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PHYSICS PARTICLES MECHANICS

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Solids And fluid Mechanics

UNIT:VII

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A matter has three state namely solid, liquid & gas.

[The fourth namely fourth state of matter is **plasma** (ionized matter)].

- Each state of matter has some different properties like ---- (I) Solid --- has --- ● **Elastic properties**.
  (II) Liquid -- ● **Surface tension and Viscosity**.
  (III) gas --- ● **Pressure diffusion**.
- Each state of matter consists of atom /molecule holds together with a force called **inter atomic force/molecular force**.

Thus,

●● All the substance (solid, liquid or, gas) has one thing in common that all of them are made up of small separate particle, which must be extremely small and are called as molecules (small mass) (Diameter 2 – 5 A^0, where $A^0 = 10^{-10}$ m.)

Most of the substances that exist in the world can be classified into one of *three phases :* solid, liquid or gas. *These three phases of matter are governed by* **interatomic and intermolecular forces** *under ordinary conditions of temperature and pressure.* When acted on by outside forces, solids tend to keep their volume and shape, liquids tend to keep their volume but not their shape and gases tend to keep neither their volume nor their shape. To properly describe a substance, we should provide a description of motion of each atom of which it is composed. Such a description would be worthless for most purposes. It would be far too complicated and detailed for the everyday uses to which the materials are put. The engineer who wishes to use a certain type of steel in construction neither wants nor requires an atomic description of the material. We are generally interested in the overall properties of a material rather than in its atomic description. In this unit, we shall discuss some of the *mechanical properties* of matter *e.g.,* elasticity, fluid pressure etc.

The forces which bind two or more atoms together to make a molecule are called **interatomic forces**. When molecules are formed as a result of interatomic forces between the atoms, there must be some intermolecular forces (*i.e.*, forces between molecules) which bind the molecules together. The interatomic and intermolecular forces are electrical in nature, arising from interactions of the electrically charged fundamental particles that make up the atoms and molecules. The gravitational forces between atoms/molecules are so weak compared with electrical forces that they are completely negligible.

Consider two atoms or molecules exerting forces of attraction on each other as shown in Fig. 18.1. If the force F on A moves it a small distance Δr (so that F can be considered constant) to the right, then work done ΔW on A is given by ;

$$\Delta W = F\,\Delta r$$

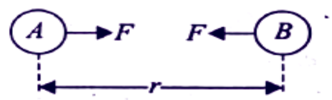


Fig. 18.1

If ΔU is the resulting change in the potential energy, then,

$$\Delta U = -\,\Delta W$$

The minus sign is present because the force is attractive and the potential energy decreases.

$\therefore$ $$\Delta U = -\,F\,\Delta r$$

$\therefore$ In the limit, $$F = -\frac{dU}{dr}$$

INTER ATOMIC FORCES

"The force which is responsible to hold together the atoms of element to form a molecule is called inter atomic force."

- The forces between atoms & molecule are not gravitational in nature but they are electrical in nature.

"The forces acting between the atom due to electrostatic interaction between the charges of the atoms are called inter atomic force."

- Inter atomic forces are active if the distance between the two atoms is of the order of atomic size = 10^{-10} m, i.e., 1 A^0.
- Size of molecules = 10^{-14} m.
- When atoms are large distance apart i.e., much more than the atomic size, no interaction takes place and therefore, interatomic force between them is zero.

1. Interatomic forces. An atom has positively charged nucleus and negatively charged electrons. The total positive charge of the nucleus is equal to the total negative charge of electrons. Therefore, an atom is electrically neutral. However, when two atoms are brought close together, there is electrical interaction between the electrons and the nuclei of the atoms. As a result, the force

between the atoms and the potential energy of the system of two atoms undergo change. Fig. 18.2 shows how the potential energy of two atoms and the force between them changes with their separation (r). *Note that interatomic forces may be attractive or repulsive depending upon the separation between the two atoms.* At large distances, the force is small and attractive and the potential energy of the system is negative. As the separation between the atoms decreases, the potential energy decreases (more negative) and the force of attraction becomes larger. The attractive force becomes maximum and then decreases to zero at an equilibrium separation r_0. At $r = r_0$, there is no net force between the atoms and their potential energy has its minimum value. When the separation is less than r_0, the force becomes repulsive and increases quite rapidly.

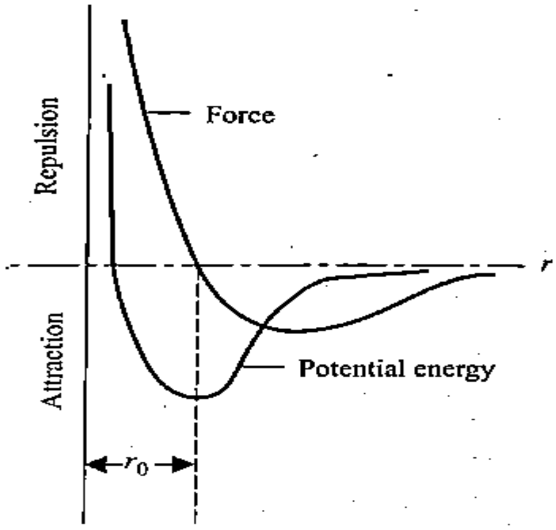


Fig. 18.2

If the two atoms have a separation of r_0, they are in their equilibrium separation. Any increase or decrease in their separation would require energy, since work would have to be done against the net attraction or the net repulsion respectively. The equilibrium is stable because an increase in r leads to an attractive force which restores r to r_0. Similarly, a decrease in r produces a repulsive force which again restores r to r_0. For NaCl, r_0 is of the order of 10^{-10} m — the same order as the distance from the centre of the atom to its outermost electrons. For small displacement from equilibrium, the force curve is linear and resembles the force exerted by a spring when compressed or extended. In solid state, NaCl exists in a regular array of Na^+ and Cl^- ions. When we try to deform the solid by pushing or pulling on it in a bid to change the separation distances between the ions, the strong forces between the ions resist compression or extension.

Characteristics of interatomic forces

(*i*) They are electrical in origin.

(*ii*) They are active over short distances.

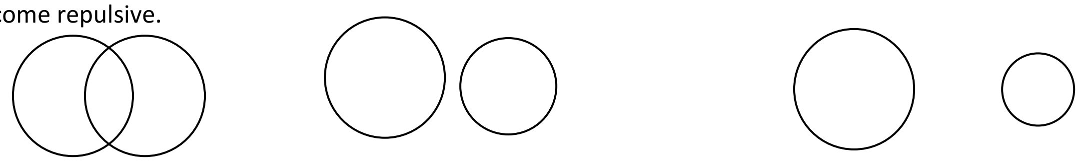
(*iii*) They are attractive upto a certain distance between the atoms and become repulsive if distance between atoms is less than this value.

(*iv*) For two atoms attracting each other, potential energy is negative.

(*v*) They are about 50 to 100 times stronger than intermolecular forces.

- When two atoms are brought closer to each other to a distance comparable with atomic size then the positive & negative charges of atoms will starts interacting with each other thereby increase electrostatic force & decreasing potential energy .

- When the distance of two atoms is very small, then the electron cloud of two atoms starts overlapping and the interatomic forces become repulsive.



over lapping of electrons cloud | Inter atomic force increases | inter atomic force are zero.

During interaction between the two atoms ----

- Electrostatic force becomes Attractive between the nucleus of one atom & electros of the other and tends to decrease the potential energy of the pair of atoms.

- Elect. force becomes repulsive between the nucleus of one atom with the nucleus of other atom & electrons of one atom with the electron.

- **The inter atomic force is equal to the negative gradient of the corresponding potential energy function.**

$$F = -\frac{dU(r)}{dr}$$

When potential energy U(r) is minimum, i.e., maximum negative, the interatomic force is Zero. (It is called state of stable equilibrium)

FORCE

"The force between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces".

➤ Inter molecular forces are active if the separation between two molecules is of order of 10^{-9} to 10^{-10} m.

The potential energy–separation and force – separation graphs for intermolecular forces are similar to those for interatomic forces. In other words, the force between two molecules changes from an attraction when the separation is large to a repulsion when the separation is small. At an equilibrium separation r_0, the intermolecular force becomes zero and the potential energy of the molecules is minimum (maximum negative). It may be noted that intermolecular forces are much weaker than interatomic forces ; say 100 times. One reason for small intermolecular attraction is that a molecule attracts many more molecules around it whereas interatomic attraction are not affected by other atoms and molecules.

In view of the attractive molecular forces, why do not all molecules eventually coalesce into matter in the liquid or solid phase? It is because the molecules are always in *motion* and have kinetic energy associated with this motion. In general, the kinetic energy increases with temperature. At very low temperatures, the average kinetic energy of a molecule may be much *less* than the maximum magnitude of potential energy—the depth of the potential well in Fig. 18.2. The molecules then condense into a liquid or solid phase with average intermolecular spacing of about r_0. But at higher temperatures, the average kinetic energy becomes larger than the depth of the potential well. The molecules can then escape the intermolecular forces and become free to move independently as in the gaseous phase of matter.

Characteristics of intermolecular forces

(*i*) They are electrical in nature.
(*ii*) They are active over short distances.
(*iii*) They are attractive upto certain distance between the molecules and become repulsive if the distance between the molecules is less than this value.
(*iv*) For two molecules attracting each other, potential energy is negative.
(*v*) They are much weaker than interatomic forces.

POLAR & NON – POLAR MOLECULES : (ELECTRIC DIPOLE AND INDUCED ELECTIC DIPOLE).

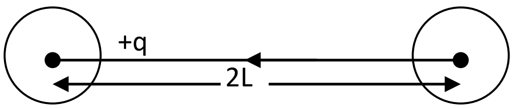
A molecule of a substance has equal no. of positive charge and negative charge and therefore is **neutral.**

➤ The negative & positive charges in a molecule may or may not have uniform distribution.

Case I: -- (non- uniform charge distribution)

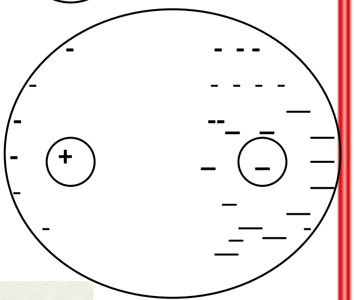
If the charge distribution is not uniform, then the centre of mass of positive charge may not coincide with the centre of mass of negative charge. Then these particular molecules **constitute 'electric dipole'.**

"A pair of equal and opposite charges separated by some distance is called an electric dipole".



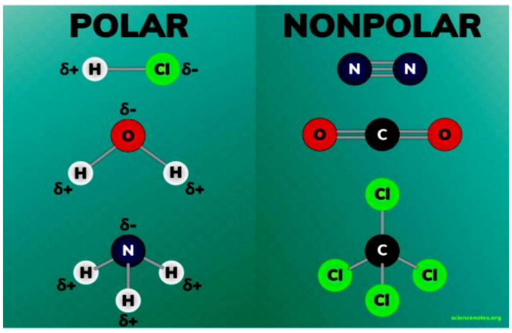
dipole moment $\vec{P} = q \times 2\vec{L}$

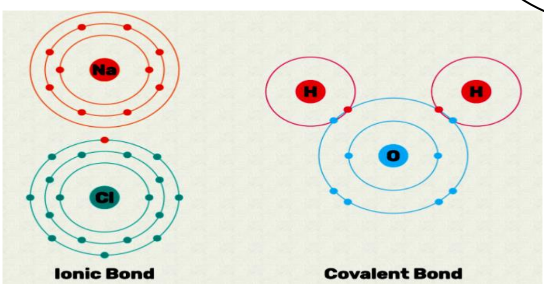
Direction --- **from -q to +q.**



∴ The molecule constituting electric dipole is called **POLAR – MOLECULES.**

Example = H_2O, CH_3OH, NH_3





Case (uniform charge distribution)

Those molecules in which the centre of mass of positive charge & negative charges coincides and they do-not give rise to any electric dipole are called non – polar molecules.

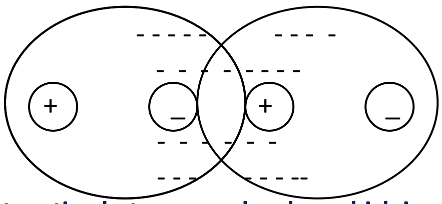
Example = O_2, CO_2, N_2.

- An Isolated non – polar molecule does not behave like an electric dipole.

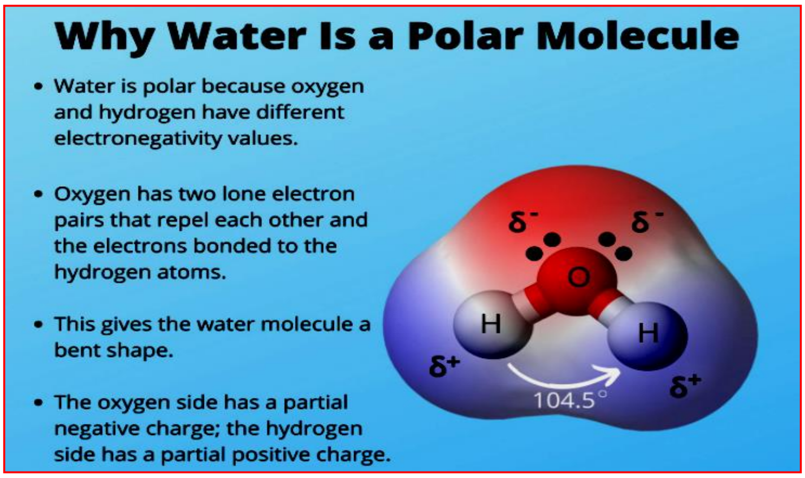
Special Case (Two Non – polar molecules come closer)

When two non – polar molecules come closer to each other, their uniform charge distribution is affected due to repulsion between like charge & also attraction between unlike charges. Due to this CM of positive & negative charges shifted. And therefore, the two non- polar molecules give rise to electric dipoles and are known as induced electric dipole.

- In case of two induced electric dipole, the CM of opposite charges face each other. Due to this the molecules are attracted towards each other. As molecules come close, their electron cloud starts overlapping.



Due to this a repulsive force starts acting between molecules, which increases rapidly with decrease in distance. It is due to this reason that the molecules do not collapse but behaves as hard sphere.



- **Variation of interatomic force with distance**: --

It has been found that the interatomic force of attraction is inversely proportional to the seventh proportional to the seventh power of the distance between the atoms.

i.e., $$f_a = \frac{-A}{r^7} \quad \rightarrow \text{Attractive}$$

Where, A = Constant & depends upon the structure of the molecule.

As the distance between the molecule is decreased and becomes equal to the order of the dimension of the molecule, they begins to repel each other. The force of repulsion between molecules between molecules changes rapidly then the attractive force.

$$\therefore \quad f_r = \frac{B}{r^9}$$

$\therefore$ **Net force acting on a molecule is** $$F_{net} = \frac{A}{r^7} + \frac{B}{r^9}$$

There is a definite distance 'r_o' (normal or equilibrium distance) between the molecule where the force of attraction (f_a) and force of repulsion (f_r) balances each other.

$$\therefore \quad F_{net} = 0$$

i.e., $$\frac{A}{r_o^7} = \frac{B}{r_o^9}$$

$$\therefore \quad r_o = \sqrt{\frac{B}{A}}$$

At this stage, the molecules are in a state of stable equilibrium.
As the intermolecular separation 'r' is decreased i.e., $r < r_o$, the repulsion component dominates the attractive component of the force. Hence the net force is repulsive in nature.

STATES OF MATTER

oils are liquids. Hydrogen, oxygen and air are gases at room temperature. The fundamental difference between these three states of matter lies in the way the forces act between their atoms or molecules. In other words, under ordinary conditions of temperature and pressure, the interatomic and intermolecular forces decide the state of matter.

(*i*) Gas. *In gases, the interatomic/intermolecular forces are practically nonexistent, allowing the atoms or molecules to move independently* except when they collide with another molecule or the walls of the container. The average separation between the molecules of a gas is large compared with their own size. Consequently, we can compress or expand a gas to fill a container of any shape or size. For this reason, a gas has neither a definite shape nor a definite volume.

(*ii*) Liquid. *In a liquid, the average separation between the molecules is comparable to their own diameter.* Individual molecules are free to move about because of the forces between them, they move so that the average separation between neighbours remains constant. As a result, a liquid is virtually incompressible and has a definite volume, although its shape can change to match the shape of its container.

(*iii*) Solid. In solids, the separations between molecules are comparable to the separations between the molecules of liquids. *However, the intermolecular forces are so strong that atoms in a solid are not free to move about.* Instead, the atoms of a solid are confined to small oscillations about fixed positions. Thus a solid has not only a definite volume but a definite shape as well.

The state in which a particular substance exists depends upon the temperature of the substance and on the external pressure surrounding it. As an example, water can have the form of ice (solid), water (liquid) or steam (gas). The change from one state to another, such as the melting of ice is usually caused by a transfer of heat.

STATES OF MATTER

IN SOLIDS --
- (1) The constituents of a substance are closely packed in a definite order.
- (2) Intermolecular space is very small, therefore solid cannot be compressed.
- (3) The constituents of solid can vibrate about their mean position but cannot move from one place to another.
- (4) Strong intermolecular force.
- (5) All the solids have the property of ***Elasticity*** (property of the body to regain its original configuration, when the deforming forces are removed).
- (6) Minimum K.E

➲ When a liquid is cooled, the KE of its molecules falls. The molecule come close to each other due to increase in intermolecular force and also in order to have a system of minimum PE. As the process continue a stage will come where PE reduces to minimum and 'r' becomes 'r_o'. At this stage, the molecules of the liquid arrange themselves symmetrically w.r.t each other and are confined to fixed position and the liquid is converted to solid.

➲ Due to thermal energy, each molecule moves about a fixed mean position with very small amplitude.

➲ Intermolecular force dominates over the thermal motion of molecules in solids.

18.3. CLASSIFICATION OF SOLIDS

In a liquid or gas, atoms and/or molecules are relatively free and can move around at will. However, in solids, they almost completely lack mobility ; all they can do is to vibrate about a mean position when energy is added to the solid. It is due to this lack of mobility that solids have definite shape and volume. The way in which a solid behaves depends upon its internal structure. Accordingly, solids can be divided into two categories *viz.*

(*i*) Crystalline solids (*ii*) Amorphous solids (*i*) **Crystalline solids.** *Solids whose constituent particles (atoms or ions or molecules) are arranged in an orderly fashion throughout in a three-dimensional pattern are called* **crystalline solids** *e.g.,* NaCl, KCl, sugar, diamond etc.

Most of solids (including metals) have crystalline structure. In a crystalline solid, the constituent particles are arranged in an orderly manner in a three-dimensional pattern. For example, NaCl has cubic structure as shown in Fig. 18.3. The positive sodium ion Na^+ (white circle) and negative chlorine ion Cl^- (black circle) occupy alternate positions in the cubic structure. It should be noted that *only ions* are present at the positions shown whereas the lines joining them are hypothetical. These lines have been drawn merely to show that NaCl has a cubic structure. It may be seen that there is regular arrangement of ions which is repeated in three dimensions. Thus the crystalline solids have a long-range order. *An important property of crystalline solid is that it has a sharp melting point e.g.,* copper has a melting point of 1084°C and aluminium has a melting point of 655°C. This is a very important property of crystalline solids. In fact, the existence of a sharp melting point of a solid, no matter what its appearance is, should lead to a strong suspicion of the crystalline internal structure.

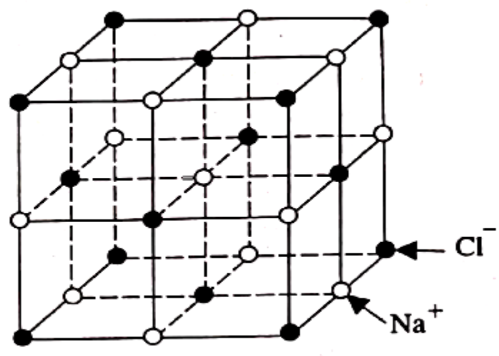
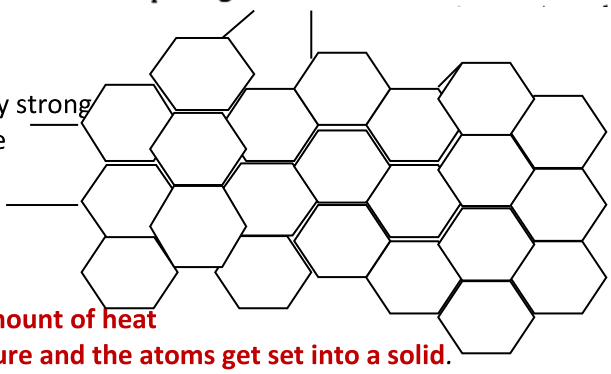


Fig. 18.3

Notes. (*a*) The manner in which the constituent particles (atoms, ions or molecules) are arranged in crystals has a marked bearing on the physical properties of the crystalline solids. For example, both graphite and diamond are crystalline forms of carbon. But their structures are different so that they have quite different physical properties.

(*b*) When constituent particles are arranged in a regular fashion, their total potential energy is less than it would be if they are packed irregularly. Therefore, it is not surprising that most solids are crystalline.

CHARECTERISTICS



- Definite external geometrical shape.
- All the bonds in ions or atoms or molecules are equally strong
- Sharp melting point, as on heating, all the gets rapture at once and at same temperature.
- Consider as true and stable solid.
  - **CHARECTERISTIC---*Melting point***

When a liquid crystallizes into a solid, a definite amount of heat energy is released in solidification at fixed temperature and the atoms get set into a solid.

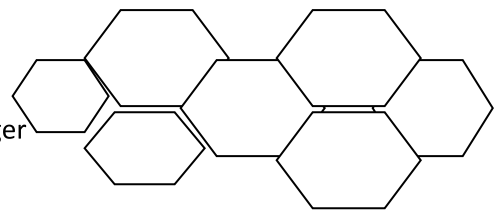
☞ EXAMPLES: -- Mica, sugar, $CuSO_4$, KI, Ca Cl_2, diamond, Rock salt etc.

(*ii*) **Amorphous solids.** *Solids whose constituent particles are not arranged in an orderly fashion are called* **amorphous solids** *e.g.,* glass, wax etc.

Although amorphous solids possess many of the properties of solids like hardness, definite shape etc., their constituent particles are not arranged in an orderly fashion as in crystalline solids. Even if an orderly arrangement exists in some amorphous solids, it extends only upto a few Å. However, a crystalline solid has well-defined geometrical pattern. *An important property of an amorphous solid is that it does not have a definite melting point.*

Note. Although amorphous solids exhibit the properties of solids, they can be regarded as liquids with very high viscosities. For example, a glass window which is in place for a few hundred years may become thicker at the bottom as a result of flow under the moderate influence of its own weight.

CHARACTERISTICS



- Not regular external shape.
- All the bonds are not equally strong.
- Do not have sharp melting point as weakest bond rapture first of all at lower temperature and stronger bonds are rapture at higher temperature.
- Consider as super cooled liquid of **high *Viscosity.***

☞ EXAMPLES: -- Talcum powder, glass, rubber, plastics, cement, paraffin wax etc

.4. DIFFERENCE BETWEEN CRYSTALLINE AND AMORPHOUS SOLIDS

S.No.	Particular	Crystalline solids	Amorphous solids
1.	Characteristic geometry	Definite and regular geometrical shapes which extend throughout.	They do not have definite geometrical shapes.
2.	Range	Long-range order of bonds.	Short-range order.
3.	Melting point	They have sharp melting point.	They do not have sharp melting point.
4.	Stability	They are more stable.	They are less stable.
5.	Directional property	They are *anisotropic i.e.*, their physical properties (e.g., refractive index, electric conductivity, thermal conductivity etc.) are different in different directions.	They are *isotropic i.e.*, their physical properties are same in all directions.

ELASTIC BEHAVIOR OF SOLIDS

So far we have assumed that the rigid bodies or solids remain undeformed when external forces act on them. In actual practice, all bodies are deformable. In other words, it is possible to change the size or shape or both of a body by the application of external forces. This deformation may not be visible to the naked eye in some cases.

Illustration. Consider a steel wire (solid) fixed to a rigid support at one end and supporting a small mass m at the other end [See Fig. 18.4]. The steel wire is under stress due to the weight of mass m. The stress pulls the molecules of the steel wire further apart, thus slightly increasing the length of the wire. When the stress is removed (*i.e.*, mass m is removed), the molecular forces restore the molecules of the steel wire to their normal spacing. Consequently, the wire regains its original length. This tendency of a solid to regain shape and size after the removal of deforming force is called its *elastic property*. The elastic property of a solid determines its behaviour under the action of deforming forces. On the removal of the deforming force, a solid body will regain its original state (*i.e.*, shape and size) if the deforming force does not exceed a certain limit called *elastic limit*. In the above case, for example, if a sufficiently large load is suspended from the wire, it is found that the wire does not regain its original length after the removal of the load. The wire is said to have permanent deformation. Elastic limit differs widely for different solid materials. It is high for a material like steel and low for a material like lead.

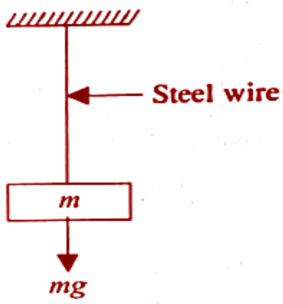


Fig. 18.4

Note. In the above case, if elastic limit is exceeded, the length of the wire will continue to increase till at a particular load, it will finally break.

ELASTICITY

The property of a body by virtue of which it tends to regain its original shape and size when deforming force is removed is called **elasticity**.

All solids show the property of elasticity. The greater the elasticity of a body, the greater is its tendency to regain its original shape and size after the removal of the deforming force/forces. The elastic property varies from solid to solid. For example, steel is very elastic. It means that steel closely returns to its original dimensions even after being subjected to relatively large forces. From this point of view, rubber is not highly elastic, even though it may be readily stretched.

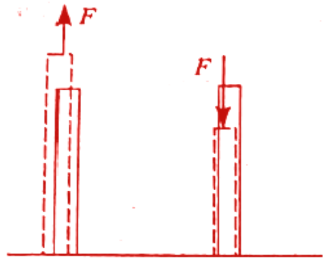
(i) **Perfectly elastic body.** *A body which regains its exact original shape and size immediately after the removal of the deforming force is called a perfectly elastic body.* Quartz and phosphor bronze are the examples of nearly perfectly elastic bodies.

No body in nature is perfectly elastic. In other words, a deforming force does produce permanent effect (even if elastic limit is not exceeded) in shape or size in every body to some or more extent. In certain cases, it may not be visible to the naked eye.

(ii) **Perfectly inelastic body.** *A body which does not have any tendency at all to regain its original shape and size on the removal of the slightest deforming force is called a perfectly inelastic or plastic body.* Wet clay and paraffin wax are the examples of nearly perfect plastic bodies. When wet clay is deformed by a force, it remains deformed. Again, no body in nature is perfectly plastic or inelastic body.

TYPES OF SOLID DEFORMATIONS

Although a solid can be deformed in many ways, there are only three basic types of deformations. All others are combinations of these three types. Fig. 18.5 shows the three basic types of deformations in solids.



➤➤➤ In fig. –(i), **Elasticity of Expansion and Compression**

The deforming forces causes a change **in Length** of the object.

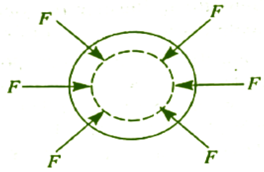
On removal of Deforming force, the object regains its original Length.

➤➤➤ In fig –(ii), **Elasticity of Volume / Bulk**

The deforming forces causes a change **in Volume** of the object.

Since the rubber ball is pressed from all sides, its volume decreases.

On removal of Pressure, the ball regains its original Volume.

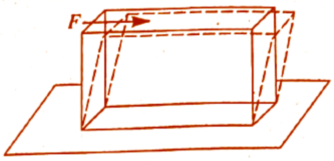


Change in volume

➤➤➤ In fig –(iii), **Elasticity of RIGIDITY/ SHAPE**

The deforming forces PRODUCES a change **in SHAPE** of the body.

The box is made of cardboard fixed at lower face. A tangential Force F is applied at the upper surface. This causes the consecutive horizontal Layers of the box to be slightly displaced or sheared relative to one body, Only the shape of the body changes.



Shear

When the deforming force is removed, the body regains its original shape.

In each of the above cases, the deforming force changes the distances between the molecules of the body, either pulling them further apart or pushing them close together. When the deforming force is removed, the molecular forces of the body restore the molecules to their normal spacing. Consequently, the body regains its original state.

Note. Other types of deformation are either a special example of one of these deformations or a combination of two or three of them. For example, bending of a beam is a combination of a stretch and a compression. The outer surface is stretched and the inner surface is compressed.

STRAIN

The change in size or shape of a body due to the deforming force/forces is called **strain.** We say that the body is strained due to stress. The strain is measured by the ratio of change in dimension to the original dimension *i.e.*,

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Since strain is a ratio of two similar quantities, it is a pure number and has no unit or dimensions.

Types of strain. Since deforming forces can produce three types of deformations (*i.e.*, change in length or volume or shape) in a body, there are three types of strain.

(*i*) **Longitudinal strain.** *When the deforming force produces change in length alone, the strain produced is called longitudinal strain.* It is measured by the ratio of change in length of the body to its original length. Thus if L is the original length of the body and ΔL is the change in length caused by stress, then,

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}$$

(*ii*) **Volumetric strain.** *When the deforming force produces change in volume, the strain is called volumetric strain.* It is measured by the ratio of change in the volume of the body to its original volume. Thus referring to Fig. 18.6, if V is the original volume of the spherical ball and ΔV is the change in volume caused by stress, then,

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

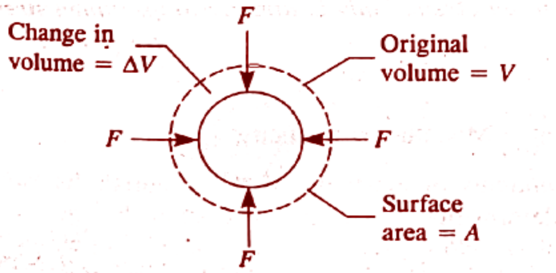


Fig. 18.6

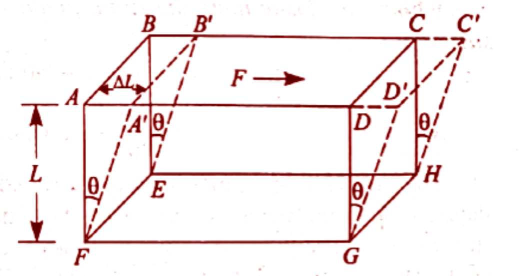


Fig. 18.7

Shearing strain

which a line originally perpendicular to the fixed face is turned due to the application of the tangential force. Consider a reactangular block fixed at its lower face *EFGH* as shown in Fig. 18.7. If a tangential force *F* is applied on its upper face *ABCD* in the direction shown, then each layer of the block parallel to face *EFGH* will be displaced laterally in the direction of force. Note that the four vertical sides have been displaced through angle θ so that there is only change in the shape of the body. The face *ABCD* is said to be sheared through an angle θ.

$$\text{Shearing strain} = \theta = \tan\theta = \frac{AA'}{AF} = \frac{\Delta L}{L}$$

Although the expression looks the same as for longitudinal strain, you should understand that the two strains refer to different physical characteristics.

ELASTIC LIMIT

On the removal of the deforming force, an elastic body will regain its original state only if the deforming force does not exceed a certain limit called elastic limit. For example, if a sufficiently large load is suspended from a copper wire, it is found that the wire does not regain its original length after removal of the load. The wire is said to have permanent deformation. This is true of volumetric or shearing strain also.

The maximum stress from which an elastic body will recover its original state after the removal of the deforming force is called **elastic limit**.

Every solid has a certain range through which it may be deformed and yet return to its original condition, before its elastic limit is reached. Elastic limit differs widely for different materials. It is very high for a substance like steel and low for a substance like lead.

HOOKE'S LAW AND MODULUS OF ELASTICITY

Robert Hooke studied the elastic behaviour of such objects as coiled springs, metal rods, metallic wires etc. He summed up his findings in a rule now known as **Hooke's law**. It is stated belows :

Provided the elastic limit is not exceeded, the extension (x) produced in a wire is directly proportional to the applied force (F) i.e.,

$$\text{Force} \propto \text{Extension}$$

or $$F \propto x \quad \text{... within elastic limit}$$

For example, if a force of magnitude *F* causes an extension *x* in the wire, then a force of 2*F* will cause an extension of 2 *x* in the wire.

Now stress is a measure of the deforming force and strain is a measure of the distortion.

Within the elastic limit, the strain produced in an elastic body is directly proportional to the stress i.e.,

$$\text{Stress} \propto \text{strain}$$

or $$\frac{\text{Stress}}{\text{Strain}} = \text{Constant} = \text{Modulus of elasticity}$$

The constant of proportionality is called modulus of elasticity of the material. Its value depends upon the material and on the type of deformation involved. Since strain has no units, modulus of elasticity has the same units as of stress *i.e.*, N/m^2. Note that if the modulus of elasticity of a material is large, it means a large stress will produce only a small strain. *Thus the greater the modulus of elasticity of a material body, the harder it is to change its size or shape and vice-versa.*

➤ Dimension -- [$ML^{-1}T^{-2}$]

- Modulus of elasticity is independent of magnitude of stress & strain as it is a constant .
- Modulus of elasticity depends on the nature of the material of the body .

Types of Modulus of Elasticity

Corresponding to three types of strains, there are three moduli of elasticity

(*i*) Young's modulus of elasticity, $Y = \dfrac{\text{Normal stress}}{\text{Longitudinal strain}}$

(*ii*) Bulk modulus of elasticity, $K = \dfrac{\text{Normal stress}}{\text{Volumetric strain}}$

(*iii*) Modulus of rigidity, $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

(I) Young's Modulus of Elasticity (Y)

It is defined as the ratio of normal stress to the longitudinal strain. It is denoted by the symbol *Y*.

$$\text{Young's modulus, } Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

Consider a wire of length *L* and area of X-section *A* fixed at one end to a rigid support as shown in Fig. 18.8. If a normal force *F* is applied to the free end, the length of the wire will change. Suppose the length increases by ΔL. Then,

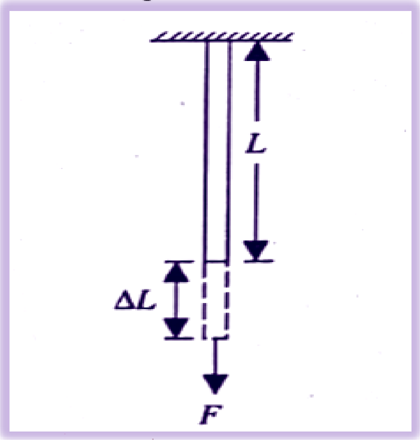
$$\text{Normal stress} = \frac{F}{A}$$

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\therefore \quad Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L}$$

If r is the radius of the wire, then $A = \pi r^2$.

$$\therefore \quad Y = \frac{FL}{\pi r^2 \Delta L}$$



S.No.	Material	Y(N/m^2)
1.	Aluminium	7.1×10^{10}
2.	Steel	20×10^{10}
3.	Glass	7×10^{10}
4.	Brass	7×10^{10}
5.	Rubber	1.0×10^{6}

steel is 10^5 times harder to stretch than the rubber.

➤ Unit - .... N/m^2 (Pascal) - (SI)
...........dyn/cm^2 –(cgs)

➤ Dimension – [$ML^{-1}T^{-2}$]

STRESS-STRAIN CURVE FOR AN ELASTIC MATERIAL

Suppose a metallic wire of uniform cross-section is suspended from a rigid support. When the load on the other end is gradually increased, the length of the wire goes on increasing. If graph between stress and strain is plotted, the shape of the curve will be as shown in Fig. 18.9.

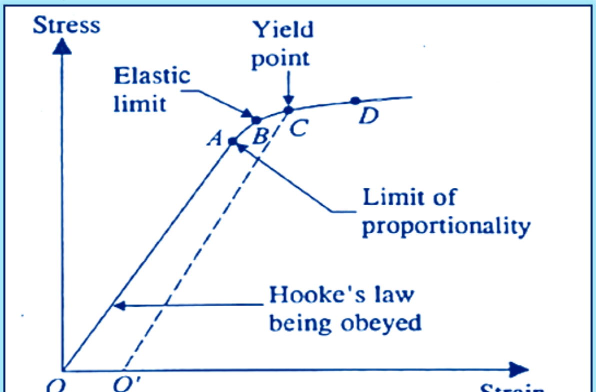


Fig. 18.9

(i) **Portion *OA*.** The portion *OA* of the graph is a straight line showing that upto point *A*, strain produced in the wire is directly proportional to the stress *i.e.*, strain ∝ stress. In this portion, the material of the wire obeys Hooke's law. The point *A* is called the **limit of proportionality**. The proportionality limit is the greatest stress a material can sustain without the departure from a linear stress-strain relation. If the applied force is removed at any point between *O* and *A*, the wire regains its original length.

(ii) **Portion *AB*.** The portion *AB* of the graph is not a straight line showing that in this region, strain is not proportional to the stress. Note that the slope of the graph is decreased ; this means that strain increases more rapidly with stress. Nevertheless, if load is removed at any point between *O* and *B*, the wire will return to its original length. The point *B* is called the **elastic limit**. *The elastic limit is the maximum stress which a body can sustain and still regain its original size and shape once the load has been removed.*

(iii) **Portion *BC*.** If the stress is increased beyond the elastic limit, a point *C* is reached at which there is marked increase in extension. This point is called **yield point**. Between *B* and *C*, the material becomes **plastic** *i.e.* if the wire is unloaded at any point between *B* and *C*, the wire does not quite come back to its original length. *The extension not recoverable after removing the load is known as* **permanent set**. However, this permanent deformation is not serious enough to be important. In practice, we must keep the stress below the yield point. Here *OO'* is the permanent set.

(iv) **Portion *CD*.** If the stress is increased beyond point *C*, the wire lengthens rapidly until we reach point *D* at the top of the curve. The point *D* is called the **utlimate strength** or **breaking stress**. Beyond point *D*, even a stress smaller than at *C* may continue to stretch the wire until it breaks.

Note. Elastic limit and limit of proportionality are very close to each other so that Hooke's law is nearly applicable upto elastic limit.

STRESS–STRAIN CURVE FOR RUBBER

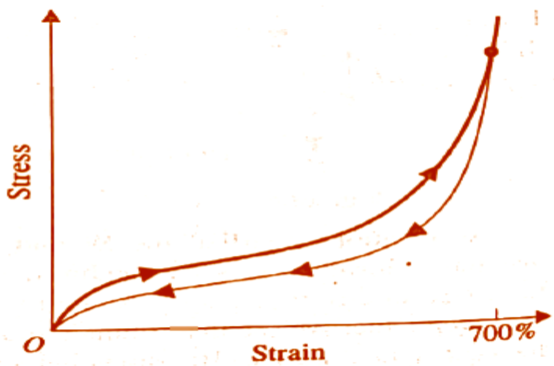


Fig. 18.10

Fig. 18.10 shows a stress-strain curve for a typical sample of vulcanised rubber that has been stretched to over seven times its original length. During *no* portion of this curve is the stress proportional to the strain *i.e.*, stress-strain curve for rubber does not obey Hooke's law. However, the substance is elastic in the sense that when the load is removed, the rubber is restored to its original length. On decreasing the load, the stress-strain curve is *not* retraced but follows the thin curve as shown in Fig. 18.10. *The lack of coincidence of the curves for increasing and decreasing stress is known as elastic hysteresis.*

When stress- strain relation exhibits this behavior, the associated forces are not **CONSERVATIVE.** Since the work done By the material in returning to its original length is less than the work required to deform it. The area bounded by the two curves (i.e., the area of hysteresis LOOP) is proportional to the energy dissipated within the material.

The larger Elastic hysteresis of some kinds of rubber makes these materials very valuable as 'vibration absorber'.

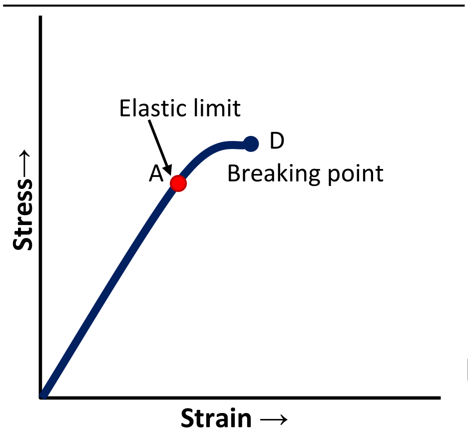
Explanation: If a block of such material is placed between a piece of machinery and the floor, elastic hysteresis Takes place during every cycle of vibration. Mechanical energy is converted to a form known as Internal energy which evidences itself by a rise in temperature. As a result, only a small amount of Vibration is transferred to the floor. Ex; Engine mounting of a car.

ELASTOMERS. The materials which can be elastically stretched to large values of strain are called elastomers. For example, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit. Its Young's modulus is very small, about 3×10^5 Nm^{-2} at slow strains. Elastic region in such cases is very large, but the material does not obey Hooke's law. In our

CLASSIFICATION OF MATERIALS ON THE BASIS OF STRESS-STRAIN CURVE

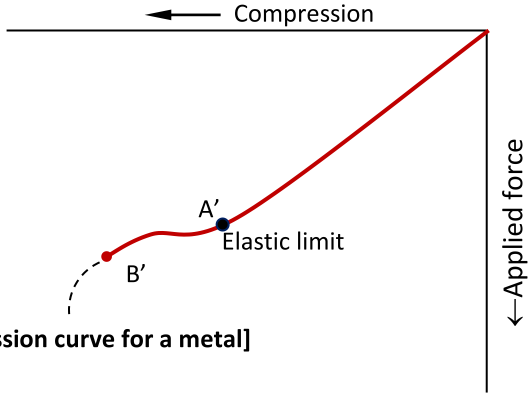
(i) **Ductile materials:** The materials which have large plastic range of extension are called ductile materials. As shown in the stress-strain curve of Fig. Their fracture point is widely separated from the elastic limit. Such materials undergo an irreversible increase in length before snapping. So, they can be drawn into thin wires. For example, copper, silver, iron, aluminum, etc.

(ii) **Brittle materials:** The materials which have very small range of plastic extension are called brittle materials. Such materials break as soon as the stress is increased beyond the elastic limit. Their breaking point lies just close to their elastic limit, as shown in Fig.
For example, cast iron, glass, ceramics, etc.



[Stress-strain curve for a brittle material]

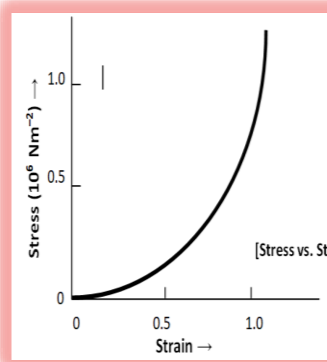
Malleability: When a solid is compressed, a stage is reached beyond which it cannot recover its original shape after the deforming force is removed. This is the elastic limit (point A') for compression. The solid then behaves like a plastic body. The yield point (B') obtained under compression is called crushing point. After this state, metals are said to be malleable i.e., they can be hammered or rolled into thin sheets. For example, gold, silver, lead, etc.



[Load-compression curve for a metal]

ELASTOMERS. **The materials which can be elastically stretched to large values of strain are called elastomers**. For example, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well-defined plastic region, rubber just breaks when pulled beyond a certain limit. Its Young's modulus is very small, about 3×10^5 Nm^{-2} at slow strains. Elastic region in such cases is very large, but the material does not obey Hooke's law. In our body, the elastic tissue of aorta (the large blood vessel carrying blood from the heart) is an elastomer, for which the stress-strain curve is shown in Fig.

Such materials can be stretched considerably and still return to their original lengths when the stresses are removed. For example, rubber can be stretched as much as 10 times its original length before elastic limit is exceeded. On the other hand, a metallic wire can be subjected to only about 1/10,000 of this extension before its elastic limit is exceeded. Fig. 18.10 shows the stress-strain curve for an elastomer (rubber).



[Stress vs. Strain curve for the elastic tissue of aorta]

Breaking force = Breaking stress × Cross-sectional area of wire.

that breaking stress has a fixed value for a material.

Elastic Fatigue

"Elastic fatigue is the property of an elastic body by virtue of which its behaviour comes less elastic under the repeated application of the deforming force."

If a material is subjected to repeated stresses over long periods of time, it becomes weaker *i.e.* the strain produced by a given amount of stress increases.

The loss of strength of the material because of repeated stresses is known as **fatigue.**

Each time the stress is applied, the internal structure of the material is changed. Each time the stress is removed, there is slight permanent change in the structure of the material. As this process continues, some portions of the material are weakened and the material may fracture. The failure of a material under these circumstances is called **fatigue failure** or **fatigue fracture.** It has been estimated that about 90% of the failures which occur in an aircraft components are due to fatigue.

CREEP

The gradual increase in strain which occurs when a material is subjected to stress for a long period of time is known as **creep.**

Unlike fatigue, it occurs even when the stress is constant. It is most marked at elevated temperatures and may be so severe that the material eventually fractures. The greater the stress, the more rapidly it happens. The turbine blades of jet engines are likely to fracture because of creep since they are under high stress and are at high temperatures. Soft metals (*e.g.* lead) and most plastics show considerable creep even at room temperature.

EXAMPLES BASED ON YOUNG'S MODULUS

- **Formulae Used**

1. Stress = $\frac{\text{Force}}{\text{Area}} = \frac{F}{A}$
2. Longitudinal strain = $\frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$
3. Young's modulus = $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$ or $Y = \frac{F/A}{\Delta l/l} = \frac{F}{A}\frac{l}{\Delta l}$
4. Percentage increase in length, $\frac{\Delta l}{l} \times 100 = \frac{F}{AY} \times 100$

- **Units Used**

Force F is in newton, area A in m^2, stress in Nm^{-2}, Young's modulus Y in Nm^{-2} or Pa, strain $\Delta l/l$ has no units

Q. 1. *A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?*

Sol. Maximum stress = $\frac{\text{Maximum load}}{\text{Area of cross-section}} = \frac{\text{Maximum load}}{\pi r^2}$

$\therefore$ Maximum load = $\pi r^2 \times$ Maximum stress = $3.142 \times (1.5 \times 10^{-2})^2 \times 10^8$ N = 7.07×10^4 N.

Q. 2. *The length of a suspended wire increases by 10^{-4} of its original length when a stress of 10^7 Nm^{-2} is applied on it. Calculate the young's modulus of the material of the wire.*

Sol. Strain = $\frac{\Delta l}{L} = 10^{-4}$, stress = 10^7 Nm^{-2}

Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{10^7 \text{ Nm}^{-2}}{10^{-4}} = 10^{11}$ Nm^{-2}

Q. 3. *A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44.500 N force, producing only elastic deformation. Calculate the resulting strain.*

Sol. Here F = 44, 500 N, A = 15.2 mm × 19.1 mm = $15.2 \times 19.1 \times 10^{-6}$ m^2

For copper, Y = 1.2×10^{11} Nm^{-2}

$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A \times \text{Strain}}$

$\therefore$ Strain = $\frac{F}{AY} = \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 1.2 \times 10^{11}}$ = 0.001277

Q. 4. *A uniform wire of steel of length 2.5 m and density 8.0 g cm^{-3} weighs 50 g. When stretched by a force of 10 kgf, the length increases by 2 mm. Calculate Young's modulus of steel.*

Sol. Here l = 2.5 m = 250 cm, Δl = 2 mm = 0.2 cm, F = 10 kg f = 10 × 9.8 N = $10 \times 9.8 \times 10^5$ dynes

Mass = Volume × density = A × l × ρ

$\therefore$ $A = \frac{\text{Mass}}{l \times \rho} = \frac{50}{250 \times 8} = 0.025$ cm^2

Young's modulus, $Y = \frac{F}{A}\frac{l}{\Delta l} = \frac{10 \times 9.8 \times 10^5 \times 250}{0.025 \times 0.2} = 4.9 \times 10^{11}$ dyne cm^{-2}

Q. 5. *A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the young's modulus, Y, of the structural steel is 2.0×10^{11} Nm^{-2}.*

Sol. Here $r = 10\ mm = 0.01\ m$, $l = 1\ m$, $F = 100\ kN = 10^5\ N$, $Y = 2.0 \times 10^{11}\ Nm^{-2}$

(a) $\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{10^5\ N}{(22/7) \times (0.01\ m)^2}$

$= 3.18 \times 10^8\ Nm^{-2}$

(b) As $Y = \frac{F}{A} . \frac{l}{\Delta l}$

∴ $\text{Elongation, } \Delta l = \frac{F}{A} . \frac{l}{Y} = \frac{3.18 \times 10^8 \times 1}{2.0 \times 10^{11}} = 1.59 \times 10^{-3}\ m = 1.59\ mm.$

(c) $\text{Strain} = \frac{\Delta l}{l} = \frac{1.59 \times 10^{-3}\ m}{1\ m} = 1.59 \times 10^{-3} = 0.16\ \%$

Q. 6. ***A steel wire of length 4.7 m and cross-section $3.0 \times 10^{-5}\ m^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-section $4.0 \times 10^{-5}\ m^2$ under a given load. What is the ratio of the young's modulus of steel to that of copper?***

Sol. For steel: $l = 4.7\ m$, $A = 3.0 \times 10^{-5}\ m^2$

For copper: $l = 3.5\ m$, $A = 4.0 \times 10^{-5}\ m^2$

Applied force F and extension Δl are same for both wires.

∴ Young's modulus of steel, $Y_S = \frac{Fl}{A. \Delta l} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l}$

Young's modulus of copper,

$Y_C = \frac{Fl}{A. \Delta l} = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l}$

$\frac{Y_S}{Y_C} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \times \frac{4.0 \times 10^{-5} \times \Delta l}{F \times 3.5} = 1.79$

Q. 7. ***What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg wt? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.***

Sol. Given $r = 1.25\ mm = 0.125\ cm$, $F = 100 \times 9.8 \times 980\ n = 98 \times 10^6$ dynes

$Y = 12.5 \times 10^{11}$ dyne cm^{-2}

As $Y = \frac{F}{A} . \frac{l}{\Delta l}$ or $\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi r^2\ Y}$

∴ The percentage increase in length is

$\frac{\Delta l}{l} \times 100 = \frac{F \times 100}{\pi r^2\ Y}$

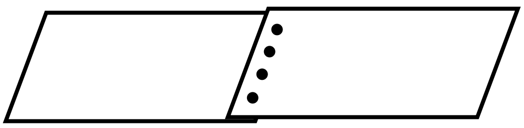
$= \frac{98 \times 10^6 \times 7 \times 100}{22 \times (0.125)^2 \times 12.5 \times 10^{11}} = 15.965 \times 10^{-2} = 0.16\ \%$

Q. 8. ***Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 2.3×10^9 Pa? Assume that each rivet is to carry one quarter of the load.***

Sol. Let the tension exerted by riveted strip = F

This tension would provide shearing force on the four rivets, which share it equally.

∴ Shearing force on each rivet $= \frac{F}{4}$



And shearing stress on each rivet

$= \frac{F/4}{A} = \frac{F}{4A}$

As the maximum shearing stress on each rivet is given to be 2.3×10^9 Pa, so we have

$\frac{F_{max}}{4A} = 2.3 \times 10^9$

or $F_{max} = 4A \times 2.3 \times 10^9 = 4 \times \pi r^2 \times 2.3 \times 10^9$

$= 4 \times \frac{22}{7} \times (3.0 \times 10^{-3})^2 \times 2.3 \times 10^9 = 260.2 \times 10^3\ N = 260\ kN.$

Q. 9. ***The breaking stress for a metal is $7.8 \times 10^9\ Nm^{-2}$. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of the metal = $7.8 \times 10^3\ kg\ m^{-3}$. Take g = 10 N kg^{-1}.***

Sol. Breaking stress = Maximum stress that the wire can withstand = $7.8 \times 10^9\ Nm^{-2}$.

When the wire is suspended vertically, it tends to break under its own weight. Let its length be l and cross-sectional area A.

Weight of wire = mg = volume × density × g = Alρg

$\text{Stress} = \frac{\text{Weight}}{A} = \frac{Al\rho g}{A} = l\rho g$

For the wire not to break, $l\rho g$ = Breaking stress = $7.8 \times 10^9\ Nm^{-2}$

∴ $l = \frac{7.8 \times 10^9}{\rho g} = \frac{7.8 \times 10^9}{7.8 \times 10^3 \times 10} = 10^5\ m.$

Q. 10. A rubber string 10 m long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is 1.5×10^3 kg m^{-3} and young's modulus for the rubber is 5×10^6 Nm^{-2}. Take g = 10 N kg^{-1}.

Sol. Let the area of cross-section of the string be A m^2. Then the weight of the string is

$$W = mg = \text{volume} \times \text{density} \times g = 10\,A \times 1.5 \times 10^3 \times 10 = 1.5 \times 10^5\ A\ N$$

$$\text{Longitudinal stress} = \frac{W}{A} = 1.5 \times 10^5\ Nm^{-2}$$

As the weight of the string acts on its centre of gravity, so its produces extension only in 5 m length of the string. If Δl be the extension in the string, then

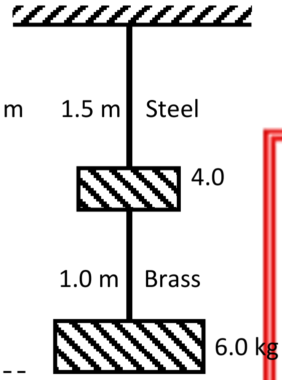
$$\text{Longitudinal strain} = \frac{\Delta l}{l} = \frac{\Delta l}{5}$$

$$\text{Young's modulus, } Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

or $$5 \times 10^6 = \frac{1.5 \times 10^5}{\Delta l/5}$$

∴ $$\Delta l = \frac{1.5 \times 10^5 \times 5}{5 \times 10^6} = 0.15\ m$$

Q. 11. Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5m and that of bass wire is 1.0 m. young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wire. (1 Pa = 1 Nm^{-2}).



Sol. For steel wire: $l_1 = 1.5$ m, $r_1 = \frac{0.25}{2}$ cm

$= 0.125 \times 10^{-2}$ m

$F_1 = 6 + 4 = 10$ kg f

$= 10 \times 9.8$ N

$Y_1 = 2.0 \times 10^{11}$ Pa

As $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$$

$$\Delta l = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$$

∴ $$\Delta l_1 = \frac{F_1 \cdot l_1}{\pi r_1^2\ Y_1}$$

$$= \frac{10 \times 9.8 \times 1.5}{3.14 \times (0.125 \times 10^{-2})^2 \times 2.0 \times 10^{11}} = 1.5 \times 10^{-4}\ m.$$

For brass wire: $l_2 = 1.0$ m, $r_2 = 0.125 \times 10^{-2}$ m

$F_1 = 6$ kg f $= 6 \times 9.8$ N, $Y_2 = 0.91 \times 10^{11}$ Pa

∴ $$\Delta l_2 = \frac{F_2 \cdot l_2}{\pi r_2^2\ Y_2}$$

$$= \frac{6 \times 9.8 \times 1.0}{3.14 \times (0.125 \times 10^{-2})^2 \times 0.91 \times 10^{11}}$$

$= 1.3 \times 10^{-4}$ m.

Q. 12. A silica glass rod has a diameter of 1 cm and is 10 cm long. The ultimate strength of glass is 50×10^6 Nm^{-2}. Estimate the largest mass that can be hung from it without breaking it. Take g = 10 N kg^{-1}.

Sol. Radius, r = ½ cm $= 0.5 \times 10^{-2}$ m, ultimate strength $= 50 \times 10^6$ Nm^{-2}.

Let M be the largest mass that can be hung. Then Ultimate strength $= \frac{Mg}{\pi r^2}$

or $$50 \times 10^6 = \frac{M \times 10}{3.14 \times (0.5 \times 10^{-2})^2}$$

or $$M = \frac{50 \times 10^6 \times 3.14 \times 0.25 \times 10^{-4}}{10} = 392.5\ kg$$

Q. 13. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.

Sol. Here $r_1 = 30$ cm $= 0.3$ m, $r_2 = 40$ cm $= 0.4$ m, $Y = 2.0 \times 10^{11}$ Pa

As the load is uniformly distributed among the four columns, hence the load on each column

$$= \frac{50,000}{4}\ kg = 12500\ kg$$

∴ $F = 12500 \times 9.8$ N

Also, A = Area of cross-section of each column $= \pi r_2^2 - \pi r_1^2 = \pi (r_2^2 - r_1^2)$

$$= \frac{22}{7}[(0.4)^2 - (0.3)^2] = \frac{22}{7} \times 0.07 = 0.22\ m^2$$

∴ $$\text{Compressional strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{F/A}{Y} = \frac{F}{AY} = \frac{12500 \times 9.8}{0.22 \times 2.0 \times 10^{11}} = 2.8 \times 10^{-6}$$

Q. 14. The maximum stress that can be applied to the material of a wire used to suspend an elevator is 1.3×10^8 Nm^{-2}. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 ms^{-2}, what is the minimum diameter of the wire?

Sol. As the elevator moves up, the tension in the wire is

$$F = mg + ma = m(g + a)$$

$$= 900 \times (9.8 + 2.2) = 10,800\ N$$

$$\text{Stress in the wire} = \frac{F}{A} = \frac{F}{\pi r^2}$$

Clearly, when the stress is maximum, r is minimum.

∴ Maximum stress = $\frac{F}{\pi r^2_{min}}$

or $r^2_{min} = \frac{F}{\pi \times \text{Maximum stress}} = \frac{10800}{3.14 \times 1.3 \times 10^8} = 0.2645 \times 10^{-4}$ m or $r_{min} = 0.5142 \times 10^{-2}$ m

Minimum diameter = 2 r_{min} = 2 × 0.5142 × 10^{-2} = 1.0284 × 10^{-2} m.

Q. 16. ***A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.***

Sol. Let T be the tension in each wire. As the bar is supported symmetrically by the three wires, the increase in length Δl of each wire should be same. Now, $Y = \frac{T}{A}\frac{l}{\Delta l}$

For all wires, we have same l, Δl and T. Hence

$Y \propto \frac{1}{A}$ or $A \propto \frac{1}{Y}$

or $\frac{\pi D^2}{4} \propto \frac{1}{Y}$ or $D \propto \frac{1}{\sqrt{Y}}$

∴ $\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \frac{1.9 \times 10^{11} \text{ Pa}}{1.1 \times 10^{11} \text{ Pa}} = 1.3$

Q. 17. ***A mass of 100 gram is attached to the end of a rubber string 49 cm long and having an area of cross-section 20 mm². The string is whirled round, horizontally at a constant speed of 40 rps in a circle of radius 51 cm. Find Young's modulus of rubber.***

Sol. When the mass is rotated at the end of rubber string, the restoring force in the string is equal to the centripetal force.

∴ $F = mr\omega^2 = mr(2\pi v)^2$

$= 100 \times 51 \times (2 \times \pi \times 40)^2$ dyne

Also, l = 49 cm, Δl = 51 – 49 = 2 cm

$A = 20 \text{ mm}^2 = 20 \times 10^{-2} \text{ cm}^2$

Hence, $Y = \frac{F \cdot l}{A \; \Delta l} = \frac{100 \times 51 \times 4 \times 9.87 \times 1600 \times 49}{20 \times 10^{-2} \times 2}$ [$\pi^2 = 9.87$] $= 3.95 \times 10^{10}$ dyne cm^{-2} = 3.95×10^9 Nm^{-2}

Q. 18. ***A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.005 cm². Calculate the elongation of the wire when the mass is at the lowest point of its path.***

Sol. The centripetal force at the lowest point is given by $mr\omega^2 = T - mg$

Where T is the tension in the wire when the mass is at the lowest point.

∴ Tension $T = mg + mr\omega^2$

$= m[g + r(2\pi v)^2] = 14.5 [9.8 + 1.0 \times 4 \times \pi^2 \times (2)^2] = 14.5 [9.8 + 16 \times 9.87] = 14.5 \times 167.72 = 2431.94$ N

Now, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l/l} = \frac{T/A}{\Delta l/l}$

∴ $\Delta l = \frac{Tl}{AY} = \frac{2431.94 \times 1.0}{0.065 \times 10^{-4} \times 2 \times 2 \times 10^{11}}$

$= 1.87 \times 10^{-3}$ m.

Example 18.6. A composite wire of diameter 1 cm consists of copper and steel wires of lengths 2.2 m and 2 m respectively. Total extension of the wire when stretched by a force is 1.2 mm. Calculate the force, given that Young's modulus for copper is 1.1 × 10¹¹ Pa and for steel is 2 × 10¹¹ Pa.

Solution. Young's modulus, $Y = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L}$

Here L is the length of the wire and ΔL its extension. The values of F and A are the same in the two cases.

For copper wire : $Y_c = \frac{F}{A} \times \frac{L_c}{\Delta L_c}$ ...(i)

For steel wire : $Y_s = \frac{F}{A} \times \frac{L_s}{\Delta L_s}$ ...(ii)

∴ $\frac{Y_c}{Y_s} = \frac{L_c}{\Delta L_c} \times \frac{\Delta L_s}{L_s}$

or $\frac{\Delta L_s}{\Delta L_c} = \frac{Y_c}{Y_s} \times \frac{L_s}{L_c} = \frac{1.1 \times 10^{11}}{2.0 \times 10^{11}} \times \frac{2.0}{2.2} = \frac{1}{2}$

∴ $\Delta L_c = 2\,\Delta L_s$ ...(iii)

Also $\Delta L_c + \Delta L_s = 1.2 \text{ mm} = 12 \times 10^{-4}$ m ...(iv) (given)

From eqs. (iii) and (iv), we have, $\Delta L_c = 8 \times 10^{-4}$ m.

Area of X-section of copper wire, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1 \times 10^{-2})^2 = 0.785 \times 10^{-4}$ m²

Putting the various values in eq. (i), we get,

$1.1 \times 10^{11} = \frac{F}{0.785 \times 10^{-4}} \times \frac{2.2}{8 \times 10^{-4}}$ ∴ $F = 3.14 \times 10^3$ N

BULK MODULUS (OR ELASTICITY IN VOLUME)

This refers to situations in which the volume (*i.e.*, bulk) of a substance is changed by the application of external normal stress. *It is defined as the ratio of normal stress to the volumetric strain.* It is denoted by the symbol K.

$$\text{Bulk modulus, } K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

Consider a sphere of volume V and surface area A. Suppose a compressive force, which is normal everywhere on the surface of the sphere, reduces its volume by ΔV (See Fig. 18.13). Then bulk modulus of the sphere material is

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

Normal stress $= F/A$

Volumetric strain $= -\Delta V/V$

The negative sign is included to indicate that the volume *decreases* with an increase in pressure.

$$\therefore \quad K = -\frac{F/A}{\Delta V/V} = -\frac{FV}{A\Delta V}$$

But $F/A = \Delta p$, the increase in pressure on the sphere.

$$\therefore \quad K = -V\frac{\Delta p}{\Delta V}$$

The SI unit of bulk modulus is N/m^2 or Pa.

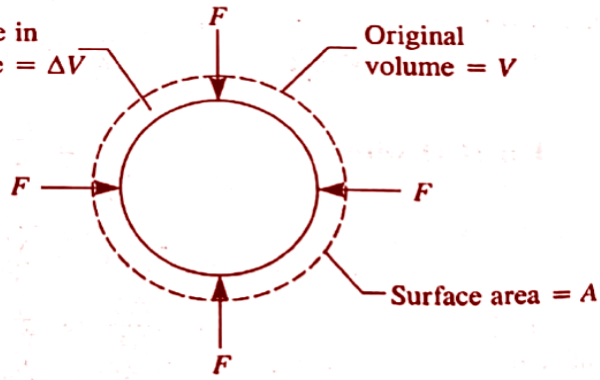


Fig. 18.13

BULK MODLUI IS POSSESSED by solids, liquids and gases. For example, the bulk modulus of steel is 16×10^{10} N/m² and that of water is 2.2×10^9 N/m². For air at normal pressure, $K = 10^5$ N/m². The bulk moduli of solids and liquids are very large indicating the fact that large forces are needed to produce even minute change in volume. Gases are more easily compressed and have correspondingly smaller bulk moduli. Thus greater the bulk modulus (K) of a material, the harder it is to change its volume.

COMPRESSIBILTY *The compressibility of a material is a measure of how easily the material is compressed.* In other words, compressibility is just the reciprocal of bulk modulus *i.e.*,

$$\text{Compressibility, } k = \frac{1}{K}$$

The SI unit of compressibility is $N^{-1}\ m^2$ or Pa^{-1} (*i.e.*, the reciprocal of unit of K).

Solids and liquids are relatively incompressible *i.e.*, they have small values of compressibility (k) or large values of bulk modulus (K) and these values are almost independent of temperature and pressure. On the other hand, gases are easily compressed (small K and large k) and the values of K and k strongly depend on the pressure or temperature.

Example 18.7. **A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 10^8 N/m². Find the bulk modulus of the material.**

Solution. Bulk modulus, $K = V\dfrac{\Delta p}{\Delta V}$

Here $\Delta p = 10^8$ N/m²; $\Delta V/V = 0.01\% = 0.01/100 = 10^{-4}$

$$\therefore \quad K = \frac{V}{\Delta V} \times \Delta p = \left(\frac{1}{10^{-4}}\right) \times (10^8) = 10^{12} \text{ N/m}^2$$

Example 18.8. **A cube is subjected to a pressure of 5×10^5 N/m². Each side of the cube is shortened by 1%. Find (*i*) the volumetric strain and (*ii*) bulk modulus of elasticity of cube material.**

Solution. Let l be the initial length of each side of the cube.

$$\text{Initial volume, } V = l^3$$

$$\text{Final length of each side } = \left(1 - \frac{1}{100}\right)l = \frac{99}{100}l$$

$$\text{Final volume} = \left(\frac{99l}{100}\right)^3$$

$$\text{Change in volume, } \Delta V = \left(\frac{99l}{100}\right)^3 - l^3 = l^3\left[\left(\frac{99}{100}\right)^3 - 1\right]$$

(*i*) $$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{l^3\left[\left(\frac{99}{100}\right)^3 - 1\right]}{l^3} = \frac{-3}{100} = -\mathbf{0.03}$$

(*ii*) $$\text{Bulk modulus, } K = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{5\times10^5}{0.03} = \mathbf{1.67 \times 10^7\ N/m^2}$$

Example 18.9. What will be the density of lead under a pressure of 2×10^8 N/m²? Density of lead = 11.4×10^3 kg/m³ and bulk modulus of lead $K = 8 \times 10^9$ N/m².

Solution. $$\text{Bulk modulus, } K = -\frac{V\Delta p}{\Delta V}$$

$$\therefore \quad \text{Change in volume, } \Delta V = -\frac{\Delta p}{K}\times V = -\frac{2\times10^8}{8\times10^9}\times V = -\frac{V}{40}$$

$$\text{New volume of lead, } V' = V + \Delta V = V - \frac{V}{40} = \frac{39}{40}V$$

If ρ' kg/m³ is the new density of lead, then its mass

$$= V'\rho' = \frac{39}{40}V\rho' \text{ kg}$$

Since the mass is a constant quantity, we have,

$$\frac{39}{40}V\rho' = V\times11.4\times10^3$$

$$\therefore \quad \rho' = \frac{40}{39}\times11.4\times10^3 = \mathbf{11.69 \times 10^3\ kg/m^3}$$

SHEAR MODULUS (OR ELASTICITY OF SHAPE)

This refers to situations in which the shape of a substance is changed by the application of tangential stress (or shear stress). *It is defined as the ratio of tangential stress to shear strain.* It is denoted by the symbol η.

$$\text{Shear modulus, } \eta = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

Consider a rectangular block whose lower face *EFGH* is fixed [See Fig. 18.14]. Let a tangential force *F* be applied on the upper face *ABCD* of the block in the direction shown. The force provides a shear stress which displaces the upper face through a small distance $AA' = \Delta L$.

If $AF = L$ and area of the upper face is *A*, then,

$$\text{Tangential stress} = \frac{F}{A}$$

$$\text{Shear strain} = \theta \simeq \tan\theta = \frac{AA'}{AF} = \frac{\Delta L}{L}$$

$$\therefore \quad \eta = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L}$$

$$\text{or} \quad \eta = \frac{FL}{A\,\Delta L}$$

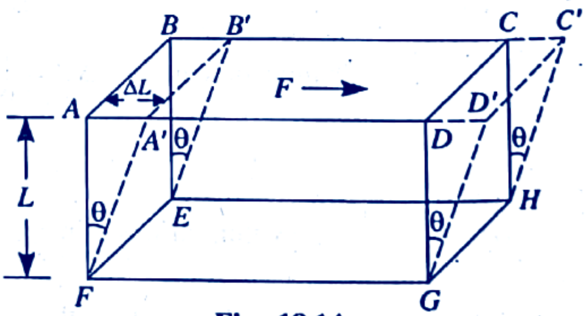


Fig. 18.14

The SI unit of shear modulus is N/m² or Pa.

The shear modulus has significance only for solid materials. A liquid or gas flows under the influence of a shear stress and cannot *permanently* support such a stress. The shear modulus is roughly one-third the value of Young's modulus for the same solid material. For example, Young's modulus for steel is 20×10^{10} N/m², whereas shear modulus for steel is 8×10^{10} N/m².

Example 18.10. An aluminium cube of each side 4 cm is subjected to a tangential force. The top face of the cube is sheared 0.012 cm *w.r.t.* the bottom. Find (*i*) shearing strain (*ii*) shear stress and (*iii*) shearing force. Given that modulus of rigidity is 2.08×10^{10} N/m^2.

Solution. $L = 4$ cm ; $\Delta L = 0.012$ cm ; $\eta = 2.08 \times 10^{10}$ N/ m^2

(*i*) Shearing strain, $\theta = \dfrac{\Delta L}{L} = \dfrac{0.012}{4} = 0.003$

(*ii*) $\eta = \dfrac{\text{Shear stress}}{\text{Shear strain}}$

$\therefore$ Shear stress $= \eta \times$ shear strain

$= (2.08\times10^{10})\times(0.003) = 6.24 \times 10^7$ N/m^2

(*iii*) Shearing force = Shear stress × area of cube face

$= (6.24 \times 10^7) \times (16 \times 10^{-4}) = 9.98 \times 10^4$ N

Example 18.11. A rubber cube of side 20 cm has one side fixed while a tangential force equal to the weight of 400 kg is applied to the opposite face. Find (*i*) shearing strain and (*ii*) the distance through which the strained side moves. Given that modulus of rigidity for rubber is 8×10^6 N/m^2.

Solution. (*i*) Shear stress $= \dfrac{F}{A} = \dfrac{400\times9.8}{(20\times10^{-2})^2} = 9.8\times10^4$ N/m^2

Modulus of rigidity, $\eta = \dfrac{\text{Shear stress}}{\text{Shear strain}}$

$\therefore$ Shear strain, $\theta = \dfrac{\text{Shear stress}}{\eta} = \dfrac{9.8\times10^4}{8\times10^6} = 0.0123$

(*ii*) Now $\theta = \dfrac{\Delta L}{L}$

$\therefore$ $\Delta L = \theta \times L = 0.0123 \times 0.2 = 0.0025$ m $= 0.25$ cm

Example 18.12. A square lead slab of side 50 cm and thickness 5.0 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor. How much is the upper edge displaced if the shear modulus of the lead is 5.6×10^9 N/m^2?

Solution. The area of cross-section of the face where the force is applied is $A = 50 \times 5 = 250$ cm^2 $= 250 \times 10^{-4}$ m^2.

Now, $\eta = \dfrac{F/A}{\theta}$

$\therefore$ Shear strain, $\theta = \dfrac{F}{A\eta} = \dfrac{9.0\times10^4}{250\times10^{-4}\times5.6\times10^9} = 6.4\times10^{-4}$

Now, $\theta = \dfrac{AA'}{AF}$

$\therefore$ Displacement of the upper edge is given by ;

$AA' = \theta\times AF = (6.4\times10^{-4})\times0.5 = 3.2 \times 10^{-4}$ m

STRAIN ENERGY

When a body is strained, work has to be done to deform the body. This work done is stored in the body in the form of " ELASTIC STRAIN ENERGY".

Consider a wire of length 'L' and area of cross-section 'A'.

Suppose on applying a Normal Force 'F' on the wire, the extension produced is '*l*'. If the extension is increased by Δl, where Δl is so small that F can be considered constant, then work done ΔW is given by ;

$$\Delta W = F\,\Delta l$$

According to Hooke's law, $F = kl$

$\therefore$ The total work done (W) in increasing the extension from 0 to l is given by ;

$$W = \int_0^l kl\,dl = \frac{1}{2}kl^2 = \frac{1}{2}(kl)l = \frac{1}{2}Fl$$

This work done is stored in the wire as elastic potential energy U (the strain energy).

$\therefore$ Strain energy, $U = \dfrac{1}{2}Fl$

We can express this relation in another useful form.

$$U = \frac{1}{2}\times\left(\frac{F}{A}\right)\times\left(\frac{l}{L}\right)\times LA$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

∴ Strain energy per unit volume, $u = \frac{1}{2} \times \text{stress} \times \text{strain}$ ...(*i*)

Therefore, the strain energy per unit volume in a deformed body is equal to one-half the product of the stress and strain. Although we have derived this expression for the case of longitudinal strain, it is applicable to any type of strain.

Example 18.13. Calculate the increase in energy of a brass bar of length 0.2 m and cross-sectional area 1 cm^2 when compressed with a force of 49 N along its length. Young's modulus of brass = 1.0 × 10^{11} N/m^2.

Solution. Initial length of bar, $L = 0.2$ m

Area of cross-section, $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$

Increase in energy of bar = W.D. in compressing

or $$U = \frac{1}{2} Fl$$

where F is the force applied and l is the decrease in length.

Now $$Y = \frac{FL}{Al} \text{ or } l = \frac{FL}{AY}$$

∴ $$U = \frac{1}{2} \times F \times \frac{FL}{AY} = \frac{F^2 L}{2AY}$$

or $$U = \frac{(49)^2 \times 0.2}{2 \times (1.0 \times 10^{-4}) \times (1.0 \times 10^{11})} = 2.4 \times 10^{-5} \text{ J}$$

Example 18.14. A 40 kg boy whose leg bones are 4 cm^2 in area and 50 cm long falls through a height of 2 m without breaking his leg bones. If the bones can stand a stress of 0.9 × 10^8 N/m^2, calculate the Young's modulus for the material of the bone. Take g = 10 ms^{-2}.

or $$40 \times 10 \times 2 = 2\left[\frac{1}{2} \times 0.9 \times 10^8 \times \text{strain} \times 2 \times 10^{-4}\right]$$

∴ $$\text{Strain} = \frac{40 \times 10 \times 2}{0.9 \times 2 \times 10^4} = \frac{2}{45}$$

∴ $$\text{Young's modulus, } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{0.9 \times 10^8}{2/45} = 2.025 \times 10^9 \text{ Nm}^{-2}$$

POISSON'S RATIO

When a wire is suspended from one end and loaded at the other end, the length of the wire increases and its diameter decreases. Thus there occurs longitudinal strain as well as lateral strain. *The ratio of lateral strain to the longitudinal strain is called* **Poisson's ratio (σ).**

Let L, D = original length and diameter respectively of the wire

$\Delta L, \Delta D$ = slight increase in length and a corresponding slight decrease in the diameter of the wire

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\text{Lateral strain} = -\frac{\Delta D}{D}$$

The negative sign shows that if length increases, the diameter decreases.

∴ $$\text{Poisson's ratio, } \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta D / D}{\Delta L / L}$$

∴ $$\sigma = \frac{-\Delta D}{D} \times \frac{L}{\Delta L}$$

Poisson's ratio is never more than 0.5, its value is usually between 0.2 and 0.4.

The relationship between Y, η and σ is

A $Y = 2\eta(1+\sigma)$

B $\eta = 2Y(1+\sigma)$

C $\sigma = \frac{2Y}{(1+\eta)}$

D $Y = \eta(1+\sigma)$

18.24. APPLICATIONS OF ELASTICITY

The primary jobs of an engineer are the design, construction and maintenance of structures, machinery etc. One of his functions as a designer is to have a good knowledge of the elastic properties of the materials he proposes to use. This will enable him to predict the behaviour of the materials under the action of deforming forces. Although there are a very large number of practical applications of elasticity, we are briefing here a few by way of illustration.

(*i*) Most parts of structures and machinery are under some kind of stress. In their design, it has to be ensured that applied stresses do not exceed the elastic limits of the materials.

(*ii*) Due to particular elastic properties, some materials are used as vibration absorbers. For example, some types of rubber have particularly large hysteresis loops and so are useful as vibration absorbers. If a block of such a rubber is placed between a piece of vibrating machinery and the floor, much of the energy of the mechanical vibrations is converted to heat energy in the rubber and so is not transmitted to the floor.

(*iii*) In a car, it is desirable that as little heat is generated as possible. For this reason, in the manufacture of car tyres that rubber is used which has small hysteresis loop.

(*iv*) Because of its strength and elastic properties, steel is used to make not only springs but in the construction of girders as well.

(*v*) The cross-section of steel girders has the form of the letter *I* as shown in Fig. 18.15. Most of the material in these *I*-beams is concentrated in the top and bottom flanges ; the piece joining the flanges, called the web, is of thinner cross-section. Thus when the beam is used horizontally in construction, the stress is predominantly in the top and bottom flanges — not in the central portion. One flange is sequeezed while the other is stretched. Between the top and bottom flanges is a stress-free region that acts principally to connect the top and bottom flanges together. This is the neutral layer where comparatively little material is needed. The flanges carry virtually all stresses in the beam. An *I*-beam is nearly as strong as a solid rectangular bar of the same overall dimensions, and its weight is considerably less. A large rectangular steel beam on a certain span might collapse under its own weight whereas an *I*-beam of the same depth would carry much added load.

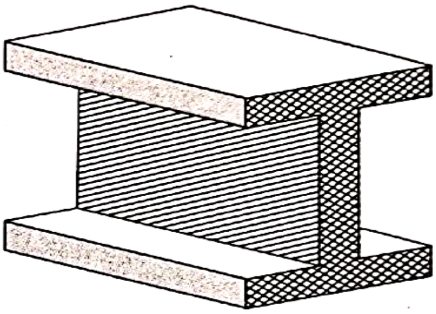


Fig. 18.15

(*vi*) A hollow shaft is stronger than a solid shaft made of same equal material. It is because the torque required to produce a given twist in a hollow shaft is greater than that required to twist a solid shaft of the same length and material through the same angle. For this reason, electric poles are made hollow.

END

CONCEPTUALS

Q.1. Matter exists in the states solids, liquids and gaseous. Can there be any other state of matter?

Ans. In addition to solids, liquids and gases, there is a fourth state of matter, the least common state in our everyday environment—**plasma** (not to be confused with the clear, formless part of blood, also called plasma). A plasma looks and behaves like a high-temperature gas but with an important difference ; it conducts electricity. The atoms and molecules that make it up are ionised, stripped of one or more electrons as a result of the more violent collisions at high temperatures. In an ideal plasma, all atoms are completely stripped off electrons and are bare nuclei. Although the electrons and the ions are themselves electrically charged, the plasma as a whole is electrically neutral.

Q.2. Solids have fixed shapes and volumes. Why?

Ans. The molecules of a solid at temperatures above absolute zero oscillate about their equilibrium positions. Because their kinetic energy is low compared with their potential energy, the molecules of solids can merely vibrate about fixed positions. For this reason, a solid has both a fixed volume and a fixed shape.

Q.3. When the interatomic distance is r_0 = 0.9 Å, the potential energy is found to be minimum. Is the force attractive or repulsive at interatomic distance of (*i*) 0.6 Å (*ii*) 1.5 Å?

Ans. When interatomic distance is r_0 (= 0.9 Å), there is no net force between two atoms and their potential energy has its minimum value. At 0.6 Å which is less than r_0, the force is repulsive. However, at 1.5 Å, the force is attractive.

Q.4. Which determines the state of matter?

Ans. The interatomic and intermolecular forces arise from two main causes :

(*i*) The *potential energy* of the atoms/molecules which is due to the interaction with the surrounding atoms/molecules. The origin is principally electrical.

(*ii*) The *thermal energy* of the atoms/molecules (*i.e.*, kinetic energy of atoms/molecules) and it depends on the temperature of the substance concerned.

The particular state in which matter appears (*i.e.*, solid, liquid or gas) and the properties it then has, are determined by the relative magnitudes of these two energies.

Q.5. A liquid has a fixed volume but no fixed shape. Why?

Ans. The molecules of liquids, like those of solids, vibrate. However, the molecules of liquids have greater kinetic energies than those of solids. The increased kinetic energy results in large amplitude of vibration. Therefore, there is more likelyhood of a molecule being able to pass through the gaps between the molecules surrounding it. Hence there is a continual molecular migration superimposed on the vibrational motion. This accounts for the ability of a liquid to adopt the shape of its container. However, the molecules are close together and a change in volume would require that intermolecular forces were overcome. Therefore, liquids have fixed volumes.

Q.6. Which is more elastic iron or rubber?

Ans. Iron is more elastic than rubber. It is because for a given stress, the strain produced in iron is much smaller than that produced in the rubber.

Q.7. A 20 kg load is hung from the end of a spring. The spring then stretches a distance of 10 cm. If, instead, a 40 kg load is hung from the spring, how much will the spring stretch?

Ans. A 40 kg load has twice the weight of a 20 kg load. According to Hooke's law, $F \propto$ extension. Two times the applied force will result in two times the stretch. So spring will stretch 20 cm.

Q.8. Why do spring balances show wrong readings after they have been used for a long time?

Ans. If a material is repeatedly stressed and unstressed, it becomes weaker *i.e.*, the strain produced by a given amount of stress increases. For this reason, the spring balances which have been used for a long time give wrong readings.

Q.9. What is ultimate strength of a material?

Ans. The maximum stress that a material can experience before fracture is called ultimate strength of the material. For example, ultimate strength of steel is about 20×10^{10} N/m^2 (compression) and that of aluminium is 7×10^{10} N/m^2.

Q.10. Why are bridges declared unsafe after long use?

Ans. A bridge is subjected to forces of varying amounts due to the flow of traffic over it. In other words, the bridge is subjected to varying stresses. As a result, it becomes weaker *i.e.*, the strain produced by a given stress increases. If the elastic limit of the bridge is exceeded, it may collapse. For this reason, bridges are declared unsafe after long use.

Q.11. Of what particles may crystalline solids be composed? Give an example of a crystalline solid composed of each kind of particle.

Ans. Most of the non-living solid materials on the earth are in the form of crystals. A crystal is a solid made up of fundamental particles (atoms, molecules or ions) arranged in a definite, regular pattern. Examples are : Atoms – metals ; molecules – sugar ; ions – sodium chloride.

Q.12. Name five primary types of crystal formation.

Ans. The five primary types of crystal formation are :

(*i*) *Ionic crystal* ; composed of alternate positive and negative ions held together by electrostatic attraction.

(*ii*) *Polar crystal* ; composed of polar molecules and held together by the electrostatic attraction by the oppositely charged ends.

(*iii*) *Atomic crystal* ; composed of atoms (all of the same element or two different elements) held together by covalent bonds.

(*iv*) *Molecular crystal* ; composed of non-polar molecules or unionised atoms held together by the weak Vander Waal's forces.

(*v*) *Metallic crystal* ; composed of metal atoms. Each atom contributes about one electron (on the average) to the swarm of electrons that binds them together.

Q.13. Name the elements that compose most of the things we see everyday.

Ans. About a dozen elements compose most of the things we see everyday. Most common substances are formed out of the combinations of two or more of these most common elements : hydrogen (H), carbon (C), nitrogen (N), oxygen (O), sodium (Na), magnesium (Mg), aluminium (Al), silicon (Si), phosphorus (P), sulphur (S), chlorine (Cl), potassium (K), calcium (Ca) and iron (Fe).

Q.14. What is fatigue limit?

Ans. Mild steel and many other ferrous metals can safely undergo an infinite number of stress cycles provided that the maximum stress is kept below a particular value known as the *fatigue limit*. There is no such limit for non-ferrous materials.

Q.15. How does fatigue failure occur?

Ans. Fatigue fractures usually start on the surface at points of high stress *e.g.* at sharp corners and around rivet holes. It is believed that each time the material is stressed, a small amount of plastic strain is produced. Since it is plastic strain, the effects of repeated stressings are cumulative and eventually produce fracture.

Q.16. What is elastic after effect?

Ans. When deforming force is removed from an elastic body, the body tends to regain original configuration. It has been found that some bodies regain their original configuration immediately while others take appreciable time to do so. This delay in regaining the original configuration by a body after the removal of the deforming force is called elastic after effect. Elastic after effect is negligible for quartz and phosphor bronze. For this reason, we use the suspensions made from quartz and phosphor bronze in galvanometers.

Q.17. The elastic limit of earth's material is 3×10^8 N/m^2 and the density of rock material is 3×10^3 kg/m^3 . What is the maximum height a mountain on earth can possess?

Ans. The maximum height of a mountain on earth can be estimated from the elastic behaviour of earth. If h is the maximum height of the mountain, then,

Pressure exerted by mountain on earth = Elastic limit of earth material

or $h\rho g = 3\times10^8 \quad \therefore \quad h = \dfrac{3\times10^8}{\rho g} = \dfrac{3\times10^8}{3\times10^3\times9.8} = 10^4$ m

It is interesting to note that the height of Mount Everest is nearly equal to this value.

Q.18. In Fig. 18.16, which material is more ductile?

Ans. The ductility of a material is the extent of plastic deformation. Clearly, it is greater for material *A*.

Q.19. In Fig. 18.16, which material is more brittle?

Ans. The plastic region for material *B* is small. Therefore, material *B* is more brittle.

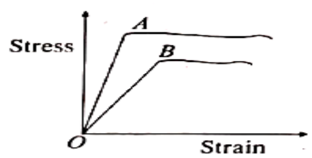


Fig. 18.16

VERY SHORT TYPE

Q.1. What is the order of (*i*) size of an atom (*ii*) size of a molecule?

Ans. (*i*) of the order of 10^{-10} m (*ii*) of the order of 3×10^{-10} m.

Q.2. Among the three states of matter which has its own shape and volume?

Ans. Solid state.

Q.3. Which forces are stronger ; interatomic or intermolecular?

Ans. Interatomic forces are stronger than intermolecular forces.

Q.4. Sand does not possess any definite shape and volume. Why is it still a solid?

Ans. Sand is divided rock. It has all the properties of solid even though its particles have indefinite volume like amorphous solids.

Q.5. What is the nature of interatomic/intermolecular forces?

Ans. These forces are electrical in nature.

Q.6. Which is the most important test for a crystalline solid?

Ans. A crystalline solid has a sharp melting point.

Q.7. What is a deforming force?

Ans. A force that produces a change in shape or size of a body is called deforming force.

Q.8. What is elasticity?

Ans. It is the property of a body to regain its shape and size when deforming force is removed.

Q.9. What is a perfectly elastic body?

Ans. If a body regains its exact shape and size on removal of the deforming force, it is called a perfectly elastic body.

Q.10. What is a perfectly plastic body?

Ans. If a body has no tendency at all to regain its original shape and size on the removal of deforming force, it is called a perfectly plastic body.

Q.11. Give two examples of nearly (*i*) perfectly elastic body (*ii*) perfectly plastic body.

Ans. (*i*) Quartz and Phosphor bronze (*ii*) Putty and Paraffin wax.

Q.12. Which is more elastic water or air and why?

Ans. Water. Bulk modulus, $K = 1/k$ where k is the compressibility of the material. Since water is less compressible, it has greater value of elasticity.

Q.13. Why are springs made of steel and not of copper?

Ans. It is because Young's modulus of steel is more than that of copper. Therefore, for a given deforming force, steel spring is stretched lesser than copper spring and regains its original state quickly on the removal of the deforming force.

Q.14. Which state of matter has volume elasticity?

Ans. All states of matter (solids, liquids and gases) have volume elasticity.

Q.15. Which state of matter has Young's modulus?

Ans. Only solids.

Q.16. What is the value of Young's modulus for a perfectly rigid body?

Ans. Young's modulus, $Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$. The deforming force produces zero strain in a perfectly rigid body so that Y is infinite.

Q.17. Why is work done in stretching a wire?

Ans. When a wire is stretched, the interatomic forces oppose the increase in length of the wire. Therefore, work has to be done against these forces.

Q.18. What happens to work done during stretching of wire?

Ans. The work done in stretching the wire is stored in it in the form of elastic potential energy.

Q.19. What is the SI unit of elasticity?

Ans. Modulus of elasticity = stress/strain. Since strain has no units, modulus of elasticity has the same units as that of stress *i.e.*, N/m^2.

Q.20. The length of a wire increases by 1% on suspending a mass of 2 kg from it. What is the linear strain in the wire?

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Ans. Linear strain $= \frac{\Delta L}{L} = 1\% = \frac{1}{100} = 0.01$.

Q.21. What is (*i*) tensile stress (*ii*) compressive stress?

Ans. (*i*) When the deforming force increases the length of the body, the stress produced is called tensile stress.

(*ii*) When the deforming force decreases the length of the body, the stress produced is called compressive stress.

Q.22. What is breaking stress?

Ans. The maximum stress that can be applied to a material without fracture (or breaking) is called breaking stress. It is also called *ultimate tensile strength*.

Q.23. Which state of matter has only bulk modulus?

Ans. Liquids and gases have only bulk modulus.

Q.24. Young's modulus of elasticity and modulus of rigidity do not exist for liquids and gases. Why ?

Ans. It is because liquids and gases cannot be deformed along one dimension only and also cannot undergo shear strain.

Q.25. Which type of modulus of elasticity is involved in (*i*) stretching coiled spring (*ii*) water lift pump (*iii*) power transmission by an automobile shaft?

Ans. (*i*) modulus of rigidity (*ii*) bulk modulus (*iii*) modulus of rigidity.

Q.26. What is the modulus of rigidity of a fluid (liquid or gas)?

Ans. Zero, because a fluid does not possess a shape of its own.

Q. 27. What is the effect of temperature on the three types of moduli?

Ans. With the rise in temperature, the values of Y, K and η decrease.

Q.28. What is the SI unit of Poisson's ratio?

Ans. It is unitless because it is ratio of two strains.

Q.29. A steel wire is stretched by a weight of 1 kg. If the radius of the wire is doubled, how will Young's modulus Y of wire be affected?

Ans. The value of Y remains unaffected because it depends upon the nature of the material.

Q.30. Write copper, steel, rubber and glass in the increasing order of their elasticity.

Ans. Rubber, glass, copper and steel.

SHORT ANSWER QUESTIONS

Q.1. What is elastic fatigue?

Ans. The loss of strength of a material because of repeated stresses is known as elastic fatigue. Due to the repeated stresses, there is a slight permanent change in the structure of the material. As this process continues, some portions of the material are weakened and material may fracture.

Q.2. How do you explain the elastic behaviour of a body?

Ans. The deforming force changes the distance between the molecules of the body, either pulling them further apart or pushing them close together. When the deforming force is removed, the molecular forces of the body restore the molecules to their normal spacing. Consequently, the body regains its original state.

Q.5. What is elastic limit? What happens if it is exceeded?

Ans. Elastic limit is the maximum stress that a body can sustain and still regain its original size and shape on the removal of the deforming force. If the elastic limit is exceeded, there will be some permanent change in size/shape of the body.

Q.6. If in a wire of Young's modulus Y, longitudinal strain X is produced, what will be potential energy stored per unit volume?

Ans. Elastic P.E. stored per unit volume in a strained body

$$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain}$$

$$= \frac{1}{2} \times Y \times (\text{strain})^2 = \frac{1}{2} YX^2$$

Q.7. A force of 1000 N stretches the length of a hanging wire by 1 mm. What force is required to stretch a wire of the same material and length but having four times the diameter by 1 mm?

Ans. $Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L}$. Since L, ΔL and Y are constant, $\frac{F}{A}$ = Constant.

$$\therefore \frac{F_2}{A_2} = \frac{F_1}{A_1} \text{ or } \frac{F_2}{D_2^2} = \frac{F_1}{D_1^2} \text{ or } \frac{F_2}{(4)^2} = \frac{1000}{(1)^2} \quad \therefore F_2 = 16000 \text{ N}$$

Q.8. The breaking force for a wire is F. What will be the breaking force for (*i*) two parallel wires of the same size (*ii*) a single wire of double thickness?

Ans. (*i*) When the two wires of the same size are suspended in parallel, the breaking force for the parallel combination $= F + F = 2F$.

(*ii*) $Y = \frac{FL}{A\,\Delta L}$ or $F = \frac{YA\,\Delta L}{L}$ $\therefore$ $F \propto D^2$

If the thickness (diameter) of the wire is doubled, the breaking force will be $4F$.

Q.9. Distinguish between interatomic and intermolecular forces.

Ans. Refer to Art. 18.1.

Q.10. The length of a metallic wire is L_1 when the tension in the wire is T_1 and is L_2 when the tension is T_2. What is the original length of the wire?

Ans. Let L and A be the length and area of cross-section of the metallic wire. Now, $Y = FL/A\Delta L$.

For the first case : $F = T_1$ and $\Delta L = L_1 - L$ $\therefore$ $Y = \frac{T_1 L}{A(L_1 - L)}$ ...(*i*)

For the second case : $F = T_2$ and $\Delta L = L_2 - L$ $\therefore$ $Y = \frac{T_2 L}{A(L_2 - L)}$ ...(*ii*)

From eqs. (*i*) and (*ii*), $\frac{T_1 L}{A(L_1 - L)} = \frac{T_2 L}{A(L_2 - L)}$

or $L(T_2 - T_1) = T_2 L_1 - T_1 L_2$ $\therefore$ $L = \frac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$

Q.11. Why is the stretching of a coil spring determined by its shear modulus?

Ans. It is because when a coil spring is stretched, there is neither a change in the length of the coil nor a change in its volume. Only change that takes place is the change in shape of the coil spring. Therefore, stretching of a coil spring is determined by its shear modulus.

Q.12. The length of a wire is cut to half. Why this change has no effect on the maximum load the wire can support?

Ans. Breaking force = Breaking stress × Area of cross-section. When the wire is cut to half, there is no change in the area of cross-section of the wire. Therefore, there is no change in the maximum load (*i.e.,* breaking force) the wire can support.

Q.13. A cable is cut to half of its original length. What will be the effect on the increase in its length under a given load?

Ans. $Y = \frac{FL}{A\,\Delta L}$. Now F, A and Y are fixed so that $\Delta L \propto L$. Therefore, change in length (ΔL) is directly proportional to the original length (L). Since the original length is halved, the increase in length will be reduced to half.

Q.14. A wire is replaced by another wire of the same length and material but of twice diameter. How does this affect elongation under a given load?

Ans. $Y = \frac{FL}{A\,\Delta L}$. Now F, L and Y are fixed so that $\Delta L \propto 1/A$. Therefore, elongation (ΔL) is inversely proportional to the area of X-section (A) of the wire. Since the diameter of the wire is doubled, the area of cross-section becomes four times so that extension will be reduced to one-fourth on replacing the wire.

Q.15. A wire of length L and area of X-section A is made of a material of Young's modulus Y. It is stretched by an amount x. What is the work done?

Ans. $Y = \frac{FL}{A\,\Delta L}$ $\therefore$ $F = \frac{YA\,\Delta L}{L}$. Average extension $= \frac{0+x}{2} = \frac{x}{2}$

$\therefore$ Work done = Force × Average extension $= \frac{YA\,\Delta L}{L} \times \frac{x}{2} = \frac{YA\,x}{L} \times \frac{x}{2} = \frac{YA\,x^2}{2L}$.

Q.16. Why do spring balances show wrong readings after they have been used for a long time?

Ans. After repeated use of a spring balance, its spring gets fatigued and there is a loss of elastic strength of the spring. Therefore, for a given load, the extension in the spring is much more than the identical new spring. As a result, the spring balance gives wrong reading.

Q.17. What will happen to the potential energy of the atoms of a solid when it is (*i*) compressed (*ii*) stretched?

Ans. (*i*) The potential energy of the atoms will increase. It is because the distances between the atoms will decrease compared to the normal spacing (minimum potential energy) of the atoms.

(*ii*) Again, the potential energy will increase because now the spacing between the atoms will increase compared to the normal spacing.

Q.18. A wire stretches by a certain amount under a load. If the load and radius are both increased to four times, find the stretch caused in the wire.

Ans. For the first case, $Y = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L}$

For the second case, $Y = \frac{4F}{16\,A} \times \frac{L}{\Delta L'}$

$\therefore$ $\frac{F}{A} \times \frac{L}{\Delta L} = \frac{4F}{16A} \times \frac{L}{\Delta L'}$ $\therefore$ $\Delta L' = \frac{\Delta L}{4}$

Q.19. A metallic wire is suspended by attaching some weight to it. If α is the longitudinal strain and Y is the Young's modulus, find the ratio between elastic potential energy and the energy density.

Ans. Elastic potential energy $= \frac{1}{2}$ × stress × strain × volume

Energy density = Elastic potential energy/volume

$\therefore$ $\frac{\text{Elastic potential energy}}{\text{Energy density}}$ = Volume of wire

THEORY – EXPLORED

❖ Introduction:

If the distance between any two points in a body remains invariable, the body is said to be a rigid body. In practice it is not possible to have a perfectly rigid body. The deformations are

(i) There may be change in length
(ii) There is a change of volume but no change in shape
(iii) There is a change in shape with no change in volume

All bodies get deformed under the action of force. The size and shape of the body will change on application of force. There is a tendency of body to recover its original size and shape on removal of this force.

Elasticity: The property of a material body to regain its original condition on the removal of deforming forces, is called elasticity. Quartz fibre is considered to be the perfectly elastic body.

Plasticity: The bodies which do not show any tendency to recover their original condition on the removal of deforming forces are called plasticity. Putty is considered to be the perfectly plastic body.

Load: The load is the combination of external forces acting on a body and its effect is to change the form or the dimensions of the body. Any kind of deforming force is known as Load.

When a body is subjected to a force or a system of forces it undergoes a change in size or shape or both. Elastic bodies offer appreciable resistance to the deforming forces. As a result, work has to be done to deform them. This amount of work is stored in body as elastic potential energy. When the deforming force is removed, its increased elastic potential energy produced a tendency in the body to restore the body to its original state of zero energy or stable equilibrium. This tendency is due to the internal forces which come into play by the deformation.

Stress: When a force is applied on a body, there will be relative displacement of the particles. Due to the property of elasticity the particles tend to regain their original position. *The restoring or recovering force per unit area set up inside the body is called stress.*

- The stress is measured in terms of the load or the force applied per unit area. Hence its units are *dynes/cm²* in CGS and *Newton/m²* in MKS.
- It has a dimension **$[ML^{-1}T^{-2}]$**. It is same as that of pressure.

There are two types of stress.

(1) Normal Stress: Restoring force per unit area *perpendicular to the surface* is called normal stress.

(2) Tangential or Shearing Stress: Restoring force per unit area *parallel to the surface* is called tangential or shearing stress.

Strain: The unit change produced in the dimensions of a body under a system of forces in equilibrium, is called strain. The strain being ratio. It has no unit.

There are following three types of strains.

(1) Longitudinal or Linear Strain: It is defined as the increase in length per unit original length of an object when the it is deformed by an external force.

The ratio of change in length to the original length is called longitudinal or elongation strain.

$$i.e.\ \ Longitudinal\ or\ Linear\ Strain = \frac{Change\ in\ length\ (l)}{Original\ length\ (L)}$$

It is also called *Elongation strain* or *Tensile strain.*

(2) Volume Strain: It is defined as change in volume per unit original volume, when an object is deformed by the external force.

The ratio of change in volume to the original volume is called volume strain.

$$i.e.\ \ Volume\ Strain = \frac{Change\ in\ volume\ (v)}{Original\ volume\ (V)}$$

(3) Shear strain: When the force applied is acting parallel to the surface of the body then the change takes place only in the shape of the body. The corresponding strain is called shear strain.

The angular deformation produced by an external force is called shear strain.

Characteristics of a Perfectly Elastic Material

If a body is perfectly elastic then

- Strain is always same for a given stress.
- Strain vanishes completely when the deforming force is removed.
- For maintaining the strain, the stress is constant.

❖ Hooke's law

This fundamental law of elasticity was proposed by Robert Hooke in 1679 and it states that "*Provided the strain is small, the stress is directly proportional to the strain*". In other words, *the ratio of stress to strain is a constant quantity for the given material* and it is called the modulus of elasticity or coefficient of elasticity.

$$Stress\ \propto Strain$$

$$\therefore\ Stress = E \times Strain$$

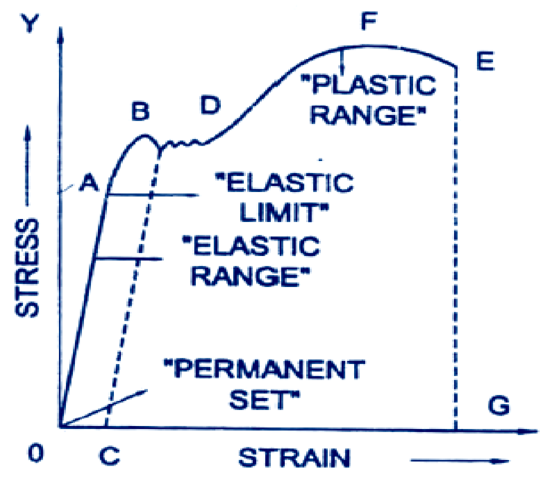
$$\therefore E = \frac{Stress}{Strain}$$

The units and dimensions of the modulus of elasticity are the same as that of stress.

❖ Elastic Limit:

When the stress is continually increased in the case of solid, a point is reached at which the strain increased more rapidly. The stress at which the linear relationship between stress and strain hold good is called elastic limit of the material.

❖ Stress-Strain Diagram:



- The direct proportionality between stress and strain is found to be true only for small values of strain as shown in the figure.
- The portion OA of the curve is a straight line showing that the stress is directly proportional to strain. It shows that Hooke's law is strictly obeyed up to the value of stress corresponding to point A. This point is called ***Elastic limit***.
- Beyond point A, the curve is not a straight line. In this region AB, the strain increases more rapidly than the stress and the behavior is partly elastic and partly plastic. If the object is unloaded at B, it does not come back to its original condition along path AO, but takes the dotted path BC. The object is said to have acquired ***permanent set***. And OC is called the ***residual strain***.
- Beyond the point B, the length of the wire starts increasing without any increase in stress. Thus, wire begins to flow after point B and it continues up to D. The point B, at which the wire begins to flow is called ***yield point***.
- Beyond the point F, the graph indicates that length of the wire increases, even if the wire is unloaded. The wire breaks ultimately at point E, called the ***breaking point*** of the wire. The portion of the graph between D and E is called the ***plastic region***.

❖ Three types of elasticity:

There are three types of strain, therefore we have three types of elasticity.

(1) Linear elasticity called ***Young's modulus***, corresponding to *linear strain*.

(2) Elasticity of volume or ***Bulk modulus***, corresponding to *volume strain*.

(3) Elasticity of shape or shear modulus or ***Modulus of Rigidity***, corresponding to *shear strain*.

(1) Young's Modulus:

- When the deforming force is applied to the body only along a particular direction, the change per unit length in that direction is called *longitudinal, linear or elongation strain*.
- The force applied per unit area of cross section is called *longitudinal or linear stress*.
- The ratio of longitudinal stress to linear strain, within the elastic limit, is called Young's modulus.
- It is denoted by Y

$$Y = \frac{Longitudinal\ stress}{Linear\ strain}$$

- Consider a wire of length L having area of cross section 'a', fixed at one end and loaded at the other end.
- Suppose that a normal force F is applied to the free end of the wire and its length increase by l.

$$Longitudinal\ stress = \frac{F}{a} \quad and\ \ linear\ strain = \frac{l}{L}$$

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}} \quad or \quad Y = \frac{FL}{al}$$

- Young's modulus can also be defined as the force applied to a wire of unit length and unit cross sectional area to produce the increase in length by unity.
- The units of Young's modulus are **Pascal or N/m²** in MKS and **dyne/cm²** in CGS system.

(2) Bulk Modulus:

- It is defined as the ratio of the normal stress to the volume strain.
- It is denoted by K. The bulk modulus is also known as the coefficient of cubical elasticity.

$$K = \frac{Normal\ stress}{Volume\ strain}$$

- Consider a cubic of volume *V* and surface area '*a*'. Suppose that a force *F* which acts uniformly over the whole surface of the cubic, produces a decrease in its volume by *v* then,

$$Normal\ stress = \frac{F}{a} \quad and \quad volume\ strain = \frac{v}{V}$$

$$\therefore K = \frac{FV}{av}$$

Now, the pressure is $P = \frac{F}{a}$

$$\therefore K = \frac{-PV}{v}$$

- If the volume increase on increasing the stress the bulk modulus given by

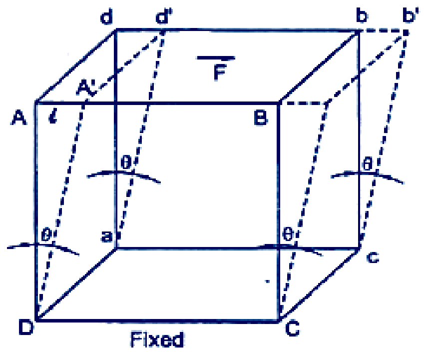
$$K = \frac{PV}{v}$$

- The units of bulk modulus are Pa or N/m² in SI.
- **Compressibility:** The reciprocal of the bulk modulus of a material is called compressibility i.e. **1/K.**

(3) Modulus of Rigidity:

- It is defined as the ratio of tangential stress to shear strain.
- It is also called shear modulus. It is denoted by η.

$$\eta = \frac{Tangential\ stress}{Shear\ strain}$$



- Consider a rectangular block, whose lower face **aDCc** is fixed and the upper face **ABbd** is subjected to tangential force *F.*
- Let '*a*' be the area of the each face and AD = *L* be the perpendicular distance between them.
- The tangential force will displace the upper face of parallelepiped by a distance AA' = *l*.

- If $\angle ADA' = \theta$, then θ is the angle of shear.

$$Tangential\ stress = \frac{F}{a} \qquad and \quad Shear\ strain = Angle\ of\ shear = \theta$$

$$\eta = \frac{\frac{F}{a}}{\theta} = \frac{F}{a\theta}$$

- For solids, angle of shear is very small, so in ΔDAA'

$$\theta \approx tan\theta = \frac{AA'}{AD} = \frac{l}{L}$$

- The distance 'l' through which the upper face has been displaced is called lateral displacement.

$$\therefore \eta = \frac{F\,L}{a\,l}$$

❖ Work done per unit volume in case of elongation strain:

- Consider a wire of length l and area of cross section 'a' suspended from a rigid support.
- Suppose that a normal force 'F' is applied at its free end and its length increases by dl.
- The work done for a small displacement dl is given by

$$dW = F\,dl \qquad \text{………… (1)}$$

- We know that,

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}}$$

$$\therefore F = \frac{Y\,a\,l}{L}$$

Substituting this value of F in above equation (1), we get

$$dW = \frac{Y\,a\,l}{L}\,dl$$

- Therefore, the total work done for the stretching a wire of length 'l' given by,

$$W = \int_0^l dW$$

$$W = \int_0^l \frac{Yal}{L}\,dl$$

$$W = \frac{Ya}{L}\int_0^l l\ dl$$

$$W = \frac{Y\,a}{L}\left(\frac{l^2}{2}\right)$$

$$W = \frac{1}{2}\frac{Y\,a\,l}{L} \times l$$

$$W = \frac{1}{2}\,F \times l$$

$$\therefore Total\ work\ done\ W = \frac{1}{2}\ straching\ force \times change\ in\ length$$

- This work done stored in form of potential energy.

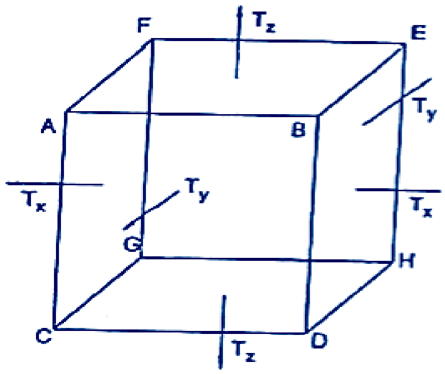
- Now, the volume of the wire = $a\,l$

$$\therefore \textit{Work done per unit volume} = \frac{1}{2}\frac{F \times l}{a \times L}$$

$$= \frac{1}{2}\left(\frac{F}{a}\right) \times \left(\frac{l}{L}\right)$$

$$\therefore \textit{Work done per unit volume of the wire} = \frac{1}{2} \times stress \times strain$$

❖ Deformation of cube –Bulk Modulus:



Let us consider a unit cube ABCDEFGHA. Suppose force T_x, T_y and T_z are acting perpendicular to the faces BEHD and AFGC, ABDC and EFGH, ABEF and DHGC respectively, as shown in figure.

Let 'α' be the increase per unit length per unit tension along the direction of the force and 'β' be the contraction produced per unit length per unit tension direction perpendicular to the force.

Due to the applied force, the elongations produce in the edges AB, BE and BD are $T_x\alpha$, $T_y\alpha$ and $T_z\alpha$ respectively. Similarly, the contraction produced in the perpendicular to these edges will be $T_x\beta$, $T_y\beta$ and $T_z\beta$.

The length of edges after elongation and contraction becomes,

$$AB = 1 + T_x\alpha - T_y\beta - T_z\beta$$

$$BE = 1 + T_y\alpha - T_x\beta - T_z\beta$$

$$BD = 1 + T_z\alpha - T_x\beta - T_y\beta$$

$$V' = AB \times BE \times BD$$

$$V' = (1 + T_x\alpha - T_y\beta - T_z\beta) \times (1 + T_y\alpha - T_x\beta - T_z\beta) \times (1 + T_z\alpha - T_x\beta - T_y\beta)$$

$$V' = 1 + (\alpha - 2\beta)\,(T_x + T_y + T_z)$$

Neglecting squares and products of α and β.

In the case of bulk modulus, the force acting uniformly in all the directions,

Hence, $T_x = T_y = T_z$

$$\therefore V' = 1 + (\alpha - 2\beta)\,3T$$

The original volume of cube is unity; therefore increase in volume of the cube

$$V' = 1 + (\alpha - 2\beta)\,3T - 1$$

$$V' = (\alpha - 2\beta)\,3T$$

If pressure P is applied instead of tension T out words, the cube compressed and the volume decreased by the amount $3P(\alpha - 2\beta)$.

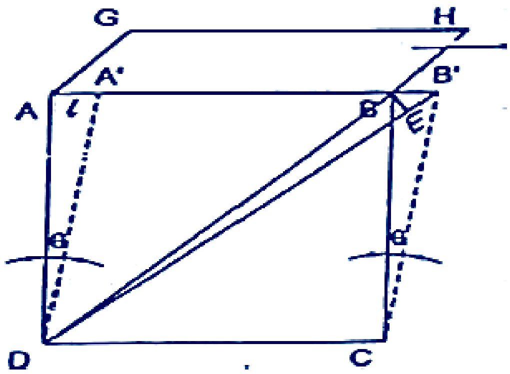
$$\textit{Volume strain} = \frac{\textit{Change in volume}}{\textit{Original volume}} = \frac{3P(\alpha - 2\beta)}{1}$$

$$\textit{Bulk modulus } K = \frac{\textit{Stress}}{\textit{Volume strain}} = \frac{P}{3P(\alpha - 2\beta)}$$

$$\therefore K = \frac{1}{3(\alpha - 2\beta)}$$

$$\textit{Compressibility} = \frac{1}{K} = \frac{1}{3(\propto - 2\beta)}$$

❖ Modulus of rigidity:



Consider a cube with an edge 'L'. Let shearing force $\vec{F}$ be applied on the tope face ABHG of a cube, which produce shear by an angle θ and linear displacement 'l'.
The face ABCD becomes A'B'CD.

$$Tensile\ stess = \frac{F}{area\ of\ face\ ABHG}$$

$$= \frac{F}{L^2} = T$$

$$Shear\ strain = \frac{l}{L}$$

$$\therefore Modulus\ of\ rigidity\ \eta = \frac{Tensile\ stess}{Shear\ strain} = \frac{T}{\theta}$$

A shearing stress along AB is equivalent to a tensile stress along DB and an equal compression stress along CA at right angles.

If α *and* β are the longitudinal and lateral strains per unit stress respectively.

Then extension along diagonal DB due to tensile stress = $DB\ T\ \alpha$ and, extension along diagonal DB due to compression stress along AC = $DB\ T\ \beta$.

Therefore, the total extension along DB = $DB\ T(\alpha + \beta)$

But, from above figure diagonal $DB = \sqrt{L^2 \times L^2}$

$$\therefore DB = \sqrt{2}L$$

Therefore, the total extension of diagonal $EB' = \sqrt{2}\,L\,T(\alpha + \beta)$ .... (1)

In Δ BB'E, $Cos\ BB'E = \frac{EB'}{BB'}$

$$\therefore EB' = BB'\ Cos\ \angle BB'E$$

$$But, BB' = l\ and\ \angle BB'E = 45^0$$

$$\therefore EB' = l\ Cos\ 45^0$$

$$\therefore EB' = \frac{l}{\sqrt{2}}$$ .... (2)

Now, comparing equation(1) and (2), we get

$$\therefore \frac{l}{\sqrt{2}} = \sqrt{2}\,L\,T(\alpha + \beta)$$

$$\therefore \frac{L\,T}{l} = \frac{1}{2(\alpha + \beta)}$$

$$\therefore \frac{T}{\frac{l}{L}} = \frac{1}{2(\alpha + \beta)}$$

$$\therefore \frac{T}{\theta} = \frac{1}{2(\alpha + \beta)}$$

$$\therefore \eta = \frac{1}{2(\alpha + \beta)}$$

❖ Young's Modulus:

Let us consider unit tension applied on the edge of the unit cube, which produces the extension 'α' linear stress =1 and linear strain = $\frac{\alpha}{1}$ = α.

$$Young's\ modulus = Y = \frac{1}{\alpha}$$

❖ Relation connecting the Elastic Constants:

We know that

$$K = \frac{1}{3(\alpha - 2\beta)}$$

and

$$\eta = \frac{1}{2(\alpha + \beta)}$$

$$(\alpha - 2\beta) = \frac{1}{3K} \qquad \dots\dots\dots\dots (1)$$

$$(\alpha + \beta) = \frac{1}{2\eta} \qquad \dots\dots\dots\dots (2)$$

Subtracting (1) and (2)

$$3\beta = \frac{1}{2\eta} - \frac{1}{3K}$$

$$3\beta = \frac{3K - 2\eta}{6\eta K}$$

$$\beta = \frac{3K - 2\eta}{18\eta K} \qquad \dots\dots\dots\dots (3)$$

Multiplying equation(2) by 2 and adding equations (1) and (2) we get,

$$3\alpha = \frac{1}{\eta} + \frac{1}{3K}$$

$$3\alpha = \frac{3K + \eta}{3K\eta}$$

$$\alpha = \frac{3K + \eta}{9K\eta} \qquad \dots\dots\dots\dots (4)$$

Form equation of young's modulus,

$$Y = \frac{1}{\alpha} \quad i.e\ \alpha = \frac{1}{Y} \qquad \dots\dots\dots\dots (5)$$

Using equation (5) in (4)

$$\frac{1}{Y} = \frac{3K + \eta}{9K\eta}$$

$$\therefore \frac{9}{Y} = \frac{3K}{K\eta} + \frac{\eta}{K\eta}$$

$$\therefore \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K} \qquad \dots\dots\dots\dots (6)$$

The above equation gives the relation connecting the three elastic constants *Y, K* and η .

❖ Poisson's Ratio:

When we stretch a wire, it becomes longer but thinner. The increase in its length is always accompanied with decrease in its cross section.

The strain produced along the direction of the applied force is called *primary or linear or tangential strain* (α) and strain produced at right angle to the applied force is called *secondary or lateral strain* (β).

Within the elastic limit, the lateral strain (β) is proportional to the linear strain (α) and the ratio between them is a constant, called Poisson's ratio (σ).

$$\sigma = \frac{Lateral\ strain}{linear\ strain} = \frac{\beta}{\alpha}$$

If the body under tension suffers no lateral strain then Poisson's ratio is zero.

❖ Limiting values of 'σ' :

We know that,

$$3K(1 - 2\sigma) = 2\eta\,(1 + \sigma)$$

Where, K and η are essentially positive quantities.

➢ Now if σ is positive, then the RHS and hence LHS must be positive.
This is true, if $1 - 2\sigma > 0$

$$\therefore 2\sigma < 1$$

$$\therefore \quad \sigma < \frac{1}{2}$$

$$\therefore \sigma < 0.5 \qquad \text{.... .... (1)}$$

➢ If σ is negative, then the LHS and hence RHS must be positive.
This is true, if $1 + \sigma > 0$

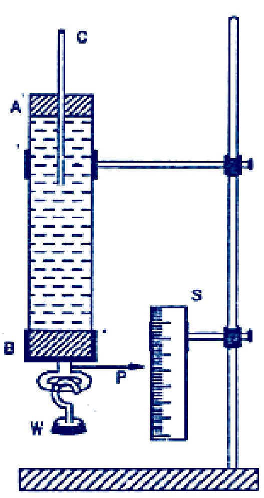
$$\therefore \quad \sigma > -1$$

$$\therefore \; -1 < \sigma \qquad \text{.... .... (2)}$$

Combining relation (1) and (2), we have

$$-1 < \sigma < 0.5 \qquad \text{.... .... (3)}$$

Thus the limiting values of σ are -1 and 0.5. In actual practice, the value of σ lie between 0.2 to 0.4.



To determine the value of σ for rubber, we take about a meter long tube AB and suspended vertically as shown in figure. Its two ends are property stopper with rubber corks and liquid glue. A glass tube C of half meter long and 1 cm in diameter is fitted vertically into the cork A through a suitable hole. A suitable weight W is then suspended from the lower end of the tube. This will increase the length and the internal volume of the tube.

It results in the fall of the level of meniscus in glass tube C. Both the increase in length (dL) and the decrease in the meniscus level (dh) are measured.

Let L, D and V be the original length, diameter and volume of the tube respectively. Then the area of cross-section of tube is

$$A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{2} \qquad \text{.... .... (1)}$$

Differentiating above equation, We have

$$dA = \frac{\pi}{4} 2D\,dD = \frac{\pi D}{2} dD$$

$$\therefore\; dA = \frac{\pi D}{2} dD \;\frac{D}{2}\;\frac{2}{D}$$

$$\therefore\; dA = \frac{\pi D^2}{4} dD \;\frac{2}{D}$$

$$\therefore\; dA = \frac{2A\,dD}{D} \qquad \text{.... .... (2)}$$

Now, the increase in length of rubber tube dL and the increase in volume dV are accompanied with the decrease in area of cross section dA.

Volume = area of cross section × length

$$V + dV = (A - dA)(L + dL)$$

$$\therefore\; V + dV = A\,L + A\,dL - dA\,L - dA \cdot dL \qquad \text{.... .... (3)}$$

Neglecting $dA \cdot dL$ being very small.

We have,

$$V + dV = A\,L + A\,dL - dA\,L$$
$$\therefore\ V + dV = V + A\,dL - dA\,L$$
$$\therefore\ dV = A\,dL - dA\,L \qquad \text{.... .... (4)}$$

Substituting the value of dA, we get,

$$dV = A\,dL - \frac{2A\,dD}{D}\,L \qquad \text{.... .... (5)}$$

Dividing by dL on both sides,

$$\frac{dV}{dL} = A - \frac{2AL}{D}\frac{dD}{dL}$$

$$\therefore\ \frac{2AL}{D}\frac{dD}{dL} = A - \frac{dV}{dL}$$

$$\therefore\ \frac{dD}{dL} = \frac{D}{2AL}\left[A - \frac{dV}{dL}\right]$$

$$\therefore\ \frac{dD}{dL} = \frac{D}{2L}\left[\frac{A}{A} - \frac{1}{A}\frac{dV}{dL}\right]$$

$$\therefore\ \frac{dD}{dL} = \frac{D}{2L}\left[1 - \frac{1}{A}\frac{dV}{dL}\right] \qquad \text{.... .... (6)}$$

Now Poisson's ratio is given by

$$\sigma = \frac{Lateral\ Strain}{Liner\ Strain} = \frac{dD/_D}{dL/_L}$$

$$\therefore\ \sigma = \frac{L}{D} \times \frac{dD}{dL} \qquad \text{.... .... (7)}$$

Substituting the value of $dD/_{dL}$ from equation (6) in (7), we get

$$\sigma = \frac{L}{D} \times \frac{D}{2L}\left[1 - \frac{1}{A}\frac{dV}{dL}\right]$$

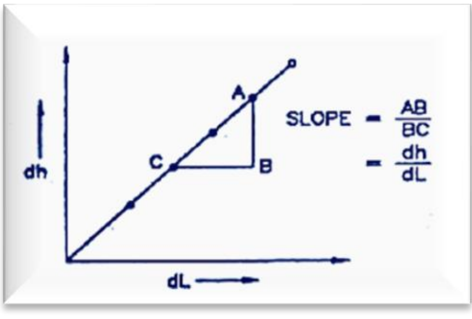
$$\therefore\ \sigma = \frac{1}{2}\left[1 - \frac{1}{A}\frac{dV}{dL}\right] \qquad \text{.... .... (8)}$$

If r be the internal radius of the tube, so that $A = \pi r^2$

$$\therefore\ \sigma = \frac{1}{2}\left[1 - \frac{1}{\pi r^2}\frac{dV}{dL}\right] \qquad \text{.... .... (9)}$$

If 'a' be the internal radius of the capillary tube, we have $dV = \pi a^2 \cdot dh$

$$\therefore\ \sigma = \frac{1}{2}\left[1 - \frac{1}{\pi r^2}\frac{\pi a^2 \cdot dh}{dL}\right]$$



$$\therefore\ \sigma = \frac{1}{2}\left[1 - \frac{a^2}{r^2}\frac{dh}{dL}\right] \qquad \text{.... .... (10)}$$

The value of 'a' and 'r' are determined by a travelling microscope and a vernier caliper respectively and the average value of $dh/_{dL}$ is obtained from the slope of the straight line graph by plotting a number of corresponding value of dh against the dL as shown in fig.

Multiple Choice Questions:

(1) Hooke's law essentially defines _____
(a) Stress (b) Strain
(c) Yield point (d) Elastic limit

(2) The dimensional formula of stress is _____
(a) $[M^0L^1T^2]$ (b) $[M^1L^{-1}T^{-2}]$
(c) $[M^1L^{-1}T^{-2}]$ (d) $[M^1L^{-1}T^{-1}]$

(3) The nearest approach to the perfectly elastic body is _____
(a) Quarts fibre (b) Putty
(c) Silver (d) Platinum

(4) _____ is the perfectly plastic material
(a) Quarts fibre (b) Putty
(c) Silver (d) Platinum

(5) The restoring force per unit area is called _____
(a) Stress (b) Strain
(c) Elasticity (d) Plasticity

(6) The change per unit dimension of the body is called _____
(a) Stress (b) Strain
(c) Elasticity (d) Plasticity

(7) The restoring force per unit area perpendicular to the surface is called _____
(a) Longitudinal Stress (b) Tangential Stress
(c) Normal Stress (d) Tensile Stress

(8) The restoring force per unit area perpendicular to the surface is called _____
(a) Longitudinal Stress (b) Lateral Stress
(c) Normal Stress (d) Tensile Stress

(c) Volume strain (d) Shear strain

(10) Compressibility of a material is reciprocal of _________
(a) Modulus of rigidity (b) Young Modulus
(c) Bulk Modulus (d) Coefficient of rigidity

(11) The work done per unit volume in stretching the wire is equal to _____
(a) Stress x strain (b) (1/2) stress x strain
(c) Stress / strain (d) Strain / stress

(12) When a body undergoes a linear tensile strain it experiences a lateral contradiction also. The ratio of lateral strain to longitudinal strain is known as:
(a) Young's modulus (b) Bulk modulus
(c) Poisson's ratio (d) Hooke's Law

(13) Theoretical value of Poisson's ratio lies between ______
(a) -1 and + 0.5 (b) - 1 and -2
(c) -0.5 and +1 (d) -1 and 0

(14) Which of the following relations is true?
(a) $3\alpha = \dfrac{3K - \eta}{3K\eta}$ (b) $3\alpha = \dfrac{3K + \eta}{3K\eta}$
(c) $\alpha = \dfrac{3K + \eta}{3K\eta}$ (d) $\alpha = \dfrac{3K - \eta}{3K\eta}$

(15) The relationship between Y, η and σ is _______
(a) $Y = 2\eta\,(1 + \sigma)$ (b) $\eta = 2Y\,(1 + \sigma)$
(c) $\sigma = 2Y / (1 + \eta)$ (d) $Y = \eta\,(1 + \sigma)$

(16) Which of the relation is true?
(a) $\sigma = \dfrac{3K - 2\eta}{6K + 2\eta}$ (b) $\sigma = \dfrac{3K + 2\eta}{6K - 2\eta}$
(c) $\sigma = \dfrac{6K + 2\eta}{6K - 2\eta}$ (d) $\sigma = \dfrac{2K + 3\eta}{2K - 6\eta}$

(17) The Poisson's ratio cannot have the value ______
(a) 0.7 (b) 0.2
(c) 0.1 (d) -0.52

(18) The Poisson's ratio can have the value _______
(a) 0.7 (b) -1.1
(c) 1.0 (d) 0.49

(19) Units of modulus of elasticity is _____
(a) dyne/cm (b) $dyne/cm^2$
(c) N/m (d) Dyne

(20) The relation between K, α and β is _______
(a) $K = \dfrac{1}{2(\alpha - 2\beta)}$ (b) $K = \dfrac{1}{3(\alpha - 2\beta)}$
(c) $K = \dfrac{1}{(\alpha - 2\beta)}$ (d) $K = \dfrac{1}{3(\alpha + 2\beta)}$

(21) In Bulk modulus, there is a change in the volume of the body but no change in ___
(a) Size (b) Shape
(c) Line (d) Angle

(22) Increase in the length of a wire is always accompanied by a decrease in ______
(a) Length (b) Breadth
(c) Cross section (d) Height

(23) The ratio of Longitudinal stress to linear strain is called ______
(a) Modulus of rigidity (b) Young Modulus
(c) Bulk Modulus (d) Coefficient of rigidity

(24) The ratio of Tensile stress to shear strain is called ______
(a) Modulus of rigidity (b) Young Modulus
(c) Bulk Modulus (d) Poisson's ratio

Answer key of MCQ:

(1)	(d)	(2)	(c)	(3)	(a)	(4)	(b)
(5)	(a)	(6)	(b)	(7)	(c)	(8)	(d)
(9)	(b)	(10)	(c)	(11)	(b)	(12)	(c)
(13)	(a)	(14)	(b)	(15)	(a)	(16)	(a)
(17)	(a)	(18)	(d)	(19)	(b)	(20)	(b)
(21)	(b)	(22)	(c)	(23)	(b)	(24)	(a)