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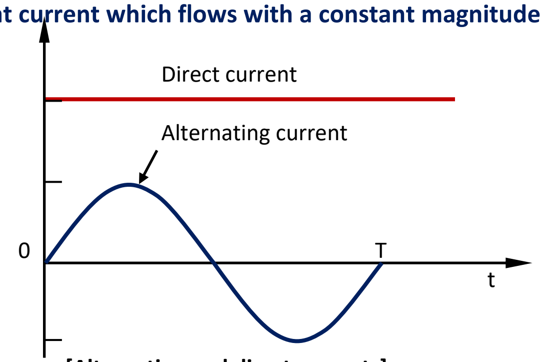
ALTERNATING CURRENT

Alternating current: (AC) "**An electric current, magnitude of which changes with time & polarity reverse periodically is called alternating currents**" (The same is true for alternating emf)

A direct current is that current which flows with a constant magnitude in the same direction, as shown in Fig.



[Alternating and direct currents]

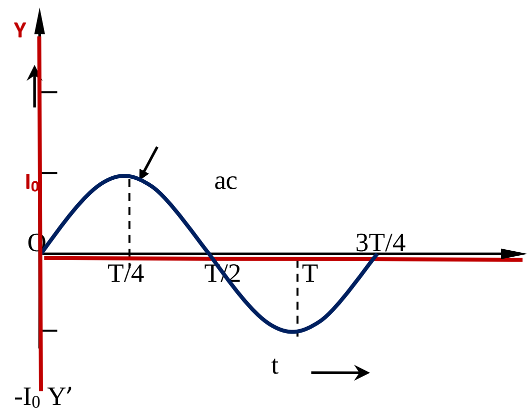
It is represented by sinusoidal wave

$$I = I_0 \sin \omega t$$
$$I = I_0 \cos \omega t$$

Where I_0 = Peak or maximum value of current (**amplitude of a c**)
I = Instantaneous value of current (mag. of current at any instant of time t)

⇒ Angular frequency of ac, $\omega = 2\pi \nu$

Where T = Time period of ac it is equal to the time taken by the ac to go through one complete cycle of variation i.e., zero to max; max to zero, zero to max. (Opposite direction) & finally max to zero.



⇒ The no. of cycle complete by ac in one second is known as ***frequency of a c***.

⇒ When a coil is rotated in a magnetic field, an alternating emf is induced in the coil. At any instant, the emf is.

or $E = E_0 \sin \omega t$
$E = E_0 \cos \omega t$

E = instantaneous value of emf
E_0 = Peak value of emf.
ωt = phase of alt. emf

Explanation: We know that when a coil is rotated in a magnetic field, an alternating emf is induced in it, which is given by the relation:

$$E = E_0 \sin \omega t$$

Suppose this emf is applied to a circuit of resistance R. Then by ohm's law, the current in the circuit will be

$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$$

or **$I = I_0 \sin \omega t$** , Thus the current in the circuit varies sinusoidally with time and is called alternating current. Here
I = instantaneous value of a.c. and is called current amplitude.
$I_0 = E_0$ = Peak or maximum value of a.c. and is called current amplitude.

Phase of alternating emf (or current) :- **"Phase of alt. emf or current may be defined as the fraction of the time period that has elapsed, since the emf (or current) last passed its zero value in positive direction".**

- If 'R'be the resistance of the circuit, then $E/R = E_0/R \sin \omega t$

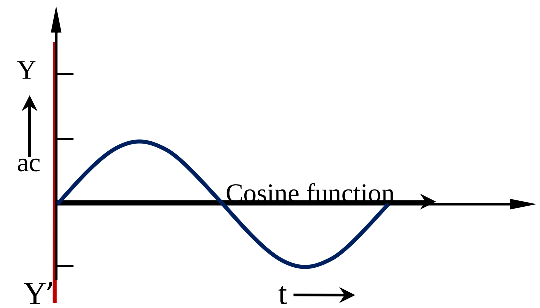
$$I = I_0 \sin \omega t$$

- **In fact, in the ac circuits, two additional circuit elements are used, there are inductor (L) and capacitor (C) i.e., current & Voltage in ac circuits are controlled by three circuit element L, C and R.**

- ∴ V = IR (When R controls I)

- V = L dI/dt (Voltage across a pure inductor)
- I = C dV/dt [∵ I = dq/dt or q = CV]
  └──→ Rate of change of potential

Ö **Inductor affects the voltage only when current I changes with time.**
Ö **Capacitor affects the current only when V changes with time.**



V and I is time dependents in case of L and C.
V and I is independent in case of R.

TRANSIENT CURRENTS:– When the electric circuit contains an inductor or a capacitor or both, the growth and decay of currents are opposed by emf induced (however, CIRCUIT containing resistance only achieve growth & decay of element of around in almost zero time). Therefore, electric current takes some time (finite) to reach it max. value (when switched on) and zero value (when switched off) Thus.

"**Electric current which vary for a small finite time while growing from zero to maximum value of while decreasing from maximum value to zero value are called transient current".**

Amplitude : *The maximum value attained by an alternating current in either direction is called its amplitude or peak value and is denoted by* I_0.

Time Period: *The time taken by an alternating current to complete one cycle of its variations is called its time period and is denoted by* **T.** This time is equal to the time taken by the coil to complete one rotation in the magnetic field.

As angular velocity of the coil is ω and its angular displacement in one complete cycle is 2π, so

Time period:= $\frac{\text{Angular displacement in a complete cycle}}{\text{Angular velocity}}$ or, $T = \frac{2\pi}{\omega}$

FREQUENCY: **The number of cycles completed per second by an alternating current is called its frequency** and is denoted by f. The frequency of an alternating current is same as the frequency of rotation of the coil in the magnetic field. Thus

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

So an alternating current be represented as $I = I_0 \sin \omega t = I_0 \sin 2\pi \nu t = I_0 \sin \frac{2\pi}{T} t$

VARIATION of alternating current with time. It rises from 0 to maximum in one direction, then falls to zero and then rises from 0 to maximum in the opposite direction and again falls to zero, thus completing one full cycle.

- The alternating current supplied to our houses has a frequency of 50 Hz.

As the alternating current is positive in one half cycle and equally negative in the other half cycle, so its mean value over a complete cycle is zero.

PROOF: The average value of alternating current over one complete cycle is zero.

Average value of a.c. over one complete cycle: The alternating current at any instant t is given by

$$I = I_0 \sin \omega t$$

Assuming the current remains constant for a small time dt, then

the amount of charge that flows through the circuit in small time dt will be **$dq = Idt = I_0 \sin \omega t . dt$**

The total charge that flows the circuit in ine complete cycle of a.c.,

$$q = \int dq = \int_0^T I_0 \sin \omega t \, dt$$

$$= I_0 \left(\frac{-\cos \omega t}{\omega}\right)_0^T = -\frac{I_0}{2\pi / T}\left(\cos \frac{2\pi}{T} t\right)_0^T$$

$$= -\frac{I_0 T}{2\pi}[\cos 2\pi - \cos 0] = -\frac{I_0 T}{2\pi}[1 - 1] = 0$$

The **average value of a.c. over one complete cycle of a.c.,**

$$I_{av} = \frac{q}{T} = 0$$

Thus, **the average value of a.c. over a complete cycle of of a.c.,**

$$I_{av} = \frac{q}{T} = 0$$

Thus **the average value of a.c. over a complete cycle of of a.c. is zero.**

. Ordinary moving coil galvanometer used for d.c. cannot be used to measure an alternating current even if its frequency is low. ***Explanation:*** Ordinary moving coil galvanometer cannot be used to measure a.c. Ordinary moving coil galvanometer is based on magnetic effect of current which , in turn, depends on direction of current. So it cannot be used to measure a.c. During one half cycle of a.c., its pointer moves in one direction and during next half cycle, it will move in the opposite direction. Now the average value of a.c. over a complete cycle is zero. Even if we measure an alternating current of low frequency, the pointer, will appear to be stationary at the zero-position due to persistence of vision.

We can measure a.c. by using a hot-wire ammeter which is based on heating effect of current and this effect is independent of the direction of current.

To measure a.c., we have to define the mean value of a.c. over half a cycle or its root mean square value.

MEAN OR AVEARAGE VALUE OF A.C.

3

Average value of a.c.: *It is defined as that value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in its half time period.*

- It is denoted by - I_{av}, I_m or I_v

RELATION BETWEEN AVERAGE VALUE AND PEAK VALUE OF A.C.:

$$I = I_0 \sin \omega t$$

This current can be assumed to ***remain constant for a small time dt.*** Then the *amount of charge that flows through the circuit insmall time dt is given by*

$$dq = I \,.\, dt = I_0 \sin \omega t \,.\, dt$$

The total charge that flows through the circuit, say in the first half cycle, i.e., from t = 0 to t = T/2 is given by

$$q = \int_0^{T/2} dq = \int_0^{T/2} I_0 \sin \omega t \, dt = I_0 \left(-\frac{\cos \omega t}{\omega}\right)_0^{T/2}$$

$$= -\frac{I_0}{2\pi / T}\left(\cos \frac{2\pi}{T} t\right)_0^{T/2}$$

$$= -\frac{I_0 T}{2\pi}[\cos \pi - \cos 0] \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$= -\frac{I_0 T}{2\pi}[-1 - 1] = \frac{I_0 T}{\pi}$$

∴ The average value of a.c. over the first half cycle is

$$I_{av} = \frac{\text{charge}}{\text{Time}} = \frac{q}{T/2} = \frac{2q}{T} = \frac{2}{T} \cdot \frac{I_0 T}{\pi} = \frac{2 I_0}{\pi} = 0.637 \, I_0$$

Thus, the mean or average value of an alternating current is $2/\pi$ or 0.637 times its peak value. The similar relation can be proved for the alternating emf, which is

$$E_{av} = \frac{2}{\pi} E_0 = 0.637 \, E_0$$

ROOT MEAN SQUARE (RMS) OR VIRTUAL OR EFFECTIVE VALUE OF A.C.

It is defined as that value of a direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time.

- It is denoted by I_{rms}, I_v or by I_{eff}.
- **Relation between the effective and peak value of a.c.:** Suppose an alternating current $I = I_0 \sin \omega t$ be passed through a circuit of resistance R. Then the amount of heat produced in small time dt will be

$$dH = I^2 R \, dt$$

If t is the time period of a.c., then heat produced in one complete cycle will be

$$H = \int_0^T I^2 R dt$$

Let I_{eff} be the effective value of a.c. Then heat produced in time T must be

$$H = I_{eff}^2 RT$$

$\therefore \quad I_{eff}^2 RT = \int_0^T I^2 R\, dt \quad$ or $\quad I^2_{eff} = \frac{1}{T}\int_0^T I^2 dt$

But $1/T \int_0^T I^2 dt$ is the mean of the squares of the instantaneous values of a.c. over one complete cycle, hence the effective or virtual or virtual value of a.c. equals its root mean square value, i.e.,

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}$$

Now $\int_0^T I^2 dt = \int_0^T I_0^2 \sin^2 \omega t\, dt = I_0^2 \int_0^T \frac{1 - \cos 2\omega t}{2} dt = \frac{I_0^2}{2}\left[\int_0^T 1\, dt - \int_0^T \cos 2\omega t\right]$

$$= \frac{I_0^2}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T \qquad \left[\because \int \cos 2\omega t = \frac{\sin 2\omega t}{\omega}\right]$$

$$= \frac{I_0^2}{2}\left((T - 0) - \frac{1}{2\omega}\left|\sin \frac{4\pi}{T} t\right|_0^T\right) \qquad [\because \sin 4\pi = \sin 0 = 0]$$

$$= \frac{I_0^2}{2}[T - 0] = \frac{I_0^2 T}{2}$$

$\therefore \quad I_{eff}$ or $I_{rms} = \sqrt{\frac{1}{T} \cdot \frac{I_0^2 T}{2}}$

or $\quad I_{eff}$ or $I_{rms} = \frac{1}{\sqrt{2}} I_0 = 0.707\, I_0 \qquad$ Thus the effective or **rms value of an a.c. is $\frac{1}{\sqrt{2}}$ time its peak value.**

ROOT MEAN SQUARE (RMS) Of AN ALTERNATING EMF

It is defined as that value of a steady voltage that produces the same amount of heat in a given resistance as is produced by the given alternating emf when applied to the same resistance for the same time.

- It is also called virtual or effective value of the alternating emf. It is denoted by E_{rms} or E_{eff} or E_v.

- **Relation between the rms value and the peak value of an alternating emf:** Suppose an alternating emf Ɛ applied to a resistance R is given by

$$E = E_0 \sin \omega t$$

Heat produced in a small time dt will be

$$dH = \frac{E^2}{R} dt = \frac{E_0^2}{R} \sin^2 \omega t\, dt$$

Let T be the time period of the alternating emf. Then heat produced in time T will be

$$H = \int dH = \int_0^T \frac{E_0^2}{R} \sin^2 \omega t\, dt$$

$$= \frac{E_0^2}{R}\int_0^T \frac{(1 - \cos 2\omega t)}{2} dt = \frac{E_0^2}{2R}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T$$

$$= \frac{E_0^2}{2R}\left((T - 0) - \frac{1}{2\omega}\left|\sin \frac{4\pi}{T} t\right|_0^T\right)$$

$$= \frac{E_0^2}{2R}\left(T - \frac{1}{2\omega}\sin(4\pi - \sin 0)\right)$$

or $\quad H = \frac{E_0^2}{2R}[T - 0] = \frac{E_0^2 T}{2R}$

If E_{rms} is the root mean square value of the alternating emf, then the amount of heat produced by it in the same resistance R in the time T will be $H = \frac{E^2_{rms} T}{R}$

From the above two equations, we get

$$\frac{E^2_{rms} T}{R} = \frac{E_0^2 T}{2R}$$

$$\therefore \quad E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707\, \mathcal{E}_0$$

Conceptual tips..

- ☑ The alternating current and voltages are generally measured and specified in terms of their rms values. When we say that the household supply is 220 a.c., we mean that its rms value is 220 V. The peak value would be
  $V_0 = \sqrt{2}.\ V_{rms} = \sqrt{2} \times 220 = 311$ V.

- ☑ Both alternating and direct currents are measured in amperes. However, it is not possible to define a.c. ampere in terms of forces between two parallel wires carrying a.c. currents, as the d.c. ampere is defined. This is because the alternating current changes direction with the source frequency and so the net force would add up to zero. To overcome this problem, we define a.c. ampere in terms of Joule heating ($H = I^2 Rt$) which is independent of the direction of current. Hence the rms value of alternating current in the circuit is one ampere of the current that produces the same average heating effect as one ampere of direct current would produce under the same conditions.

- ☑ Alternating currents and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. As the working of these instruments is based on the heating effect of current, so they are called hot-wire instruments.

Examples based on Induced EMF in a Rotating Coil

◆ **Formula Used**

1. Instantaneous value of a.c., $I = I_0 \sin \omega t$,
2. Average or mean value of a.c. over half cycle,
   $I_{av} = \frac{2}{\pi} I_0 = 0.637\, I_0$
3. Effective or rms or virtual value of a.c.,
   I_{eff} or I_{rms} or $I_v = \frac{1}{\sqrt{2}} I_0 = 0.707\, I_0$
4. For alternating voltages, we have
   $E = E \sin \omega t$, $\quad E_{av} = 0.637\, E_0$, $\quad E_{rms} = \frac{1}{\sqrt{2}} E_0$

◆ **Units Used** : Current I, I_0 and I_{rms} are in ampere, voltages E, E_0 and E_{rms} are in volt.

Q. 1. ***The electric mains in a house are marked 220 V, 50 Hz. Write down the equation for instantaneous voltage.***

Sol. Here $E_{rms} = 220$ V, f = 50 Hz

Instantaneous voltage is given by

$$E = E_0 \sin \omega t = \sqrt{2}\, E_{rms} \sin 2\pi f t = 1.414 \times 220 \sin (2 \times 3.14 \times 50\, t) = 311 \sin 314\, t \text{ volt.}$$

Q. 2. ***An electric bulb operates 12 V d.c. If this bulb is connected to an a.c. source and gives normal brightness, what would be the peak value of the source?***

Sol. For normal brightness of the bulb, $E_{rms} = 12$ V$\therefore$ $\quad E_0 = 12$ V $= 1.414 \times 12 = 17$ V

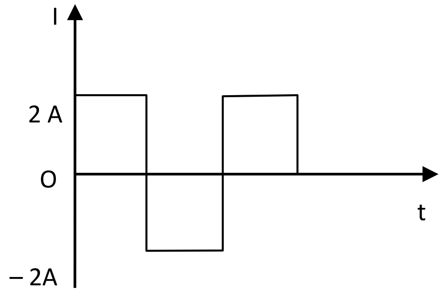
Q. 3. ***The peak value of an alternating voltage applied to a 50 Ω resistance is 10 V. Find the rms current. If the voltage frequency is 100 Hz, write the equation for the instantaneous current.***

Sol. Here R = 50 Ω, $E_0 = 10$ V, f = 100 Hz

$$I_0 = \frac{E_0}{R} = \frac{10}{50} = \frac{1}{5} \text{ A} = 200 \text{ mA}$$

$I_{rms} = 0.707\, I_0 = 0.707 \times 200 = 141.4$ mA. The instantaneous current is given by , $I = I_0 \sin 2\pi ft = 200 \sin 200\, \pi\, t$ mA.

Q. 4. ***Calculate the rms value of the alternating current shown in Fig.***



Sol. $I_{rms} = \sqrt{\dfrac{I_1^2 + I_2^2 + I_3^2}{3}}$

$= \sqrt{\dfrac{2^2 + (-2)^2 + 2^2}{3}} = \mathbf{2A}$

Q. 5. ***The electric current in a circuit is given by $I = i_0 (t/\tau)$ for some time. Calculate the rms current for the period t = 0 to t = τ.***

Sol. The mean square current for the rms current for the period t = 0 to t = τ is given by

$$\overline{i}^2 = \frac{1}{\tau}\int_0^{\tau} i_0^2 \left(\frac{t}{\tau}\right)^2 dt$$

$$= \frac{i_0^2}{\tau^3}\int_0^{\tau} t^2\, dt = \frac{i_0^2}{\tau^3}\left(\frac{t^3}{3}\right)_0^{\tau} = \frac{i_0^2}{\tau^3}\cdot\frac{\tau^3}{3} = \frac{i_0^2}{3}$$

$\therefore$ $i_{rms} = \sqrt{\overline{i}^2} = \sqrt{\dfrac{i_0^2}{3}} = \dfrac{i_0}{\sqrt{3}}$

Q. 6. ***If the effective value of current in 50 Hz a.c. circuit is 5.0 A, what is (i) the peak value of current (ii) the mean value of current over half a cycle and (iii) the value of current 1/300 s after it was zero?***

Sol. Here $I_{eff} = 5$ A, f = 50 Hz

(i) $I_0 = \sqrt{2}\, I_{eff} = \sqrt{2} \times 5 = 7.07$ A.

(ii) $I_m = \dfrac{2}{\pi} I_0 = 0.637 \times 7.07 = 4.5$ A.

(iii) At t = 1/300 s,

$I = I_0 \sin 2\pi ft = 7.07 \sin\left(2\pi \times 50 \times \dfrac{1}{300}\right)$

$= 7.07 \sin\dfrac{\pi}{3} = 7.07 \times \dfrac{\sqrt{3}}{2} = 6.12$ A

Q. 7. ***The instantaneous value of an alternating voltage in volts is given by the expression $\mathcal{E}_t = 140 \sin 300t$, where t is in second. What is (i) peak value of the voltage, (ii) it rms value and (iii) frequency of the supply? Take $\pi = 3$, $\sqrt{2} = 1.4$.***

Sol. Comparing the equation: $\mathcal{E}_t = 140 \sin 300t$

with the standard equation: $E = E_0 \sin \omega t$, we get

(i) Peak voltage, $E_0 = 140$ V.

(ii) rms value of voltage, $E_{rms} = \dfrac{E_0}{\sqrt{2}} = \dfrac{140}{1.4} = 100$ V

(iii) Angular frequency, $\omega = 300$ $\therefore$ Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{300}{2 \times 3} = 50$ Hz

Q. 8. ***A resistance of 40 Ω is connected to an a.c. source of 220 V, 50 Hz. Find (i) the rms current (ii) the maximum instantaneous current in the resistor and (iii) the time taken by the current to change from its maximum value to the rms value.***

Sol. **(i)** $E_{rms} = 220$ V, R = 40 Ω

$\therefore$ $I_{rms} = \dfrac{E_{rms}}{R} = \dfrac{220}{40} = 5.5$ A.

(ii) Maximum instantaneous current,

$I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 5.5 = 7.8$ A.

(iii) Let the alternating current be given by

$I = I_0 \sin \omega t$,

Let the a.c. take its maximum and rms values at instants t_1 and t_2 respectively. Then $I_0 = I_0 \sin \omega t_1$,

Which implies $\omega t_1 = \dfrac{\pi}{2}$ and $I_{rms} = I_0 = \dfrac{I_0}{\sqrt{2}} = I_0 \sin \omega t_2$

Which implies $\omega t_2 = \dfrac{\pi}{2} + \dfrac{\pi}{4}$

$\therefore$ $t_2 - t_1 = \dfrac{\pi}{4\omega} = \dfrac{\pi}{4 \times 2\pi f} = \dfrac{\pi}{4 \times 2\pi \times 50} = \dfrac{1}{400}$ s = 2.5 ms

PHASORS AND PHASOR DIAGRAMS

A rotating vector that represents a sinusoidally varying quantity is called a phasor.

- This vector **is imagined** to rotate with angular velocity equal to the angular frequency of that quantity.
- Its length represents the amplitude of the quantity and its projection upon a fixed axis gives the instantaneous value of the quantity.
- The phase angle between two quantities is shown as the phase angle between their phasors.
- The study of a.c. circuits is greatly simplified if we treat alternating currents and voltages as phasors.

A diagram that represents alternating current and voltage of the same frequency as rotating vectors (phasors) along with proper phase angle between them is called a phasor diagram.

Suppose the alternating emf and current in a circuit are given by

$E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi.)$

where is the phase angle between E and I.

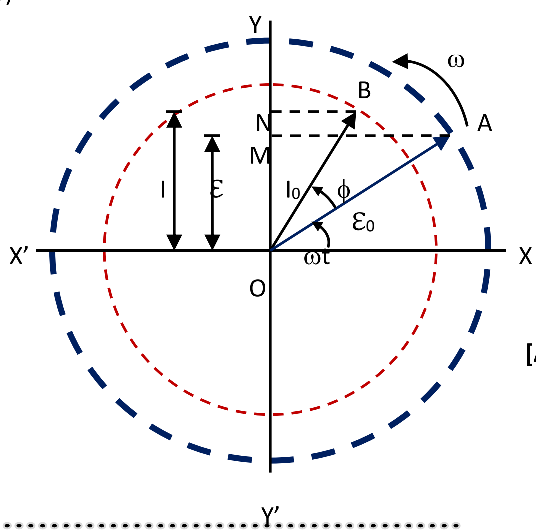
Representation: To represent these quantities as phasors, we draw circles of radii $\mathcal{E}_0$ and I_0 as shown in Fig.

Let $\angle AOX = \omega t$

and $\angle BOX = \omega t + \phi$.

then vector $\vec{OA}$ represents phasor $\vec{E}$ of magnitude E_0 and vector $\vec{OB}$ represents phasor $\vec{I}$ of magnitude I_0, both rotating with the same angular velocity ω in the anticlockwise direction. The projection OM (= E) of $\vec{OA}$ on the vertical axis represents the instantaneous value of the alternating emf. the projection ON (= I) of $\vec{OB}$ on the vertical axis represents the instantaneous value of the alternating current. The angle $\phi = \angle AOB$ represents the phase angle between the phasors $\vec{E}$ and $\vec{I}$. In the present case, the current leads the emf by phase angle ϕ. If the current lags behind the emf,

$I = I_0 \sin(\omega t - \phi)$



[A phasor diagram for an alternating emf and current]

For Your Knowledge...

☑ Through in phasor diagram, we represent alternating current and voltage as rotating vectors, these quantities are not really vectors themselves. These are scalar quantities. In fact, the amplitudes and phases of the harmonically varying scalars combine mathematically in the same way as do the projections of rotating vectors of corresponding magnitudes and directions. Thus, the representation of the harmonically varying quantities as rotating vectors enables us to use the laws of vector addition for adding these quantities.

☑ In an a.c. circuit, the current may lag behind or lead the voltage, depending the type of the circuit through which the current flows. This concept is analogous to two cars running at the same speed, with one following the other at a distance. More appropriately, it is like two pendulums of the same frequency which start their motions at different instants of time.

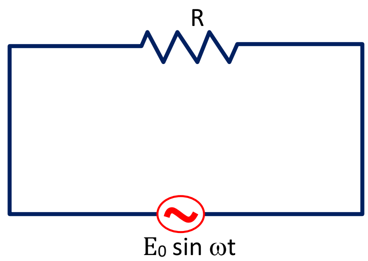
A.C. CIRCUIT CONTAINING ONLY A RESISTOR

✱ ***The voltage and current always vary in the same phase in an a.c. circuit containing resistance only.***

suppose a resistor of resistance R is connected to a source of alternating emf E given by

$$E = E_0 \sin \omega t \qquad \text{... (1)}$$

Such a circuit is known as a purely resistive circuit.



[A.C. through a resistor]

If I be the current in the circuit at instant t, then the potential drop across R will be IR. According to Kirchhoff's loop rule,

Instantaneous emf of the source = Instantaneous p.d. across R

or $E_0 \sin \omega t = IR$

or $I = \dfrac{E_0}{R} \sin \omega t$

or $I = I_0 \sin \omega t \qquad \text{... (2)}$

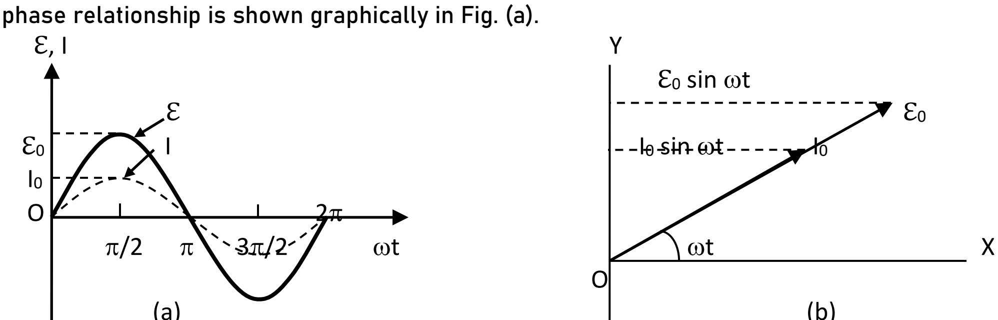
where $I_0 = \dfrac{E_0}{R}$ = the maximum or peak value of a.c.

Comparing $I_0 = E_0 / R$ with ohm's law equation, i.e., **current = voltage / resistance**, (resistance to a.c. is represented by R-which is the value of resistance to d.c.)

- Hence, **behaviour of R in d.c. and a.c. circuits is the same. R can reduce a.c. as well as d.c. equally effectively.**

From equation (1) and (2), we note that both E and I are functions of sin ωt. Hence the emf E and current I are in same phase in a purely resistive circuit.

- This means that both E and I attain their zero, minimum and maximum values at the same respective times.
  - This phase relationship is shown graphically in Fig. (a).



[(a) Graph of Eand I versus ωt and (b) Phasor diagram, for a resistive a.c. circuit.]

- Both the phasors E and I are in the same direction, making same angle ωt with x – axis.
- The phase angle between E and I is zero.
- Though **voltage and current in an a.c. circuit are represented by phasors i.e. rotating vectors**, they are not vectors themselves they are **scalar quantities**.
- The vector diagram [(ii) & (iii)] representing phase relationship between alternating current and alternating e.m.f. In this diagram **peak values of alternating current (I_0) and alternating e.m.f. (E_0) are represented by arrows called phasors, rotating in the anticlock wise direction.**
- **The length of the arrow represents the maximum value of the quantity.**
- The **projection of the arrow** on any axis **represents the instantaneous value of the quantity**. In the **sine form, projection is taken on the vertical axis** and in the **cosine form; projection is taken on the horizontal axis.**

The phase difference between the two alternating quantities is represented by the angle between the two vectors $\vec{E_0}$ & $\vec{I_0}$.

In the **a.c. circuit containing R** only, current and voltage are in the same phase. Therefore, both phasors I_0, E_0 are in the same direction making and angle (ωt) with OX. This is so for all times. Their projections on YOY' represent the instantaneous values of alternating current and voltage i.e.

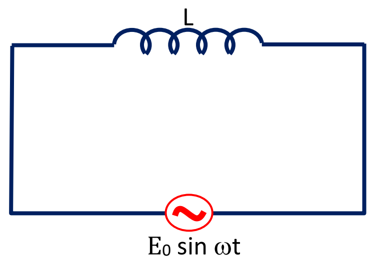
$$I = I_0 \sin \omega t \quad \& \quad E = E_0 \sin \omega t.$$

A.C. CIRCUIT CONTAINING ONLY AN INDUCTOR

A sinusoidal emf is applied to a circuit containing an inductor only. The current lags behind the voltage by $\pi/2$ radian.

An inductor of inductance L connected to a source of alternating emf $\mathcal{E}$ given by $E = E_0 \sin \omega t$ ... (1)

We assume that the inductor has negligible resistance. Thus the circuit is purely inductive a.c. circuit.

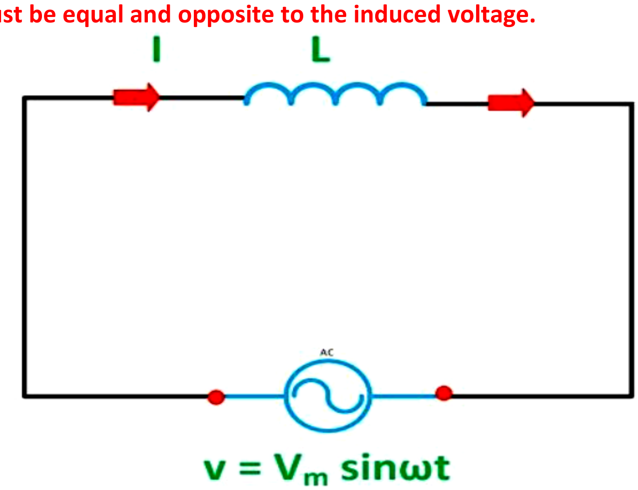


[A.C. through an inductor]

Let a source of alternating e.m.f. be connected to a circuit containing a pure inductance L only. Suppose the alternating e.m.f. supplied is represented by **$E = E_0 \sin \omega t$** -------------------------- (i)

If dI/dt is the rate of change of current through L at any instant, then induced e.m.f. in the inductor at the same instant is = -dI/dt. The ***negative sign*** indicates that ***induced e.m.f. opposes*** the ***change of current***.

--------**To maintain the flow of current, the applied voltage must be equal and opposite to the induced voltage.**



i.e. $E = -L\left(\frac{dI}{dt}\right) = E_0 \sin \omega t$

or $dI = \frac{E_0 \sin \omega t \, dt}{L}$

Integration both sides, we get

$I = \frac{E_0}{L} \int \sin \omega t \, dt$

$= \frac{E_0}{L}\left[\frac{-\cos \omega t}{\omega}\right] = -\frac{E_0}{\omega L}\cos \omega t$

$= -\frac{E_0}{\omega L} \sin\left[\frac{\pi}{2} - \omega t\right]$

or $I = \frac{E_0}{\omega L} \sin(\omega t - \pi/2)$ ------------------ (ii)

--------------*The current will be maximum i.e. $I = I_0$, when $\sin(\omega t - \pi/2)$ = maximum = 1*

From (ii), $I_0 = \frac{E_0}{\omega L} \times 1$ ---------------------- (iii)

putting in (ii) , we get $I = I_0 \sin(\omega t - \pi/2)$ ------------- **[iv]**

Comparing (i) & (iv), we find that in an **a.c. circuit containing L only**, current **I lags behind the voltage E by a phase angle of 90^0.**

The vector diagram [(ii) & (iii)] represents the vector diagram or the phasor diagram of **a.c. circuit containing L only**. The vector representing **$\vec{E_0}$ makes an angle (ωt) with OX**. As **current lags behind the e.m.f. be 90^0**, therefore, phasor representing I_0 is turned clock wise through 90^0 from direction of E_0.

The **projection of these phasors on YOY'** give **instantaneous values E and I.**
Comparing (iii) with ohm's law equation, i.e., Current = voltage/Resistance, we find that (ωL) represents the effective **resistance offered by inductance L**. This is called **inductive reactance** and is denoted by **X_L**.

Thus, $X_L = \omega L = 2\pi v L$

(Where **v** is the frequency of a.c. supply.)

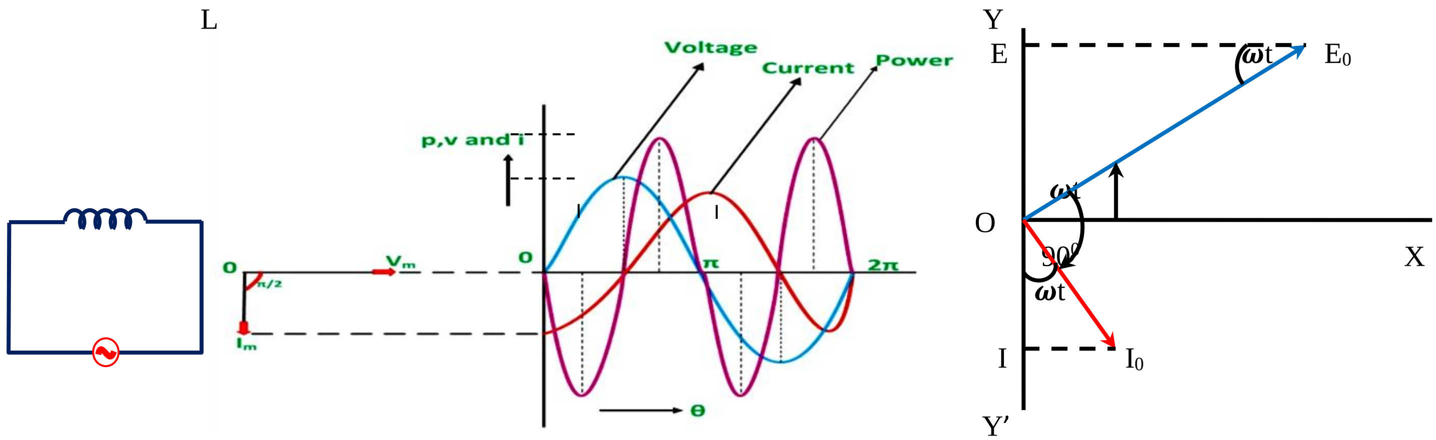
In d.c. circuits, v = 0, ∴ $X_L = 0$
i.e., a pure inductance offers zero resistance to d.c. Further, $X_L \propto v$ i.e., higher the frequency of a.c., more is the inductive reactance.

The units of inductive reactance.

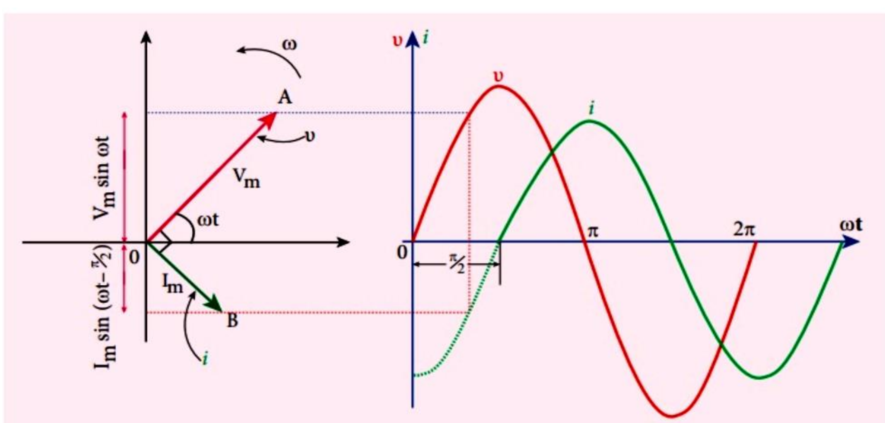
$X_L = \omega L = 1$ Henry

$= \dfrac{1 \text{ volt}}{\text{Sec amp/sec}}$ = **Ohm.** Hence X_L is measured on ohm, just as resistance is measured in ohm.

In a circuit containing R, opposition arises on account of obstruction to the passage of electrons through the resistor. In an inductive circuit, it is the self-induced e.m.f. that opposes the growth of current.



(i) (ii) (iii)



Explanation:
IN an inductive a.c. circuit, the voltage is ahead of the current in phase by 90° or the current lags behind the voltage in phase by 90°. This means that the voltage E attains its maximum value (E_0) a quarter of cycle (time T/4) earlier than the current I, or the current attains its peak value (I_0) a quarter of cycle later than the voltage Ɛ.

Inductive reactance: Comparing equation $I_0 = E_0/\omega L$ with the ohmic relation $I_0 = E_0/R$, we find that **ωL** plays the same role here as the resistance R in resistive case. It is a measure of the effective resistance or opposition offered by the inductor to the flow of a.c. through it. Such a non-resistive opposition to the flow of current is called ***reactance***. In this case, it is called ***inductive reactance*** and is denoted by X_L.

$$\therefore \quad X_L = \omega L = 2\pi \nu L$$

where f is the frequency of a.c. supply. The SI unit of inductive reactance is ohm (Ω).

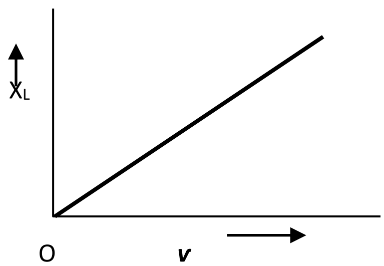
For a.c., $X_L \propto \nu$

For d.c., $\nu = 0$, so $X_L = 0$

Thus, an **inductor allows d.c. flow through it easily but opposes the flow of a.c. through it**. Obviously,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{\omega L\sqrt{2}} = \frac{E_{ms}}{\omega L} = \frac{E_{rms}}{X_L}$$

Variation of X_L with frequency: As $X_L \propto \nu$, so the graph of X_L versus ν is a straight line with a positive slope. As f increases, X_L also increases.

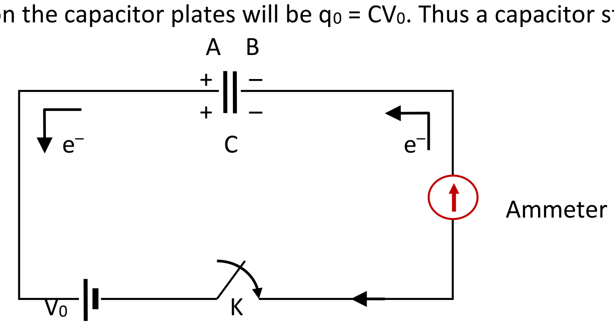


[Graph of X_L versus f]

A.C. CIRCUIT CONTAINING ONLY A CAPACITOR

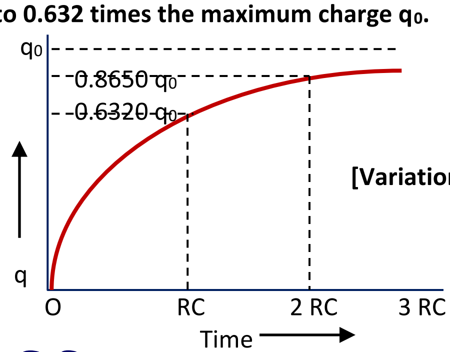
Effects of a capacitor in a d.c. circuit: A capacitor of capacitance C connected to a battery through a tapping key K.

As the circuit is closed, electrons start flowing from the plate A to the positive terminal of the battery and from the negative terminal to the plate B of a capacitor. The plates A and B start acquiring positive and negative charges respectively. The capacitor gets **progressively charged until the potential difference across the plates A and B becomes equal to the p.d. across the terminals of the battery**. As soon as this happens, the charging of the capacitor stops. Thus, during the capacitor is being charged, an electric current does flow through the rest of the circuit, as is clear from the momentary deflection in the ammeter. The maximum charge on the capacitor plates will be $q_0 = CV_0$. Thus a capacitor stops a d.c.



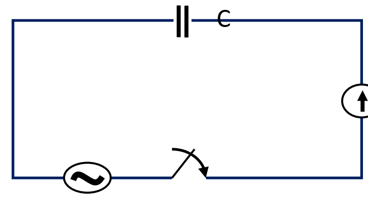
[A capacitor in a d.c. circuit]

- **If a resistance R is also included in series with the capacitor**, the process of charging of the capacitor gets slowed down and the capacitor takes longer time to get fully charged. Fig. shows the variation of charge q with time t. Clearly, the charge grows exponentially from zero to the maximum value q_0. **We may define the time constant of the RC-circuit as the time in which the capacitor gets charged to 0.632 times the maximum charge q_0.**



[Variation of charge q with time t during the charging of a capacitor]

Effect of capacitor in an a.c. circuit: Capacitor of capacitance C connected to a source of alternating emf. Due to the alternating voltage of the source, the capacitor gets charged in one direction in the first half cycle, then discharged, and then charged in the opposite direction during the second half cycle and again discharged and so on. As a result, there is a continuous, though alternating, current in the circuit. Thus, a capacitor provides an easy path for a.c.



[A capacitor in an a.c. circuit]

◘ Alternating emf Applied to a capacitor: -

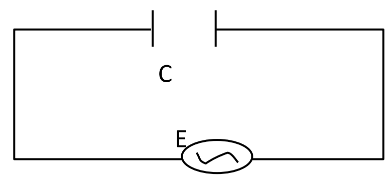
Consider an ideal capacitor of capacitance 'C' connected to source of ac supply

The alternating emf applied **is** $E = E_0 \sin \omega t$................... (i)

Due to this applied emf, an alternating current flow in the circuit. The two plates of the capacitor become appositively charged.

Let q be the charge on the capacitor at any instant.

Therefore, **potential difference across the capacitor**, $V_c = q/C$



But $V_C = E$ or $E = q/C$

or $E_0 \sin \omega t = q/C$

$$q = E_0 C \sin \omega t$$

Now, **Instantaneous current**, $I = dq/dt$

$$I = \frac{d}{dt}(E_0 C \sin \omega t),$$

$$I = E_0 C \frac{d}{dt}(\sin \omega t)$$

$$I = \omega E_0 C (\cos \omega t)$$

$$I = \frac{E_0}{1/C\omega}(\cos \omega t)$$

$$I = \frac{E_0}{1/C\omega}(\sin \omega t + \pi/2) \quad \text{........................ (ii)}$$

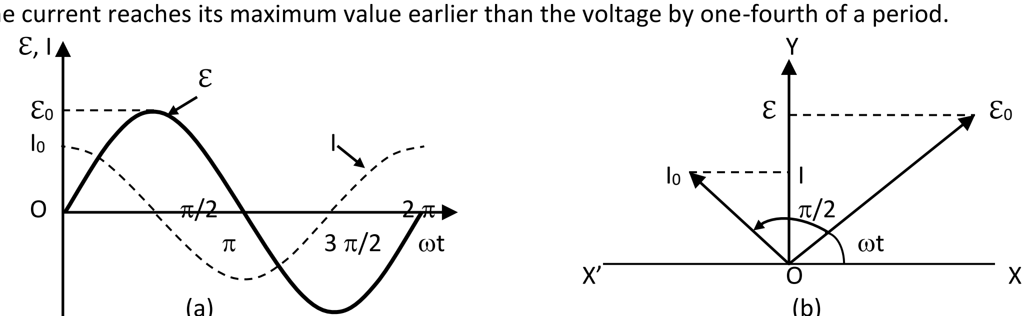
⇒ The value of I will be maximum i.e. I_0, if $\sin(\omega t + \pi/2) = 1$ = maximums

Therefore, $I_0 = E_0/1/C\omega$

∴ from (ii), $I = I_0 \sin(\omega t + \pi/2)$ ....(iii)

Comparing (i) & (ii), we observe that current leads the emf by an angle ***π/2 in a purely capacitive circuit***.

◘ Phase relationship between Ɛ and I. On comparing equations (1) and (2), we find that in a capacitive a.c. circuit, the current leads the voltage or the voltage lags behind the current in phase by π/2 radian. The phase relationship between Ɛ and I is shown graphically in Fig. (a). We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.



(a) Graph of Ɛ and I versus ωt and (b) phasor diagram, for a capacitance a.c. circuit]

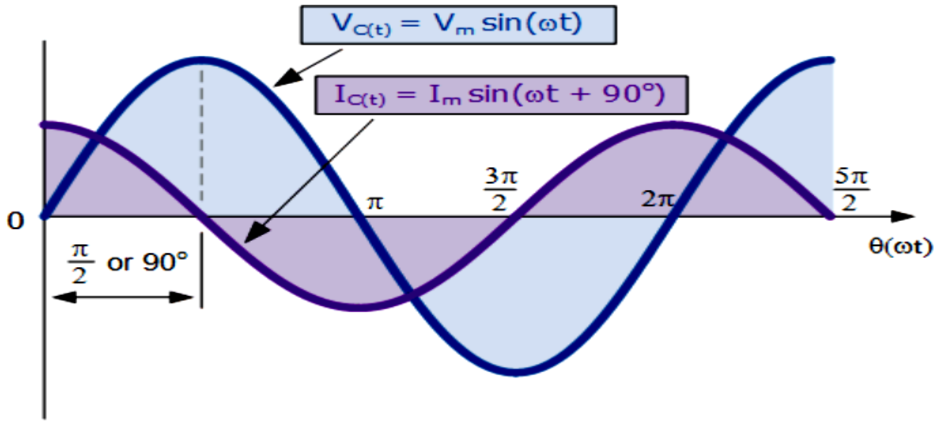


Fig. (b) shows the phasor diagram for a capacitive a.c. circuit. The phasor Ɛ makes an angle ωt with X-axis in anticlockwise direction. As the current leads the emf in phase by π/2 rad, so the current phasor I makes an angle π/2 rad with phasor E in anticlockwise direction.

Capacitive reactance: Comparing the relation,

$$I_0 = \frac{E_0}{1/\omega C}$$

with the ohmic relation $I_0 = E_0$, we find that the factor 1 is the effective resistance or opposition offered by the capacitor to the flow of a.c. through it. It is called capacitive reactance and is denoted by X_C. Thus

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

The SI unit of capacitive reactance is ohm (Ω).

- For a.c., $X_C \propto \frac{1}{\nu}$

- For d.c., $\nu = 0$ ∴ $X_C = \infty$

Thus, **a capacitor allows a.c. to flow through it easily but offers infinite resistance to the flow of d.c.,**

- i.e., a capacitor block d.c. Obviously,

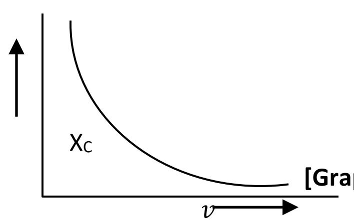
$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{1/\omega C.\sqrt{2}} = \frac{E_{rms}}{1/\omega C} = \frac{E_{rms}}{X_C}$$

Variation of capacitive reactance with frequency: Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

i.e., $X_C \propto \frac{1}{\nu}$

Thus the capacitive reactance varies inversely with the frequency. As f increases, X_C decreases. Fig. shows the variation of X_C with ν.



[Graph of X_C versus f]

Capacitative Reactance ($X_C = 1/C\omega$)

"The capacitative reactance is the effective resistance offered by a capacitor to the flow of current in the circuit.

∴ $X_C = 1/C\omega = 1/C\,2\pi\nu$

- **For DC** ν (frequency) = 0, Therefore, $X_C = \infty$

  Capacitance offers infinite resistance to the flow of dc so dc cannot pass through a capacitor.

- **For AC** ν = finite, Therefore, X_C = 1/finite value = smaller value.

  Capacitor offers small opposition to the flow of ac can pass through a capacitor easily.

Unit ⇒ $X_C = 1/C\omega$ = 1/farad × sec = sec/ C/V = Sec × volt / amp × sec = ohm.

Numerical Examples based on (i) Inductive reactance (ii) Capacitive reactance

◆ **FORMULA USED**

1. For an a.c. circuit containing inductor only,

(i) Inductive reactance, $X_L = \omega L = 2\pi \nu L$

(ii) Current amplitude, $I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\mathcal{E}_0}{\omega L}$

(iii) Effective current, $I_{rms} = \frac{\mathcal{E}_{rms}}{X_L} = \frac{\mathcal{E}_{rms}}{\omega L} = \frac{\mathcal{E}_0}{\sqrt{2}.\,\omega L}$

2. For an a.c. circuit containing capacitor only,

(i) Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$

(ii) Current amplitude, $I_0 = \frac{\mathcal{E}_0}{X_C} = \frac{\mathcal{E}_0}{1/\omega C}$

(iii) Effective current, $I_{rms} = \frac{\mathcal{E}_{rms}}{X_C} = \frac{\mathcal{E}_{rms}}{1/\omega C} = \frac{\mathcal{E}_0}{\sqrt{2}\,.\,1/\omega C}$

◆ **UNITS USED**

Inductance L is in Henry, capacitance C in farad, reactances X_L and X_C in ohm, currents I_0 and I_{rms} in ampere and voltages $\mathcal{E}_0$ and $\mathcal{E}_{rms}$ in volt.

Q. 1. A 100 Hz a.c. is flowing in a 14 mH coil. Find its reactance.

Sol. Here $\nu = 100$ Hz, L = 14 mH = 14×10^{-3} H

Reactance, $X_L = 2\pi\nu L = 2 \times 22/7 \times 100 \times 14 \times 10^{-3} = 8.8\ \Omega$

Q. 2. A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Sol. Here, L = 25.0 mH = 25.0×10^{-3} H, $\mathcal{E}_{rms} = 220$ V, $\nu = 50$ Hz

$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25.0 \times 10^{-3} = 7.85\ \Omega$

$I_{rms} = \frac{\mathcal{E}_{rms}}{X_L} = \frac{220}{7.85} = 28.03$ A

Q. 3. Find the maximum value of current when an inductance of one henry is connected to an a.c. source of an inductance of one henry is connected to an a.c. source of 200 volts, 50 Hz.

Sol. Here L = 1 H, $\mathcal{E}_{eff} = 200$ V, $\nu = 50$ Hz

Maximum current, $I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\sqrt{2} \times \mathcal{E}_{eff}}{2\pi\nu L} = \frac{\sqrt{2} \times 200}{2 \times 3.14 \times 50 \times 1} = 0.9$ A

Q. 4. A coil has an inductance of 1 H. (i) At what frequency will it have a reactance of 3142 Ω? (ii) What should be the capacity of a capacity which has the same reactance at that frequency?

Sol. (i) Here L = 1 H, $X_L = 3142\ \Omega$

∴ Frequency, $\nu = \frac{X_L}{2\pi L} = \frac{3142}{2 \times 3.142 \times 1} = 500$ Hz [∵ $X_L = 2\pi\nu L$]

(ii) $X_C = X_L = 3142\ \Omega$

But $X_C = \frac{1}{2\pi\nu C}$

∴ $C = \frac{1}{2\pi\nu X_C} = \frac{1}{2 \times 3.142 \times 500 \times 3142} = 0.11 \times 10^{-6}\ F = 0.11\ \mu F$

Q. 5. An a.c. circuit consists of only an inductor of inductance 2 H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference (V_{eff}) across the inductor. ($\pi = 3.14$)

Sol. Here L = 2 H, $I_0 = 0.25$ A, f = 60 Hz

Inductive reactance, $X_L = \frac{V_{eff}}{I_{eff}}$

∴ $V_{eff} = X_L . I_{eff} = 2\pi f L . \frac{I_0}{\sqrt{2}}$

$= 2 \times 3.14 \times 60 \times 2 \times \frac{0.25}{1.414}$ V = 133.2 V

Q. 6. **Alternating emf, $\mathcal{E} = 220 \sin 100\pi t$ is applied to a circuit containing an inductance of $1/\pi$ H. Write an equation for instantaneous current through the circuit. What will be the reading of an a.c. ammeter if connected in the circuit**

Sol. Alternating emf, $\mathcal{E} = 220 \sin 100\pi t$

Comparing with $\mathcal{E} = \mathcal{E}_0 \sin 2\pi ft$, we get $\mathcal{E}_0 = 220$ V, f = 50 Hz

Current amplitude, $I_0 = \frac{\mathcal{E}_0}{\omega L} = \frac{\mathcal{E}_0}{2\pi fL} = \frac{220}{2\pi \times 50 \times 1/\pi} = 2.2$ A

Since the current in an inductive circuit lag behind the emf in phase by $\pi/2$ radian, therefore, instantaneous current through the circuit is $I = I_0 \sin(100\pi t - \pi/2) = 2.2 \sin(100\pi t - \pi/2)$

The a.c. ammeter will read the rms value of current, $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2.2}{\sqrt{2}} = 1.556$ A

Q. 7. ***An inductor of inductance 200 mH is connected to an a.c. source of peak emf 210 V and frequency 50 Hz. Calculate the peak current. What is the instantaneous voltage of the source when the current is at its peak value?***

Sol. Here L = 200 mH = 0.2 H, $\mathcal{E}_0 = 210$ V, f = 50 Hz

Peak current, $I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\mathcal{E}_0}{2\pi fL} = \frac{210}{2 \times 3.14 \times 50 \times 0.2} = 3.3$ A

As in an inductive a.c. circuit, current lags behind the emf by $\pi/2$, so the voltage is zero when the current is at its peak value.

Q. 8. ***A 1.50 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?***

Sol. Here C = 1.50 μF = 1.50×10^{-6} F, $\mathcal{E}_{rms} = 220$ V, f = 50 Hz

Capacitive reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 1.50 \times 10^{-6}} = 212\ \Omega$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{X_C} = \frac{220}{212} = 1.04 \text{ A}$$

Peak current, $I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 1.04 = 1.47$ A.

The current in the circuit oscillates between + 1.47 A and – 1.47 A and is ahead of emf by 90°. Now, $X_C \propto \frac{1}{f}$

If frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

Q. 9. ***A capacitor of 1 μF is connected to an a.c. source of emf $\mathcal{E} = 250 \sin 100\pi t$. Write an equation for instantaneous current through the circuit and give reading of a.c. ammeter connected in the circuit.***

Sol. Here C = 1 μF = 10^{-6} F, $\mathcal{E}_0 = 250$ V, $\omega = 100\pi$ rad s^{-1}.

The instantaneous current through the circuit,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = \omega C\mathcal{E}_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

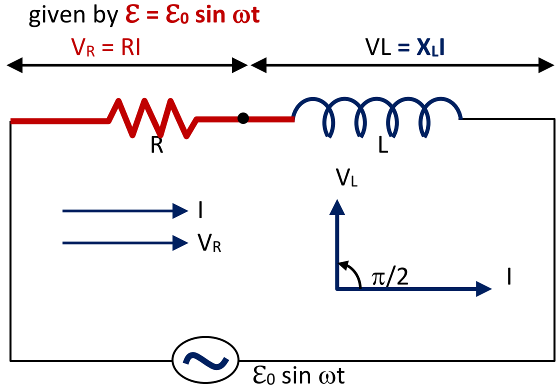
$$= 2 \times 3.14 \times 50 \times 10^{-6} \times 250 \sin\left(100\pi t + \frac{\pi}{2}\right)$$

$$= 0.0786 \sin\left(100\pi t + \frac{\pi}{2}\right)$$

Reading of the a.c. ammeter is $I_{rms} = 0.707\, I_0 = 0.707 \times 0.0786 \simeq 0.06$ A.

A.C. CIRCUIT WITH RESISTANCE AND INDUCTANCE IN SERIES

Consider a resistor R and inductance L connected in series to a source of alternating emf $\mathcal{E}$ given by $\mathcal{E} = \mathcal{E}_0 \sin \omega t$



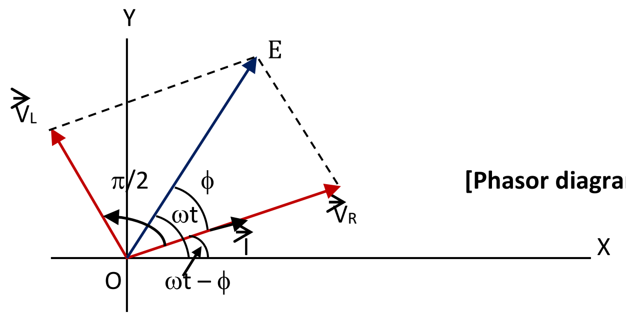
[A series LR – circuit]

Let I be the current through the series circuit at any instant. Then

◆1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance R will be in phase with current $\vec{I}$. So phasors $\vec{V}_R$ and $\vec{I}$ are in same direction, as shown in Fig. The amplitude of $\vec{V}_R$ is

◆2. Voltage $\vec{V}_L = X_L \vec{I}$ across the inductance L is ahead of current $\vec{I}$ in phase by π/2 rad. So phasor $\vec{V}_L$ lies π/2 rad anticlockwise w.r.t. the phasor $\vec{I}$. Its amplitude is , $V_0^L = I_0 X_L$
where X_L is the inductive reactance.

By parallelogram law of vector addition, $\vec{V}_R + \vec{V}_L = \vec{E}$



[Phasor diagram for series LR-circuit]

Using Pythagorean theorem, we get

$$E_0^2 = (V_0^R)^2 + (V_0^L)^2 = (I_0R)^2 + (I_0X_L)^2 = I_0^2 (R^2 + X_L^2)$$

or $$I_0 = \frac{E_0}{\sqrt{R^2 + X_L^2}}$$

Clearly, $\sqrt{R^2 + X_L^2}$ is the effective resistance of the series LR circuit which opposes or impedes the flow of a.c. through it. It is called its impedance and is denoted by Z. Thus

The effective opposition offered by the LR series combination to ac is called impedance (Z) of LR circuit.

Therefore, I = E/Z.............................. (ii)

From (i) & (ii) $$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

◆The phase angle ϕ between the resultant voltage and current is given by

$$\tan\phi = \frac{V_0^L}{V_0^R} = \frac{I_0X_L}{I_0R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

◆The phasor diagram that the current lags behind the emf by phase angle ϕ, so the instantaneous value of current is given by

$$I = I_0 \sin(\omega t - \phi)$$

◆◆Numerical Examples based on series LR-circuit

◆ **FORMULA USED**

1. Impedance, $Z = E_{rms} = \sqrt{R^2 + L^2} = \sqrt{R^2 + \omega^2 L^2}$

2. Current, $I_{rms} = \frac{E_{rms}}{Z}$

3. Phase angle ϕ given by $\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R}$ or $\cos\phi = \frac{R}{Z}$

4. Instantaneous current, $I = I_0 \sin(\omega t - \phi)$

◆ **UNITS USED** : R, X_L and Z are all in ohm, inductance L in henry and angular frequency ω in rad s^{-1}.

Q. 1. When an inductor L and a resistor R in series are connected across a 12 V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by π/3 radian. Calculate the value of R.

Sol. Here, $\mathcal{E}_{rms} = 12$ V, f = 50 Hz, $I_{rms} = 0.5$ A, ϕ = π/3 rad

Impedance, $Z = \frac{\mathcal{E}_{rms}}{I_{rms}} = \frac{12}{0.5} = 24\ \Omega$

As $\cos\phi = \frac{R}{Z}$ ∴ $R = Z\cos\phi = 24 \cos \pi/3 = 24 \times ½ = 12\ \Omega$

Q. 2. A bulb of resistance 10 Ω, connected to an inductor of inductance L, is in series with an a.c. source marked 100 V, 50 Hz. If the phase angle between the voltage and current is π/4 radian, calculate the value of L.

Sol. Here R = 10 Ω, f = 50 Hz, ϕ = π/4 rad

As $\tan\phi = \frac{X_L}{R} = \frac{2\pi f L}{R}$

∴ $L = \frac{R\tan\phi}{2\pi f} = \frac{10 \times \tan \pi/4}{2 \times 3.142 \times 50} = 0.0318$ H.

Q. 3. ***A coil of resistance 300 Ω and inductance 1.0 H is connected across an alternating voltage of frequency 300/2 π Hz. Calculate the phase difference between the voltage and current in the circuit.***

Sol. Here R = 300 Ω, L = 1.0 H, f = $\frac{300}{2\pi}$ Hz

$\tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R} = \frac{2\pi \times 300 \times 1.0}{2\pi \times 300} = 1$ ∴ Phase difference, $\phi = 45°$

Q. 4. ***A coil 'when connected across a 10 V d.c. supply draws a current of 2 A. When it is connected across a 10 V – 50 Hz a.c. supply, the same coil draws a current of 1 A. Explain why it draws lesser current in the second case. Hence determine the self inductance of the coil.***

Sol. The coil draws lesser current in the second case because of the reactance offered by the inductor.

In case of d.c., V = 10 V, I = 2 A

∴ $R = \frac{V}{I} = \frac{10}{2} = 5\ \Omega$

In case of a.c., $\mathcal{E}_{eff} = 10$ V, $I_{eff} = 1$ A

∴ $Z = \frac{\mathcal{E}_{eff}}{I_{eff}} = \frac{10}{1} = 10\ \Omega$

Inductive reactance,

$X_L = \sqrt{Z^2 - R^2} = \sqrt{10^2 - 5^2} = 5\sqrt{3}\ \Omega$ or $2\pi f L = 5\sqrt{3}$

∴ $L = \frac{5\sqrt{3}}{2\pi f} = \frac{5\sqrt{3}}{2 \times 3 \times 50} = 0.0288$ H.

Q. 5. ***An 80 V, 800 W heater is to be operated on a 100 V, 50 Hz supply. Calculate the inductance of the choke required.***

Sol. As P = VI ∴ $I = \frac{P}{V} = \frac{800}{80} = 10$ A and $R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$

As the choke is connected in series with the heater, the current should remain same for the impedance adjusted.

∴ $I_{eff} = \frac{V_{eff}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{V_{eff}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$

or $10 = \frac{100}{\sqrt{8^2 + 4\pi^2 \times 50^2 \times L^2}}$

or $64 + 10000\,\pi^2 L^2 = 100$

or $L^2 = \frac{36}{10000\,\pi^2}$ or $L = \underline{\quad 6 \quad} = 0.019$ H

Q. 6. ***A student connects a long air core coil of manganin wire to a 100 V d.c. source and records a current of 1.5 A. When the same coil is connected across 100 V, 50 Hz a.c. source the current reduces to 1.0 A.***
(i) Give reason for this observation. ***(ii) Calculate the value of the reactance of the coil.***

Sol. (i) For d.c. circuit, resistance

$R = \frac{V}{I} = \frac{100}{1.5} = \frac{200}{3} = 66.7\ \Omega$

For a.c. circuit, impedance

$Z = \frac{V_{eff}}{I_{eff}} = \frac{100}{1} = 100\ \Omega$ As the effective resistance of the coil is greater for a.c. than for d.c. so the current decreases in a.c. circuit.

(ii) As $Z = \sqrt{R^2 + X_L^2}$

∴ $X_L = \sqrt{Z^2 - R^2} = 100^2 - \left(\frac{200}{3}\right)^2$

$= \frac{100\sqrt{5}}{3} = \frac{100 \times 2.2361}{3} = 74.53\ \Omega$

Q. 7. ***When 200 volts d.c. are applied across a coil, a current of 2 ampere flows through it. When 200 volts a.c. of 50 cps are applied to the same coil, only 1.0 ampere flows. Calculate the resistance, impedance and inductance of the coil.***

Sol. (i) For d.c circuit, V = 200 V, I = 2 A

∴ Resistance, $R = \frac{V}{I} = \frac{200}{2} = 100\ \Omega$

(ii) For a.c. circuit, $\mathcal{E}_{eff} = 200$ V, $I_{eff} = 1.0$ A, f = 50 Hz.

(iii) Let L be the inductance of the coil. Then

$\omega^2 L^2 = Z^2 - R^2 = 200^2 - 100^2 = 30{,}000$ [∵ $Z = \sqrt{R^2 + \omega^2 L^2}$]

or $\omega L = 100\sqrt{3}\ \Omega$

∴ $L = \frac{100\sqrt{3}}{\omega} = \frac{100\sqrt{3}}{2\pi f} = \frac{100\sqrt{3}}{2 \times 3.14 \times 50} = 0.55$ H

Q. 8. ***A 60 – 10 W electric lamp is to be run on 100 V – 60 Hz mains. (i) Calculate the inductance of the choke required. (ii) If a resistor is to be used in place of choke coil to achieve the same result, calculate its value.***

Sol. Here $\mathcal{E}_{eff} = 60$ V, P = 10 W

Resistance of the lamp, $R = \frac{\mathcal{E}^2_{eff}}{P} = \frac{60 \times 60}{10} = 360\ \Omega$

Current through the lamp, $I_{eff} = \frac{P}{\mathcal{E}_{eff}} = \frac{10}{60} = \frac{1}{6}$ A

(i) $\mathcal{E}'_{eff} = 100$ V, f = 60 Hz

Required impedance, $Z = \frac{\mathcal{E}'_{eff}}{I_{eff}} = \frac{100}{1/6} = 600\ \Omega$

Required impedance, $Z = \frac{\mathcal{E}'_{eff}}{I_{eff}} = \frac{100}{1/6} = 600\ \Omega$

Reactance of required choke = $\sqrt{Z^2 - R^2}$

or $X_L = \sqrt{600^2 - 360^2} = 480\ \Omega$

Inductance of required choke,

$$L = \frac{X_L}{2\pi f} = \frac{480}{2 \times 31.4 \times 60} = 1.273 \text{ H}$$

(ii) Value of resistance required in place of choke

= 600 – 360 = 240 Ω

Q. 9. ***A 12 Ω resistance and an inductance of 0.05/π H are connected in series. Across the ends of the circuit is connected a 130 V a.c. supply of 50 Hz. Calculate (i) the current in the circuit and (ii) phase difference between the current and voltage.***

Sol. Here R = 12 Ω, $L = \frac{0.05}{\pi}$ H, $\mathcal{E}_{rms} = 130$ V, f = 50 Hz.

Impendence of the LR-circuit

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$$

$$= \sqrt{12^2 + 4\pi^2 \times 2500 \times \frac{25 \times 10^{-4}}{\pi^2}}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13\ \Omega$$

(i) Current in the circuit,

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{130}{13} = 10 \text{ A}$$

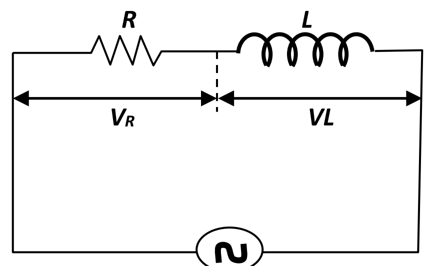
(ii) Phase difference ϕ is given by

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi f l}{R} = \frac{2\pi \times 50 \times 0.05}{12 \times \pi} = 0.4167$$

∴ $\phi = \tan^{-1}(0.4167) = 22.6°$

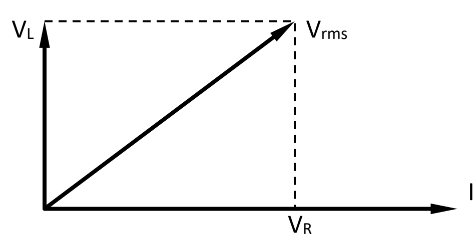
Here the voltage leads the current by a phase angle of 22.6°.

Q. 10. ***The a.c. circuit shown in Fig., has a choke L and a resistance R. The potential difference across the resistance R is V_R = 160 V] and that across the choke is V_L = 120 V. Find the virtual value of the applied voltage. If the virtual current in the circuit be 1.0 A, then calculate the total impedance of circuit. If the direct current be passed in the circuit, then what will be the potential difference in the circuit?***



[$\mathcal{E} = \mathcal{E}_0 \sin\omega t$]

Sol. As V_R is in phase with current I and V_L is 90° ahead of current I in phase, so the phase difference between V_R and V_L is 90°, as shown in Fig.

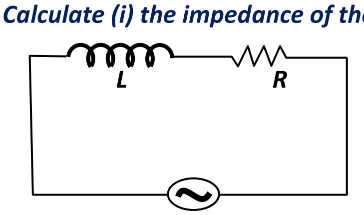


∴ $V_{rms} = \sqrt{V^2_R + V^2_L} = \sqrt{160^2 + 120^2} = 200$ V

Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \frac{200}{1.0} = 200\ \Omega$

When direct current ($\omega = 0$) is passed, reactance ωL becomes zero. ∴ P.D. in the circuit = P. D. across R = 160 V

Q. 11. ***In the circuit shown in Fig. the potential difference across the inductor L and resistor R are 120 V and 90 V respectively and the rms value of current is 3 A. Calculate (i) the impedance of the circuit and (ii) the phase angle between the voltage and current.***



Sol. (i) The voltages across L and C are 90° out of these. Their resultant voltage is

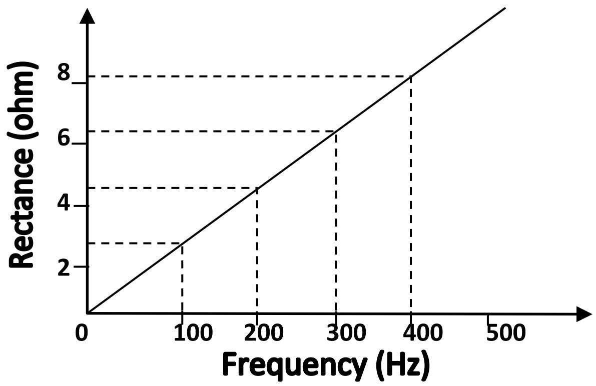
$V_{rms} = \sqrt{V_R^2 + V_L^2} = \sqrt{90^2 + 120^2} = 150$ V

$I_{rms} = 3$ A

∴ Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \frac{150}{3} = 50\ \Omega$

(ii) $\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{120}{90} = \frac{4}{3}$ ∴ Phase angle, $\phi = \tan^{-1} 4/3 = 53.1°$

Q. 12. ***Fig. shows how the reactance of an inductor varies with frequency.***
(i) Calculate the value of the inductance of the inductor using the information given in the graph.
(ii) If this inductor is connected in series to a resistor of 8 ohm, find what would be the impedance at 300 Hz?



Sol. (i) Inductance, $L = \frac{X_L}{2\pi f}$

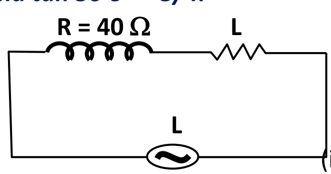
$= \frac{1}{2\pi} \times$ slope of $X_L - f$ graph

$= \frac{1}{2\pi} \times \frac{8-0}{400-0} = \frac{1}{100\pi} = 3.18 \times 10^{-3}$ H

(ii) From the given graph, when f = 300 Hz, $X_L = 6\ \Omega$

∴ Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{8^2 + 6^2} = 10\ \Omega$

Q. 13. ***In the circuit shown in Fig., the current is found to lag behind the voltage by an angle of 36.9°. Calculate the***
(i) Inductive reactance, ***(ii) impedance of the circuit,***
(iii) Current flowing in the circuit, and ***(iv) frequency of the applied emf.***
Take L = 0.1 H, cos 36.9° = 4/5 and tan 36-9° = 3/4.



Sol. (i) As $\tan\phi = \frac{X_L}{R}$

∴ $X_L = R \tan\phi = 40 \tan 36.9° = 40 \times ¾ = 30\ \Omega$

(ii) Impedance, $Z = \sqrt{R^2 + X_L^2}$

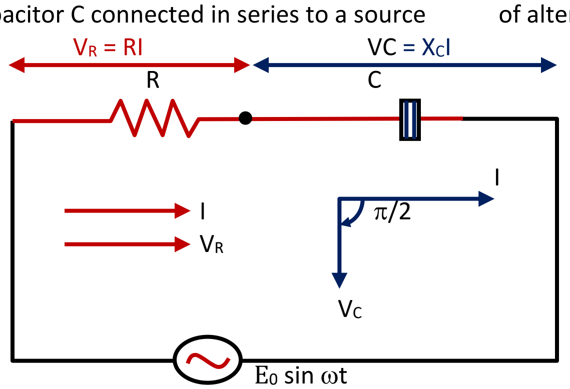
$= \sqrt{40^2 + 30^2} = 50\ \Omega$

(iii) $I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{110}{50} = 2.2$ A

(iv) Frequency, $f = \frac{X_L}{2\pi L} = \frac{30}{2\pi \times 0.1} = 47.75$ A

A.C. CIRCUIT WITH RESISTANCE AND CAPACITOR IN SERIES

Consider a resistor R and capacitor C connected in series to a source of alternating emf E given by $E = E_0 \sin \omega t$



[Series LR – circuit]

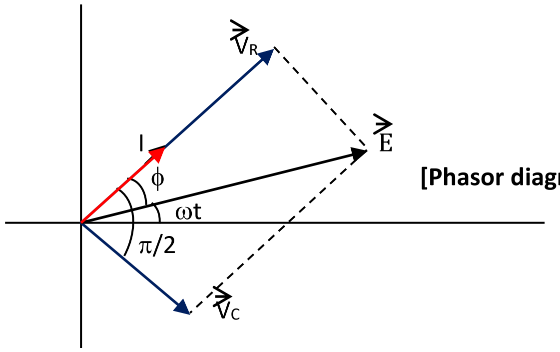
Let I be the current through the series circuit at any instant. Then

◆1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance R will be in phase with current $\vec{I}$. So phasors $\vec{V}_R$ and $\vec{I}$ are in same direction, as shown in Fig. The amplitude of $\vec{V}_R$ is $\mathbf{V_0^R = I_0^R}$

◆2. Voltage $\vec{V}_C = X_C\vec{I}$ across the capacitance C lags behind the current $\vec{I}$ in phase by π/2 rad. So phasor $\vec{V}_C$ lies π/2 clockwise w.r.t. the phasor $\vec{I}$. Its amplitude is

$\mathbf{V_0^C = I_0 X_C}$ where X_C is the capacitive reactance.

By parallelogram law of vector addition, $\vec{V}_R + \vec{V}_L = \vec{E}$



[Phasor diagram for a series CR-circuit]

By parallelogram law of vector addition,

$\vec{V}_R + \vec{V}_C = \vec{E}$

Using Pythagorean theorem, we get

$E_0^2 = (V_0^R)^2 + (V_0^C)^2 = (I_0R)^2 + (I_0X_C)^2$

$= I_0^2\,(R^2 + X_C^2)$

or $I_0 = \dfrac{E_0}{\sqrt{R^2 + X_C^2}}$

Clearly, $\sqrt{R^2 + X_C^2}$ **is the effective resistance of the series CR circuit which opposes or impedes the flow of current through it and is called its impedance and is denoted by Z.** Thus

$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + 1/\omega^2C^2}$ [∵ $X_C = 1/\omega C$]

The phase angle ϕ between the resultant voltage and current is given by

$\tan\phi = \dfrac{V_0^C}{V_0^R} = \dfrac{I_0X_C}{I_0R} = \dfrac{X_C}{R} = \dfrac{1/\omega C}{R}$

◆ From the phasor diagram that the current ahead of emf by phase angle ϕ, so the instantaneous value of current is $I = I_0 \sin(\omega t + \phi)$

◆◆Numerical Examples based on series LR-circuit

◆ **FORMULA USED**

1. Impedance, $Z = \mathcal{E}_{rms} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$

2. Current, $I_{rms} = \dfrac{\mathcal{E}_{rms}}{Z}$

3. Phase angle ϕ is given by

$\tan\phi = \dfrac{X_C}{R} = \dfrac{1/\omega C}{R}$ or $\cos\phi = \dfrac{R}{Z}$

4. Instantaneous current, $I = I_0 \sin(\omega t + \phi)$

◆ **UNITS USED**

R, X_C and Z are all in ohm, capacitance C in farad and angular frequency ω in rad s^{-1}.

Q. 1. What is the value of current in the a.c. circuit containing R = 10 Ω, C = 50 μC in series across 200 V, 50 Hz a.c. source?

Sol. Here R = 10 Ω, C = 50 μF = 50 × 10^{-6} F, V_{eff} = 200 V, f = 50 Hz

$Z = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}}$

$= \sqrt{10^2 + \dfrac{1}{4\pi^2 \times (50)^2 \times (50 \times 10^{-6})^2}}$

$= \sqrt{100 + 4053} = 64.4\ \Omega$

Current, $I_{eff} = \dfrac{V_{eff}}{Z} = \dfrac{200}{64.4}$ A = 3.10 A

Q. 2. ***When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current flows through the circuit but it leads the applied voltage by π/2 radian. (i) Name the device X and Y. (ii) Calculate the current flowing in the circuit, when same voltage is applied across the series combination of X and Y.***

Sol. (a) Device X is a resistor and Y is a capacitor.

(b) Here $R = X_C = \frac{\mathcal{E}_{eff}}{I_{eff}} = \frac{220}{0.5} = 440\ \Omega$

When X and Y are connected in series, their impedance becomes

$Z = \sqrt{R^2 + X^2 C} = \sqrt{440^2 + 440^2}$

$= \sqrt{387200} = 622.25\ \Omega$

Current, $I_{eff} = \frac{\mathcal{E}_{eff}}{Z} = \frac{220}{622.25} = 0.35$ A

Q. 3. ***A series circuit contains a resistor of 20 Ω, a capacitor and an ammeter of negligible resistance. It is connected to a source of 220 V – 50 Hz. If the reading of the ammeter is 2.5 A, Calculate the reactance of the capacitor.***

Sol. Here $R = 20\ \Omega$, $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz, $I_{rms} = 2.5$ A

Impedance, $Z = \frac{\mathcal{E}_{rms}}{I_{rms}} = \frac{220}{2.5} = 88\ \Omega$

But, $Z = \sqrt{R^2 + X_C^2}$

∴ $X_C = \sqrt{Z^2 - R^2} = \sqrt{88^2 - 20^2}$

$= \sqrt{(88 + 20)(80 - 20)} = \sqrt{108 \times 68} = 85.7\ \Omega$

Q. 4. ***An alternating current of 1.5 mA rms and angular frequency w = 100 rad s⁻¹ flows through a 10 k Ω resistor and 0.50 μF capacitor in series. Calculate the value of rms voltage across the capacitor and the impedance of the circuit.***

Sol. Here $\omega = 100$ rad s^{-1}, $I_{rms} = 1.5$ mA $= 1.5 \times 10^{-3}$ A

$R = 10$ k $\Omega = 10^4\ \Omega$, $C = 0.50\ \mu F = 0.5 \times 10^{-6}$ F

Impedance, $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$= \sqrt{(10^4)^2 + \frac{1}{(100)^2 \times (0.5 \times 10^{-6})^2}}$

$= \sqrt{10^8 + 4 \times 10^8} = \sqrt{5 \times 10^8} = 2.23 \times 10^4\ \Omega.$

The rms voltage across the capacitor is, $V'_{rms} = X_C I_{rms} = \frac{1}{\omega C} \times I_{rms}$

$= \frac{1}{100 \times 0.5 \times 10^{-6}} \times 1.5 \times 10^{-3}$ V $= 30$ V

Q. 5. ***A 20 V – 5 W lamp is to run on 200 V – 50 Hz a.c. mains. Find the capacitance of a capacitor required to run the lamp.***

Sol. Current rating of the lamp,

$I = \frac{P}{V} = \frac{5}{20} = 0.25$ A

Resistance of the lamp, $R = \frac{V}{I} = \frac{20}{0.25} = 80\ \Omega$

In order to run the lamp on 200 V – 50 Hz a.c. mains, a capacitor of capacitance C must be connected in series to increase the effective resistance so that current through the lamp does not exceed 0.25 A. Then

$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2} = \sqrt{80^2 + \left(\frac{1}{314\ C}\right)^2}$

As $I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$

∴ $0.25 = \frac{200}{\sqrt{80^2 + \frac{1}{314\ C}^2}}$

or $80^2 + \frac{1}{(314\ C)^2} = \left(\frac{200}{0.25}\right)^2 = 800^2$

or $\frac{1}{(314\ C)^2} = 800^2 - 80^2 = 880 \times 720$

or $\frac{1}{314\ C} = \sqrt{880 \times 720} = 796$

∴ $C = \frac{1}{314 \times 796} = 4.0 \times 10^{-6}$ F $= 4.0\ \mu F$

Q. 6. ***A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source. (a) Calculate the current in the circuit: (b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.***

Sol. Here, $R = 200\ \Omega$, $C = 15.0\ \mu F = 15.0 \times 10^{-6}$ F, $V_{rms} = 220$ V, $f = 50$ Hz

(a) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 50 \times 15.0 \times 10^{-6}} = 212.3\ \Omega$

$Z = \sqrt{R^2 + X^2 C} = \sqrt{(200)^2 + (212.3)^2} = 291.5\ \Omega$

Therefore, the current in the circuit is

$I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{220\text{ V}}{291.5\ \Omega} = 0.755$ A

(b) As the current is same throughout the series circuit, we have

$V^R{}_{rms} = I_{rms}.\ R = 0.755 \times 200 = 151$ V

$V^C{}_{rms} = I_{rms}.\ X_C = 0.755 \times 212.3 = 160.3$ V

The algebraic sum of the two voltages, V_R and V_C is 311.3 V which is more than the source voltage of 220 V. These two voltages are 90° out of phase. These cannot be added like ordinary numbers. The voltages is obtained by using Pythagoras theorem,

$V_R + C = \sqrt{V^2{}_R + V^2{}_C} = \sqrt{(151)^2 + (160.3)^2} = 220$ V

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

$V_R + C = \sqrt{V^2{}_R + V^2{}_C} = \sqrt{(151)^2 + (160.3)^2} = 220$ V

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

Q. 7. ***In a series R-C circuit, R = 300 Ω, C = 0.25 μF, V = 100 V and ω = 10,000 rad s⁻¹. Find the current in the circuit and calculate the voltage across the resistor and the capacitor.***
Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Sol. Here $R = 30\ \Omega$, $C = 0.25 \times 10^{-6}$ F, $V_{rms} = 100$ V, $\omega = 10{,}000$ rad s^{-1}

$X_C = 1 = \dfrac{1}{10^4 \times 0.25 \times 10^{-6}} = 400\ \Omega$

$Z = \sqrt{R^2 + X^2{}_C} = \sqrt{30^2 + 400^2} = \sqrt{160900} = 401.1\ \Omega$

$I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{100}{401.1} = 0.25$ A

$V^R{}_{rms} = I_{rms}.\ R = 0.25 \times 30 = 7.5$ V

$V^C{}_{rms} = I_{rms}.\ X_C = 0.25 \times 400 = 100$ V

Yes, the algebraic sum of the voltages across R and C is more than the source voltage of 100 V. This is due to the fact that these voltages are not in the same phase.

Q. 8. ***An a.c. circuit consists of a series combination of circuit elements 'X' and 'Y'. The current is ahead of the voltage in phase by π/4. If element 'X' is a pure resistor of 100 Ω, (i) name the circuit element 'Y' and (ii) Calculate the rms value of current, if rms value of voltage is 141 V.***

Sol. (i) The circuit element 'Y' is a capacitor. (ii) Phase angle $\phi = \dfrac{\pi}{4}$

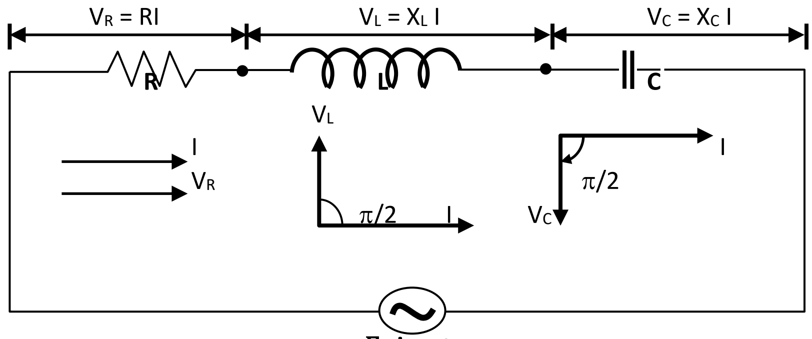
But $\cos\phi = \dfrac{R}{Z}$ ∴ $\cos \pi = \underline{100\ \Omega}$

or $Z = \dfrac{100}{1/\sqrt{2}} = 100\sqrt{2} = 100 \times 1.414 = 141.4\ \Omega$

$I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{141\text{ V}}{141.4\ \Omega} = 1$ A

◆◆ SERIES LCR - CIRCUIT

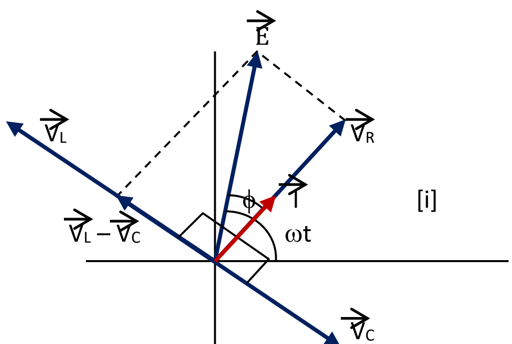
Suppose a resistance R, an inductance L and capacitance C are connected in series to a source of alternating emf Ɛ given by **E= E_0 sin ωt**



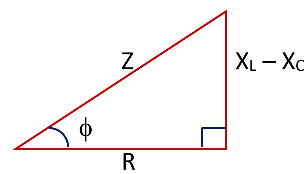
Let I be the current in the series circuit at any instant. Then

◆1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance R will be in phase with current $\vec{I}$. So, phasors $\vec{V}_R$ and $\vec{I}$ are in same direction,

■ The amplitude of $\vec{V}_R$ is **$V_0^R = I_0R$**



[Phasor diagram for a series LCR circuit when $V_L > V_C$]



[Impedance triangle when $X_L > X_C$]

◆2. **Voltage $\vec{V}_L = X_L\vec{I}$** across the inductance L is ahead of current I in phase by π/2 rad. So, phasor $\vec{V}_L$ lies π/2 rad anticlockwise w.r.t. the phasor $\vec{I}$. ■ Its **amplitude is** **$V_0^L = I_0 X_L$**

◆3. **Voltage $\vec{V}_C = X_C\vec{I}$** across the capacitance C lags behind the current I in phase by π/2 rad. So, phasor $\vec{V}_C$ lies π/2 clockwise w.r.t. the phasor $\vec{I}$. ■ **Its amplitude is** **$V_0^C = I_0 X_C$**

As $\vec{V}_L$ and $\vec{V}_C$ are in opposite directions, their resultant is $(\vec{V}_L - \vec{V}_C)$. By parallelogram law, the resultant of $\vec{V}_R$ and $(\vec{V}_L - \vec{V}_C)$ must be equal to the applied emf $\vec{E}$, given by the diagonal of the parallelogram.

Using Pythagorean theorem, we get

$$E_0^2 = (V_0^R)^2 + (V_0^L - V_0^C)^2$$
$$= (I_0 R)^2 + (I_0 X_L - I_0 X_C)^2$$
$$= I_0^2 [R^2 + (X_L - X_C)^2]$$

or $$I_0 = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Clearly, $\sqrt{R^2 + (X_L - X_C)^2}$ is the effective resistance of the series LCR circuit which opposes or impedance the flow of current through it and is called its ***impedance***. It is denoted by Z and its SI unit is ohm (Ω). Thus

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The relationship between the resistance R, inductive reactance X_L, capacitive reactance X_C and the impedance Z is shown in Fig. The right angled Δ OAP is called the ***impedance triangle***.

■ SPECIAL CASES:

⬢1. When **$X_L > X_C$ or $V_L > V_C$,** we see from Fig.[i] that emf if ahead of current by phase angle ϕ which is given by

$\tan\phi = \frac{X_L - X_C}{R}$ or $\cos\phi = \frac{R}{Z}$

The instantaneous current in the circuit will be $I = I_0 \sin(\omega t - \phi)$

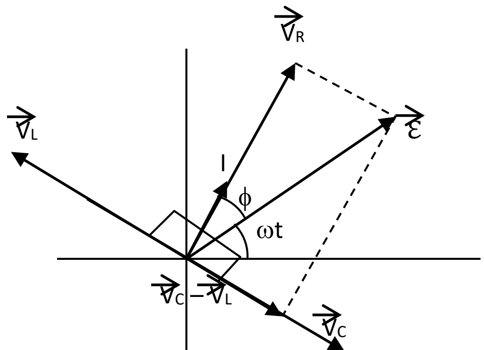
The series LCR-circuit is said to be inductive.

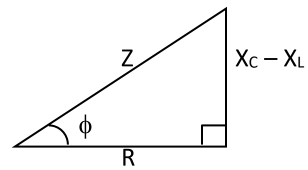
2. When $X_L < X_C$ or $V_L < V_C$, we see from Fig. that current is ahead of emf by phase angle ϕ which is given by

$\tan\phi = \frac{X_C - X_L}{R}$ or $\cos\phi = \frac{R}{Z}$

The instantaneous current in circuit will be $I = I_0 \sin(\omega t + \phi)$

The series LCR-circuit is said to be capacitive.





[Impedance triangle when $X_L > X_C$]

[Phasor diagram for a series LCR circuit when $V_L > V_C$]

3. When $X_L = X_C$ or $V_L = V_C$, $\phi = 0$, the emf and current will be in the same phase.

The series LCR-circuit said to be purely resistive.

$I_0 = \frac{E_0}{Z}$ or $\frac{I_0}{\sqrt{2}} = \frac{E_0}{\sqrt{2}\,Z}$ or $I_{rms} = \frac{E_{rms}}{Z}$

Susceptance: The reciprocal of the reactance of an a.c. circuit is called its susceptance. Its SI unit is ohm^{-1} or mho.

Admittance:

Reciprocal of impedance of a circuit is called Admittance of the circuit. i.e. Admittance (A) = 1/Z

---- Unit of impedance (Z) of the circuit is ohm.

---- Unit of admittance of the circuit is ohm^{-1} i.e., mho or seimen.

Impedance and Admittance...

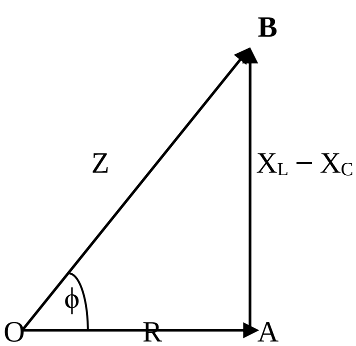
The total effective opposition offered by LCR circuit to alternating current is known as Impedance. In general, impedance (Z) comprises of three parts i.e., resistance (R), inductive reactance (X_L) and capacitive reactance (X_C), where X_L and X_C are opposite to each other. In series LCR circuit, the total reactance is taken as ± (X_L – X_C) reciprocal of reactance is known as susceptance. Impedance (Z) of LCR circuit can be represented diagrammatically by Impedance triangle as shown in fig.

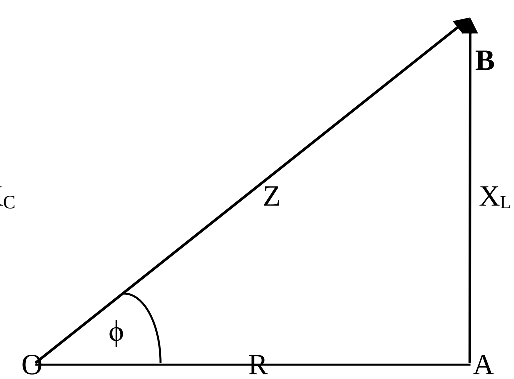
$$Z = \sqrt{R^2 + \left[L\omega - \frac{1}{C\omega}\right]^2} = \sqrt{R^2 + (X_C - X_L)^2}$$

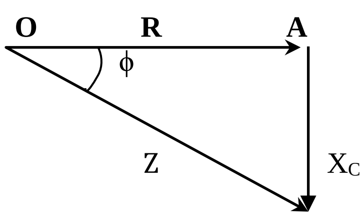
Impedance of LR circuit is given by

$$Z = \sqrt{R^2 + (L\omega)^2} = \sqrt{R^2 + X_L^2}$$

And is represented by fig.







The phase difference between current and voltage is

$$\tan\phi = \frac{L\omega}{R} = \frac{X_L}{R}$$

This shows that current leads the voltage by an angle of ϕ

Impedance of CR circuit is given by

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} = \sqrt{R^2 + X_C^2}$$

The voltage lags behind the current by an angle $\tan\phi = \frac{\frac{1}{C\omega}}{R} = \frac{1}{C\omega R} = \frac{X_C}{R}$

RESONANCE CONDITION OF A SERIES LCR-CIRCUIT

A series LCR circuit is said to be in the resonance condition when the current through it has its maximum value.

The current amplitude I_0 for a series LCR-circuit is given by

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Clearly, I_0 becomes zero both for $\omega \to 0$ and $\omega \to \infty$. The value of I_0 is maximum when

$$\omega L - \frac{1}{\omega C} = 0 \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

Then impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R$

….. the impedance is minimum. **The circuit is purely resistive.** The current and voltage are in the same phase and the current in the circuit is maximum. **This condition of the LCR-circuit is called resonance condition.**

The frequency at which the current amplitude I_0 attains a peak value is called natural or resonant frequency of the LCR-circuit and is denoted by ν_r.

Determination of resonant frequency: $\omega_r = 2\pi\nu_r = \frac{1}{\sqrt{LC}}$

or $\nu_r = \frac{1}{2\pi\sqrt{LC}}$

The current amplitude at resonant frequency will be

$$I_0 = \frac{E_0}{R}$$

IMPORTANT CHARACTERISTICS OF THE SERIES RESONANT CIRCUIT.

1. **Resonance occurs in a series LCR-circuit when** $X_L = X_C$.
2. **Resonant frequency,** $\nu_r = \frac{1}{2\pi\sqrt{LC}}$
3. **The impedance is minimum and purely resistive.**
4. **The current has a maximum value of** (E_0/R) **at resonant condition.**
5. **The power dissipated in the circuit is maximum and is equal to** E^2_{rms}/R.
6. **The current is in phase with the voltage or the power factor is unity** ($\cos\phi = 1$ when $\phi = 0$)
7. **Series resonance can occur at all values of resistance R.**
8. **The voltage across R is equal to the applied emf.**
9. **The voltages across L and C are equal and have a phase difference of 180° and so their resultant is zero.**
10. **The voltages across L and C are very high as compared to the applied voltage. Hence a series LCR-circuit is used to obtain a large magnification of a.c. voltage.**

◆11. *The series resonant circuit is also called an acceptor circuit. When a number of frequencies are fed to it, it accepts only one frequency ν_r and rejects the other frequencies. The current is maximum for this frequency.*

Conceptual tips.................................

☑ Resonance occurs in a series LCR-circuit when $X_L = X_C$ or $\omega_r = 1/\sqrt{LC}$. For resonance to occur, the presence of both L and C elements in the circuit is essential. Only then the voltages L and C (being 180° out of phase) will cancel each other and current amplitude will be E_0/R i.e., the total source voltage will appear across R. So we cannot have resonance in LR-and LC-circuits.

❑❑ QUALITY FACTOR OF RESONANCE CIRCUIT:

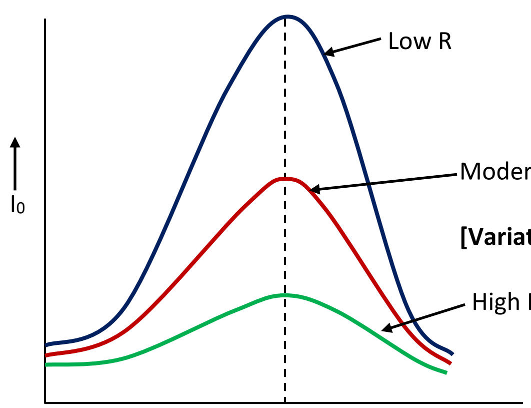
Q factor of series LCR circuit is defined as 2π times the ratio of the energy stored in the circuit to the energy dissipated in resistance per cycle of a.c. supply.

i.e. $$Q = \frac{2\pi \times \text{energy stored in the circuit per cycle}}{\text{energy dissipated per cycle}} \quad \text{------------------------(i)}$$

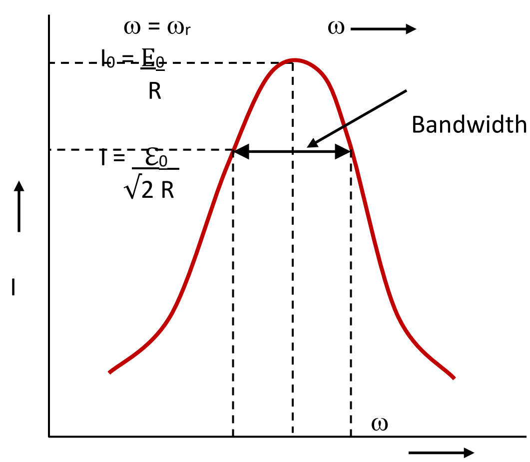
• It measures the ability of the circuit to differentiate between different frequencies of nearly equal magnitude. It is proportion to the sharpness of the resonance curve. Sharper the resonance curve, larger is the Q factor.

direction. **The resonant frequency is independent of R**, but the **sharpness of peak depends on R.** The peak is higher for smaller values of R. **Thus the resonance is sharp for small R and a flat one for large R.**

❑ *The sharpness of resonance is measured by a coefficient called the quality or Q-factor of the circuit.*



[Variation of current amplitude with frequency in an LCR-circuit]



EXPRESSION: ---

In an LCR circuit, maximum energy is stored in the inductor when the current through it is maximum i.e. at resonance. On the other hand, maximum energy is stored in the capacitor when voltage across it is maximum.

Thus, the total energy stored in the circuit remains the same.

Maximum energy stored = $\frac{1}{2} L I_0^2$ ----------------------- (ii)

Now, energy dissipated per cycle at resonance is in the form of heat energy produced in the resistance R in time period T.

Therefore, **Energy dissipated per cycle = $I^2_{rms} RT$** ----------------------- (iii)

Using (ii) & (iii) in eqn. (i), we get

$$Q = \frac{2\pi \times \frac{1}{2} LI_0^2}{I^2_{rms} RT} = \frac{\pi LI_0^2}{I^2_{rms} RT} \quad \text{------------------------ (iv)}$$

Now $T = \frac{1}{\nu_r}$, where ν_r is resonant frequency and $I_{rms} = \frac{I_0}{\sqrt{2}}$

Therefore, equation (iv) becomes

$$Q = \frac{\pi LI_0^2}{\left(\frac{I_0^2 R}{2}\right) \times \frac{1}{\nu_r}} = \frac{2\pi \nu_r L}{R} \quad \text{------------------------ (v)}$$

Putting $2\pi \nu_r = \omega_0$

Therefore,

$$\boldsymbol{Q = \frac{\omega_0 L}{R}}$$

Since $\omega_0 = \frac{1}{\sqrt{LC}}$

Therefore, Eqn. (v) can be written as

$$Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

✱Q factor of series resonance circuit is also referred to as **voltage multiplier** of the circuit ***because it can also be defined in terms of voltages. It is the ratio of voltage across the capacitor or inductor to the voltage across resistor at resonance.***

i.e. $Q = \dfrac{\text{Voltage across C or L}}{\text{Voltage across R}}$

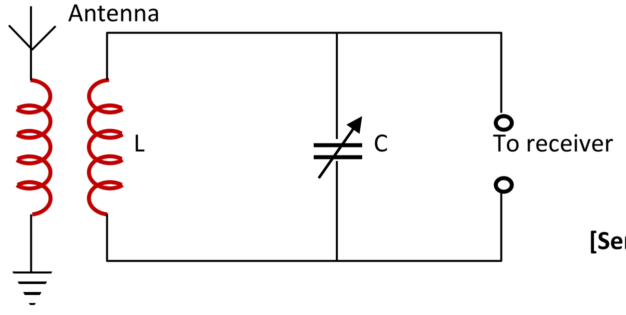
Values of Q: Being ratio of same quantities, Q is just a number. It normally varies from 10 to 100. In VHF circuits, its value may be very large

Importance of Q: Circuits having large factors are more selective and have numerous applications in electronics e.g., the tuning of a radio set to a particular frequency. There are many signals in air whose frequencies are very close to each other.

A radio set is tuned to a station by turning the tuning knob. When we turn the tuning knob of the radio, we basically change the value of the capacitance of the capacitor of LC circuit. Thus, the natural frequency of the LC circuit is adjusted till it matches the frequency (υ) of the desired signal and the radio catches desired station. Hence, we can select the desired form a large number of signals of nearly same frequencies.

✱ We see that if Q-factor is large i.e., if R is low or L is large, that bandwidth 2 $\Delta\omega$ is small. This means that the resonance is sharp or the series resonant circuit is more selective.

✱ Tuning of radio receiver: The tuning circuit of a radio or TV is an example of LCR resonant circuit. Signals are transmitted by different stations at different frequencies. These frequencies are picked up by the antenna and corresponding to these frequencies, a number of voltages appear across the series LCR-circuit. But maximum current flows through the circuit for that a.c. voltage which has frequency equal to $\nu_r = \frac{1}{2\pi\sqrt{LC}}$. If Q-value of the circuit is large, the signals of the other stations will be very weak. By changing the value of the adjustable capacitor C, the signal from the desired station can be tuned in.



[Series resonant circuit]

Examples based on series LCR-circuit, its Resonance and Q-Factor

◆ **FORMULA USED**

1. Impedance of a series LCR-circuit

$$Z = E_{rms} = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

2. Phase angle ϕ between current and voltage is given by

$$\tan\phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \cos\phi = \frac{R}{Z}$$

3. Resonant frequency of LCR-series circuit (when $X_L = X_C$), $f_r = \frac{1}{2\pi\sqrt{LC}}$

4. Q-Factor $= \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

where ω_1 and ω_2 are the frequencies at which current falls to $1/\sqrt{2}$ times its resonant value.

◆ **UNITS USED**

R, X_L, X_C and Z are all in ohm, inductance L in henry, capacitance C in farad, angular frequencies ω_1, ω_2 and ω_r in rad s^{-1}.

Q. 1. ***Determine the impedance of a series LCR-circuit if the reactance of C and L are 250 Ω and 220 Ω respectively and R is 40 Ω.***

Sol. Here $X_C = 250\ \Omega$, $X_L = 220\ \Omega$, $R = 40\ \Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{40^2 + (220 - 250)^2} = \sqrt{1600 + 900} = 50\ \Omega$

Q. 2. ***A resistor of 50 ohm, an inductor of (20/π) H and a capacitor of (5/π) μF are connected in series to a voltage source 230 V, 50 Hz. Find the impedance of the circuit.***

Sol. Here $R = 50\ \Omega$, $L = \frac{20}{\pi}$ H,

$C = \frac{5}{\pi}\ \mu F = \frac{5}{\pi} \times 10^{-6}$ F

$E_{eff} = 230$ V, f = 50 Hz

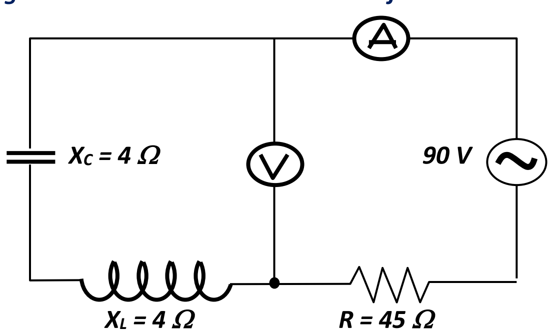
$X_L = 2\pi f L = \frac{20}{\pi} \times 2 \times \pi \times 50 = 2000\ \Omega$

$$X_C = \frac{1}{C \times 2\pi f} = \frac{1}{5/\pi \times 10^{-6} \times 2 \times \pi \times 50} = 2000\ \Omega$$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(50)^2 + (2000 - 2000)^2} = \sqrt{2500}\ \Omega = 50\ \Omega$

Q. 3. ***What will be the readings in the voltmeter and ammeter of the circuit shown in Fig.?***



Sol. Impedance of the circuit is

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{45^2 + (4-4)^2} = 45\ \Omega$

Reading of the ammeter $= I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{90}{45} = 2$ A

Reading of the voltmeter $= (X_L - X_C)\, I_{rms} = (4 - 4) \times 2 = 0$

Q. 4. ***A 0.3 H inductor, 60 μF capacitor and a 50 Ω resistor are connected in series with a 120 V, 60 Hz supply. Calculate***
(i) Impedance of the circuit ***(ii) Current flowing in the circuit***

Sol. Here L = 0.3 H, C = 60 μF = 60×10^{-6} F, R = 50 Ω, $V_{eff} = 120$ V, f = 60 Hz

(i) Inductive reactance, $X_L = 2\pi f L = 2 \times 3.14 \times 60 \times 0.3 = 113.04\ \Omega$

Capacitive reactance, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 60 \times 60 \times 10^{-6}}$

Net reactance $= X_L - X_C = 113.04 - 44.23 = 68.81\ \Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (68.81)^2}$

$= \sqrt{2500 + 4734.8} = \sqrt{7234.8} \simeq 85\ \Omega$

(ii) Current in the circuit is $I_{eff} = \dfrac{V_{eff}}{Z} = \dfrac{120}{85} = 1.41$ A.

Q. 5. A resistor of 12 ohm, a capacitor of reactance 14 ohm and a pure inductor of inductance 0.1 henry are joined in series and placed across a 200 volt, 50 Hz a.c. supply. Calculate: (i) The current in the circuit and (ii) The phase angle between the current and the voltage. Take π = 3 for purpose of calculations.

Sol. Here $R = 12\ \Omega$, $X_C = 14\ \Omega$, $L = 0.1$ H

$\mathcal{E}_{eff} = 200$ V, f = 50 Hz

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and, $X_L = \omega L = 2\pi f L = 2 \times 3 \times 50 \times 0.1 = 30\ \Omega$

$\therefore$ $Z = \sqrt{400} = 20\ \Omega$

(i) The current in the circuit,

$I_{eff} = \dfrac{\mathcal{E}_{eff}}{Z} = \dfrac{200}{20} = 10$ A

(ii) The phase angle ϕ between the current and voltage is given by

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{30 - 14}{12} = \dfrac{16}{12} = \dfrac{4}{3} = 1.3333$

$\therefore$ $\phi = \tan^{-1}(1.3333) \simeq 53.1°$

Q. 6. A 100 mH inductor, a 20 μF capacitor and a 10 ohm resistor are connected in series to a 100 V, 50 Hz a.c. source. Calculate: (i) Impedance of the circuit at resonance (ii) Current at resonance (iii) Resonant frequency

Sol. Here L = 100 mH = 0.1 H, f = 50 Hz, C = 20 μF = 2×10^{-5} F, R = 10 Ω, $\mathcal{E}_{rms} = 100$ V

(i) Impedance at resonance, $Z = R = 10\ \Omega$

(ii) Current at resonance,

$I_{rms} = \dfrac{\mathcal{E}_{rms}}{Z} = \dfrac{100}{10} = 10$ A

(ii) Resonant frequency

$f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2 \times 3.14 \times \sqrt{0.1 \times 2 \times 10^{-5}}} = 112.6$ Hz.

Q. 7. A series LCR circuit consists of a resistance of 10 Ω, a capacitor of reactance 60 Ω and an inductor coil. The circuit is found to resonate when put across 300 V, 100 Hz supply. Calculate (i) The inductance of the coil (ii) Current in the circuit at resonance.

Sol. Here $R = 10\ \Omega$, $X_C = 60\ \Omega$, $V_{eff} = 300$ V, f = 100 Hz

(i) At resonance, $X_L = X_C$ or $2\pi f L = 60$

$\therefore$ Inductance, $L = \dfrac{60}{2\pi f} = \dfrac{60}{2 \times 3.14 \times 100} = 0.095$ H

(ii) Current in the circuit at resonance is

$I_{eff} = \dfrac{V_{eff}}{R} = \dfrac{300}{10} = 30$ A

Q. 8. A resistance of 2 ohms, a coil of inductance 0.01 H are connected in series with a capacitor, and put across a 200 volt, 50 Hz supply. Calculate (i) The capacitance of the capacitor so that the circuit resonates. (ii) The current and voltage across the capacitor at resonance. (Take π = 3)

Sol. Here $R = 2\ \Omega$, L = 0.01 H, $\mathcal{E}_{eff} = 200$ V, f = 50 Hz

(i) Resonance frequency, $f = \dfrac{1}{2\pi\sqrt{LC}}$

$\therefore$ $C = \dfrac{1}{4\pi^2 f^2 L} = \dfrac{1}{4 \times (3)^2 \times (50)^2 \times (0.01)}$

$= \dfrac{1}{4 \times 9 \times 2500 \times 0.01} = \dfrac{1}{900}$

$= 0.0011\ F = 11 \times 10^{-4}$ F.

(ii) $I_{eff} = \dfrac{\mathcal{E}_{eff}}{2} = \dfrac{200}{2} = 100$ A

$\therefore$ $V_C = I_{eff} X_C = I_{eff} \dfrac{1}{2\pi f C} = \dfrac{100}{2 \times 3 \times 50 \times 11 \times 10^{-4}}$

$= \dfrac{100 \times 10^4}{3300} = 303.03$ V.

Q. 9. An inductor coil joined to a 6 V battery draws a steady current of 12 A. This coil is connected in series to a capacitor and a.c. source of alternating emf 6 V. If the current in the circuit is in phase with the emf, find the rms current.

Sol. Resistance of the coil,

$R = \dfrac{V}{I} = \dfrac{6}{12} = 0.5\ \Omega$

In the a.c. circuit, the current is in phase with the emf. $\therefore$ Impedance, $Z = R = 0.5\ \Omega$, $I_{rms} = \dfrac{\mathcal{E}_{rms}}{Z} = \dfrac{6}{0.5} = 12$ A

Q. 10. A radio wave of wavelength 300 m can be transmitted by a transmission centre. A condenser of capacity 2.4 μ F is available. Calculate the inductance of the required coil for resonance.

Sol. Frequency of the radio wave, $v = \frac{c}{\lambda} = \frac{3 \times 10^8}{300} = 10^6$ Hz

For resonance,

$$v = \frac{1}{2\pi\sqrt{LC}}$$

or $V^2 = \frac{1}{4\pi^2 LC}$

$\therefore$ Inductance, $L = \frac{1}{4\pi^2 v^2 C}$

$$= \frac{1}{4 \times 9.87 \times (10^6)^2 \times 2.4 \times 10^{-6}}$$

$= 1.055 \times 10^8$ H.

Q. 11. A 25.0 mF capacitor, a 0.10 henry inductor and a 25.0-ohm resistor are connected in series with an A.C. source whose emf is given by $\mathcal{E}$ = 310 sin 314 t (volt). (i) What is the frequency of the emf? (ii) What is the reactance of the circuit? (iii) What is the impedance of the circuit? (iv) What is the current of the circuit? (v) What is the phase angle of the current by which it leads or lags the applied emf? (vi) What is the expression for the instantaneous value of current in the circuit? (vii) What are the effective voltages across the capacitor, the inductor and the resistor? (viii) Construct a vector diagram for these voltages. (ix) What value of inductance will make the impedance of circuit minimum?

Sol. (i) Given $\mathcal{E} = 310 \sin 314\, t$ (volt)

Comparing it with $\mathcal{E} = \mathcal{E}_0 \sin 2\pi ft$, we get

$2\pi f = 314$ or $f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = 50$ Hz

(ii) $X_C = \frac{1}{2\pi fC} = \frac{1 \times 7}{2 \times 22 \times 50 \times 25 \times 10^{-6}} = 127.3\ \Omega$ [$\because$ 1 μF = 10^{-6} F]

$X_L = 2\pi f L = 2 \times \frac{22}{7} \times 50 \times 0.1 = 31.4\ \Omega$

As X_L and X_C are out of phase by 180°, therefore, Net reactance = $X_C - X_L$ = 127.3 – 31.4 = 95.9 Ω and it is capacitive.

(iii) Impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(25)^2 + (95.9)^2}$

$= \sqrt{625 + 9196.81} = \sqrt{9821.81} = 99.1\ \Omega$

(iv) Effective current, $I_{eff} = \frac{\mathcal{E}_{eff}}{Z}$

But $\mathcal{E}_{eff} = \frac{\mathcal{E}_0}{\sqrt{2}} = \frac{310}{\sqrt{2}} = 220$ V

$\therefore$ $I_{eff} = \frac{220}{99.1} = 2.22$ A

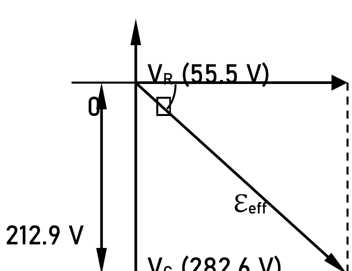
(v) The phase and ϕ is given by

$\tan\phi = \frac{X_C - X_L}{R} = \frac{95.9}{25} = 3.84$

Hence ϕ = 75.4° or 1.31 rad.

As the circuit is capacitive, the current leads the voltage by 75.4°.

(viii) Vector diagram of voltages is shown in Fig.



(vi) The instantaneous current is given by

$I = I_0 \sin(2\pi ft + \phi)$

But $I_0 = I_{eff}\sqrt{2} = 2.22\sqrt{2} = 3.13$ A

$\therefore$ $I = 3.13 \sin(314\, t + 1.31)$

(vii) Effective voltage across the capacitor is

$V_C = I_{eff} = X_C = 2.22 \times 127.3 = 282.6$ V

Effective voltage across the inductor is

$V_L = I_{eff} X_L = 2.22 \times 31.4 = 69.7$ V

Effective voltage across the resistor

$VR = I_{eff} R = 2.22 \times 25 = 55.5$ V

(ix) Impedance, $Z = \sqrt{R + (X_L - X_C)^2}$

Z is minimum, if $X_L = X_C$

or if $2\pi f L = \frac{1}{2\pi f C}$

or $L = \frac{1}{4\pi^2 f^2 C} = \frac{7 \times 7}{4\,(22)^2\,(50)^2 \times 25 \times 10^{-6}}$

= 0.405

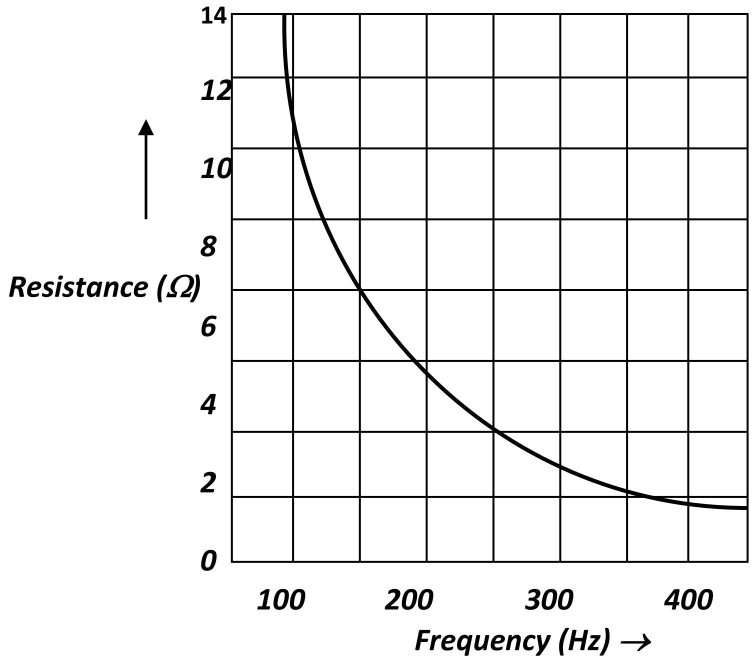
Q. 12. Fig. given below shows how the reactance of a capacitor varies with frequency.

(i) Use the information on graph to calculate the value of capacity of the capacitor.

(ii) An inductor of inductance 'L' has the same reactance as the capacitor at 100 Hz. Find the value of L.

(iii) Using the same axes, draw a graph of reactance against frequency for the inductor given in part (ii).

(iv) If this capacitor and inductor were connected in series to a resistor of 10 Ω, what would be the impedance of the combination at 300 Hz?



Sol. (i) For f = 100 Hz, $X_C = 6\ \Omega$

As $X_C = \dfrac{1}{2\pi f C}$

∴ $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi \times 100 \times 6} = 2.65 \times 10^{-4}$ F

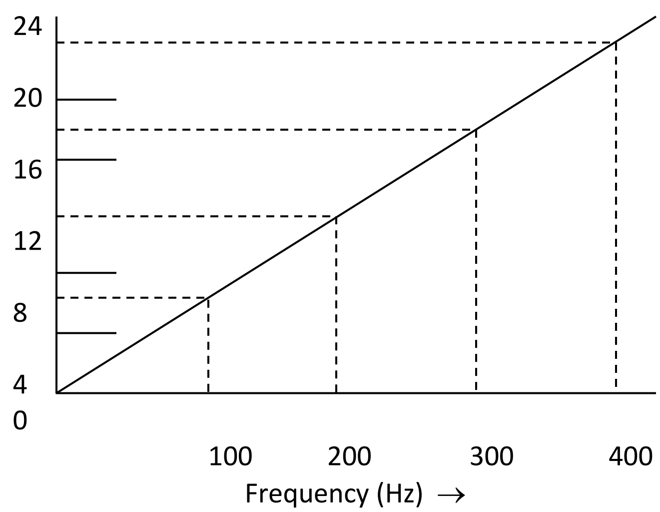
(ii) For f = 100 hz, $X_L = X_C = 6\ \Omega$

As $X_L = 2\pi f L$

∴ $L = \dfrac{X_L}{2\pi f} = \dfrac{6}{2\pi \times 100} = 9.459 \times 10^{-3}$ H

(iii) As $X_L \propto f$, so values of X_L at difference values of f are as follows:

f (Hz)	100	200	300	400
X_L (Ω)	6	12	18	24



(iv) Now f' = 300 Hz

$X_C' = \dfrac{f}{f'} .\ X_C = \dfrac{100}{300} \times 6 = 2\ \Omega$

$X_L' = \dfrac{f}{f} - X_L = \dfrac{300}{} \times 6 = 18\ \Omega$

$Z = \sqrt{R^2 + (XL' - XC')^2} = \sqrt{10^2 + (18-2)^2} = \sqrt{3.56} = 18.87\ \Omega$

Q. 13. A 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an a.c. source. What should be the frequency of the a.c. source, for which the current drawn in the circuit is maximum? If the peak value of emf of the source is 200 V, find for maximum current: (i) The inductance and capacitor reactances of the circuit, (ii) total impedance of the circuit (iii) peak value of current in the circuit, (iv) the phase difference between voltages across inductor and resistor, and (v) the phase difference between voltages across inductor and capacitor.

Sol. Here $C = 2\ \mu F = 2 \times 10^{-6}$ F, $R = 100\ \Omega$, $L = 8$ H, $\mathcal{E}_0 = 200$ V

The current drawn in the circuit will be maximum when the frequency of the a.c. source is equal to the resonant frequency f_r of the circuit.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8 \times 2 \times 10^{-6}}} = \frac{10^3}{8\pi} = 39.8 \text{ Hz}$$

(i) X_L and X_C at resonant frequency,

$$= 2\pi f_r L = 2\pi \times \frac{10^3}{8\pi} = 2000\ \Omega$$

(ii) Total impedance at resonance,

$$Z = R = 100\ \Omega$$

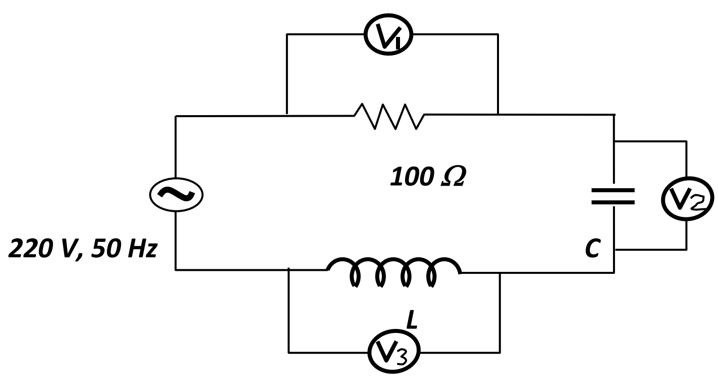
(iii) Peak value of current,

$$I_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{R} = \frac{200}{100} = 2 \text{ A}$$

(iv) Phase difference between voltages across inductor and resistor = 90°

(v) Phase difference between voltages across inductor and capacitor = 180°

Q. 14. A series LCR-circuit is connected to an a.c. source (220 V – 50 Hz), as shown in Fig. If the voltages of the three voltmeters V_1, V_2 and V_3 are 65 V, 541 V and 204 V respectively, calculate:
(i) the current in the circuit, (ii) the value of the inductor L,
(iii) the value of the capacitor C, and (iv) the value of C (for the same L) required to produce resonance.



Sol. Here $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz, $R = 100\ \Omega$, $V_R = 65$ V, $V_C = 415$ V, V_L 204 V

(i) If I_{rms} be the current in the circuit, then

$$V_R = I_{rms} R$$

or $$I_{rms} = \frac{V_R}{R} = \frac{65}{100} = 0.65 \text{ A}$$

(ii) As $V_L = I_{rms} X_L$

∴ $$X_L = \frac{V_L}{I_{rms}} = \frac{204}{0.65} = 313.85\ \Omega$$

or $$2\pi f L = 313.85\ \Omega$$

or $$L = \frac{313.85}{2\pi f} = \frac{313.85}{2\pi \times 50} = 1.0 \text{ H}$$

(iii) As $V_C = I_{rms} X_C$

∴ $$X_C = \frac{V_C}{I_{rms}} = \frac{415}{0.65} = 638.46\ \Omega$$

But $$X_C = \frac{1}{2\pi f C}$$

∴ $$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 638.46} = 5 \times 10^{-6} \text{ F} = 5\ \mu\text{F}.$$

(iv) Suppose a capacitor of capacitance C′ produces resonance with an inductor of 1.0 H. Then

$$2\pi f L = \frac{1}{2\pi f C'}$$

∴ $$C' = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 1.0} = 5 \times 10^{-6} \text{ F} = 5\ \mu\text{F}.$$

Q. 15. In a series LCR-circuit, the resonant frequency is 800 Hz. The half power points are obtained at frequencies 745 and 855 Hz. Calculate the Q-factor of the circuit. Also calculate the bandwidth.

Sol. Here $f_r = 800$ Hz, $f_1 = 745$ Hz, $f_2 = 855$ Hz

(i) $$Q = \frac{f_r}{f_2 - f_1} = \frac{800}{855 - 745} = \frac{800}{110} = 7.27$$

(ii) Bandwidth $= f_2 - f_1 = 855 - 745 = 110$ Hz.

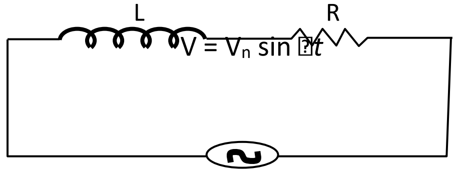
◆◆CHOKE COIL

◆**Choke coil:** A choke coil is simply an inductor with large inductance which is used to reduce current in a.c. circuits without much loss of energy.

◆**Principle:** The working of a choke is based on the fact that when a.c. flows through an inductor, current lags behind the emf by phase angle of $\pi/2$ rad.

◆**Construction:** It made of thick insulated copper wire wound closely in a large number turns over a soft-iron laminated core. Choke coil offers a large current $X_L = 2\pi\nu L$ to the flow of a.c. and hence current is reduced. Laminated core reduces losses due to eddy currents.

◆**Working:** As shown in Fig. a choke is put in series across an electrical appliance of resistance R and is connected to an a.c. source. This forms an LR-circuit. **[Chock coil]**



Average power dissipated per cycle in the circuit is

$$P_{av} = V_{eff} I_{eff} \cos\phi = V_{eff} I_{eff} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

Inductance L of the choke coil is very large so that $R << \omega L$. Then

$$\text{Power factor,} \quad \cos\phi = \frac{R}{\omega L} = 0$$

i.e., ◆ Average power dissipated by the coil is very small. As $Z = \sqrt{R^2 + \omega^2 L^2}$ IS large, so current is reduced without appreciable wastage of power.

◆**Preference of choke coil over the ohmic resistance:** A choke coil reduces current in a.c. circuit without consuming any power. When an ohmic resistance is used, current reduces but energy losses occur due to heating, So a choke coil is preferred.

Uses: The most common use of choke coil is in the fluorescent tubes with a.c. mains. If the tube is connected directly across 220 V source, it would draw large currents which would damage the tube. With the used of choke coil, the voltage is recued to an appropriate value, without wasting any power. Choke coils are also used in various electronic circuits, mercury lamp and in sodium vapour lamp.

◆◆POWER IN AN A.C. CIRCUIT

The rate at which electric energy is consumed in an electric circuit is called its power. In a d.c. circuit, power is given by the product of voltage and current. But in an a.c. circuit, both voltage E and current I vary sinusoidally with time and are generally not phase. So for an a.c. circuit, we define instantaneous power as the product of the instantaneous voltage and instantaneous current.

Suppose in an a.c. circuit, the voltage and current at any instant are given by

$$E = E_0 \sin\omega t$$

and $\quad I = I_0 \sin(\omega t - \phi)$ where ϕ is the phase angle by which the voltage $\mathcal{E}$ leads the current I.

The instantaneous power is given by

$$P = EI = E_0 I_0 \sin\omega t \cdot \sin(\omega t - \phi)$$
$$= E_0 I_0 \sin\omega t\,[\sin\omega t \cos\phi - \cos\omega t \sin\phi]$$
$$= E_0 I_0 [\sin^2\omega t \cos\phi - \sin\omega t \cos\omega t \sin\phi)$$
$$= \frac{E_0 I_0}{2} [(\mathbf{1 - \cos 2\omega t}) \cos\phi - \mathbf{\sin 2\omega t} \sin\phi]$$
$$= \frac{E_0 I_0}{2} [\cos\phi - (\cos 2\omega t \cos\phi + \sin 2\omega t \sin\phi)]$$
$$= \frac{E_0 I_0}{2} [\mathbf{\cos\phi - \cos(2\omega t - \phi)}]$$

If we assume the instantaneous power to remain constant for a small time dt, the work done during this time is

$$dW = Pdt = EI\,dt$$

Total work done over a complete cycle (i.e., from t = 0 to t = T) is

$$W = \int_0^T EI\,dt$$

Hence average power dissipated in the circuit over a complete cycle is

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T EI\,dt$$

$$= \frac{E_0 I_0}{2T}\left[\int_0^T \cos\phi\, dt - \int_0^T \cos(2\omega t - \phi)\, dt\right]$$

$$= \frac{E_0 I_0}{2T}\left[\cos\phi \,|\,t\,|_0^T - 0\right] = \frac{E_0 I_0}{2T}[\cos\phi\,(T-0)]$$

or $P_{av} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cdot \cos\phi$

or $\mathbf{P_{av} = E_{rms} I_{rms} \cos\phi = E_{rms} I_{rms} \cdot \frac{R}{Z}}$

Power Consumed in a series LCR circuit (Predominantly inductive)

Let in series LCR circuit, the phase difference between current and voltage be ϕ.
The instantaneous values of voltage and current in LCR circuit are given by

$E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

Therefore, instantaneous power input to LCR circuit are given by

$P_i = EI = E_0 I_0 \sin\omega t \sin(\omega t + \phi)$

$= E_0 I_0 \sin\omega t\,[\sin\omega t \cos\phi + \cos\omega t \sin\phi]$

[Since, sin (A+B) = sin A cos B + cos A sin B]

$= E_0 I_0 [\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi]$

$= E_0 I_0 \left\{\sin^2\omega t \cos\phi + \frac{2\sin\omega t \cos\omega t \sin\phi}{2}\right\}$

or $P_i = E_0 I_0 \left\{\sin^2\omega t \cos\phi + \frac{\sin 2\omega t \sin\phi}{2}\right\}$ -------------------- (i)

[Since, 2 sin A cos A = sin 2A]

The average power over a complete cycle of a.c. through LCR circuit is given by

$$P = \int_0^T \frac{P_i\, dt}{T}$$

Using eqn. (i) in eqn. (ii), we get

$$P = \frac{1}{T}\int_0^T E_0 I_0 \left\{\sin^2\omega t \cos\phi + \frac{\sin 2\omega t \sin\phi}{2}\right\} dt$$

$$= \frac{E_0 I_0}{T}\left\{\int_0^T \sin^2\omega t \cos\phi\, dt + \int_0^T \frac{\sin 2\omega t \sin\phi}{2}\, dt\right\}$$

$$= \frac{E_0 I_0}{T}\left\{\cos\phi \int_0^T \sin^2\omega t\, dt + \frac{\sin\phi}{2}\int_0^T \sin 2\omega t\, dt\right\} \quad \text{------------------------(iii)}$$

Now $\int_0^T \sin^2\omega t\, dt = \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt = \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right]$

$= \frac{1}{2}[T - 0] = \frac{T}{2}$ ---------------------------- (iv)

and $\int_0^T \sin 2\omega t\, dt = 0$ ---------------------------- (v)

Using eqns. (iv) & (v) in eqn. (iii). we get

$$P = \left\{\frac{E_0 I_0}{T} \cos\phi \times \frac{T}{2} + \sin\phi \times \frac{0}{2}\right\} = \frac{E_0 I_0 T}{2T}\cos\phi = \frac{E_0 I_0 \cos\phi}{2} = \frac{E_0}{\sqrt{2}}\,\frac{I_0}{\sqrt{2}}\cos\phi = E_{rms} I_{rms} \cos\phi$$

■ The quantity **'Cos ϕ'** is called the ***power factor***.

SPECIAL CASES:

1. **Pure resistive circuit:** Here the voltage and current are in same phase, i.e., $\phi = 0$ and $\cos\phi = 1$.

$\therefore$ $P_{av} = E_{rms} \cdot I_{rms} \times 1 = E_{rms} \cdot I_{rms} = \frac{E^2_{rms}}{R}$

2. **Pure inductive circuit:** Here the voltage leads the current in phase by $\frac{\pi}{2}$, i.e., $\phi = \frac{\pi}{2}$ $\quad\therefore\quad P_{av} = E_{rms}. I_{rms} \cos\frac{\pi}{2} = 0$

Thus, the average power consumed in an inductive circuit over a complete cycle is zero.

3. **Pure capacitive circuit:** Here the voltage lags behind the current in phase by $\frac{\pi}{2}$, i.e., $\phi = -\frac{\pi}{2}$

$\therefore \quad P_{av} = E_{rms}. I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0$

Thus, the average power consumed in a capacitive circuit over a complete cycle is also zero.

4. **Series LCR-circuit:** For a series LCR-circuit, $P_{av} = E_{rms} I_{rms} \cos\phi$, where $\phi = \tan^{-1}\frac{X_L - X_C}{R}$. Some, ϕ may have a non-zero value for series LR-, LC- and LCR-circuits. So power is consumed in such circuits, but only in the resistor R.

5. **Power dissipated at resonance in LCR-circuit.** At resonance, $X_L = X_C$. and $\phi = 0$.
So $\cos\phi = 1$, and $P_{av} = \mathcal{E}_{rms} I_{rms}$. **That is maximum power is dissipated in the circuit (through R) at resonance.**

◆■POWER FACTOR:

The average power of an a.c. circuit is given by $\mathbf{P_{av} = E_{rms} . I_{rms} \cos\phi}$

Average power = Virtual emf × Virtual current × cos ϕ

Ratio of true power and apparent power (virtual power) in an a.c. circuit is called as power factor of the circuit.

i.e. Power factor, $\cos\phi = \frac{P}{E_{rms} I_{rms}} = \frac{P}{P_{rms}}$

◆**Power factor ($\cos\phi$) is always positive and not more than 1.**

(i) **For circuit having pure resistor**, $\cos\phi = 1$ (Since, $\phi = 0$)

(ii) **For circuit having pure inductor or pure capacitor, $\cos\phi = 1$** (Since, $\phi = \pi/2$)

(iii) **For RC circuit**, $\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

(iv) For LR circuit, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$

✱**The product $\mathcal{E}_{rms}$. I_{rms} does not give the actual power and is called apparent power**. It gives actual or true power only when multiplied by factor cos ϕ. The factor cos ϕ is called the power factor of an a.c. circuit.

$\therefore$ **True power = Apparent power × Power factor.**

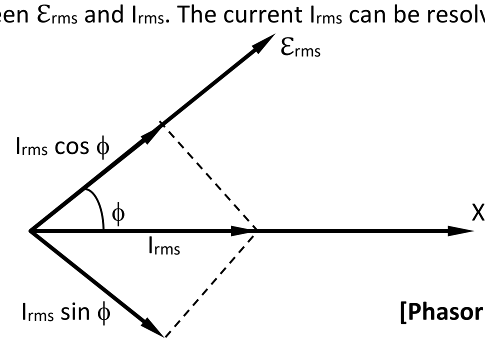
Thus, power factor may be defined as the ratio of the true power to the apparent power of an a.c. circuit. Its value varies from 0 to

◆■WATTLESS CURRENT

The current in a.c. circuit is said to be wattles if the average power consumed in the circuit is zero. The average power of an a.c. circuit is given by $P_{av} = E_{rms} I_{rms} \cos\phi$

■ ***Wattless current is that component of the circuit current due to which to power consumed in the circuit is zero.***

✱The phase and ϕ between $\mathcal{E}_{rms}$ and I_{rms}. The current I_{rms} can be resolved into two components:



[Phasor diagram]

(a) Component $I_{rms} \cos \phi$ along $\mathcal{E}_{rms}$. As the phase angle between $I_{rms} \cos \phi$ and $\mathcal{E}_{rms}$ is zero, therefore

$$P_{av} = \mathcal{E}_{rms} (I_{rms} \cos \phi) \cos 0 = \mathcal{E}_{rms} I_{rms} \cos \phi$$

(b) Component $I_{rms} \sin \phi$ normal to $\mathcal{E}_{rms}$. As the phase angle between $I_{rms} \sin \phi$ and $\mathcal{E}_{rms}$ is $\frac{\pi}{2}$, therefore

$$P_{av} = \mathcal{E}_{rms} (I_{rms} \sin \phi) \cos \frac{\pi}{2} = 0$$

The component I_{rms} sin ϕ as the idle or wattless current because it does not consume any power in a.c. circuit.

This happens in a purely inductive or capacitive circuit in which the voltage and current differ by a phase angle $\frac{\pi}{2}$, i.e., $\phi = \pm \frac{\pi}{2}$, so that

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos (\pm \pi / 2) = 0$$

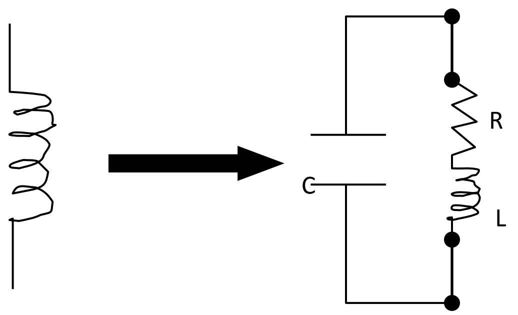
Thus, the current in the circuit has no power. It flows sometimes along the voltage and sometimes against the voltage, so that the net work done per cycle is zero. For example, when the secondary of a transformer is open, the current in the primary is almost wattless.

■ **Wattful current is that component of the circuit current due which the power is consumed in the circuit.**

◆■ Behaviour of Real or Ideal Resistor, Inductor and Capacitor:

(A) **Real resistors**. An ideal resistor has only ohmic resistance. But the real resistor, say a metallic wire possesses wire some inductance and capacitance in addition to resistance.

When current is passes through the metallic wire (resistor), magnetic field is set up around the wire, thus it has some inductance. Two current carrying parallel wires in the circuit possess some capacitance also.



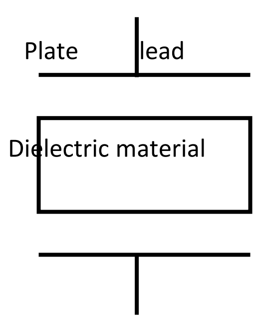
[i]

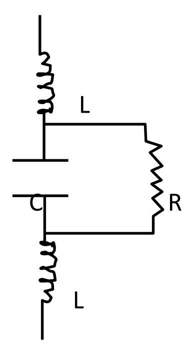
Thus, a metallic wire not only acts as a resistor but also as an inductor and capacitor.

(B) **Real inductors**. An ideal inductor has only inductance. But the real inductor consists of a conducting wire wound in the form of a coil. The conducting wire possesses some ohmic resistance. Each turn of the coil has some capacitance also. So a real inductor is equivalent to a LCR circuit as shown in fig. {i}

(C) **Real Capacitor**. A capacitor consists of two parallel plates separated by a dielectric. The dielectric has high resistance and the leads connected with the plates of the capacitor has some inductance.

So a real capacitor is equivalent to LCR circuit as shown in fig. {ii}





[ii]

• In an actual practice, the inductor is not pure but has some resistance may be very small. Due to this small value of resistance of an inductor, power is dissipated in the form of heart.

• Similarly, some power is also dissipated in the form of heat produced in a capacitor.

AVERAGE POWER ASSOCIATED WITH A RESISTOR

An ideal resistor dissipated power of V^2_{rms} / R in an a.c. circuit.

In case of a pure resistor, the voltage and current are always in same phase. So we can write the instantaneous values of voltage and current as :

$$V = V_0 \sin \omega t \quad \text{and} \quad I = I_0 \sin \omega t$$

Work done in small time dt will be

$$dW = P\,dt = VI\,dt = V_0 I_0 \sin^2 \omega t\,dt$$

$$= \frac{V_0 I_0}{2}(1 - \cos 2\omega t)\,dt$$

The average power dissipated per cycle in the resistor will be

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW = \frac{V_0 I_0}{2T}\int_0^T (1 - \cos 2\omega t)\,dt$$

$$= \frac{V_0 I_0}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T = \frac{V_0 I_0}{2T}[(T - 0) - 0]$$

$$= \frac{V_0 I_0}{2} = \frac{V_0^2}{2R}$$

or $P_{av} = \frac{V_0 I_0}{\sqrt{2}\sqrt{2}} = V_{rms} I_{rms} = \frac{V^2_{rms}}{R}$ $\left(\because \frac{V_0}{\sqrt{2}} = V_{rms}\right)$

ENERGY AND AVERAGE POWER ASSOCIATED WITH A PURE INDUCTOR

When an inductor is connected to a source of emf, the current starts growing through it. An induced emf is set up in the inductor which opposes the growth of current through it. The external source has to expend energy in building up the current through the inductor against the induced emf. The energy is stored in the inductor as magnetic field energy.

Let I be the current through the inductor L at any instant t. The current rises at the rate dI/dt. So the induced emf is

$$E = -L\frac{dI}{dt}$$

The work done against the induced emf in small time dt is

$$dW = P\,dt = -E\,I\,dt = +L\frac{dI}{dt}.I\,dt = LI\,dI$$

The total work done in building up the current from 0 to I_0 is

$$W = \int dW = \int_0^{I_0} LI\,dI = L\left[\frac{I^2}{2}\right]_0^{I_0} = \frac{1}{2}LI_0^2$$

This work done is stored as the magnetic field energy U in the inductor ∴ $U = \frac{1}{2} LI_0^2$

An ideal inductor connected to an a.c. source does not dissipate any power.

Average power associated with an inductor: When a.c. is applied to an ideal inductor, current lags behind the voltage in phase by $\pi/2$ radian. So, we can write the instantaneous values of voltage and current as follows:

$$V = V_0 \sin \omega t$$

and

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -I_0 \sin\left(\frac{\pi}{2} - \omega t\right) = -I_0 \cos \omega t$$

Work done in small time dt is $dW = P\,dt = -V_0 I_0 \sin \omega t \cos \omega t\,dt$

$$= -\frac{V_0 I_0}{2}\sin 2\omega t\,dt$$

Thus, the average power dissipated per cycle in an inductor is zero.

The average power dissipated per cycle in the inductor is

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW = -\frac{V_0 I_0}{2T}\int \sin 2\omega t\,dt$$

$$= +\frac{V_0 I_0}{2T}\left(\frac{\cos 2\omega t}{2\omega}\right)_0^T = \frac{V_0 I_0}{4T\omega}\left(\cos \frac{4\pi t}{T}\right)_0^T$$

$$= \frac{V_0 I_0}{4T\omega}[\cos 4\pi - \cos 0] = \frac{V_0 I_0}{4T\omega}[1 - 1]$$

Conceptual tips..

- The energy stored in an inductor resides in the region of its magnetic field.
- The average power consumed per cycle in an inductor connected to an a.c. source is zero. The physical meaning of this result is as follows. During the first quarter to each current cycle, as the current increases, the magnetic flux through the inductor builds up and energy is stored in the inductor from the external source. In the next quarter of cycle as the current decreases, the flux decreases and the stored energy is returned to the source. Thus, in half cycle, no net power is consumed by the inductor.

ENERGY AND AVERAGE POWER ASSOCIATED WITH A PURE CAPACITOR

Consider a capacitor of capacitance C. Suppose the displacement of charge q from one plate to another sets up a potential difference V between its plates. Then $V = \frac{q}{C}$

Suppose now a small additional charge dq be displaced from one place to another. Then work done is

$$dW = V\,dq = \frac{q}{C}\,dq$$

∴ Total work done in displacing a charge q from one plate to another is

$$W = \int_0^q dW = \int_0^q \frac{q}{C}\,dq = \frac{1}{2}\frac{q^2}{C}$$

This energy is stored as the electrostatic energy U in the capacitor.

∴ $U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2$ [∵ q = CV]

An ideal capacitor connected to an a.c. source does not dissipate any power.

Average power associated with a capacitor: When an a.c. is applied to a capacitor, the current leads the voltage in phase by $\pi/2$ radian. So we write the expressions for instantaneous voltage and current as follows:

$$V = V_0 \sin \omega t$$

and $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos \omega t$

Work done in the circuit in small time dt will be

$$dW = P\,dt = VI\,dt = V_0 I_0 \sin \omega t \cos \omega t\,dt = \frac{V_0 I_0}{2} \sin 2\omega t\,dt$$

The average power dissipated per cycle in the capacitor is

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW = \frac{V_0 I_0}{2T}\int_0^T \sin 2\omega t\,dt$$

$$= \frac{V_0 I_0}{2T}\left[-\frac{\cos 2\omega t}{2\omega}\right]_0^T$$

$$= -\frac{V_0 I_0}{4T\omega}\left[\cos \frac{4\pi t}{T}\right]_0^T$$

$$= -\frac{V_0 I_0}{4T\omega}[\cos 4\pi - \cos 0]$$

$$= -\frac{V_0 I_0}{4T\omega}[1 - 1] = 0$$

Thus the average power dissipated per cycle in a capacitor is zero.

Concetual tips......................................

- Energy stored in a capacitor resides in the region of its electric field.
- The external source has to supply an energy ½ CV^2 to charge a capacitor to a p.d. V but this energy is returned back during the discharging process. When the capacitor is connected across an a.c. source, it absorbs energy from the source for a quarter cycle as it is charged. It returns energy to source in the next quarter cycle as it is discharged. Thus, in a half cycle, no net power is consumed by the capacitor.

Examples based on Energy and power associated with A.C. Circuits

◆ Formula Used

1. Average power consumed per cycle in any a.c. circuit, $P_{av} = \mathcal{E}_{rms} I_{rms} \cos \phi$, $\mathcal{E}_{rms} I_{rms}$ is the apparent power
2. Power factor, $\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$
3. Average power consumed per cycle in a pure resistive circuit, $P_{av} = \frac{\mathcal{E}_0^2}{2R} = \mathcal{E}_{rms}.\ I_{rms} = \frac{\mathcal{E}^2_{rms}}{R}$
4. Energy stored in an inductor, $U = \frac{1}{2} LI^2$
5. Average power consumed per cycle in pure inductive circuit = 0
6. Energy stored in a capacitor, $U = \frac{1}{2} CV^2 = \frac{1}{2}\frac{Q^2}{C}$
7. Average power consumed per cycle in a pure capacitive circuit = 0
8. For an LCR-circuit in resonance, $X_L = X_C$ and $f_r = \frac{1}{2\pi\sqrt{LC}}$

◆ Units Used

Power P_{av} is in watt, current I_{rms} in ampere, voltage $\mathcal{E}_{rms}$ in volt, inductance L in henry, capacitance C in farad, energy U in joule and R, X_L, X_C and Z are all in ohm.

Q. 1. **A light bulb is rated at 100 W for a 220 V supply. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the rms current through the bulb.**

Sol. Here, $P_{av} = 100$ W, $V_{rms} = 220$ V

(a) $R = \frac{V^2_{rms}}{P_{av}} = \frac{(220)^2}{100} = 484\ \Omega$

(b) $V_0 = \sqrt{2}\, V_{rms} = 1.414 \times 220 = 311$ V

(c) $I_{rms} = \frac{Pav}{V_{rms}} = \frac{100}{220} = 0.45$ A

Q. 2. **A capacitor and a resistor are connected in series with an a.c. source. If the potential differences across C, R are 120 V, 90 V respectively and if the r.m.s. current of the circuit is 3 A, calculate the (i) impedance, (ii) power factor of the circuit.**

Sol. $\mathcal{E}_{rms} = \sqrt{V^2_R + V^2C} = \sqrt{90^2 + 120^2}$

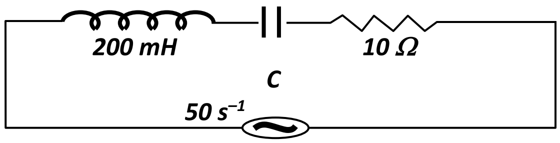
$= \sqrt{22500} = 150$ V

$I_{rms} = 3$ A

(i) Impedance, $Z = \frac{\mathcal{E}_{rms}}{I_{rms}} = \frac{150}{3} = 50\ \Omega$

(ii) Power factor, $\cos\phi = \frac{V_R}{\mathcal{E}_{rms}} = \frac{90}{150} = 0.6$

Q. 3. **In the following circuit, calculate (i) the capacitance 'C' of the capacitor, if the power factor of the circuit is unity, and (ii) also calculate the Q-factor of the circuit.**



Sol. (i) Power factor, $\cos\phi = \frac{R}{Z} = 1$

or $Z = R$

∴ $X_C = X_L$ or $\frac{1}{2\pi fC} = 2\pi fL$

or $C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 200 \times 10^{-3}}$ $= 5 \times 10^{-5}$ F $= 50\ \mu$F

(ii) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{5 \times 10^{-5}}} = 6.32$

Q. 4. **An alternating voltage $\mathcal{E}$ = 200 sin 300 sin 300 t is applied across a series combination of R = 10 Ω and an inductor of 800 mH. Calculate: (i) impedance of circuit (ii) peak value of current in the circuit (iii) power factor of the circuit.**

Sol. Given $\mathcal{E} = 200 \sin 300\, t$

Comparing with equation, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, we find that

$\mathcal{E}_0 = 200$ V, $\omega = 300$ rad s^{-1}

(i) Impedance, $Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{10^2 + (300)^2 \times (800 \times 10^{-3})^2} = 240.2\ \Omega$

(ii) Peak value of current, $I_0 = \frac{\mathcal{E}_0}{Z} = \frac{200}{240.2} = 0.832$ A

(iii) Power factor, $\cos\phi = \frac{R}{Z} = \frac{10}{240.2} = 0.041$

Q. **5.** **A 200 V variable frequency a.c. source is connected to a series combination of L = 5 H, C = 80 μF and R = 40 Ω. Calculate (i) angular frequency of the source to get maximum current in the circuit, (ii) the current amplitude at resonance and (iii) the power dissipated in the circuit.**

Sol. Here $\mathcal{E}_{rms} = 200$ V, $L = 5$ H, $C = 80\ \mu$F, $R = 40\ \Omega$

(i) Resonant angular frequency, $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50$ rad s^{-1}

(ii) At resonance, $Z = R = 40\ \Omega$

The current amplitude at resonance,

$I_0 = \frac{\mathcal{E}_0}{R} = \frac{\sqrt{2}\,\mathcal{E}_{rms}}{R} = \frac{1.414 \times 200}{40} = 7.07$ A

(iii) Power dissipated in the circuit

$= \frac{\mathcal{E}^2_{rms}}{R} = \frac{(200)^2}{40} = 1000$ W

Q. 6. **A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which R = 3 Ω, L = 25.48 mH, and C = 796 μF. Find (a) the impedance of the circuit, (b) the phase difference between the voltage across the source and the currents, (c) the power dissipated in the circuit, and (d) the power factor.**

Sol. Here $\mathcal{E}_0 = 283$ V, f = 50 Hz, R = 3 Ω, L = 25.48 × 10^{-3} H, C = 796 × 10^{-6} F

(a) $X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8\ \Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\ \Omega$

(b) Phase difference ϕ is given by

$\tan\phi = \frac{X_L - X_C}{R} = \frac{8-4}{3} = \frac{4}{3}$

∴ $\phi = \tan^{-1}\frac{4}{3} = 53.1°$

(c) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\mathcal{E}_0}{\sqrt{2}R} = \frac{1}{1.414} = \times \frac{283}{5} = 40$ A

Power dissipated in the circuit,

$P_{av} = I^2_{rms} R = (40)^2 \times 3 = 4800$ W

(d) Power factor = cos ϕ = cos 53.1° = 0.6

Thus, the current in the circuit lags behind the voltage across the source by a phase angle of 53.1°.

Q. 7. Suppose the frequency of the source in the previous example can be varied. (a) What is the frequency of the source at which resonance occurs? (b) Calculate the impedance, the current and the power dissipated at the resonant condition.

Sol. (a) Resonant frequency of the source,

$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}}$

$= \frac{221.1}{2 \times 3.14} = 35.4$ Hz

(b) At resonance, the impedance is

Z = R = 3 Ω

The rms current at resonance,

$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{0.707\ \mathcal{E}_0}{R} = \frac{0.707 \times 283}{3} = 66.7$ A.

The power dissipated at resonance is $P_{av} = I^2_{rms} R = (66.7)^2 \times 3W = 13.35$ kW.

Obviously, the power dissipated at resonance is more than the power dissipated in the non-resonant condition of the above example.

Q. 8. A virtual current of 4 A flows in a coil when it is connected in a circuit having alternating current of frequency 50 Hz. Power consumed in the coil is 240 W. Calculate the inductance of the coil if the virtual potential difference across it is 100 V.

Sol. Here $I_{eff} = 4$ A, f = 50 Hz, $V_{eff} = 100$ V, P = 240 W

$P = I^2_{eff} R$ ∴ 240 = 16 R

or $R = \frac{240}{16} = 15\ \Omega$; $Z = \frac{V_{eff}}{I_{eff}} = \frac{100}{4} = 25\ \Omega$

But $Z = \sqrt{R^2 + \omega^2 L^2}$ or $Z^2 = R^2 + \omega^2 L^2$

∴ $L = \frac{\sqrt{Z^2 - R^2}}{\omega} = \frac{\sqrt{25^2 - 15^2}}{2\pi \times 50}$

$= \frac{20}{100\pi} = \frac{1}{5\pi}$ H [∵ ω = 2 πf]

Q. 9. A circuit draws a power of 550 W from a source of 220 V, 50 Hz. The power factor of the circuit is 0.8. The circuit lags behind the Voltage. Show that the capacitor of about 1/42 π × 10^{-2} F will have to be connected to bring its power factor to unity.

Sol. As $P_{av} = V_{eff}.\ I_{eff} \cos\phi$

∴ $I_{eff} = \frac{P_{av}}{V_{eff}\cos\phi} = \frac{550}{220 \times 0.8} = \frac{25}{8}$ A

$R = \frac{P_{av}}{I^2_{eff}} = \frac{550 \times 8 \times 8}{25 \times 25} = \frac{22 \times 64}{25}\ \Omega$ [∵ $P_{av} = I^2_{eff} R$]

$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{1-(0.8)^2}{0.8} = \frac{0.6}{0.8} = \frac{3}{4}$

But $\tan\phi = \frac{X_L}{R}$

∴ $X_L = \tan\phi.\ R = \frac{3}{4} \times \frac{22 \times 64}{25} = 42\ \Omega$

For power factor to be unity,

$X_L = X_C$ or $\omega L = \frac{1}{\omega C}$

or $C = \frac{1}{\omega^2 L} = \frac{1}{\omega X_L} = \frac{1}{2\pi f} \cdot \frac{1}{X_L} = \frac{1}{100\pi} \cdot \frac{1}{42}$ [∵ $\omega L = X_L$] or $C = \frac{1}{42\pi} \times 10^{-2}$ F

Q. 11. ***Show that if a coil of self-inductance L and resistance R is connected to a source of emf, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, the average power consumed is $\frac{1}{2} \mathcal{E}_0^2 R / (R^2 + \omega^2 L^2)$.***

Sol. Given $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

$\therefore \quad I = I_0 \sin (\omega t - \phi)$, where $\tan \phi = \frac{\omega L}{R}$

The power is consumed only across the resistance and not across the inductance. So average power consumed per cycle is

$$P_{av} = \frac{1}{T} \int_0^T I^2 R\, dt = \frac{1}{T} \int_0^T I_0^2 \sin^2 (\omega t - \phi) R\, dt$$

$$= \frac{I_0^2 R}{2T} \int_0^T 2 \sin^2 (\omega t - \phi)\, dt$$

$$= \frac{I_0^2 R}{2T} \int [1 - \cos 2 (\omega t - \phi)]\, dt$$

$$= I_0^2 R\, [T - 0] = \frac{\mathcal{E}_0^2 R}{2(R^2 + \omega^2 L^2)}$$

$$\left(\because I_0 = \frac{\mathcal{E}_0}{2 (R^2 + \omega^2 L^2)}\right)$$

LC-OSCILLATIONS

"When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC-oscillations".

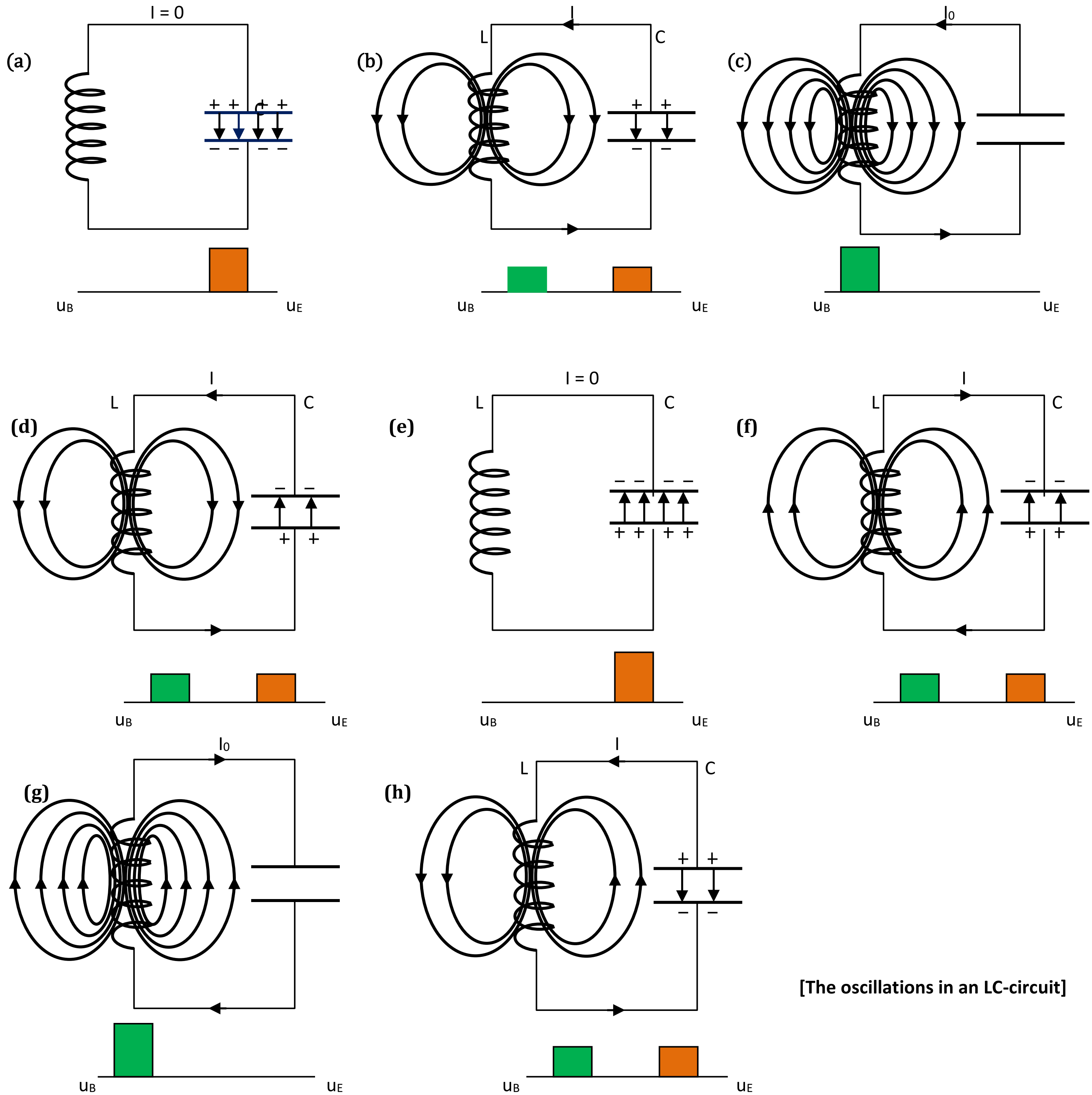
Qualitative explanations for the production of LC-oscillations:

- Fig. (a) shows a capacitor with initial charge q_0 connected to an ideal inductor. The electrical energy stored in the charged capacitor is **$U_E = \frac{1}{2} \frac{q_0^2}{C}$. As there is no current in the circuit, the energy stored in the magnetic field of the inductor is zero.**

- As the circuit is closed [Fig (b)], the capacitor begins to discharge itself through the inductor, causing a current I. As the current I increases, it builds up magnetic field around the inductor. **A part of electric energy of the capacitor gets stored in the inductor in the form of magnetic energy,** **$U_B = \frac{1}{2} LI^2$**

- At the later instant [Fig. (c)], **the capacitor gets fully discharged** and **p.d. across its plates becomes zero**. The current reaches its maximum value I_0, the energy stored in the magnetic field is $\frac{1}{2} LI_0^2$. **Thus the entire electrostatic energy of the capacitor has been converted into the magnetic field energy of the inductor.**

- After the discharge of the capacitor is complete, the magnetic flux linked with the inductor decreases, inducting a current in the same direction (Lenz's law) as the earlier current, as shown in Fig. (d). The current thus persists, though with decreasing magnitude, and charged the capacitor in the opposite direction. The magnetic energy of the inductor begins to change into the electrostatic energy of the capacitor. this process continues till the capacitor is fully charge [Fig. (e)]. But it is charge with a polarity opposite to that in its initial state [Fig. (a)]. Thus the entire energy is again stored as $\frac{1}{2} q_0^2/C$ in the electric field of the capacitor.

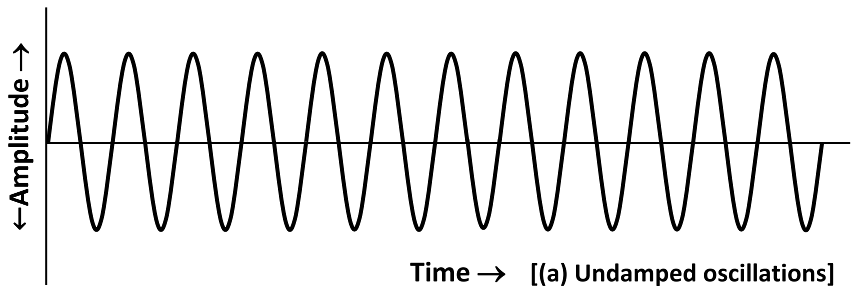


[The oscillations in an LC-circuit]

The capacitor begins to discharge again, sending current in opposite direction [Fig. (f)]. The energy is once again transferred to the magnetic field of the inductor. Thus, the process repeats in the opposite direction [Fig. (g) and (h)]. The circuit eventually returns to the initial state [Fig (a)].

Thus, the energy of system continuously surges back and forth between the electric field of the capacitor and the magnetic field of the inductor. This produces electrical oscillations of a definite frequency ν_0. These are called LC-oscillations.

If there is no loss of energy, the amplitude of the oscillations remains constant as shown in Fig. (a). Such oscillations are called undamped oscillations.



Time → [(a) Undamped oscillations]

□ **LC-oscillations are usually damped** [reasons]:

□1. Every inductor has some resistance. This causes energy loss as heat. The amplitude of oscillations goes on decreasing and the oscillations finally die out.

□2. Even if the resistance were zero, the total energy of the system would not remain constant. It is radiated away in the form of electromagnetic waves. In fact the working of radio and TV transmitted is based on such radiations.

- **(i) *In LC circuit, resistance* of the circuit plays the role of *friction* which *decreases* the *amplitude of the oscillations*.**
- **(ii) As energy in the LC circuit is dissipated in the form of heat, so LC circuit becomes warmer.**
- **(iii) With the rise in temperature, the resistance of the LC circuit increases and hence the dissipation of energy becomes faster. As a result of this, the amplitude of LC oscillations decreases rapidly.**
- **An electric circuit containing an inductor of inductance (L) and a capacitor of capacity (C) connected in parallel is called as tank circuit.**

Examples based on LC-Oscillations

◆ **FORMULA USED**

1. Angular frequency of free oscillations of an LC-circuit, $\omega = \frac{1}{\sqrt{LC}}$
2. Frequency of free oscillations of an LC-circuit, $f = \frac{1}{2\pi\sqrt{LC}}$
3. Instantaneous charge on the capacitor, $q = q_0 \cos \omega t$
4. Instantaneous current in the LC-circuit, $I = -dq = I_0 \sin \omega t$, where $I_0 = \omega q_0$
5. Electrical energy stored in the capacitor at any instant, $U_E = \frac{1}{2} \cdot \frac{q^2}{C}$

   $$U_E^{max} = \frac{1}{2} \cdot \frac{q_0^2}{C}$$

6. Magnetic energy stored in the inductor at any instant, $U_B = \frac{1}{2} LI^2$

   $$U_B^{max} = \frac{1}{2} LI_0^2$$

7. Total energy stored in the LC-circuit, $U = U_E + U_B = \frac{1}{2} \cdot \frac{q_0^2}{C} = \frac{1}{2} LI_0^2$

◆ **UNITS USED** :Charges q and q_0 are in coulomb, current I and I_0 in ampere, inductance L in henry, capacitance C in farad, angular frequency ω in rad s^{-1}, and energies U, U_E and U_B are in joule.

Q. 1. ***Calculate the wavelength of radio waves radiated out by a circuit consisting of 0.02 μF capacitor and 8 μF inductor in series.***

Sol. Here $C = 0.02\ \mu F = 0.02 \times 10^{-6}$ F, $L = 8\ \mu F = 8 \times 10^{-6}$ H

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02 \times 10^{-6} \times 8 \times 10^{-6}}}$$

$$= 3.98 \times 10^5 \text{ Hz}$$

The wavelength of the radio waves produced is $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3.98 \times 10^5} = 7.54 \times 10^2$ m

Q. 2. ***An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC-circuit is set oscillating at its natural frequency. Let q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that maximum value of charge q is 200 μC.***

(a) When q = 100 μC, what is the value of $\frac{dI}{dt}$?

(b) When q = 200 μC, what is the value of I? ***(c) Find the maximum value of I.***
(d) When I is equal to one-half its maximum value, what is the value of q?

Sol. Here L = 2.0 mH = 2.0×1^{-3} H, C = 5.0 μF = 5.0×10^{-6} F

The natural frequency of LC-oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 10^{-3} \times 5.0 \times 10^{-6}}} = 10^4 \text{ rad s}^{-1}$$

The charge on the capacitor at any instant t during LC-oscillations,

$q = q_0 \cos \omega t$

∴ $I = -\frac{dq}{dt} = q_0 \omega \sin \omega t = I_0 \sin \omega t$

and $dI/dt = q_0 \omega^2 \cos \omega t = \omega^2 q$

(a) When q = 100 μC = 10^{-4} C,

$\frac{dI}{dt} = \omega^2 q = 10^8 \times 10^{-4} = 10^4 \text{ As}^{-1}$

(b) Here q = 200 μC. Also q_0 = 200 μC

As $q = q_0 \cos \omega t$ ∴ $200 = 200 \cos \omega t$

or $\cos \omega t = 1$ or $\omega t = 0°$

∴ $I = q_0 \omega \sin 0° = 0$

This can also be followed from the fact that when q = 200 μC = q_0, the capacitor is fully charged. At this instant, the current in the LC-circuit is zero.

(c) $I_0 = \omega q_0 = 10^4 \times 200 \times 10^{-6} = 2.0$

(d) $I = I_0 \sin \omega t$

When $I = I_0/2$, we have

$I_0 = I_0 \sin \omega t$ or $\sin \omega t = 0.5$

∴ $\omega t = 30°$

Here $q = q_0 \cos \omega t = 200 \times 10^{-6} \times \cos 30°$

$= 200 \times 10^{-6} \times 0.866$

$= 173.2 \times 10^{-6}$ C = 173.2 μC.