated in volts and the charge flown through the coil in coulombs.

Sol. R = 10 Ω, N = 1000, $\phi_1 = 5.5 \times 10^{-4}$ Wb, $\phi_2 = 0.5 \times 10^{-4}$ Wb, t = 0.1 s

Induced emf,

$$e = -N\frac{\phi_2 - \phi_1}{t}$$

$$= -\frac{1000 \times 0.5 \times 10^{-4} - 5.5 \times 10^{-4}}{0.1} = 5\text{ V}$$

Current through 10 Ω resistance coil is

$$I = \frac{e}{R} = \frac{5}{10} = 0.5\text{ A}$$

$\therefore$ The charge passing through the coil in 0.1 s is

$q = It = 0.5 \times 0.1 = 0.05$ C

Q. 7. A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T (tesla). If the induced emf is 11 V when the magnetic field is changed to 1000 T in 4 s, what is the number of turns in the coil? 8

Sol. Radius of coil, $r = \frac{0.02}{2} = 0.01$ m

$B_1 = 6000$ T, $B_2 = 1000$ T, t = 4s, $\mathcal{E} = 11$ V

Now, $|e| = N\frac{\phi_2 - \phi_1}{t} = NA\frac{B_2 - B_1}{t}$

$= N.\pi r^2.\frac{B_2 - B_1}{t}$

$\therefore \quad 11 = N.\frac{22}{7} \times (0.01)^2 \frac{6000 - 1000}{4}$

Number of turns, $N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = 28$

Q. 8. A wire 88 cm long bent into a circular loop is placed perpendicular to the magnetic field of flux density 2.5 Wbm^{-2}. Within 0.5 s, the loop is changed into 22 cm square and flux density is increased to 3.0 Wbm^{-2}. Calculate the value of the emf induced.

Sol. For circular loop, $2\pi r = 88$ cm

or $r = \frac{88 \times 7}{2 \times 22} = 14$ cm = 0.14 m

$\therefore$ Initial area of loop,

Final area of loop, $A_2 = (0.22)^2 = 0.0484$ m^2

$B_1 = 2.5$ Wbm^{-2}, $B_2 = 3.0$ Wbm^{-2}, t = 0.5 s

Induced emf,

$$e = -\left(\frac{\phi_2 - \phi_1}{t}\right) = -\left(\frac{B_2 A_2 - B_1 A_1}{t}\right)$$

$$= -\left(\frac{3 \times 0.0484 - 2.5 \times 0.0616}{0.5}\right)$$

$$= -\left(\frac{0.1452 - 0.154}{0.5}\right) = \left(\frac{0.0088}{0.5}\right) = 0.0176 \text{ V}$$

Q. 9. A coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned through 180° in 1 / 10 second. Calculate the average induced emf.

Sol. Here $A = 500$ cm^2 $= 500 \times 10^{-4}$ m^2, N = 1000, B = 0.4 G = 0.4×10^{-4} T, t = 1/10 s

When the coil is held perpendicular to the field, the normal to the plane of the coil makes an angle of 0° with the field B.

$\therefore$ Initial flux, $\phi_1 = BA \cos 0° = BA$

Final flux, $\phi_2 = BA \cos 180° = -BA$

Average induced emf,

$$e = -N\left(\frac{\phi_2 - \phi_1}{t}\right) = -N\left(\frac{-BA - BA}{t}\right)$$

$$= \frac{2NBA}{t} = \frac{2 \times 1000 \times 0.4 \times 10^{-4} \times 500 \times 10^{-4}}{1/10} \text{ V} = 0.04 \text{ V}$$

Q. 10. A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.025 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is 3.0×10^{-5} T.

Sol. Here $A = \pi(0.10)^2 = \pi \times 10^{-2}$ m^2, R = 2 Ω, N = 500, t = 0.25 s, $B = 3.0 \times 10^{-5}$ T

Initial flux through each turn of the coil,

$\phi_1 = BA \cos\theta_1 = 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \cos 0° = 3\pi \times 10^{-7}$ Wb

Final flux through each turn of the coil,

$\phi_2 = BA \cos\theta_2$

$= 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \cos 180°$

$= -3\pi \times 10^{-7}$ Wb

Estimated value of emf in the coil,

$e = -N\frac{\phi_2 - \phi_1}{t}$

$= -500 \frac{-3\pi \times 10^{-7} - 3\pi \times 10^{-7}}{0.25}$

$= \frac{500 \times 6\pi \times 10^{-7}}{0.25} = 3.8 \times 10^{-3}$ V

Current induced in the coil,

$I = \frac{e}{R} = \frac{3.8 \times 10^{-3} \text{ V}}{2\,\Omega} = 1.9 \times 10^{-3}$ A

Q. 11. A coil of cross-sectional area A lines in a uniform magnetic field B with its plane perpendicular to the field. In this position the normal to the coil makes an angle of 0° with the field. The coil rotates at a uniform rate to complete on rotation in time T. Find the average induced emf in the coil during the interval when the coil rotates:

(i) From 0° to 90° position

(ii) From 90° to 180° position

(iii) From 180° to0 270° and

(iv) From 270° to 360°

Sol. (i) For rotation from 0° to 90°

$\phi_1 = BA \cos 0° = BA$, $\phi_2 = BA \cos 90° = 0$, t = T/4

$\therefore$ Average induced emf,

$e = -\frac{\phi_2 - \phi_1}{t} = -\frac{0 - BA}{T/4} = \frac{4\,BA}{T}$

(ii) For rotation from 90° to 180°

$\phi_1 = BA \cos 90° = 0$,

$\phi_2 = BA \cos 180° = -BA$,

t= T/4

$\therefore \quad e = -\frac{-BA - 0}{T/4} = \frac{4\,BA}{T}$

(iii) For rotation from 180° to 270°

$\phi_1 = BA\cos 180° = -BA$, $\phi_2 = BA\cos 270° = 0$, $t = T/4$

$\therefore \quad e = -\dfrac{0 + BA}{T/4} = -\dfrac{4BA}{T}$

(iv) For rotation from 270° to 360°

$\phi_1 = BA\cos 270° = 0$,

$\phi_2 = BA\cos 360° = BA$, $t = T/4$

$\therefore \quad e = -\dfrac{BA - 0}{T/4} = -\dfrac{4BA}{T}$

As the sense of the induced emf in the second half rotation is opposite to that in the first half rotation, the induced current will change its direction after first half rotation.

*****Q. 12.A conducting of the induced emf in the second half rotation is opposite to that in the first half rotation. the induced current will change its direction after first half rotation.**

Sol. Suppose r is the radius of the loop at time t. Then magnetic flux linked with the loop is

$\phi = \pi r^2 B$

$\therefore \quad |e| = \dfrac{d\phi}{dt} = 2\pi r B \dfrac{dr}{dt}$

Here $dr = 1.0$ mm/s $= 1.0 \times 10^{-3}$ ms^{-1}

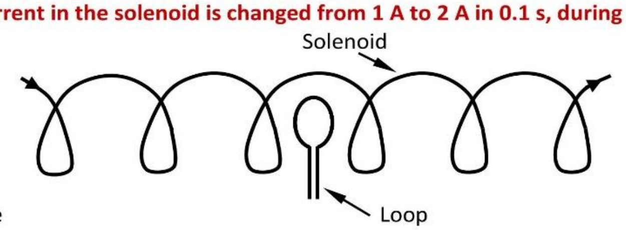
When $r = 2.0$ cm $= 2.0 \times 10^{-2}$ m,

the magnitude of induced emf is

$|e| = 2 \times 3.14 \times 2.0 \times 10^{-2} \times 0.02 \times 1.0 \times 10^{-3}$

$= 2.5 \times 10^{-6}$ V $= 2.5\ \mu$V

Q. 13. A long solenoid of 10 turns/cm has a small loop of area 1 sq. cm placed inside with the normal of the loop parallel to the axis. Calculate the voltage across the small loop if the current in the solenoid is changed from 1 A to 2 A in 0.1 s, during the duration of this change.



Sol. Here n = 10 turns/ cm

$= 10 \times 100$ turns/m $= 1000$ turns/ m

$A = 1$ cm$^2 = 10^{-4}$ m^2, $dI = 2 - 1 = 1$ A, $dt = 0.1$ s

The magnetic field inside the solenoid is given by

$B = \mu_0 nI$

$\therefore$ Rate of change of magnetic field with time will be

$\dfrac{dB}{dt} = \dfrac{d}{dt}(\mu_0 NI) = \mu_0 n \dfrac{dI}{dt}$

Hence magnitude of the induced voltage is

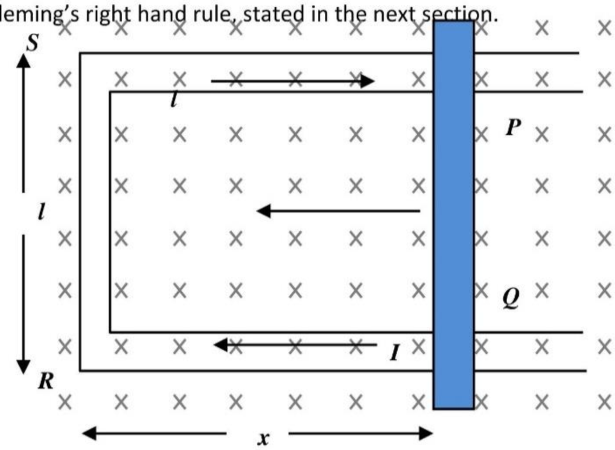
$e = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(BA) = A\dfrac{dB}{dt} = A.\ \mu_0 n \dfrac{dI}{dt}$

$= 10^{-4} \times 4\pi \times 10^{-7} \times 1000 \times \dfrac{1}{0.1}$ V

$= 12.56 \times 10^{-7}$ V.

MOTIONAL EMF FROM FARADAY'S LAWS

Motional emf from Faraday's law: Induced emf by change of area of the coil linked with the magnetic field: The emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf. As shown in Fig., consider a conductor PQ of length l free to move on U-shaped conducting rails situated in a uniform and time independent magnetic field B, directed normally into the plane of paper. The conductor PQ is moved inwards with a speed v. As the conductor slides towards left, the area of the rectangular loop PQRS decreases. This decreases the magnetic flux linked with the closed loop. Hence an emf is set up across the ends of conductor PQ because of which an induced current flows in the circuit along the path PQRS. The direction of induced current can by determined by using Fleming's right hand rule, stated in the next section.



Suppose a length x of the loop lies inside the magnetic field at any instant of time t. Then the magnetic flux linked with the rectangular loop PQRS is $\phi = BA = Blx$

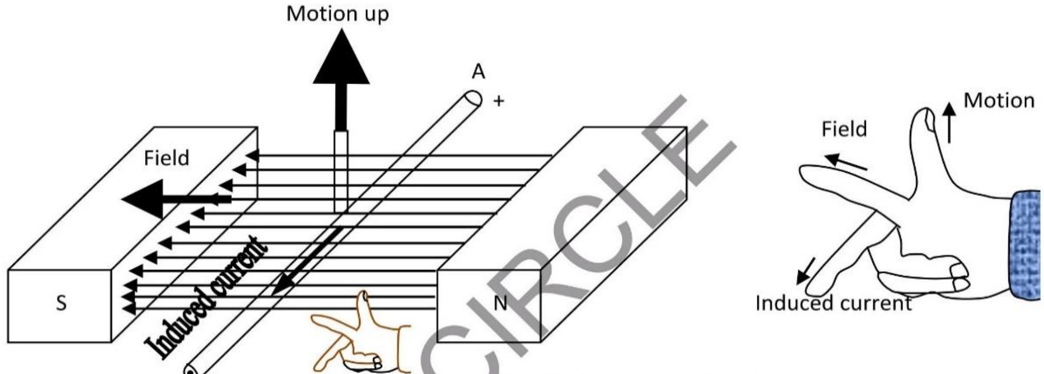
According to Faraday's law of electromagnetic induction, the induced emf is

$e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(Blx) = -Bl\dfrac{dx}{dt}$ or $e = Blv$

Where $dx/dt = -v$, because the velocity v is in the decreasing direction of x. The induced emf Blv is called motional emf because this emf is induced due to the motion of a conductor in a magnetic field.

FLEMING'S RIGHT HAND RULE

Fleming's right hand rule: This rule gives the direction of induced current set up in the conductor moving perpendicular to a magnetic field and can be stated as follows:



[Fleming's right hand rule]

If we stretch the thumb and the first two figures of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor; then the central finger points in the direction of current induced in the conductor.

This rule is also called the dynamo or generator rule and has wide practical applications.

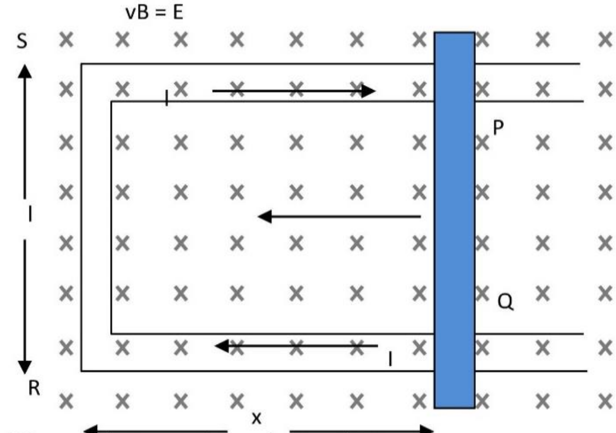
MOTIONAL EMF FROM LORENTZ FORCE AND ENERGY CONSIDERATION

Motional emf from Lorentz force: A conductor has a large number of free electrons. When it moves through a magnetic field, a Lorentz force acting on the free electrons can set up a current. Fig. shows a rectangular conductor in which arm PQ is free to move. It is placed in a uniform magnetic field B, directed normally into the plane of paper. As the arm PQ is moved towards left with a speed v, the free electrons of PQ also move with the same speed towards left. The electrons experience a magnetic Lorentz force, $F_m = qvB$. According to Fleming's left hand rule, this force acts in the direction QP and hence the free electrons will move towards P. A negative charge accumulates at P and a positive charge at Q. An electric field E is set up in the conductor from Q to P. This field exerts a force, $F_e = qE$ on the free electrons. The accumulation of charges at the two ends continues till these two forces balance each other, i.e.,

$$F_m = F_e$$

or $$qvB = qE$$

or $$vB = E$$



[Motional emf]

The potential difference between the ends Q and P is

$$V = El = vBl$$

Clearly, it is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf,

$$e = Blv$$

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

Current induced in the loop: Let R be the resistance of the movable arm PQ of the rectangular loop PQRS shown in Fig. Suppose the total resistance of the remaining arms QR, RS and SP is negligible compared to R. Then the current in the loop will be

$$I = \frac{e}{R} = \frac{Blv}{R}$$

Force on the movable arm: The conductor PQ of length l and carrying current I experiences force F in the perpendicular magnetic field B. The force is given by ,

$$F = IlB \sin 90^\circ = \left(\frac{Blv}{R}\right) lB = \frac{B^2 l^2 v}{R}$$

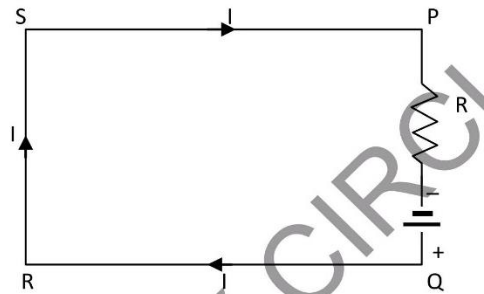
This force (due to induced current) acts in the outward direction opposite to the velocity of the arm in accordance with Lenz's law. Hence to move the arm with a constant velocity v, it should be pulled with a constant force F.

Power delivered by the external force: The power supplied by the external force to maintain the motion of the movable arm is

$$P = Fv = \frac{B^2 l^2 v^2}{R}$$

Power dissipated as Joule loss: The power dissipated in the loop as Joule heating loss is

$$P_J = I^2 R = \left(\frac{Blv}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}$$



[Equivalent electrical circuit of the electromagnetic set up of Fig. (Above)]

Clearly, $P_J = P$. Thus, the mechanical energy expanded to maintain the motion of the movable arm is first converted into electrical energy (the induced emf) and then to thermal energy. This is consistent with the law of conservation of energy. This fact allows us to represent the electromagnetic set up of Fig. by the equivalent electrical circuit shown in Fig.

RELATION BETWEEN INDUCED CHARGE AND CHANGE IN MAGNETIC FLUX

According to Faraday's law, the magnitude of induced emf,

$$|e| = \frac{\Delta \phi}{\Delta t}$$

If R is the total resistance of the closed loop, then the induced current will be

$$I = \frac{e}{R} \quad \text{or} \quad \frac{\Delta q}{\Delta t} = \frac{\Delta \phi}{\Delta t} \cdot \frac{1}{R}$$

Hence the charge induced in the circuit in time Δt,

$$\Delta q = \frac{\Delta \phi}{R} = \frac{\text{Net change in magnetic flux}}{\text{Resistance}}$$

Clearly, the induced charge depends on the net change in the magnetic flux and not on the time interval Δt of the flux change. Thus the induced charge does not depend on the rate of change of magnetic flux.

Examples based on Motional EMF

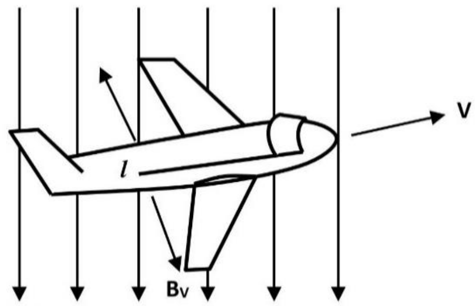
- **FORMULA USED**

  1. The emf induced in a conductor of length l moving with velocity v perpendicular to field B, $\mathcal{E} = Blv$.
  2. Induced emf developed between the two ends of rod rotating at its one end in perpendicular magnetic field, $\mathcal{E} = \frac{1}{2} Bl^2 \omega$

- **UNIT USED**

  Field B is in tesla or Wbm^{-2}, length l in metre, velocity v in ms^{-1}, angular speed ω in rad s^{-1} and emf $\mathcal{E}$ in volt.

Q. 1. An aircraft with a wing span of 40 m flies with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is 1.75×10^{-5} T. Find the emf that emf that develops between the tips of the wings.



Sol. The metallic part between the wing tips can be treated as a single conductor cutting flux lines due to vertical component of earth's magnetic field. So emf is induced between the tips of its wings.

Here $l = 40$ m, $B_V = 1.75 \times 10^{-5}$ T

$v = 1080\ kmh^{-1} = \frac{1080 \times 1000}{3600}\ ms^{-1}$

$= 300\ ms^{-1}$

$\therefore \quad e = B_V l v = 1.75 \times 10^{-5} \times 40 \times 300 = 0.21$ V.

Q. 2. **A jet plane is travelling west at 450 ms^{-1}. If the horizontal component of earth's magnetic field at that place is 4×10^{-4} tesla and the angle of dip is 30°, find the emf induced between the ends of wings having a span of 30 m.**

Sol. Here $l = 30$ m, $v = 450\ ms^{-1}$, $\delta = 30°$, $B_H = 4 \times 10^{-4}$ T

The wings of a airplane cut the flux-lines of the vertical component of earth's magnetic field, which is given by

$B_V = B_H \tan\delta = 4 \times 10^{-4} \tan 30°$

$= 4 \times 10^{-4} \times 0.577$ T

The emf induced across the tips of the wings is

$e = B_V lv = 4 \times 10^{-4} \times 0.577 \times 30 \times 450 = 3.12$ V

Q. 3. **A railway track running north-south has two parallel rails 1.0 m apart. Calculate the value of induced emf between the rails when a train passes at a speed of 90 kmh^{-1}. The horizontal component of earth's magnetic field at that place is 0.3×10^{-4} Wbm^{-2} and angle of dip is 60°.**

Sol. Here $l = 1.0$ m, $B_H = 0.3 \times 10^{-4}$ Wb m^{-2}, $\delta = 60°$

$B_V = B_H \tan\delta = 0.3 \times 10^{-4} \tan 60°$

$= 0.3 \times 10^{-4} \times 1.732 = 0.52 \times 10^{-4}\ Wbm^{-2}$.

$v = 90\ kmh^{-1} = \frac{90 \times 1000}{3600} = 25\ ms^{-1}$.

$\therefore$ $e = B_V lv = 0.52 \times 10^{-4} \times 1.0 \times 25 = 1.3 \times 10^{-3}$ V.

Q. 4. **A conductor of length 1.0 m falls freely under gravity from a height of 10 m so that it cuts the lines of force of the horizontal component of earth's magnetic field of $3 \times 10^{-5}\ Wbm^{-2}$. Find the emf induced in the conductor.**

Sol. The velocity v attained by the conductor as it falls through a height of 10 m is given by

$v^2 = u^2 + 2gs = 0 + 2 \times 9.8 \times 10 = 4 \times 49$

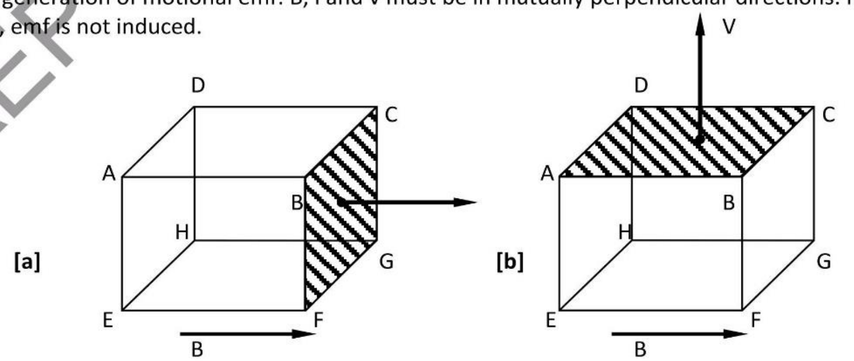
$\therefore$ $v = 2 \times 7 = 14\ ms^{-1}$

Induced emf,

$e = B_H lv = 3 \times 10^{-5} \times 1.0 \times 14 = 4.2 \times 10^{-4}$ V

Q. 5. **Twelve wires of equal lengths (each 10 cm) are connected in the form of a skeleton-cube (i) If the cube is moving with a velocity of 5 ms^{-1} in the direction of a magnetic field of 0.05 Wbm^{-2}, find the emf induced in each arm of the cube. (ii) If the cube moves perpendicular to the field, what will be the induced emf in each arm?**

Sol. For the generation of motional emf: B, l and v must be in mutually perpendicular directions. If any two of these quantities are parallel, emf is not induced.



(i) In Fig. (a), the velocity of any conductor is parallel to the field B, so no emf is induced in any conductor.

(ii) In Fig. (b), the arms AE, BF, CG and DH are parallel to the velocity v, no emf is induced in these arms. Also, the arms AB, DC, EF and HG are parallel to the field B, so no emf is induced in these arms.

The arms AD, BC, EH and FG are perpendicular to both B and v. Hence emf is induced in each of these arms and is given by

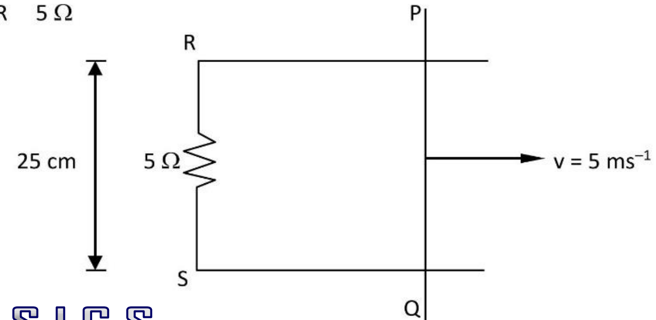
$e = Blv = 0.05 \times 10 \times 10^{-2} \times 5 = 2.5 \times 10^{-2}$ V.

Q. 6. **Fig. shows a conducting rod PQ in contact with metal rails RP and SQ which are 25 cm apart in a uniform magnetic field of flux density 0.4 T acting perpendicular to the plane of the paper. Ends R and S are connected through a 5 Ω resistance. What is the emf when the rod moves to the right with a velocity of 5 ms^{-1}? What is the magnitude and direction of the current through 5 Ω resistances? If the rod moves to the left with the same speed, what will be the new current and its direction**

Sol. Here $B = 0.4$ T, $v = 5\ ms^{-1}$, $l = 25$ cm $= 0.25$ m

Induced emf, $e = Blv = 0.4 \times 0.25 \times 5 = 0.5$ V

Current, $I = \frac{e}{R} = \frac{0.5\ V}{5\ \Omega} = 0.1$ A



Applying Fleming right hand rule, the induced current flows from Q to P, i.e., from the end R to S through the 5 Ω resistance.

If the rod moves to the left with the same speed, then the current of 0.1 A will flow through 5 Ω resistance from the end S to R.

Q. 7. A metallic rod of length L is rotated at an angular speed ω normal to a uniform magnetic field B. Derive expressions for the (i) emf induced in the rod (ii) current induced and (iii) heat dissipation, if the resistance of the rod is R.

Sol. Suppose the rod completes one revolution in time T. Then change in flux

$= B \times \text{Area swept} = B \times \pi L^2$

Induced emf $= \dfrac{\text{Change in flux}}{\text{Time}}$

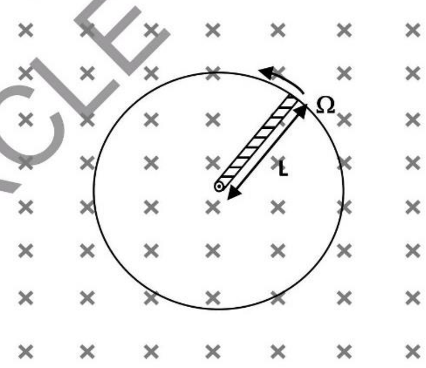
or $\mathcal{E} = B \times \pi L^2 = B\pi L^2 f$ $[\because T = 1/f]$

As $f = \dfrac{\omega}{2\pi}$, therefore

$\mathcal{E} = B \pi L^2 . \dfrac{\omega}{2\pi} = \dfrac{1}{2} BL^2 \omega$

Induced current, $I = \dfrac{\mathcal{E}}{R} = \dfrac{1}{2}\dfrac{BL^2 \omega}{R}$

Heat dissipation in time t, $Q = \dfrac{\mathcal{E}^2 t}{R} = \dfrac{1}{4}\dfrac{B^2 L^2 \omega}{R}$



Q. 8. A metal disc of radius R rotates with an angular velocity ω about an axis perpendicular to its plane passing through its centre in a magnetic field B acting perpendicular to the plane of the disc. Calculate the induced emf between the rim and the axis of the disc.

Sol. Consider a disc of radius R rotating in a transverse magnetic field B with frequency f. In time period T, the disc completes one revolution.

∴ Change in flux $= B \times \text{Area swept} = B \times \pi R^2$

Induced emf $= \dfrac{\text{change in flux}}{\text{Time}}$

$\mathcal{E} = \dfrac{B \pi R^2}{T} = B\pi R^2 f$ $\left(\because \dfrac{1}{f} = T\right)$

As $f = \dfrac{\omega}{2\pi}$, $\therefore \mathcal{E} = B \times \pi R^2 . \dfrac{\omega}{2\pi} = \dfrac{1}{2} BR^2 \omega$

Q. 9. A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field B_H at a place. If B_H = 0.4 G at the place, what is the induced emf between the axle and the rim of the wheel? Note that 1 G = 10^{-4} T.

Sol. Here L = 0.50 m, $B = 0.40\text{ G} = 0.40 \times 10^{-4}$ T

$f = 120 \dfrac{\text{rev}}{\text{min}} = \dfrac{120}{60} \dfrac{\text{rev}}{\text{sec}} = 2$ rps

Induced emf,

$\mathcal{E} = B \pi L^2 f = 0.40 \times 10^{-4} \times 3.14 \times (0.50)^2 \times 2$

$= 6.28 \times 10^{-5}$ V.

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

Q. 10. When a wheel with metal spokes 1.2 m long rotates in a magnetic field of flux density 5×10^{-5} T normal to the plane of the wheel, an emf of 10^{-2} V is induced between the rim and the axle of the wheel. Find the rate of revolution of the wheel.

Sol. Here $L = 1.2$ m, $B = 5 \times 10^{-5}$ T, $\mathcal{E} = 10^{-2}$ V, f = ?

As $\mathcal{E} = B \pi L^2 f$

∴ $f = \dfrac{\mathcal{E}}{B \pi L^2}$

$= \dfrac{10^{-2}}{5 \times 10^{-5} \times 3.14 \times (1.2)^2} = 44.2$ rps.

Q. 11. A circular copper disc 10 cm in radius rotates at 20 π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc. (i) Calculate the potential difference developed between the axis of the disc and the rim. (ii) What is the induced current, if the resistance of the disc is 2 ohm?

Sol. Here R = 10 cm = 0.10 m, $\omega = 20 \pi$ rad s^{-1}, B = 0.2 T

(i) P.D. developed between the axis and the rim

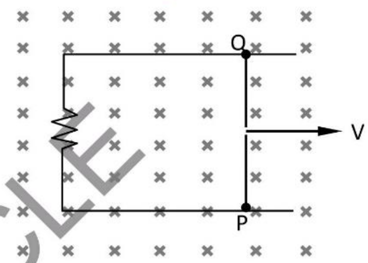
$\mathcal{E} = ½ BR^2 \omega$

$= ½ \times 0.2 \times (0.10)^2\ 20 \pi = 0.0628$ V

(ii) Induced current,

$I = \dfrac{\mathcal{E}}{R} = \dfrac{0.0628}{2} = 0.0314$ A

Q. 12. **A 0.5 m long metal rod PQ completes the circuit as shown in Fig. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. if the resistance of the total circuit is 3 Ω, calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 ms^{-1}.** 14



Sol. Here $e = Blv$

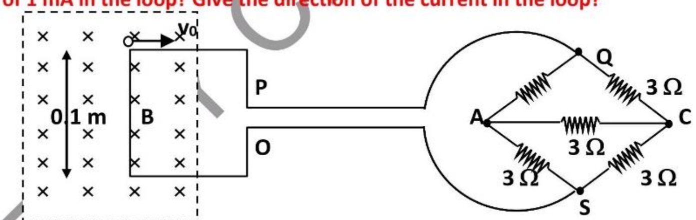
and $I = \frac{e}{R} = \frac{Blv}{R}$

$F = IlB \sin 90° = \frac{Blv}{R} . lB = \frac{B^2 l^2 v}{R}$

But, l = 0.5 m, B = 0.15 T, R = 3 Ω, v = 2 ms^{-1}

∴ $F = \frac{(0.15)^2 \times (0.5)^2 \times 2}{3} = 0.00375$ N.

Q. 13. **A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 Wb m^{-2} as shown in Fig. The magnetic lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to the network of resistors each of value 3 Ω. The resistance of the loop wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in the loop? Give the direction of the current in the loop?**



Sol. The network of resistance is a balanced Wheatstone's bridge. So the resistance AC is ineffective. The equivalent resistance R' of the network is given by

$\frac{1}{R'} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ or $R' = 3\,\Omega$

As the resistance of the loop is 1 Ω, therefore, effective resistance of the circuit,

$R = 3 + 1 = 4\,\Omega$

emf induced in the loop, $e = Blv_0$

∴ Current in the loop, $I = \frac{e}{R} = \frac{Bl\,v_0}{R}$

Hence speed of the loop, $v_0 = \frac{IR}{Bl}$

Given I = 1 mA = 10^{-3} A, l = 0.1 m, B = 2 Wbm^{-2} ∴ $v_0 = \frac{10^{-3} \times 4}{2 \times 0.1} = 2 \times 10^{-2}\ ms^{-1} = 2\ cms^{-1}$

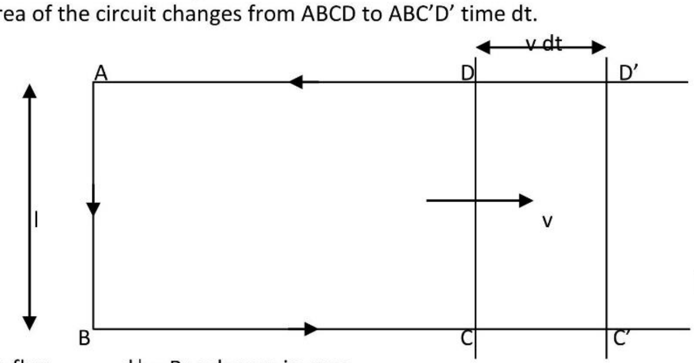
METHODS OF GENERATING INDUCED EMF

Methods of generating induced emf: An induced emf can be produced by changing the magnetic flux linked with a circuit. The magnetic flux, $\phi = BA \cos\theta$ can be changed by one of the following methods:

1. Changing the magnetic field B, 2. Changing the area A of the coil, and 3. Changing the relative orientation θ of B and A.

1. Induced emf by changing the magnetic field B: We have already learnt before how an induced emf is set up in a coil on changing the magnetic magnetic flux through it by (i) moving a magnet towards a stationary coil, (ii) moving a coil towards a stationary magnet and (iii) varying current in the neighbouring coil.

2. Induced emf by changing the area of the coil: Consider a conductor CD of length l moving with a velocity v towards right on U-shaped conducting rails situated in a magnetic field B, as shown in Fig. The field is uniform and points into the plane of the paper. As a conductor slides, the area of the circuit changes from ABCD to ABC'D' time dt.



[Induced emf by changing area of the loop]

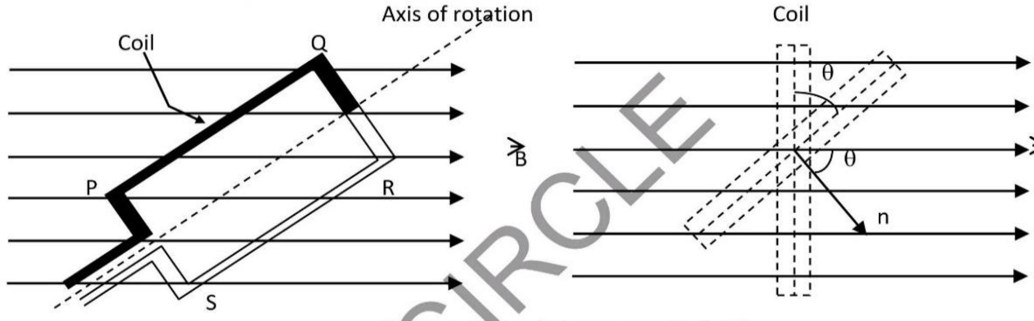
The increase in flux, $d\phi = B \times$ change in area

$= B \times$ area CDD' C' = B. l. vdt

This sets up induced emf in the loop of magnitude, $|e| = \frac{d\phi}{dt} = Blv$

According to Fleming's right hand rule, the induced current flows in the anticlockwise direction.

3. Induced emf by changing relative orientation of the coil and the magnetic field: Theory of AC generator: Consider a coil PQRS free to rotate in the uniform magnetic field $\vec{B}$. The axis of rotation of the coil is perpendicular to the field $\vec{B}$. The flux through the coil, when its normal makes an angle θ with the field, is given by $\phi = BA\cos\theta$ Where A is the face area of coil.



[(a) Rotating coil in a magnetic field]

If the coil rotates with an angular velocity ω and turns through an angle θ in time t, then $\theta = \omega t$

∴ $\phi = BA\cos\omega t$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t) = BA\omega\sin\omega t$$

If the coil has N turns, then the total induced emf will be $e = NBA\omega\sin\omega t$

Thus the induced emf varies sinusoidally with time t. The value of induced emf is maximum when sin ωt = 1 or ωt = 90°, i.e., when the plane of the coil is parallel to the field B. Denoting this maximum value by $\mathcal{E}_0$, we have

$e_0 = NBA\omega$

∴ $e = e_0 \sin\omega t = e_0 \sin 2\pi ft$

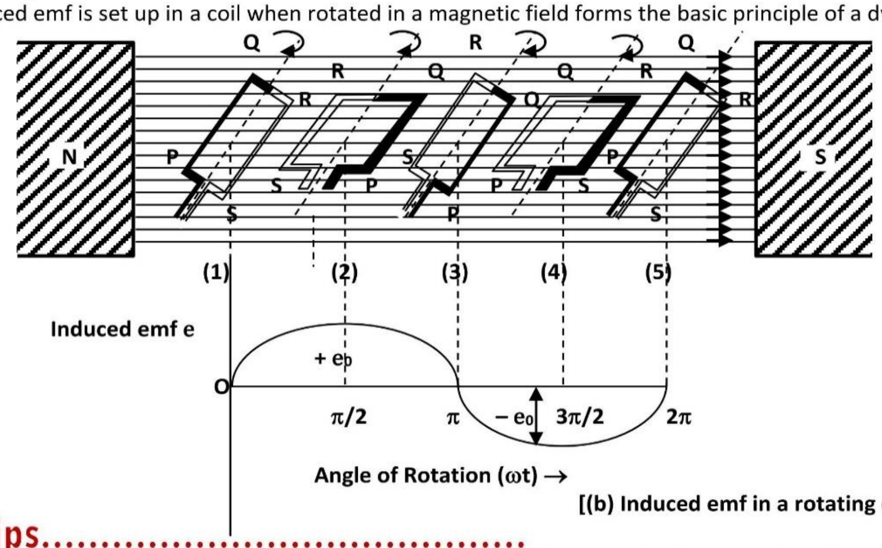
where f is the frequency of rotation of the coil.

Fig. (b) shows how the induced emf $\mathcal{E}$ between the two terminals of the coil varies with time. We consider the following special cases:

1. When ωt = 0°, the plane of the coil is perpendicular to $\vec{B}$, sin ωt = sin 0° = 0 so that e = 0
2. When ωt = π/2, the plane of the coil is parallel to field $\vec{B}$, sin ωt = sin π/2 = 1, so that $e = e_0$
3. When ωt = π, the plane of the coil is again perpendicular to B, sin ωt = sin π = 0, so that e = 0
4. When ωt = 3π/2 the plane of the coil is again parallel to $\vec{B}$, sin ωt = sin 3π/2 = – 1 so that $e = -e_0$
5. When ωt = 2π, the plane of the coil again becomes perpendicular to B after completing one rotation, sin ωt = sin 2π = 0 so that $\mathcal{E}$ = 0

As the coil continues to rotate in the same sense, the same cycle of changes repeats again and again, As shown in Fig. (b), the graph between emf $\mathcal{E}$ and time t is a sine curve. Such an emf is called sinusoidal or alternating emf. Both the magnitude and direction of this emf change regularly with time.

The fact that an induced emf is set up in a coil when rotated in a magnetic field forms the basic principle of a dynamo or a generator.



[(b) Induced emf in a rotating coil]

Conceptual Tips..

- ☑ **The magnetic flux linked with a surface is maximum when it is held perpendicular to the direction of the magnetic field and the flux linked is zero when the surface is held parallel to the direction of the magnetic field.**
- ☑ **Induced emf is set up wherever the magnetic flux linked with a circuit changes even if the circuit is open. However, the induced current flows only when the circuit is closed.**
- ☑ **No emf is induced when a coil and a magnet move with the same velocity in the same direction.**
- ☑ **No emf is induced when a magnet is rotated about its own axis. However, emf is induced when a magnet is rotated about an axis perpendicular to its length.**

- ☑ **No emf is induced when a closed loop moves totally inside a uniform magnetic field.**
- ☑ **Just as a changing magnetic field produced an electric field, a changing electric field also sets up a magnetic field.**
- ☑ **The electric fields created by stationary charges have vanishing and path independent loop integrals. Such fields are called conservative fields.**

$$\oint \vec{E}\cdot d\vec{l} = 0$$

- ☑ **The electric fields created by time-varying magnetic fields have non-vanishing loop integrals and are called non-conservative fields. Their loop integrals are path dependent.**

$$\oint \vec{E}\cdot d\vec{l} = \frac{d\phi}{dt}$$

- ☑ **Electric potential is meaningful only for electric fields produced by stationary charges. It has no meaning for electric fields set up by magnetic induction.**
- ☑ **The heart beating induces a.c. in the surrounding tissues. The detection and study of these currents is called electrocardiography which provides valuable information regarding the pathology of the heart.**
- ☑ **Migration of birds: Every winter birds from Siberia fly unerringly to water spots in the Indian subcontinent. It is believed that migratory birds make use of earth's magnetic field to determine their direction. As birds contain no ferromagnetic material, so electromagnetic induction appears to be the only mechanism to determine direction. However, very small emfs induced across the bodies of these birds create a doubt about the validity of this hypothesis. So the migration pattern of birds is still a mystery.**

Examples based on Induced EMF in a Rotating Coil

◆ **FORMULA USED**

1. $\mathcal{E} = \mathcal{E}_0 \sin \omega t$
2. $\mathcal{E}_0 = NBA\omega$, where $\omega = 2\pi f$
3. Maximum induced current, $I_0 = \frac{\mathcal{E}_0}{R}$

◆ **UNITS USED**

The induced emf $\mathcal{E}$ and maximum induced emf $\mathcal{E}_0$ are in volt, field B in tesla, area A in m^2, angular frequency ω in $rads^{-1}$, current I_0 in ampere, resistance R in ohm.

Q. 1. A circular coil of area 300 cm^2 and 25 turns rotates about its vertical diameter with an angular speed of 40 s^{-1} in a uniform horizontal magnetic field of magnitude 0.05 T. Obtain the maximum voltage induced in the coil.

Sol. Here $A = 300\ cm^2 = 300 \times 10^{-4}\ m^2$, $N = 25$, $\omega = 40\ s^{-1}$, $B = 0.05\ T$

The maximum voltage induced in the coil is

$$\mathcal{E}_0 = NBA\omega = 25 \times 0.05 \times 300 \times 10^{-4} \times 40 = 1.5\ V.$$

Q. 2. A rectangular coil of length 1 n and width 0.5 m, and 10 turns is rotated at 50 revolutions per second. the magnetic field within which the coil is rotated is B = 0.5 T. Calculated the peak value of the voltage generated across the ends of the coil.

Sol. Here $A = 1\ m \times 0.5\ m = 0.5\ m = 0.5\ m^2$, $N = 10$, $f = 50$ rps, $B = 0.5\ T$

Peak voltage $\mathcal{E}_0 = NBA\omega = NBA \times 2\pi f$

$= 10 \times 0.5 \times 0.5 \times 2 \times 3.14 \times 50$

$= 785\ V.$

Q. 3. A flat coil of 500 turns, each of area $5 \times 10^{-3}\ m^2$, rotates in a uniform magnetic field of 0.14 T at an angular speed of 150 rad s^{-1}. The coil has a resistance of 5 Ω. the induced emf is applied to an external resistance of 10 Ω. Calculate the peak current through the resistance.

Sol. Here $A = 5 \times 10^{-3}\ m^2$, $B = 0.14\ T$, $\omega = 150\ rad\ s^{-1}$

Total resistance, $R = 5 + 10 = 15\ \Omega$

Peak value of induced emf, $\mathcal{E}_0 = NBA\omega = 1 \times 0.14 \times 5 \times 10^{-3} \times 150 = 52.5\ V$

Peak current, $I_0 = \frac{\mathcal{E}_0}{R} = \frac{52.5}{15} = 3.5\ A$

Q. 4. A rectangular coil of wire has dimensions 0.2 m × 0.1 m. The coil has 2000 turns. The coil rotates in a magnetic field about an axis parallel to its length and perpendicular to the magnetic field of 0.02 Wb m^{-2}. The speed of rotation of the coil is 4200 r.p.m. Calculative (i) the maximum value of the induced emf in the coil (ii) the instantaneous value of induced emf when the plane of the coil has rotated through an angle of 30° from the initial position.

Sol. Here $A = 0.2 \times 0.1\ m^2 = 2 \times 10^{-2}\ m^2$, $N = 2000$, $B = 0.02\ Wbm^{-2}$

$f = 4200$ r.p.m. $= \frac{4200}{60}$ rps $= 70$ rps

$\omega = 2\pi f = 2\pi \times 70 = 140\pi\ rad\ s^{-1}$

(i) The maximum value of the induced emf in the coil is $\mathcal{E}_0 = NBA\omega$

$= 2000 \times 0.2 \times 2 \times 10^{-2} \times 140 \times \frac{22}{7}\ V = 3520\ V$

(ii) The instantaneous value of induced emf when the coil has turned through 30° is

$e = e_0 \sin \omega t$

$= e_0 \sin 30° = 3520 \times ½ \text{ V} = 1760 \text{ V}$

Q. 5. A rectangular coil of 200 turns of wire, 15 cm × 40 cm makes 50 revolutions/second about an axis perpendicular to the magnetic field of 0.08 weber/m². What is the instantaneous value of induced emf when the plane of the coil makes an angle with the magnetic lines of (i) 0° (ii) 60° and (iii) 90°?

Sol. Here $N = 200$, $A = 15 \text{ cm} \times 40 \text{ cm} = 15 \times 40 \times 10^{-4} \text{ m}^2$

$= 6 \times 10^{-2} \text{ m}^2$

$B = 0.08 \text{ Wb m}^{-2}$, ω

$= 2\pi \times 50 = 100\pi \text{ rad s}^{-1}$

Induced emf at any instant is given by

$e = e_0 \sin \omega t = NBA\omega \sin \omega t$

If the plane of the coil makes an angle α with the magnetic lines of force, then the angle between the normal to the plane of the coil and the magnetic field will be

$\omega t = 90° - \alpha$

(i) When $\alpha = 0°$, $\omega t = 90° - 0° = 90°$

$\therefore$ $e = NBA\omega \sin 90°$

$= 200 \times 0.08 \times 6 \times 10^{-2} \times 100\pi \times 1 \text{ V}$

(ii) When $\alpha = 60° = 30°$

$\therefore$ $e = NBA\omega \sin 30° = 301.6 \times ½ = 150.8 \text{ V}$.

(iii) When $\alpha = 90°$, $\omega t = 90° - 90° = 0°$

$\therefore$ $e = NBA\omega \sin 0° = 0$.

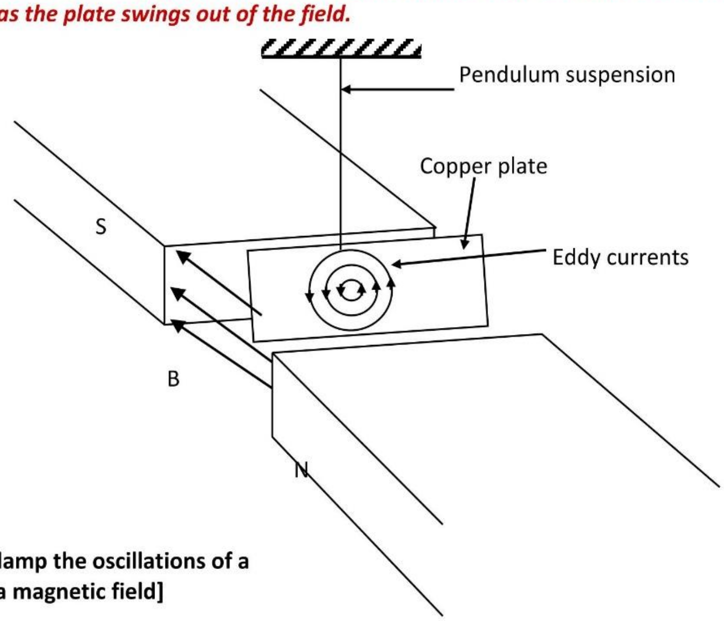
EDDY CURRENTS

Eddy currents: Currents can be induced, not only in conducting coils, but also in conducting sheets or blocks. Whenever the magnetic flux linked with a metal sheet or block changes, an emf is induced in it. The induced currents flow in closed paths in planes perpendicular to the lines of force throughout the body of the metal. These currents look like eddies or whirlpools in water and so they are knows as **eddy currents**. As these currents were first discovered by Foucault in 1895, so eddy currents are also known as **Foucault currents**.

- Eddy currents are the currents induced in solid metallic masses when the magnetic flux threading through them changes.
- Eddy currents also oppose the change in magnetic flux, so their direction is given by Lenz's law.

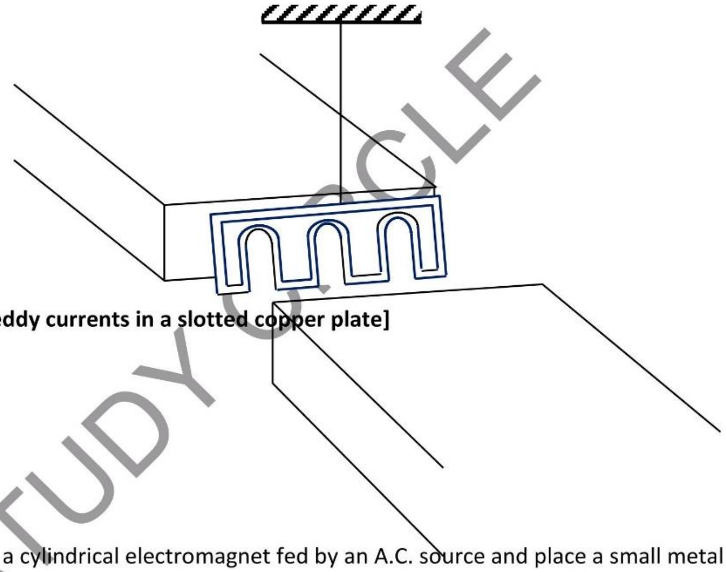
Experimental demonstration of eddy currents:

Experiment 1: Take a pendulum having its bob in the form of a flat copper plate. As shown in Fig. (a), it is free to oscillate between the pole pieces of an electromagnet. In the absence of any magnetic field, the pendulum swings freely. As the electromagnet is switched on, the oscillations of the pendulum get highly damped and soon it comes to rest. This is because as the copper plate moves in between the pole pieces of the magnet, magnetic flux threading through it changes. ***So eddy currents are set up in it which according to Lenz's law, opposes the motion of the copper plate in the magnetic field. Eddy currents flow anticlockwise as the plate swings into the field and clockwise as the plate swings out of the field.***



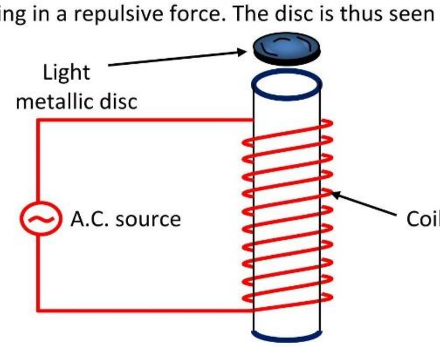
[(a) Eddy currents damp the oscillations of a copper plate in a magnetic field]

Experiment 2: Now take the pendulum of a flat copper plate with narrow slots cut across it, as shown in Fig. (b).
As the electromagnet is switched on, eddy currents are set up in the plate. ***But this plate swings for longer duration than the plate without slots. This is because the loop has much larger paths for the electrons to travel. Larger paths offer more resistance to electrons and so the eddy currents are sufficiently reduced. As a result, the opposition to the oscillations becomes very small.***



[(b) Reduced eddy currents in a slotted copper plate]

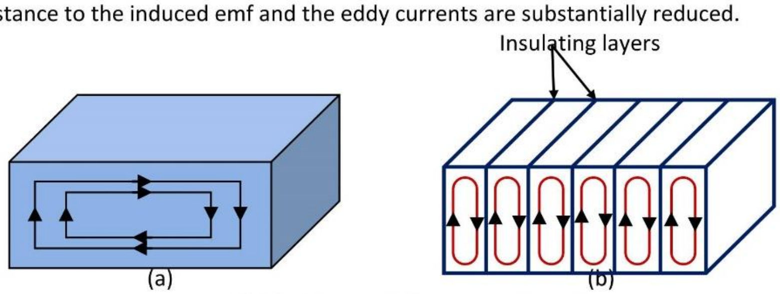
Experiment 3: Take a cylindrical electromagnet fed by an A.C. source and place a small metal disc over its top. As the current is switched on, the magnetic field at the disc rises from zero to a finite value, setting up eddy currents which effectively convert it into a small magnet. If initially the top end of the electromagnet acquires N-polarity by Lenz's law, the lower face of the small magnetic disc will also have N-polarity, resulting in a repulsive force. The disc is thus seen to be thrown up as the current in the electromagnet is switched on.



[A light metal disc on top of an electromagnet is thrown up as the current is switched on]

Undesirable effects of eddy currents: Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos, and also in the cores of transformers, which experience flux changes, when they are in use. Eddy currents cause unnecessary heating and wastage of power. The heat produced by eddy currents may even damage the insulation of coils.

Minimisation of eddy current: The eddy currents can be reduced by using laminated core which instead of a single solid mass consists of thin sheets of metal, insulated from each other by a thin layer of varnish, as shown in Fig. The planes of the sheets are placed perpendicular to the direction of the currents that would be set up by the emf induced in the material. The insulation between the sheets then offers high resistance to the induced emf and the eddy currents are substantially reduced.



[(a) Solid core, (b) Laminated core]

Applications of eddy currents: Although eddy currents are undesirable, still they find applications in the following devices:

1. Induction furnace: If a metal specimen is placed in a rapidly changing magnetic field (produced by high frequency a.c.), very large eddy currents are set up. The heat produced is sufficient to even melt the metal. This process is used on the extraction of some metal from their ores.

2. Electromagnetic damping: When a current is passed through a galvanometer, its coil suffers few oscillations before coming to rest in the final position. As the coil moves in the magnetic field, induced current is set in the coil which opposes its motion.

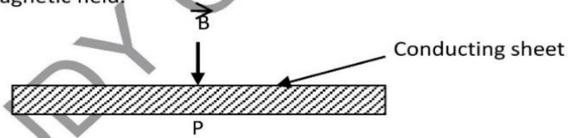
The oscillations of the coil which opposes its motion. The oscillations of the coil are damped. This is called electromagnetic damping. The electromagnetic damping can be further increased by winding the coil on a light copper or aluminium frame. As the frame moves in the magnetic field, eddy currents are set up in the frame which resists the motion of the coil. This is how a galvanometer is rendered dead beat, i.e., the coil does not oscillate – it deflects and stays in the final position immediately.

3. Electric brakes: A strong magnetic field is applied to the rotating drum attached to the wheel. Eddy currents set up in the drum exert a torque on the drum so as the stop the train.

4. Speedometers: In a speedometer, a magnet rotates with the speed of the vehicle. The magnet is placed inside an aluminium drum which is carefully pivoted and held in position by a hair spring. As the magnet rotates, eddy currents are set up in the drum which opposes the motion of the magnet. A torque is exerted on the drum in the opposite direction which deflects the drum through an angle depending on the speed of the vehicle.

5. Induction motor: In an a.c. induction motor, a rotating magnetic field is produced by two single phase alternating currents having a phase difference of 90°. A metallic rotor is placed in the magnetic field. The eddy currents set up in the rotor tend to oppose the relative motion between the rotating magnetic field and the door. As a result, the rotor also stars rotating about its axis.

6. Electromagnetic shielding: Eddy currents may be used for electromagnetic shielding. As shown in Fig. when a magnetic field B, directed towards a metallic sheet is suddenly switched on, large eddy currents are produced in the sheet. The change in the magnetic field is only partially detected at points (such as P) on the other side of the sheet. The higher the conductivity of the sheet, the better the shielding of the transient magnetic field.



[Electromagnetic shielding]

7. Inductothermy: Eddy currents can be used to heat localized tissues of the human body. This branch is called Inductothermy.

8. Energy meters: In energy meters: In energy meters used for measuring electric energy, the eddy currents induced in an aluminium disc are made use of.

Conceptuals tips......................................

☑ **Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it charges.**

☑ **Eddy currents tend to follow the path of least resistance inside a conductor. So they form irregularly shaped loops. However, their directions are not random, but guided by Lenz's law.**

☑ **Eddy currents have both undesirable effects and practically useful applications.**

☑ **Eddy currents can be induced in biological tissues. For example, the cavity of the eye is filled with a conducting fluid. A large transient magnetic field of 1 T alternating at a frequency of 60 Hz then induced such a large current in the retina that it produced a sensation of intense brightness.**

◘ SELF INDUCTION (Inertia of electricity)

"Self induction is the property of a coil by virtue of which; the coil opposes any changes in the strength of current flowing through it by inducing an emf in itself".

(for this reason self induction is also called the **inertia of electricity**)

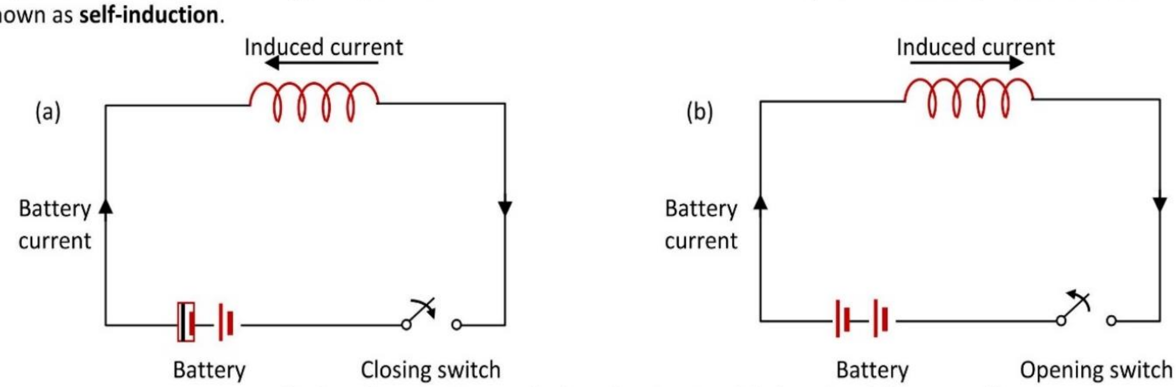
or

"Self induction is the property by which if opposes the growth or decay of the current flowing through it".

or

"The phenomenon, according to which an opposing induced emf is produced in a coil as result of change in current of magnetic flux linked with the coil, is called self induction".

■ When a current flow in a coil, it gives rise to a magnetic flux through the coil itself. As the strength of current changes, the linked magnetic flux changes and an opposing emf is induced in the coil. This emf is called **self-induced emf** or **back emf** and the phenomenon is known as **self-induction**.



[Induced current in a coil when the circuit is (a) closed and (b) opened]

Self-induction is the phenomenon of production of induced emf in a coil when a changing current passes through it.

Fig. (a) shows a battery and a tapping switch connected in series to a coil. ***As the switch is closed, the current increases and hence the magnetic flux through the coil increases from zero to a maximum value and the induced current flows in the opposite direction of the battery current***. In Fig. (b), as the tapping switch is opened, ***the current and hence the magnetic flux through the coil decreases from a maximum value to zero and the induced current flows in the same direction as that of the battery current.***

- The increasing magnetic flux set up an induced emf in the coil in accordance with faraday's law of electromagnetic induction.
- The direction of the included emf in the coil, according to LENZ"S law , should be such as to oppose the growth of current in the coil .
- Thus the growth of current delayed in the coil . When the current in the coil attains maximum value, constant (maximum) so, there is no induced or back emf.

- It is clear that the current does not attains its maximum value immediately but takes sometime to do so.
- When the key k is released, the currents being to decay from maximum to zero value. The magnetic flux linked with the coil also begins to decrease. The decreasing magnetic flux set up an induced emf in the coil. The direction of the induced emf is such as to oppose the decay of current.

- **At the time of make (growth) the direction of induced current is opposite to the Battery current.**
- **At the time of break (decay), the direction of induced current/emf is the same as the direction of Battery current.**

Coefficient of Self Induction :- (*or self inductance*)

Let, ϕ = magnetic flux linked with the coil when current I flows through it.

Then, $\phi \propto I$ or $\phi = LI$

•• *The value of 'L' depends upon the no. of turns, area of cross section and permeability of the material of the core (if any) on which the coil is wound.* ••

"The self inductance of a coil is the magnetic flux linked with the coil when unit current flows through it".

Definition. (With respect to time)

$$\frac{d\phi}{dt} = L\frac{dI}{dt}$$

But, *induced emf* $\Rightarrow e = -d\phi/dt$

$\therefore$ $\boxed{e = -L\, dI/dt}$

If dI/dt = 1, then L= -e (numerically)

"Coefficient of self induction of a coil is equal to the emf induced (numerically) in the coil when rate of change of current through the coil is unity".

(i) ***from*** $\phi = IL$

1 Henry = 1 Wb/sec   H = Wb/sec

(ii) ***from*** $e = -L\, dI/dt$

$\therefore L = -e/dI/dt$

1 Henry = 1 volt / 1 Amp/ sec = volt. Sec/Amp

"The Self inductance of coil is said to be 1 H when a current changing at the rate of 1 A/sec causes an induced emf equal to 1V in the coil".

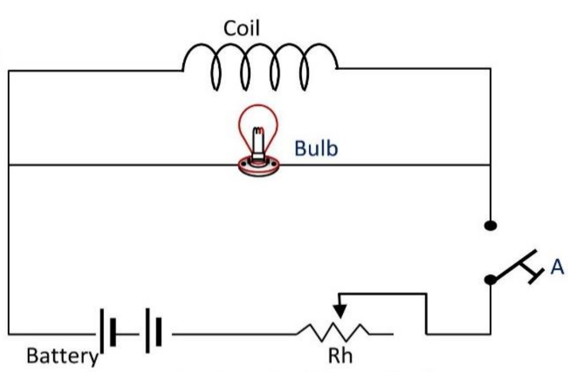
- **Smaller units**:- 1 mH = 10^{-3} H; 1 μH = 10^{-6} H
- **Dimensional formula**:-

$$e = L\, dI/dt$$

$$\therefore L = \frac{e}{dI/dt} = \frac{W/dq}{dI/dt} = \frac{W}{I\, dI}$$

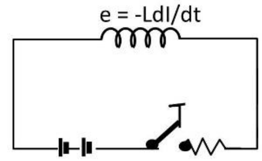
$$[L] = \frac{[ML^2T^{-2}]}{[A^2]} = [ML^2T^{-2}A^{-2}]$$

Experiment to demonstrate self-induction: Take a solenoid having a large number of turns of insulated wire wound over a soft iron core. Such a solenoid is called a choke coil. Connect the solenoid in series with a battery, a rheostat and a tapping key. Connect a 6 V bulb in parallel with the solenoid. Press the tapping key and adjust the current with the help of rheostat so that the bulb just glows faintly. As the tapping key is released, the bulb glows brightly for a moment and then goes out. This is because as the circuit is broken suddenly, the magnetic flux linked with the coil suddenly vanished, i.e., the rate of change of magnetic flux linked with the coil is very large. Hence large self-induced emf and current are produced in the coil which makes the bulb glow brightly for a moment.



[Demonstration of self-induction]

Energy stored is an inductor:- Consider an inductor of inductance L connected across a battery. When current flows through the inductor, an emf is induced in it.



$\therefore$ $e = -L\,dI/dt$

Magnitude $e = L\,dI/dt$

Let an infinitesimally small change '*dq*' be driven through the inductor. So work done by the external voltage is

$$dw = e\,dq, \quad dw = L\frac{dI}{dt}\,dq = L\,dI\left(\frac{dq}{dt}\right)$$

But, $dq/dt = I$

$\therefore$ $dW = L\,I\,dI$

$\therefore$ Total workdone to maintain the maximum value of current I_0 through the inductor is

$$\int dW = \int_0^{I_0} LI\,dI$$

$$W = L\left[\frac{I^2}{2}\right]_0^{I} = L\left[\frac{I_0^2}{2}\right]$$

$$\mathbf{W = \tfrac{1}{2} L I_0^2}$$

Hence, **Energy stored in the inductor is**, $\boxed{U = \frac{1}{2} LI^2}$

$\therefore$ Total magnetic flux linked with the solenoid, $\phi = \mu_0 \frac{N}{l} I \times \pi r^2 \times N$

But, $\phi = LI$

$\therefore$ $LI = \mu_0 \frac{N^2 I}{l} \pi r^2$

$$\boxed{L = \mu_0 \frac{N^2 \pi r^2}{l}} \qquad \boxed{L = \mu_0 \frac{N^2 A}{l}} \qquad (A = \pi r^2)$$

Soft induction of an air cored solenoid L depends on :-

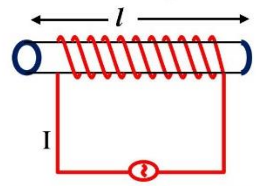
-------- i) The total no. of turns (N)

-------- ii) Length of the solenoid (*l*)

-------- iii) Area of cross section (A) of the solenoid.

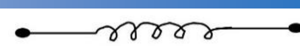
■ **Solenoid wound on magnetic material**: - When a solenoid is wound on a rod of magnetic material of permeability **μ**.

$\therefore$ Self induction, $L = \frac{\mu N^2 A}{l}$ or $\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$



i.e., *Self inductance of the coil increases of the air core of the coil is replaced by the iron core..*

▶ **Inductor**: - " **A coil having a sufficiently high inductance is called 'Inductor'** ".



(Symbol)

◙ **An ideal inductor has high value of self inductance and zero ohmic resistance.**

Self - induction of a plane coil :- Consider a plane coil of radius 'r' meter through which a current I ampere is flowing. Let 'N' be the no. of turns of the coil.

$\therefore$ Magnetic field at the centre of the coil.

$$B = \frac{\mu_0 NI}{2r} \quad \text{Tesla}$$

Magnetic flux per turn of coil = B A (Where $A = \pi r^2$, area of cross section of the coil.)

Total magnetic flux linked with N turns, $\phi = \frac{\mu_0 N^2 I}{2r} A$ Tesla

$$= \frac{\mu_0 N^2 I}{2r} \times \pi r^2 = \frac{\mu_0 N^2 I}{2} \pi r$$

$$\phi = \left(\frac{\mu_0 \pi N^2 r}{2}\right) I$$

But $\phi = LI$

$\therefore$ $\boxed{L = \frac{\mu_0 \pi N^2 r}{2}}$ **Henry**

SELF-INDUCTION OF A LONG SOLENOID

Self-inductance of a long solenoid: Consider a long solenoid of length 'l' and radius r with r << l and having n turns per unit length. If a current I flows through the coil, then the magnetic field inside the coil is almost constant and is given by

$B = \mu_0 nI$

Magnetic flux linked with each turn = BA = μ_0 nIA

Where A = πr^2 = the cross-sectional area of the solenoid.

∴ Magnetic flux linked with the entire solenoid is ϕ = Flux linked with each turn × total number of turns

$= \mu_0 nIA \times nl = \mu_0 n^2 l AI$

But $\phi = LI$

∴ Self-inductance of the long solenoid is

$L = \mu_0 n^2 lA.$

If N is the total number of turns in the solenoid, then $n = N/l$ and so

$$L = \frac{\mu_0 N^2 A}{l}$$

If the coil is wound over a material of high relative magnetic permeability μ_r (e.g. soft iron), then

$$L = \mu_r \mu_0 n^2 lA = \frac{\mu_r \mu_0 N^2 A}{l}$$

✱Factors on which self-inductance depends: The self-inductance of a solenoid depends on its geometry and magnetic permeability of the core material.

1. Number of turns: Larger the number of turns in the solenoid, larger is its self-inductance.

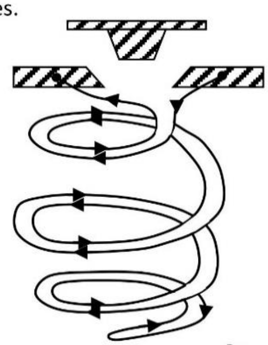
2. Area of cross-section: Larger the number of turns in the solenoid, larger is its self-inductance.

3. Permeability of the core material: The self inductance of a solenoid increase μ_r times if it is wound over an iron core of relative permeability μ_r.

PHENOMENA ASSOCIATED WITH SELF-INDUCTION

1. Sparking: The break of a circuit is very sudden. When the circuit is switched off, a large self induced emf is set up in the circuit in the same direction as the original emf. This causes a big spark across the switch.

2. Non-inducting winding: In resistance boxes and post office boxes, different resistance coils have to be used. Here the wire is first doubled over itself and then wound in the form of a coil over a bobbin. Due to this, the currents in the two halves of the wire flow in opposite direction as shown in Fig. The inductive effects of the two halves of the wire, being in opposite directions, cancel each other. The net self-inductance of the coil is minimum. Such a winding of coils is called non-inductive winding. The resistance coils having no self-inductance are called non-inductive resistances.



[Non-inductive winding of resistance coil]

Conceptuals tips..

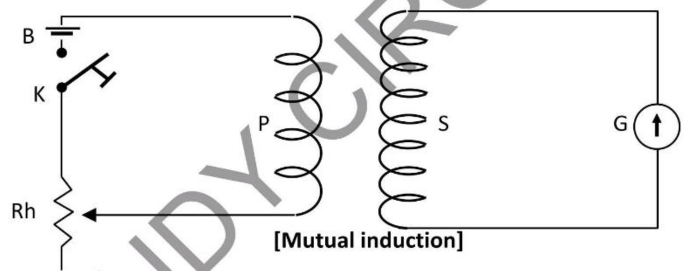
- ☑ **Inductance is a measure of the ratio of induced flux ϕ to the current I. It is a scalar quantity having the dimension of magnetic flux divided by current. Its dimensions in terms of the fundamental quantities are $[ML^2T^{-2}A^{-2}]$. Its SI unit is Wb A^{-1} or VsA^{-1} which is called henry (H). It is named in honour of Joseph Henry who discovered electromagnetic induction in USA independently of Faraday in England.**
- ☑ **Inductance plays the role of electrical inertia. The analogue of self-inductance in mechanics is mass.**
- ☑ **A solenoid made from a thick wire has a negligible resistance but a sufficiently large self-inductance. Such an element is called ideal inductor, denoted by**
- ☑ **A wire itself cannot act as an inductor because the magnetic flux linked with the wire of negligible cross-sectional area is zero. Only a wire bent into the form of a coil can act as an inductor. Moreover, the self-induced emf appears only during the time the current through it is changing.**
- ☑ **The inductance of a coil depends on its geometry and the intrinsic properties of the material that fill up the space inside it. In this sense, it bears similarity to capacitance and resistance. The capacitance of a parallel plate capacitor depends on the plate area and plate separation (geometry) and the dielectric constant κ of the interposing medium (intrinsic material property). Similarly, the resistance of a conductor depends on its length and cross sectional area (geometry) and resistivity (intrinsic material property).**
- ☑ **The capacitance, resistance, inductance and diode (describe in chapter 14) constitute the four passive elements of an electrical circuit. In fact, these are the four alphabets of electrical/electronic engineering.**

MUTUAL INDUCTION

"Mutual induction is the phenomenon of production of induced emf in one coil due to varying current in current in the neighbouring coil".

- ***Mutual induction is the property of this coil of virtue of which each apposes any change in the strength of current flowing through theother by developing an emf (induced).***

consider two coils P and S placed close to each other. The coil P is connected in series to a battery B and a rheostat Rh through a tapping key K. The coil S is connected to a galvanometer G. When a current flows through coil P, it produces a magnetic field which produces a magnetic flux through coil S. If the current in the coil S changes which induced an emf and hence a current in it, as is seen from the deflection in the galvanometer. The coil P is called the primary coil and coil S, the secondary coil, because it is the former which causes an induced emf in the latter. The two circuits are said to be **coupled circuit**.



[Mutual induction]

- **Coefficient of mutual induction**: At any instant,

Magnetic flux linked with the secondary coil $\propto$ current in the primary coil

i.e., $\phi \propto I$

or $\phi = MI$ ... (1)

The proportionality constant M is called the mutual inductance or coefficient of mutual induction of the two coils.

Any change in the current I sets up an induced emf in the secondary coil which is given by

$$e = -\frac{d\phi}{dt} = -M.\frac{dI}{dt} \quad ... (2)$$

If in equation (1) I = 1, then $\phi = M$

Thus **the mutual inductance of two coils is numerically equal to the magnetic flux linked with one coil when a unit current passes through the other coil.**

Again, from equation (2), if

$\frac{dI}{dt} = 1$, then $e = -M$

The **mutual inductance of two coils may be defined as the induced emf set up in one coil when the current in the neighbouring coil changes at the unit rate.**

- **Unit of mutual inductance:** From equation (2), we have

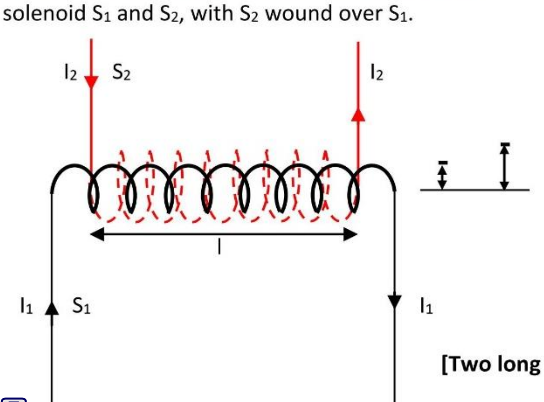
$$M = \frac{e}{\frac{dI}{dt}}$$

$\therefore$ SI unit of $M = \frac{1\ V}{1\ As^{-1}} = 1\ VsA^{-1} = 1$ henry (H)

The mutual inductance of two coils is said to be one henry if an induced emf of one volt is set up in one coil when the current in the neighbouring coil changes at the rate of 1 ampere per second.

MUTUAL INDUCTANCE OF TWO LONG SOLENOIDS

consider two long co-axial solenoid S_1 and S_2, with S_2 wound over S_1.



[Two long coaxial solenoids of same length l]

Let l = **length** of each solenoid, r_1, r_2 = **radii** of the two solenoids

$A = \pi r_1^2$ = **Area** of cross-section of inner solenoid S_1 N_1, N_2 = **number of turns** in the two solenoids

First we pass a **time varying current I_2** through S_2.

- The magnet field set up inside S_2 due to I_2 is , $\mathbf{B_2 = \mu_0 n_2 I_2}$

  where $n_2 = N_2/l$ = the number of turns per unit length of S_2.
- Total magnetic flux linked with the inner solenoid S_1 is

  $$\phi_1 = B_2 AN_1 = \mu_0 n_2 I_2 . AN_1$$

∴ Mutual inductance of ***coil 1 with respect to coil 2*** is

$$M_{12} = \frac{\phi_1}{I_2} = \mu_0 n_2 A N_1 = \frac{\mu_0 N_1 N_2 A}{l}$$

Consider the flux linked with the outer solenoid S_2 due to the current I_1 in the inner solenoid S_1.

- The field B_1 due to I_1 is constant inside S_1 but zero in the annular region between the two solenoid. Hence

  $$B_1 = \mu_0 n_1 I_1$$

  where $n_1 = N_1/l$ = the number of turns per unit length of S_1.

∴ Total flux linked with the outer solenoid S_2 is

$$\phi_2 = B_1 AN_2 = \mu_0 n_1 I_1 . AN_2 = \frac{\mu_0 N_1 N_2 A I_1}{l}$$

∴ Mutual inductance of ***coil 2 with respect to coil 1*** is

$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Clearly, $M_{12} = M_{21} = M$ (say)

∴ $M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l = \mu_0 n_1 n_2 \pi r_1^2 l$

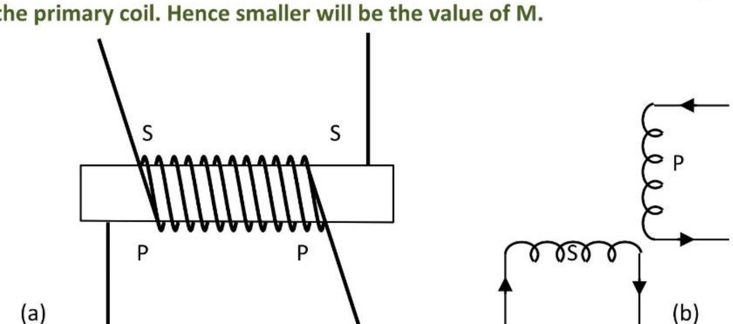
Thus, the mutual inductance of two coils is the property of their combination.

It does not matter which one of them functions as the primary or the secondary coil. This fact is known as RECIPROCITY THEOREM.

◻ Factors on which mutual inductance depends: The mutual inductance of two solenoids depends on their geometry and the magnetic permeability of the core material.

1. **Number of turns:** Larger the number of turns in the two solenoids, larger will be their mutual inductance.

   $$M \propto N_1 N_2$$
2. **Common cross-sectional area:** Larger the common cross-sectional area of two solenoids, larger will be their mutual inductance.
3. **Relative separation:** Larger the distance between two solenoids, smaller will be the magnetic flux linked with the secondary coil due to current in the primary coil. Hence smaller will be the value of M.



[(a) M is maximum when primary envelops secondary, (b) M is minimum when primary is perpendicular to secondary]

4. **Relative orientation of the two coils:** M is maximum when the entire flux of the primary linked with the secondary, i.e., when the primary coil completely envelops the secondary coil. M is minimum when the two coils are perpendicular to each other, as shown in Fig.
5. **Permeability of the core material:** If the two coils are wound over an iron core of relative permeability μ_r, their mutual inductance increases μ_r times.

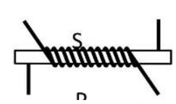
◻ Coefficient of coupling: The coefficient of coupling of two coils givens a measure of the manner in which the two coils are coupled together. If L_1 and L_2 are the self-inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

The value of K lies between 0 and 1.

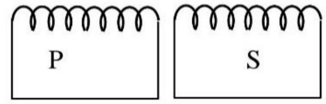
Case I:- When the two coils P & S are wound on each other K = Max When the coupling is perfect i.e., the entire flux of primary is linked with the secondary, M is maximum and K = 1.

∴ M = Max

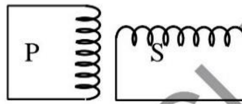


K is further increases when the two are wound on a soft iron are (as μ) is very large.

Case II:- K is large but less then that of in 1st case. M is also large but less than that of in 1st case.



Case III:- They are perpendicular to each other K = Min^m



When there is no coupling, M = 0 and K = 0.

Conceptual tips......................................

☑ When two coils are inductively coupled, in addition to the emf produced due to mutual induction, induced emf is set up in the two coils due to self-induction also.

☑ The mutual inductance of two coils is a property of their combination. The value of M remains unchanged irrespective of the fact that current is passed through one coil or the other.

☑ While calculating the mutual inductance of two long co-axial solenoids, the cross-sectional area of the inner solenoids, the cross-sectional area of the inner solenoid is to be considered.

☑ While calculating the mutual inductance of two co-axial solenoids of different lengths, the length of the larger solenoid is to be considered.

GROUPING OF INDUCTANCES

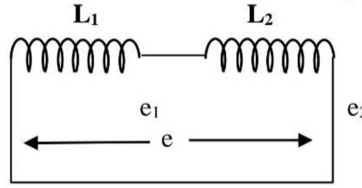
[A]**Grouping of coils**:-{ ***mutual inductance is negligible}***

1) **Coils in series**:- When two coil of inductance L_1 and L_2 are connected in series and the coefficient of coupling K = 0 than as in series current through coil is same and potential is divided.

$\therefore\ e = e_1 + e_2$

$L_S \frac{dI}{dt} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$

$\therefore\ \mathbf{L_s = L_1 + L_2}$

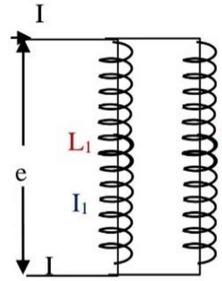


2) **Coils in parallel**:- $I = I_1 + I_2$

$dI/dt = dI_1/dt + dI_2/dt$

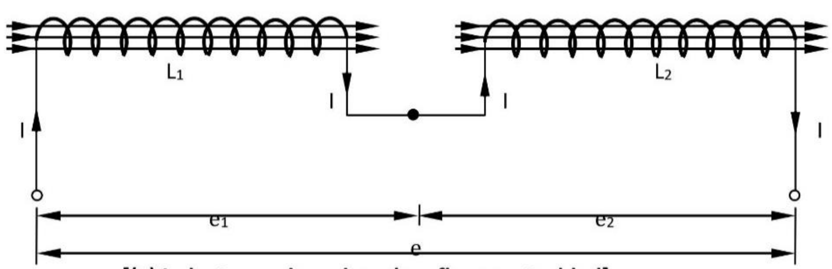
$e/Lp = e/L_1 + e/L_2$

$\therefore\ \frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$



***Aliter* = [B]**

Inductors in series: (i) Let the series connection be such that the current flows in the same sense in the two coils as shown in Fig. (a)



[(a) Inductances in series when fluxes get added]

Let L_{eq} be the equivalent inductance of the two self-inductances L_1 and L_2 connected in series. For the series combination, the emfs induced in the two coils get added up. Thus

$e_{eq} = e_1 + e_2$

If the rate of change of current in the series circuit is $\frac{dI}{Dt}$, then

$e_1 = -L_1 \frac{dI}{dt} - M\frac{dI}{dt}$, $e_2 = -L_2 \frac{dI}{dt} - M\frac{dI}{dt}$

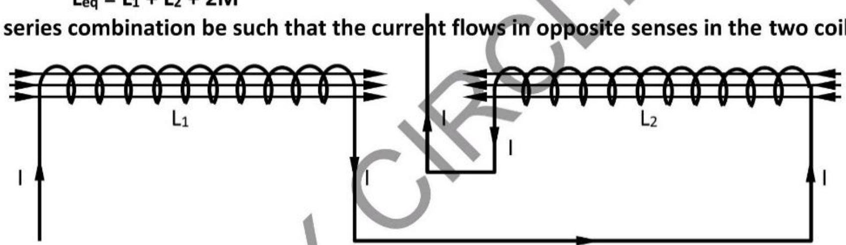
and, $e_{eq} = -L_{eq}\frac{dI}{dt}$

The negative sign throughout indicates that both self and mutual induced emf's are opposing the applied emf. Using the above equations, we have $e_{eq} = e_1 + e_2$

or $-L_{eq} . \frac{dI}{dt} = -(L_1 + M + L_2 + M)\frac{dI}{dt}$

or $\mathbf{L_{eq} = L_1 + L_2 + 2M}$

(ii) Let the series combination be such that the current flows in opposite senses in the two coils, as shown …



[(b) Inductances in series when fluxes get subtracted]

The emfs induced in the two coils will be

$e_1 = -L_1\frac{dI}{dt} + M\frac{dI}{dt}$, $e_2 = -L_2\frac{dI}{dt} + M\frac{dI}{dt}$

Here the mutual emfs act in the direction of applied emf and hence positive. For this series combination also, the emfs induced in the two coils get added up.

Hence $e_{eq} = e_1 + e_2 = -[L_1 - M + L_2 - M]\frac{dI}{dt}$

But $e_{eq} = -L_{eq}\frac{dI}{dt}$

∴ $-L_{eq}\frac{dI}{dt} = -[L_1 + L_2 - 2M]\frac{dI}{dt}$

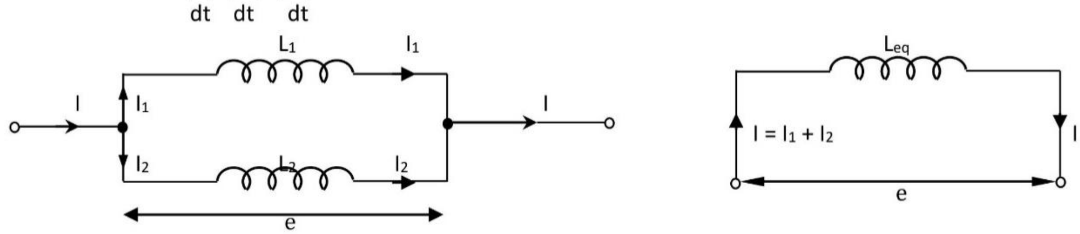
or $L_{eq} = L_1 + L_2 - 2M$

◘. Two inductors of self-inductances L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible………………………………………………………………….

Inductances in parallel: For the parallel combination, the total current I divide up through the two coils as

$I = I_1 + I_2$

∴ $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$



[Inductances in parallel]

For parallel combination, induced emf across the combination is equal to the induced emf across each inductance. Thus

$e = -L_1\frac{dI_1}{dt}$ or $e = -\frac{dI_1}{dt}$

$e = -L_2\frac{dI_2}{dt}$ or $e = -\frac{dI_2}{dt}$

This is because the mutual inductance M is negligible. If L_{eq} is the equivalent inductance of the parallel combination, then

$e = -L_{eq} . \frac{dI}{dt} = -L_{eq}\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right)$

$= L_{eq} - \left(\frac{dI_1}{dt} - \frac{dI_2}{dt}\right)$ or $\frac{e}{L_{eq}} = \left(\frac{e}{L_1} + \frac{e}{L_2}\right)$

or $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

⬢ If there is any mutual inductance M between the coils, then

$L = \frac{L_1L_2 - M^2}{L_1 + L_2 \pm M}$

Examples based on Self-Induction and Mutual induction

FORMULA USED

1. For self-induction, $\phi = LI$
2. Self induced emf, $e = -L\frac{dI}{dt}$
3. For Mutual induction, $\phi = MI$
4. Mutual induced emf, $e = -M\frac{dI}{dt}$
5. Self-inductance of long solenoid,
$L = \mu_0 N_2 A = \mu_0 n^2 Al$, where $n = \frac{N}{l}$
6. Mutual inductance of two closely wound solenoids,
$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 Al$,
where $n_1 = \frac{N_1}{l}$, $n_2 = \frac{N_2}{l}$

UNITS USED

Flux ϕ is in weber, inductances L and M in henry, emf 'e' in volt, current I in ampere, cross-sectional area A in m^2, number of turns per unit length n, n_1 and n_2 are in m^{-1}.

Q. 1. What is the self-inductance of a coil, in which magnetic flux of 40 milliweber is produced when 2 A current flows through it?

Sol. Here $\phi = 40$ mWb $= 40 \times 10^{-3}$ Wb, I = 2 A

Self-inductance,

$$L = \frac{\phi}{L} = \frac{40 \times 10^{-3}}{2} = 2 \times 10^{-2} \text{ Wb}$$

Q. 2. A 200 turn coil of self-inductance 20 mH carries a current of 4 mA. Find the magnetic flux linked with each turn of the coil.

Sol. Let ϕ be the magnetic flux linked with each of the N turns of the coil. Then

$N\phi \propto I$ or $N\phi = LI$

$$\therefore \quad \phi = \frac{LI}{N} = \frac{20 \times 10^{-3} \times 4 \times 10^{-3}}{200} = 4 \times 10^{-7} \text{ Wb.}$$

Q. 3. If a rate of change of current of 4 As^{-1} induces an emf of 10 mV in a solenoid, what is the self-inductance of the solenoid?

Sol. Here dI = 4 As^{-1}, $|e| = 20$ mV $= 20 \times 10^{-3}$ V

As $|e| = L\frac{dI}{dt}$

$$\therefore \quad L = \frac{|e|}{dI/dt} = \frac{20 \times 10^{-3}}{4} = 5 \times 10^{-3} \text{ H} = 5 \text{ mH.}$$

Q. 4. A 12 V battery connected to a 6 Ω, 10 H coil through a switch drives a constant current through the circuit. The switch is suddenly opened. If it takes 1 ms to open the switch, find the average emf induced across the coil.

Sol. Steady-state current $= \frac{12 \text{ V}}{6\ \Omega} = 2$ A

Final current = 0

$$\therefore \quad \frac{dI}{dt} = \frac{(0-2)\text{ A}}{1 \text{ ms}} = -2 \times 10^{-3} \text{ As}^{-1}$$

Induced emf,

$$e = -L\, dI = -10 \times (-2 \times 10^3) = 20,000 \text{ V}$$

Such a high emf usually causes sparks across the open switch.

Q. 5. An inductor of 5 H inductance carries a steady current of 2 A. How can a 50 V self-induced emf be made of appear in the inductor?

Sol. Suppose the current reduces to zero in time t second. Thus

$L = 5$ H, $dI = 0 - 2 = -2$ A, $\mathcal{E} = 50$ V

As $e = -L\frac{dI}{dt}$

$$\therefore \quad 50 = -5 \times \frac{-2}{t} \quad \text{or} \quad t = 0.2 \text{ s}$$

Hence an induced emf of 50 V can be generated by reducing the current to zero in 0.2 s.

Q. 6. What is the self-inductance of an air core solenoid 50 cm long and 2 cm radius if it has 50 turns?

Sol. Here $L = 50$ cm $= 0.50$ m,
$r = 2$ cm $= 2 \times 10^{-2}$ m
$N = 500$, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

Self-inductance of the solenoid is

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (500)^2 \times \pi \times (2 \times 10^{-2})^2}{0.50} = 7.89 \times 10^{-4} \text{ H.}$$

Q. 7. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced cross the ends of the ends of the open switch in the circuit?

Sol. $l = 30$ cm = 0.30 m, $A = 25\ cm^2 = 25 \times 10^{-4}\ m^2$
$N = 500$, $dt = 10^{-3}$ s, $dI = 0 - 2.5 = -2.5$ A

Back emf $= -L\dfrac{dI}{dt} = -\dfrac{\mu_0 N^2 A}{l} \cdot \dfrac{dI}{dt}$

$= -\dfrac{4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 10^{-4} \times (-2.5)}{6.30 \times 10^{-3}} = 6.642$ V.

Q. 8. **A large circular coil, of radius R, and a small circular coil, of radius r, is put in vicinity of each other. If the coefficient of mutual induction, for this pair, equals 1 MH, what would be the flux linked with the larger coil when a current of 0.5 A flows through the smaller coil? When the current in the smaller coil falls to zero, what would be its effect in the larger coil?**

Sol. Here M = 1 mH, I = 0.5 A

Flux, $\phi = MI = 10^{-3}\ H \times 0.5\ A = 5 \times 10^{-4}$ Wb.

When the current in the smaller coil falls to zero, an induced emf is set up in the larger coil due to the decrease in the linked flux.

Q. 9. **What is the mutual inductance of a pair of coils if a current change of six ampere in one coil causes the flux in the second coil of 2000 turns to change by 12×10^{-4} Wb per turn?**

Sol. Here N = 2000, I = 6 A

Flux per turn = 12×10^{-4} Wb

Total flux, $\phi = 2000 \times 12 \times 10^{-4} = 2.4$ Wb

As $\phi = MI$

$\therefore$ $M = \dfrac{\phi}{I} = \dfrac{2.4}{6} = 0.4$ H.

Q. 10. **An emf of 0.5 V is developed in the secondary coil, when current in primary coil changes from 5.0 A to 2.0 A in 300 millisecond. Calculate the mutual inductance of the two coils?**

Sol. Here $e = 0.5$ V, $dI = 2 - 5 = -3$ A, $dt = 300$ ms $= 300 \times 10^{-3}$ s

As $e = -M\dfrac{dI}{dt}$ $\therefore$ $0.5 = -M \times \dfrac{-3}{300 \times 10^{-3}}$

or $M = 0.05$ H

Q. 11. **If the current in the primary circuit of a pair of coils changes from 5 A to 1 A in 0.02 s, calculate (i) induced emf in the secondary coil if the mutual inductance between the two coils is 0.5 H and (ii) the change of flux per turn in the secondary, if it has 200 turns.**

Sol. (i) $e = -M\dfrac{dI}{dt} = -0.5 \times \dfrac{(1-5)}{0.02}$

$= \dfrac{2}{0.02} = 100$ V

(ii) $e = -N\dfrac{d\phi}{0.02}$

$\therefore$ Change in flux per turn,

$d\phi = -\dfrac{100 \times 0.02}{200} = -0.01$ Wb.

The negative sign shows a decrease of magnetic flux.

Q. 12. **Over a solenoid of 50 cm length and 2 cm radius and having 500 turns, is wound another wire of 50 turns near the centre. Calculate the (i) mutual inductance of the two coils (ii) induced emf in the second coil when the current in the primary changes from 0 to 5 A in 0.02 s.**

Sol. Here $N_1 = 500$, $N_2 = 50$, $r = 2$ cm $= 0.02$ m

$l = 50$ cm $= 0.50$ m, $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$

(i) The mutual inductance of the two coils is

$M = \dfrac{\mu_0 N_1 N_2 A}{l} = \dfrac{\mu_0 N_1 N_2 . \pi r^2}{l}$

$M = \dfrac{4\pi \times 10^{-7} \times 500 \times 50 \times \pi \times (0.02)^2}{0.5}$

$= 78.96 \times 10^{-6}\ H = 78.96\ \mu H$.

(ii) The emf induced in the second coil is

$e = -M\dfrac{dI}{dt} = -78.96 \times 10^{-6} . \dfrac{(5-0)}{0.02}$

$= -19.74 \times 10^{-3}\ V = -19.74$ mV.

The negative sign indicates a back emf.

Q. 13. **A solenoidal coil has 50 turns per centimetre along its length and a cross-sectional area of $4 \times 10^{-4}\ m^2$. 200 turns of another wire are wound round the first solenoid coaxially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils. Given $\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$.**

Sol. Here n_1 50 turns per cm = 5000 turns per metre, $n_2 l = 200$ turns, $A = 4 \times 10^{-4}\ m^2$

$M = \mu_0 n_1 n_2 A l = \mu_0 n_1 (n_2 l) A = 4\pi \times 10^{-7} \times 5000 \times 200 \times 4 \times 10^{-4} = 5.027 \times 10^{-4}$ H

Q. 14. **A solenoid of length 50 cm with 20 turns per cm and area of cross-section 40 cm² completely surrounds another co-axial solenoid of the same length, area of cross-section 25 cm² with 25 turns per cm. Calculate the mutual-induction of the system.**

Sol. Here $l = 50$ cm $= 0.50$ m

Total no. of turns in outer-solenoid, $N_1 = n_1 l = 20 \times 50 = 1000$

Area of cross-section of outer solenoid, $A_1 = 40\ cm^2 = 40 \times 10^{-4}\ m^2$

Total no. of turns in inner solenoid, $N_2 = n_2 l = 25 \times 50 = 1250$

Area of cross-section of inner solenoid, $A_2 = 25\ cm^2 = 25 \times 10^{-4}\ m^2$

To determine mutual inductance, we take area of cross-section of inner solenoid.

$\therefore$ $M = \dfrac{\mu_0 N_1 N_2 A_2}{l} = \dfrac{4\pi \times 10^{-7} \times 1000 \times 1250 \times 25 \times 10^{-4}}{0.50} = 7.85 \times 10^{-3}\ H = 7.85$ mH.

Q. 15. (a) A solenoid with an air-core has an average radius of 15 cm, area of cross-section 12 cm² and 1200 turns. Obtain the self-inductance of the toroid. Ignore field variation across the cross – section of the toroid.
(b) A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2.0 A in 0.05 s, obtain the induced emf in the second coil.

Sol. (a) The uniform magnetic field set up inside a solenoid is given by

$$B = \mu_0 nI = \frac{\mu_0 N}{2\pi r} \cdot I \qquad \left(\because n = \frac{N}{2\pi r}\right)$$

∴ Total flux linked with the N turns is

$$\phi = NBA = N \cdot \frac{\mu_0 NI}{2\pi r} \cdot A = \frac{\mu_0 N^2 IA}{2\pi r}$$

∴ Self-inductance of the toroid is

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times (1200)^2 \times 12 \times 10^{-4}}{2\pi \times 0.15} = 2.304 \times 10^{-3}\text{ H} = 2.3\text{ mH}$$

(b) Here $N_1 = 1200$, $N_2 = 300$

$dt = 0.05$ s, $dI = 2.0 - 0 = 2.0$ A

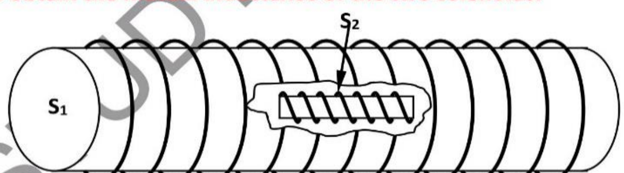
∴ The emf induced in the second coil is

$$e = M\frac{dI}{dt} = \frac{\mu_0 N_1 N_2 A}{l_1}\frac{dI}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times 1200 \times 300 \times 12 \times 10^{-4}}{2\pi \times 0.15} \times \frac{2.0}{0.05}$$

$= 0.023$ V $\qquad [\because l = 2\pi r]$

Q. 16. Fig. shows a short solenoid of length 4 cm, radius 2.0 cm and number of turns 100 lying inside on the axis of a long solenoid, 80 cm length and number of turns 1500. What is the flux through the long solenoid if a current of 3.0 A flows through the short solenoid? Also obtain the mutual inductance of the two solenoids.



Sol. As the short solenoid produces a complicated magnetic field, so it is difficult to calculate mutual inductance and flux through the outer solenoid. For this purpose we make use of the principle of reciprocity of mutual inductance i.e.,

$$M_{12} = M_{21}$$

Suppose S_1 represents the long solenoid and S_2, the short solenoid. Then

$l_1 = 80$ cm $= 0.80$ m, $N_1 = 1500$ $\qquad l_2 = 4$ cm $= 0.04$ m,
$R_2 = 2.0$ cm $= 0.02$ m $\qquad N_2 = 100$, $I_2 = 3.0$ A

The uniform magnetic field inside the long solenoid is given by

$$B_1 = \frac{\mu_0 N_1 I_1}{l_1}$$

Since the short solenoid lies completely inside the long solenoid, the flux linked with it is given by

$$\phi = N_2 A_2 B_1 = N_2 A_2 \cdot \frac{\mu_0 N_1 I_1}{l_1}$$

∴ Flux through each turn of short solenoid

$$= \frac{\phi_2}{N_2} = \frac{\mu_0 N_1 I_1}{l_1} \cdot A_2$$

$$= \frac{\mu_0 N_1 I_1}{l_1} \cdot \pi R_2^2$$

By definition, $\phi_2 = M_{21} I_1$

or $\qquad N_2 \cdot \frac{\mu_0 N_1 I_1 \cdot \pi R_2^2}{l_1} = M_{21} I_1$

From the symmetry of mutual inductance, we have

$$M_{12} = M_{21} = \frac{\mu_0 \pi R_2^2 \cdot N_1 N_2}{l_1}$$

$$= \frac{4\pi \times 10^{-7} \times \pi \times (0.02)^2 \times 1500 \times 100}{0.80}\text{ H}$$

$= 2.96 \times 10^{-4}$ H

The total flux linked with the long solenoid is

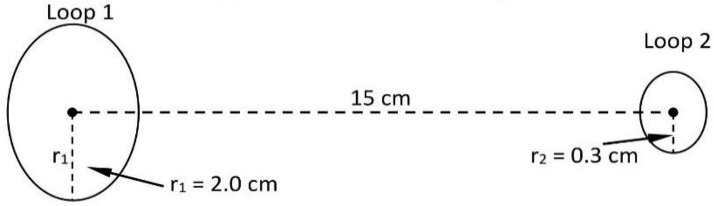
$N_1\phi_1 = M_{12} I_2 = 2.96 \times 10^{-4} \times 3.0$ Wb $= 8.88 \times 10^{-4}$ Wb $= 8.9 \times 10^{-4}$ Wb.

Q. 17. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. the distance between their centres is 15 cm. (a) What is the flux linking the bigger loop if a current of 2.0 A flows through the smaller loop? (b) Obtain the mutual induction of the two loops.

Sol. We use here the idea of symmetry of mutual inductance between two coils, i.e.,

$$M_{12} = M_{21}$$

Here 1 refers to the bigger loop and 2, the smaller loop.
Let us calculate the flux through the smaller loop due to a current I_1 in the bigger loop. The area of the smaller loop is so small that we can use formula for field B at any point on the axis of the bigger loop.



∴ Field B_2 at 2 due to current I_1 in 1 is

$$B_2 = \frac{\mu_0 I_1 r_1^2}{2(x^2 + r_1^2)^{3/2}}$$

Where x = the distance between the centres of the two loops. Flux linked with loop 2,

$$\phi = B_2 \times \text{area} = B_2 \pi r_2^2 = \frac{\pi \mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}} . I_1$$

$$\therefore \quad M_{21} = \frac{\phi_2}{I_1} = \frac{\pi \mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}} = M_{12}$$

Thus flux linked with loop 1,

$$\phi_1 = M_{12} I_2 = \frac{\pi \mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}} . I_2$$

Given r_1 = 20 cm = 0.20 m, r_2 = 0.3 cm = 3×10^{-3} m

x = 15 cm = 0.15 m, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

$$\therefore \quad M_{21} = \frac{\pi \times 4\pi \times 10^{-7} \times (0.2)^2 \times (3 \times 10^{-3})^2}{2[(0.15)^2 + (0.2)^2]^{3/2}}$$

$$= \frac{144\pi^2 \times 10^{-9}}{2 \times (625)^{3/2}} = 4.55 \times 10^{-11} \text{ H.}$$

Flux linking the bigger loop is

$\phi_1 = M_{12} . I_2 = M_{21} . I_1 = 4.55 \times 10^{-11} \times 2$ Wb

$= 9.1 \times 10^{-11}$ Wb.

∴ Field B_2 at 2 due to current I_1 in 1 is

$$B_2 = \frac{\mu_0 I_1 r_1^2}{2(x^2 + r_1^2)^{3/2}}$$

Where x = the distance between the centres of the two loops. Flux linked with loop 2,

$$\phi = B_2 \times \text{area} = B_2 \pi r_2^2 = \frac{\pi \mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}} . I_1$$

$$\therefore \quad M_{21} = \frac{\phi_2}{I_1} = \frac{\pi \mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}} = M_{12}$$

Thus flux linked with loop 1,

$$\phi_1 = M_{12} I_2 = \frac{\pi \mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}} . I_2$$

Given r_1 = 20 cm = 0.20 m, r_2 = 0.3 cm = 3×10^{-3} m

x = 15 cm = 0.15 m, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

$$\therefore \quad M_{21} = \frac{\pi \times 4\pi \times 10^{-7} \times (0.2)^2 \times (3 \times 10^{-3})^2}{2[(0.15)^2 + (0.2)^2]^{3/2}}$$

$$= \frac{144\pi^2 \times 10^{-9}}{2 \times (625)^{3/2}} = 4.55 \times 10^{-11} \text{ H.}$$

Flux linking the bigger loop is

$\phi_1 = M_{12} . I_2 = M_{21} . I_1 = 4.55 \times 10^{-11} \times 2$ Wb

$= 9.1 \times 10^{-11}$ Wb.

Q. 18. Three inductances are connected as shown in Fig. Fin the equivalent inductance.

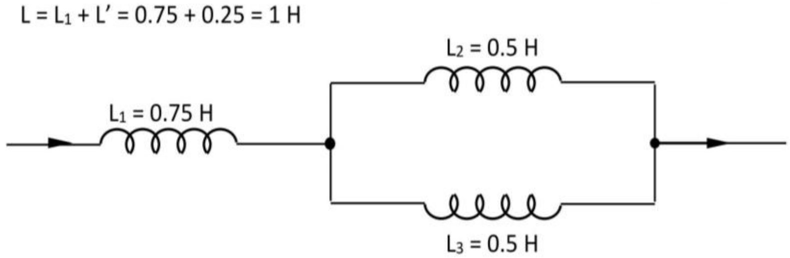
Sol. The equivalent inductance L′ of L_2 and L_3 is given by

$$\frac{1}{L'} = \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{0.5} + \frac{1}{0.5} = 4$$

or $L' = ¼ = 0.25$ H

Now, L_1 and L′ form a series combination, their equivalent inductance is given by

$L = L_1 + L' = 0.75 + 0.25 = 1$ H



... END.