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ELECTROMAGNETIC INDUCTION
AND ALTERNATING CURRENT

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ELECTROMAGNETIC INDUCTION

ALTERNATING CURRENT

REVIEW OF BASIC CONCEPTS

1. Magnetic Flux

The magnetic flux through any surface placed in a magnetic field is determined by the number of field lines that cut through that surface. The magnetic flux through a coil of area A in a uniform magnetic field B is defined as

$$\phi = \mathbf{B} \cdot \mathbf{A} = B\,A\cos\theta$$

where θ is the angle between the normal to the plane of the coil and the magnetic field. If the coil has N turns, the magnetic flux through the coil is given by

$$\phi = N\,B\,A\cos\theta$$

The SI unit of flux is called weber (Wb).

For a curved surface,

$$\phi = \int \mathbf{B}\cdot\mathbf{dA}$$

2. Faraday's Laws of Electromagnetic Induction

The magnitude and direction of induced emf can be determined by the application of two l... of electromagnetic induction: (i) Faraday's la... Lenz's law.

Faraday's Law of Electromagnetic Induc...

On the basis of various experiments, Faraday f...

1. whenever magnetic flux linked with a circuit changes, an induced emf is produced in the circuit,
2. the induced emf lasts as long as the change in the magnetic flux is taking place, and
3. the magnitude of the induced emf is directly proportional to the rate of change of magnetic flux, i.e.

$$e \propto \frac{d\phi}{dt}$$

Lenz's Law

According to Lenz's law, the direction of the induced emf is such that it always opposes the cause that has produced it.

Thus

$$e = -k\,\frac{d\phi}{dt}$$

where k is a positive constant whose value depends on the system of units. In SI system of units, $k = 1$ and one can write

$$e = -\frac{d\phi}{dt}$$

Magnitude of induced emf is $|e| = \left|\frac{d\phi}{dt}\right|$

If ϕ is the flux through one turn of a coil, then for a coil of N turns

$$|\,e\,| = N\left|\frac{d\phi}{dt}\right|$$

The magnitude of the induced current is given by

$$i = \frac{\text{induced emf}}{\text{total resistance of circuit}} = \frac{1}{R}\left|\frac{d\phi}{dt}\right|$$

The direction of induced current is obtained by Lenz's law.

Flow of Induced Charge

When a current is induced in a circuit due to change in magnetic flux, induced charge q flows through the circuit.

$$q = \int i\,dt = \int \frac{1}{R}\left|\frac{d\phi}{dt}\right| dt = \frac{1}{R}\int |d\phi| = \frac{\text{change in flux}}{\text{Resistance}}$$

Heat Dissipation

Heat dissipated due to induced current is

$$H = \int e\,idt = \int \left|\frac{d\phi}{dt}\right| idt = i\int |d\phi|$$

$$= \text{induced current} \times \text{change in flux}$$

Fleming's Right Hand Rule

This rule gives the direction of the induced emf when a conductor moves at right angles to a magnetic field. Hold the thumb and the first two fingers of your right hand mutually perpendicular to each other. Then, if the first finger points in the direction of the magnetic field and the thumb points in the direction of the motion of the conductor, then the second finger gives the direction of the induced emf (and hence of the induced current).

Applications of Lenz's Law

(i) If the magnet is moved towards the coil or coil is moved towards the magnet, the induced current i is anticlockwise Fig. 14.1. The current is clockwise if the magnet is moved away from the coil.

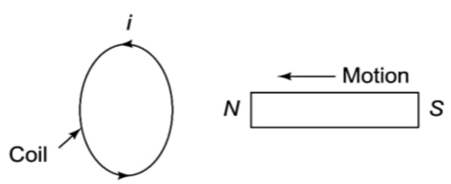


Fig. 14.1

(ii) The induced current i in the coil is anticlockwise if (Fig. 14.2)

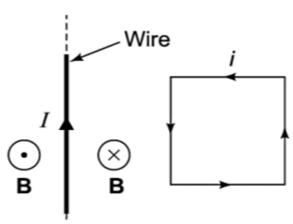


Fig. 14.2

(a) the coil is moved towards the long wire carrying current I

or

(b) the current I increases with time.

The current I is clockwise if

(a) the coil is moved away from wire

or

(b) the current I decreases with time.

(iii) Two coils carrying currents I_1 and I_2 placed with their planes parallel approach each other (Fig. 14.3).

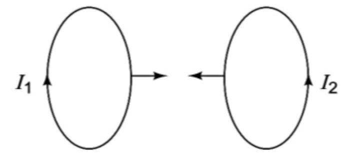


Fig. 14.3

(a) If I_1 and I_2 are both clockwise (or anticlockwise), then both I_1 and I_2 will decrease.

(b) If the currents I_1 and I_2 are in opposite sense, both the currents will increase.

3. Expression for Induced EMF

(i) **Change in flux due to change in magnetic field (B).**

If **B** increases with time, the induced current i is anticlockwise so that it produces a magnetic field pointing outwards (opposite to **B**). The induced emf is (Fig. 14.4)

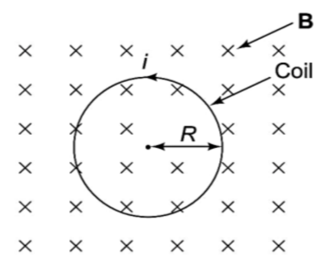


Fig. 14.4

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = \pi R^2 \frac{dB}{dt}$$

If **B** decreases with time, I will be clockwise.

If **B** remains constant but the radius of the coil increases at a rate $\frac{dR}{dt}$, then

$$|e| = B\frac{d}{dt}(\pi R^2) = B \times 2\pi R\frac{dR}{dt}.$$

(ii) **Change in flux due to change in area (A)**

If a rectangular coil $PQRS$ is moved out of a region of uniform magnetic field **B** with a velocity v, the emf induced is (Fig. 14.5(a))

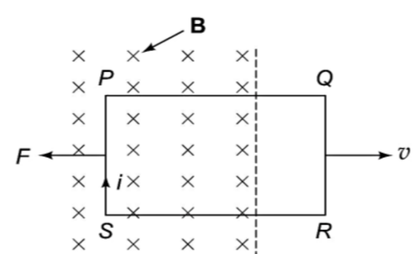


Fig. 14.5(a)

$|e| = Blv$ where $l = PS = QR$

Induced current i is clockwise. If R is the resistance of the coil,

$$i = \frac{e}{R} = \frac{Blv}{R}$$

Force F required to pull the coil out with constant velocity v is

$$F = Bil = \frac{B^2l^2v}{R}$$

Power needed is $P = Fv = \dfrac{B^2l^2v^2}{R}$

= heat dissipated

The current will be anticlockwise, if the coil is pushed into the region of magnetic field.

Note

(a) If the coil is moved within the region of uniform magnetic field, no change in flux takes place and hence no emf is induced.

(b) If the magnetic field is non-uniform and the coil is kept stationary in it, no change in flux occurs and hence no emf is induced.

The above results also hold in the case of rod XY sliding on metallic rails $PQRS$ to the right as shown in Fig. 14.5(b).

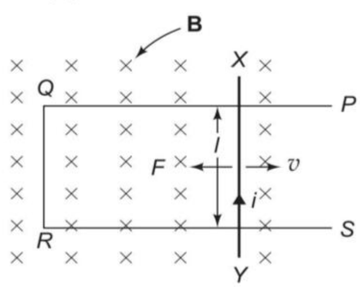


Fig. 14.5(b)

(iii) **Change in flux due to change in orientation (θ) (A.C. generator)**

If a coil of area A, consisting of N turns is rotated in a magnetic field B with angular velocity ω, the emf induced in it is given by

$$e = e_0 \sin\theta = e_0 \sin\omega t$$

where $e_0 = NBA\omega$ is the amplitude (peak value). Thus an alternating emf is produced.

4. Motional Emf

(i) When a rod (or wire) of length l is moved with a velocity v in a magnetic field **B** as shown in Fig. 14.6(a), the emf induced between the ends P and Q of the rod is given by

$$e = Blv$$

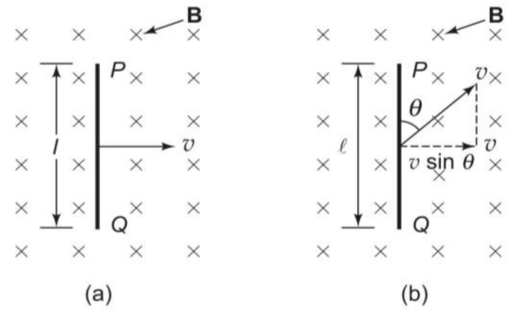


Fig. 14.6

If the rod is moved as shown in Fig. 14.6(b), then

$$e = Blv \sin\theta$$

(ii) When a semicircular rod (or wire) of radius R is moved with a velocity v in a magnetic field **B** as shown in Fig. 14.7, the emf induced between the ends P and Q of the rod is given by $e = Bv\,(2R) = 2BvR$

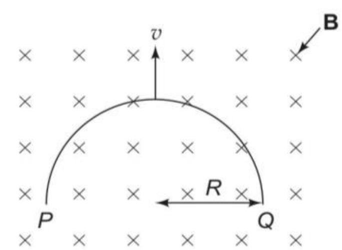


Fig. 14.7

(iii) When a rod PQ of length l pivoted at one end P is rotated with angular velocity ω in a magnetic field **B** as shown in Fig. 14.8(a), the emf induced between its ends is given by

$$e = \frac{1}{2}B\omega l^2$$

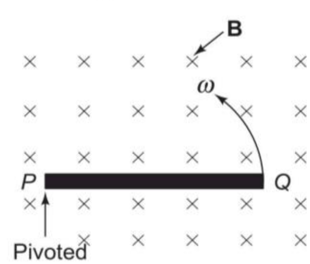


Fig. 14.8(a)

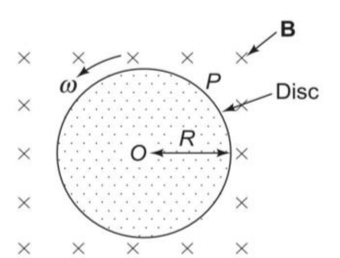


Fig. 14.8(b)

(iv) When a disc of radius R is rotated about its centre with angular velocity ω in a magnetic field **B** as shown in Fig. 14.8(b), the emf induced between its centre O and a point P on its rim is given by

$$e = \frac{1}{2} B\omega R^2$$

5. Electric Motor

When a current is passed through a coil placed in a magnetic field by connecting its end to a source of voltage V, it experiences a torque which rotates it. As a result, an emf e is induced in it. This emf is called back emf as it opposes the applied voltage V (from Lenz's law). If R is the resistance of the coil, then current in it is

$$i = \frac{V-e}{R}$$

Input power = Vi and heat loss = i^2R. Hence output power = $Vi - i^2R = ei$.

Efficiency of motor $\eta = \frac{ei}{Vi} = \frac{e}{V}$

Some important points about a d.c. motor

(i) Back emf e and hence current i vary sinusoidally even if the source of voltage V is a d.c. (battery).

(ii) When output power is maximum, $e = \frac{V}{2}$ and $\eta = 50\%$.

(iii) Initially, i.e. when the motor is switched on, $e = 0$ and initial current = V/R which is very large. So, for safety, a starter is used.

(iv) At full speed, back emf is maximum and current i is minimum.

EXAMPLE 1 A straight metal wire of length L, cross-sectional area a and resistivity ρ is made into a square frame. A uniform magnetic field B is perpendicular to the plane of the frame and is changing at a constant rate dB/dt. The current induced in the frame is

(a) $\frac{La}{\rho}\frac{dB}{dt}$ (b) $\frac{La}{4\rho}\frac{dB}{dt}$

(c) $\frac{La}{8\rho}\frac{dB}{dt}$ (d) $\frac{La}{16\rho}\frac{dB}{dt}$

SOLUTION Side of frame = $\frac{L}{4}$. Area of frame is $A = L^2/16$. Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(AB) = A\frac{dB}{dt} = \frac{L^2}{16}\frac{dB}{dt}$$

Resistance of frame is $R = \frac{\rho L}{a}$.

$\therefore$ Induced current $= \frac{|e|}{R} = \frac{La}{16\rho}\frac{dB}{dt}$

EXAMPLE 2 A square metal frame $PQRS$ of side 15 cm and resistance 1.0 Ω is moved with a speed of 4/3 cm s^{-1} in a uniform magnetic field B = 2.0 T which is perpendicular to the plane of the frame as shown in Fig. 14.9. The frame is connected to a network of resistances as shown. The current induced in the frame is

(a) 1 mA (b) 2 mA

(c) 3 mA (d) 4 mA

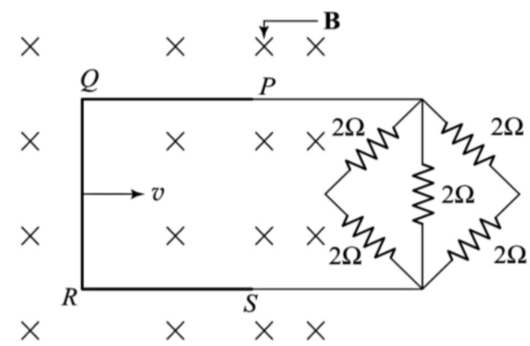


Fig. 14.9

SOLUTION Equivalent resistance of the network between P and S is given by

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} \Rightarrow R_{eq} = 1\ \Omega$$

Total resistance $R = 1 + 1 = 2\ \Omega$

Magnitude of emf induced in the frame is

$$|e| = B\,l\,v$$

$$= 2.0 \times 0.15 \times \left(\frac{4}{3} \times 10^{-2}\right) = 4 \times 10^{-3}\ \text{V}$$

$\therefore$ Induced current $= \frac{|e|}{R} = \frac{4 \times 10^{-3}\ \text{V}}{2\ \Omega} = 2 \times 10^{-3}$ A

$= 2$ mA

EXAMPLE 3 A square coil $PQRS$ of resistance 2 Ω, 100 turns and side 10 cm is placed in a magnetic field B = 2.0 T. The direction of the magnetic field is perpendicular to the plane of the coil as shown in Fig. 14.10. The work done is pulling the coil completely out of the region of magnetic field in 2.0 s without any acceleration is

(a) 0.01 J (b) 0.1 J

(c) 1.0 J (d) 10 J

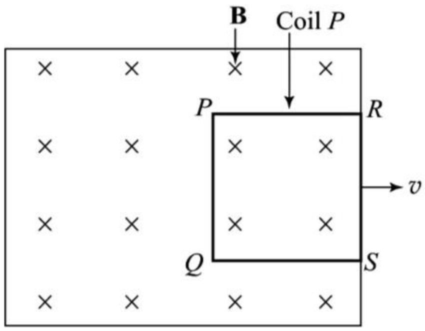


Fig. 14.10

SOLUTION Speed of coil is $v = \frac{100 \text{ cm}}{2.0 \text{ s}} = 5 \text{ cm s}^{-1}$ $= 5 \times 10^{-2} \text{ ms}^{-1}$

The emf induced in arm PQ of the coil is

$$|e| = N B l v = 100 \times 2.0 \times 0.1 \times 5 \times 10^{-2} = 1.0 \text{ V}$$

Induced current in the coil is

$$I = \frac{|e|}{R} = \frac{1.0 \text{ V}}{2 \,\Omega} = 0.5 \text{ A}$$

Force on PQ due to magnetic field is

$$F = B I l = 2.0 \times 0.5 \times 0.1 = 0.1 \text{ N}$$

From Lenz's law, this force acts to the left on arm PQ. To pull the coil to the right without acceleration, an external force $F = 0.1$ N must be applied to the right. Therefore, work done to pull the coil completely is

$$W = 0.1 \text{ N} \times 0.1 \text{ m} = 0.01 \text{ J}$$

So the correct choice is (a).

EXAMPLE 4 A wire PQ of mass $m = 10$ g and length $l = 25$ cm can freely slide on horizontal, smooth and parallel rails placed in a uniform magnetic field $B = 2.0$ T as shown in Fig. 14.11. The ends of the rails are connected by a capacitor $C = 2$ mF. A constant force $F = 1.2 \times 10^{-3}$ N is applied as shown. If the resistance of the rails is zero ohm, the acceleration of wire PQ will be

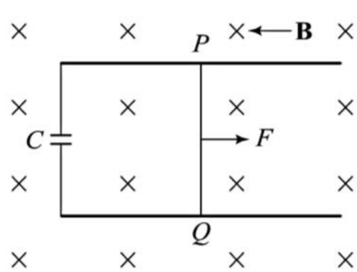


Fig. 14.11

(a) 0.1 ms^{-2} (b) 0.5 ms^{-2}
(c) 1.0 ms^{-2} (d) 2.0 ms^{-2}

SOLUTION Let v be the velocity of the wire at time t. The induced emf is $e = B\,l\,v$. The charge on the capacitor at time t is

$$q = Ce = C B l v$$

$$\therefore \quad \text{Current} \quad I = \frac{dq}{dt} = C B l \frac{dv}{dt} = C B l a$$

where a = acceleration of the wire. The direction of a (from Lenz's law) is to the left. Force on PQ due to this current is

$$f = B I l = CB^2 l^2 a \text{ towards left.}$$

$\therefore$ Net force on wire is $F_{\text{net}} = F - f = F - CB^2 l^2 a$. The acceleration of wire by this force is

$$a = \frac{F_{\text{net}}}{m} = \frac{F - CB^2 l^2 a}{m}$$

$$\Rightarrow \quad a = \frac{F}{m + CB^2 l^2} = \frac{1.2 \times 10^{-3}}{10^{-2} + 2 \times 10^{-3} \times (2.0)^2 \times (0.25)^2} = 0.1 \text{ m s}^{-2}$$

EXAMPLE 5 A square coil of resistance 2 Ω, 100 turns and side 10 cm is placed with its plane making an angle of 30° with a uniform magnetic field of 0.1 T. In 0.05 s the coil rotates until its plane becomes parallel to the magnetic field. Calculate the current induced in the coil.

SOLUTION $R = 2\,\Omega$, $N = 100$, $A = 0.1 \times 0.1 = 10^{-2} \text{ m}^2$,

$$\theta_1 = 90° - 30° = 60°,$$

$$\theta_2 = 90° - 0 = 90°, B = 0.1 \text{ T and } t = 0.05 \text{ s}$$

$$\text{Change in flux} = NBA(\cos\theta_2 - \cos\theta_1) = 100 \times 0.1 \times 10^{-2} \times (\cos 90° - \cos 60°) = 0.1 \times \left(0 - \frac{1}{2}\right) = 0.05 \text{ Wb}$$

$$\text{Induced emf } e = \frac{\text{change in flux}}{\text{time}} = \frac{0.05}{0.05} = 1 \text{ V}$$

$$\text{Induced Current } i = \frac{e}{R} = \frac{1}{2} = 0.5 \text{ A}$$

EXAMPLE 6 A solenoid of diameter 0.2 m has 500 turns per metre. At the centre of this solenoid, a coil of 100 turns is wrapped closely around it. If the current in the solenoid changes from zero to 2 A in 1 millisecond, calculate the induced emf developed in the coil.

SOLUTION The magnetic field due to current I in a solenoid having n turns per unit length is

$$B = \mu_0 n I$$

This is the magnetic field threading the coil. The direction of the field is parallel to the axis of the solenoid. Hence angle between the normal to the plane of the coil and the magnetic field is $\theta = 0°$. If A is the cross-sectional area of the coil and N the number of turns in it, then the magnetic flux threading the coil is

$$\phi = NAB\cos\theta = NAB\cos 0° = NAB = N\pi r^2 \mu_0 nI$$

Since the coil is wrapped closely around the solenoid, the radius of the coil (r) = radius of solenoid = 0.1 m.

Change in flux if the current change from $I_1 = 0$ to $I_2 = 2$ A is

$$\Delta\phi = N\pi r^2 \mu_0 n (I_2 - I_1)$$
$$= 100 \times (\pi \times 0.1^2) \times (4\pi \times 10^{-7}) \times 500 \times (2 - 0)$$
$$= 4\pi^2 \times 10^{-4} \text{ Wb}$$

Induced emf $e = \left|\frac{\Delta\phi}{\Delta t}\right| = \frac{4\pi^2 \times 10^{-4}}{10^{-3}} = 0.4\pi^2 = 3.95$ V

EXAMPLE 7 The magnetic flux through a coil of resistance 6.5 Ω placed with its plane perpendicular to a uniform magnetic field varies with time t (in second) as

$$\phi = (3t^2 + 5t + 2) \text{ milliweber}$$

Find the induced current in the coil at $t = 10$ s.

SOLUTION $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 5t + 2)$

$$= (6t + 5) \text{ MV}.$$

At $t = 10$ s, $e = (6 \times 10 + 5)$ mV $= 65 \times 10^{-3}$ V

Induced current at $t = 10$ s is

$$I = \frac{e}{R} = \frac{65 \times 10^{-3}}{6.5} = 10^{-2} \text{A}$$

EXAMPLE 8 A metal wheel with 8 metallic spokes, each 60 cm long is rotated at a speed of 100 rev./min in a plane perpendicular to earth magnetic field of 0.3×10^{-4} T. Find the magnitude of the induced emf between the axle and the rim of the wheel.

SOLUTION $\nu = 100$ rev./min. $= \frac{100}{60} = \frac{5}{3}$ rev./s

$l = 0.6$ m. The emf developed between the ends of a spoke is (as $\omega = 2\pi\nu$)

$$e = \frac{1}{2} Bl^2 \omega$$
$$= \frac{1}{2} \times (0.3 \times 10^{-4}) \times (0.6)^2 \times (2\pi \times \frac{5}{3})$$
$$= 1.8\pi \times 10^{-5} \text{ V} = 5.65 \times 10^{-5} \text{ V}$$

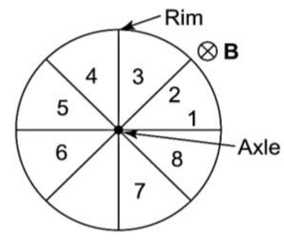


Fig. 14.12

The same emf is induced between the ends of each spoke. It is clear from Fig. 14.12 that the spokes are joined in parallel. Hence the emf between rim and axle = emf across each spoke = 5.65×10^{-5} V.

EXAMPLE 9 An aircraft with a wing span of 50 m is flying with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude at a place where the vertical component of earth's magnetic field is 2×10^{-5} T. Find the emf developed between the tips of the wing.

SOLUTION $v = 1080$ kmh^{-1} $= 300$ ms^{-1}

$$e = Blv = (2 \times 10^{-5}) \times 50 \times 300 = 0.3 \text{ V}$$

EXAMPLE 10 A circular coil of mean radius r and having N turns is kept in a horizontal plane. A magnetic field B exists in the vertical direction as shown in Fig. 14.13(a). Find the emf induced in the loop

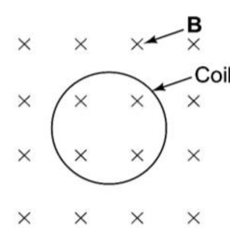


Fig. 14.13(a)

(a) if it is held stationary and the magnetic field is uniform,

(b) if it is held stationary and the magnetic field is non-uniform,

(c) if it is rotated with an angular velocity ω about an axis passing through its centre and perpendicular to its plane, and the magnetic field is uniform.

(d) if it is rotated with an angular velocity ω about its diameter. Assume that the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field at time $t = 0$.

(e) if the coil is a square of side L and is rotated about its diameter.

SOLUTION The emf is induced if the magnetic flux through the coil changes with time.

(a) In this case there is no change in magnetic flux with time, hence no emf induced.

(b) In this case also the magnetic flux through the coil does not change with time, hence no emf induced.

(c) In this case, the number of field lines through the coil does not change with time, hence the magnetic flux does not change with time. So no emf is induced in the coil (see Fig. 14.13(b))

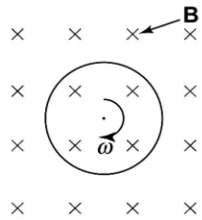


Fig. 14.13(b)

(d) If the coil is rotated about a diameter, as shown in Fig. 14.13(c), there is a change in magnetic flux with time. Hence emf will be induced in the coil. Area of the coil is $A = \pi r^2$. If the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field, at times $t = 0$ then at time t, $\theta = \omega t$. The magnetic flux at this time is

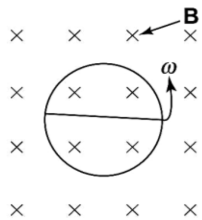


Fig. 14.13(c)

$$\phi = NBA\cos\theta = NBA\cos\omega t$$

∴ Induced emf is

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

$$= \omega NBA\sin\omega t$$

$$= \omega NB \times \pi r^2 \sin\omega t$$

$$= \pi N r^2 B\omega\sin\omega t$$

(e) If a square coil of side L and N turns is rotated about a diameter as shown in 14.13(d), there is a change in magnetic flux with time. Hence emf will be induced in the coil. Area of the coil is $A = L^2$. If the normal to the plane of the coil makes an angle $\theta = 0$ with the magnetic field at time $t = 0$, then at time t, $\theta = \omega t$. The magnetic flux at this time is

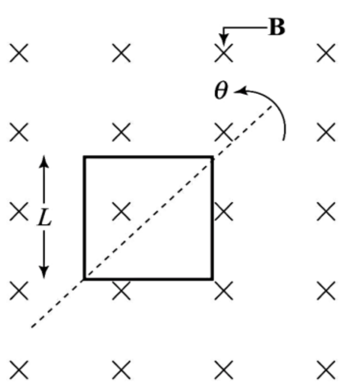


Fig. 14.13(d)

$$\phi = NBA\cos\theta$$

$$= NBL^2\cos(\omega t)$$

∴ Induced emf is

$$e = -\frac{d\phi}{dt} = NBL^2\omega\sin(\omega t)$$

Thus, as in case (d) above, an alternating emf is induced in the coil

EXAMPLE 11 A metal rod PQ of length L is moved with a velocity v making an angle θ with a uniform magnetic field B as shown in Fig. 14.14. Obtain the expression for the emf induced between the ends of the rod.

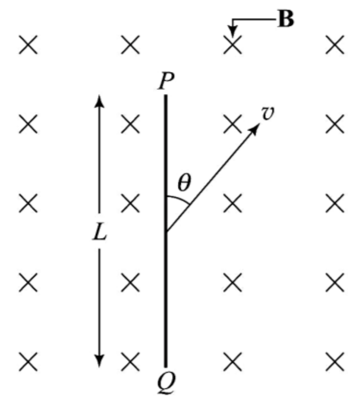


Fig. 14.14

SOLUTION The component of velocity v perpendicular to the length of the rod is

$$v_\perp = v\sin\theta$$

Only the perpendicular component induces an emf in the rod. Since the magnetic field is perpendicular to the plane of motion, the emf induced between the ends of the rod is

$$e = B\,l\,v_\perp = B\,l\,v\sin\theta$$

EXAMPLE 12 A metal rod PQ moves with a velocity v parallal to a very long straight wire CD carrying a current I as shown in Fig. 14.15. The ends P and Q of the rod are at distances a and b from the wire as shown. Obtain the expression for the emf induced between the ends of the rod.

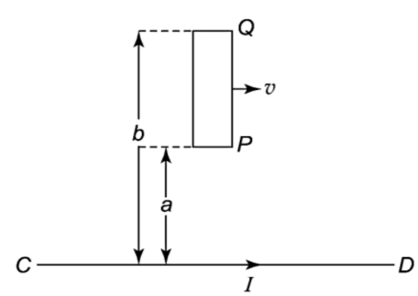


Fig. 14.15

SOLUTION Divide the rod into a large number of very small elements, each of length dx. Consider one such element at distance x from wire CD as shown in Fig. 14.16.

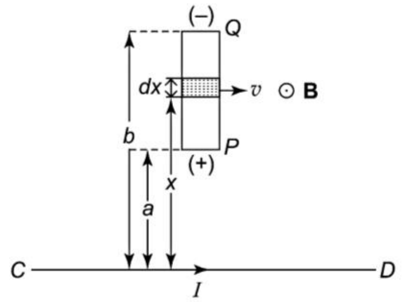


Fig. 14.16

The magnetic field at the element due to current I in wire CD is

$$B = \frac{\mu_0 I}{2\pi x}$$

The direction of the field is upwards perpendicular to velocity v. The magnitude of magnetic field is different at different points on the rod PQ. From Fleming's L.H. rule, the free electrons in the rod will experience force in the direction P to Q. So free electrons move from P to Q. Hence end P acquires a positive charge (due to loss of electrons) and end Q acquires a negative charge (due to gain of electrons).

Force on the element is

$$dF = qvB = qv \times \frac{\mu_0 I}{2\pi x}$$

Therefore, electric field set up in the element is

$$dE = \frac{dF}{q} = \frac{qv\mu_0 I}{2\pi xq} = \frac{\mu_0 Iv}{2\pi x}$$

Now $\quad dE = -\dfrac{dV}{dx}$

$\Rightarrow \quad dV = -dE \times dx = -\dfrac{\mu_0 Iv}{2\pi x}dx$

where dV is the voltage induced in the element. The voltage induced in the rod PQ is

$$|V| = \frac{\mu_0 Iv}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)$$

EXAMPLE 13 A metal rod PQ of length l slides with a velocity v on two parallel rails AB and CD parallel to a long straight wire XY carrying a current I as shown in Fig. 14.17. A resistance R is connected between the rails as shown. The velocity of rod PQ is kept constant by applying force.

(a) Obtain the expression for the current induced in resistance R.

(b) Obtain the expression for the force to be applied on rod PQ to keep its velocity constant at v.

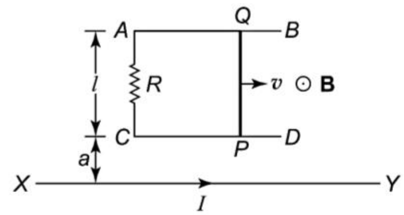


Fig. 14.17

SOLUTION In this case the induced emf is due to change in magnetic flux which is due to the change in the area of $ACPQ$ with time.

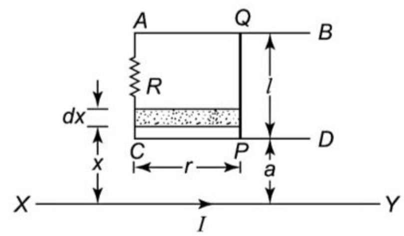


Fig. 14.18

Magnetic flux through an infinitesimal area element of width dx at a distance x from PQ is [Fig. 14.18]

$$d\phi = BdA = Brdx = \frac{\mu_0 Ir}{2\pi x}dx$$

where r is the position of PQ at an instant of time t. Magnetic flux through $ACPQ$ is

$$\phi = \int d\phi = \int_a^{(a+l)} \frac{\mu_0 Ir}{2\pi x}dx$$

$$= \frac{\mu_0 Ir}{2\pi}\int_a^{(a+l)}\frac{dx}{x} = \frac{\mu_0 Ir}{2\pi}\ln\left(\frac{a+l}{a}\right)$$

$\therefore$ Induecd emf $e = \left|\dfrac{d\phi}{dt}\right| = \dfrac{\mu_0 I}{2\pi}\ln\left(\dfrac{a+l}{a}\right)\dfrac{dr}{dt}$

$$= \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{a+l}{a}\right) \qquad \left(\because v = \frac{dr}{dt}\right)$$

(a) Current induced in R is

$$i = \frac{e}{R} = \frac{\mu_0 Iv}{2\pi R}\ln\left(\frac{a+l}{a}\right)$$

(b) This current will exert a force on the element of width dx which is given by

$$dF = Bidx = \frac{\mu_0 I}{2\pi x}idx$$

$\therefore$ Force to be applied on PQ is

$$F = \int dF = \frac{\mu_0 I i}{2\pi}\int_a^{(a+l)}\frac{dx}{x}$$

$$= \frac{\mu_0 I}{2\pi}\times\frac{\mu_0 I v}{2\pi R}\ln\left(\frac{a+l}{a}\right)\int_a^{(a+l)}\frac{dx}{x}$$

$$= \frac{v}{R}\left[\frac{\mu_0 I}{2\pi}\ln\left(\frac{a+l}{a}\right)\right]^2$$

EXAMPLE 14 A metal rod PQ of length l slides on two parallel rails AB and CD, each rail having a resistance k per unit length. The rod and the rails are in a region of a uniform magnetic field B directed into the plane of the paper as shown in Fig. 14.19. A resistance R is connected between the rails. A variable force F is applied to PQ so that it is accelerated to the right. Obtain the expression for the velocity v of rod PQ when it is at a distance x from R.

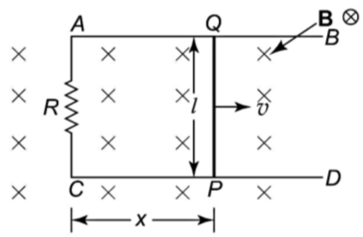


Fig. 14.19

SOLUTION Magnetic flux through $ACPQ$ when the rod is at a distance x from R is

$$\phi = BA = Blx$$

Induced emf at that instant is

$$e = \left|\frac{d\phi}{dt}\right| = Bl\,\frac{dx}{dt} = Blv$$

Resistance of $ACPQ = R + 2kx$. Therefore, current induced in the circuit is

$$i = \frac{e}{(R+2kx)} = \frac{Blv}{(R+2kx)}$$

$$\Rightarrow \quad v = \frac{i(R+2kx)}{Bl}$$

EXAMPLE 15 A metal rod PQ of mass m and of negligible resistance slides on two parallel metal rails AB and CD separated by a distance l. The rails have negligible resistance and have a resistance R connected between them as shown in Fig. 14.20. The rod and the rails are located in a region of uniform magnetic field direction into the plane of the loop $ACPQ$. The rod is given an initial velocity u. Obtain the expression for the distance x covered by the rod before it comes to rest. Neglect friction between the rod and the rails.

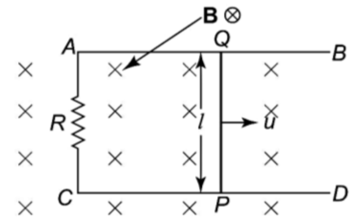


Fig. 14.20

SOLUTION Induced currnet $i = \dfrac{Bvl}{R}$

$$\text{Force } F = Bil = -ma = -m\,\frac{dv}{dt}$$

The negative sign shows that force opposes the acceleration (Lenz's law)

$$\therefore \quad m\,\frac{dv}{dt} = -B\times\frac{Bvl}{R}\times l$$

$$= -\frac{B^2vl^2}{R}$$

$$\Rightarrow \quad \frac{dv}{v} = -\frac{B^2l^2}{mR}\,dt = -kdt$$

where $\quad k = \dfrac{B^2l^2}{mR}$

Intergrating $\displaystyle\int_u^v\frac{dv}{v} = -k\int_0^t dt$

$$\Rightarrow \quad \ln\left(\frac{v}{u}\right) = -kt$$

$$\frac{v}{u} = e^{-kt}$$

$$\Rightarrow \quad v = ue^{-kt}$$

$$\therefore \quad \frac{dx}{dt} = ue^{-kt}$$

The rod comes to rest when $t = \infty$.
Integrating

$$\int_0^x dx = u\int_0^\infty e^{-kt}\,dt$$

$$x = -\frac{u}{k}\left|e^{-kt}\right|_0^\infty = -\frac{u}{k}\,(0-1) = \frac{u}{k}$$

$$\therefore \quad x = \frac{umR}{B^2l^2}$$

EXAMPLE 16 A circular coil of radius r has N turns and a resistance R. It is placed with its plane at right angles to a uniform magnetic field B. Find the expression for the amount of charge Q which passes through the coil when it is rotated through an angle of 180° in its plane.

SOLUTION Area of the coil $(A) = \pi r^2$

Since the plane of the coil is normal to the magnetic field, the magnetic flux through the coil $= NBA \cos 0° = NBA$. When the coil is rotated through 180°, the magnetic flux through it will be $= NBA \cos 180° = -NBA$. Therefore, change in flux is

$$\phi = NBA - (-NBA) = 2NBA$$

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt}$$

$\therefore$ Induced current is $i = \frac{|e|}{R} = \frac{d\phi}{dt} \times \frac{1}{R}$

$$\Rightarrow \quad iR = \frac{d\phi}{dt}$$

$$\frac{dq}{dt} R = \frac{d\phi}{dt}$$

$$\Rightarrow \quad dqR = d\phi$$

$$\Rightarrow \quad dq = \frac{d\phi}{R}$$

$$\Rightarrow \quad Q = \frac{\phi}{R} = \frac{2NBA}{R} = \frac{2NB \times \pi r^2}{R}$$

EXAMPLE 17 Two circular coils A and B of radii a and b respectively (with $b > a$) have their planes perpendicular to the plane of the page. They are separated co-axially by a distance $x = \sqrt{3}\,b$ as shown in Fig. 14.21. A transient current I flows through coil B for a very short time interval. If the resistance of coil A is R obtain the expression for the charge that flows through coil A during the short time interval.

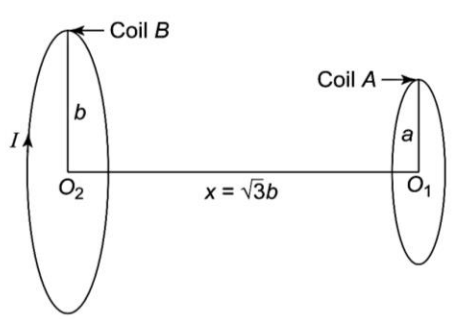


Fig. 14.21

SOLUTION Magnetic field at the centre of coil A due to current I in coil B is

$$B_{AB} = \frac{\mu_0 I b^2}{2(b^2 + x^2)^{3/2}} = \frac{\mu_0 I}{16b} \quad (\because x = \sqrt{3}\,b)$$

Since the magnetic field is along the axis of coil A, it is perpendicular to the plane of A, hence $\theta = 0°$. Therefore, magnetic flux through A is

$$\phi = B_{AB} \times \text{area of coil } A \times \cos 0° = \frac{\mu_0 I \times \pi a^2}{16b}$$

$\therefore$ Induced emf is $|e| = \left|\frac{d\phi}{dt}\right|$

$$\Rightarrow \quad IR = \frac{d\phi}{dt}$$

$$\Rightarrow \quad IRdt = d\phi$$

$$\therefore \quad \int I dt = \frac{1}{R}\int d\phi = \frac{\phi}{R}$$

$$\text{or} \quad Q = \frac{\phi}{R} = \frac{\mu_0 I \times \pi a^2}{16bR}$$

EXAMPLE 18 A thin non-conducting disc of radius R and mass M is held horizontally and is capable of rotation about an axis passing through its centre and perpendicular to its plane. A charge Q is distributed uniformly over the surface of the disc.

A time-varying magnetic field $B = kt$ (where k is a constant and t is the time) directed perpendicular to the plane of the disc is applied to it. If the disc is stationary initially (i.e. at $t = 0$). Find

(a) The torque acting on the disc.

(b) The angular velocity acquired by the disc as a function of t.

SOLUTION

(a) Area of disc $= \pi R^2$

Charge per unit area $= \frac{Q}{\pi R^2}$

Area of a small element of width dx at a distance x from the centre of the disc $= 2\pi x dx$. Therefore, charge of the element is [Fig. 14.22]

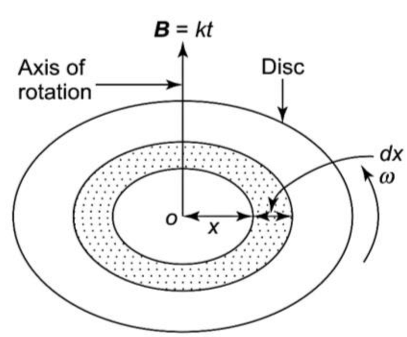


Fig. 14.22

$$dq = \frac{Q}{\pi R^2} \times 2\pi x dx$$

A time-varying magnetic field gives rise to an electric field E. Since

$$E = -\frac{dV}{dl}$$

$$\int dV = -\int E dl = -E \times 2\pi x$$

$$\Rightarrow \quad V = -E \times 2\pi x \quad (1)$$

where V is emf induced in the element, which is given by

$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$$

$$= -\frac{d}{dt}(kt \times \pi x^2) = -\pi k x^2 \quad (2)$$

From (1) and (2) we get

$$-E \times 2\pi x = -\pi k x^2$$

$$\Rightarrow \quad E = \frac{kx}{2} \quad (3)$$

Force acting on the element is

$$dF = dq \times E = \frac{Q}{\pi R^2} \times 2\pi x dx \times \frac{kx}{2}$$

$$= \frac{kQ}{R^2} x^2 dx$$

Torque acting on the disc is

$$\tau = \int_0^R x dF = \frac{kQ}{R^2}\int_0^R x^3 dx = \frac{kQR^2}{4}$$

(b) $\tau = I\alpha$, where I is the moment of inertia of the disc about the axis of rotation and α is the angular acceleration

$$I = \frac{1}{2}MR^2 \text{ and } \alpha = \frac{d\omega}{dt}\text{. Hence}$$

$$\frac{kQR^2}{4} = \frac{1}{2}MR^2 \times \frac{d\omega}{dt}$$

$$\Rightarrow \quad d\omega = \frac{kQ}{2M}dt$$

$$\Rightarrow \quad \int_0^\omega d\omega = \frac{kQ}{2M}\int_0^t dt$$

$$\Rightarrow \quad \omega = \frac{kQt}{2M}$$

6. Mutual Inductance

If the current in a coil is i then the flux linked with a neighbouring coil is $\phi = Mi$ where M is the coefficient of mutual inductance. If current i is changing with time, the emf induced in the neighbouring coil is

$$e = -M\frac{di}{dt}$$

Expressions of M in some situations

(i) A small coil of length l, number of turns N_1 wound closely on a long coil of N_2 turns.

$$M = \frac{\mu_0 N_1 N_2 A}{l}; A = \text{common cross-sectional area}$$

(ii) Two coplanar and concentric coils of radii R and r ($R >> r$) Fig. (14.23)

$$M = \frac{\mu_0 \pi r^2}{2R}$$

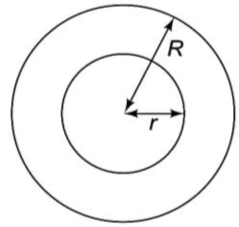


Fig. 14.23

(iii) A small circular coil of radius r at the centre of a large rectangular coil of sides a and b with $a, b >> r$ (Fig. 14.24)

$$M = \frac{2\mu_0 r^2\sqrt{a^2+b^2}}{ab}$$

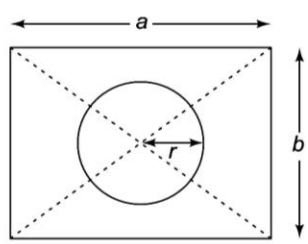


Fig. 14.24

(iv) A rectangular loop of sides a and b placed at a distance x from a long straight wire (Fig. 14.25)

$$M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{x+b}{x}\right)$$

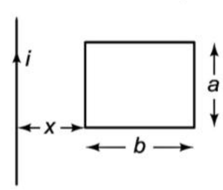


Fig. 14.25

> **Note** If the medium is different from air, μ_0 in above expressions is replaced by $\mu = \mu_0 \mu_r$, where μ_r is the relative permeability of the medium.

7. Self Inductance

If i is the instantaneous current in a coil, flux $\phi = Li$, where L is the self inductance of the coil. Induced emf $e = -L\dfrac{di}{dt}$.

(i) The self inductance of a coil of N turns, cross-sectional area A and length l is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

(ii) Direction of induced emf is such that it opposes the change in current (Fig. 14.26(a) and (b))

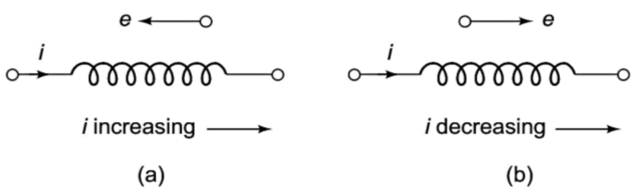


Fig. 14.26

(iii) Energy stored in the inductor $U = \dfrac{1}{2}\ Li^2$.

(iv) Inductors in series (Fig. 14.27)

Equivalent inductance is

(a) $L = L_1 + L_2$ (when the flux linked with one coil is not linked with the other, i.e. $M = 0$)

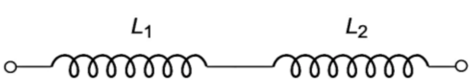


Fig. 14.27

(b) $L = L_1 + L_2 + 2M$ (when flux of one coil is in the same direction as that of the other coil)

$L = L_1 + L_2 - 2M$ (when the fluxes oppose each other)

(v) Inductors in parallel

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{(when } M = 0\text{)}$$

(vi) $M = k\sqrt{L_1 L_2}$; k = coefficient of coupling.

8. Growth and Decay of Current in a d.c L—R Circuit (Fig. 14.28)

If switch S_1 is closed at $t = 0$, with switch S_2 open, no current flows in the beginning (as the inductor behaves as open switch) [Fig. 14.28]. The current starts increasing and at time t, $i = i_o\,(1 - e^{-t/\tau})$, where $\tau = \dfrac{L}{R}$ is the time constant. (see Fig. 14.29)

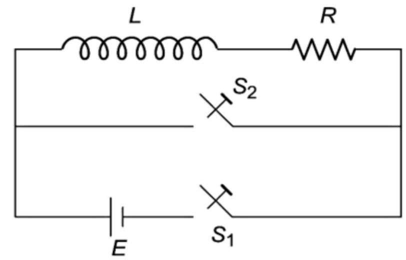


Fig. 14.28

After a long time ($t = \infty$), the current attains a steady value $i_o = E/R$ (now the ideal inductor behaves as a closed switch).

At $\quad t = \tau,\ i = i_o\left(1 - \dfrac{1}{e}\right) = 0.632\ i_0.$

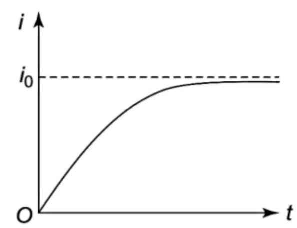


Fig. 14.29

Decay of current: At time $t = 0$, let $i_0 = E/R$ be the current in the circuit. If S_2 is closed (with S_1 open), the current decays as (see Fig. 14.30)

$$i = i_0\, e^{-t/\tau}$$

At $\quad t = \tau,\ i = \dfrac{i_0}{e} = 0.368 i_0.$

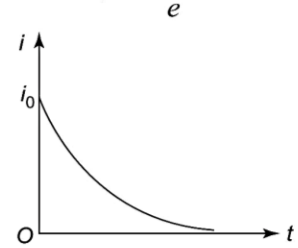


Fig. 14.30

9. Energy Stored in an Inductor

If the current in a coil of self inductance L is increased from zero to a steady value I, the energy stored in the magnetic field of the coil is

$$U = \frac{1}{2}\ LI^2$$

EXAMPLE 19 A magnetic flux of 5 μWb is linked with a coil when a current of 1 mA flows through it. Find the self inductance of the coil.

SOLUTION $\phi = LI \Rightarrow L = \dfrac{\phi}{I} = \dfrac{5\times10^{-6}}{1\times10^{-3}}$

$$= 5\times10^{-3}\ \text{H} = 5\ \text{mH}$$

EXAMPLE 20 An emf of 1 mV is induced in a coil when the current in it changes steadily from 2 A to 4 A in 0.1 s. Find the self inductance of the coil.

SOLUTION $\dfrac{dI}{dt} = \dfrac{4-2}{0.1} = 20\ \text{AS}^{-1}$

$$|e| = L\frac{dI}{dt}$$

$$1\times10^{-3} = L\times20 \Rightarrow L = 5\times10^{-5}\ \text{H} = 50\ \mu\text{H}$$

EXAMPLE 21 A solenoid 1.0 m long and 0.05 m diameter has 700 turns. Another solenoid of 50 turns is tightly wound over the first solenoid. Find the emf induced in the second solenoid when the current in the first solenoid changes from 0 to 5 A in 0.01 s.

SOLUTION Mutual inductance $M = \dfrac{\mu_0 A N_1 N_2}{l}$

$$= \frac{4\pi\times10^{-7}\times\pi(0.025)^2\times700\times50}{1.0}$$

$$= 8.6\times10^{-5}\ \text{H}$$

$$|e| = M\frac{dI}{dt} = 8.6\times10^{-5}\times\frac{5-0}{0.01} = 4.3\times10^{-2}\ \text{V}$$

EXAMPLE 22 An ideal inductor of inductance 5 H and a pure resistor of resistance 100 Ω are connected in series to a battery of emf 6 V of negligible internal resistance through a switch. The switch is closed at time $t = 0$

(a) Find the maximum (or steady) value of the current.

(b) What is the time constant τ of the circuit?

(c) How long does it take for the current to rise to 50% of the maximum value?

(d) Find the potential difference across the inductor at $t = 0.1$ s. Given $e^{-2} = 0.135$.

SOLUTION $i = i_0\,(1 - e^{\,t/\tau})$; i_0 = maximum value of i

(a) When $t\to\infty$, $i = i_0 = \dfrac{E}{R} = \dfrac{6}{100} = 0.06$ A

(b) Time constant $\tau = \dfrac{L}{R} = \dfrac{5}{100} = 0.05$ s

(c) $0.5\,i_0 = i_0\,(1 - e^{-t/\tau})$

$$\Rightarrow \quad \frac{1}{2} = 1 - e^{-t/\tau}$$

$$\Rightarrow \quad e^{-t/\tau} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \quad e^{\,t/\tau} = 2 \Rightarrow \frac{t}{\tau} = \ln(2) = 0.693$$

$$\therefore \quad t = 0.693\,\tau = 0.693\times0.05 = 0.0346\ \text{s}$$

(d) $V_L = -L\dfrac{di}{dt} = -L\dfrac{d}{dt}[i_0(1 - e^{-t/\tau})]$

$$= -Li_0\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= \frac{Li_0}{\tau}e^{-t/\tau}$$

$$= \frac{5\times0.06}{0.05}\times e^{-0.1/.05} = 6\times e^{-2}$$

$$= 6\times0.135 = 0.8\ \text{V}$$

EXAMPLE 23 An inductor of inductance 10 H and a resistor of resistance 16 Ω are connected to a 12 V dc source.

(a) Find the final steady current.

(b) How much energy is consumed to attains this steady current?

(c) What is the power dissipated in the resistor at this current?

SOLUTION

(a) $i_0 = \dfrac{E}{R} = \dfrac{12}{16} = \dfrac{3}{4}\ \text{A} = 0.75\ \text{A}$

(b) $U = \dfrac{1}{2}Li_0^2 = \dfrac{1}{2}\times10\times\left(\dfrac{3}{4}\right)^2 = 2.8\ \text{J}$

(c) $P = i_0E = \dfrac{3}{4}\times12 = 9\ \text{W}$

or $P = i_0^2R = \left(\dfrac{3}{4}\right)^2\times16 = 9\ \text{W}$

EXAMPLE 24 An inductor of inductance 100 mH and a resistor of resistance 50 Ω are connected in series to a 2 V battery. After some time the current attains a steady value. The battery is now short circuited. Calculate the time required for the current to fall to half the steady value.

SOLUTION L = 100 mH = 0.1 H, R = 50 Ω, E = 2 V.

$$i_0 = \frac{E}{R} = \frac{2}{50} = 0.04\ \text{A}$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 0.002\ \text{s}$$

$$\frac{i}{i_0} = e^{-t/\tau}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/\tau} \Rightarrow 2 = e^{t/\tau} \Rightarrow \ln 2 = \frac{t}{\tau}$$

$$\therefore \quad t = \tau \ln 2 = 0.002 \times 0.693 = 1.386 \times 10^{-3}\ \text{s}$$

10. Transformer

The transformer is a device used for converting a low ac voltage into a high ac voltage and vice versa. The former is called the step-up transformer and the latter the step-down transformer.

A transformer consists of two coils each of which is wound on an iron core. One of the coils is connected to a source of alternating emf. This coil is called the *primary* of the transformer while the other is called the *secondary* of the transformer. Any of the two coils can act as primary while the other as secondary. The alternating emf in one coil induces an alternating emf in the second coil. The presence of an iron core in the primary and secondary makes the flux linkage between the two coils very large. The alternating emf in the coil makes the magnetic flux in the iron also vary periodically. This varying magnetic flux in iron induces an alternating emf in the secondary.

If the magnetic field lines remain confined to the core, then all the field lines threading the primary also go across the secondary. Then the magnetic fluxes across the secondary and primary will be simply proportional to the number of turns in them, i.e.

$$\frac{\phi_s}{\phi_p} = \frac{N_s}{N_p} \quad \text{or} \quad \phi_s = \left(\frac{N_s}{N_p}\right)\phi_p$$

where N_s is the number of turns in the secondary and N_p is the number of turns in the primary. Now from Faraday's law the emf induced across the secondary is $e_s = -(d\phi_s/dt)$ and that across the primary is $e_p = -(d\phi_p/dt)$.

Thus

$$e_s = -\frac{d}{dt}\left(\frac{N_s}{N_p}\cdot\phi_p\right) = -\frac{N_s}{N_p}\frac{d\phi_p}{dt}$$

or

$$e_s = \frac{N_s}{N_p}e_p$$

From this equation, it follows that if $N_s > N_p$, then $e_s > e_p$, i.e. the voltage across the secondary is greater than the input primary voltage. Such a transformer in which the number of turns in the secondary is more than in the primary is called a *step-up* transformer. But if $N_s < N_p$, then $e_s < e_p$. Such a transformer is called a *step-down* transformer. The former are used in TV, high-voltage power supplies and the latter in radio transmitter sets, battery eliminators, etc.

Usually, there are a number of energy losses in actual transformers. These are: (i) Joule heating (I^2R) losses in the primary and secondary coils due to their resistance (generally, these losses are minimized by using wires of large diameters so that resistance is low); and (ii) the losses in the iron core which include the heating of the core due to eddy currents and power loss due to hysteresis. The eddy currents can be minimized by using laminated iron.

In an ideal transformer, the entire power in the primary is transferred to the secondary. For an ideal transformer,

input power = output power

or $$e_s I_s = e_p I_p$$

Therefore, $$\frac{e_s}{e_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

where I_p and I_s are the currents in the primary and the secondary of the transformer.

EXAMPLE 25 A step down transformer is used to reduce the main supply of 220 V to 10 V. If the primary draws 5 A and secondary 88 A current, calculate the efficiency of the transformer.

SOLUTION $e_p = 220$ V, $e_s = 10$ V, $I_p = 5$ A and $I_s = 88$ A

Input power $(P_i) = e_p \times I_p = 220 \times 5 = 1100$ W

Output power $(P_o) = e_s \times I_s = 10 \times 88 = 880$ W

$$\text{Efficiency } \eta = \frac{P_o}{P_i} = \frac{880}{1100} = 0.8 \text{ or } 80\%$$

EXAMPLE 26 A transformer has an efficiency of 75%. The power input is 4 kW at 100 V. If the secondary voltage is 200 V, calculate the currents in the primary and secondary.

SOLUTION

$P_i = e_p I_p = 4$ kW $= 4000$ W, $e_p = 100$ V and $e_s = 200$ V

$$I_p = \frac{P_i}{e_p} = \frac{4000}{100} = 40\ \text{A}$$

$$\eta = \frac{P_o}{P_i} \Rightarrow P_o = \eta P_i = 0.75 \times 4000 = 3000\ \text{W}$$

$$\therefore \quad I_s = \frac{P_o}{e_s} = \frac{3000}{200} = 15\ \text{A}$$

EXAMPLE 27 The primary of a transformer has 400 turns while the secondary has 2000 turns. The power output from the secondary at 1000 V is 12 kW.

(a) Calculate the primary voltage.

(b) If the resistance of the primary is 0.9 Ω and that of the secondary is 5 Ω and the efficiency of the transformer is 90%, calculate the power loss in the primary coil and in the secondary coil.

SOLUTION $N_p = 400, N_s = 2000, P_o = 12000$ W, $e_s = 1000$ V

(a) $\frac{e_p}{e_s} = \frac{N_p}{N_s} \Rightarrow e_p = \frac{N_p}{N_s} \times e_s = \frac{400}{2000} \times 1000$

$= 200$ V

(b) $\eta = \frac{P_o}{P_i} = \frac{P_o}{e_p I_p}$

$$\Rightarrow \quad I_p = \frac{P_o}{\eta e_p} = \frac{12000}{0.9 \times 200} = \frac{200}{3} \text{ A}$$

$$I_s = \frac{P_o}{e_s} = \frac{12000}{1000} = 12 \text{ A}$$

Power loss in primary $= I_p^2 \times R_p = \left(\frac{200}{3}\right)^2 \times 0.9$

$= 4000$ W

Power loss in secondary $= I_s^2 \times R_s = (12)^2 \times 5$

$= 720$ W

EXAMPLE 28 A power transformer is used to step up an emf of 220 V to 4.4 kV to transmit 6.6 kW of power. If the primary has 1000 turns, find (a) number of turns in the secondary and (b) the current rating of the secondary. Assume that the efficiency of the transformer is 80%.

SOLUTION

(a) $N_s = \frac{e_s}{e_p} \times N_p = \frac{4400}{220} \times 1000 = 20{,}000$

(b) $I_s = \frac{P_o}{e_s} = \frac{\eta P_i}{e_s} = \frac{0.8 \times 6600}{4400} = 1.2$ A

I_s is called the current rating of the secondary.

11. Alternating Current

If an alternating voltage $V = V_0 \sin \omega t$ is applied across a resistance R, the current I in the circuit is

$$I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t \qquad (1)$$

where $I_0 = V_0/R$, is the *maximum* or *peak value* of the current. It is clear from Eq. (1) that the current I varies sinusoidally with time; its magnitude changes continuously with time and its direction is reversed periodically. A *sinusoidally varying current whose magnitude changes continuously with time and whose direction reverses periodically is called an alternating current* (or simply ac).

The angular frequency ω of an alternating current is related to its time period T and frequency ν as

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

where ω is expressed in radians per second (rad s^{-1}), T in seconds (s) and ν in hertz. (Hz). In terms of T, Eq. (1) reads

$$I = I_0 \sin\left(\frac{2\pi t}{T}\right)$$

Root Mean Square Voltage and Current

The mean value of a periodic function $X(t)$ of time period T over one time period is defined as

$$\overline{X} = \frac{\int_0^T X(t)dt}{\int_0^T dt} = \frac{1}{T}\int_0^T X(t)dt$$

(i) Mean or average value of alternating voltage $V = V_0 \sin(\omega t)$ is

$$\overline{V} = \frac{1}{T}\int_0^T V_0 \sin(\omega t)dt$$

$$= \frac{V_0}{T}\int_0^T \sin(\omega t)dt = -\frac{V_0}{T\omega}\left|\cos(\omega t)\right|_0^T$$

$$= -\frac{V_0}{T\omega}\left|\cos\left(\frac{2\pi t}{T}\right)\right|_0^T$$

$$= \frac{V_0}{2\pi}(\cos 2\pi - \cos 0) = 0$$

Similarly mean value of alternating current $I = I_0 \sin(\omega t)$ over one time period is $\overline{I} = 0$

(ii) Mean square value of alternating voltage $V = V_0 \sin(\omega t)$ is

$$\overline{V^2} = \frac{1}{T}\int V_0^2 \sin(\omega t)dt$$

$$= \frac{V_0^2}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)dt$$

$$= \frac{V_0^2}{T}\left[\frac{1}{2}\int_0^T dt - \frac{1}{2}\int_0^T \cos(2\omega t)dt\right]$$

$$=\frac{V_0^2}{T}\left[\frac{T}{2}-\frac{1}{2}\left|\frac{\sin(2\omega t)}{2\omega}\right|_0^T\right]$$

$$=\frac{V_0^2}{T}\left[\frac{T}{2}-0\right]=\frac{V_0^2}{2}$$

Root mean square (rms) value of the alternating voltage is

$$V_{rms}=\sqrt{\overline{V^2}}=\frac{V_0}{\sqrt{2}}=\frac{1}{\sqrt{2}}\times \text{peak value of } V$$

Similarly, root mean square (rms) value of alternating current $I = I_0 \sin(\omega t)$ is

$$I_{rms}=\sqrt{\overline{I^2}}=\frac{I_0}{\sqrt{2}}=\frac{1}{\sqrt{2}}\times \text{peak value of } I$$

EXAMPLE 29 An alternating voltage V (in volt) varies with time t (in second) as

$$V = 100 \sin(50\,\pi t)$$

Find the peak value, rms value and frequency of the alternating voltage.

SOLUTION Comparing the given equation with

$$V = V_0 \sin(\omega t)$$

We get $V_0 = 100$ V,

$$V_{rms}=\frac{V_0}{\sqrt{2}}=\frac{100}{\sqrt{2}}=70.7 \text{ V}$$

and $\omega = 50\pi$

or $2\pi\nu = 50\pi \Rightarrow \nu = 25$ Hz

EXAMPLE 30 A 100 Ω electric iron is connected to a 200 V, 50 Hz a.c. supply. Find (a) rms value of voltage, (b) peak value of voltage, (c) rms value of current and (d) peak value of current.

SOLUTION If an alternating supply is given to be 200 V, 50 Hz, it implies that the rms value of voltage is 200 V and the frequency is 50 Hz.

(a) $V_{rms} = 200$ V

(b) $V_0 = \sqrt{2}\,V_{rms} = \sqrt{2}\times 200 = 282.8$ V.

(c) $I_{rms}=\frac{V_{rms}}{R}=\frac{200}{100}=2$ A

(d) $I_0=\frac{V_0}{R}=\frac{282.8}{100}=2.828\simeq 2.83$ A

12. Series LCR Circuit

The applied voltage V divides into three parts, V_L (across L), V_C (across C) and V_R (across R) such that (Fig. 14.31).

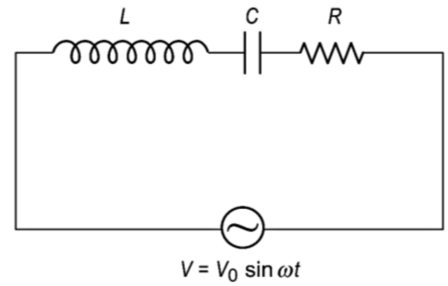


Fig. 14.31

$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$

The impedance of the circuit is

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

The current in the circuit is

$$I = I_0 \sin(\omega t - \phi)$$

$$\tan\phi=\frac{\left(\omega L-\frac{1}{\omega C}\right)}{R}$$

(i) If $\omega L > \frac{1}{\omega C}$, i.e. $\omega > \frac{1}{\sqrt{LC}}$, then $\tan\phi$ is positive and voltage leads the current.

(ii) If $\omega L < \frac{1}{\omega C}$, i.e. $\omega < \frac{1}{\sqrt{LC}}$, then voltage lags behind current.

(iii) If $\omega L = \frac{1}{\omega C}$, i.e. $\omega = \frac{1}{\sqrt{LC}}$, then $\phi = 0$

This is the case of resonance. Voltage and current are in phase. $Z = R$ (minimum) and current is maximum.

Special Cases

(a) A.C. circuit containing only a pure resistor (Fig. 14.32)

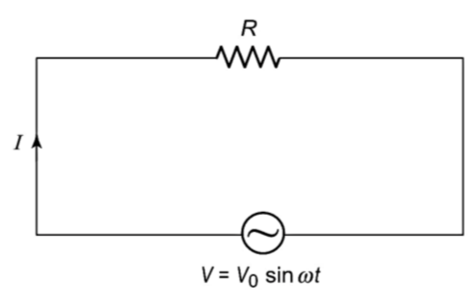


Fig. 14.32

$$V_R = V_0 \sin \omega t$$

$$I = I_0 \sin \omega t$$

where $$I_0 = \frac{V_0}{R}$$

The voltage across R is always in phase with the current in the circuit.

(b) A.C. circuit containing only an ideal inductor (Fig. 14.33)

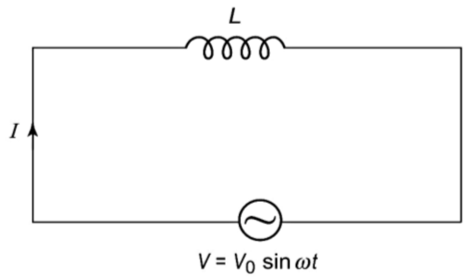


Fig. 14.33

$$V_L = V_0 \sin \omega t$$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $$I_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega L}$$

$X_L = \omega L$ is called inductive reactance. The voltage across the inductance leads the current in the circuit by a phase angle of $\pi/2$.

(c) A.C. circuit containing only an ideal capacitor (Fig. 14.34)

$$V_C = V_0 \sin \omega t$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $$I_0 = \frac{V_0}{X_C} = \omega C V_0$$

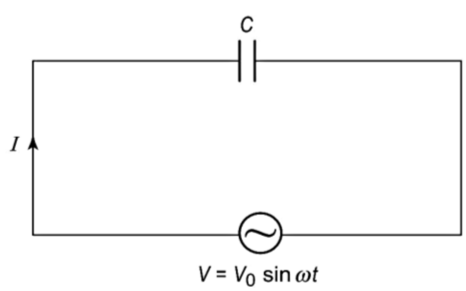


Fig. 14.34

$X_C = \dfrac{1}{\omega C}$ is called capacitative reactance. The voltage across the capacitor lags behind the current in the circuit by a phase angle of $\pi/2$.

(d) A.C. circuit containing an ideal inductor and a pure resistor (Fig. 14.35)

$$V_0 = I_0 Z$$

Where $$V_0 = \sqrt{V_R^2 + V_L^2}$$

$$V_R = IR, \; V_L = IX_L$$

and $Z = \dfrac{V_0}{I_0} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$ is called impedance.

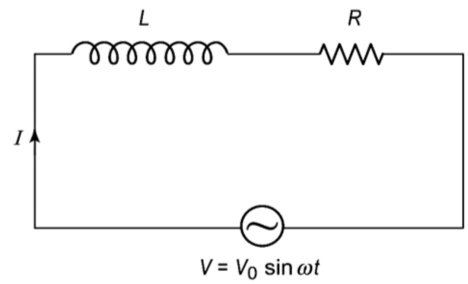


Fig. 14.35

$$I = I_0 \sin(\omega t - \phi)$$

where $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$ is the phase angle between the voltage and current in the circuit.

(e) A.C. circuit containing an ideal capacitor and a pure resistor (Fig. 14.36)

$$V_0 = I_0 Z$$

where $$V_0 = \sqrt{V_R^2 + V_C^2}$$

$$V_R = IR, \; V_C = IX_C$$

and $$Z = \frac{V_0}{I_0} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

is called impedance.

$$I = I_0 \sin(\omega t + \phi)$$

where $\phi = \tan^{-1}\left(\dfrac{1}{R\omega C}\right)$ is the phase angle between the voltage and current in the circuit.

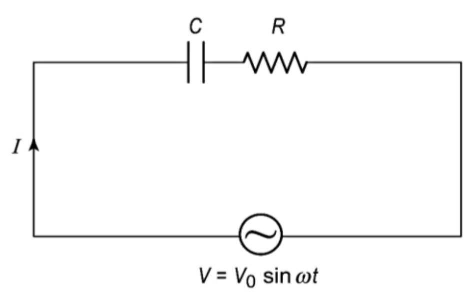


Fig. 14.36

13. Power in LCR Circuit

In a series LCR circuit driven by an alternating voltage $V = V_0 \sin \omega t$, the current in the circuit is

$$I = I_0 \sin(\omega t \pm \phi)$$

depending upon whether $X_L < X_C$ or $X_L > X_C$.

$$I_0 = \frac{V_0}{Z};\ Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Instantaneous power supplied to the circuit by the A.C. source is

$$P(t) = VI = V_0 \sin \omega t \times I_0 \sin(\omega t \pm \phi)$$
$$= V_0 I_0 \sin \omega t \times \sin(\omega t \pm \phi)$$

∴ Average power supplied by the source in one complete cycle is

$$\overline{P} = \frac{1}{T}\int_0^T P(t)\,dt$$

$$= \frac{1}{T} \times V_0 I_0 \int_0^T \sin \omega t\,(\sin \omega t \cos\phi \pm \cos \omega t \sin \phi)dt$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \int_0^T \sin^2(\omega t)dt \pm \sin\phi \int_0^T \sin(\omega t) \times \cos(\omega t)dt\right]$$

$$= \frac{V_0 I_0}{T}\left[\cos\phi \times \frac{T}{2} \pm 0\right]$$

$$= \frac{V_0 I_0}{2}\cos\phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}\cos\phi$$

or $\overline{P} = V_{rms} I_{rms} \cos \phi$

Power Factor of an A.C. Circuit

The power supplied by the source depends not only on V_{rms} and I_{rms} but also on $\cos \phi$. The quantity $\cos \phi$ is called the power factor of the A.C. circuit. Now

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

and $$\cos \phi = \frac{1}{(1 + \tan^2 \phi)^{1/2}}$$

$$= \frac{1}{1 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}$$

$$= \frac{R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$$= \frac{R}{Z}$$

$$\therefore \text{Power factor} = \frac{\text{resistance}}{\text{impedance}}$$

Special Cases

(a) *For an A.C. circuit containing only a resistor,*

$Z = R$ and $\cos \phi = \frac{R}{R} = 1 \Rightarrow \phi = 0$ and $\overline{P} = V_{rms} I_{rms}$

(b) *For an A.C. circuit containing only an inductor or a capacitor,*

$\phi = 90°$. Hence $\overline{P} = 0$

(c) At resonance, $\phi = 0$ for an LCR circuit. Hence $\overline{P}$ = maximum, i.e. maximum power is delivered to the circuit form A.C. source.

Wattless Current

For an A.C. circuit containing only a pure inductor or an ideal capacitor $\phi = 90°$. Hence

$$\overline{P} = V_{rms} I_{rms} \cos 90° = 0$$

Such an A.C. circuit consumes no power. The current flowing through the inductor or capacitor consumes no power and is called wattless current.

Bandwidth and Quality Factor of LCR Circuit

For an LCR circuit driven by an alternating voltage $V = V_0 \sin \omega t$, the peak (amplitude) value of the current is given by

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

$$\therefore \quad (I_0)_{max} = \frac{V_0}{R} \quad \Rightarrow \quad V_0 = (I_0)_{max} R$$

In terms of $(I_0)_{max}$, I_0 is given by

$$I_0 = \frac{(I_0)_{max} R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

Figure 14.37 shows the variation of I_0 versus ω.

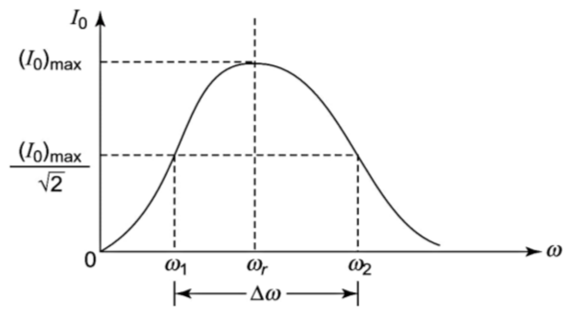


Fig. 14.37

Let ω_1 and ω_2 be the values of ω when $I_0 = \frac{(I_0)_{max}}{\sqrt{2}}$, i.e. when

$$\frac{(I_0)_{max} R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}} = \frac{(I_0)_{max} R}{\sqrt{2}}$$

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\Rightarrow \quad \omega L - \frac{1}{\omega C} = \pm R$$

Case 1: $\omega L - \frac{1}{\omega C} = +R \Rightarrow \omega^2 - \frac{1}{LC} = \frac{R\omega}{L}$

$$\Rightarrow \quad \omega^2 - \frac{R\omega}{L} - \omega_r^2 = 0, \text{ where } \omega_r = \frac{1}{\sqrt{LC}}$$

The positive root of this quadratic equation is

$$\omega_2 = \frac{R}{2L} + \left(1 + \frac{4\omega_r^2 L^2}{R^2}\right)^{1/2}$$

Case 2: $\omega L - \frac{1}{\omega C} = -R$

$$\Rightarrow \quad \omega^2 + \frac{R\omega}{L} - \omega_r^2 = 0$$

The positive root of this quadratic equation is

$$\omega_1 = -\frac{R}{2L} + \left(1 + \frac{4\omega_r^2 L^2}{R^2}\right)^{1/2}$$

Bandwidth $\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$

Quality factor (or Q factor) of LCR circuit is defined as

$$Q = \frac{\text{resonant frequency}}{\text{bandwidth}} = \frac{\omega_r}{\Delta\omega}$$

$$= \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q is a dimensionless number. Figure 14.38 shows the graph of $\overline{P}$ versus ω for some values of Q.

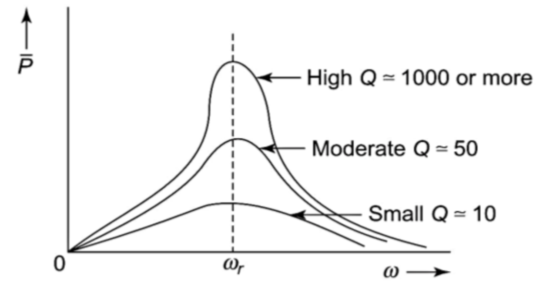


Fig. 14.38

The power peak is sharp for high Q. The resonance is then said to be sharp. Higher the value of Q, the sharper is the resonance and greater is the power absorbed from the source.

EXAMPLE 31 A coil of inductance 0.5 H and a resistor of resistance 100 Ω are connected in series to a 240 V, 50 Hz supply.

(a) Find the maximum current in the circuit.

(b) What is the time lag between voltage maximum and current maximum?

SOLUTION Given $V_{rms} = 240$ V, $\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$ rad s^{-1}, $L = 0.5$ H and $R = 100\ \Omega$.

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin(\omega t - \phi);$$

$$I_0 = \frac{V_0}{(R^2 + \omega^2 L^2)^{1/2}},$$

$$\tan \phi = \frac{\omega L}{R}$$

(a) $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 240$ V

$$\therefore I_0 = \frac{\sqrt{2} \times 240}{[(100)^2 + (100\pi \times 0.5)^2]^{1/2}} = 1.82 \text{ A}$$

(b) V is maximum when $\sin \omega t = +1 \Rightarrow \omega t_1 = \frac{\pi}{2}$

$$\Rightarrow t_1 = \frac{\pi}{2\omega}$$

I is maximum when $\sin(\omega t - \phi) = +1 \Rightarrow \omega t_2 - \phi = \frac{\pi}{2}$

$$\Rightarrow t_2 = \frac{\pi}{2\omega} - \frac{\phi}{\omega}$$

$\therefore$ Time lag between voltage maximum and current maximum is

$$\Delta t = t_1 - t_2 = \frac{\pi}{2\omega} - \left(\frac{\pi}{2\omega} - \frac{\phi}{\omega}\right) = \frac{\phi}{\omega}$$

Now $\quad \tan \phi = \frac{\omega L}{R} = \frac{100\pi \times 0.5}{100} = 1.57$

$$\Rightarrow \qquad \phi = 57.5° = \frac{57.5\times\pi}{180} \text{ rad}$$

$$\therefore \qquad \Delta t = \frac{\phi}{\omega} = \frac{57.5\times\pi}{180\times100\pi} = 3.2\times10^{-3}\text{ s}$$

EXAMPLE 32 A capacitor of capacitance 100 μF and a resistor of resistance 40 Ω are connected in series to a 110 V, 60 Hz supply.

(a) Find the maximum current in the circuit.

(b) What is the time lag between current maximum and voltage maximum?

SOLUTION Given $C = 100\ \mu F = 100\times10^{-6}$ F, $R = 40\ \Omega$, $V_{rms} = 110$ V, $\omega = 2\pi\nu = 2\pi\times60 = 120$ rad s^{-1}

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin(\omega t + \phi),$$

$$I_0 = \frac{V_0}{\left(R^2 + \frac{1}{\omega^2 C^2}\right)^{1/2}}, \tan\phi = \frac{1}{\omega CR}$$

(a) $$I_0 = \frac{\sqrt{2}\times110}{\left[\left((40)^2 + \frac{1}{(120\times10^{-4})^2}\right)\right]^{1/2}} = 3.24\text{ A}$$

(b) V is maximum when $\sin\omega t = +1 \Rightarrow \omega t_1 = \frac{\pi}{2}$

$\Rightarrow t_1 = \frac{\pi}{2\omega}$

I is maximum when $\sin(\omega t + \phi) = +1$

$$\Rightarrow \omega t_2 + \phi = \frac{\pi}{2} \Rightarrow t_2 = \frac{\pi}{2\omega} - \frac{\phi}{\omega}$$

∴ Time lag between current maximum and voltage maximum is

$$\Delta t = t_1 - t_2 = \frac{\phi}{\omega}$$

Now $$\tan\phi = \frac{1}{\omega CR} = \frac{1}{120\pi\times10^{-4}\times40} = 0.663$$

$$\Rightarrow \qquad \phi = 33.5° = \frac{33.5\times\pi}{180} \text{ rad}$$

$$\therefore \qquad \Delta t = \frac{\phi}{\omega} = \frac{33.5\times\pi}{180\times120\pi} = 1.55\times10^{-3}\text{ s}$$

EXAMPLE 33 A series LCR circuit with $L = 5$ H, $C = 80\ \mu$F and $R = 40\ \Omega$ is connected to a variable frequency 230 V a.c. source.

(a) What is the source frequency which drives the circuit at resonance?

(b) What is the impedance of the circuit at resonance?

(c) Find peak value of the current at resonance.

(d) Find the rms potential differences across L, C and R at resonance.

(e) What is the total potential difference across the combination of L and C at resonance.

(f) Find the maximum power transferred to the circuit from the source in one complete cycle.

SOLUTION

(a) $$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times80\times10^{-6}}} = 50\text{ rad s}^{-1}$$

$$\therefore\ \nu_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = 7.96\text{ Hz}$$

(b) $$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

$$= R \qquad \left(\because\ \omega L = \frac{1}{\omega C} \text{ at resonance}\right)$$

$$= 40\ \Omega$$

(c) $$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2}\times230}{40} = 8.1\text{ A}$$

(d) $$(V_L)_{rms} = I_{rms}\times X_L = I_{rms}\times\omega_r L$$

$$= \frac{230}{40}\times50\times5 = 1437.5\text{ V}$$

$$(V_C)_{rms} = I_{rms}\times X_C = I_{rms}\times\frac{1}{\omega_r C}$$

$$= \frac{230}{40}\times\frac{1}{80\times10^{-6}\times50} = 1437.5\text{ V}$$

$$(V_R)_{rms} = I_{rms}\times R = \frac{230}{40}\times40 = 230\text{ V}$$

(e) $$(V_{L,C})_{rms} = I_{rms}\times\left(\omega_r L - \frac{1}{\omega_r C}\right)$$

$$= 1437.5 - 1437.5 = 0$$

(f) $$P_{max} = V_{rms}\times I_{rms}\times\cos\phi$$

$$= V_{rms}\times I_{rms} \qquad (\because\ \phi = 0 \text{ at resonance})$$

$$= \frac{V_{rms}^2}{R} = \frac{230\times230}{40} = 1322.5\text{ W}$$

EXAMPLE 34 When an alternating voltage of 220 V is applied across a device A, a current of 0.5 A flows through the circuit and it is in phase with the applied voltage. When the same voltage is applied across a device

B, again the same current flows in the circuit but it leads the voltage by $\pi/2$. (a) Name devices A and B. (b) Compute the current when the same voltage is applied across a series combination of A and B.

SOLUTION

(a) Device A is a resistor and B is a capacitor.

(b) Given $V_{rms} = 220$ V, $I_{rms} = 0.5$ A

Resistance of A is $R = \dfrac{V_{rms}}{I_{rms}} = \dfrac{220}{0.5} = 440\ \Omega$

Reactance of B is $X_C = \dfrac{V_{rms}}{I_{rms}} = \dfrac{220}{0.5} = 440\ \Omega$

Impedance when A and B are connected in series is

$$Z = \sqrt{R^2 \times X_C^2} = \sqrt{(440)^2 + (440)^2} = 622.3\ \Omega$$

$$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{622.3} = 0.35 \text{ A}$$

14. LC Oscillations

In an electrical circuit consisting of an inductance L and a capacitance C, the charge (and hence current) oscillates harmonically with an angular frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

and time period $T = 2\pi\sqrt{LC}$

The charge and current in the circuit vary with time as

$$q = q_0 \sin(\omega t + \phi) \text{ and } I = I_0 \cos(\omega t + \phi)$$

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. An ideal solenoid of cross-sectional area $10^{-4}\,m^2$ has 500 turns per metre. At the centre of this solenoid, another coil of 100 turns is wrapped closely around it. If the current in the coil changes from 0 to 2 A in 3.14 millisecond, the emf developed in the second coil is
   (a) 1 mV (b) 2 mV
   (c) 3 mV (d) 4 mV
2. A rectangular loop of sides 8 cm and 2 cm with a small break in it is moving out of a region of uniform magnetic field of 0.3 T, directed normal to the loop. What is the emf developed across the break if the velocity of the loop is 1 cms^{-1} in a direction normal to the longer side of the loop?
   (a) 0.06 mV (b) 0.12 mV
   (c) 0.18 mV (d) 0.24 mV
3. In Q. 2, for how long does the induced emf last?
   (a) 2 s (b) 4 s
   (c) 6 s (d) 8 s
4. In Q. 3, what is the emf developed across the break if the velocity of the loop is 1 cm s^{-1} in a direction normal to the shorter side of the loop?
   (a) 0.06 mV (b) 0.12 mV
   (c) 0.18 mV (d) 0.24 mV
5. In Q. 4, how long does the induced emf last?
   (a) 2 s (b) 4 s
   (c) 6 s (d) 8 s
6. A pair of coils has a mutual inductance of 2 H. If the current in the primary changes from 10 A to zero in 0.1 s, the induced emf in the secondary will be
   (a) 100 V (b) 200 V
   (c) 300 V (d) 400 V
7. A motor having an armature of resistance 2 Ω is designed to operate at 220 V mains. At full speed, it develops a back emf of 210 V. What is the current in the armature when the motor is running at full speed?
   (a) 2.5 A (b) 5.0 A
   (c) 7.5 A (d) 10 A
8. In Q. 7, when the motor was switched on, what was the current in the armature if no starter was used?
   (a) zero (b) 5.0 A
   (c) 110 A (d) 220 A
9. In Q. 7, the efficiency of the motor at full speed is very nearly equal to
   (a) 65% (b) 75%
   (c) 85% (d) 95%

10. The resistance of the armature of a generator is 0.2 Ω. It yields an emf of 220 V in an open circuit and a potential difference of 210 V at full load. The current at full load is

(a) 30 A (b) 40 A
(c) 50 A (d) 60 A

11. In Q. 10, the power delivered to the external circuit is

(a) 9.0 kW (b) 9.5 kW
(c) 10 kW (d) 10.5 kW

12. The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1000 V is 12 kW, what is the primary voltage?

(a) 200 V (b) 300 V
(c) 400 V (d) 500 V

13. In Q. 12, if the resistance of the primary is 0.2 Ω and that of the secondary is 2 Ω and the efficiency of the transformer is 80%, the power loss in the primary is

(a) 1.125 kW (b) 2.25 kW
(c) 3.375 kW (d) 4.5 kW

14. In Q. 13, the power loss in the secondary is

(a) 72 W (b) 144 W
(c) 216 W (d) 288 W

15. The magnitude of the induced emf produced in a coil when a magnet is inserted into it does NOT depend upon the

(a) number of turns in the coil
(b) resistance of the coil
(c) magnetic moment of the magnet
(d) speed of approach of the magnet

16. A coil of wire is held with its plane horizontal to the earth's surface and a small bar magnet dropped vertically down through it. The magnet will fall with a

(a) constant acceleration equal to g
(b) constant acceleration greater than g
(c) constant acceleration less than g
(d) non-uniform acceleration less than g

17. An electron moves along the line PQ which lies in the same plane as a circular loop of conducting wire, as shown in Fig. 14.39. What will be the direction of the induced current, if any, in the loop?

(a) Anti-clockwise
(b) Clockwise
(c) Alternating
(d) No current will be induced in the loop.

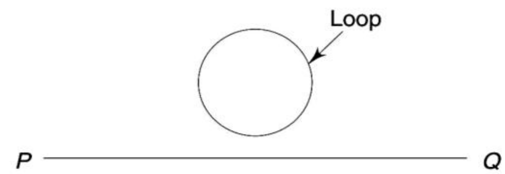


Fig. 14.39

18. A magnet is moved in the direction indicated by an arrow between two coils AB and CD as shown in Fig. 14.40. What is the direction of the induced current in each coil?

(a) A to B in coil X and C to D in coil Y
(b) A to B in coil X and D to C in coil Y
(c) B to A in coil X and C to D in coil Y
(d) B to A in coil X and D to C in coil Y.

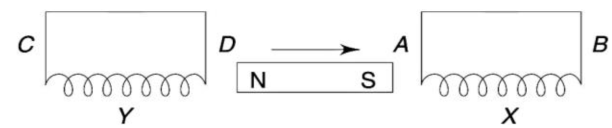


Fig. 14.40

19. Figure 14.41 shows two coils P and Q placed close to each other. When the circuit of coil P is suddenly broken by lifting the key K,

(a) a current flows from X to Y in coil Q
(b) a current flows from Y to X in coil Q
(c) no current flows in coil Q
(d) an alternating current flows in coil Q.

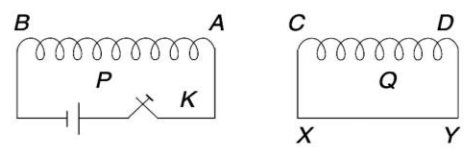


Fig. 14.41

20. A transformer has 200 windings in the primary and 400 windings in the secondary. The primary is connected to an ac supply of 110 V and a current of 10 A flows in it. The voltage across the secondary and the current in it, respectively, are

(a) 55 V, 20 A (b) 440 V, 5 A
(c) 220 V, 10 A (d) 220 V, 5 A

21. The magnitude of the emf across the secondary of a transformer does NOT depend upon

(a) the magnitude of the emf applied across the primary
(b) the number of the turns in the primary
(c) the number of turns in the secondary
(d) the resistances of the primary and the secondary.

22. Electrical energy generated at a power house is delivered to distant places over long transmission cables at a very high ac voltage of about 33,000 volts. The reason for this is that

(a) at high voltages energy is delivered much faster than at low voltages
(b) there is less wastage of energy at high voltages
(c) the high voltage prevents theft of costly transmission cables
(d) it is much easier to generate large amounts of energy at high voltages.

23. Flux ϕ (in weber) in a closed circuit of resistance 10 Ω varies with time t (in seconds) according to the equation

$$\phi = 6t^2 - 5t + 1$$

The magnitude of the induced current in the circuit at $t = 0.25$ s is
(a) 0.2 A (b) 0.6 A
(c) 0.8 A (d) 1.2 A

24. A circuit has a self inductance of 1 henry and carries a current of 2 A. To prevent sparking when the circuit is broken, a capacitor which can withstand 400 volts is used. The least capacitance of the capacitor connected across the switch must be equal to
(a) 12.5 μF (b) 25 μF
(c) 50 μF (d) 100 μF

25. An aeroplane is moving north horizontally, with a speed of 200 ms^{-1}, at a place where the vertical component of the earth's magnetic field is 0.5×10^{-4} T. What is the induced emf set up between the tips of the wings if they are 10 m apart?
(a) 0.01 V (b) 0.1 V
(c) 1 V (d) 10 V

26. A step-down transformer is employed to reduce the main supply of 220 V to 11 V. The primary draws 5 A of current and the secondary 90 A. What is the efficiency of the transformer?
(a) 20% (b) 40%
(c) 70% (d) 90%

27. A 25 kW dc generator produces a potential difference of 250 V. If the resistance of the transmission line is 1 Ω, what percentage of the original power is lost during transmission?
(a) 40% (b) 50%
(c) 60% (d) 75%

28. Two resistances of 10 Ω and 20 Ω and an ideal inductor of inductance 5 H are connected to a 2 V battery through a key K, as shown in Fig. 14.42. The key is inserted at $t = 0$. What is the final value of current in the 10 Ω resistor?
(a) 2/3 A (b) 1/3 A
(c) 1/6 A (d) zero

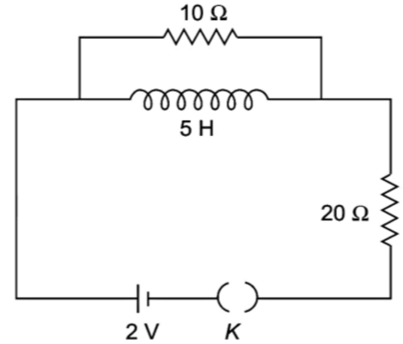


Fig. 14.42

29. What is the final value of the current through the 20 Ω resistor shown in Fig. 14.7.
(a) zero (b) 0.1 A
(c) 2/3 A (d) 1/3 A

30. A 10 Ω electric heater is connected to a 200 V, 50 Hz mains supply. What is the peak value of the potential difference across the heater element?
(a) 220 V (b) $220/\sqrt{2}$ V
(c) 110 V (d) $220\sqrt{2}$ V

31. A choke is used as a resistance in
(a) dc circuits
(b) ac circuits
(c) both ac and dc circuits
(d) full-wave rectifier circuits

32. At resonance, the value of the power factor in an LCR series circuit is
(a) zero (b) 1/2
(c) 1 (d) not definite

Level B

33. An ac series circuit contains 4 Ω resistance and 3 Ω inductive reactance. What is the impedance of the circuit?
(a) 1 Ω (b) 5 Ω
(c) 7 Ω (d) $\frac{7}{\sqrt{2}}$ Ω

34. An inductive coil has a resistance of 100 Ω. When an ac signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?
(a) 10 mH (b) 12 mH
(c) 16 mH (d) 20 mH

35. An ac source of variable frequency f is connected to an LCR series circuit. Which one of the graphs in Fig. 14.43 represents the variation of current I in the circuit with frequency f?

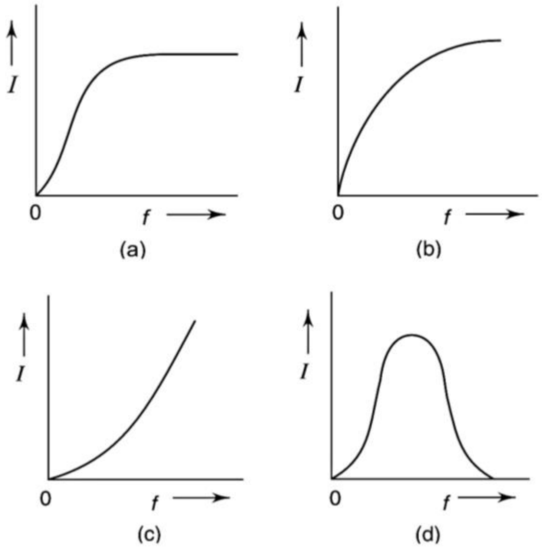


Fig. 14.43

36. Choose the correct statement. In the case of ac circuits, Ohm's law holds for
(a) peak values of voltage and current
(b) effective values of voltage and current
(c) instantaneous values of voltage and current
(d) all the above.

37. Two circuits 1 and 2 are connected to identical dc sources each of emf 12 V. Circuit 1 has a self inductance L_1 = 10 H and circuit 2 has a self inductance L_2 = 10 mH. The total resistance of each circuit is 48 Ω. The ratio of steady currents in circuits 1 and 2 is
(a) 1000 (b) 100
(c) 10 (d) 1

38. In Q. 37, what is the ratio of energy consumed in circuits 1 and 2 to build up the current to the steady state value?
(a) 1000 (b) 100
(c) 10 (d) 1

39. In Q. 37, what is the ratio of the power dissipated by circuits 1 and 2 after the steady state is reached?
(a) 1000 (b) 100
(c) 10 (d) 1

40. An inductor of self inductance 100 mH and a resistor of resistance 50 Ω are connected to a 2 V battery. The time required for the current to fall to half its steady value is
(a) 2 millisecond
(b) 2 ln (0.5) millisecond
(c) 2 ln (1) millisecond
(d) 2 ln (2) millisecond

41. Figure 14.44 shows a series *LCR* circuit connected to a variable frequency 200 V source. L = 5 H, C = 80 μ F and R = 40 Ω. What is the source frequency which drives the circuit at resonance?
(a) 25 Hz (b) $\frac{25}{\pi}$ Hz
(c) 50 Hz (d) $\frac{50}{\pi}$ Hz

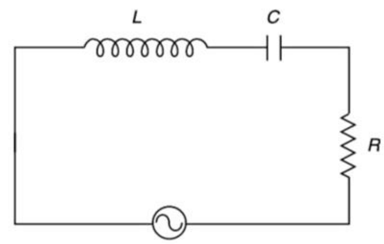


Fig. 14.44

42. In Q. 41, what is the impedance of the circuit at resonance?
(a) 20 Ω (b) 40 Ω
(c) 60 Ω (d) 80 Ω

43. In Q. 41, the current amplitude at resonance is
(a) $\frac{5}{\sqrt{2}}$ A (b) 5 A
(c) $5\sqrt{2}$ A (d) 10 A

44. In Q. 41, the rms potential drop across the inductor at resonance is
(a) 1 kV (b) 1.25 kV
(c) 1.5 kV (d) 1.75 kV

45. In Q. 41, the rms potential drop across the capacitor at resonance is
(a) 1 kV (b) 1.25 kV
(c) 1.5 kV (d) 1.75 kV

46. In Q. 41, the rms potential drop across the resistor at resonance is
(a) $\frac{100}{\sqrt{2}}$ V (b) 100 V
(c) $\frac{200}{\sqrt{2}}$ V (d) 200 V

47. In an *LCR* circuit, if V is the effective value of the applied voltage, V_R is the voltage across R, V_L is the effective voltage across L, V_C is the effective voltage across C, then
(a) $V = V_R + V_L + V_C$
(b) $V^2 = V_R^2 + V_L^2 + V_C^2$
(c) $V^2 = V_R^2 + (V_L - V_C)^2$
(d) $V^2 = V_L^2 + (V_R - V_C)^2$

48. L, C and R, respectively represent inductance, capacitance and resistance. Which one of the following combinations has the dimensions of frequency?
(a) $1/RC$ (b) $1/LC$
(c) L/R (d) C/L

49. The network shown in Fig. 14.45 is part of a circuit.

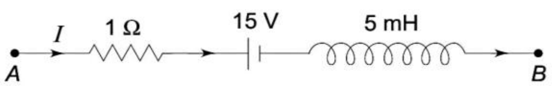


Fig. 14.45

What is the potential difference $(V_B - V_A)$ when current I is 5μ A and is decreasing at a rate of 10^{-3} As^{-1}?
(a) 5 V (b) 10 V
(c) 15 V (d) zero

50. A capacitor of capacitance 2 μF is charged to a potential difference of 12 V. The charging battery is then removed and the capacitor is connected across an inductor of self inductance 0.6 mH. The current in the circuit at a time when the potential difference across the capacitor is 6 V is
(a) 0.3 A (b) 0.6 A
(c) 0.9 A (d) 1.2 A

51. In an inductor, the current I (in ampere) varies with time t (in second) as
$$I = 5 + 16t$$
If the emf induced in the inductor is 10 mV, what is its self inductance?
(a) 6.25×10^{-4} H (b) 6.25×10^{-3} H
(c) 7.5×10^{-4} H (d) 7.5×10^{-3} H

52. In Q. 51 above, the power supplied to the inductor at $t = 1$ s is
(a) 0.021 W (b) 0.21 W
(c) 2.1 W (d) 21 W

53. A wire in the form of a circular loop of radius r lies with its plane normal to a magnetic field B. If the wire is pulled to take a square shape in the same plane in time t, the emf induced in the loop is given by
(a) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{10}\right)$ (b) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{8}\right)$
(c) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{6}\right)$ (d) $\frac{\pi Br^2}{t}\left(1-\frac{\pi}{4}\right)$

54. A uniformly wound solenoid coil of self-inductance 1.8×10^{-4} H and resistance 6 Ω is broken up into two identical coils. These identical coils are then connected across a 12 V battery of negligible resistance. The time constant for the current in the circuit is
(a) 0.3×10^{-4} s (b) 0.3×10^{-3} s
(c) 0.3×10^{-2} s (d) 0.3 μs

55. In Q. 54 above, the steady current through the battery is
(a) 8 μA (b) 8 mA
(c) 0.8 A (d) 8 A

56. A square loop of side l, mass m and resistance R falls vertically into a uniform magnetic field directed perpendicular to the plane of the coil. The height h through which the loop falls so that it attains terminal velocity on entering the region of magnetic field is given by
(a) $\frac{mgR}{2Bl}$ (b) $\frac{m^2gR^2}{2B^2l^2}$
(c) $\frac{mgR^2}{4B^3l^3}$ (d) $\frac{m^2gR^2}{2B^4l^4}$

57. The mutual inductance between two planar concentric rings of radii r_1 and r_2 (with $r_1 > r_2$) placed in air is given by
(a) $\frac{\mu_0\pi r_2^2}{2r_1}$ (b) $\frac{\mu_0\pi r_1^2}{2r_2}$
(c) $\frac{\mu_0\pi (r_1+r_2)^2}{2r_1}$ (d) $\frac{\mu_0\pi (r_1+r_2)^2}{2r_2}$

58. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field B = 2T as shown in Fig. 14.46. The magnetic field is perpendicular to the plane of the loop and directed into the paper. The loop is connected to a network of resistors, each equal to 3 Ω. What should be the speed of the loop so as to have a steady current of 1 mA in the loop?
(a) 1 cm s^{-1} (b) 2 cm s^{-1}
(c) 3 cm s^{-1} (d) 4 cm s^{-1}

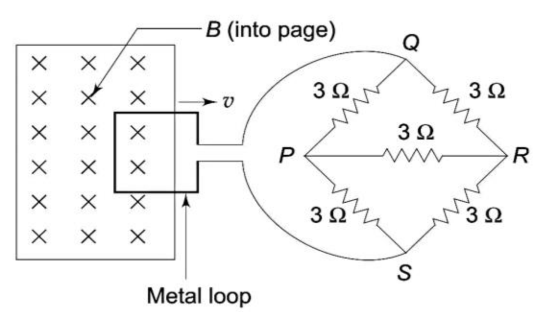


Fig. 14.46

59. If a coil of metal wire is kept stationary in a non-uniform magnetic field,
(a) an emf and current are both induced in the coil
(b) a current but no emf is induced in the coil
(c) an emf but no current is induced in the coil
(d) neither emf nor current is induced in the coil

60. In an ac circuit the potential differences across an inductance and a resistance connected in series are respectively 16 V and 20 V. The total potential difference across the circuit is
(a) 20.0 V (b) 25.6 V
(c) 31.9 V (d) 53.5 V

61. An alternating voltage $V = V_0 \sin \omega t$ is applied across a circuit. As a result a current $I = I_0 \sin(\omega t - \pi/2)$ flows in it. The power consumed per cycle is
(a) zero (b) $0.5\ V_0 I_0$
(c) $0.707\ V_0 I_0$ (d) $1.414\ V_0 I_0$

62. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic field B, constant in space and time, pointing perpendicular and into the plane of the loop exists everywhere as shown in Fig. 14.47. The current induced in the loop is
(a) BLv/R clockwise
(b) BLv/R anticlockwise
(c) $2BLv/R$ anticlockwise
(d) zero

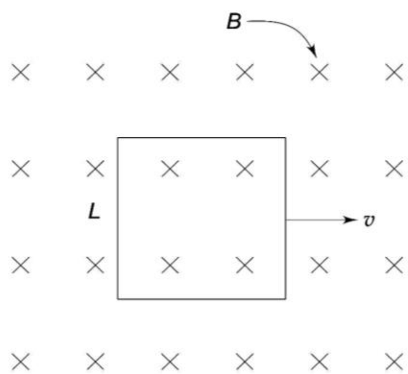


Fig. 14.47

63. A thin circular ring of area A is held perpendicular to a uniform magnetic field B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is
(a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$
(c) ABR (d) $\frac{B^2 A}{R^2}$

64. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic field B (Fig. 14.48). At the position MNQ the speed of the ring is v and the potential difference across the ring is
(a) zero
(b) $\frac{1}{2} Bv\,\pi R^2$ and M is at higher potential
(c) $\pi R\,Bv$ and Q is at higher potential
(d) $2\,RBv$ and Q is at higher potential.

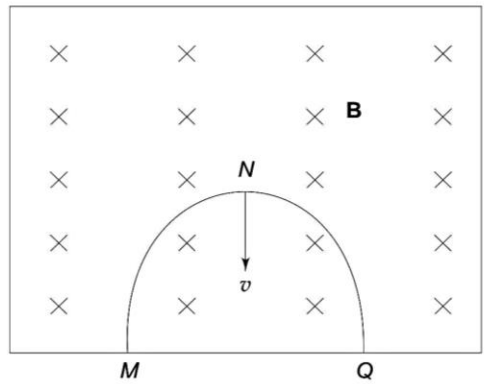


Fig. 14.48

65. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L >> l$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to
(a) $\frac{l}{L}$ (b) $\frac{l^2}{L}$
(c) $\frac{L}{l}$ (d) $\frac{L^2}{l}$

66. A circular loop of radius R, carrying current I, lies in the x-y plane with its centre at the origin. The total magnetic flux through the x-y plane is
(a) directly proportional to I
(b) directly proportional to R
(c) inversely proportional to R
(d) zero

67. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time
(a) 500 s (b) 20 s
(c) 35 ms (d) 1 ms

68. A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in Fig. 14.49. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region
(a) is zero (b) decreases as $\frac{1}{r}$
(c) increases as r (d) decreases as $\frac{1}{r^2}$

75. Two parallel wires PQ and ST placed a distance w apart are connected by a resistor R as shown in Fig. 14.54 and placed in a magnetic field B which is perpendicular to the plane containing the wires. A rod CD connects the two wires. The power spent to slide the rod CD with a velocity v along the wires is (neglect the resistance of the wires and the rod)

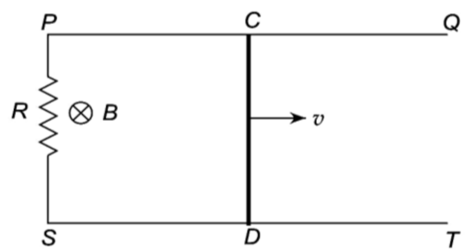


Fig. 14.54

(a) $\frac{Bwv}{R}$ (b) $\frac{Bwv}{R^2}$

(c) $\frac{(Bwv)^2}{R}$ (d) $\left(\frac{Bwv}{R}\right)^2$

76. An air plane, with 20 m wingspread is flying at 250 ms^{-1} parallel to the earth's surface at a place where the horizontal component of earth's magnetic field is 2×10^{-5} T and angle of dip is 60°. The magnitude of the induced emf between the tips of the wings is

(a) $\frac{1}{10}$V (b) $\frac{\sqrt{2}}{10}$V

(c) $\frac{\sqrt{3}}{10}$V (d) $\frac{1}{5}$V

77. A metallic wheel with 8 metallic spokes each of length r is rotating at an angular frequency ω in a plane perpendicular a magnetic field B. The magnitude of the induced emf between the axle and the rim of the wheel is

(a) $\frac{1}{2}\omega r^2 B$ (b) $2\,\omega r^2 B$

(c) $4\,\omega r^2 B$ (d) $8\,\omega r^2 B$

78. A solenoid of inductance L and resistance R is connected to a battary. The time taken for the magnetic energy to reach $\frac{1}{4}$ of its maximum value is

(a) $\frac{L}{R}\log_e(1)$ (b) $\frac{L}{R}\log_e(2)$

(c) $\frac{L}{R}\log_e(3)$ (d) $\frac{L}{R}\log_e(4)$

79. An LCR series circuit with $R = 100\ \Omega$ is connected to a 200 V, 50 Hz a.c. source. When only the capacitance is removed, the voltage leads the current by 60°. When only the inductance is removed, the current leads the voltage by 60°. The current in the circuit is

(a) $\frac{2}{\sqrt{3}}$ A (b) $\frac{\sqrt{3}}{2}$ A

(c) 1 A (d) 2 A

80. A coil of metal wire is kept stationary with its plane perpendicular to a uniform magnetic field directed along the positive x-axis. The current induced in the coil.

(a) circulates in anti-clockwise direction when viewed from the x-axis.
(b) circulates in clockwise direction when viewed from the x-axis.
(c) is perpendicular to the direction of the magnetic field
(d) is zero

81. A uniform metal rod is moving with a uniform velocity v parallel to a long straight wire carrying a current I. The rod is perpendicular to the wire with its ends at distances r_1 and r_2 (with $r_2 > r_1$) from it. The emf induced in the rod is

(a) zero (b) $\frac{\mu_0 I v}{2\pi}\log_e\left(\frac{r_2}{r_1}\right)$

(c) $\frac{\mu_0 I v}{2\pi}\log_e\left(\frac{r_1}{r_2}\right)$ (d) $\frac{\mu_0 I v}{4\pi}\left(1-\frac{r_1}{r_2}\right)$

82. The current in a coil of self inductance 2.0 H is increasing according to the equation $I = 2\sin(t^2)$ ampere. The amount of energy spent during the period when the current changes from zero to 2 A is

(a) 2 J (b) 4 J
(c) 8 J (d) 16 J

83. In a car spark coil, an emf of 40,000 volts is induced in its secondary when the current in its primary changes from 4 A to zero is 10 μs. The mutual inductance between the primary and the secondary windings of the spark coil is

(a) 0.1 H (b) 0.2 H
(c) 0.3 H (d) 0.4 H

84. A rectangular wire loop of sides a and b is placed in a non-uniform magnetic field which varies with x as $B = kx$ where k is a constant. The magnetic field is directed perpendicular to the plane of the coil as shown in Fig. 14.55. The magnetic flux through the coil is

(a) zero (b) kab^2

(c) $\frac{1}{2}kab^2$ (d) $\sqrt{2}\,kab^2$

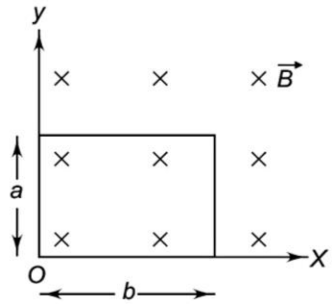


Fig. 14.55

85. A capacitor of capacitance 2 μF is charged to 50 V. The charging battery is then disconnected and a coil of inductance 5 mH is connected across it. Assuming that the coil has negligible resistance, the peak value of the current in the circuit will be
(a) 1 A (b) 2 A
(c) 3 A (d) 4 A

86. Figure 14.56 shows two situations in which a magnet falls downwards through a horizontal loop.

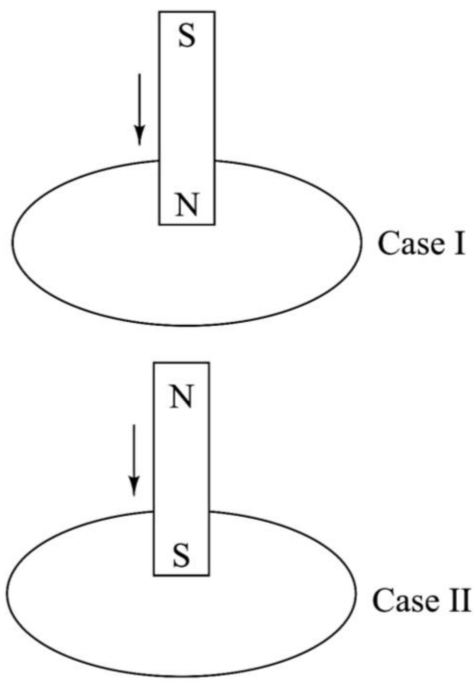


Fig. 14.56

As seen from above, what is the direction of the induced current in cases I and II?
(a) Counterclockwise in case I and clockwise in case II
(b) Clockwise in case I and counterclockwise in case II
(c) Counterclockwise in both the cases I and II
(d) Clockwise in both the cases I and II.

87. Figure 14.57 shows two situations in which a magnet is pulled upward with a constant velocity and enters a coil at time $t = 0$. The magnet moves completely through the coil at time t.

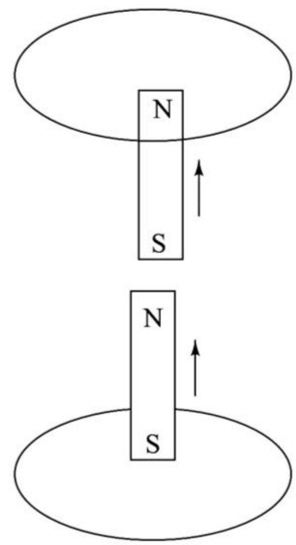


Fig. 14.57

Choose the correct statement from the following. As seen from above, the direction of the induced current is
(a) always clockwise
(b) always counterclockwise
(c) first clockwise and then counterclockwise
(d) first counterclockwise and then clockwise.

88. A small square wire loop of side a = 3 cm has 6 turns and has a resistant of R = 3 m Ω. It is placed at a distance of r = 60 cm from a long straight wire as shown in Fig. 14.58. If the current I in the wire decreases steadily from 7A to 2A in 1 ms, the induced current in the loop is

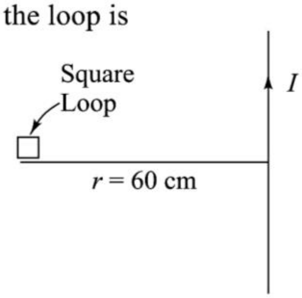


Fig. 14.58

(a) 3.14 mA in the counterclockwise sense
(b) 3.14 mA in the clockwise sense
(c) 3.0 mA in the counterclockwise sense
(d) 3.0 mA in the clockwise sense.

89. In the circuit shown in Fig. 14.59, the current through the 10 Ω resistor is I_1 when the switch S is open and I_2 when S has been closed for a long time. L is an ideal inductor of inductance of 5 mH. The ratio I_2/I_1 is

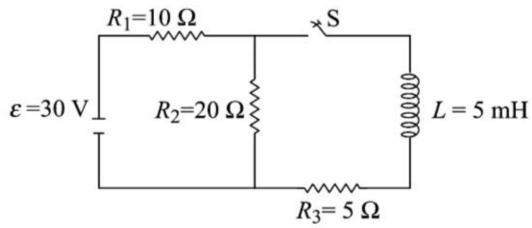


Fig. 14.59

(a) 1 (b) $\frac{5}{3}$

(c) $\frac{15}{7}$ (d) $\frac{30}{11}$

90. Figure 14.60 shows a small circular wire loop of radius r placed on an insulating stand inside a hollow solenoid of radius R. The solenoid has n turns per unit length and carries a current which decreases at a steady rate $\frac{dI}{dt}$. The induced emf in the coil is ε.

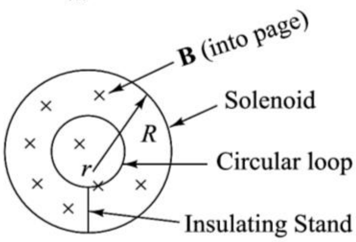


Fig. 14.60

Choose the correct statement from the following.

(a) $\varepsilon = \mu_0 n\pi r^2 \frac{dI}{dt}$; induced current is clockwise

(b) $\varepsilon = \mu_0 n\pi r^2 \frac{dI}{dt}$; induced current is counter clockwise.

(c) $\varepsilon = \mu_0 n\pi R^2 \frac{dI}{dt}$; induced current is clockwise

(d) $\varepsilon = \mu_0 n\pi R^2 \frac{dI}{dt}$; induced current is counter clockwise.

91. A square wire loop, of side a has a long straight wire carrying a current I that passes through the centre of the loop and perpendicular to its plane as shown in Fig. 14.61.

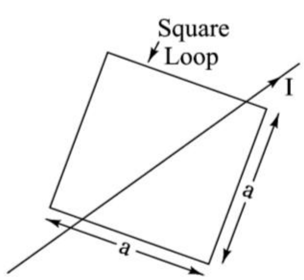


Fig. 14.61

The current induced in the square loop is

(a) $\frac{\mu_0 Ia}{\pi\sqrt{2}}$ (b) $\frac{\mu_0 Ia}{\pi}$

(c) $\frac{\sqrt{2}\,\mu_0 Ia}{\pi}$ (d) zero

92. Figure 14.62 shows a solenoid of inductance L, a resistor of resistance R and a battery of terminal voltage V.

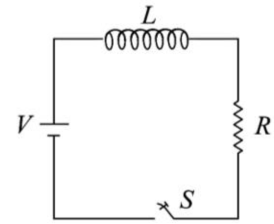


Fig. 14.62

At $t = 0$, the switch S is closed. After how long will the current attain of its final steady state value.

(a) $(\ln 2)\left(\frac{L}{R}\right)$ (b) $2(\ln 2)\left(\frac{L}{R}\right)$

(c) $\left(\ln\frac{3}{4}\right)\left(\frac{L}{R}\right)$ (d) $\left(\ln\frac{1}{4}\right)\left(\frac{L}{R}\right)$

93. In Q. 92 above, if N is the number of turns in the solenoid, what is the magnetic flux per turn of the solenoid when the current attains its maximum value?

(a) $\frac{LV}{NR}$ (b) $\frac{RV}{NL}$

(c) $\frac{NLV}{R}$ (d) $\frac{NRV}{L}$

94. Figure 14.63 shows an electrical circuit containing a two-way switch S. Initially the switch is open. Then terminal 1 is connected to terminal 3. When the current in R_1 attains a maximum (steady-state) value, terminal 1 is disconnected from terminal 2 and quickly connected to terminal 3. The potential difference across 3Ω resistor immediately after 1 is connected to 3 is

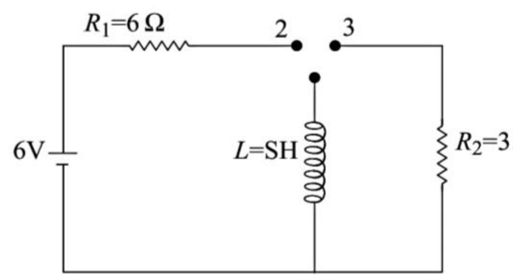


Fig. 14.63

(a) 6V (b) 3V
(c) 2V (d) zero

95. A capacitor of capacitance 2μF is charged to a potential difference of $12\sqrt{2}$ V. The charging battery is then removed and the cpacitor is connected to an inductor of inductance of 5mH. At the time when the potential difference across the capacitor drops to 12V, the current in the circuit is
(a) 0.06 A (b) 0.12A
(c) 0.18 A (d) 0.24 A

96. An inductor of inductance 100 mH is connected in series with a resistor of resistance 0.2Ω. This combination is connected across a 2V battery as shown in Fig. 14.64. The energy stored in the inductor will reduce to 1/9 of its maximum value in time

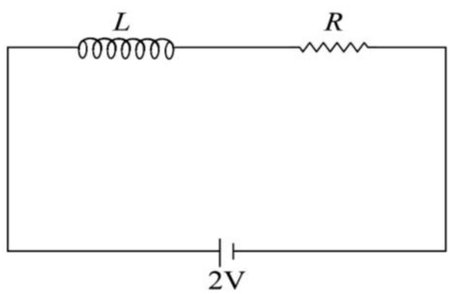


Fig. 14.64

(a) $\frac{1}{2}\ln\left(\frac{3}{2}\right)$ second (b) $\frac{1}{2}\ln(3)$ second
(c) $\frac{1}{9}\ln(2)$ second (d) $2\ln(9)$ second

97. A small circular wire loop of raidus r is placed inside a large square loop $ABCD$ of side a (will $a >> r$). The loops lie in the x-y plane with their centres at the origin O. The mutual inductance of the system is proportional to
(a) $\frac{r^2}{a}$ (b) $\frac{a^2}{R}$
(c) $a\ln\left(\frac{r}{a}\right)$ (d) $r\ln\left(\frac{a}{r}\right)$

98. A small circular loop of radius r is placed inside a large circular loop of radius R (with $R>>r$). The loops are coplanar with their centres coinciding. The mutual inductance of the system is proportional to
(a) $\frac{r}{R}$ (b) $\frac{R}{r}$
(c) $\frac{r^2}{R}$ (d) $\frac{R^2}{r}$

99. A small square loop of side l is placed inside a large square loop of side L (with $L>>l$). The loops are coplanar with their centres coinciding. The mutual inductance of the system is proportional to
(a) $\frac{l}{L}$ (b) $\frac{L}{l}$
(c) $\frac{l^2}{L}$ (d) $\frac{L^2}{l}$

100. A short bar magnet is at rest at time $t = 0$. It is moved towards a solenoid with a constant velocity v up to time $t = t_0$ after which it is moved away from the solenoid [see Fig. 14.65].

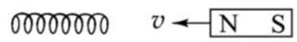


Fig. 14.65

Which of the graphs shown in Fig. 14.66 best represents the variation of induced emf e in the solenoid with time t?

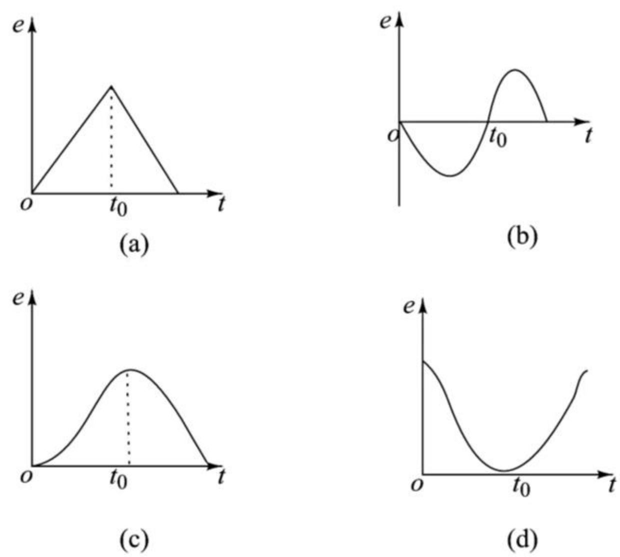


Fig. 14.66

101. An AC voltage source of a fixed peak value V_0 and variable angular frequency ω is connected in series with an inductor of inductance L and a bulb of resistance R (inductance zero). When ω is increased, the brightness of the bulb will
(a) increase (b) decrease
(c) remain unchanged (d) become zero

102. In the circuit shown in Fig. 14.67, the AC voltage source has r.m.s. voltage 20V and angular frequency 100 rad s^{-1}. The peak value of the voltage across the 100 Ω resistor is

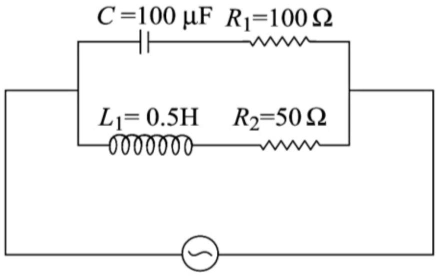


Fig. 14.67

(a) $5\sqrt{2}$ V (b) 10 V

(c) $10\sqrt{2}$ V (d) 20 V

103. In Q. 102 above, what is the peak value of the current through the 50 Ω resistor?
(a) 0.4A (b) 0.6A
(c) 0.8A (d) 1.0A

104. A circuit has an inductance of 100 mH and carries a current of 3A. To prevent sparking when the circuit is switched off, a capacitor is connected across the switch. If the capacitor can withstand a maximum voltage of 300 V, the minimum capacitance it must have should be equal to
(a) 10 μF (b) 20 μF
(c) 60 μF (d) 90 μF

105. When a current is started in the primary of a transformer, an ammeter connected to the secondary shows an instantaneous current *I*. Now if the primary is suddenly rotated through 180º, the instantaneous current
(a) becomes *I*/2
(b) becomes 2*I*
(c) remains unchanged
(d) becomes equal to zero

106. A rod *XY* is connected to the plates of a capacitor. The rod is placed in a region of uniform magnetic field ***B*** directed into the page as shown in fig. 14.68.

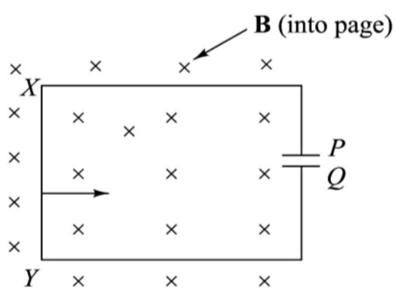


Fig. 14.68

If the rod is pulled out of the region of the magnetic field with a certain velocity *v*, then
(a) plate *P* acquires a positive charge and plate *Q* acquires an equal negative charge
(b) plate *P* acquires a negative charge and plate *Q* acquires an equal positive charge.
(c) both the plates acquire similar charges.
(d) no plate acquires any charge.

107. The variation of magnetic flux ϕ through a coil varies with time *t* as shown in Fig. 14.69.

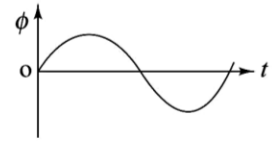


Fig. 14.69

Which graph shown in Fig. 14.70 best represents the variation of induced emf *e* in the coil with time *t*.

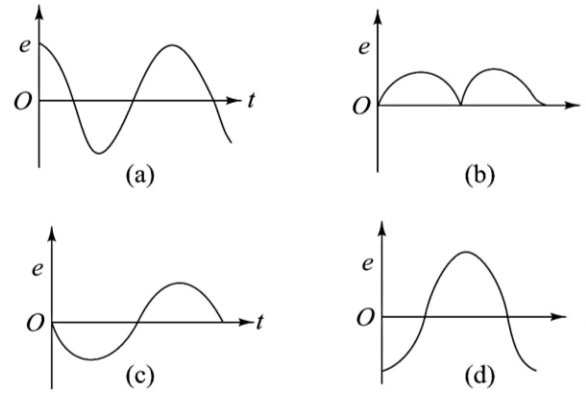


Fig. 14.70

108. Two inductors of inductances L_1 and L_2 are placed far apart so that the mutual inductance between them is negligible. If they are connected in parallel, The equivalent inductance of the combination is

(a) $L_1 + L_2$ (b) $\sqrt{L_1^2 + L_2^2}$

(c) $\sqrt{L_1 L_2}$ (d) $\frac{L_1 L_2}{L_1 + L_2}$

109. In the circuit shown in Fig. 14.71, terminal 1 is connected to terminal 2 until steady state is reached. Terminal 1 then disconnected from 2 and connected with terminal 3. The total heat produced in R_2 is
(a) 4 J (b) 8J
(c) 12J (d) 16J

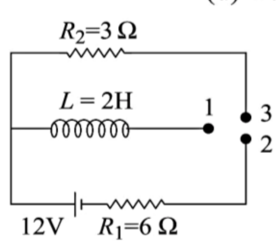


Fig. 14.71

110. Figure 14.72 shows an *L*-*R* circuit connected to a battery of a constant emf *E*. The switch S is closed at time *t* = 0. If *e* denotes the induced emf across the inductor and *i* the current in the circuit at any time *t*,

which of the graphs shown in Fig. 14.73 represents the variation of e with i?

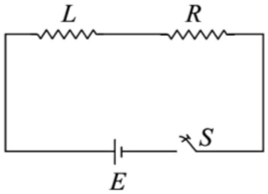


Fig. 14.72

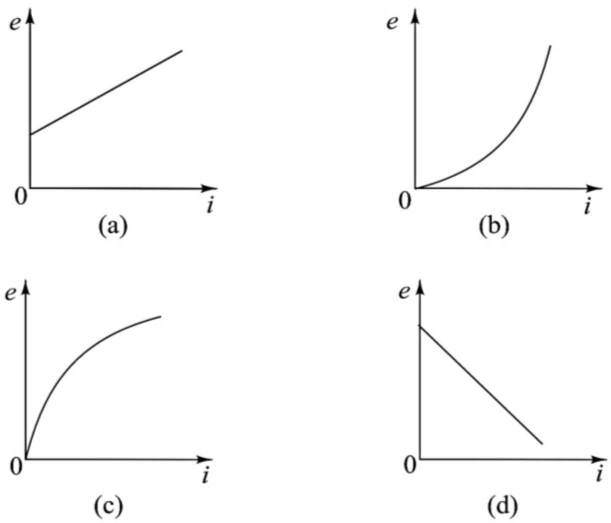


Fig. 14.73

111. A straight wire PQ of length 50 cm and resistance 0.8 Ω slides on parallel metal rails CD and EF with a velocity of 4 cms^{-1} in a uniform magnetic field of 2 T directed into the page as shown in Fig. 14.74. Two resistances 3 Ω and 2 Ω are connected as shown in the figure. The external force to be applied to PQ to keep it moving at a constant velocity of 4 cms^{-1} is

(a) 4×10^{-1} N (b) 2×10^{-2} N
(c) 4×10^{-3} N (d) 2×10^{-4} N

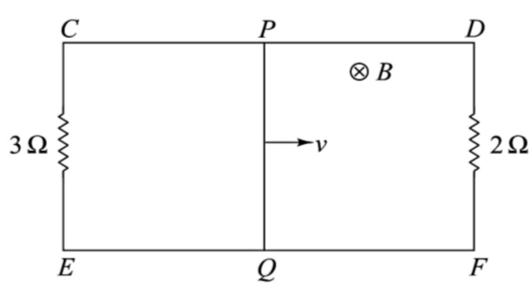


Fig. 14.74

112. A metal wire PQ of resistance 6 Ω is connected along a diameter of metal ring of radius 100 cm lying in the x-y plane. A uniform magnetic field $B = 4$T exists directed along the positive z-axis as shown in Fig. 14.75. The ring is rotated in the x-y plane about an axis passing through its centre O and perpendicular to its plane at an angular frequency of 20 rad s^{-1}. An external resistance of 1Ω is connected between the centre of the ring and its rim. The current through the external resistance is

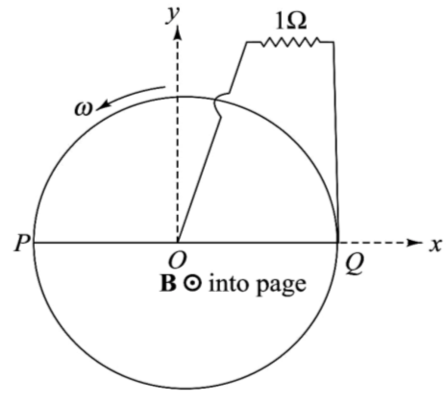


Fig. 14.75

(a) 4A (b) 3A
(c) 2.5A (d) 1.0A

113. The voltage V of an ac source varies will time t as

$$V = 220 \sin(50\pi t) \cos(50\pi t)$$

where V is in volt and t in second. The rms voltage is very nearly equal to

(a) 78 V (b) 89 V
(c) 110 V (d) 155 V

114. In Q.113 above, the frequency of the ac source is

(a) 25 Hz (b) 50 Hz
(c) 100 Hz (d) 200 Hz

115. In mutual inductance between two solenoids is 10 mH. The current I in one solenoid changes with time t as

$$I = 5 \sin(50\pi t)$$

where I is in ampere and t in second. The maximum value of emf (in volt) induced in the other solenoid is

(a) 2.5π (b) 5π
(c) 7.5π (d) 10π

116. In Q. 115 above, the phase between the current in the first solenoid and the emf induced in the second solenoid is

(a) zero (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

117. An inductor and a capacitor are connected to an a.c. voltage source as shown in Fig. 14.76. If I_L is the current through L at a certain instant of time and I_C

is the current through C at that instant, the current drawn from the source at that instant is $(I_L > I_C)$

(a) $I_L + I_C$ (b) $I_L - I_C$

(c) $\dfrac{I_L I_C}{I_L + I_C}$ (d) $\dfrac{I_L I_C}{I_L - I_C}$

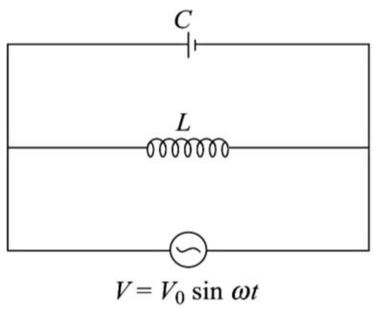


Fig. 14.76

118. In Q. 117, what is the current drawn from the source if C and L were connected to the source as shown in Fig. 14.77?

(a) zero (b) $I_L + I_C$

(c) $I_L - I_C$ (d) $\sqrt{I_L I_C}$

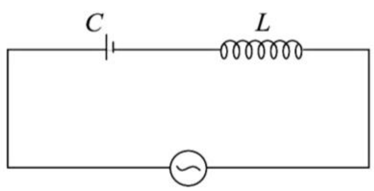


Fig. 14.77

Answers

Level A

1. (d)	2. (d)	3. (a)	4. (a)
5. (d)	6. (b)	7. (b)	8. (c)
9. (d)	10. (c)	11. (d)	12. (a)
13. (a)	14. (d)	15. (b)	16. (d)
17. (a)	18. (d)	19. (b)	20. (d)
21. (d)	22. (b)	23. (a)	24. (b)
25. (b)	26. (d)	27. (a)	28. (d)
29. (b)	30. (d)	31. (b)	32. (c)

Level B

33. (b)	34. (c)	35. (d)	36. (d)
37. (d)	38. (a)	39. (d)	40. (d)
41. (b)	42. (b)	43. (c)	44. (b)
45. (b)	46. (d)	47. (c)	48. (a)
49. (c)	50. (b)	51. (a)	52. (b)
53. (d)	54. (a)	55. (d)	56. (d)
57. (a)	58. (b)	59. (d)	60. (b)
61. (a)	62. (d)	63. (b)	64. (d)
65. (b)	66. (d)	67. (d)	68. (b)
69. (a)	70. (d)	71. (d)	72. (b)
73. (b)	74. (a)	75. (c)	76. (c)
77. (a)	78. (b)	79. (d)	80. (d)
81. (b)	82. (b)	83. (c)	84. (c)
85. (a)	86. (a)	87. (c)	88. (d)
89. (c)	90. (a)	91. (d)	92. (b)
93. (a)	94. (b)	95. (d)	96. (a)
97. (a)	98. (c)	99. (c)	100. (b)
101. (b)	102. (d)	103. (a)	104. (a)
105. (b)	106. (a)	107. (d)	108. (d)
109. (a)	110. (d)	111. (b)	112. (a)
113. (a)	114. (b)	115. (a)	116. (c)
117. (b)	118. (a)		

Solutions

Level A

1. The magnetic field due to the solenoid is

$$B = \mu_0 n I$$

where n is the number of turns per unit length. Since the second coil is wrapped closely around the solenoid, the cross-sectional area of the coil can be taken to be equal to that of the solenoid. Since the initial current and hence the initial magnetic field is zero, the change of flux for a single turn is $\mu_0 nIA$, where A is the cross-sectional area. Therefore, induced emf for a single turn is

$$|e| = \frac{\mu_0 n I A}{t} = \frac{4 \times 3.14 \times 10^{-7} \times 500 \times 2 \times 10^{-4}}{3.14 \times 10^{-3}} = 4 \times 10^{-5}\ \text{V}$$

$\therefore$ Induced emf for 100 turns $= 4 \times 10^{-5} \times 100 = 4 \times 10^{-3} = 4$ mV. Hence the correct choice is (d).

2. $B = 0.3$ T, $v = 1\ \text{cms}^{-1} = 0.01\ \text{ms}^{-1}$, $l = 8$ cm $= 0.08$ m and $b = 2$ cm $= 0.02$ m.

$e = Blv = 0.3 \times 0.08 \times 0.01$
$= 2.4 \times 10^{-4}$ V $= 0.24$ mV

3. The time during which the emf lasts is the time taken by the breadth ($b = 2$ cm) of the coil to move out of the field. Since the speed of coil is 1 cms^{-1}, this time will be 2 s.

4. $E = Bbv = 0.3 \times 0.02 \times 0.01$

$$= 0.6 \times 10^{-4} \text{ V} = 0.06 \text{ m V}$$

5. Time during which this emf lasts is the time taken by the length ($l = 8$ cm) to move out of the field = 8 s.

6. Induced emf in the secondary is

$$e_s = -M\frac{dI}{dt} = (-2) \times \left(-\frac{10}{0.1}\right) = 200 \text{ V}$$

Hence the correct choice is (b).

7. $R_a = 2\ \Omega$, $E_a = 220$ V, $E_b = 210$ V. Current at full speed is

$$I_a = \frac{E_a - E_b}{R_a} = \frac{220-210}{2} = 5 \text{ A}$$

Hence the correct choice is (b).

8. When the motor was switched on, $E_b = 0$ since the initial speed of the armature is zero (it is initially at rest). If no starter is used, the starting current in the armature is

$$I_s = \frac{E_a}{R_a} = \frac{220}{2.0} = 110 \text{ A}$$

which is ruinously large and it will burn the windings of the armature. Hence a starter must be inserted in series with the armature when the motor is first switched on (or started).

9. Efficiency = $\dfrac{\text{power output}}{\text{power input}}$

Power input is $IE_a = 5.0 \times 220 = 1100$ W. Power loss due to heating of the armature is $I^2R_a = (5.0)^2 \times 2.0 =$ 50 W. Therefore, power output is 1100 – 50 = 1050 W. Hence efficiency is 1050/1100 ≅ 0.95 or 95%.

10. Resistance of armature is 0.2 Ω, potential difference in open circuit is 220 V and potential difference at full load is 210 V.

Current in the circuit is

$$I = \frac{220-210}{0.2} = 50 \text{ A}$$

11. Power delivered = 210 × 50 = 10.5 kW, which is choice (d)

12. $N_p = 400$, $N_s = 2000$ and $E_s = 1000$ V. Now

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}$$

or

$$E_p = \frac{E_s \times N_p}{N_s} = \frac{1000 \times 400}{2000} = 200 \text{ V}$$

Hence the correct choice is (a).

13. Since the efficiency is 80%, the input power is

$$12 \text{ kW} \times \frac{100}{80} = 15 \text{ kW}$$

∴ Current in the primary is

$$I_p = \frac{\text{input power}}{\text{input voltage}} = \frac{15\,\text{kW}}{200\,\text{V}}$$

$$= \frac{15 \times 1000}{200} = 75 \text{ A}$$

∴ Power loss in primary $= I_p^2 R_p = (75)^2 \times 0.2 =$ 1125 W = 1.125 kW. Hence the correct choice is (a).

14. Current in the secondary is

$$I_s = \frac{\text{output power}}{\text{output voltage}} = \frac{12\,\text{kW}}{1000\,\text{V}}$$

$$= \frac{12 \times 1000}{1000} = 12 \text{ A}$$

∴ Power loss in secondary $= I_s^2 R_s = (12)^2 \times 2 =$ 288 W. Hence the correct choice is (d).

15. The correct choice is (b).

16. The falling magnet induces a current in the coil. From Lenz's law, the direction of the current is such that its magnetic field opposes the motion of the magnet. Hence the correct choice is (d).

17. Since the charge of electron is negative, the moving electron constitutes a current in the direction opposite to the direction of motion of the electron, i.e. the direction of the current will be from Q to P. The magnetic field threading the coil due to the motion of the electron will be directed into the plane of the page, i.e. perpendicular to the plane of coil directed into the page. To oppose this, the current in the coil must be anticlockwise, in accordance with Lenz's law. Hence the correct choice is (a).

18. As the magnet is moving towards coil AB, the magnetic flux linked with it increases. By Lenz's law, the current must flow in the coil in a direction which would tend to oppose the increase in flux, i.e. the current should produce a magnetic field in a direction opposite to the field of the magnet. Thus the end of the coil closer to the magnet should become a

south pole. Hence the current must be clockwise in the coil and flow from B to A.

As the magnet is moving away from coil CD, the magnetic flux linked with it decreases. The current must oppose this decrease and produce a field in the same direction as that of the magnet. The induced current must flow from D to C.

19. The current in the coil P is flowing in an anticlockwise direction. Hence end A is the north pole and end B the south pole. When the key K is lifted, the current in coil P starts decreasing leading to a decrease in magnetic flux through coil Q. By Lenz's law, the induced current in this coil must oppose the decrease in flux. This can happen if the direction of the induced current is such that a south pole is produced at end C and a north pole at end D. Therefore the induced current must flow in clockwise direction, i.e. from Y to X.

20. Voltage across secondary is

$$E_s = E_p \times \frac{N_s}{N_p} = \frac{110 \times 400}{200} = 220 \text{ V}$$

Current in secondary is

$$I_s = I_p \times \frac{E_p}{E_s} = \frac{10 \times 110}{220} = 5 \text{ A}$$

Hence the correct choice is (d).

21. The correct choice is (d).

22. The correct choice is (b).

23. The rate of change of flux gives the induced emf. Thus

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(6t^2 - 5t + 1) = -12t + 5$$

At $t = 0.25$ s, $E = -12 \times 0.25 + 5 = -3 + 5 = 2$ V

$\therefore$ Induced current $I = \dfrac{E}{R} = \dfrac{2}{10} = 0.2$ A, which is choice (a).

24. The least capacitance is such that the energy stored in the capacitor is equal to that stored in the inductor, i.e.

$$\frac{1}{2} CV^2 = \frac{1}{2} LI^2$$

or $$C = \frac{LI^2}{V^2} = \frac{1 \times (2)^2}{(400)^2} = 25 \times 10^{-6} \text{ F}$$

$$= 25 \ \mu\text{F}$$

Hence the correct choice is (b).

25. $E = Blv = 0.5 \times 10^{-4} \times 10 \times 200 = 0.1$ V which is choice (b)

26. Power output $= E_s I_s = 11 \times 90 = 990$ W. Power input $= E_p I_p = 220 \times 5 = 1100$ W. Therefore,

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}} = \frac{990}{1100} = \frac{9}{10}$$

$$= 0.9 \text{ or } 90\%$$

Hence the correct choice is (d).

27. Current in the transmission line is

$$I = \frac{\text{power}}{\text{voltage}} = \frac{25000}{250} = 100 \text{ A}$$

$\therefore$ Power loss $= I^2 R = (100)^2 \times 1 = 10000$ W.

Therefore, the percentage of original power lost is

$$\frac{10000}{25000} \times 100 = 40\%$$

Hence the correct choice is (a).

28. Since the resistance of an ideal inductor is zero, the final value of the current in the 10 Ω resistor is zero.

29. Since the resistance of the inductor is zero, the total resistance of the circuit is $R = 20$ Ω. Therefore, current $= V/R = 2/20 = 0.1$ A.

30. Peak value of voltage is $V_0 = \sqrt{2}\ V_{\text{rms}} = \sqrt{2} \times 220$ V. Hence the correct choice is (d).

31. A choke is an inductor. Hence it is used only in ac circuits.

32. The correct choice is (c).

Level B

33. Given $R = 4$ Ω and $\omega L = 3$ Ω. The impedance is

$$Z = (R^2 + \omega^2 L^2)^{1/2} = (16 + 9)^{1/2} = 5 \ \Omega$$

Hence the correct choice is (b).

34. We know that $\tan \delta = \dfrac{\omega L}{R}$. Therefore

$$L = \frac{R \tan \delta}{\omega} = \frac{100 \times \tan 45°}{2\pi \times 1000}$$

$$\simeq 15.9 \times 10^{-3} \simeq 16 \text{ mH}$$

Hence the correct choice is (c).

35. The current in an LCR circuit is given by

$$I = \frac{V}{\left[R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}}$$

where $\omega = 2\pi f$. Thus I increases with increase in ω upto a value of $\omega = \omega_c$ given by

$$\omega_c L = \frac{1}{\omega_c C}$$

or $$\omega_c = \frac{1}{\sqrt{LC}}$$

when I becomes maximum. At $\omega > \omega_c$, I decreases with increase in ω. Hence the correct graph is (d).

36. The correct choice is (d).

37. $E = 12$ V, $L_1 = 10$ H, $L_2 = 10 \times 10^{-3}$ H and $R = 48\ \Omega$. Steady state current is $I_0 = E/R$ and is independent of the inductance. Hence, the value of the steady state current is the same for both circuits.

$$I_0 = \frac{E}{R} = \frac{12}{48} = 0.25\ \text{A}$$

38. The energy consumed by the circuit to build up the current I_0 is

$$E = \frac{1}{2} L I_0^2$$

For circuit 1,

$$E_1 = \frac{1}{2} L_1 I_0^2 = \frac{1}{2} \times 10 \times (0.25)^2$$
$$= 3.125 \times 10^{-1}\ \text{J}$$

For circuit 2,

$$E_2 = \frac{1}{2} L_2 I_0^2 = \frac{1}{2} \times 10 \times 10^{-3} \times (0.25)^2$$
$$= 3.125 \times 10^{-4}\ \text{J}$$

$\therefore \quad \frac{E_1}{E_2} = 1000$. Hence the correct choice is (a)

39. Power dissipated at current I_0 is $I_0^2 R$. Since I_0 and R are the same for both the circuits, they dissipate the same power which is

$$P = I_0^2 R = (0.25)^2 \times 48 = 3.0\ \text{W}$$

40. The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{50} = 2 \times 10^{-3}\ \text{s}$$
$$= 2\ \text{millisecond.}$$

Current at time t is given by

$$I = I_0\, e^{-t/\tau}$$

where I_0 is the steady current. Therefore, time for I to fall to $I_0/2$ is

$$e^{-t/\tau} = \frac{1}{2} \text{ or } e^{t/\tau} = 2 \text{ or } t = \tau \ln(2).$$

Hence the correct choice is (d).

41. The resonant angular frequency is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\left(5.0 \times 80 \times 10^{-6}\right)^{1/2}}$$
$$= 50\ \text{rad s}^{-1}$$

Therefore, the resonant frequency is

$$\nu_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}\ \text{Hz}$$

42. The impedance is given by

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

When $\quad \omega = \omega_r = 1/\sqrt{LC}$

(i.e. at resonance), $\omega L = 1/\omega C$, and therefore

$$Z = R = 40\ \Omega$$

43. Current amplitude at resonance is

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{R} = \frac{\sqrt{2}\, V_{\text{rms}}}{R} = \frac{\sqrt{2} \times 200}{40} = 5\sqrt{2}\ \text{A.}$$

44. The rms current in the circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{200}{40} = 5\ \text{A}$$

$\therefore$ The rms potential drop across L is

$$V_{\text{rms}} = I_{\text{rms}} \times \omega_r \times L = 5 \times 50 \times 5 = 1250\ \text{V} = 1.25\ \text{kV}$$

45. The rms potential drop across C is

$$V_{\text{rms}} = I_{\text{rms}} \times \frac{1}{\omega_r C} = 5 \times \frac{1}{50 \times 80 \times 10^{-6}} = 1.25\ \text{kV}$$

46. The rms potential drop across R is

$$V_{\text{rms}} = I_{\text{rms}} \times R = 5 \times 40 = 200\ \text{V}$$

47. The correct choice is (c).

48. The dimensions of RC are those of ohm × charge / voltage, i.e.

$$\frac{\text{voltage}}{\text{current}} \times \frac{\text{charge}}{\text{voltage}} = \frac{\text{charge}}{\text{charge/time}} = \text{time}$$

Hence the dimensions of $1/RC$ are those of frequency.

49. Since inside the cell, the current is taken to flow from the negative to the positive terminal, we have

$$V_A - IR + E - L\frac{dI}{dt} = V_B$$

or $$V_B - V_A = -IR + E - L\frac{dI}{dt}$$

Since I is decreasing with t, $\frac{dI}{dt}$ is negative. Hence

$$V_B - V_A = -5\times10^{-6}\times1 + 15 - (5\times10^{-3})\times(-10^{-3})$$
$$= 15\text{ V}$$

Thus the correct choice is (c).

50. We know that $Q = CV$ and $Q = Q_0 \cos \omega t$. Also $Q_0 = CV_0$.

$$\therefore \quad \cos\omega t = \frac{Q}{Q_0} = \frac{V}{V_0} = \frac{6}{12} = \frac{1}{2} \text{ or } \omega t = \frac{\pi}{3}.$$

Now ω is given by

$$\omega = \frac{1}{\sqrt{LC}} \quad (1)$$

Given $L = 0.6\times10^{-3}$ H and $C = 2\times10^{-6}$ F. Using these values in Eq. (1) we get $\omega = \frac{10^5}{2\sqrt{3}}$ rad s^{-1}.

$$\text{Now} \quad I = \frac{dQ}{dt} = \frac{d}{dt}(Q_0\cos\omega t)$$
$$= -Q_0\,\omega\sin\omega t$$
$$\therefore \quad |I| = Q_0\,\omega\sin\omega t = CV_0\,\omega\sin\omega t$$
$$= (2\times10^{-6})\times12\times\frac{10^5}{2\sqrt{3}}\sin\left(\frac{\pi}{3}\right)$$
$$= 0.6\text{ A}$$

Hence the correct choice is (b).

51. Now $|e| = L\frac{dI}{dt}$

or $\quad 10\times10^{-3} = L\times\frac{d}{dt}(5+16t) = L\times(16)$

or $\quad L = 6.25\times10^{-4}$ H.

Hence the correct choice is (a).

52. Power $= VI = 10\times10^{-3}\times(5+16t)$

At $t = 1$ s, power $= 10\times10^{-3}\times(5+16) = 0.21$ W

Hence the correct choice is (b).

53. Induced emf (e)

$$= \frac{\text{magnetic field}\times\text{change in area}}{\text{time}} = \frac{B\Delta A}{t}$$

Since the circumference of the circular loop $= 2\pi r$, the side of the square loop $= \frac{2\pi r}{4} = \frac{\pi r}{2}$.

Therefore,

$$\Delta A = \pi r^2 - \left(\frac{\pi r}{2}\right)^2 = \pi r^2\left(1-\frac{\pi}{4}\right)$$

$$\therefore \quad e = \frac{B(\pi r^2)}{t}\left(1-\frac{\pi}{4}\right)$$

Hence the correct choice is (d).

54. Self inductance of each coil $= \frac{1}{2}\times(1.8\times10^{-4}) = 0.9\times10^{-4}$ H. Resistance of each coil $= \frac{6}{2} = 3\ \Omega$.

When two such coils are connected in parallel, the self-inductance of the combination is $L = 0.45\times10^{-4}$ H and the resistance of the combination is $R = 1.5\ \Omega$.

$$\therefore \text{Time constant} = \frac{L}{R} = \frac{0.45\times10^{-4}}{1.5} = 0.3\times10^{-4}\text{ s}$$

Hence the correct choice is (a).

55. Steady current $I_0 = \frac{V}{R} = \frac{12}{1.5} = 8$ A which is choice (d).

56. Velocity $v = \sqrt{2gh}$. Induced emf $e = Blv = Bl\sqrt{2gh}$.

Therefore, the induced current in the loop is

$$I = \frac{Bl\sqrt{2gh}}{R}$$

$$\therefore \text{Force} \quad F = BIl = \frac{B^2l^2\sqrt{2gh}}{R}$$

The loop will attain terminal velocity if this force equals mg, i.e. if

$$\frac{B^2l^2\sqrt{2gh}}{R} = mg$$

which gives $h = \frac{m^2gR^2}{2B^4l^4}$

Hence the correct choice is (d).

57. Magnetic field due to the larger coil at its centre is

$$B = \frac{\mu_0 I}{2r_1}$$

where I is the current in the larger coil. Flux through the inner coil is

$$\phi = B\times\pi r_2^2 = \frac{\mu_0 I}{2r_1}\times\pi r_2^2$$

But $\phi = MI$. Therefore

$$M = \frac{\mu_0\pi r_2^2}{2r_1}$$

Hence the correct choice is (a).

58. The network *PQRS* is a balanced Wheatstone's bridge. Hence the resistance of 3 Ω between *P* and *R* is ineffective. The net resistance of the network, therefore, is 3 Ω. Total resistance $R = 3\ \Omega + 1\ \Omega = 4\ \Omega$. Now, induced emf is $e = Blv = 2 \times 0.1 \times v = 0.2\,v$.

$\therefore$ Induced current $I = \dfrac{e}{R} = \dfrac{0.2v}{4}$.

Given $I = 1 \times 10^{-3}$ A

Hence $\quad 1 \times 10^{-3} = \dfrac{0.2v}{4}$

which gives $v = 2 \times 10^{-2}\ \text{ms}^{-1} = 2\ \text{cm s}^{-1}$, which is choice (b).

59. If a coil is not moved in a magnetic field, the magnetic flux does not change. Hence no emf or current is induced in the coil. Hence the correct choice is (d).

60. Since the voltage leads the current by a phase angle of 90°, the total potential difference across the circuit is

$$V = (V_R^2 + V_L^2)^{1/2} = (20 \times 20 + 16 \times 16)^{1/2} = 25.6\ \text{V},$$

which is choice (b).

61. The phase angle between voltage *V* and current *I* is $\pi/2$. Therefore, power factor $\cos\phi = \cos(\pi/2) = 0$. Hence the power consumed is zero, which is choice (a).

62. Since the magnetic field is constant in time and space and exists everywhere, there is no change in magnetic flux when the loop is moved in it. Hence no current is induced, which is choice (d).

63. Induced charge $q = \dfrac{\text{change of flux}}{\text{resistance}} = \dfrac{\phi_f - \phi_i}{R}$. But final area = 0, therefore, $\phi_f = 0$. Numerically, $\phi_i = BA$. Therefore, $q = BA/R$, which is choice (b).

64. As the ring falls with a velocity v the decrease in area with time is

$$\frac{dA}{dt} = -(2R)v$$

$\therefore$ Induced emf, $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(BA) = -B\dfrac{dA}{dt} = 2RBv$.

From Lenz's law, the induced current in the ring must produce magnetic field in the upward direction. Hence *Q* is at higher potential. Hence the correct choice is (d).

65. Refer to Fig. 14.78. The magnetic field due a current *I* in the large loop at its centre is

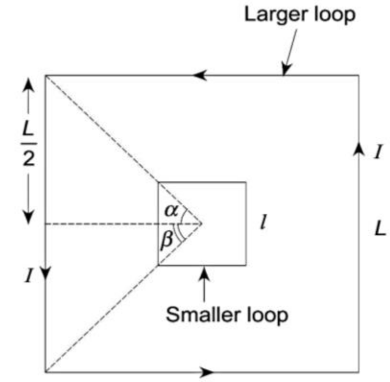


Fig. 14.78

$$B = 4 \text{ times that due to one side}$$

$$= 4 \times \frac{\mu_0}{4\pi}\frac{I}{(L/2)}(\cos\alpha + \cos\beta)$$

$$= \frac{2\mu_0 I}{\pi L}(\cos 45° + \cos 45°)$$

$$= \frac{2\sqrt{2}\mu_0 I}{\pi L} \quad (\because\ \alpha = \beta = 45°)$$

The magnetic flux that links the larger loop with the smaller loop of side l ($l << L$) is

$$\phi_{12} = Bl^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}$$

$\therefore$ Mutual inductance $M_{12} = \dfrac{\phi_{12}}{I} = \dfrac{2\sqrt{2}\mu_0}{\pi}\left(\dfrac{l^2}{L}\right)$

i.e. $M_{12} \propto \dfrac{l^2}{L}$, which is choice (b).

66. Figure 14.79 shows the field lines (shown as broken curves) of the magnetic field due to the current flowing in the loop. It is clear from the figure that the magnetic flux in the *x*-*y* plane will be zero. Hence the correct choice is (d).

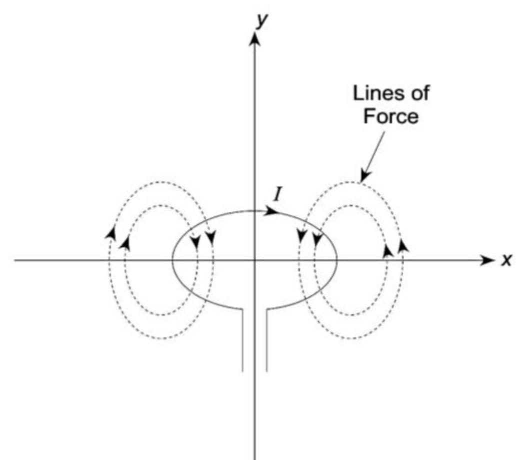


Fig. 14.79

67. The current in the inductor is given by

$$I = \frac{V}{R}\,(1 - e^{-t/\tau}), \text{ where } \tau = L/R.$$

$$\text{Given, } \tau = \frac{L}{R} = \frac{8.4\text{ mH}}{6\,\Omega} = 1.4\text{ ms (millisecond)}$$

$$\therefore\ 1.0 = \frac{12}{6}\,(1 - e^{-t/1.4\text{ ms}})$$

$$\text{or } e^{-t/1.4\text{ ms}} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{or } \quad -\frac{t}{1.4\text{ ms}} = \log_e\left(\frac{1}{2}\right) = -0.693$$

$$\text{or } \quad t = 0.693 \times 1.4\text{ ms} = 0.97\text{ ms}$$

Hence the correct choice is (d).

68. A time varying magnetic field produces an electric field. The magnitude of the electric field at a distance r from the centre of a circular region of radius a where a time varying field B exists is given by

$$E = \frac{a^2}{2r}\frac{dB}{dt}$$

At $r = a$, $E = (a/2)\,dB/dt$, which the value of E at the edge of the circular region. For $r > a$, E decreases as $1/r$. Hence the correct choice is (b).

69. The mutual inductance between the two coils in orientation (A) is the maximum since the flux linkage in (A) is the maximum as shown in Fig. 14.80.

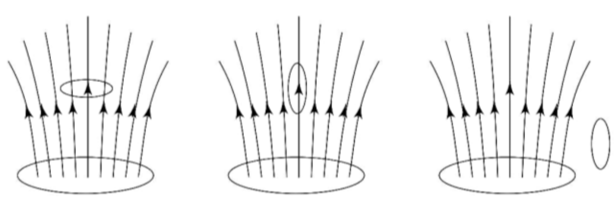


Fig. 14.80

70. Electric field will be induced in both AD and BC, since both are moving perpendicular to the direction of the magnetic field and the flux linked with them is changing with time. Hence the correct choice is (d).

71. Let the switch be closed at time $t = 0$. The current I_p flowing in P grows for a time, say t_0, after which it becomes steady. During this time the magnetic field (due to I_p) (from left to right) increases at the location of circuit Q. According to Lenz's law, the induced current $(I_Q)_1$ should be such that it tries to decrease the magnetic field. Therefore, the magnetic field due to this current must be from right to left. Hence this induced current $(I_Q)_1$ should be anticlockwise (opposite to the direction of I_P). After the switch S is opened, the current I_P takes a finite time to decay to zero and the reverse of the above phenomenon is observed. Hence the induced current $(I_Q)_2$ should be clockwise. Thus the correct choice is (d).

72. The magnitude of the induced voltage is proportional to the rate of change of magnetic flux which, in turn, depends on the number of turns in the coil, i.e. $V \propto n$. The resistance of a wire is given by

$R = \dfrac{\rho l}{\pi r^2}$ or $R \propto \dfrac{l}{r^2}$. Here ρ is the resistivity of the material of the wire.

$$\therefore \text{ Power } \quad P = \frac{V^2}{R} \propto \frac{n^2}{l/r^2} \text{ or } P \propto \frac{(nr)^2}{l}$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{l_1}{l_2}\right) \qquad (1)$$

Now, if a wire of length l_1 and radius r_1 is stretched to a length l_2 such that its radius reduced to r_2, then (since the mass of the wire remains constant)

$$m = \pi r_1^2\, l_1\, d = \pi r_2^2\, l_2\, d \qquad (d \text{ is the density})$$

or $\quad \dfrac{l_1}{l_2} = \left(\dfrac{r_2}{r_1}\right)^2$. Using this in Eq. (1), we get

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^4$$

Given $\quad \dfrac{n_2}{n_1} = 4$ and $\dfrac{r_2}{r_1} = \dfrac{1}{2}$.

Using these values, we get

$$\frac{P_2}{P_1} = (4)^2 \times \left(\frac{1}{2}\right)^4 = 1,$$

which is choice (b).

73. Induced emf is $|e| = n\dfrac{\Delta\Phi}{\Delta t}$. Now

$$\Delta q = I\,\Delta t$$

$$= \frac{e}{R}\,\Delta t = \frac{n\Delta\Phi}{R\Delta t} \times \Delta t = \frac{n\Delta\Phi}{R}$$

$$= \frac{n(\Phi_2 - \Phi_1)}{R}$$

Hence the correct choice is (b).

74. Given $E = E_0 \sin(100\,t)$. Comparing this with $E = E_0 \sin \omega t$, we have $\omega = 100$ rad s^{-1}. It follows from the figure that the current leads the e.m.f. which is true only for *R-C* circuit, and not for *R-L* circuit. Hence the circuit does not contain an inductor. Thus choices (c) and (d) are not possible. For *R-C* circuit, the phase difference between *E* and *I* is given by

$$\tan \phi = \frac{1}{\omega RC} \quad \text{(i)}$$

Given $\phi = \pi/4$. Also $\omega = 100$ rad s^{-1}. Using these values in (i), we get

$$\tan\left(\frac{\pi}{4}\right) = \frac{1}{100\,RC} \quad \text{or} \quad RC = \frac{1}{100}$$

This relation between *R* and *C* is satisfied by choice (a) and not choice (b). Hence the correct choice is (a).

75. When wire *CD* is made to slide on wires *PQ* and *ST*, the flux linked with the circuit changes with time and hence an emf is induced in the circuit, which is given by

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = B\frac{dA}{dt}$$

If wire *CD* moves a distance *dx* is time *dt*, then $A = wdx$ (here $w = CD$) and

$$|e| = B\frac{d}{dt}(wdx) = Bw\frac{dx}{dt} = Bwv$$

The induced current is

$$I = \frac{e}{R} = \frac{Bwv}{R}$$

This current is caused by the motion of wire *CD*. From Lenz's law, the current *I* opposes the motion of wire *CD*. Therefore, work has to be done to slide the wire *CD*. Now, the magnetic force on wire *CD* (of length *w*) is

$$F = BIw = B\left(\frac{Bwv}{R}\right)w = \frac{B^2w^2v}{R} \quad (1)$$

Work done is sliding wire *CD* through a small distance *dx* in time *dt* is

$$dW = Fdx$$

Therefore, the work done per second is

$$P = \frac{dW}{dt} = F\frac{dx}{dt} = Fv$$

Using (1), we get

$$P = \frac{B^2w^2v^2}{R}$$

Hence the correct choice is (c).

76. As the air plane is flying horizontally parallel to the earth's surface, the flux linked with it will be due to the vertical component B_V of the earth's field.

Now

$$B_V = B_H \tan\theta = 2\times10^{-5}\times\tan 60°$$
$$= 2\sqrt{3}\times10^{-5}\ \text{Wbm}^{-2}$$

$\therefore$ Induced emf is $|e| = B_V lv = 2\sqrt{3}\times10^{-5}\times20\times250$

$= \dfrac{\sqrt{3}}{10}$ V, which is choice (c).

77. Refer to Fig. 14.81. Let ν be the frequency of rotation. The time taken for 1 full rotation is $T = 1/\nu$. Therefore, rate of change of area is

$$\frac{A}{T} = \frac{\pi r^2}{T} = \pi r^2 \nu$$

Now, the emf induced between the axle and rim is $e = B\times$ rate of change of area

$$= B\times\pi r^2\nu = \frac{1}{2}Br^2\omega \quad (\because\ \omega = 2\pi\nu)$$

Since the same emf is produced between the ends of each spoke, and these emfs are in parallel as is evident from Fig. 14.81, the net emf between the axle and the rim of the wheel will be the same as that across each spoke. We notice that all the eight spokes are connected with one end at the rim and the other at the axle. Hence the magnitude of the net emf between the axle and the rim is independent of the number of spokes.

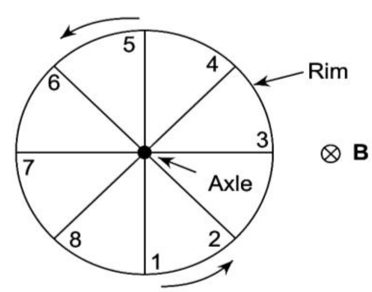


Fig. 14.81

78. The growth of current in an *LR* circuit is given by

$$I = I_0(I - e^{-Rt/L}) \quad (1)$$

where I_0 is the maximum current. The energy stored at time *t* is

$$U = \frac{1}{2}LI^2$$

We are required to find the time at which the energy stored is one-forth the maximum value, i.e. when

$$U = \frac{U_0}{4} \text{ where}$$

$$U_0 = \frac{1}{2} LI_0^2$$

i.e. $\frac{1}{2} LI^2 = \frac{1}{4}\left(\frac{1}{2} LI_0^2\right)$ or $I = \frac{I_0}{2}$

Using this in Eq. (1), we get

$$\frac{I_0}{2} = I_0 (1 - e^{-Rt/L}) \quad \text{or} \quad \frac{1}{2} = 1 - e^{-Rt/L}$$

or $e^{-Rt/L} = \frac{1}{2}$ or $-\frac{Rt}{L} = \log_e\left(\frac{1}{2}\right)$

or $t = \frac{L}{R} \log_e(2)$, which is choice (b).

79. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \quad \text{or} \quad \omega L = R\tan\theta = 100 \tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100 \tan 60°$

The impedance of the LCR circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$$= R = 100\ \Omega$$

The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2 \text{ A}.$$

Hence the correct choice is (d).

80. If the coil is not moved in a magnetic field, the magnetic flux linked with the coil does not change. Hence no emf or current is induced in the coil. Thus the correct choice is (d).

81. An emf is induced in the rod because it cuts through the lines of force of the magnetic field of the current carrying wire. Consider a small element of length dr of the rod at a distance r from the wire. The magnetic field due to a current I in the wire at a distance r from it is (Fig. 14.82)

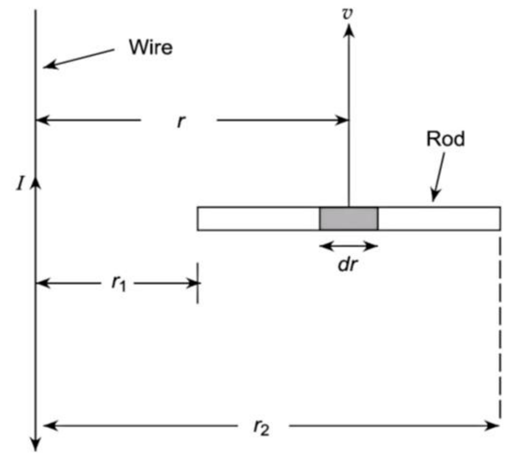


Fig. 14.82

$$B = \frac{\mu_0 I}{2\pi r}$$

The emf induced in the element of length dr is

$$de = Bvdr = \frac{\mu_0 I v}{2\pi} \frac{dr}{r}$$

$\therefore$ The emf induced in the whole rod is

$$e = \frac{\mu_0 I v}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I v}{2\pi} \left| \log_e r \right|_{r_1}^{r_2}$$

or $e = \frac{\mu_0 I v}{2\pi} \log_e\left(\frac{r_2}{r_1}\right)$,

which is choice (b).

82. $W = \frac{1}{2} LI_0^2$, where I_0 = peak value of I = 2 A. Thus

$$W = \frac{1}{2} \times 2.0 \times (2)^2 = 4 \text{ J}$$

Hence the current choice is (b).

83. $e = -M\frac{\Delta I}{\Delta t}$

or $M = -\frac{e\Delta I}{\Delta I} = -\frac{40{,}000 \times (10 \times 10^{-6})}{(-4-0)}$

$$= 0.1 \text{ H}$$

The correct choice is (a).

84. Since the magnetic field varies with x, we find the flux by considering a small element of the loop of width dx and length a at a distance x from O, as shown in Fig. 14.83.

The total magnetic flux is

$$\phi = \int B dA = \int_0^b kx(adx)$$

$$= ka\int_0^b x dx = \frac{1}{2}kab^2$$

so the correct choice is (c).

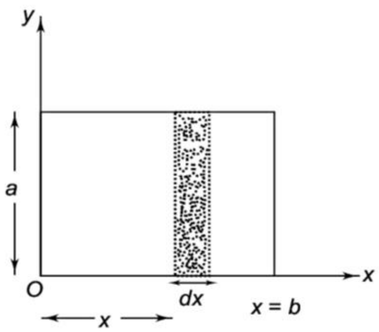


Fig. 14.83

85. $\frac{1}{2}LI_0^2 = \frac{1}{2}CV^2$ ($\because$ there is no loss of energy due to joule heating as $R = 0$). Hence

$$I_0 = V\sqrt{\frac{C}{L}} = 50\times\sqrt{\frac{2\times10^{-6}}{5\times10^{-3}}} = 1\text{ A}$$

which is choice (a).

86. From Lenz's law, the direction of the induced current in the coil must be such that it opposes the downward gravitational force of the falling magnet. This can happen if the upper face of the coil in case I develops north polarity so that the magnet is repelled. From clock rule, Ⓝ the current in case I must be counter clockwise. By the same logic, the current in case II must be clockwise. So the correct choice is (a). This can also be explained as follows.

In case I, the magnetic field of the magnet at the centre of the coil (which lies on the axial line of the axis) is directed downwards. By the right hand screw rule, the induced current must be counter-clockwise because then it will produce a magnetic field upwards.

87. At time t = 0, the north pole N of the magnet is moved upward towards the loop. From Lenz's law, the lower face of the coil (as seen from above) must acquire north polarity so that the magnet is repelled downwards. Hence the upper face of coil must acquire south polarity. From clock rule, the current induced in the coil must be clockwise Ⓢ. At time t, the magnet moves out of the coil. Since the south pole S of the magnet is going upwards, the upper face of the coil must acquire north polarity to attract it downwards. Hence, as seen from above, the current in the coil is now counter-clockwise Ⓝ. So the correct choice is (c).

88. Since the wire is very small (a = 3 cm = 0.03 m) and the straight wire is at a very large distance (r = 60 cm = 0.6 m) from it, the magnetic field due to the current in the straight wire can be taken to be uniform through the loop and is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnetic flux through the squere loop is

$\phi = N(\mathbf{B.A}) = NBA\cos\theta$

where $A = a^2$. Since the magnetic field B at the loop is out of the plane of the loop (from right hand thumb rule) and the direction of area vector $\boldsymbol{A}$ is also normal to the plane of the loop, $\theta = 0°$. So

$\phi = NBA\cos 0° = NBA = NBa^2$

$$\phi = \frac{Na^2\mu_0 I}{2\pi r}$$

Therefore, the induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{Na^2\mu_0}{2\pi r}\frac{dI}{dt}$$

$$= \frac{6\times(0.03)^2\times(4\pi\times10^{-7})\times(2-7)}{2\pi\times(0.6)\times(1\times10^{-3})}$$

$$= 9\times10^{-6}\text{V}$$

$\therefore$ Induced current is

$$I = \frac{\varepsilon}{R} = \frac{9\times10^{-6}}{3\times10^{-3}} = 3\times10^{-3}\text{A} = 3\text{ mA}$$

The magnetic field B is out of the page. From Lenz's law, the direction of the induced current in the loop must be directed into the page. This is possible if the current in the coil is clockwise. So the correct choice is (d).

89. When switch S is open, the current in the circuit is

$$I_1 = \frac{\varepsilon}{(R_1+R_2)} = \frac{30}{(10+20)} = 1\text{A}$$

Since R_1 and R_2 are in series, this is also the current through R_1.

When switch S is closed, R_2 and R_3 are in parallel and their combined resistance is ($\because$ L is an ideal inductor)

$$R' = \frac{R_2\times R_3}{(R_2+R_3)} = \frac{20\times5}{(20+5)} = 4\,\Omega$$

Therefore, the current in the circuit now is

$$I_2 = \frac{\varepsilon}{(R_1 + R')} = \frac{30}{(10+4)} = \frac{15}{7}\text{ A}$$

$\therefore \quad \frac{I_2}{I_1} = \frac{15}{7}$, which is choice (c).

90. $B = \mu_0 nI$ directed into the page. Magnetic flux through the loop is

$$\phi = BA = (\mu_0 nI) \times \pi r^2$$

Induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -\mu_0 n\pi r^2\left(-\frac{dI}{dt}\right)$$

$$= \mu_0 n\pi r^2 \frac{dI}{dt}$$

Since the magnetic field due to the solenoid is into the page and is decreasing (because I is decreasing with time) from Lenz's law, the direction of the current in loop should be such that it produces a magnetic field into the page. Hence the induced current in the loop must be clockwise. So the correct choice is (a).

91. The current in the wire produces a magnetic field which produces magnetic flux through the square loop. Since current does not change with time, the magnetic field (and hence the magnetic flux) does not change with time. Hence no emf (and hence no current) is induced in the loop. So the correct choice is (d).

92. $I = I_0(1 - e^{-t/\tau})$

where I_0 is the maximum or final steady-state current and $\tau = \frac{L}{R}$ is the time constant of the $L - R$ circuit.

$I = \frac{3}{4}I_0$ at time t given by

$$\frac{3}{4}I_0 = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad e^{-t/\tau} = -\frac{1}{4}$$

$$\Rightarrow \quad e^{t/\tau} = 4$$

$$\frac{t}{\tau} = \ln 4 = 2(\ln 2)$$

So the correct choice is (b).

93. Maximum current is

$$I_0 = \frac{V}{R}$$

If ϕ is the magnetic flux per unit turn of the solenoid, the total flux is $\Phi = N\phi$ which is equal to $L\,I_0$, i.e.

$$N\phi = LI_0$$

$$\phi = \frac{LI_0}{N} = \frac{LV}{NR}$$

So the correct choice is (a).

94. At the instant when 1 is connected to 2, the inductor L prevents the change in current. At this instant, therefore, the current in the inductor and the rest of the circuit is zero. After the steady state is reached, the inductor has no emf (since $\frac{dI}{dt} = 0$), so the steady state current I_0 is obtained from Ohm'a law,

$$I_0 = \frac{V}{R_1} = \frac{6\text{V}}{6\Omega} = 1\text{A}$$

When 1 is connected to 3 at, say, $t = 0$, the current in the inductor does not decay instantly; it is equal to I_0 =1A at t = 0. The current through the inductor decays with time t as

$$I = I_0\, e^{-t/\tau}$$

where $\tau = \frac{L}{R_2}$. Notice that at $t = 0$, $I = I_0$.

The emf induced in the inductor is

$$\varepsilon = -L\frac{dI}{dt}$$

$$= -L\frac{dI}{dt}\left[I_0 e^{-t/\tau}\right]$$

$$= -L \times \left(-\frac{1}{\tau}\right) I_0 e^{-t/\tau}$$

At $t = 0$, $e^{-t/\tau} = e^{-0} = 1$. Hence

$$\varepsilon = \frac{LI_0}{\tau}$$

$$= \frac{L}{L/R_2} \times I_0$$

$$= R_2 I_0 = 3\Omega \times 1\text{A} = 3\text{V}$$

Since R_2 is in parallel with L, the potential across $R_2 = \varepsilon$ =3V. So the correct choice is (b).

95. $Q = CV$ and $Q_0 = CV_0$. Also $Q = Q_0 \sin \omega t$ = Thus

$$\sin \omega t = \frac{Q}{Q_0} = \frac{CV}{CV_0} = \frac{V}{V_0} = \frac{12}{12\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Now ω is given by

$$\omega = \frac{L}{\sqrt{LC}}$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}[Q_0 \sin \omega t]$$

$$= \omega\, Q_0 \cos \omega t$$

Now $\sin \omega t = \frac{1}{\sqrt{2}}$ because $\cos \omega t = \frac{1}{\sqrt{2}}$. Also $Q_0 = CV_0$.

$$\therefore \quad I = \frac{1}{\sqrt{LC}} \times CV_0 \times \frac{1}{\sqrt{2}}$$

$$= \sqrt{\frac{C}{L}} \times \frac{V_0}{\sqrt{2}}$$

$$= \sqrt{\frac{2\times10^{-6}}{5\times10^{-3}}} \times \frac{12\sqrt{2}}{\sqrt{2}} = 0.24 \text{ A}$$

So the correct choice is (d).

96. Let I_0 be the maximum current and I be the current at time t when the energy stored in inductor becomes $\frac{1}{9}$ of the maximum energy, then

$$\frac{1}{2}LI^2 = \frac{1}{9}\times\frac{1}{2}LI_0^2 \quad \Rightarrow \quad I = \frac{I_0}{3} \Rightarrow I_0 = 3I$$

Time constant $\tau = \frac{L}{R} = \frac{100\times10^{-3}}{0.2} = \frac{1}{2}$ second

Now $\quad I = I_0(1 - e^{-t/\tau})$

$\Rightarrow \quad I = 3I(1 - e^{-t/\tau})$

$\Rightarrow \quad e^{-t/\tau} = \frac{2}{3}$

$\Rightarrow \quad e^{t/\tau} = \frac{3}{2}$

$\Rightarrow \quad \frac{t}{\tau} = \ln\left(\frac{3}{2}\right)$

$\Rightarrow \quad t = \tau\ln\left(\frac{3}{2}\right) = \frac{1}{2}\ln\left(\frac{3}{2}\right)$ second

So the correct choice is (a).

97. Refer to Fig. 14.84. Let I be the current in the square loop.

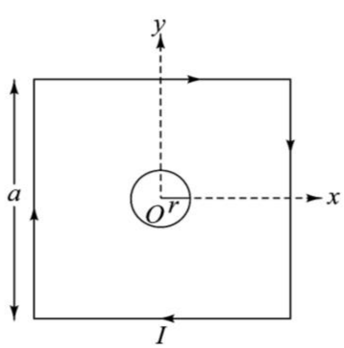


Fig. 14.84

The magnetic field at centre O due to current I in the square loop side a is [see page 13.2 of Chapter 13]

$$B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

Since $r << a$, the magnetic field can be assumed to be constant throughout the inner circular loop. Therefore, magnetic flux through the circular loop is

$$\phi = BA = B \times \pi r^2$$

$$\phi = \frac{2\sqrt{2}\,\mu_0 I}{\pi a} \times \pi r^2 = \frac{2\sqrt{2}\,\mu_0 I r^2}{a}$$

By definition $\phi = MI$. Hence

$$M = \frac{2\sqrt{2}\,\mu_0 r^2}{a}$$

Thus $M \propto \frac{r^2}{a}$ which is choice (a).

98. Refer to Fig. 14.86. Let I be the current in the outer (larger) loop

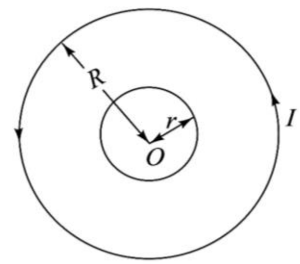


Fig. 14.85

Magnetic field at O due to current I in the outer loop is

$$B = \frac{\mu_0 I}{2R}$$

Magnetic flux through the inner loop is

$$\phi = BA = \frac{\mu_0 I}{2R} \times \pi r^2 \qquad (1)$$

Now $\phi = MI \qquad (2)$

Comparing (1) and (2) we get

$$M = \frac{\mu_0 \pi}{2}\left(\frac{r^2}{R}\right)$$

Thus $M \propto \frac{r^2}{R}$. So the correct choice is (c).

99. Refer to Fig. 14.86. Let I be the current in the outer (larger) loop.

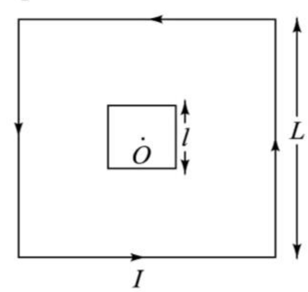


Fig. 14.86

Magnetic field at O due to current I in the outer loop is $B = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}$

Magnetic flux through the inner loop is

$$\phi = BA = \frac{2\sqrt{2}\,\mu_0 I}{\pi L} \times l^2 \quad \textbf{(1)}$$

Now $\phi = MI$ (2)

Comparing (1) and (2) we find that $M \propto \frac{l^2}{L}$. So the correct choice is (c).

100. As the bar magnet is moved towards the solenoid, the induced emf $e = -d\phi/dt$ is negative and as it is moved away from the solenoid, induced emf is positive. So the correct graph is (b).

101. Impedance $z = \sqrt{R^2 + (\omega L)^2}$ and $I_0 = \frac{V_0}{Z}$. As ω is increased, Z increases. Hence current I_0 decreases. As a result the brightness of the bulb will decrease. So the correct choice is (b).

102. $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 20$ V.

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times (100 \times 10^{-6})} = 100\ \Omega.$$

Let I_1 be the peak value of the current in the upper branch of the circuit. Then

$$I_1 = \frac{V_0}{Z_1} = \frac{V_0}{(x_c^2 + R_1^2)^{1/2}}$$

$$= \frac{20\sqrt{2}}{[(100)^2 + (100)^2]^{1/2}}$$

$$= \frac{1}{5}\text{A}$$

∴ Peak value of voltage across 100 Ω resistor is

$$V_1 = I_1 R_1 = \frac{1}{5} \times 100 = 20\text{V}$$

So the correct choice is (d).

103. $X_L = \omega L = 100 \times 0.5 = 50\,\Omega$

Let I_2 be the peak value of the current in the lower branch of the circuit. Then

$$I_2 = \frac{V_0}{Z_2} = \frac{V_0}{(X_L^2 + R_1^2)^{1/2}}$$

$$= \frac{20\sqrt{2}}{[(50)^2 + (50)^2]^{1/2}} = 0.4\text{A}$$

So the correct choice is (a).

104. To Prevent sparking, the energy stored in the induction must at least be equal to that stored in the capacitor, *i.e.*

$$\frac{1}{2} L I^2 = \frac{1}{2} C V^2$$

$$C = \frac{LI^2}{V^2} = \frac{0.1 \times (3)^2}{(300)^2} = 10 \times 10^{-6}\text{F} = 10\mu\text{F}$$

So the correct choice is (a).

105. Before the primary is rotated, the current through it changes from zero to i. When the primary is rotated through 180°, the current changes from i, to $-i$. Hence the current now changes by $2i$. Since the change in current is doubled, its rate of change is also doubled. Hence emf induced across the secondary is doubled and the induced current is also doubled. So the correct choice is (b).

106. From Fleming's left hand rule, the free electrons in rod XY will move from X to Y. Hence and X acquires a positive charge (due to loss of electrons) and end Y acquires an equal negative charge (due to gain of electrons). Therefore, the correct choice is (a).

107. The graph shown in Fig. 14.69 is a sine curve. Hence

$$\phi = \phi_0 \sin \omega t = \phi_0 \sin\left(\frac{2\pi t}{T}\right)$$

Induced emf is

$$e = -\frac{d\phi}{dt} = -\phi_0 \times \frac{2\pi}{T} \cos\left(\frac{2\pi t}{T}\right) = -e_0 \cos\left(\frac{2\pi t}{T}\right)$$

It is clear that the correct choice is (*d*).

108. The inductive reactances are

$X_1 = \omega L_1$ and $X_2 = \omega L_2$

If L_{eq} is the equivalent inductance, then $X_{eq} = \omega L_{eq}$. since the reactances X_1 and X_2 are in parallel,

$$\frac{1}{X_{eq}} = \frac{1}{X_1} + \frac{1}{X_2}$$

$$\Rightarrow \quad \frac{1}{\omega L_{eq}} = \frac{1}{\omega L_1} + \frac{1}{\omega L_2}$$

$$\Rightarrow \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

So the correct choice is (*d*).

109. In the steady state, the current through L is

$$I = \frac{V}{R_1} = \frac{12\text{V}}{6\Omega} = 2\text{A}$$

∴ Energey stored in $L = \frac{1}{2} L I^2$

$$= \frac{1}{2} \times 2 \times (2)^2 = 4\text{J}$$

So the correct choice is (a).

110. $i = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$ (1)

where $\tau = \frac{L}{R}$

$$\therefore \frac{di}{dt} = \frac{E}{R} \times \left(-\frac{1}{\tau}\right) e^{-t/\tau} = -\frac{E}{L} e^{-t/\tau}$$

Induced emf is

$$e = -L\frac{di}{dt} = E e^{-t/\tau} \quad (2)$$

From (2), $e^{-t/\tau} = \frac{e}{E}$. Using this in (1), we have

$$i = \frac{E}{R}\left(1 - \frac{e}{E}\right) = \frac{E}{R} - \frac{e}{R}$$

or $e = E - iR$ (3)

Equation (3) shows that the graph of e against i is a straight line with negative slope and positive intercept. So the correct graph is (d).

111. Motional emf induced between the ends of PQ is
$e = Blv = 2 \times 0.5 \times (4 \times 10^{-2}) = 4 \times 10^{-2}$ V

From Fleming's right hand rule, the direction of the induced current in PQ is from Q to P. The circuit can be assumed to have a cell of emf $e = 4 \times 10^{-2}$ V and internal resistance $r = 0.8\Omega$ connected as shown in Fig. 14.87(a).

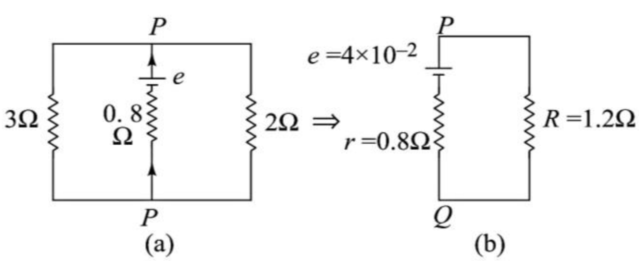


Fig. 14.87

Resistances 3Ω and 2Ω are in parallel. Their combined resistance is

$$R = \frac{3 \times 2}{3 + 2} = 1.2\,\Omega$$

The circuit can be redrawn as shown in Fig. 14.87 (b). Current in the circuit is

$$I = \frac{e}{R + r} = \frac{4 \times 10^{-2}}{1.2 \times 0.8} = 2 \times 10^{-2}\text{A}$$

Therefore, force acting on wire PQ is

$F = BIL = 2 \times (2 \times 10^{-2}) \times 0.5 = 2 \times 10^{-2}$ N

So the correct choice is (b).

112. The emf inducted between OQ or OP is

$$e = \frac{1}{2} B\omega l^2$$

$$= \frac{1}{2} \times 4 \times 20 \times (0.5)^2 = 10\text{ V}$$

The circuit can be redrawn as shown in Fig. 14.88 (because OP and OQ are in parallel).

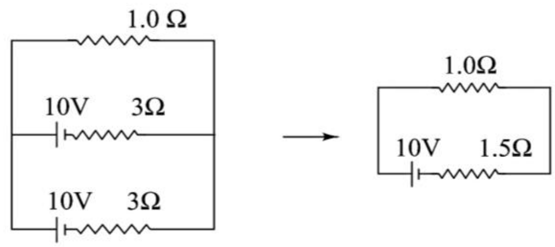


Fig. 14.88

$$\therefore I = \frac{10}{1.5 + 1.0} = 4\text{A}$$

So the correct choice is (*a*).

113. $V = 220 \sin(50\pi t) \cos(50\pi t)$
$= 110 \times [2 \sin(50\pi t) \cos(50\pi t)]$
or $V = 110 \sin(100\pi t)$ (1)

Peak value of voltage is $V_0 = 110$ V. The r.m.s value is

$V_{rms} = \frac{110}{\sqrt{2}} = 77.8 \simeq 78$ V, which is choice (a).

114. From Eq. (1) above,

$$\omega = 100\pi$$

$$\Rightarrow 2\pi\nu = 100\pi$$

$$\Rightarrow \nu = 50\text{ Hz}$$

So the correct choice is (b).

115. Given $I = 5 \sin(50\pi t)$ (1)

The emf induced in the second solenoid is

$$E = -M\frac{dI}{dt}$$

$$= -M \times 5 \times 50\pi \times \cos(50\pi t)$$

$$= -E_0 \cos(50\pi t)$$

$$E = E_0 \sin\left(50\pi t - \frac{\pi}{2}\right) \quad (2)$$

Maximum value of E is

$$E_0 = M \times 5 \times 50\pi$$

$$= (10 \times 10^{-3}) \times 5 \times 50\pi = 2.5\pi \text{ volt}$$

Hence the correct choice is (a).

116. From Eqs. (1) and (2) it follows that the phase difference between I and E is $\frac{\pi}{2}$. So the correct choice is (c).

117. I_L lags behind V by $\pi/2$ and I_c leads V by $\pi/2$. Hence phase difference between I_L and I_c is 180º, *i.e.* they are in opposite phases. So the current drawn from the source is $I = I_L - I_c$, which is choice (b).

118. In Fig. 11.77, L and C are in series. Hence $I_L = I_c$ but they differ in phase by 180°. So $I = I_L - I_C = 0$ which is choice (a).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

Two long parallel horizontal rails, distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see Fig. 14.89). There is a uniform magnetic field B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that as the rod moves, constant current flows through R.

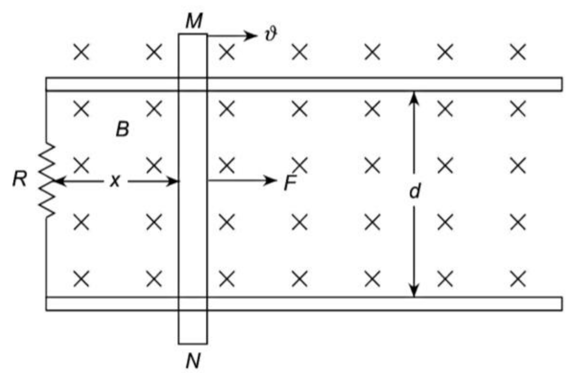


Fig. 14.89

1. The magnitude of the emf induced in the loop is

(a) $Bvd\left(\frac{2\lambda x}{R}\right)$ (b) $Bvd\left(\frac{R}{2\lambda x}\right)$

(c) Bvd (d) $\frac{1}{2}Bvd$

2. The current in the loop is

(a) $\frac{Bvd}{R}$ (b) $\frac{Bvd}{2\lambda x}$

(c) $\frac{2Bvd}{(R+2\lambda x)}$ (d) $\frac{Bvd}{(R+2\lambda x)}$

3. The velocity of the rod is

(a) $\frac{B^2d^2}{2\lambda m}\left(1+\frac{2\lambda x}{R}\right)$

(b) $\frac{B^2d^2}{R}\left(1-\frac{R}{2\lambda x}\right)$

(c) $\frac{B^2d^2}{2\lambda m}\log_e\left(1-\frac{R}{2\lambda x}\right)$

(d) $\frac{B^2d^2}{2\lambda m}\log_e\left(1+\frac{2\lambda x}{R}\right)$

Solutions

1. Let the distance from R to MN be x. Then the area of the loop between MN and R is xd and the magnetic flux linked with the loop is Bxd. As the rod moves, the emf induced in the loop is given by

$$|e| = \frac{d}{dt}(Bxd) = Bd\,\frac{dx}{dt} = Bvd$$

where v = velocity of MN.
So the correct choice is (c).

2. The total resistance of the loop between R and MN is $R + 2\lambda x$. The current in the loop is given by

$$I = \frac{|e|}{R+2\lambda x} = \frac{Bvd}{R+2\lambda x}$$

The correct choice is (d).

3. Force acting on the rod,

$$F = IBd = \frac{B^2d^2v}{R+2\lambda x}$$

$$\therefore \quad m\frac{dv}{dt} = \frac{B^2d^2}{R+2\lambda x}\cdot\frac{dx}{dt}$$

or $$dv = \frac{B^2d^2}{m} \times \frac{dx}{(R+2\lambda x)}$$

Integrating, we have

$$\int_0^v dv = \frac{B^2d^2}{m}\int_0^x \frac{dx}{(R+2\lambda x)}$$

or $$v = \frac{B^2d^2}{2\lambda m}\log_e\left(\frac{R+2\lambda x}{R}\right)$$

Hence the correct choice is (d).

Questions 4 to 6 are based on the following passage.

Passage II

A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails without friction. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string, hangs vertically. A constant magnetic field B exists perpendicular to the table. The system is released from rest. (Fig. 14.90)

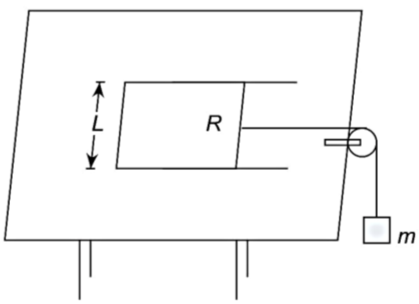


Fig. 14.90

4. The acceleration of the mass m moving in the downward direction is

(a) g (b) $\frac{B^2L^2v}{mR}$

(c) $\left(g - \frac{B^2L^2v}{mR}\right)$ (d) $\left(g + \frac{B^2L^2v}{mR}\right)$

5. The terminal velocity acquired by the rod is

(a) g (b) $\sqrt{gR}$

(c) $\frac{\sqrt{mgR}}{BL}$ (d) $\frac{mgR}{B^2L^2}$

6. The acceleration of mass m when the velocity of the rod is half the terminal velocity is

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{3}$ (d) $\frac{g}{4}$

Solutions

4. Refer to Fig. 14.91. Let v be the velocity of the rod along the positive x-direction at an instant of time and let the magnetic field B act perpendicular to the table along the positive y-direction. The emf induced in the rod is $e = BLv$. Therefore, the induced current is

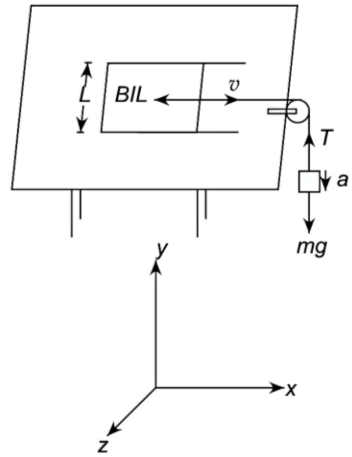


Fig. 14.91

$$I = \frac{e}{R} = \frac{BLv}{R} \quad (1)$$

The rod of length L carrying a current I in magnetic field will experience a force

$$F = BIL \quad (2)$$

along the negative x-direction. Since the rod is massless, this force will also be equal to the tension T in the string acting along the positive x-direction, i.e. $T = F = BIL$.

Let a be the acceleration of mass m moving in the downward direction, then

ma = net force acting on $m = mg - T = mg - F$

or $$a = g - \frac{F}{m} \quad (3)$$

Using (1) and (2) in (3), we have

$$a = g - \frac{BIL}{m} = g - \frac{B \times BL^2v}{mR}$$

$$= g - \frac{B^2L^2v}{mR} \quad (4)$$

So the correct choice is (c).

5. The rod will acquire terminal velocity v_t when $a = 0$. Putting $a = 0$ and $v = v_t$ in Eq. (4) we have

$$0 = g - \frac{B^2L^2v_t}{mR} \quad \text{or } v_t = \frac{mgR}{B^2L^2}$$

The correct choice is (d).

14. When the velocity of the rod is half the terminal velocity, i.e. when

$$v = \frac{v_t}{2} = \frac{mgR}{2B^2L^2},$$

then from Eq. (4), we have

$$a = g - \frac{B^2L^2v_t/2}{mR}$$

$$= g - \frac{B^2L^2}{2mR} \times \frac{mgR}{B^2L^2} = g - \frac{g}{2} = \frac{g}{2}$$

Thus the correct choice is (b).

Questions 7 to 10 are based on the following passage.

Passage III

An infinitesimally small bar magnet of dipole moment **M** is pointing and moving with a speed v in the x-direction. A small closed circular conducting loop of radius a and negligible self inductance lies in the y-z plane with its centre at $x = 0$, and its axis coinciding with the x-axis.

7. The magnitude of magnetic field at a distance x on the axis of the short bar manget is

(a) $\frac{\mu_0 M}{2\pi x}$ (b) $\frac{\mu_0 M}{2\pi x^2}$

(c) $\frac{\mu_0 M}{2\pi x^3}$ (d) $\frac{\mu_0 M}{2\pi x^4}$

8. If $x = 2a$, the magnetic flux through the loop is

(a) $\mu_0 M$ (b) $\frac{\mu_0 M}{2}$

(c) $\frac{\mu_0 M}{4a}$ (d) $\frac{\mu_0 M}{16a}$

9. If $x = 2a$, the emf induced in the loop is

(a) $\frac{3}{16}\frac{\mu_0 M v}{a^2}$ (b) $\frac{3}{32}\frac{\mu_0 M v}{a^2}$

(c) $\frac{1}{8}\frac{\mu_0 M v}{a^2}$ (d) $\frac{1}{16}\frac{\mu_0 M v}{a^2}$

10. If $x = 2a$, the magnetic moment of the loop is

(a) $\frac{3\pi \mu_0 M v}{32R}$ (b) $\frac{3\pi \mu_0 M v}{8R}$

(c) $\frac{\pi \mu_0 M v}{2R}$ (d) $\frac{3\pi \mu_0 M v}{4R}$

Solutions

7. Refer to Fig. 14.92. The magnetic field at a distance x on the axis of a magnet of length $2l$ and dipole moment M is given by

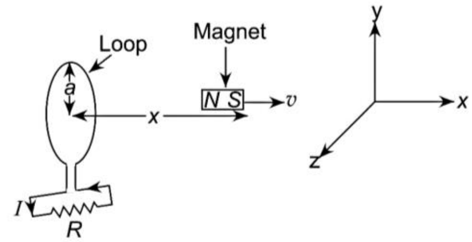


Fig. 14.92

$$B = \frac{\mu_0}{2\pi}\cdot\frac{Mx}{(x^2 - l^2)^2}$$

Since $x >> l$, we have

$$B = \frac{\mu_0 M}{2\pi x^3}$$

So the correct choice is (c).

8. Due to B, the flux through the loop is

$$\phi = BA = B(\pi a^2) = \frac{\mu_0 M}{2\pi x^3} \times \pi a^2 = \frac{\mu_0 M a^2}{2x^3}$$

If $x = 2a$, we find that the correct choice is (d).

9. Induced emf in the loop is

$$e = -\frac{d\phi}{dt} = -\frac{dx}{dt}\frac{d\phi}{dx} = -v\frac{d\phi}{dx}$$

$$= -\frac{\mu_0 M a^2 v}{2}\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{3}{2}\frac{\mu_0 M a^2 v}{x^4}$$

Putting $x = 2a$, we get $e = \frac{3\mu_0}{32}\frac{Mv}{a^2}$, which is choice (b).

10. Induced current in the loop is

$$I = \frac{e}{R} = \frac{3}{2}\frac{\mu_0 M a^2 v}{x^4 R}$$

Magnetic moment of the loop is

$$M_0 = I \times \text{area enclosed by the loop} = I(\pi a^2)$$

$$= \frac{3\pi}{2}\frac{\mu_0 M a^4 v}{x^4 R}$$

Putting $x = 2a$, we find that the correct choice is (a).

Questions 11 to 14 are based on the following passage.

Passage IV

Two resistances of 10 Ω and 20 Ω and an ideal inductor of inductance 5 H are connected to a battery of 2 V through a key K as shown in Fig. 14.93. If at $t = 0$, K is inserted.

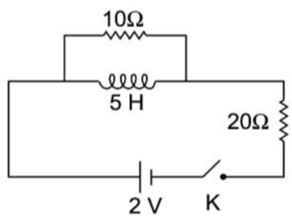


Fig. 14.93

11. The initial current through the battery is

(a) $\frac{1}{15}$ A (b) $\frac{2}{15}$ A

(c) 0.2 A (d) 0.4 A

12. The initial potential drop across the inductor is

(a) $\frac{1}{6}$ V (b) $\frac{1}{3}$ V

(c) $\frac{2}{3}$ V (d) $\frac{4}{3}$ V

13. The final current through the 10 Ω resistor is

(a) $\frac{1}{15}$ A (b) 0.2 A

(c) 0.1 A (d) zero

14. The final current through the 20 Ω resistor is

(a) 0.1 A (b) 0.2 A

(c) 0.3 A (d) zero

Solutions

11. As soon as K is inserted, i.e. $t = 0$, dI/dt is maximum, which implies that the opposing emf $L\ dI/dt$ is high and the inductor will be have as a very large resistor. So the current will flow through both the resistances only. The current through the battery $I(0)$ at $t = 0$ is

$$I(0) = \frac{2.0}{10+20} = \frac{1}{15}\ \text{A}$$

The correct choice is (a).

12. Since the 10 Ω resistor and inductor are in parallel, the potential drop across the inductor is the same as that across the 10 Ω resistor. Hence, the initial potential drop is

$$I(0) \times 10 = \frac{10}{15}\ \text{V}$$

So the correct choice is (c).

13. When the current has attained a constant value, the opposing emf across the inductor is zero. The inductor would behave as a short and the whole current will pass through it. The final current through the 10 Ω resistor is, therefore, zero. The correct choice is (d).

14. Since the inductor behaves as a short or zero resistance, the total resistance of the circuit is also 20 Ω (for 10 Ω is in parallel with the inductor). The final current in the circuit is therefore, 2.0/20 = 0.1 A. Hence the correct choice is (a).

Questions 15 to 17 are based on the following passage.

Passage V

An LCR series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/sec. When only the capacitance is removed, the current leads the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°.

15. The impedance of the LCR circuit is

(a) 100 Ω (b) $100\sqrt{2}$ Ω

(c) 200 Ω (b) $200\sqrt{2}$ Ω

16. The current in the circuit is

(a) $\sqrt{2}$ A (b) 2 A

(c) $2\sqrt{2}$ A (d) 1 A

17. The power dissipated in the circuit is

(a) 200 W (b) 400 W

(c) 800 W (d) 100 W

Solutions

15. When capacitance is removed, the circuit contains only inductance and resistance. Phase difference θ between the current and voltage is then given by

$$\tan\theta = \frac{\omega L}{R} \text{ or } \omega L = R\tan\theta = 100\tan 60°$$

When the circuit contains only capacitance and resistance, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$$\therefore \quad \frac{1}{C\omega} = R\tan\phi = 100\tan 60°$$

The impedance of the LCR circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + (100\tan 60° - 100\tan 60°)^2}$$

$= R = 100$ Ω, which is choice (a).

16. The current is given by

$$I = \frac{V}{R} = \frac{200}{100} = 2\ \text{A}$$

The correct choice is (b).

17. The power dissipated in the circuit is
$P = I^2R = 4 \times 100 = 400$ W
So the correct choice is (b).

Questions 18 to 20 are based on the following passage.

Passage VI

An LCR circuit consists of an inductor, a capacitor and a resistor driven by a battery and connected by two switches S_1 and S_2 as shown in Fig. 14.94.

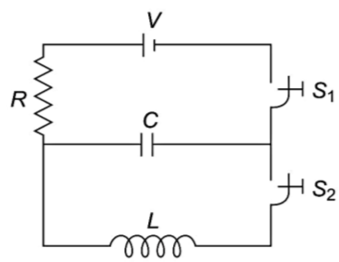


Fig. 14.94

18. At time $t = 0$ switch S_1 is closed and S_2 is left open. The maximum charge the capacitor plate can hold is q_0 and τ is the time constant of the RC circuit. Then
(a) at time $t = \tau$, the charge on the capacitor plates is $q = q_0/2$.
(b) at $t = 2\tau$, $q = q_0(1 - e^{-2})$
(c) at $t = 2\tau$, $q = q_0(1 - e^{-1})$
(d) work done by the battery is half the energy dissipated in the resistor.

19. At time $t = 0$ when the charge on the capacitor plates is q, switch S_1 is opened and S_2 is closed.
The maximum charge the capacitor can hold is q_0. Choose the correct statement from the following.
(a) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right)$
(b) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} - \frac{\pi}{2}\right)$
(c) $q = -LC \frac{d^2q}{dt^2}$
(d) $q = -\frac{1}{\sqrt{LC}} \frac{d^2q}{dt^2}$

20. At an instant of time $t = 0$ when the capacitor has been charged to a voltage V, switch S_1 is opened and S_2 is closed. Then
(a) at $t = 0$, the energy is stored in the magnetic field of the inductor.
(b) at $t > 0$, there is no exchange of energy between the capacitor and the inductor.
(c) at $t > 0$, the current in the circuit flows only in one direction.
(d) the maximum value of the current in the circuit is $\sqrt{\frac{C}{L}}V$.

Solutions

18. In an RC circuit, the charge on the capacitor plates at a time t is given by
$$q = q_0(1 - e^{-t/\tau})$$
where $\tau = RC$ is the time constant. At $t = 2\tau$, we have
$$q = q_0(1 - e^{-2})$$
Hence the correct choice is (b).

19. When S_2 is closed and S_1 is open, the charge oscillates in the LC circuit at an angular frequency given by
$$\omega = \frac{1}{\sqrt{LC}} \quad (1)$$
Now $q \neq 0$ at $t = 0$. Hence choices (a) and (b) are wrong. The charge q varies with time t as
$$q = q_0\cos(\omega t + \phi) \quad (2)$$
where ϕ is not equal to $\pi/2$. Differentiating Eq. (2) twice with respect to t, we get
$$\frac{d^2q}{dt^2} = -\omega^2 q_0 \cos(\omega t + \phi) = -\omega^2 q$$
$$q = -\frac{1}{\omega^2}\frac{d^2q}{dt^2} = -LC\frac{d^2q}{dt^2} \quad \text{[use Eq. (1)]}$$
Hence the correct choice is (c).

20. At $t = 0$, the energy is stored in the electric field in the space between the capacitor plates. As time passes (i.e. at $t > 0$), there is an exchange of energy between the capacitor and the inductor. The charge q varies with time t as
$$q = q_0 \cos\omega t, \text{ where } \omega = \frac{1}{\sqrt{LC}}$$
The current in the circuit is given by
$$I = \frac{dq}{dt} = \frac{d}{dt}(q_0 \cos \omega t) = -\omega q_0 \sin \omega t$$
which is alternating and not unidirectional. The maximum value of current is
$$I_{max} = \omega q_0 = \frac{1}{\sqrt{LC}} \times CV \quad (q_0 = CV)$$
$$= \sqrt{\frac{C}{L}}V$$
Hence the correct choice is (d).

3 SECTION Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four options out of which only *one* option is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

No induced emf is developed across the ends of a conductor if it is moved parallel to a magnetic field.

Statement-2

No force acts on the free electrons of the conductor.

2. Statement-1

No current is induced in a metal loop if it is rotated in an electric field.

Statement-2

The electric flux through the loop does not change with time.

3. Statement-1

A rectangular loop and a circular loop are moved with a constant velocity from a region of magnetic field out into a field-free region. The field is normal to the loops. Then a constant emf will be induced in the circular loop and a time-varying emf will be induced in the rectangular loop.

Statement-2

The induced emf is constant if the magnetic flux changes at a constant rate.

4. Statement-1

A magnetised iron bar is dropped vertically through a hollow region of a thick cylindrical shell made of copper. The bar will fall with an acceleration less than g, the acceleration due to gravity.

Statement-2

The emf induced in the bar causes a retarding force to act on the falling bar.

5. Statement-1

A coil is connected in series with a bulb and this combination is connected to a d.c. source. If an iron core is inserted in the coil, the brightness of the bulb will increase.

Statement-2

The reactance offered by the coil to d.c. current is zero.

6. Statement-1

A coil is connected in series with a bulb and this combination is connected to an a.c. source. If an iron core is inserted in the coil, the brightness of the bulb will be reduced.

Statement-2

When an iron core is inserted in the coil, its inductance decreases.

7. Statement-1

A variable capacitor is connected in series with a bulb and this combination is connected to an a.c. source. If the capacitance of the variable capacitor is decreased, the brightness of the bulb is reduced.

Statement-2

The reactance of the capacitor increases if the capacitance is reduced.

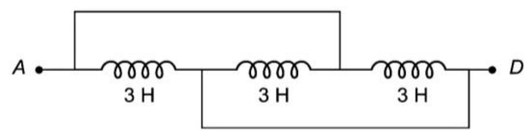
Solutions

1. The correct choice is (a). Let $\vec{v}$ be the velocity of the conductor in a magnetic field $\vec{B}$. Since the free electrons in the conductor are moving with it, force $\vec{F} = e(\vec{v} \times \vec{B})$ is zero because $\vec{v}$ is parallel to $\vec{B}$. Consequently, no induced emf is developed between the ends of the conductor.
2. The correct choice is (b). A current is induced in a loop only if magnetic flux linked with the coil changes.
3. The correct choice is (d). The induced emf is constant in the case of rectangular coil because the rate of change of area is constant. But in the case of the circular coil, the rate of change of area (and hence the rate of change of magnetic flux) keeps varying as the loop is moving towards the field-free region.
4. The correct choice is (a). The retarding force is caused by the eddy currents and according to Lenz's law, the induced emf must oppose the cause. The cause is the falling bar.

5. The correct choice is (d). A coil offers no reactance to d.c. currents. Hence there will be no change in the brightness of the bulb when an iron core is inserted in the coil.
6. The correct choice is (c). If an iron core is inserted in the coil, its inductance L increases. Hence its reactance ωL increases, causing a decrease in the current in the circuit. As a result, the brightness of the bulb will reduce.
7. The correct choice is (a). The reactance of a capacitor is $1/\omega C$. Hence if C is decreased, the reactance will increase and as a result the current in the circuit is decreased causing a decrease in the brightness of the bulb.

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The inductance between points A and D is



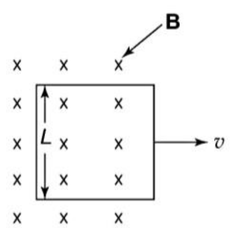
(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H **[2002]**

2. The power factor of an AC circuit having a resistance R and inductance L (connected in series) is (ω is the angular frequency of the AC source)

(a) $\frac{R}{\omega L}$ (b) $\frac{R}{\left[R^2+\omega^2L^2\right]^{1/2}}$
(c) $\frac{\omega L}{R}$ (d) $\frac{R}{\left[R^2-\omega^2L^2\right]^{1/2}}$ **[2002]**

3. In a transformer, the number of turns in the primary is 140 and that in the secondary is 280. If the current in the primary is 4A, then the current in the secondary is
(a) 4A (b) 2A
(c) 6A (d) 10A **[2002]**

4. A conducting square loop of side L and resistance R moves in a plane with a uniform velocity v perpendicular to one of its sides. A uniform magnetic field **B** pointing perpendicular and into the plane of the loop exists everywhere with half the loop outside the field. The induced emf is



(a) zero (b) RvB
(c) $\frac{vBL}{R}$ (d) vBL **[2002]**

5. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which the currents are changed in the two coils
(b) the relative position and orientation of the two coils
(c) the material of the wires of the coils
(d) the currents in the two coils. **[2003]**

6. When the current in a coil changes from 2A to – 2A in 0.05s, an emf of 8V is induced in the coil. The self inductance of the coil is
(a) 0.2H (b) 0.4H
(c) 0.8H (d) 0.1H **[2003]**

7. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is
(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$
(c) $\frac{Q}{\sqrt{2}}$ (d) Q **[2003]**

8. The core of a transformer is laminated so as to
(a) reduce energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage. **[2003]**

9. Alternating current cannot be measured by DC ammeter because
(a) AC cannot pass through DC ammeter
(b) AC changes direction
(c) the average value of current for complete cycle is zero
(d) DC ammeter will get damaged. **[2004]**

10. In an LCR series circuit, the voltage across L, C and R is 50V each. The voltage across the LC combination will be
(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) zero **[2004]**

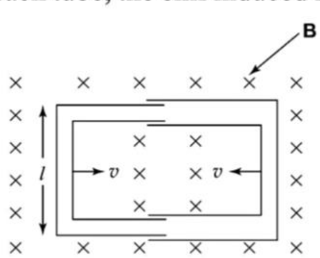
11. A coil having n turns and resistance R is connected to a galvanometer of resistance $4R$. The combination is moved in a region of magnetic field. If the magnetic flux through each turn of the coil changes from ϕ_1 to ϕ_2 in time t, the induced current in the circuit is
(a) $\frac{(\phi_2-\phi_1)}{5Rnt}$ (b) $-\frac{n(\phi_2-\phi_1)}{5Rt}$
(c) $-\frac{(\phi_2-\phi_1)}{Rnt}$ (d) $-\frac{n(\phi_2-\phi_1)}{Rt}$ **[2004]**

12. In a uniform magnetic field B, a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is
(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{(B\pi r^2\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$ **[2004]**

13. In an LCR circuit, the capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to
(a) $4L$ (b) $2L$
(c) $\frac{L}{2}$ (d) $\frac{L}{4}$ **[2004]**

14. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, the emf developed between the two ends of the conductor is
(a) 5 μV (b) 50 μV
(c) 5 mV (d) 50 mV **[2004]**

15. One conducting U-tube can slide inside another as shown in the Figure, always maintaining electrical contact between them. A uniform magnetic field **B** is perpendicular to the plane of the figure. Each tube moves towards each other with a velocity v. If l is the width of each tube, the emf induced in circuit will be



(a) Blv (b) $-Blv$
(c) zero (d) $2Blv$ **[2005]**

16. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in
(a) 0.05s (b) 0.1s
(c) 0.15s (d) 0.3s **[2005]**

17. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitor of capacitance
(a) 4 μF (b) 8 μF
(c) 1 μF (d) 2 μF **[2005]**

18. A circuit has a resistance of 12Ω and an impedance of 15Ω. The power factor of the circuit is
(a) 0.8 (b) 0.4
(c) 1.25 (d) 0.125 **[2005]**

19. The phase difference between the alternating current and emf is π/2. Which of the following cannot be the constituent of the circuit?
(a) C alone (b) R, L
(c) L, C (d) L alone **[2005]**

20. The flux linked with a coil at any instant t is given by

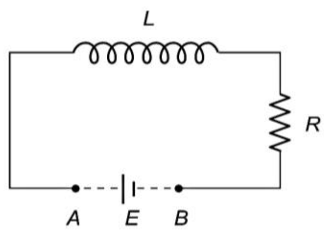
$$\phi = 10t^2 - 50t + 250$$

The induced emf at $t = 3$ s is
(a) 10 V (b) 190 V
(c) – 190 V (d) – 10 V **[2006]**

21. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1$ kΩ with $C = 2$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is
(a) 250 V (b) 4×10^{-3} V
(c) 2.5×10^{-2} V (d) 40 V **[2006]**

22. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is
(a) $NABR$ (b) ωNAB
(c) $\omega NABR$ (d) NAB **[2006]**

23. An inductor ($L = 100$ mH), a resistor ($R = 100$ Ω) and a battery ($E = 100$ V) are initially connected in series as shown in the Figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is



(a) 0.1 A (b) 1 A
(c) $1/e$ A (d) eA **[2006]**

24. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

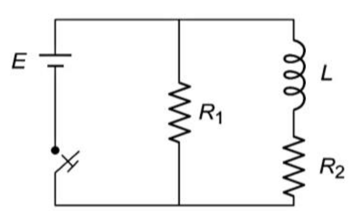
(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (d) $P =$ zero

(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}\ E_0 I_0$ **[2007]**

25. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5V. 2 seconds after the connection is made, the current flowing in amperes in the circuit is
(a) $(1 - e)$ (b) e
(c) e^{-1} (d) $(1 - e^{-1})$ **[2007]**

26. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ cm^2$ and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(a) $2.4\ \pi \times 10^{-4}$ H (b) $2.4\ \pi \times 10^{-5}$ H
(c) $4.8\ \pi \times 10^{-4}$ H (d) $4.8\ \pi \times 10^{-5}$ H
[2008]

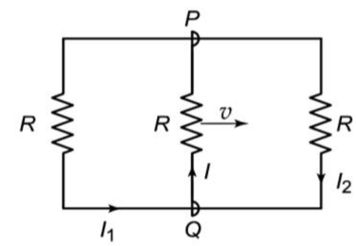
27. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop (in volt) across L as a function of time t is



(a) $6\,[1 - e^{-t/0.2}]$ (b) $12\,e^{-5t}$

(c) $6\,e^{-5t}$ (d) $\frac{12}{t}\,e^{-3t}$ **[2009]**

28. A rectangular loop has a sliding connector PQ of length l and resistance R and it is moving with a speed v as shown in the figure. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



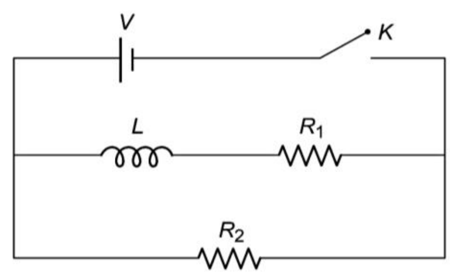
(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ **[2010]**

29. In the circuit shown in the figure, the key K is closed at $t = 0$. The current through the battery is



(a) $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ **[2010]**

30. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind

the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the *LCR* circuit is

(a) 242 W (b) 305 W
(c) 210 W (d) zero W **[2010]**

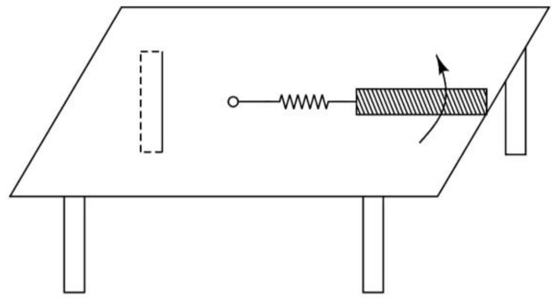
31. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance *C* and an electric bulb is resistance *R* (inductance zero). When ω is increased
(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases **[2010]**

32. A fully charged capacitor *C* with initial charge q_0 is connected to a coil of self inductance *L* at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is
(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$
(c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$ **[2011]**

33. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1}m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is
(a) 1 mV (b) 0.75 mV
(c) 0.50 mV (d) 0.15 mV **[2011]**

34. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to
(a) development of air current when thee plate is placed
(b) induction of electrical charge on the plate.
(c) shielding of magnetic line of force as aluminium is a paramagnetic material.
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping. **[2012]**

35. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table ith one end of the string fixed. If there is a vertical magnetic field '*B*' in the region, the e.m.f. induced across the ends of the rod is



(a) $\frac{3B\omega\ell^2}{2}$ (b) $2B\omega\ell^2$
(c) $\frac{5B\omega\ell^2}{2}$ (d) $B\omega\ell^2$ **[2013]**

36. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. the distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is
(a) 6×10^{-11} weber (b) 3.3×10^{-11} weber
(c) 6.6×10^{-9} weber (d) 9.1×10^{-11} weber **[2013]**

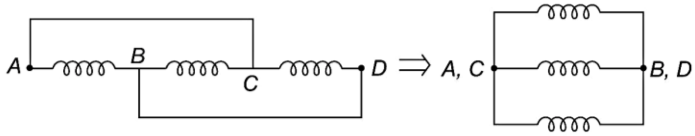
37. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The average power consumption in the circuit is
(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P =$ zero
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}E_0 I_0$ **[2014]**

Answers

1. (d)	2. (b)	3. (b)	4. (d)
5. (b)	6. (d)	7. (c)	8. (a)
9. (c)	10. (d)	11. (b)	12. (b)
13. (c)	14. (b)	15. (d)	16. (b)
17. (c)	18. (a)	19. (b)	20. (d)
21. (a)	22. (b)	23. (c)	24. (b)
25. (d)	26. (a)	27. (b)	28. (b)
29. (b)	30. (a)	31. (b)	32. (b)
33. (d)	34. (d)	35. (c)	36. (d)
37. (b)			

Solutions

1. The three inductors are in parallel as shown in the Figure.



∴ Equivalent inductance between A and D is

$$\frac{1}{L_{eq}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1 \Rightarrow L_{eq} = 1\text{ H}$$

2. Power factor $= \dfrac{R}{\left[R^2+\omega^2L^2\right]^{1/2}} = \dfrac{\text{resistance}}{\text{impedance}}$

3. Assuming that the transformer is ideal, i.e. there is no power loss, output power = input power

$\therefore E_p I_p = E_s I_s$. Also $\dfrac{E_p}{E_s} = \dfrac{N_p}{N_s}$

Therefore $\dfrac{I_p}{I_s} = \dfrac{E_s}{E_p} = \dfrac{N_s}{N_p} = \dfrac{280}{140} = 2$

$\Rightarrow I_s = \dfrac{I_p}{2} = \dfrac{4}{2} = 2\text{A}$

4. The induced emf is due to change in magnetic flux due to change in area and is given by $e = vBL$.

5. The correct choice is (*b*).

6. Induced emf is

$$e = -L\frac{dI}{dt}$$

Change in current dI = final current – initial current = – 2A – 2A = – 4A

Also $dt = 0.05$ s.

$$8 = -L \times \frac{-4}{0.05} \Rightarrow L = 0.1\text{ H}$$

7. The maximum energy stored in the electric field between capacitor plates is

$(U_e)_{max} = \dfrac{Q^2}{2C}$ ($\because Q$ = maximum charge)

At this time, the energy stored in the magnetic field of the inductor is zero. Let q be the charge on the capacitor when the energy is equally shared between electric and magnetic fields, then

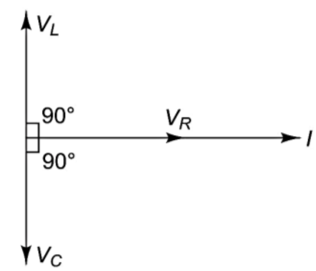
$$U_e = \frac{q^2}{2C} = \frac{1}{2}(U_e)_{max} = \frac{Q^2}{4C}$$

$$\Rightarrow q = \frac{Q}{\sqrt{2}}$$

8. The correct choice is (a).

9. A complete cycle of an alternating current consists of two half cycles. In one half cycle the current is positive and in the next half cycle, the current is negative. Hence, in one complete cycle, the average value of current is zero. So a DC ammeter will read zero.

10. Given $V_L = V_C = V_R = 50$ V. In a series LCR circuit, the voltage V_L across L leads the current I by 90° and the voltage V_C across C lags behind the current I by 90°. Since $V_L = V_C$, the voltage across the LC combination will be zero.



11. Induced emf is

$$e = -n\frac{d\phi}{dt} = -n\,\frac{(\text{final flux} - \text{initial flux})}{\text{time}}$$

$$= -n\left(\frac{\phi_2-\phi_1}{t}\right)$$

Total resistance in the circuit $R' = R + 4R = 5R$

$\therefore$ Induced current $= \dfrac{e}{R'} = \dfrac{n(\phi_2-\phi_1)}{5Rt}$

12. Magnetic flux through the coil is

$$\phi = BA\cos\theta$$

Here $A = \dfrac{1}{2}(\pi r^2)$ and $\theta = \omega t$. Therefore,

$$\therefore \quad \phi = \frac{1}{2}(B\pi r^2)\cos\omega t$$

The induced emf is

$$e = -\frac{d\phi}{dt} = \left(\frac{B\pi r^2\omega}{2}\right)\sin\omega t$$

$$\text{Power } P = \frac{e^2}{R} = \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$$

Average power in one time period is

$$\langle P\rangle = \frac{(B\pi r^2\omega)^2}{4R}\langle\sin^2\omega t\rangle$$

$$= \frac{(B\pi r^2\omega)^2}{8R} \qquad \left(\because \langle\sin^2\omega t\rangle = \frac{1}{2}\right)$$

13. Resonant frequency is

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}$$

If C is doubled, L must be halved so that ν_r remains unchanged.

14. $e = \frac{1}{2}\ B\omega l^2$

$= \frac{1}{2} \times (0.2 \times 10^{-4}) \times 5 \times (1)^2$

$= 0.5 \times 10^{-4}$ V

$= 50 \times 10^{-6}$ V

$= 50$ μV

15. Suppose each tube moves a small distance dx in time dt. The change in area = $l \times 2dx$. The rate of change in area is

$$\frac{dA}{dt} = 2l\frac{dx}{dt} = 2lv$$

Magnitude of induced emf is

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = B\frac{dA}{dt} = 2Blv$$

16. At an instant of time t, the current is given by

$I = I_0\ (1 - e^{-Rt/L})$; I_0 = steady state current

At $t = 0$, $I = 0$. Let $I = \frac{I_0}{2}$ at time t. Then

$$\frac{I_0}{2} = I_0\ (1 - e^{-Rt/L})$$

$$\frac{1}{2} = 1 - e^{-Rt/L}$$

$\Rightarrow \quad e^{-Rt/L} = \frac{1}{2}$

$\Rightarrow \quad e^{Rt/L} = 2$

$\Rightarrow \quad \frac{Rt}{L} = \ln(2) = 0.693$

$\therefore \quad t = \frac{0.693 \times L}{R}$

$= \frac{0.693 \times (300 \times 10^{-3})}{2}$

$\simeq 0.1$ s

17. The maximum power is imparted at resonance, i.e. when

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \nu = \frac{1}{2\pi\sqrt{LC}}$$

which gives

$$C = \frac{1}{4\pi^2\nu^2 L} = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 10}$$

$\simeq 1 \times 10^{-6}$ F = 1 μF

18. Power factor = $\frac{R}{Z} = \frac{12}{15} = 0.8$

19. If an AC circuit contains only capacitance C or only inductance L or a series combination of resistance C and inductance L, the phase difference between the current and voltage is $\pi/2$. But if the circuit contains R and L, the phase difference between the current and voltage can have any value between 0 and $\pi/2$ depending on the values of L and R. Hence the correct choice is (b).

20. Induced emf is $E = -\frac{d\phi}{dt}$

$= -\frac{d}{dt}\ (10t^2 - 50t + 250)$

$= -20t + 50$

$\therefore E$ (at $t = 3$s) $= -20 \times 3 + 50 = -10$ V,

21. The resonant angular frequency is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

which gives $\quad L = \frac{1}{C\omega^2} = \frac{1}{(2 \times 10^{-6})(200)^2}$

$= \frac{100}{8} = 12.5$ H

At resonance, impedance $Z = R$. Therefore, the current in the circuit is

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{100\text{V}}{1000\Omega} = 0.1 \text{ A}$$

$\therefore$ Voltage across inductor $(VL) = IX_L = I\omega L$

$= 0.1 \times 200 \times 12.5$

$= 250$ V

22. The induced emf is given by

$$E = \omega NAB \sin \omega t$$

$\therefore \quad E_{max} = \omega NAB.$

23. The decay of current in an LR circuit is given by

$$I = \frac{E}{R}\ e^{-Rt/L}$$

Substituting the values of E, R, L and t we get

$$I = \frac{1}{e} \text{ ampere}$$

24. Phase difference between voltage E and current I is $\phi = \frac{\pi}{2}$. The time-averaged power consumption is given by

$<P> = I_{rms} E_{rms} \cos\phi = 0 \quad \left[\because \phi = \frac{\pi}{2}\right]$

25. $L = 10$ H, $R = 5\ \Omega$, $V = 5$ V, $t = 2$s

The steady state current is

$$I_0 = \frac{V}{R} = \frac{5\text{V}}{5\Omega} = 1\text{ A}$$

The current at time $t = 2$ s in the L–R circuit is

$$I = I_0(1 - e^{-Rt/L})$$
$$= 1 \times (1 - e^{-5\times 2/10})$$
$$= (1 - e^{-1}), \text{ which is choice (d)}$$

26. $A = 10\text{ cm}^2 = 10 \times 10^{-4}\text{ m}^2 = 10^{-3}\text{ m}^2$, $l = 20$ cm $= 0.2$ m

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$
$$= \frac{(4\pi \times 10^{-7}) \times 300 \times 400 \times 10^{-3}}{0.2} = 2.4\,\pi \times 10^{-4}\text{ H}$$

27. Time constant $\tau = \frac{L}{R_2} = \frac{400 \times 10^{-3}}{2} = 0.2$ s.

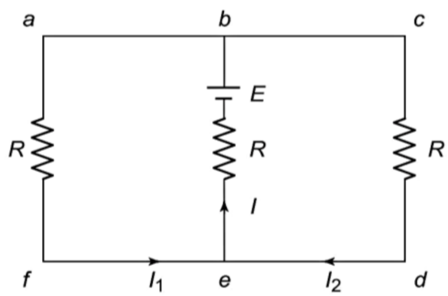
Current through the inductance is

$$I = \frac{E}{R_2}(1 - e^{-t/\tau})$$
$$= \frac{12}{2}(1 - e^{-t/0.2}) = 6(1 - e^{-5t})$$

$\therefore$ Potential drop across L is

$$e = -L\frac{dI}{dt} = -0.4 \times \frac{d}{dt}\left[6(1 - e^{-5t})\right]$$
$$= 0.4 \times 6 \times 5 \times e^{-5t} = 12e^{-5t}\text{ volt}$$

28. When a wire of length l moves with a velocity v perpendicular to a magnetic field B, an emf $E = Blv$ is induced between its ends. Hence the equivalent circuit is as shown in the figure. From Kirchhoff's junction rule,



$$I_1 + I_2 = I$$

Applying Kirchhoff's loop rule to loops *abefa* and *bcdeb* we have

$$E - IR - I_1 R = 0 \quad (1)$$

and $$I_2 R + IR - E = 0 \quad (2)$$

Adding (1) and (2), we get $I_1 = I_2$

Subtracting (1) and (2), we get

$$2E - 2IR - (I_1 + I_2)R = 0$$
$$\Rightarrow \quad 2E - 2IR - IR = 0$$
$$\Rightarrow \quad I = \frac{2E}{3R} = \frac{2Blv}{3R}$$
$$\therefore \quad I_1 = I_2 = \frac{I}{2} = \frac{Blv}{3R}$$

29. In a d.c. circuit consisting of L and R, the current I grows with time t as

$$I = I_0(1 - e^{-t/\tau})$$

where $\tau = \frac{L}{R}$ is the time constant and I_0 is the final steady current. At $t = 0$, $I = I_0(1 - 1) = 0$. Thus no current flows in the branch containing L and R_1 at $t = 0$ and the inductor behaves as open switch (infinite resistance). Hence at $t = 0$,

$$I = \frac{V}{R_2}$$

At $t = \infty$, $I = I_0(1 - 0) = I_0$ (= final steady current). Hence at $t = \infty$, the inductor behaves as a closed switch (zero resistance). So at $t = \infty$, the equivalent resistance of the circuit is $R = \frac{R_1 R_2}{R_1 + R_2}$ and the current is

$$I = \frac{V}{R} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

30. In a series LCR circuit the phase angle ϕ between voltage and current is given by

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

If capacitor is taken out, $\tan\phi = \frac{\omega L}{R} \Rightarrow \tan 30° = \frac{\omega L}{R}$. If inductor is taken out, $\tan\phi = -\frac{1}{\omega CR} \Rightarrow -\tan 30° = -\frac{1}{\omega CR}$. Hence

$$\frac{\omega L}{R} = \frac{1}{\omega CR}$$
$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

This is the resonance condition. At resonance, impedance $Z = R$. Hence

$$P = \frac{V^2}{R} = \frac{(220)^2}{200} = 242\text{ W}$$

31. Impedance $Z = \left[R^2 + \frac{1}{(\omega C)^2}\right]^{1\backslash 2}$, $I_0 = \frac{V}{Z}$. As ω increases, Z decreases. Hence the current I_0 increases.

Therefore, the bulb glows brighter, which is choice (b).

32. Let q be the charge on the capacitor when $U_e = U_m$ where $U_e = \dfrac{q^2}{2C}$ and $U_m = \dfrac{1}{2}Li^2$. Given $U_e = U_m$, i.e.

$$\frac{q^2}{2C} = \frac{1}{2}Li^2 = \frac{1}{2}L = \left(\frac{dq}{dt}\right)^2$$

$$\Rightarrow \quad \frac{dq}{dt} = \pm\frac{q}{\sqrt{LC}} \quad (1)$$

The angular frequency of an LC circuit is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

The charge oscillates between the capacitor plates simple harmonically. Since $q = q_0$ at $t = 0$,

$$q = q_0 \cos \omega t \quad (2)$$

$$\therefore \ \frac{dq}{dt} = -\omega q_0 \sin \omega t = -\frac{q_0}{\sqrt{LC}}\sin \omega t \quad (3)$$

Using (2) and (3) in (1) we get

$$-\frac{q_0}{\sqrt{LC}}\sin \omega t = +\frac{q_0 \cos \omega t}{\sqrt{LC}}$$

$$\Rightarrow \tan \omega t = +1$$

$$\omega t = +\frac{\pi}{4}$$

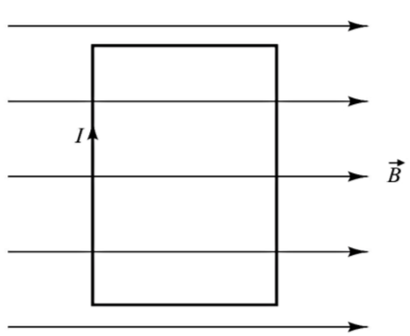
$$\Rightarrow \ \frac{t}{\sqrt{LC}} = +\frac{\pi}{4} = t = +\frac{\pi}{4}\sqrt{LC}.$$

33. $\varepsilon = B\ell v$

$= (5.0 \times 10^{-5}) \times 2 \times 1.5$

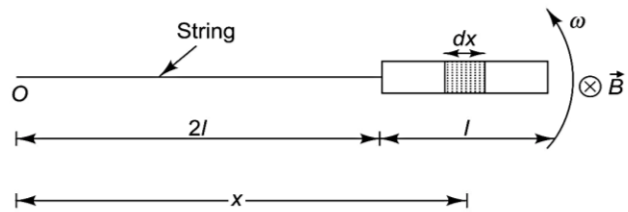
$= 15 \times 10^{-5}\text{ V} = 0.15\text{ mV}$

34. When a matel plate is placed near an oscillating coil, eddy currents are set up in the metal plate. From Lenz's law, these current will oppose the current in the coil. Hence the coil quickly stops. This phenomenon is called electromagnetic damping. So the correct shoice is (d).

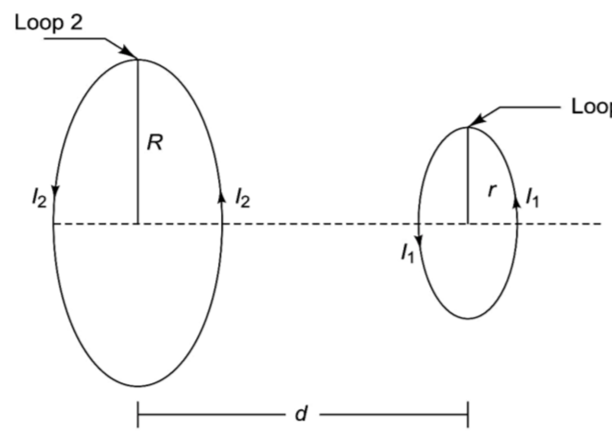


35. Divide the rod into a large number of very small elements each of length dx. Consider one such element at a distance x from end O where the string is fixed. Linear speed of the element is $v = \omega\, dx$. The magnitude of the emf induced in the element $= Bxv = B\omega\, xdx$. Therefore, emf induced between the ends of the rod is

$$\varepsilon = \int_{2\ell}^{3\ell} B\omega\, xdx = B\omega \int_{2\ell}^{3\ell} xdx = \frac{5B\omega\ell^2}{2}$$



36.



If M_{12} is the coefficient of mutual inductance between loops 1 and 2, the magnetic flux linked will loop 1 is

$$\phi_{12} = M_{12}I_2 = \frac{\mu_0 I_2 R^2}{2(d^2 + R^2)^{3/2}} \times \pi r^2$$

$$M_{12} = \frac{\mu_0 R^2}{2(d^2 + R^2)^{3/2}} \times \pi r^2$$

By symmetry, the magnetic flux linked will coil 2 is

$$\phi_2 = M_{21}I_1 = M_{12}\, I_1$$

$$= \frac{\mu_0 R^2 \times \pi r^2 I_1}{2(d^2 + R^2)^{3/2}} \quad (1)$$

Now $\mu_0 = 4\pi \times 10^{-7}\text{ TmA}^{-1}$, $R = 0.2\text{m}$, $r = 0.3 \times 10^{-2}$ m, $d = 0.15\text{m}$ and $I_1 = 2.0$ A. Substituting these values in (1), we get $\phi_2 = 9.1 \times 10^{-11}$ weber.

37. The phase difference between voltage E and current I is $\phi = \dfrac{\pi}{2}$. The time-averaged power consumption is $P = I_{\text{rms}}\, E_{\text{rms}} \cos \phi = 0$ $\quad \left(\because \ \cos\dfrac{\pi}{2} = 0\right)$