AEP STUDY CIRCLE

SINCE 2001...

IIT-NEET-CBSE

ACCENTS EDUCATIONAL PROMOTERS

ONLINE-OFFLINE LEARNING ACADEMY

24 YEARS OF LEGACY

X CBSE

02 COORDINATE GEOMETRY

COORDINATE GEOMETRY-CBSE:X-MATHEMATICS

YOUR GATEWAY TO EXCELLENCE IN

IIT-JEE, NEET AND CBSE EXAMS

IIT-JEE

NEET

CBSE

COORDINATE GEOMETRY

CONTACT US:

+91-9939586130

+91-9955930311

aepstudycircle@gmail.com

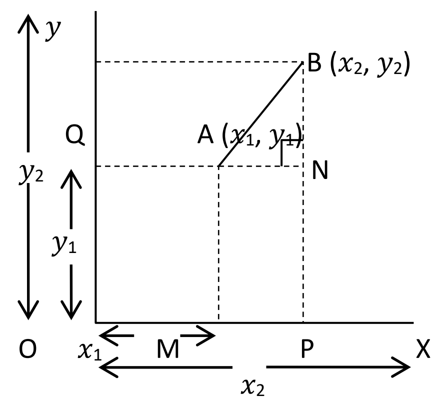
www.aepstudycircle.com

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

CO-ORDINATE GEOMETRY

CONCEPTS

1. If the coordinates of a point is its perpendicular distance from y – axis.
2. If is evident from the above discussion that:
   (i) The abscissa of a point is its perpendicular distance from y – axis.
   (ii) The ordinate of a point is its perpendicular distance from x – axis.
   (iii) The abscissa of every point situated on the right side of y – axis is positive and the abscissa of every point situated on the left side of y-axis is negative.
   (iv) The ordinate of every point situated above x-axis is positive and that of every point below x-axis is negative.
   (v) The abscissa of every point on y-axis is zero.
   (vi) The ordinate of every point on x-axis is zero.
   (vii) Coordinates of the origin are O(0, 0).



Distance Formula:

Given: A (x, y) and B (x, y) be the co-ordinate

To prove: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Proof: Let the co-ordinate of point A be (x, y) and co-ordinate of point B be $(x_2\ y_2)$.

Also, AN = QN = QA

= OP – MP (∵ QN = OP and QA = OM)

$= x_2 - x_1$

And BN = BP – NP

= YO – OQ (∵ YO = BP and NP = OQ)

$= y_2 - y_1$

Now, In rt. Δ BAN

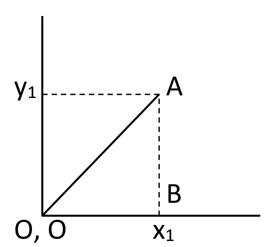
$AB^2 = BN^2 + AN^2$ (By Pythagoras theorem)

$AB = \sqrt{(BN)^2 + AN^2}$

$$AB = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Hence proved.

Corollary: Distance of a point from the origin



$$AB = \sqrt{x_1^2 + y_1^2}$$

-------*Tips n' technique*---------

For on Isosceles triangle	→ Prove that at least two sides are equal.
For on equilateral triangle	→ Prove that all the three sides are equal.
For a right Δ	→ The sum of square of two sides is equal to square of third side.
For a square	→ Prove that the four sides and the diagonal are equal.
For a rectangle	→ Prove that the opp. pair of sides and two diagonals are equal.
For a rhombus	→ Prove that all the adjacent sides are equal.
For a parallelogram	→ Prove that *opp*. sides are equal & parallel.
For collinear point	→ Prove that the sum of distance between two-point pair is equal to the third pair of point.

1. In which of the following quadrilateral do the following point lies?

(a) (5, 3); (b) (-2, 4); (c) (-4, -7); (d) (8, -3)

Sol. a) (5, 3)

Since, abscissa i.e., 5 greater than zero, (o) and ordinate i.e., 3 greater than zero (0).

∴ point (5, 3) lies on 1st co-ordinate.

b) (2, 4)

Since, abscissa i.e., -2 is not greater than zero, (o), and ordinate i.e., 4 is greater than zero (0).

∴ point (-2, 4) lies on 2nd co-ordinate

c) (-4, -7)

Since abscissa i.e., -4 is not greater than zero (o) and ordinate i.e., -7 also is not greater than zero (0)

∴ point (-4, 7) lies on 3rd co-ordinate

d) (8, -3)

Since, abscissa i.e., 8 greater than, zero (0) and ordinate i.e., -3 is not greater than zero (0)

Therefore, point (8, -3) lies on 4th co-ordinate.

2. Find the distance between the point: -

A. A (2, 3)
B (-1, 7)

B. A (-3, -4)
B (3, 0)

C. A (5, -12)
B (9, -9)

D. A (0, 0)
B (-5, 12)

E. P (7, 13)
Q (10, 9)

F. A $(x + y, x - y)$
B $(x - y, -x - y)$

G. A $(at_1^2, 2at)$; B $(at_2^2, 2at_2)$

Sol. A) A (2, 3)
B (-1, 7)

Here, $x_1 = 2$, $x_2 = -1$ and $y_1 = -3$, $y_2 = 7$

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1 - 2)^2 + (7 - 3)^2}$

$= \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ units

B) A (-3, -4)
B (3, 0)

Here, $x_1 = -3$, $x_2 = 3$ and $y_1 = -4$, $y_2 = 0$

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[(3 - (-3)]^2 + [0 - (-4)]^2}$

$= \sqrt{(3 + 3)^2 + (0 + 4)^2}$

$= \sqrt{(6)^2 + (4)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$ units

C) A (5, -12)
B (9, -9)

Here, $x_1 = 5$, $x_2 = 9$ and $y_1 = -12$, $y_2 = -9$

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9 - 5)^2 + [(-9 - (-12)]^2}$

$= \sqrt{(4)^2 + (-9 + 12)^2} = \sqrt{(4)^2 + (3)^2}$

$= \sqrt{(6)^2 + (4)^2} = \sqrt{36 + 16} = 52 = 2\sqrt{13}$ units

D) A (0, 0)
B (-5, 12)

Here, $x_1 = 0$, $x_2 = -5$ and $y_1 = 0$, $y_2 = 12$

∴ Magnitude of AB = $|AB| = \sqrt{(x_2^2 + y_2^2}$

$= \sqrt{(-5)^2 + (12)^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$ units

E) P (7, 13)
Q (10, 9)

Here, $x_1 = 7$, $x_2 = 10$ and $y_1 = 13$, $y_2 = 9$

∴ Magnitude of PQ = |PQ| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(10 - 7)^2 + (9 - 13)^2}$

$= \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units

F) A $(x + y, x - y)$

B $(x - y, -x - y)$

Here, $x_1 = x + y$, $x_2 = x - y$ and $y_1 = x - y$, $y_2 = -x - y$

∴ Magnitude of AB = |AB| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[(x - y) - (x + y)]^2 + [(-x - y) - (x - y)]}$

$= \sqrt{(x - y - x - y)^2 + (-x - y) - x + y)^2}$

$= \sqrt{(-2y)^2 + (-2x)^2} = \sqrt{4y^2 + 4x^2}$

$= \sqrt{4(x^2 + y^2)} = \sqrt{4}(x^2 + y^2) = 2\sqrt{x^2 + y^2}$ units

G) A $(at_1^2, 2at)$; B $(at_2^2, 2at_2)$

Here, $x_1 = at_1^2$, $x_2 = at_2^2$ and $y_1 = 2at$, $y_2 = 2at_2$

∴ Magnitude of AB = |AB| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(at^2_2 - at^1_2)^2 + (2at^2 - 2at^1)^2}$

$= \sqrt{a^2 (t_2^2 - t_1^2)^2 + 4a^2 (t_2 - t_1)^2}$

$= \sqrt{a^2 [(t_2^2 - t_1^2)^2 + 4 (t_2 - t_1)^2]}$

$= \sqrt{a^2 [\{(t_2 + t_1) (t_2 - t_1)\}^2 + 4 (t_2 - t_1)^2]}$

$= \sqrt{a^2 (t_2 - t_1)^2 [(t_2 + t_1)^2 + 4]}$

$= a (t_2 - t_1) \sqrt{(t_2 + t_2)^2 + 4}$ units

3. Find the distance of point P (7, -7) from the origin.

Sol. Magnitude of PO |PO| $= \sqrt{x_1^2 + y_1^2}$

$= \sqrt{(7)^2 + (-7)^2}$

$= \sqrt{49 + 49}$

$= \sqrt{98} = 7\sqrt{2}$ units.

4. A (7, 0), B (0, -24) and 0(0, 0) are the vertices of the triangle. Calculate the length of the hypotenuse of ΔABC.

Sol. Here, $x_1 = 7$, $x_2 = 0$ and $y_1 = 0$, $y_2 = -24$

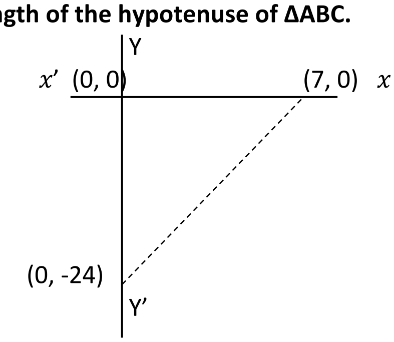
Magnitude of AB = |AB| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 7)^2 + (24 - 0)^2}$

$= \sqrt{(-7)^2 + (-24)^2}$

$= \sqrt{49 + 576}$

$= 625 = 25$ units



5. Find the distance of the point A (6, -6) from the origin.

Sol. Magnitude of AB = |AB| = $\sqrt{x_1^2 + y_1^2}$

$= \sqrt{(6)^2 + (-6)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$ units

6. Find the distance between the point A (a sin α, a cos α) and B (a cos α, -a sin α).

Sol. Here, x_1 = a sin α, x_2 = a cos α and y_1 = a cos α, y_2 = - a sin α

Magnitude of AB = |AB| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a \cos\alpha - a \sin\alpha)^2 + (-a \sin\alpha - a \cos\alpha)^2}$

$= \sqrt{a^2 (\cos\alpha - \sin\alpha)^2 + (-a^2) (\sin\alpha + \cos\alpha)^2}$

$= \sqrt{a^2[\cos^2 A + \sin^2\alpha - 2 \cos\alpha . \sin\alpha + \sin^2\alpha + \cos^2\alpha + 2 \cos\alpha . \sin\alpha]}$

$= a\sqrt{2} (\cos^2 A + \sin^2\alpha$

$= a\sqrt{2}$ units

7. A distance between A (1, 3) and B (x, 7) is 5 unit. Calculate the possible values of x.

Sol. A (1, 3) and B (x, 7) are the point such that AB = 5 units.

Magnitude of AB = |AB| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(x - 1)^2 + (7 - 3)^2}$

=> $(5)^2 = x^2 + 1 - 2x + 16$
=> $25 - 16 - 1 = x^2 - 2x$
=> $8 = x^2 - 2x$
=> $x^2 - 2x - 8 = 0$
=> $x^2 - 4x + 2x - 8 = 0$
=> $(x - 4) + 2(x - 4) = 0$
=> $(x + 2)(x - 4) = 0$

Either, $x + 2 = 0$, $x = -2$ | or, $x - 4 = 0$, $x = 4$

8. A is the point of Y axis whose ordinate is 5 and B is the point whose co-ordinate is (-3, 1). Calculate the length of AB.

Sol. Here, $x_1 = 0$, $x_2 = -3$; $y_1 = 5$, $y_2 = 1$

Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-3)^2 + (1 - 5)^2}$
$= \sqrt{(-3)^2 + (-4)^2}$
$= \sqrt{(9) + (16)}$
$= \sqrt{25}$
$= \sqrt{5}$ units

9. Find the value of x for which the distance between the points P (3, -5) and Q (x, 2) is $\sqrt{58}$ units.

Sol. Here, $x_1 = 3$, $x_2 = x$; $y_1 = -5$, $y_2 = 2$

Magnitude of PQ = $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\sqrt{58} = \sqrt{(x - 3)^2 + [(2 - (-5)]^2}$
$(\sqrt{58})^2 = x^2 + 9 - 6x + 49$
$58 - 49 - 9 = x^2 - 6x$
$0 = x^2 - 6x$
$x^2 - 6x = 0$
$x(x - 6) = 0$
$(x - 6) = 0$
$x = 6$

10. A point 'A' is at the distance of $\sqrt{10}$ units from the point B (4, 3). Find the co-ordinate of point 'A' If its ordinate is twice the abscissa.

Sol. Let the abscissa of point A be x

And the ordinate of point B be $2x$

Here, $x_1 = x$, $x_2 = 4$; $y_1 = 2x$, $y_2 = 3$

Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\sqrt{10} = \sqrt{(4 - x)^2 + (3 - 2x)^2}$
$(\sqrt{10})^2 = 16 + x^2 - 8x + 9 + 4x^2 + 12x$
$10 - 16 - 9 = x^2 - 8x + 4x^2 - 12x$
$-15 = 5x^2 - 20x$
$5x^2 - 20x + 15 = 0$
$5x^2 - 15x - 5x + 15 = 0$
$5x(x - 3) - 5x + 15 = 0$
$(5x - 5)(x - 3) = 0$

Either, $5x - 5 = 0$, $5x = 5$, $x = 1$ | or $x - 3 = 0$, $x = 3$

If $x = 1$
∴ abscissa of point A = 1
& Ordinate of point A = 2
∴ Co-ordinate o point A = (1, 2)

If $x = 3$
∴ abscissa of point A = 3
& ordinate of point A = 6
∴ Co-ordinate of point A = (3, 6)

11. A point P (2, -1) is equidistant from the point A (α, -7) and B (-3, α). Find the value of α.

Sol. Since point P is equilateral from point A & point B

∴ Magnitude of PA = Magnitude of PB

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(\alpha - 2)^2 + (-7 - 1)^2} = \sqrt{(-3 - 2)^2 + (\alpha + 1)^2}$

=> $\sqrt{\alpha^2 + 4 - 4\alpha + 64} = \sqrt{25 + \alpha^2 + 1 + 2\alpha}$

=> $\sqrt{\alpha^2 - 4\alpha + 68} = \sqrt{2 + 2\alpha + 26}$

=> $\cancel{\alpha^2} - 4\alpha + 68 = \cancel{\alpha^2} + 2\alpha + 26$

=> $4\alpha + 2\alpha + 26 - 68 = 0$

=> $6\alpha - 42 = 0$

=> $\alpha = 74$

∴ Value of $\alpha = 7$

12. Show that the point (α, α); (-α, α) and (-α√3, α√3) are the vertices of an equilateral Δ.

Sol. Let, the vertices of Δ of point P (α, α), Q (-α, -α) and R (-α√3, α√3)

∴ Magnitude of PQ = $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-\alpha - \alpha)^2 + (-\alpha - \alpha)^2}$

$= \sqrt{(-2x)^2 + (-2x)^2}$

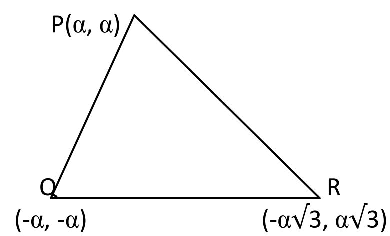
$= \sqrt{4\alpha^2 + 4\alpha^2 = 8\alpha 2} = 2\alpha\sqrt{2}$ units ... (i)

∴ Magnitude of QR = $|QR| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-x\sqrt{3} + \alpha)^2 + (\alpha\sqrt{3} + \alpha)^2}$

$= \sqrt{3\alpha^2 + \alpha^2 - 2 \times \alpha\sqrt{3} + \alpha^2 3 + \alpha^2 + 2 \times \alpha\sqrt{3}}$

$= \sqrt{8\alpha^2} = 2\alpha\sqrt{2}$ units ... (ii)



or Magnitude of RP = $|RP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(\alpha + \alpha\sqrt{3})^2 + (\alpha - \alpha\sqrt{3})^2}$

$= \sqrt{\alpha^2 + 3\alpha^2 + 2 \times \sqrt{3\alpha^2} + \alpha^2 + 3\alpha^2 - 2 \times \sqrt{3\alpha^2}}$

$= \sqrt{8\alpha^2} = 2\alpha\sqrt{2}$ units ... (iii)

From (i), (ii) and (iii)

PQ = QR = RP ∴ ΔPQR is an equilateral Δ.

13. Prove that point A (1, -3); B (-3, 0) and C (4, 1) are the vertices of the isosceles of Δ also find the area of Δ.

Sol. Let the vertices of Δ of point A (1, -3); B (-3, 0) and C (4, 1)

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

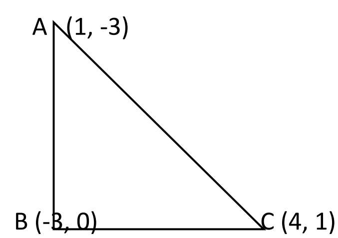
$= \sqrt{(-3 - 1)^2 + (0 + 3)^2}$

$= \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16 + 4}$

$= \sqrt{25}$

= 5 units ... (i)



Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 - 4)^2 + (-3 - 1)^2}$

$= \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ units ... (ii)

From (i) & (ii) AB = AC

i.e., ΔABC its two sides are equal .... (A)

Now, Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3 - 4)^2 + (0 - 1)^2}$

$= \sqrt{(-7)^2 + (-1)^2}$

$= \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$ units

Now,

$BC^2 = AB^2 + AC^2$ [By Pythagoras theorem]

$(5\sqrt{2})^2 = (5)^2 + (5)^2$

$50 = 50$

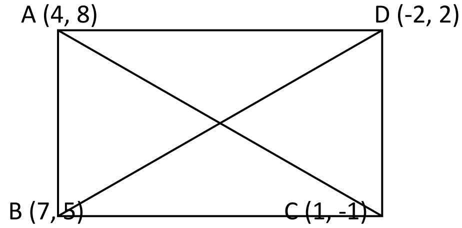
i.e., the sum of square of two equal sides is equal to the square of hypotenuse ... (B)

From (A) and (B)

ΔABC is rt. Angle isosceles Δ

Now, Area of isosceles of Δ = ½ × base × altitude = ½ × 5 × 5 = 25/2 units^2

14. Show that the point (4, 8); (7, 5); (1, -1) and (-2, 2) from a rectangle.



Sol. Let the vertices of a quad^r of point A (4, 8), B (7, 5), C (1, -1) and D (-2, 2)

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-4)^2 + (5-8)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{9+9} = \sqrt{18}$ units ... (i)

Magnitude of CD = $|CD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-1)^2 + (2+1)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{18}$ units ... (ii)

From (i) and (ii) AB = AC ... (A)

Now,

Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1-7)^2 + (-1-5)^2}$

$= \sqrt{(-6)^2 + (6)^2}$

$= \sqrt{36+36} = \sqrt{72}$ units ... (iii)

Also,

Magnitude of AD = $|AD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4+2)^2 + (8-2)^2}$

$= \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36+36} = \sqrt{72}$ units ... (iv)

From (iii) and (iv) BC = AD ... (B)

From (A) & (B) AB = CD and BC = AD

i.e., Δ ABCD its opp. sides are equal ... (C)

Now,

Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1-4)^2 + (-1-8)^2}$

$= \sqrt{(-3)^2 + (-9)^2}$

$= \sqrt{9+81} = \sqrt{90}$ units ... (v)

Magnitude of BD = $|BD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-7)^2 + (2-5)^2}$

$= \sqrt{(-9)^2 + (-3)^2}$

$= \sqrt{81+9} = \sqrt{90}$ units ... (vi)

From (v) & (vi) AC = BD

i.e., its diagonal is also equal ... (D)

From (C) & (D)

□ ABCD is a rectangle **hence proved.**

15. Show that the point (0, -1); (-2, 3); (6, 7) and (8, 3) are the vertices of rectangle. Find the area of the rectangle.

Sol. Let the vertices of a point A (0, 1), B (-2, 3), C (6, 7) and D (8, 3)

Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-0)^2 + (3+1)^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units ... (i)

Magnitude of CD = $|CD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-6)^2 + (3-7)^2}$

$= \sqrt{(2)^2 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units ... (ii)

From (i) and (ii) AB = CD ... (A)

Also, Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6+2)^2 + (7-3)^2}$

$= \sqrt{(8)^2 + (4)^2}$

$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$ units ... (iii)

Magnitude of DA = | DA | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 8)^2 + (-1 - 3)^2}$

$= \sqrt{(-8)^2 + (-4)^2}$ $= \sqrt{64 + 16} = \sqrt{80} = \sqrt{5}$ units ... (iv)

From (iii) and (iv) BC = AD ... (B)

i.e., From (A) and (B)

AB = CD and BC = AD

i.e., □ ABCD its opp. sides are equal ... (C)

Now,

Magnitude of AC = | AC | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - 0)^2 + (7 + 1)^2}$

$= \sqrt{(6)^2 + (8)^2}$

$= \sqrt{36 + 64} = 100 = 10$ units ... (v)

Magnitude of BD = | BD | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 + 2)^2 + (3 - 3)^2}$

$= \sqrt{(10)^2 + (0)^2}$

$= \sqrt{100} = 10$ units ... (vi)

From (v) & (vi) BD = AC

i.e., its diagonal are also equal ... (D)

From (C) and (D)

□ ABCD is a rectangle **Hence proved.**

Now,

Area of rectangle = $l \times b$

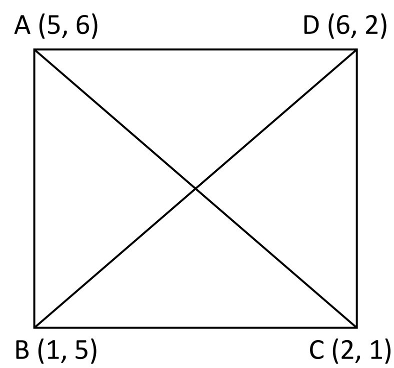
$= 4\sqrt{5} \times 2\sqrt{5}$

$= 8 \times 5 = 40$ units

i.e. Area of rectangle = 40 units

16. Show that the points A(5, 6); B(1, 5); C(2, 1) and D(6, 2) are the vertices of quad^r ABCD.

Sol. Let, the vertices of quad^r ABCD be A (5, 6), B (1, 5); C (2, 1) and D (6, 2)



Magnitude of AB = | AB | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 - 5)^2 + (5 - 6)^2}$

$= \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{16 + 1} = \sqrt{17}$ units ... (i)

Magnitude of CD = | CD | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - 1)^2 + (1 - 5)^2}$

$= \sqrt{(1)^2 + (-4)^2}$

$= \sqrt{1 + 16} = \sqrt{17}$ units ... (iii)

Magnitude of DA = | DA | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - 6)^2 + (6 - 2)^2}$

$= \sqrt{(-1)^2 + (4)^2}$

$= \sqrt{1 + 16} = \sqrt{17}$ units ... (iv)

From (i), (ii), (iii) and (iv)

AB = BC = CD = DA

i.e., its all sides are equal ... (A)

Now, Magnitude of AC = | AC | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - 5)^2 + (1 - 6)^2}$

$= \sqrt{(-3)^2 + (-5)^2}$

$= \sqrt{9 + 25} = \sqrt{34}$ units ... (v)

Magnitude of BD = | BD | $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - 1)^2 + (2 - 5)^2}$

$= \sqrt{(5)^2 + (-3)^2}$ $= \sqrt{25 + 9} = \sqrt{34}$ units ... (vi)

From (v) & (vi) Diagonal AC = Diagonal BD ... (B)

From (A) & (B) All sides of □ ABCD are equal & their diagonals are also equal.

i.e., Quadrilateral ABCD is a square. **Hence Proved**

17 Show that (-3, 2); (-5, -5); (2, -3) and (4, 4) are the vertices of a rhombus.

Sol. Let the vertices of quad^r ABCD of point A (-3, 2) BC (-5, -5), 6(2, -3), D (4, 4)

Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5 + 3)^2 + (-5 - 2)^2}$

$= \sqrt{(-2)^2 + (-7)^2}$

$= \sqrt{4 + 49} = \sqrt{53}$ units ... (i)

Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 + 5)^2 + (-3 + 5)^2}$

$= \sqrt{(7)^2 + (2)^2}$

$= \sqrt{49 + 4} = \sqrt{53}$ units ... (ii)

Magnitude of CD = $|CD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - 2)^2 + (4 + 3)^2}$

$= \sqrt{(2)^2 + (7)^2}$

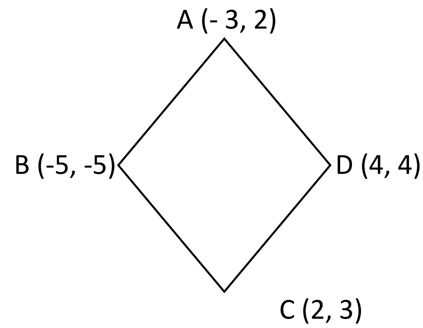
$= \sqrt{4 + 49} = \sqrt{53}$ units ... (iii)

Magnitude of DA = $|DA| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3 - 4)^2 + (2 - 4)^2}$

$= \sqrt{(-7)^2 + (-2)^2}$

$= \sqrt{49 + 4} = \sqrt{53}$ units ... (iv)



From (i), (ii), (iii) and (iv) AB = BC = CD = DA

i.e., its all sides of □ ABCD are equal.

∴ □ABCD is a rhombus.

18. (-4, -1); (-1, 1); (-1, -2) and (l, m) are the vertices of the square. Find *l*, m and length of the diagonal.

Sol. Let the vertices of ABCD of a point A (-4, -1), B (-1, -1), C, (-1, -2) and D (l, m)

Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1 + 4)^2 + (-2 + 1)^2}$

$= \sqrt{(3)^2 + (-1)^2}$

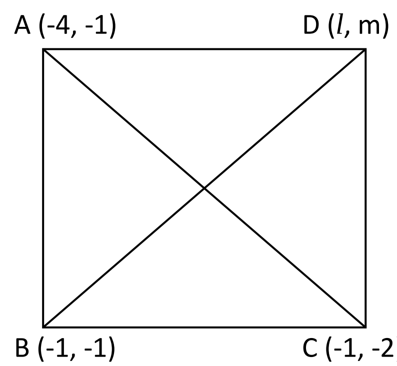
$= \sqrt{9 + 1} = \sqrt{10}$ units

Magnitude of BD = $|BD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{10} = \sqrt{(l + 1)^2 + (m + 1)^2}$

$\sqrt{10} = \sqrt{l^2 + 1 + 2l + m^2 + l + 2m}$

$10 - 2 = l^2 + m^2 + 2l + 2m$ ... (i)



Now,

AB = CD

Magnitude of AB = Magnitude of CD

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(-1 + 4)^2 + (-1 + 1)^2} \quad \sqrt{(l + 1)^2 + (m + 2)^2}$

$(+3)^2 + (0)^2 = (l + 1)^2 + (m + 2)^2$

$9 = l^2 + 1 + 2l + m^2 + 4 + 4m$

$9 - 5 = l^2 + m^2 + 4m + 2l$

$4 = l^2 + m^2 + 4m + 2l$ ... (ii)

Subtracting eq. (ii) from (i)

$8 - 4 = l^2 + m^2 + 2l + 2m - l^2 - m^2 - 4m - 2l$

$4 = -2m$

$m = -2$

Putting the value of m in eq. (ii)

$4 = l^2 + (-2)^2 + 4 \times -2 + 2l$

$4 = l^2 + 4 - 8 + 2l$

$8 = l^2 + 2l \quad \Rightarrow \quad l^2 + 2l - 8 = 0$

$l^2 + 4l - 2l - 8 = 0 \quad \Rightarrow \quad l(l + 4) - 2(l + 4) = 0$

$(l - 2)(l + 4) = 0$

Either, $\boxed{l = 2,}$ or $\boxed{l = -4}$

19. Find the radius of the circle if the end point of the diameter are (7, 5) and (-11, 3)

Sol. Magnitude of AC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-11 - 7)^2 + (3 - 5)^2}$

$= \sqrt{(18)^2 + (-2)^2}$

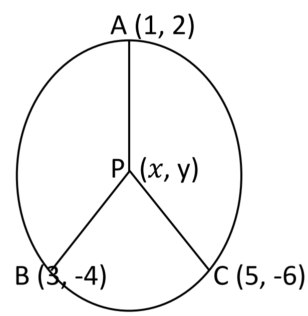
$= \sqrt{324 + 4} = \sqrt{328} = 2\sqrt{82}$ units

$\therefore$ Radius $= \frac{2\sqrt{8}}{2} = \sqrt{82}$

20. (1, 2); (3, -4); (5, -6) are three points on a circle. Find the co-ordinate of its centre.

Sol. Let P (x, y) be the radius of circle having points A (1, 2), B (3, -4) and C (5, -6)

$\therefore$ PA = PB = PC (Radii of same circle)



$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

From 1st & 2nd term

$\Rightarrow \sqrt{(1 - x)^2 + (2 - y)^2} = \sqrt{(3 - x)^2 + (-4 - y)^2}$

$\Rightarrow \sqrt{1 + x^2 - 2x + 4 + y^2 - 4y} = \sqrt{9 + x^2 - 6x + 16 + y^2 + 8y}$

$\Rightarrow x^2 + y^2 - 2x - 4y + 5 = x^2 + y^2 - 6x + 8y + 25$

$\Rightarrow -2x + 6x - 4y - 8y = 25 - 5$

$\Rightarrow 4x - 12y = 20$

$\Rightarrow 4(x - 3y) = 20$

$\Rightarrow x - 3y = 5 \quad \ldots$ (i)

From (ii) and last term

$\Rightarrow \sqrt{(3 - x)^2 + (-4 - y)^2} = \sqrt{(5 - x)^2 + (-6 - y)^2}$

$\Rightarrow 9 + x^2 - 6x + 16 + y^2 + 8y = 25 + x^2 - 10x + 36 + y^2 + 12y$

$\Rightarrow -6x + 8y = -10x + 12y + 36$

$\Rightarrow 4x - 4y = 36$

$\Rightarrow 4(x - y) = 36$

$\Rightarrow x - y = 9 \quad \ldots$ (ii)

Subtracting eq. (i) from (ii)

$y - y - x + 3y = 9 - 5 \quad \Rightarrow \quad +2y = 4$

$\therefore \boxed{y = 2}$

Putting the value of y in eq. (ii)

$y - 2 = 9$

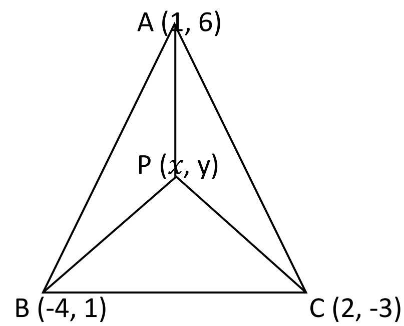
$y = 9 + 2 \quad \boxed{x = 11}$

21. Find the circumcentre of Δ form a (1, 6); (-4, 1) and (2, -3).

Sol. Let the vertices of Δ of a point a (1, 6), B (-4, 1), C (3, -3) PA = PB = PC

From 1st and 2nd term

Magnitude of PA = Magnitude of PB



$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{(x - 1)^2 + (y - 6)^2} = \sqrt{(x + 4)^2 + (y - 1)^2}$

$\Rightarrow x^2 + 1 + 2x + y^2 + 36 - 12y = x^2 + 16 + 8x + y^2 + 1 - 2y$

$\Rightarrow 8x + 2x - 2y + 12y = 36 - 16$

$\Rightarrow 10x + 14y = 20$

$\Rightarrow 10(x + 7y) = 20\,2 \quad \Rightarrow \quad x + y = 2 \quad \Rightarrow \quad x = 2 - y$

From 2nd and 3rd term

Magnitude of PB = Magnitude of PC

=> $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
=> $\sqrt{(x_2+4)^2+(y-1)^2} = \sqrt{(x-2)^2+(y+3)^2}$
=> $x^2+16+8x+y^2+1-2y = x^2+4+4x+y^2+9+6y$
=> $8x+4x-2y-6y = 13-17$ => $12x-8y = -4$ => $4(3x-2y) = -4$
=> $3x-2y+1 = 0$ => $3x-2y+1$ ... (i)

Putting the value of x in equation (i)

$3\times(2-y)-2y = -1$
$6-3y-2y = -1$ => $\cancel{-}5x = \cancel{-}7$ => $y = 7/5$
∴ $x = 2-7/5 = 3/5$

22. (-2, 2); (x, 8); (6, x) are three concyclic points whose centre is (2, 5). Find all possible value of x and y.

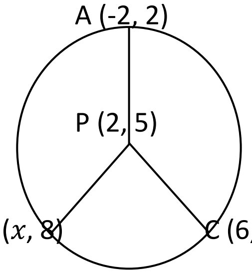
Sol. Let, B (x, 8), A (-2, 2) and C (6, y) are three concyclic point and P (2, 5) be the centre.

∴ PA = PB = PC [radii of the same circle]

From 1st and 2nd term

=> $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
=> $\sqrt{(-2-2)^2+(2-5)^2} = \sqrt{(x-2)^2+(8-5)^2}$
=> $\sqrt{(-4)^2+(-3)^2} = \sqrt{x^2+4-4x+(3)^2}$
=> $16+\cancel{9} = x^2-4x+\cancel{9}+4$
=> $x^2-4x+4-16 = 0$ => $x^2-4x-12 = 0$
=> $x^2-6x+2x-12 = 0$
=> $x(x-6)+2(x-6) = 0$
=> $(x-6)(x+2) = 0$

Either, $x = 6$ | or $x = -2$



10

From 1st and last term

=> $\sqrt{(-2-2)^2+(2-5)^2} = \sqrt{(6-2)^2+(y-5)^2}$
=> $\sqrt{(-4)^2+(-3)^2} = \sqrt{(4)^2+y^2+25-10y}$
=> $16+9 = \cancel{16}-y^2+25-10y$ => $y^2-10y+25-9 = 0$ => $y^2-10y+16 = 0$
=> $\cancel{y^2}-8y-2y+16 = 0$ => $y(y-8)-2(y-8) = 0$ => $(y-8)(y-2) = 0$

Either, $y = 8$ or $y = 2$

23. Find the co-ordinate of the circumcentre of a Δ whose vertices are (4, 6); (0, 4); (6, 2) Also, find its circumradius.

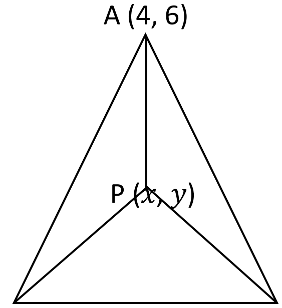
Sol. Let, A (4, 6); B (0, 4); C(6, 2) be the vertices of given Δ and P (x, y) be the circumcentre of ΔABC.

PA = PB = PC

From 1st & last term

Magnitude of PA = Magnitude of PB

=> $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
=> $\sqrt{(x-4)^2+(y-6)^2} = \sqrt{(x-0)^2+(y-4)^2}$
=> $x^2+\cancel{16}-8x+\cancel{y^2}+36-12y = \cancel{x^2}+\cancel{y^2}+16-8y$
=> $8x+12y-8y = 36$
=> $4(2x+y) = 36$
=> $2x+y = 9$ ... (i)



From 2nd & last term

Magnitude of PB = Magnitude of PC

=> $\cancel{\sqrt{}}(x_2-x_1)^2+(y_2-y_1)^2 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
=> $\sqrt{(x-0)^2+(y-4)^2} = \sqrt{(x-6)^2+(y-2)^2}$
=> $x^2+y^2+16-8y = x^2+36-12x+y^2+4-4y$
=> $12x-8y+4y = 36+4-16$ => $12x-4y = 24$
=> $3x-y = 6$ ... (ii)

Adding eq. (i) and (ii)

$2x+\cancel{y}+3x-\cancel{y} = 15$
$\cancel{5}x = \cancel{15}$ => $x = 3$

Putting x = 3 in eq (ii)

$3 \times 3 - y = 6$ => $-y = 6 - 9$

$\cancel{-}y = \cancel{-}3$ => $y = 3$ ∴ circumcentre of Δ (3, 3)

Now, Radius PA = PB = PC = Magnitude of PA

= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-3)^2 + (6-3)^2}$

= $\sqrt{(1)^2 + (3)^2} = \sqrt{1+9} = \sqrt{10}$ units.

24. Find the co-ordinate of the centre of circle passing through the point (2, 1); (5, 8); (2, -9) also, find the radius.

Sol. Let A (2, 1); B (5, 8); C (2, -9) be the point of a circle and P (x, y) be the centre.

PA = PB = PC [Radii of same circle)

From (i) & last term

Magnitude of PA = Magnitude of PB

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-5)^2 + (y-8)^2}$

=> $\cancel{x^2} + 4 - 4x + \cancel{y^2} + 1 - 2y = \cancel{x^2} + 25 - 10x + \cancel{y^2} + 64 + 16y$

=> $-4 + 10x - 2y - 16y = 25 + 64 - 5$

=> $6x - 18y = 84$

=> $\cancel{6}(x - 3y) = \cancel{84}\ 14$

=> $x - 3y = 14$ => $x = 14 + 3y$

From 2nd & last term

Magnitude of PB = Magnitude of PC

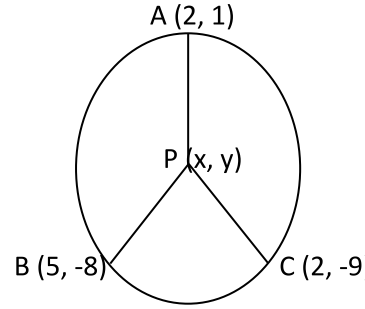
=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x-5)^2 + (y+8)^2} = \sqrt{(x-2)^2 + (y+9)^2}$

=> $\cancel{x^2} + 25 - 10x + \cancel{y^2} + 64 + 16y = \cancel{x^2} + 4 - 4x + \cancel{y^2} + 81 + 18y$

=> $10x - 4x - 16y + 18y = 25 + 64 - 85$ => $6x + 2y = 4$ => $2(2x + y) = 4$

=> $3x + y = 2$ ... (ii)



Putting the value of x in eq (ii)

=> $3 \times (14 + 3y) + y = 2$ => $42 + 9y + y = 2$ => $10y = 2 - 42$

=> $10y = -40$ => $Y = -4$

Putting y = -4 in eq (i)

=> $3x - 4 = 2$ => $3x = 6$ => $x = 2$ ∴ Centre of circle = (2, -4)

Now,

Radius = PA = PB = PC 11

Magnitude of PA = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(2-2)^2 + (1+4)^2}$ = $\sqrt{(0)^2 + (5)^2}$ = $\sqrt{25}$ = 5 units

25. If two vertices of an equilateral Δ be (0, 0) and (3, √3). Find the third vertex.

Sol. Here, A (3, √3), O (o, o) be the vertices of an equilateral Δ

Let, B (x, y) be the third vertex.

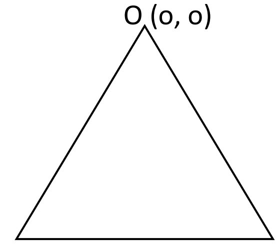
∴ Magnitude of OA = Magnitude of OB

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(3-0)^2 + (\sqrt{3}-0)^2} = \sqrt{(x-o)^2 + (y-o)^2}$

=> $\sqrt{9+3} = \sqrt{x^2 + y^2}$

=> $x^2 + y^2 = 12$ ... (i)



Now,

Magnitude of OB = Magnitude of AB

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x-o)^2 + (y-o)^2} = \sqrt{(3-x)^2 + (\sqrt{3}-y)^2}$

=> $\cancel{x^2} + y^2 = 9 + \cancel{x^2} - 6x + 3 + y^2 + 2\sqrt{3}y$

=> $6x + 2\sqrt{3}y = 12$

=> $\cancel{2}(3x + \sqrt{3}y) = 1\cancel{2}$

=> $3x + \sqrt{3}y = 6$

=> $\sqrt{3}y = 6 - 3$ => $y = \frac{6 - 3x}{\sqrt{3}}$ ... (ii)

Putting $y = \frac{6 - 3x}{\sqrt{3}}$ in eq. (i)

=> $x^2 + (6 - 3x/\sqrt{3})^2 = 12$

=> $x^2 + 36 + 9x^2 - 36x/3 = 12$

=> $3x^2 + \cancel{36} + 9x^2 - 36x = \cancel{36}$

=> $12x^2 - 36x = 0$

=> $12x\,(x - 3) = 0$ => $x - 3 = 0$ ∴ $x = 3$

Putting x = 3 in eq. (ii)

$y = 6 - 3x/\sqrt{3}$

$= 6 - 3 \times 3/\sqrt{3}$

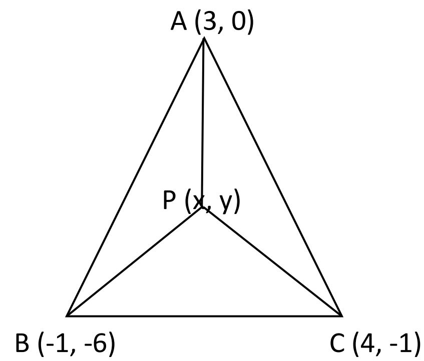
$= 6 - 9/\sqrt{3}$

$= -3/\sqrt{3}$

∴ Third vertices be $(3, -\sqrt{3})$..

26. Find the co-ordinate of the circum-centre of Δ ABC with vertices at A (3, 0); B (-1, -6) and C (4, -1). Find the circum radius.

Sol. Let, A (3, 0); B (-1, -6) and C (4, -1) are the vertices of a Δ ABC and P (x, y) be the centre.



PA = PB = PC

From 1st & 2nd term

Magnitude of PA = Magnitude of PB

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x - 3)^2 + (y - 0)^2} = \sqrt{(x + 1)^2 + (y + 6)^2}$

=> $x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$

=> 6x + 2x + 12y = 9 – 36 – 1

=> 8x + 12y = - 28

=> $\cancel{4}(2x + 3y) = -\cancel{28}\ 7$

=> 2x + 3y = - 7 ... (i)

Now,

Magnitude of PB = Magnitude of PC

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x + 1)^2 + (y + 6)^2} = \sqrt{(x - 3)^2 + (y - 0)^2}$

=> $\cancel{x^2} + \cancel{1} + 2x + \cancel{y^2} + 36 + 12y = \cancel{x^2} + 16 - 8x + \cancel{y^2} + \cancel{1} + 2y$

=> $2x + 8x + 12y - 2y = -36 + 16$ => $10x + 10y = -20$

=> $10\,(x + y) = -20$ => $x + y = -2$

=> $x = -2 - y$ ... (ii)

Putting x in eq (i)

=> $2\,(-2 - y) + 3y = -7$

=> $-4 - 2y + 3y = -7$

=> $y = -7 + 4$

=> $y = -3$

Putting y = -3 in eq. (ii)

$x = -2 + 3 = +1$

∴ circumcentre = (1, -3)

Now, Radius = PA = PB = PC = Magnitude of PA

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ => $\sqrt{(½ - 3)^2 + (-5/2 - 0)^2}$ => $\sqrt{(+1 + 3)^2 + (-3 - 0)^2}$

=> $\sqrt{(4)^2 + (-3)^2}$ => $\sqrt{6 + 9} = 5$ units

27. Find co-ordinate of the centre of the circle passing through the point (1, 2); (3, 4) and (5, -6). Also find the radius of this circle.

Sol. Let, A (1, 2); B (3, 4) and C (5, -6) be the vertices of circle and P (x, y) be the centre.

PA = PB = PC [Radii of some circle]

From 1st & 2nd term

Magnitude of PA = Magnitude of PB

=> $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

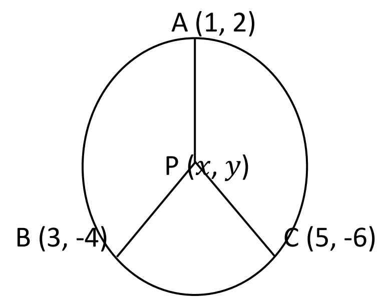
=> $\sqrt{(x-1)^2+(y-2)^2} = \sqrt{(-3+x)^2+(y+4)^2}$

=> $x^2+1-2x+y^2+4-4y = \cancel{x^2}+9-6x+\cancel{y^2}+16+8y$

=> $-2x+6x-4y-8y = 25-5$ => $4x-4y = 20$

=> $4(x-3y) = 20$

=> $x-3y = 5$ ... (i)



Now,

=> $\sqrt{(x-3)^2+(y+4)^2} = \sqrt{(x-5)^2+(y+6)^2}$

=> $x^2+9-6x+y^2+16+8y = \cancel{x^2}+25-10x+y^2+36+12y$

=> $\cancel{-6x}+10x+8\cancel{y}-12y = 61-25$ => $4x-4y = 36$ => $4(x-y) = 36$

=> $x-y = 9$ ... (ii)

Subtracting eq. (i) from (ii)

=> $\cancel{x}-y-\cancel{x}+3y = 9-5$

=> $2y = 4$

=> $y = 2$

Putting y = 2 in eq. (ii)

$x-2 = 9$

$x = 9+2$

$x = 11$

∴ Co-ordinate of P = (11, 2)

Now, Radius circle = PA = PB = PC

∴ Magnitude of PA = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(11-1)^2+(2-2)^2} = \sqrt{(10)^2+(0)^2} = \sqrt{100} = 10$ units

28. Find the co-ordinate of point whose abscissa is 10 and which is at the distance of 10 units from (2, -3).

Sol. Let the point be (10, y) and other point be (2, -3). Also, AB = 10 units

∴ Magnitude of AB = $|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$10 = \sqrt{(2-10)^2+(-3-y)^2}$ => $100 = (-8)^2+(-3-y)^2$

$100 = 64+9+y^2+6y$ => $y^2+6y = 100-73$

=> $y^2+6y-27$ => $y^2+9y-3y-17 = 0$ => $(y+9)-3(y+9) = 0$

$(y-3)(y+9) = 0$

Either, $y-3 = 0$ ∴ $\boxed{y = 3}$ or $y+9 = 0$ ∴ $\boxed{y = -9}$

29. Find the point on x – axis which is equidistance from (5, 4); (-2, 3).

Sol. Let, the point on x -axis be (x, 0)

∵ AB = AC

∴ Magnitude of AB = Magnitude of AC

=> $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

=> $\sqrt{(x-5)^2+(0-4)^2} = \sqrt{(x+2)^2+(0-3)^2}$

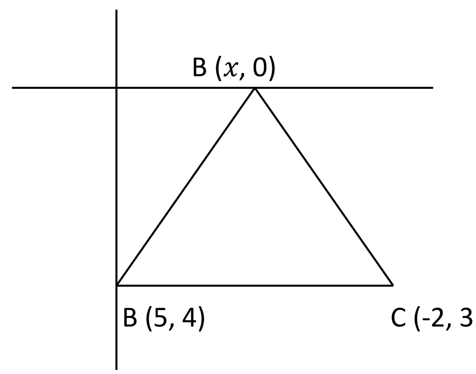
=> $x^2+25-10x+(-4)^2 = x^2+4+4x+(-3)^2$

=> $\cancel{x^2}+25-10x+16 = \cancel{x^2}+4+4x+9$

=> $10x+4x = 25+16-13$

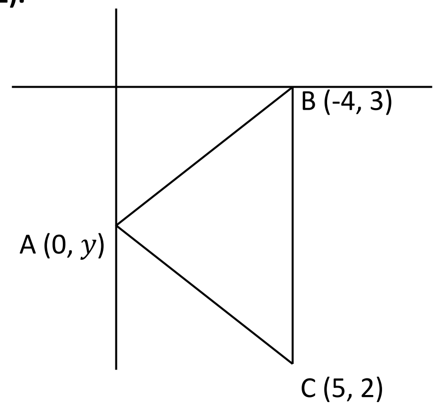
=> $14x = 28$ ∴ $x = 2$

∴ $\boxed{\text{Co-ordinate of A be (2, 0)}}$



30. Find the point on Y-axis which is equidistance from (-4, 3) and (5, 2).

Sol. Let, the point on Y-axis be (0, y)



∵ AB = AC

∴ Magnitude of AB = Magnitude of AC

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(0 + 4)^2 + (y - 3)^2} = \sqrt{(0 - 5)^2 + (y - 2)^2}$

=> $(4)^2 + y^2 + 9 - 6y = (-5)^2 + y^2 + 4 - 4y$

=> $16 + y^2 + 9 - 6y = 25 + y^2 + 4 - 4y$

=> $6y - 4y = -4$

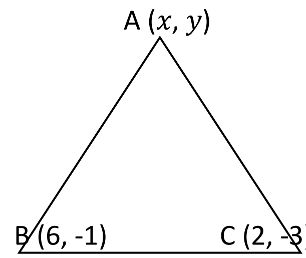
=> $2y = -4$

=> $y = -2$

∴ **Co-ordinate on Y-axis be (0, -2)**

31. If (x, y) is equidistance from (6, -1) and (2, -3). Find the relation between X and Y.

Sol. Let, A (x, y) be the point which is equidistance from B (6, -1) & C (2, -3).



∵ AB = AC

∴ Magnitude of AB =Magnitude of AC

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x - 6)^2 + (y + 1)^2} = \sqrt{(x - 2)^2 + (y + 3)^2}$

=> $x^2 + 36 - 12y + y^2 + 1 + 2y = x^2 + 4 - 4x + y^2 + 9 + 6y$

=> $12x - 4x - 2y + 6y = 37 - 13$

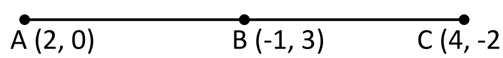
=> $8x + 4y = 24$ => $4(2x + y) = 24$

=> $2x + y = 6$

∴ **$2x + y - 6 = 0$**

32. i) Show that A (2, 0), B (-1, 3); C (4, -2) are collinear.

Sol. Let, A (2, 0), B (-1, 3); C (4, -2) are three point



∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 + 1)^2 + (0 - 3)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$ units

Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 + 1)^2 + (-2 - 3)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$ units

Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - 2)^2 + (-2 - 0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = 2\sqrt{2}$ units

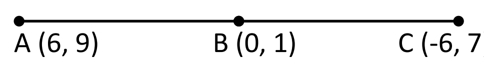
∴ BC = AB + AC

$5\sqrt{2} = 3\sqrt{2} + 2\sqrt{2}$

$5\sqrt{2} = 5\sqrt{2}$ ∴ Point A, B, C, D are collinear.

ii) Show that (6, 9); (0, 1) & C (-6, -7) are collinear.

Let, point A (6, 9); B (0, 1) & C (-6, -7) are three point.



∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - 0)^2 + (9 - 1)^2}$

$= \sqrt{36 + 64} = \sqrt{100} = 10$ units

Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 + 6)^2 + (1 + 7)^2}$

$= \sqrt{36 + 64} = \sqrt{100} = 10$ units

Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 + 6)^2 + (9 + 7)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$ units

∴ AC = AB + BC

20 = 10 + 10 => 20 = 20

∴ Point A, B & C are collinear.

(iii)Show that (-1, -1); (+2, +3) & (8, 11) are collinear.

Let, A (-1, -1); B (+2, +3) & C (8, 11)

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 + 1)^2 + (3 + 1)^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units

Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 2)^2 + (11 - 3)^2}$

$= \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ units

Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(11 + 1)^2 + (8 + 1)^2}$

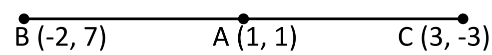
$= \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15$ units

∴ AC = AB + BC

15 = 5 + 10 = 15 ∴ Point AB & C are collinear.

(iv) Show that (1, 1); (-2, 7) & (3, -3) are collinear.

Let, A (1, 1); B (-2, 7) & C (3, -3) are three points.



∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 + 2)^2 + (1 - 7)^2}$

$= \sqrt{(3)^2 + (-6)^2}$

$= \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$ units

Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 + 2)^2 + (-3 - 7)^2}$

$= \sqrt{(5)^2 + (-10)^2}$

$= \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$ units

Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - 1)^2 + (-3 - 1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$ units

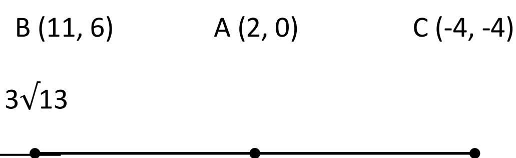
∴ BC = AB + AC

$5\sqrt{5} = 3\sqrt{5} + 2\sqrt{5}$

$5\sqrt{5} = 5\sqrt{5}$ ∴ Point A, B & C are collinear.

(v) Show that (2, 0); (11, 6) & (-4, -4) are collinear.

Let, A (2, 0); B (11, 6) & C (-4, -4) are three points.



∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(11 - 2)^2 + (6 - 0)^2}$

$= \sqrt{(9)^2 + (6)^2} = \sqrt{81 + 36} = \sqrt{117} = 3\sqrt{13}$

∴ Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 11)^2 + (-4 - 6)^2} = \sqrt{(-15)^2 + (-10)^2}$

$= \sqrt{225 + 100} = \sqrt{325} = 5\sqrt{13}$

∴ Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 2)^2 + (-4 - 0)^2} = \sqrt{(-6)^2 + (-4)^2}$

$= \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$ units

∴ Point A, B & C are collinear.

(vi) Show that (1, 0); (3, 5) and (6, 3) are collinear.

Let, A (1, 0); B (3, 5) and C (6, 9) are three points.

∴ Magnitude of AB = $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - 1)^2 + (5 - 0)^2} = \sqrt{(2)^2 + (5)^2} = \sqrt{4 + 25} = \sqrt{29}$

∴ Magnitude of BC = $|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - 3)^2 + (9 - 5)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units

∴ Magnitude of AC = $|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

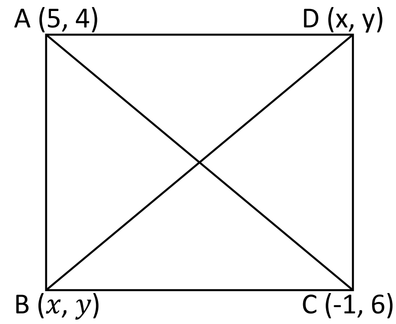
$= \sqrt{(6 - 1)^2 + (9 - 0)^2} = \sqrt{(5)^2 + (9)^2} = \sqrt{25 + 81} = \sqrt{106}$

∴ Point A, B & C are not collinear.

33. **If two vertices of square are (5, 4) and (-1, 6). Find the co-ordinate ordinate of remaining vertices.**

Sol. Let, the co-ordinate of vertices of □ ABCD be A (5, 4); C (-1, 6)

Also, let, the vertices of B = (x, y) & D (x, y)



Magnitude of BC = Magnitude of AB

=> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=> $\sqrt{(x + 1)^2 + (y - 6)^2} = \sqrt{(x - 5)^2 + (y - 4)^2}$

=> $\sqrt{x^2 + 1 + 2x + y^2 + 36 - 12y} = \sqrt{x^2 + 25 - 10x + y^2 + 16 - 8y}$

=> $\cancel{x^2} + 37 + \cancel{y^2} - 12y + 2x = \cancel{x^2} + \cancel{y^2} + 41 - 10x - 8y$

=> $10x + 2x - 12y + 8y = 41 - 37$

=> $12x - 4y = 4$

=> $3x - y = 1$ => $y = 3x - 1$

Now,

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

=> $(5 + 1)^2 + (4 - 6)^2 = (x - 5)^2 + (y - 4)^2 + (x + 1)^2 + (y - 6)^2$

=> $\cancel{36} + 4 = x^2 + 25 - 10x + y^2 + 16 - 8y + x^2 + 1 + 2x + y^2 + \cancel{36} - 12y$

=> $4 = 2x^2 + 2y^2 - 8x - 20y + 42$

=> $-38 = 2x^2 + 2(3x - 1)^2 - 8x - 20(3x - 1)$

=> $-38 = 2x^2 + 2[9x^2 + 1 - 6x] - 8x - 60x + 20$

=> $-58 = 2x^2 + 18x^2 + 2 - 12x - 8x - 60x$

=> $-60 = 20x^2 - 80x$

=> $-\cancel{60}\,3 = \cancel{20}(x^2 - 4x)$ => $-3 = x^2 - 4x$

=> $x^2 - 4x + 3 = 0$ => $x^2 - 3x - x + 3 = 0$

=> $(x - 3)(x - 1) = 0$

Either,		or,
$x - 3 = 0$	=> $x = 3$	$x - 1 = 0$
If $x = 3$	∴ $y = 3 - 1 = 2$	$x = 1$
∴ $y = 9 - 1 = 8$	

∴ Co-ordinate of B = (3, 8) or (1, 2)

<u>INTERNAL DIVISIONS</u>: A point R between P and Q on line PQ is said to divide PQ internally in the ratio of $m_1 = m_2$ if

$$\frac{PR}{RQ} = \frac{m_1}{m_2}$$

<u>FORMULA FOR INTERNAL DIVISION</u>:

To find co-ordinates of R this divides internally the straight line joining the given point in a given point in a given ratio.

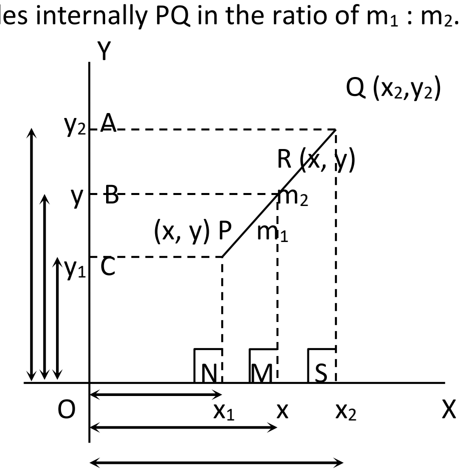
Let P (x_1, y_1) and Q (x_2, y_2) be the given points. Also let R (x, y) divides internally PQ in the ratio of $m_1 : m_2$.

We draw PN, RM and QS are ⊥ar on the same line OX.

∴ They are parallel to each other.

i.e., PN || RM || QS

Now, PN || RM || QS and PQ and NS are the transversal.



∴ $\frac{PR}{RQ} = \frac{NM}{MS}$

=> $m_1/m_2 = OM - ON/OS - ON$

=> $m_1/m_2 = x - x_1/x_2 - x$

=> $m_1 x_2 = m_1 x = m_2 x - m_2 x_1$

=> $m_2 x + m_1 x = m_1 x_2 + m_2 x_1$

=> $x(m_2 + m_1) = m_1 x_2 + m_2 x_1$

=> $\boxed{x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}}$

||ly, we can prove that $\boxed{y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}}$

Mid-Point Formula:

Corollary: Show that the mid – point M of a line segment with end point P(x, y), Q (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Let, M be the midpoint of the line-segment joining P (x, y) and Q (x_2, y_2). Here, M divides PQ in the ratio of 1:1

∴ Co – ordinate of mid-point M are..

=> $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

=> $\left(\frac{1 \times x_2 + 1 \times x_1}{1 + 1}, \frac{1 \times y_2 + 1 \times y_1}{1 + 1}\right)$

=> $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ **Hence proved**.

Let P (x, y) and Q (x_2, y_2) be the given points also,

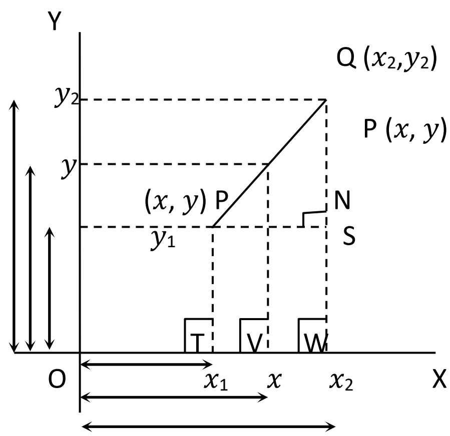
Let R (x, y) be the mid-point of line segment joining PQ.

∴ $m_1/m_2 = 1/1$

i.e., $m_1 : m_2$

Draw PT, RV and PW the ⊥ar to OX

∴ PT || RV || QW



Now,

In rt Δ PQN, RS || QN

∴ PR/RQ = PS/SN

=> m_1/m_2 = PS/SN

=> 1/1 = PS/SN

=> PS = SN

=> $x - x_1 = x_2 - x$

=> $x + x = x_2 + x_1$

=> $2x = x_1 + x_2$

=> $\boxed{x = \frac{x_1 + x_2}{2}}$

Again,

In rt. Δ PQN

$\frac{PR}{PQ} = \frac{RS}{QN}$ [∵ Δ PRS ~ Δ PQN (By AA similarity)]

=> $m_1/m_1 + m_2$ = RS/QN

=> $m_1/2m_1$ = RS/QN [∵ $m_1 = m_2$]

=> ½ = RS/QN

=> 2RS = QN

=> $2(y - y_1) = y_2 - y_1$ => $2y - 2y_1 = y_2 - y_1$

=> $2y = y_2 - y_1 + 2y_1$ => $2y = y_2 + y_1$

=> $\boxed{y = \frac{y_1 + y_2}{2}}$ **Hence proved**.

1. Calculate the co-ordinate of point 'P' which divides the line-segment joining.

(i) A (1, 3); B (5, 9) in the ratio of 1:2

Here, $x_1 = 1$; $x_2 = 5$ and $y_1 = 3$; $y_2 = 9$

Also, $m_1 = 1$, $m_2 = 2$

$x = m_1x_2 + m_2x_1/m_1 + m_2$

$= \frac{1 \times 5 + 2 \times 1}{1 + 2} = \frac{5 + 2}{3} = \frac{7}{3}$

$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 9 + 2 \times 3}{1 + 2} = \frac{9 + 6}{3} = \frac{15}{3} = 5$

∴ $\boxed{\text{Co-ordinate of P} = [7/3, 5]}$

(ii) A (-4, 6); B (3, -5) in the ratio of 3:2

Here $x_1 = -4$; $x_2 = 5$ and $y_1 = 6$; $y_2 = -5$

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

= $\left(\frac{3 \times 3 + 2 \times -4}{3 + 2}, \frac{3 \times -5 + 2 \times 6}{3 + 2}\right)$

= $\left(\frac{9 + -8}{5}, \frac{-15 + 12}{5}\right)$

= [1/5, -3/5]

∴ Co-ordinate of P = [1/5, -3/5]

(iii) A (0, 3); B (4, -1) in the ratio of 1:2

Here, $x_1 = 0$; $x_2 = 4$ and $y_1 = 3$; $y_2 = -1$

$x = \frac{m_1x_2 + m_2 x_1}{m_1 + m_2}$

$= \frac{1 \times 4 + 2 \times 0}{1 + 2}$

$= \frac{4 + 0}{3} = \frac{4}{3}$

$y = \frac{m_1y_2 + m_2 y_1}{m_1 + m_2}$

$= \frac{1 \times -1 + 2 \times 3}{1 + 2}$

$= \frac{-1 + 6}{3} = +\frac{5}{3}$

∴ Co-ordinate of P = [4/2, 5/3]

(iv) A (1, -3); B (-5, 9) in the ratio of 2:15

Here, $x_1 = 1$; $x_2 = -5$ and $y_1 = -3$; $y_2 = 9$

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

= $\left(\frac{2 \times -5 + 15 \times 1}{2 + 15}, \frac{2 \times 9 + 15 \times -3}{2 + 15}\right)$

= $\left(\frac{-10 + 15}{17}, \frac{18 - 45}{17}\right)$

= [5/17, 27/17]

∴ Co-ordinate of P = [5/17, 27/17]

(v) A (5, -2); B (1, 6) in the ratio of 3:1

Here, $x_1 = 5$; $x_2 = 9$ and $y_1 = -2$; $y_2 = 6$

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

= $\left(\frac{3 \times 9 + 1 \times 5}{3 + 1}, \frac{3 \times 6 + 1 \times -2}{3 + 1}\right)$

= $\left(\frac{27 + 5}{4}, \frac{18 - 2}{4}\right)$

= [32/4, 16/4]

= 8, 4

∴ Co-ordinate of P = [8, 4]

(vii) A (2, 1); B (5, 8) in the ratio of 1:2

Here, $x_1 = 2$; $x_2 = 5$ and $y_1 = 1$; $y_2 = 8$ also, $m_1 = 1$; $m_2 = 2$

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{1 \times 5 + 2 \times 2}{2 + 1}, \frac{1 \times 8 + 2 \times 1}{2 + 1}\right)$

$= \left(\frac{5 + 4}{3}, \frac{8 - 2}{3}\right)$

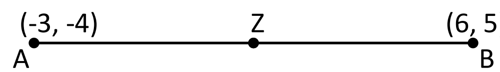
= [9/3, 10/3]

= [3, 10/3]

∴ Co-ordinate of P = [3, 10/4]

2. Let A (-3, -4) and B (6, 5) be two given points in a plane. Find the co-ordinate of the point which divides AB in the ratio of 5:4.

Sol. Here $x_1 = 3$; $x_2 = 6$ and $y_1 = -4$; $y_2 = 5$ and also, $m_1 = 5$; $m_2 = 4$



Co-ordinate of Z = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{5 \times 6 + 4 \times -3}{5 + 4}, \frac{5 \times 5 + 4 \times -4}{5 + 4}\right)$

$= \left(\frac{30 - 12}{9}, \frac{25 - 16}{9}\right)$

= [18/9, 9/9]

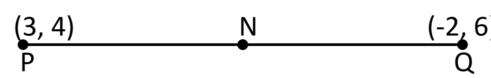
= [2, 1]

∴ Co-ordinate of Z = [2, 1]

3. Let P (3, -4) and Q (-2, 6) be two given point in a plane. Find the co-ordinate of the point which divides PQ in the ratio of 7:3.

Sol. Here, $x_1 = 3$; $x_2 = -2$ and $y_1 = -4$; $y_2 = 6$

Also, $m_1 = 7$; $m_2 = 3$



Co-ordinate of N = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{7 \times -2 + 3 \times -3}{7 + 3}, \frac{7 \times 6 + 3 \times -4}{7 + 3}\right)$

$= \left(\frac{-14 + 9}{10}, \frac{42 - 12}{10}\right)$

= [-5/10, 30/10]

= [1/2, 3]

∴ Co-ordinate of N = [-1/2, 3]

4. In what ratio is the line-segment joining (2, -3); (5, 6) divides by x – axis.

Sol. Let, x-axis divided P(2, -3) and B (5, 6) in ratio K:1

By section formula,

Co-ordinate of Z = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{5K + 2}{7 + 3}, \frac{6K - 3}{7 + 3}\right)$

Since Z lies on x-axis

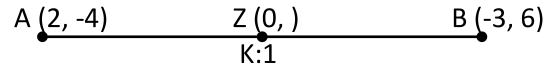
∴ $\frac{6K - 3}{K + 1} = 0$

=> $6K - 3 = 0$

=> $6K = 3$ => $K = ½$ ∴ Required ratio = [½, 1] = (1:2)

5. In what ratio is the line joining (2, -4) and (-3, 6) divided by y-axis.

Sol. Let y- axis divides A (2, -4) and (-3, 6) in ratio K:1



By section formula,

$$\text{Co-ordinate of Z} = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{-3K + 2}{K + 1}, \frac{6K - 4}{K + 1}\right)$$

Since Z lies on y-axis

∴ $\frac{-3K + 2}{K + 1} = 0$

=> $-3K + 2 = 0$

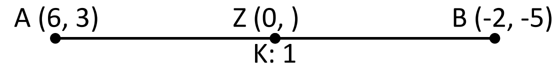
=> $3K = 2$

=> $K = 2/3$

∴ Required ratio = [2/3, 1]= (2:3)

6. In what ratio is the line-segment joining A (6, 3); B (-2, -5) divided by x-axis.

Sol. Let x-axis divides A (6, 3) & B (-2, -5) in ratio K:1



By section formula,

$$\text{Co-ordinate of Z} = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{-2K + 6}{K + 1}, \frac{-5K - 3}{K + 1}\right)$$

Since Z lies on x-axis

∴ $\frac{-5K + 3}{K + 1} = 0$

=> $-5K + 3 = 0$

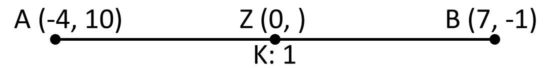
=> $5K = 3$

=> $5K = 3/5$

∴ Required ratio = [3/5, 1]= (3:5)

7. Find the ratio in which y-axis divides the joining of A (-4, 10); B (7, -1).

Sol. Let y-axis divides A (-4, 10); B (7, -1) in ratio K:1



By section formula,

$$\text{Co-ordinate of Z} = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{7K - 4}{K + 1}, \frac{-K - 10}{K + 1}\right)$$

Since Z lies on y-axis

∴ $\frac{7K - 3}{K + 1} = 0$

=> $7K - 4 = 0$

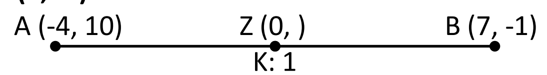
=> $7K = 4$

=> $K = 4/7$

∴ Required ratio = [4/7, 1]= (4:7)

8. In what ratio does the point (1, a) divides the join of (-1, 4) and (4, -1). Also find the value of a.

Sol. Let x-axis divides P (4, -1) & Q (4, -1) in ratio K:1



$$\text{Co-ordinate of Z} = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$= \left(\frac{4K - 1}{K + 1}, \frac{-K + 4}{K + 1}\right)$$

∴ $\frac{7K - 3}{K + 1} = 1$

=> 4K - 1 = K + 1 => 3K = 2 => K = 2/3

Putting K = 2/3

$\frac{-K + 4}{K + 1} = a$

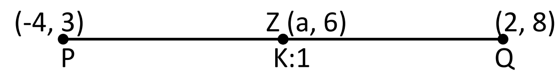
$\frac{-2/3 + 4}{2/3 + 1} = a$ => 2/3 + 4 = 2/3 a + a

=> -2 + 12/3 = 2a + 3a/3

=> $2\ \cancel{10}/\cancel{3} = \cancel{5}a/\cancel{3}$ =>a = 2

9. In what ratio does the point (a, 6) divides the join of (-4, 3) & (2, 8) also find the value of a.

Sol. Let x-axis divides P (-4, 3) & Q (2, 8) in ratio K:1



Co-ordinate of Z = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{2K - 4}{K + 1}, \frac{8K + 3}{K + 1}\right)$

∴ $\frac{8K + 3}{K + 1} = 6$

=> 8K + 3 = 6K + 6 => 2K = 3 => K = 3/2

Putting K = 3/2

$\frac{2K - 4}{K + 1} = a$

$\frac{2 \times 3/2 - 4}{3/2 + 1} = a$ => 6/2 – 4/3 + 2/2 = a

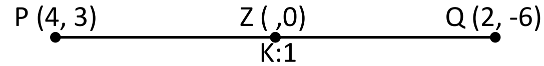
=> $\frac{6 - 8}{2/\frac{5}{2}} = a$

=> $\frac{2/\cancel{2}}{5/\cancel{2}} = a$ => -2 = 5a => a = -2/5

10. In what ratio the join of (4, 3) and (2, -6) divided by x-axis also find the co-ordinate of point of intersection.

Sol. Let x-axis divide P (4, 3) and Q (2, -6) in ratio K:1

By section formula,



Co-ordinate of Z = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{2K + 4}{K + 1}, \frac{-6K + 3}{K + 1}\right)$

Since Z lies on x-axis

∴ $\frac{-6K + 3}{K + 1} = 0$

=> -6K + 3 = 0 => K = ½

∴ Required ratio = [½; 1] = (1:2)

∴ Co-ordinate of Z = $\left(\frac{2K + 4}{K + 1}, 0\right)$

$= \left(\frac{2 \times ½ + 4}{½ + 1}, 0\right)$

$= \left(\frac{5 \times 2}{1 + 2}, 0\right)$

= [10/3, 0]

11. Find the ratio in which the join of (-4, 6) and (3, 0) divided by y-axis. Also find the co-ordinate of the point of the intersection.

Sol. Let y-axis divides P(-4, 6) & Q (3, 0) in ratio K:1

Co-ordinate of Z = $\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$= \left(\frac{3K - 4}{K + 1}, \frac{K \times 0 + 6}{K + 1}\right)$

Since Z lies on x-axis

∴ $\frac{3K - 4}{K + 1} = 0$

=> 3K - 4 = 0 => 3K = 4 => K = 4/3

∴ Required ratio = [4/3, 1] = (4:3)

∴ Co-ordinate of Z = $\left(0, \frac{0 + 6}{K + 1}\right)$

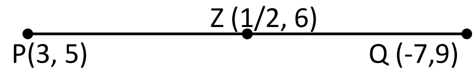
$= \left(0, \frac{6}{4/3 + 1}\right)$

$= \left(0, \frac{6}{4 + 3/3}\right)$

= [0, 18/7]

12. In what ratio does the point (½, 6) divides the line joining (3, 5) and (-7, 9).

Sol. Let P (3, 5) & Q (-7, 9) divides into K:1



Co-ordinate of Z = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{-7K + 3}{K + 1}, \frac{9K + 5}{K + 1}\right)$

=> $\frac{-7K + 3}{K + 1} = ½$

=> (-7K + 3)2 = K + 1

=> -14 K + 6 = K + 1

=> 15 K = 5

=> K = 1/3

$\frac{9K + 5}{K + 1} = 6$

9K + 5 = 6K + 6

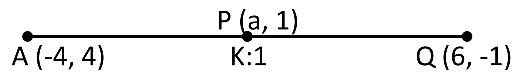
3K = 1

K = 1/3

∴ Required ratio = [1/3:1] = [1:3]

13. Find the ratio in which P (a, 1) divides the join of (-4, 4) and (6, -1) and hence, find the value of a.

Sol. Let divides the line segment N (-4, 4) and Q (6, -1) in ratio K:1



Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{6K - 4}{K + 1}, \frac{-K + 4}{K + 1}\right)$

=> $\frac{-K + 4}{K + 1} = 1$

=> -K + 4 = K + 1

=> 2K = 3 => $\boxed{K = 3/2}$

Putting K = 3/2

$\frac{6K - 4}{K + 1} = a$ => 6K – 4 = a K + a

=> **3** ~~6~~ × 3/~~2~~ – 4 = a × 3/2 + a

=> 9 – 4 = 3a + 2a / 2

=> ~~5~~ = ~~5~~a/2

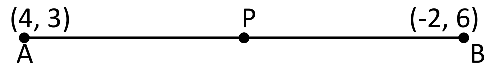
=> $\boxed{a = 2}$

14. P is a point on line-segment A (4, 3) and B (-2, 6) such that 5AP = 2BP. Find co-ordinate of P.

Sol. Since 5AP = 2BP

AP/BP = 2/5

∴ AP : BP = 2:5



Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{2 \times -2 + 5 \times 4}{2 + 5}, \frac{2 \times 6 + 5 \times 3}{2 + 5}\right)$

$= \left(\frac{-4 + 20}{7}, \frac{12 + 15}{7}\right)$

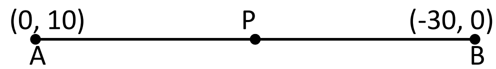
= [16/7, 27/7]

∴ Co-ordinate of P = [16/7, 27/7]

15. Given two fix point A (0, 10); B (-30, 0). Calculate the co-ordinate of P which lies on AB.

(i) 2AP = 3PB (ii) 3AP = AB

(iii) 7BP = AB



Sol. (i) 2AP = 3PB

AP/PB = 3/2

∴ AP / PB = 3:2

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{3 \times -30 + 2 \times 0}{3 + 2}, \frac{3 \times 0 + 2 \times 10}{3 + 2}\right)$

$= \left(\frac{-90}{5}, \frac{20}{5}\right)$

= [-18, 5]

∴ Co-ordinate of P = [-18, 5]

(ii) 3AP = AB => 3AP = AP + PB => 2AP = AP + PB

2AP = PB

AP/PB = ½ => AP:PB = 1:2

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\frac{1 \times -30 + 2 \times 0}{1 + 2}, \frac{1 \times 0 + 2 \times 10}{1 + 2}\right)$

$= \left(\frac{-30}{3}, \frac{20}{3}\right)$

= [-10, 20/3]

∴ Co-ordinate of P = [-10, 20/3]

(iii) 7BP = AB

7BP = AP + PB

6PB = AP

AP/PB = 6/16

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

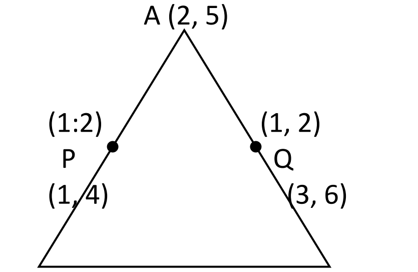
$= \left(\frac{6 \times -30 + 1 \times 0}{6 + 7}, \frac{6 \times 0 + 1 \times 10}{6 + 7}\right)$

$= \left(\frac{-180}{7}, \frac{10}{7}\right)$

= [-180/7, 10/7]

∴ Co-ordinate of P = [-180/7, 10/7]

16. **A (2, 5); B (-1, 2); C (5, 8) are the co-ordinate of the vertices of ΔABC. Point P & Q lies on AB & AC such that AP/BP = AQ/QC = 1/2**
1. Calculate the co-ordinate of P & Q
2. Show that PQ = 1/3 BC



(i) $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \dfrac{-1 + 4}{3} = 1$

$y = \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \dfrac{2 + 10}{3} = \dfrac{12}{3} = 4$

Co-ordinate of Q = $\left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\dfrac{5 + 4}{3}, \dfrac{8 + 10}{3}\right)$

$= \left(\dfrac{9}{3}, \dfrac{18}{3}\right)$

= [3, 6]

∴ Co-ordinate of Q = [3, 6]

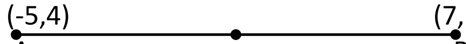
(ii) By distance formula.

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(3 - 1)^2 + (6 - 4)^2}$
$= \sqrt{4 + 4}$
$= 2\sqrt{2}$

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5 + 1)^2 + (8 - 2)^2}$
$= \sqrt{36 + 36}$
$= 6\sqrt{2}$

L.H.S. = PQ $= 2\sqrt{2}$
R.H.S. = 1/3 BC
$= 1/3 \times 6\sqrt{2} = 2\sqrt{2}$ ∴ L.H.S. = R.H.S. **Hence proved**

17. **Find the co-ordinate of mid-point of line-joining (-5, 4) & (7, 8).**



Sol. Mid-point of AB = $x = \dfrac{x_1 + x_2}{2}$

$\Rightarrow \dfrac{-5 + 7}{2} = 2/2 = 1$

And $y = \dfrac{y_1 + y_2}{2} = \dfrac{4 + 8}{2} = \dfrac{12}{2} = 6$ ∴ Co-ordinate of mid-point of AB = (1,6)

18. **Find the co-ordinate of point of tri-section of line segment joining (3, 2); (6, -7).**

Sol. Let P and Q trisect line-segment AB.

∴ AP/PB = ½ also, AQ/QB = 2/1

Co-ordinate of P = $\left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\dfrac{1 \times 6 + 2 \times 3}{1 + 2}, \dfrac{1 \times -7 + 2 \times 2}{1 + 2}\right)$

$\Rightarrow \left(\dfrac{6 + 6}{3}, \dfrac{-7 + 4}{3}\right)$ = [12/3, -3/3] => 4, -1

∴ Co-ordinate of P = [4, -1]

Co-ordinate of Q = $\left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$= \left(\dfrac{2 \times 6 + 1 \times 3}{2 + 1}, \dfrac{2 \times -7 + 1 \times 2}{2 + 1}\right)$

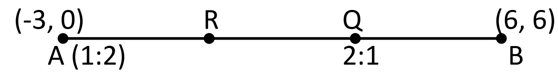
$\Rightarrow \left(\dfrac{12 + 3}{3}, \dfrac{-14 + 2}{3}\right)$ = [15/3, -12/3]

=> 5, -4

∴ Co-ordinate of Q = [5, -4]

19. Find the co-ordinate of point of trisection of line-segment A (-3, 0) and B (6, 6).

Sol. Let P and Q trisects line-segment AB



∴ AP/PB = ½ and AQ/QB = 2/1

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

= $\left(\frac{1 \times 6 + 2 \times -3}{1 + 2}, \frac{1 \times 6 + 2 \times 0}{1 + 2}\right)$

=> $\left(\frac{6 - 6}{3}, \frac{6}{3}\right)$ = [0, 2]

∴ **Co-ordinate of P = (0, 2)**

Also,

Co-ordinate of Q = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

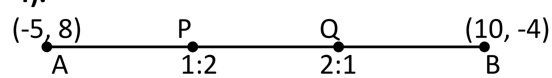
= $\left(\frac{2 \times 6 + 1 \times -3}{2 + 1}, \frac{2 \times 6 + 1 \times 0}{2 + 1}\right)$

=> $\left(\frac{12 - 3}{3}, \frac{12 + 0}{3}\right)$ = [3, 4]

∴ **Co-ordinate of Q = [3, 4]**

20. Find the co-ordinate of trisection of line segment A (-5, 8); B (10, -4).

Sol. Let P and Q tri divides the line-segment AB.



∴ AP/PB = ½ and AQ/QB = 2/1

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

= $\left(\frac{1 \times 10 + 2 \times -5}{1 + 2}, \frac{1 \times 4 + 2 \times 8}{1 + 2}\right)$

=> $\left(\frac{10 - 10}{3}, \frac{-4 + 16}{3}\right)$ = [0, -4]

∴ **Co-ordinate of P = [0, -4]**

Also,

Co-ordinate of Q = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

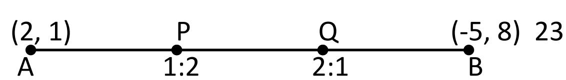
= $\left(\frac{2 \times 10 + 1 \times -5}{2 + 1}, \frac{2 \times -4 + 1 \times 8}{2 + 1}\right)$

=> $\left(\frac{20 - 5}{3}, \frac{-8 + 8}{3}\right)$ = [3, 0]

∴ **Co-ordinate of Q = [5, 0]**

21. Find the co-ordinate of trisection of line segment A (2, 1); B (-5, 8).

Sol. Let P and Q trisect the line segment AB.



∴ AP/PB = ½ and AQ/QB = 2/1

Co-ordinate of P = $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

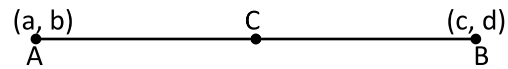
= $\left(\frac{1 \times -5 + 2 \times 2}{1 + 2}, \frac{1 \times 8 + 2 \times 1}{1 + 2}\right)$

=> $\left(\frac{-5 + 4}{3}, \frac{8 + 2}{3}\right)$ = [-1/3, 10/3]

∴ **Co-ordinate of P = [-1/3, 10/3]**

22. **'C' divides A (a, b) and B (c, d) in the ratio of 3:2. Find the co-ordinates of C.**

Sol. Here, $x_1 = a$, $x_2 = C$ and $y_1 = b$ & $y_2 = d$ $m_1 = 3$ and $m_2 = 2$



Co-ordinate of C = $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

= $\left(\dfrac{3 \times c + 2 \times a}{3 + 2}, \dfrac{3 \times d + 2 \times b}{3 + 2}\right)$

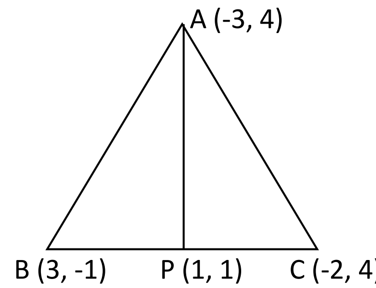
=> $\left(\dfrac{3c + 2a}{5}, \dfrac{3d + 2b}{5}\right)$

∴ **Co-ordinates of C =** $\left(\dfrac{3c + 2a}{5}, \dfrac{3d + 2b}{5}\right)$

23. **A (-3, 4) ; B (3, -1) and C (-2, 4) are the vertices of Δ ABC. Find the length of line segment AP where 'P' lies on BC such that BP/PC = 2/3.**

Sol. Since, BP/PC = 2/3

∴ BP : PC = 2 : 3



Co-ordinate of P = $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

= $\left(\dfrac{2 \times -2 + 3 \times 3}{2 + 3}, \dfrac{2 \times 4 + 3 \times -1}{2 + 3}\right)$

=> $\left(\dfrac{-4 + 9}{5}, \dfrac{8 - 3}{5}\right)$ = [1, 1]

Length of AP = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-3 - 1)^2 + (4 - 1)^2}$

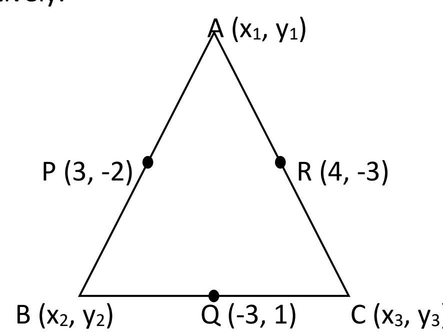
= $(-4)^2 + (3)^2$ = 5 units

Hence, Co-ordinate of P = [1, 1] And Length of AP = 4 units.

24. **If the co-ordinate of mid-point of side of A Δ be (3, -2), (-3, 1) and (4, -3). Find the co-ordinates of its vertex.**

Sol. Let A (3, -2); B (-3, 1) and C (4, -3) be the co-ordinate of the vertices of Δ ABC.

Also, let (3, -2); Q (-3, 1) & (4, -3) be the mid-point of AB, BC & AC respectively.



∴ $3 = \dfrac{x_1 + x_2}{2}$ => $x_1 + x_2 = 6$ ... (i) [By Mid-point]

$-2 = \dfrac{y_1 + y_2}{2}$ => $y_1 + y_2 = -4$ ... (ii) [By mid-point theorem]

And,

$-3 = \dfrac{x_2 + x_3}{2}$ => $x_2 + x_3 = -6$ ... (iii) [By mid-point theorem]

$1 = \dfrac{y_2 + y_3}{2}$ => $y_2 + y_3 = 2$ ... (iv) [same reason]

Also,

$4 = \dfrac{x_1 + x_3}{2}$ => $x_1 + x_3 = 8$ ... (v) [,, ,,]

And $-3 = \dfrac{y_1 + y_3}{2}$ => $y_1 + y_3 = -6$ ... (vi) [,, ,,]

Adding eq (i), (iii) & (v) ∴ $x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 6 - 6 + 8$

=> $2 (x_1 + x_2 + x_3) = 8$ => $x_1 + x_2 + x_3 = 4$

Since $x_1 + x_2 = 6$ [Proved above]

∴ $6 + x_3 = 4$ => $x_3 = -2$

also, $x_2 + x_3 = -6$ [Proved above]

∴ $-6 + x_1 = 4$ => $x_1 = 10$

And, $x_1 + x_3 = 8$ [Proved above]

∴ $8 + x_2 = 4$ => $x_2 = -4$

Now, adding eq (ii), (iv) & (vi)

∴ $y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = -4 + 2 - 6$ => $2(y_1 + y_2 + y_3) = -8$

=> $y_1 + y_2 + y_3 = -8$

Since, $y_1 + y_2 = -4$ ∴ $-4 + y_3 = -4$

$y_3 = -0$

Also, $y_2 + y_3 = 2$ ∴ $2 + y_1 = -4$

$y_1 = -6$

And, $y_1 + y_3 = 6$ [Proved above] ∴ $-6 + y_2 = -4$

$y_2 = +2$

∴ Co-ordinate of A = (10, -6) Co-ordinate of B = (-4, 2) Co-ordinate of C = (-2, 0)

25. The co-ordinates of one end point of a diameter of a circle are (3, 5). If the co-ordinate of the centre is (6, 6), Find the co-ordinate of the other end of the diameter.

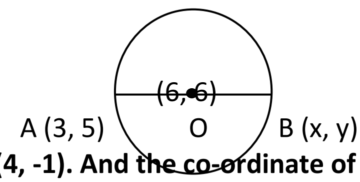
Sol. Let A (3, 5) be the co-ordinate of one end of circle and (6, 6) be the co-ordinate of the centre of circle and also B (x, y) be the co-ordinate of the centre.

∴ $6 = \frac{3 + x}{2}$ [By Mid-point theorem] => $3 + x = 12$ => $x = 9$

Also, $6 = \frac{5 + y}{2}$

=> $5 + y = 12$ => $y = 7$

∴ Co-ordinate of other end of circle = (9, 7)



26. The co-ordinate of one point of a diameter of a circle are (4, -1). And the co-ordinate of the centre is (1, -3). Find the co-ordinate of the other end of the diameter.

Sol. Let, A (4, -1) and B (x, y) and B (x, y) be the co-ordinate of the end of the diameter of a circle.

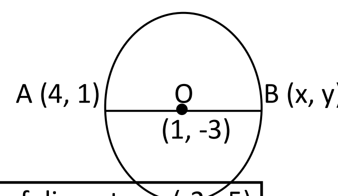
Also, 0(1, -3) be the co-ordinate of its centre.

∴ $1 = \frac{4 + x}{2}$ [By mid-point theorem]

=> $4 + x = 2$ => $x = -2$

And $-3 = \frac{-1 + y}{2}$ [By mid-point theorem]

=> $-6 = -1 + y$ => $y = -5$ ∴ Co-ordinate of the other end of diameter = (-2, -5)



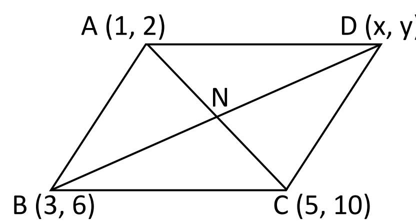
27. Three vertices of a parallelogram taken in order are (1, -2); (3, -6) & (5, 10). Find the co-ordinate of fourth vertices.

Sol. Let A (1, -2); B (3, -6), C (5, 10) & D (x, y) be the vertices of ||gm ABCD.

Also, AC & BD are two diagonals.

∴ They bisect each other.

i.e., N is the mid-point of AC & BD



∴ Co-ordinate of N = $\left(\frac{1 + 5}{2}, \frac{-2 + 10}{2}\right)$

= [6/2, 8/2] = (3, 4)

Also, Co-ordinator of N = $\left(\frac{3 + x}{2}, \frac{6 + y}{2}\right)$

∴ $\frac{3 + x}{2} = 3$ | and, $6 + y/2 = 4$

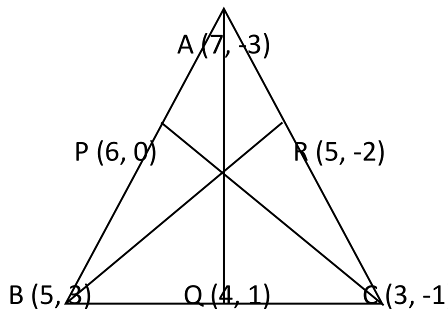
=> $3 + x = 6$ | => $6 + y = 8$

=> $x = 3$ | ∴ $y = 2$

∴ Co-ordinate of fourth vertex i.e, D = (3, 2)

28. Find the length of the medium of Δ ABC whose vertices are A (7, -3); B (5, 3); C (3, 1).

Sol. Let A (7, -3); B (5, 3) & C (3, -1) be the co-ordinate of vertices of Δ ABC



∴ Mid-point of AB = x = 7 + 5/2 = 12/12 = 6
=> Y = -3 + 3/2 = 0/2 = 0
∴ Mid-point of AB = (6, 0)
And, Mid-point of BC = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [5 + 3/2, 3 – ½] = [8/2, 2/2] = [4, 1]
∴ Mid-point of BC = (4, 1)
And, Mid-point of AC = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [7 + 3/2, -3 – ½] = [10/2, -4/2] = [5, 2]
∴ Mid-point of AC = (5, -2)
Now,

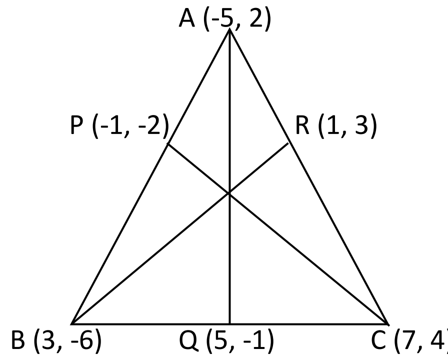
Length of AQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(7-4)^2 + (-3-1)^2}$
$= \sqrt{(3)^2 + (-4)^2} = \sqrt{5}$ units.

Length of PC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(7-4)^2 + (0+1)^2}$
$= \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$ units.

Length of BR = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(5-5)^2 + (3+2)^2}$
$= \sqrt{(0)^2 + (5)^2} = \sqrt{5}$ units.

29. (-5, 2); (3, -6) and (7, 4) are the vertices of Δ ABC. Find the length of its all medium.

Sol. Let A (-5, 2); B (3, -6) and C (7, 4) are the vertices of Δ ABC.
Also, P, Q & R be its medium



∴ Mid-point of AB = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [- 5 + 3/2, 2 – 6/2] = [-2/2, -4/2] = (-1, -2)
∴ Mid-point of AC = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [-5 + 7/2, 2 + 4/2] = [2/2, 6/2] = (1, 3)
& Mid-point of BC = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [3 + 7/2, -6 + 4/2] = [10/2, -2/2] = (5, -1)

Now,

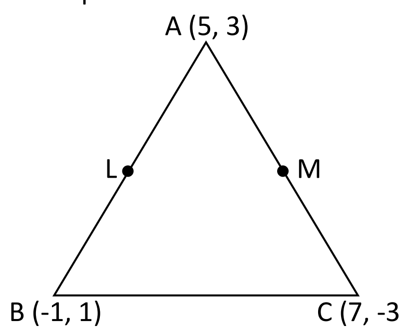
Length of AQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(-5-5)^2 + (2+1)^2}$
$= \sqrt{(-10)^2 + (3)^2} = \sqrt{100+9} = \sqrt{109}$ units.

Length of PC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(7+1)^2 + (4+2)^2}$ $= \sqrt{(8)^2 + (6)^2} = \sqrt{48+36} = \sqrt{84}$ units.

Length of BR = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(3-1)^2 + (-6-3)^2}$ $= \sqrt{(2)^2 + (-9)^2} = \sqrt{85}$ units.

30. A (5, 3); B (-1, 1) & C (7, -3) are the vertices of ∆ ABC. If ∠ is the mid-point of AB & M is the mid-point of AC prove that ∠M = ½ BC.

Sol. Let, A (5, 3); B (-1, 1) & C (7, -3) be the vertices of ∆ ABC and L & M be the mid-point.



∴ Mid-point of L = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [5 – 1/2, 3 + 1/2] = [4/2, 4/2] = (2, 2)
Midpoint of M = $[x_1 + x_2/2, y_1 + y_2/2]$
=> [5 + 7/2, 3 – 3/2] = (6, 0)

By Distance formula,
LM = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6-2)^2 + (0-2)^2}$
$= \sqrt{(4)^2 + (-2)^2}$
$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ units.

By Distance formula,
BC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(7+1)^2 + (-3-1)^2}$
$= \sqrt{64+16} = \sqrt{80}$
$= 4\sqrt{5}$ units.

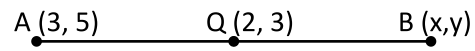
Now,

L.H.S. = LM = $2\sqrt{5}$ units

R.H.S. = ½ BC = 1/~~2~~ × **2** ~~4~~$\sqrt{5}$ = $2\sqrt{5}$ units. ∴ L.H.S. = R.H.S. **Hence proved**

31. Point A and B have the co-ordinates (3, 5) and (x, y) resp. The mid-point of AB is (2, 3). Find the value of x and y.

Sol. Let A (3, 5) and B (x, y) be the co-ordinate of AB also P (2, 3) be the co-ordinate of mid-point of AB.



∴ Co-ordinate of P = [3 + x/2, 5 + y/2]

∴ 3 + x/2 = 2 | Also, 5 + y/2 = 3

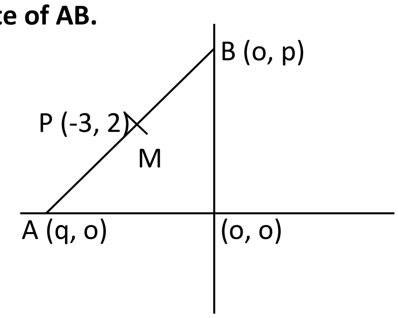
=> 3 + x = 4 | => 5 + y = 6

=> x = 1 | => y = 1

∴ Value of x = 1 & y = 1 i.e., Co-ordinate of B = (x, y) = (1, 1)

32. P (-3, 2) lies on the mid-point of AB as show in the fig. Find the co-ordinate of AB.

Sol. Let A (q, o) and B (o, p) are the co-ordinate of AB.



Co-ordinate of M = [q + 0/2, o + p/2]

∴ q + o/2 = -3 | Also, o + p/2 = 2

=> q + 0 = -6 | o + p = 4

=> q = -6 | => p = 4

∴ Co-ordinate of B = (0, 4)

& Co-ordinate of A = (0, -6)

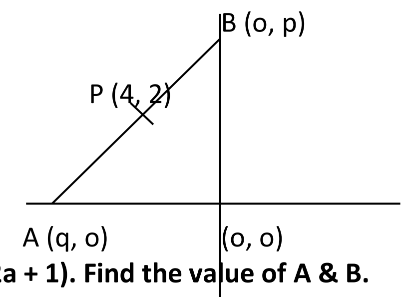
33. In the following fig P (4, 2) is the point on the line AB. Find the co-ordinate of A & B.

Sol. Let P (4, 2) lies on AB also P be the mid-point of A (q, o) & B (o, p) also O be the origin.

∴ By mid-point relation

Co-ordinate of q_1 = q + o/2 => 4 + q + o/2 => q = 8

Co-ordinate of A = (q, o) = (8, 0)



Co-ordinate of P = P + o/2 => 2 – P + o/2 => P= 4

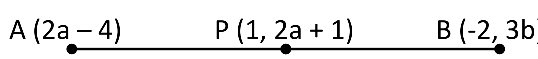
∴ Co-ordinate of B = (O, P)

= (O, 4)

34. The mid-point of the line AB joining (29, 4) and (-2, 3b) is (1, 2a + 1). Find the value of A & B.

Sol. Let A (29, 4); B (-2, 3b) and P (1, 2a + 1) be the co-ordinate of A, B, P resp.

∴ Co-ordinate of P = [2a – 2/2, 4 + 3b/2]



∴ 2a – 2/2 = 1 | Also, 4 + 3b/2 = 2a + 1

=> 2a – 2 = 2 | => 4 + 3b = 4a + 2

=> 2a = 4 | => 3b = 4 × 2 – 2

=> a = 2 | => 3b = 8 – 2

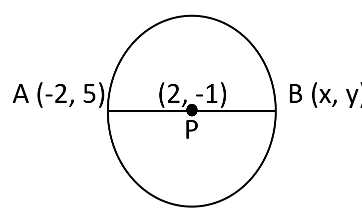
| => 3b = 6 => b = 2

∴ Co-ordinate of A (2 × 2, 4) = (4, 4) & Co-ordinate of B (-2, 3b) = (-2, 6)

35. One end of diameter of a circle is (-2, 5). Find the co-ordinate of the other end if the centre of the circle is (2, -1).

Sol. Let A (-2, 5) and B (x, y) be the co-ordinate of diameter AB and P (2, -1) be the co-ordinate of its centre.

∴ Co-ordinate of P = [-2 + x/2, 5 + y/2]



∴ -2 + x/2 = 2 | Also, 5 + y/2 = -1

=> -2 + x = 4 | => 5 + y = -2

=> x = 6 | => y = -7

∴ Co-ordinate of B = (6, -7)

36. A (-1, 0), B (1, 3), D (3, 5) are the vertices of parallelogram ABCD. Find the co-ordinate of the vertex 'C'.

Sol. A (-1, 0); B (1, 3) and D (3, 5) also C (x, y) be the co-ordinate of vertices of ||gm ABCD.

∴ Mid-point of BD = [3 + 1/2, 5 + 3/2]
= [4/2, 8/2]
= (2, 4)

Also,

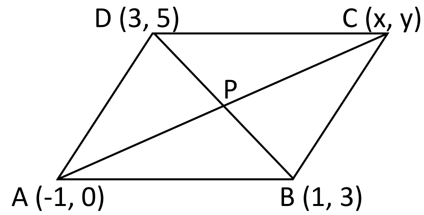
Co-ordinate of P = [-1 + x/2, 0 + y/2]

∴ -1 + x/2 = 2 Also, 0 + y/2 = 4

=> -1 + x = 4 => y = 8

=> x = 5

∴ Co-ordinate of C = (5, 8)



37. Given FB = BC = CD, B (0, 5) and C (1, 8). Find the co-ordinate of A & D.

Sol. Let A (x, y); B (0, 3); C (1, 8) & D (x, y) be the co-ordinate o line ABCD.

Since AB = BC

∴ B is the mid-point of AC

∴ Co-ordinate of B = [x + 1/2, y + 8/2]

=> x + 1/2 = 0 | Also, y + 8/2 = 3

=> x = -1 | => y = 6 – 8 => y = -2

∴ Co-ordinate of A = (-1, -2)

Now,

BC = CD

∴ C is the mid-point of BD

∴ Co-ordinate of C = [0 + x_1/2, 3 + y_1/2]

=> 0 + x_1/2 = 1 3 + y_1/2 = 8

=> 0 + x_1 = 2 3 + y_1 = 13

∴ x_1 = 2 y_1 = 13

∴ Co-ordinate of D (2, 13)

38. P (4, 2) ; q (-1, 5) are the vertices of ||gm PQRS. And (-3, 2) are the co-ordinate of point of intersection of its diagonal. Find the co-ordinate of R & S.

Sol. Let P (4, 2) ; q (-1, 5), R (x, y) & S (x, y_1) be the co-ordinate of ||PQRS.

Also, A (-3, 2) be the co-ordinate of point of intersection of its diagonal.

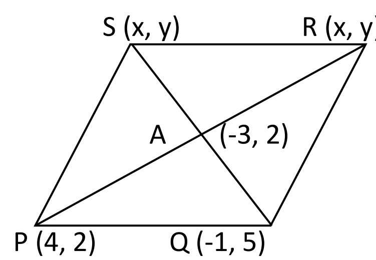
∴ Co-ordinate of A = [4 + x/2, 2 + y/2]

∴ 4 + x/2 = -3 | Also, 5 + y/2 = 2

=> 4 + x = -6 | => 5 + y = 4

=> x = -2 | => y = -1

∴ Co-ordinate of S = (-5, -1)



27

CENTROID

1. Find the co-ordinate of centroid of ΔABC whose vertices are A (-3, 0); B (5, -2) & (-8, 5).

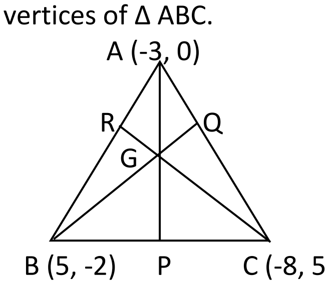
Sol. Let A (-3, 0); B (5, -2) & C (-8, 5) be the co-ordinate of its vertices of Δ ABC.

∴ Co-ordinate of centroid G =

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{-3 + 5 - 8}{3}, \frac{0 - 2 + 5}{3}\right)$$

= [-6/3, 3/3] = (-2, 1)



2. Find the co-ordinate of centroid of ΔABC whose vertices are A (6, -2); B (4, -3) & (-1, -4).

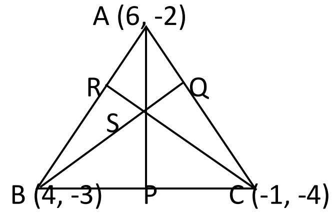
Sol. Let A (6, -2); B (4, -3) & (-1, -4) be the co-ordinate of its vertices of Δ ABC.

∴ Co-ordinate of centroid G =

$= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\dfrac{6 + 4 - 1}{3}, \dfrac{-2 - 3 - 4}{3}\right)$

= [9/3, -9/3] = (3, -3)



3. Find the co-ordinate of centroid of ΔABC whose vertices are A (-7, 5); B (-1, -3) & (5, 7).

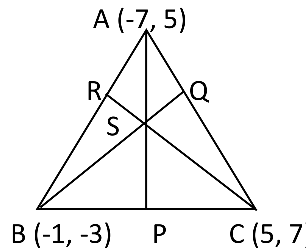
Sol. Let A (-7, 5); B (-1, -3) & (5, 7) be the co-ordinate of its vertices of Δ ABC.

∴ Co-ordinate of centroid G =

$= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\dfrac{-7 - 1 + 5}{3}, \dfrac{5 - 3 + 7}{3}\right)$

= [-3/3, 9/3] = (-1, 3)



4. If G (-2, 1) is the centroid of Δ ABC and two of its vertices are A (-1, 6) & B (-5, 2). Find the 3rd vertices of Δ.

Sol. Let A (-1, 6); B (-5, 2) & C (x, y) be the co-ordinate of vertices of Δ ABC also G (-2, 1) be its centroid.

∴ Co-ordinate of its centroid

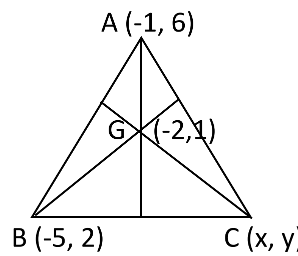
$= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

∴ $\dfrac{-1 - 5 + x}{3} = -2$ | Also, $\dfrac{6 + 2 + y}{3} = 1$

=> -6 + x = -6 | => 8 + y = 3

=> x = 0 | => y = -5

∴ Co-ordinate of C = (0, -5)



5. Two vertices of Δ are (-5, 4) & (3, 7). If its centroid is (1,-2), Find 3rd vertex.

Sol. Let A (-5, 4); B (3, 7) & C (x, y) be the co-ordinate of vertices of Δ ABC.

Also, G (1, -2) be its centroid

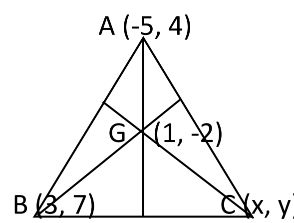
∴ Co-ordinate of its centroid = $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

∴ -5 + 3 + x/3 = 1 | => 4 + 7 + y/3 = -2

=> -2 + x = 3 | => 11 + y = -6

=> x = 5 | => y = -17

∴ Co-ordinate of C = (-5, -17)



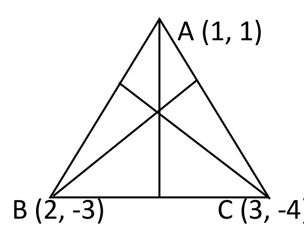
6. P (6, 3); Q (-2, 5), R (-1, 7), Find centroid.

Sol. Let P (6, 3); Q (-2, 5) & R (-1, 7) be the co-ordinate of vertices of Δ PQR.

∴ Co-ordinate of centroid G = $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

= [6 + -2 – 1/3, 3 + 5 + 7/3]

= [3/3, 15/3] = (1, 5)



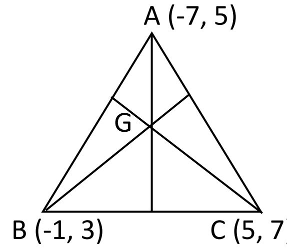
7. **A (-7, 5); B (-1, 3); C (5, 7), Find the centroid.**

Sol. Let A (-7, 5); B (-1, 3) and C (5, 7) be the vertices of Δ ABC

∴ Co-ordinate of centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

= [-7 – 1 + 5/3, 5 + 3 + 7/3]

= [-3/3, 15/3] = (-1, 5)



8. **The co-ordinate of vertices of Δ are (4, -3); (-5, 2) and (x, y). If the centroid of Δ is at the origin show that x = y = 1**

Sol. Let A (4, -3); B (-5, 2) and C (x, y) be the co-ordinate of the vertices of Δ ABC and let (0, 0) be the co-ordinate of centroid.

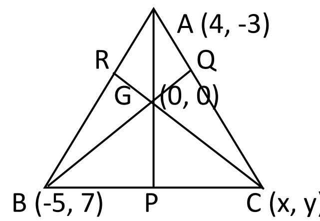
Co-ordinate of G = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

∴ $\frac{x_1 + x_2 + x_3}{3} = 0$ => 4 – 5 + x = 0

=> **x = 1**

And $\frac{y_1 + y_2 + y_3}{3} = 0$

∴ Co-ordinate of C = (1, 1) i.e., x = 1, y = 1



◘ IN CENTRE

1. **Find the co-ordinate of in centre of Δ whose vertices are (-8, 0); (7, -36) & (7, 20).**

Sol. Let (x_1, y_1); B (x_2, y_2) & C (x_2, y_3) are the co-ordinate of vertex of ΔABC.

Magnitude of a = |BC| = $\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$

= $\sqrt{(7 - 7)^2 + (-36 - 20)^2}$

= $\sqrt{(-56)^2}$

= $\sqrt{56 \times 56}$ = 56 units

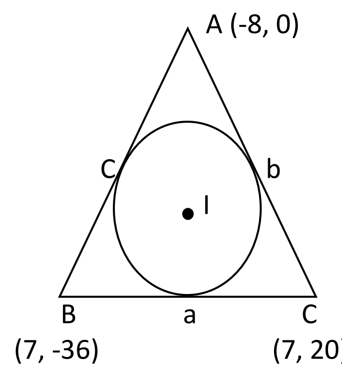
Magnitude of b = |AC| = $\sqrt{(x_3 - x_1)^2 + (x_3 - x_1)^2}$

= $\sqrt{(7 + 8)^2 + (20 - 0)^2}$

= $\sqrt{(15)^2 + (20)^2}$

= 25 units.

Magnitude of C = |AB| = $\sqrt{(7 + 8)^2 + (-36 - 0)^2}$ = 39 units.



∴ Co-ordinate of Incentre of I = $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

= $\left(\frac{56 \times -8 + 25 \times 7 + 39 \times 7}{56 + 39 + 25}, \frac{56 \times 0 + -36 \times 25 + 20 \times 39}{56 + 39 + 25}\right)$ = [0, -120/120] = [0, -1]

2. **If the co-ordinate of the mid-point of side of the Δ (1, 1), (2, -3) and (3, 4), Find its incentre.**

Sol. Let A (x_1, y_1); B (x_2, y_2) & C (x_3, y_3) are the co-ordinate of vertices of Δ ABC.

Also, C (1, 1); b (3, 4) & a (2, -3) are co-ordinate of mid-point of side of Δ.

∴ Co-ordinate of C = $[x_1 + x_2/2, y_1 + y_2/2]$

∴ $x_1 + x_2/2 = 1$ | Also, $y_1 + y_2/2 = 1$

∴ $x_1 + x_2 = 2$ ... (i) | ∴ $y_1 + y_2 = 2$ ... (ii)

Co-ordinate of a = $[x_2 + x_3/2, y_2 + y_3/2]$

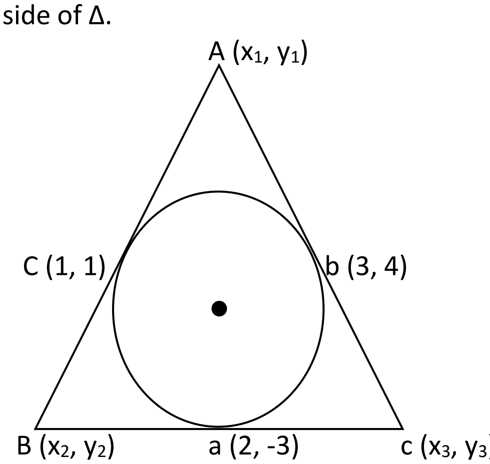
∴ $x_2 + x_3/2 = 2$ | Also, $y_2 + y_3/2 = -3$

i.e., $x_2 + x_3 = 4$ ... (iii) | i.e., $y_2 + y_3 = -6$ ... (iv)

Co-ordinate of b = $[x_2 + x_1/2, y_1 + y_3/2]$

∴ $x_1 + x_3/2 = 3$ | Also, $y_1 + y_3/2 = 4$

i.e., $x_1 + x_3 = 6$ ... (v) | i.e., $y_1 + y_3 = 8$ ... (vi)



Adding eq (i), (iii) & (v), we get,

=> $x_1 + x_2 + x_2 + x_3 + x_3 + x_1 = 2 + 4 + 6$

=> $2(x_1 + x_2 + x_3) = 12$

=> $x_1 + x_2 + x_3 = 6$

But $x_2 + x_3 = 4$ [from (iii)] | Also, $x_1 + x_2 = 2$ [from (i)]

∴ $x1 + 4 = 6$ | ∴ $x_3 + 2 = 6$

i.e., $x_1 = 2$ | i.e., $x_3 = 4$

And, $x_3 + x_1 = 6$ [from (v)]

∴ $x_2 = 6 – 6$ i.e., $x_2 = 0$

Now, adding eq (ii), (iv) & (vi), we get

=> $y_1 + y_2 + y_3 + y_1 + y_2 + y_3 = 2 – 6 + 8$

=> $2 (y_1 + y_2 + y_3)= 4$

=> $y_1 + y_2 + y_3 = 2$

But, $y1 + y2 = 2$ [from (ii)] | Also, $y_2 + y_3 = -6$ [from (iv)]

i.e., $y_3 = 0$ | i.e., $y_1 = 8$

And, $y_3 + y_1 = 8$ [from (vi)]

$y_2 + 8 = 2$

$y_2 = -6$

Magnitude of a = |BC| = $\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$ = $\sqrt{(10 - 4)^2 + (-6 - 0)^2}$= $\sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$ units

Magnitude of b = |AC| = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

= $\sqrt{(4 - 2)^2 + (0 - 8)^2}$= $\sqrt{(2)^2 + (-8)^2}$ = $\sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$ units

Magnitude of c = |AB| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = $\sqrt{(0 - 2)^2 + (-6 - 8)^2}$

= $\sqrt{(-2)^2 + (-14)^2}$ = $\sqrt{4 + 196} = \sqrt{200} = 10\sqrt{2}$ units

∴ Co-ordinate of Incentre of I = $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

= $\left(\frac{2\sqrt{3} \times 2 + 2\sqrt{17} \times 0 + 10\sqrt{2} \times 4}{2\sqrt{3} + 2\sqrt{17} + 10\sqrt{2}}, \frac{2\sqrt{3} \times 8 + 2\sqrt{17} \times -6 + 10\sqrt{2} \times 0}{2\sqrt{3} + 2\sqrt{17} + 10\sqrt{2}}\right)$

= $\left(\frac{4\sqrt{13} + 40\sqrt{2}}{2\sqrt{3} + 2\sqrt{17} + 10\sqrt{2}}, \frac{16\sqrt{13} - 12\sqrt{17}}{2\sqrt{3} + 2\sqrt{17} + 10\sqrt{2}}\right)$

= $\left(\frac{2\sqrt{13} + 20\sqrt{2}}{\sqrt{13} + \sqrt{17} + 5\sqrt{2}}, \frac{8\sqrt{13} - 6\sqrt{17}}{\sqrt{13} + \sqrt{17} + 5\sqrt{2}}\right)$

PROBLEM BASED ON CONDITION OF COLLINEARITY OF THREE POINT

1. Show that the points (-1, 1); (5, 7) & (8, 10) are collinear.

Sol. Let A (-1, 1); B (5, 7) & C (8, 10) are given point.

Here, $x_1 = -1$ Also, $y_1 = 1$

$x_2 = 5$ $y_2 = 7$

& $x_3 = 8$ $y_3 = 10$

∴ $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$ = $-1 (7 - 10) + 5 (10 - 1) + 8 (1 - 7)$

= $-1 (7 - 10) + 5 (10 - 1) + 8 (1 - 7)=-1 \times -3 + 5 \times 9 + 8 \times -6$

= $3 + 45 - 48$ = $48 - 48$ = 0

∴ $x_1 (y_2 + y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

i.e., point A (-1, 1); B (5, 7) & C (8, 10) are collinear. [**Hence proved**]

2. Show that the point A (a, b + c); B (b, c + a) & C (c, a + b) are collinear.

Sol. Let A (a, b + c); B (b, c + a) & C (c, a + b) are given point.

Here, $x_1 = a$ Also, $y_1 = b + c$

$x_2 = b$ $y_2 = c + a$

& $x_3 = c$ $y_3 = a + b$

∴ $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$

=> $a (c + \cancel{a} - \cancel{a} - b) + b (a + \cancel{b} - \cancel{b} - c) + c (b + \cancel{c} - \cancel{c} - a)$

=> $a (c - b) + b (a - c) + c (b - a)$

=> $\cancel{ac} - \cancel{ab} + \cancel{ba} - \cancel{bc} + \cancel{bc} - \cancel{ac}$ = 0

∴ $x_1 (y_2 + y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

i.e., given point are collinear. [**Hence proved**]

3. **Show that the point (-5, 1); (5, 5) & (10, 7) are collinear.**

Sol. Let A (-5, 1); B (5, 5) & C (10, 7) are given point.

Here, $x_1 = -5$ Also, $y_1 = 1$

$x_2 = 5$ $y_2 = 5$ & $x_3 = 10$ $y_3 = 7$

∴ $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = -5 (5 - 7) + 5 (7 - 1) + 10 (1 - 5)$

$= -5 \times -2 + 5 \times 6 + 10 \times -4 = 10 + 30 - 40 = 0$

∴ $x_1 (y_2 + y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

i.e., given point are collinear. [**Hence proved**]

4. **For what value of x are the point of (-3, 12); (7, 6) & (x, 9) collinear?**

Sol. Let A (-3, 12); B (7, 6) & C (x, 9) are the collinear point.

Here, $x_1 = -3$ Also, $y_1 = 12$

$x_2 = 7$ $y_2 = 6$

& $x_3 = x$ $y_3 = 9$

∴ $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$ => $-3 (6 - 9) + 7 (9 - 12) + x (12 - 6) = 0$

=> $-3 \times -3 + 7 \times -3 + x \times 6 = 0$ => $9 - 21 + 6x = 0$

=> $6x = 12$

∴ $\boxed{x = 2}$ ∴ The value of x be 2.

5. **For what value of y are the point (1, 4); (3, y); (-3, 16) collinear.**

Sol. ∴ $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

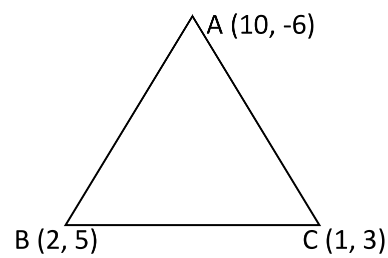
=> $1 (y - 16) + 3 (16 - 4) + (-3) (4 - y) = 0$ => $y - 16 + 36 - 12 + 3y = 0$

=> $4y = -8$ ∴ $\boxed{y = -2}$

6. **Find the ar. Of Δ whose vertices are:-**

i) **(10, -6) (2, 5) & (1, 3)**

Sol. Let A (10, -6); B (2, 5) & C (1, 3) are vertices of Δ



Here, $x_1 = 10$ Also, $y_1 = -6$

$x_2 = 2$ $y_2 = 5$

& $x_3 = 1$ $y_3 = 3$

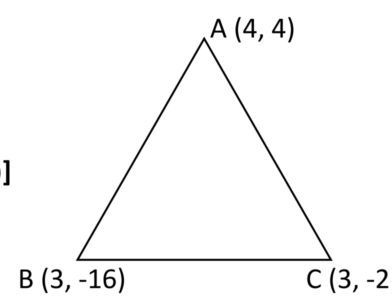
∴ Area of Δ ABC = ½ $[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

= ½ $[10 (5 - 3) + 2 (3 + 6) + 1 (-6 - 5)]$

= ½ $[20 + 18 - 11]$ = ½ × 27 = 13.5 sq. unit.

ii) **(4, 4); (3, -16) & (3, -2)**

Sol. Let A (4, 4); B (3, -16) & C (3, -2) are vertices of Δ



Here, $x_1 = 4$ Also, $y_1 = 4$

$x_2 = 3$ $y_2 = 16$

& $x_3 = 3$ $y_3 = -2$

∴ **Area of Δ ABC = ½ $[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$**

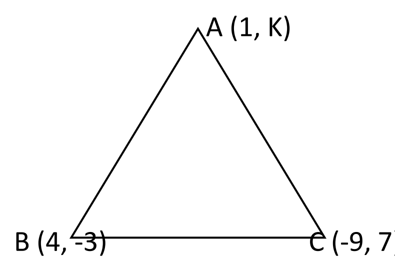
= ½ $[4 (-16 + 2) + 3 (-2 - 4) + 3 (4 + 16)]$

= ½ $[4 \times (-14) + 3 (-6) + 3 \times 20]$

= ½ $[-56 - 18 + 60]$ = ½ × -14 = -7 sq. unit.

7. **If the vertices of Δ are (1, K); (4, -3); (-9, 7) & its area is 15 sq. unit. Find the value of 'K'.**

Sol. Let A (1, K); B (4, -3) & C (-9, 7) are vertices of Δ



Here, also Area of Δ = 15 sq. units.

& $x_1 = 1$ Also, $y_1 = K$

$x_2 = 47$ $y_2 = -3$

$x_3 = -9$ $y_3 = 7$

∴ Area of Δ = ½ $[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

=> $15 = ½ [1 (-3 - 7) + 4 (7 + K) + (-9) (K + 3)]$

=> $15 = ½ [-10 + 28 - 4K - 9K - 27]$

=> $15 = ½ \times -9 - 13 K$

=> $13K = -30 - 9$

=> $K = -39/13 = -3$ ∴ $\boxed{K = -3}$

8. If the point (x, y); (-5, 7) & (-4, 5) are collinear then shown that 2x + y + 3 = 0.

Sol. Let A (x, y); B (-5, 7) & C (-4, 5) are the collinear point.

Here, $x_1 = x$ and, $y_1 = y$

$x_2 = -5$ $y_2 = 7$

$x_3 = -4$ $y_3 = 5$

Since given point be collinear.

∴ $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$

=> $x (7 - 5) + (-5) (5 - y) + (-4) (y - 7) = 0$

=> $2x - 25 + 5y - 4y + 28 = 0$

=> $\boxed{2x + y + 3 = 0}$ [**Hence proved**]