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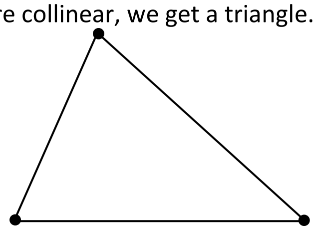
1. Quadrilateral

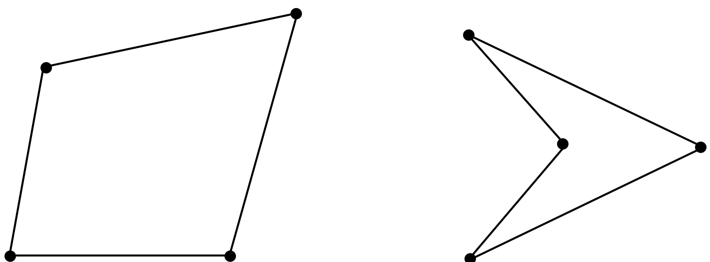
We know that the figure obtained on joining three non-collinear points in pairs is a triangle. If we mark four points and join them in some order, then there are three possibilities for the figure obtained:

(i) If all the points are collinear (in the same line), we obtain a line segment.

(ii) If three out of four points are collinear, we get a triangle.



(iii) If no three points out of four are collinear, we obtain a closed figure with four sides.

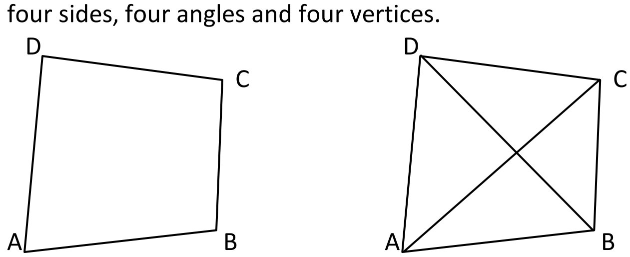


(i) (ii)

Each of the figure obtained by joining four points in order, as in case (iii), is called a quadrilateral. (quad means four and lateral for sides).

Constituents of a quadrilateral

A quadrilateral has four sides, four angles and four vertices.



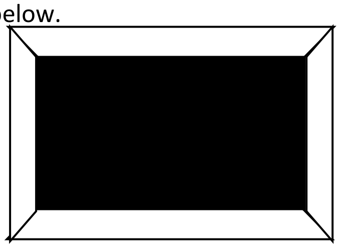
(i) (ii)

In quadrilateral ABCD, AB, BC, CD and DA are the four sides: A, B, C and D are the four vertices and ∠A, ∠B, ∠C and ∠D are the four angles formed at the vertices.

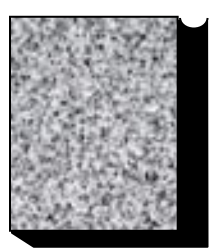
If we join the opposite vertices A to C and B to D, then AC and BD are the two diagonals of the quadrilateral ABCD.

3. Quadrilaterals in Practical Life

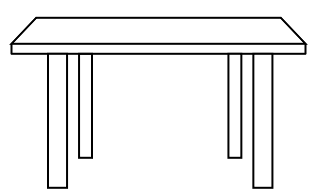
We find so many objects around us which are of the shape of a quadrilateral – the floor, walls, ceiling, windows of our classroom, the blackboard, each face of the duster, each page of our mathematics book, the top of our study table, etc. Some of these are given below.



Blackboard



Book



Table

4. Angle Sum Property of a Quadrilateral

The sum of the angles of a quadrilateral is 360°.

Verification:

Let ABCD be a quadrilateral and AC be a diagonal.

In ∆ADC,

$\angle DAC + \angle ACD + \angle D = 180^\circ$ ... (1) [Sum of all the angles of a triangle is 180°]

In ΔABC,

$\angle CAB + \angle ACB + \angle B = 180^\circ$ ... (2) [Sum of all the angles of a triangle is 180°]

Adding (1) and (2), we get

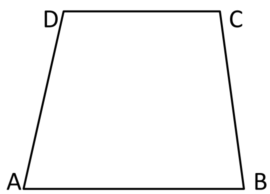
$\angle DAC + \angle ACD + \angle D + \angle CAB + \angle ACB + \angle B = 180^\circ + 180^\circ = 360^\circ$

Also, $\angle DAC + \angle CAB = \angle A$ and $\angle ACD + \angle ACD + \angle ACB = \angle C$

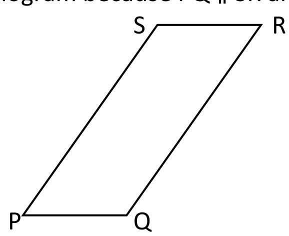
So, $\angle A + \angle D + \angle B + \angle C = 360^\circ$ **i.e., the sum of the angles of a quadrilateral is 360°.**

5. Types of Quadrilaterals

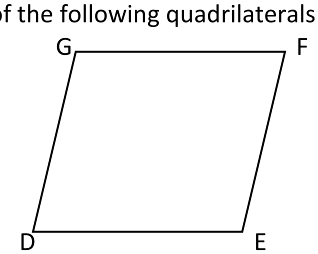
(i) Trapezium: A quadrilateral, in which exactly one pair of opposite sides is parallel, is called a trapezium. In the adjacent figure, the quadrilateral ABCD is a trapezium because exactly one pair of its opposite sides namely, AB and CD are parallel.

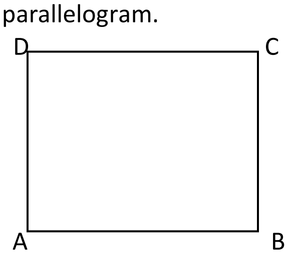


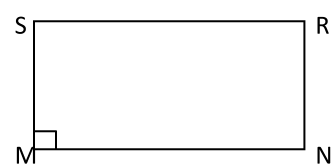
(ii) Parallelogram: A quadrilateral, in which both pairs of opposite sides are parallel, is called a parallelogram. In the adjacent figure, the quadrilateral PQRS is a parallelogram because PQ ∥ SR and PS ∥ QR.



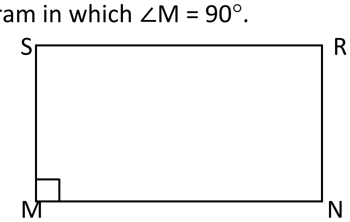
Similarly, each of the following quadrilaterals is a parallelogram.



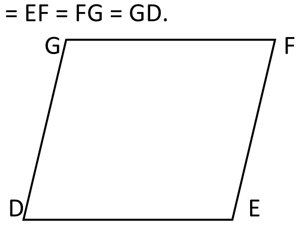




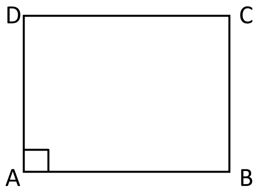
(iii) Rectangle: It is a special parallelogram whose one angle is a right angle. In the following figure, the quadrilateral MNRS is a rectangle because MNRS is a parallelogram in which $\angle M = 90^\circ$.



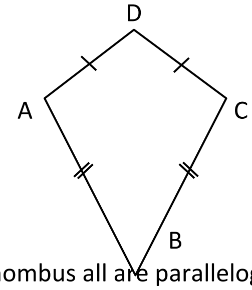
(iv) Rhombus: It is a special parallelogram whose all sides are equal. In the following figure, the quadrilateral DEFG is a rhombus because DEFG is a parallelogram in which DE = EF = FG = GD.



(v) Square: It is a special parallelogram in which all sides are equal and one angle is right angle. In the following figure, the quadrilateral ABCD is a square because ABCD is a parallelogram in which AB = BC = CD = DA and $\angle A = 90^\circ$.



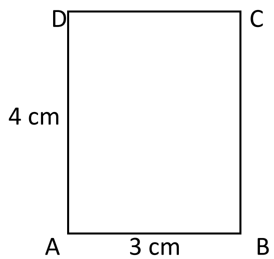
(vi) Kite: A quadrilateral, in which two pairs of adjacent sides are equal, is called a kite. In the following figure, quadrilateral ABCD is a kite because AB = BC and CD = AD.



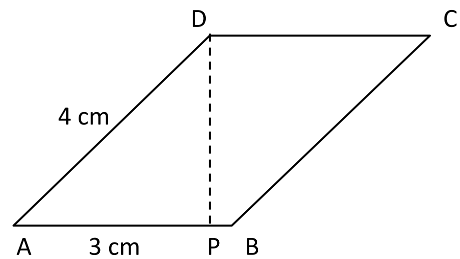
●●●: (i) A square, rectangle and rhombus all are parallelograms.
●●●: (ii) A square is a rectangle and also a rhombus.
●●●: (iii) A parallelogram is a trapezium.
●●●: (iv) A kite is not a parallelogram.
●●●: (v) A trapezium is not a parallelogram (as only one pair of opposite sides is parallel in a trapezium and we require both pairs to be parallel in a parallelogram).

●●●:Out of rectangle and a parallelogram with same perimeter, the area of the parallelogram is less than the area of the rectangle. For example: Consider a rectangle and a parallelogram with same perimeter 14 cm as shown below:

(i)



(ii)

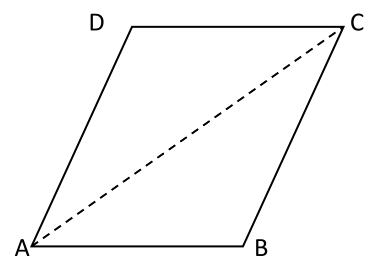


Area of the parallelogram = DP × AB
Area of the rectangle = AB × AD
∵ DP < AD ∴ Area of the parallelogram < area of the rectangle

7. Properties of a Parallelogram

●●●:*(i)* A diagonal of a parallelogram divides it into two congruent triangles

Verification by activity



Cut out a parallelogram from a sheet of paper and cut it along a diagonal. We obtain two triangles

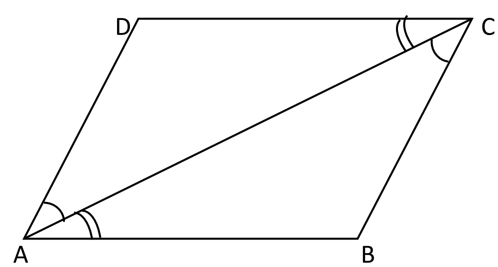
Place one triangle over the other. Turn one around, if necessary.

We observe that the two triangles are congruent to each other.

Repeat this activity with some more parallelograms. Each time we will observe that each diagonal divides the parallelogram into two congruent triangles.

Verification by exhaustion

Given: ABCD is a parallelogram and AC is a diagonal. The diagonal AC divides parallelogram ABCD into two triangles, namely ΔABC and ΔCDA.



To prove: ΔABC ≅ ΔCDA

Proof: In ΔABC and ΔCDA,

∵ BC ∥ AD

an AC is a transversal.

∴ ∠BCA = ∠DAC ... (1) [Pair of alternate angles]

∵ AB ∥ DC

an AC is a transversal.

∴ ∠BAC = ∠DCA ... (2) [Pair of alternate angles]

AC = CA ... (3) [Common]

∴ ∆ABC ≅ ∆CDA [ASA Rule]

=> Diagonal AC divides parallelogram ABCD into two congruent triangles ABC and CDA.

●●●: *(ii)* In a parallelogram, opposite sides are equal

Proof: ∆ABC ≅ ∆CDA [Proved above]

∴ AB = CD [c.p.c.t.]

and BC = DA [c.p.c.t.]

●●●: *(iii)* If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram

In ∆ABC and ∆CDA,

AB = CD [Given]

BC = DA [Given]

AC = CA [Common]

∴ ∆ABC ≅ ∆CDA [SSS Rule]

∴ ∠BAC = ∠DCA [c.p.c.t.]

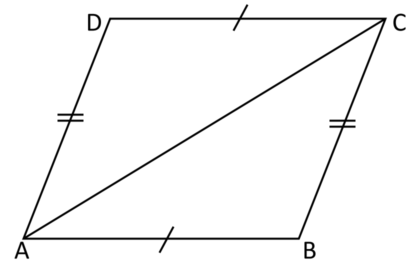
and ∠BCA = ∠BAC [c.p.c.t.]

These give respectively

AB ∥ DC

and AD ∥ BC

∴ Quadrilateral ABCD is a parallelogram [A quadrilateral is a parallelogram if its pairs of opposite sides are parallel]



●●●: *(iv)* In a parallelogram, opposite angles are equal

Verification by activity

Draw a parallelogram and measure its angles. We observe that each pair of opposite angles is equal. Repeat this with some more parallelogram. We arrive at the same result.

Verification by exhaustion

∠BAC = ∠DCA [Proved above]

∠BCA = ∠DAC

∴ ∠BAC + ∠DAC = ∠DCA + ∠BCA

=> ∠A = ∠C Similarly, we can prove that ∠B = ∠D

●●●: *(v)* If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

Proof: ∠A = ∠C

=> ½ ∠A = ½ ∠C [Halves of equals are equal]

=> ∠BAC = ∠DCA

But these are alternate angles

∴ AB ∥ DC

Similarly, we can prove that AD ∥ BC [taking ∠B = ∠D]

∴ ABCD is a parallelogram

●●●: *(vi)* The diaGonals of a parallelogram bisect each other

Verification by activity

Draw a parallelogram ABCD and draw both its diagonal intersecting at the point O.

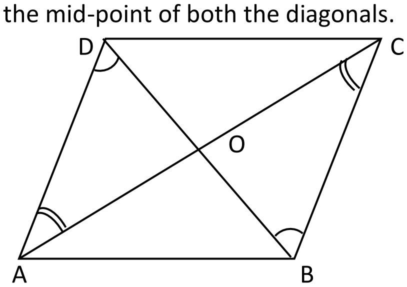
Measure the lengths of OA, OB, OC and OD.

We observe that OA = OC and OB = OD

or, O is the mid-point of both the diagonals.

Repeat this activity with some more parallelograms.

Each time we will find that O is the mid-point of both the diagonals.



Verification by exhaustion

In ∆AOD and ∆COB,

∠DAO = ∠BCO [Alt. Int. ∠S ∵ AD ∥ BC and AC intersects them

AD = CB [Opp. sides of ∥gm]

$\angle ADO = \angle CBO$ [Alt. Int. ∠S ∵ AD ∥ BC and BD intersect them

∴ $\Delta AOD \cong \Delta COB$ [ASA Rule]

∴ OA = OC [c.p.c.t.]

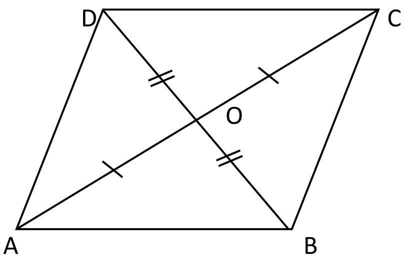
and OD = OB [c.p.c.t.]

●●●: (vii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram

Given: In quadrilateral ABCD, OA = OC and OB = OD

To prove: ABCD is a ∥gm.

Proof: In ΔAOD and ΔCOB



AO = CO [Given]

OD = OB [Given]

$\angle AOD = \angle OCB$ [Vert. Opp. ∠s]

∴ $\Delta AOD \cong \Delta COB$ [SAS Rule]

∴ $\angle OAD = \angle OCB$ [c.p.c.t.]

But these are alternate interior angles and they are equal.

∴ AD ∥ BC

Similarly, we can prove that

AB ∥ CD Therefore ABCD is a parallelogram.

●●●: (*viii*) Another condition for a quadrilateral to be a parallelogram

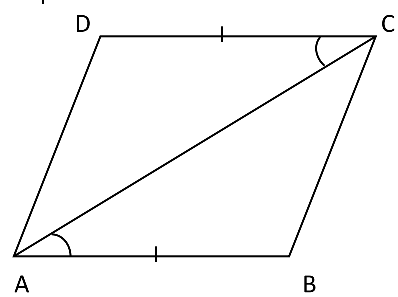
Theorem: A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Given: ABCD is a quadrilateral in which AB = CD and AB ∥ CD.

To Prove: Quadrilateral ABCD is a parallelogram.

Construction: Draw a diagonal AC.

Proof: In ΔABC and ΔCDA,



AB = CD ... (1) [Given]

AC = CA ... (2) [Common]

$\angle BAC = \angle DCA$ ... (3)

[Alt. Int. ∠s ∵ AB ∥ DC and AC intersects them]

∴ $\Delta ABC \cong \Delta CDA$ [SAS Rule]

∴ $\angle BCA = \angle DAC$

But these are alternate interior angles and they are equal

∴ AD ∥ BC

Thus, AB ∥ CD and AD ∥ BC

Therefore, quadrilateral ABCD is a parallelogram.

a e p'S. ILLUSTRATIVE EXAMPLES (Exercise – 1)

●● *Q. 1. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.*

Sol. Let ABCD be a quadrilateral in which

$\angle A : \angle B : \angle C : \angle D = 3 : 5 : 9 : 13$

Sum of the ratios = 3 + 5 + 9 + 13 = 30

Also, $\angle A + \angle B + \angle C + \angle D = 360^\circ$ [Sum of all the angles of a quadrilateral is 360°]

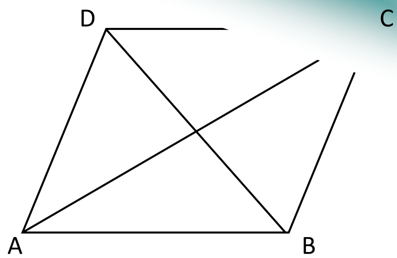
∴ $\angle A = \frac{3}{30} \times 360^\circ = 36^\circ$

$\angle B = \frac{5}{30} \times 360^\circ = 60^\circ$

$\angle C = \frac{9}{30} \times 360^\circ = 108^\circ$ and $\angle D = \frac{13}{30} \times 360^\circ = 156^\circ$

●● *Q. 2. If the diagonals of a parallelogram are equal, then slow that it is a rectangle.*

Sol. **Given:** In parallelogram ABCD, AC = BD



To prove: ∥gm ABCD is a rectangle.

Proof: In ΔACB and ΔBDA,

AC = BD [Given]

AB = BA [Common]

BC = AD [Opposite sides of ∥gm ABCD]

∴ ΔACB ≅ ΔBDA [SSS Rule]

∴ ∠ABC = ∠BAD ... (1) [c.p.c.t.]

Again, ∵ AD ∥ BC [Opp. sides of ∥gm ABCD and transversal AB intersects them.]

∴ ∠BAD + ∠ABC = 180° ... (2) [Sum of consecutive interior angles on the same side of the transversal is 180°]

From (1) and (2),

∠BAD = ∠ABC = 90°

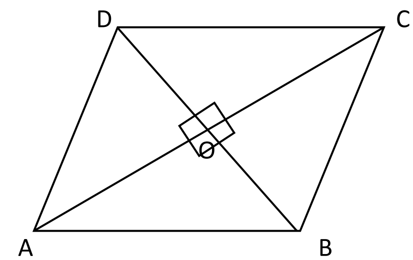
∴ ∠A = 90 ∴ ∥ gm ABCD is a rectangle.

Q. 3. ***Show that if the diagonals of a quadrilateral bisect each at right angles, then it is a rhombus.***

Sol. **Given:** ABCD is a quadrilateral where diagonals AC and BD intersect each other at right angles at O.

To prove: Quadrilateral ABCD is a rhombus.

Proof: In ΔAOB and ΔAOD,



AO = AO [Common]

OB = OD [Given]

∠AOB = ∠AOD [Each = 90°]

∴ ΔAOB ≅ ΔAOD [SAS Rule]

∴ AB = AD ... (1) [c.p.c.t.]

Similarly, we can prove that

AB = BC ... (2)

BC = CD ... (3)

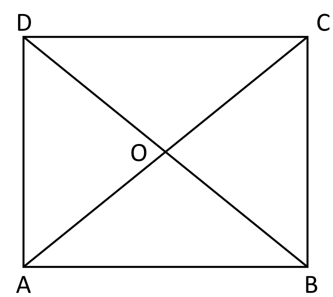
CD = AD ... (4)

In view of (1), (2), (3) and (4), we obtain AB = BC = CD = DA ∴ Quadrilateral ABCD is a rhombus.

Q. 4. ***Show that the diagonals of a square are equal and bisect each other at right angles.***

Sol. **Given:** ABCD is a square.

To prove: (i) AC = BD



(ii) AC and BD bisect each other at right angles.

Proof: (i) In ΔABC and ΔBAD,

AB = BA [Common]

BC = AD [Opp. sides of square ABCD]

∠ABC = ∠BAD [Each = 90° (∵ ABCD is a square)]

∴ ΔABC ≅ ΔBAD [SAS Rule]

∴ AC = BD [c.p.c.t.]

(ii) In ΔOAD and ΔOCB,

AD = CB [Opp. sides of square ABCD]

∠OAD = ∠OCB ∵ AD ∥ BC and transversal AC interests them

∠ODA = ∠OBC ∵ AD ∥ BC and transversal BD intersects them

∴ ΔOAD ≅ ΔOCB [ASA Rule]

∴ OA = OC ... (1)

Similarly, we can prove that

OB = OD ... (2) In view of (1) and (2), AC and BD bisect each other

Again, in ΔOBA and ΔODA,

OB = OD [From (2) above]

BA = DA [Opp. sides of square ABCD]

OA = OA [Common]

∴ ΔOBA ≅ ΔODA [SSS Rule] ∴ ∠AOB = ∠AOD [c.p.c.t.]

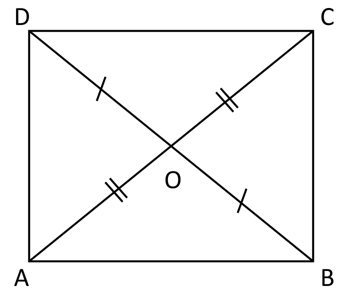
But ∠AOB + ∠AOD = 180° [Linear Pair Axiom]

∴ ∠AOB = ∠AOD = 90° ∴ AC and BD bisect each other at right angles

Q. 5. ***Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.***

Sol. **Given:** The diagonals AC and BD of a quadrilateral ABCD and equal and bisect each other at right angles.

To prove: Quadrilateral ABCD is a square.



Proof: In ∆OAD and ∆OCB,

OA = OC [Given]
OD = OB [Given]
∠AOD = ∠COB [Vertically Opposite angles]
∴ ∆OAD ≅ ∆OCB [SAS rule]
∴ AD = CB [c.p.c.t.]
∠ODA = ∠OBC [c.p.c.t.]
∴ ∠ODA = ∠OBC
∴ AD ∥ BC

Now, ∵ AD = CB and AD ∥ CB ∴ Quadrilateral ABCD is a ∥gm.

In ∆ AOB and ∆AOD,

AO = AO [Common]
OB = OD [Given]
∠AOB = ∠AOD [Each = 90° (Given)]
∴ ∆AOB ≅ ∆AOD [SAS Rule]
∴ AB = AD

Now, ∵ ABCD is a parallelogram and AB = AD ∴ ABCD is a rhombus.

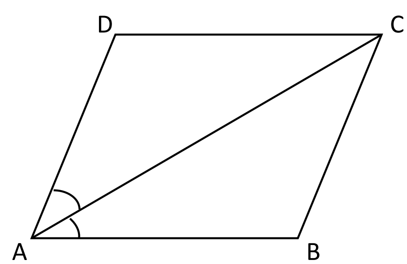
Again, in ∆ABC and ∆BAD, AC = BD [Given]
BC = AD [∵ ABCD is a rhombus]
AB = BA [Common]
∴ ∆ABC ≅ ∆BAD [SSS Rule]
∴ ∠ABC = ∠BAD [Opp. sides of ∥ gm ABCD and transversal AB intersects them.]
∴ ∠ABC + ∠BAD = 180° [Sum of consecutive interior angles on the same side of the transversal is 180°]
∴ ∠ABC = ∠BAD = 90°
Similarly, ∠BCD = ∠ADC = 90° ∴ ABCD is a square.

●●Q. 6. *Diagonal AC of a parallelogram ABCD bisects ∠A (see figure). Show that:*
(i) it bisects ∠C also. (ii) ABCD is a rhombus.

Sol. **Given:** Diagonal AC of a parallelogram ABCD bisects ∠A.

To Prove: (i) it bisects ∠C also.

(ii) ABCD is a rhombus.



Proof: (i) In ∆ADC and ∆CBA,

AD = CB [Opp. sides of ∥gm ABCD]
CA = CA [Common]
DC = BA [Opp. sides of ∥gm ABCD]
∴ ∆ADC ≅ ∆CBA [SSS Rule]
∴ ∠ACD = ∠CAB [c.p.c.t.]
and ∠DAC = ∠BCA [c.p.c.t.]
But ∠CAB = ∠DAC [Given]
∴ ∠ACD = ∠BCA ∴ AC bisects ∠C also.

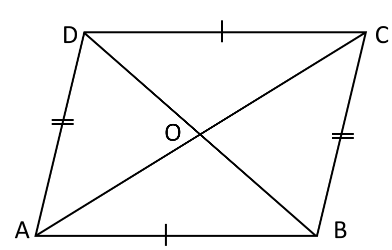
(ii) From above, ∠ACD = ∠CAD
∴ AD = CD [Opposite sides of equal angles of a triangle are equal]
∴ AB = BC = CD = DA [∵ ABCD is a ∥gm] ∴ ABCD is a rhombus.

●●Q. 7. *ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.*

Sol. **Given:** ABCD is a rhombus.

To prove: (i) Diagonal AC bisects ∠A as well as ∠C.

(ii) Diagonal BD bisects ∠B as well as ∠D.



Proof: ABCD is a rhombus

∴ AD = CD
∴ ∠DAC = ∠DCA ... (1)
[Angles opposite to equal sides of a triangle are equal]

Also, CD ∥ AB
and transversal AC intersects them

∴ ∠DAC = ∠BCA ... (2) [Alt. Int. ∠S]

From (1) and (2)

∠DCA = ∠BCA

=> AC bisects ∠C

Similarly, AC bisects ∠A (ii) proceeding similarly as in (i) above, we can prove that BD bisects ∠B as well as ∠D.

Q. 8. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that (i) ABCD is a square (ii) diagonal BD bisects ∠B as well as ∠D.

Sol. **Given:** ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.

To prove: (i) ABCD is a square.

(ii) diagonal BC bisects ∠B as well as ∠D.

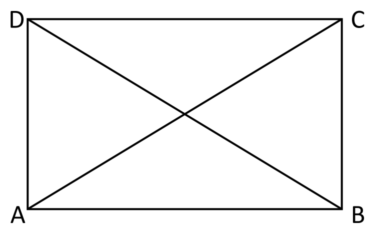
Proof: (i) ∵ AB ∥ DC and transversal AC intersects them.

∴ ∠ACD = ∠CAB [Alt. Int. ∠S]

But ∠CAB = ∠CAD ∴ ∠ACD = ∠CAD

∴ AD = CD [Sides opposite to equal angles of a triangle are equal]

∴ ABCD is a square



(ii) In ΔBDA and ΔDBC,

BD = DB [Common]

DA = BC [Sides of a square ABCD]

AB = DC [Sides of a square ABCD]

∴ ΔBDA ≅ ΔDBC [SSS Rule]

∴ ∠ABD = ∠CDB [c.p.c.t.]

But ∠CDB = ∠CBD [∵ CB = CD (sides of a square ABCD)]

∴ ∠ABD = ∠CBD

∴ BD bisects ∠B

Now, ∠ABD = ∠CBD

∠ABD = ∠ADB [∵ AB = AD]

∠ABD = ∠ADB [∵ CB = CD]

∴ ∠ADB = ∠CDB

∴ BD bisects ∠D.

Q. 9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see figure). Show that:

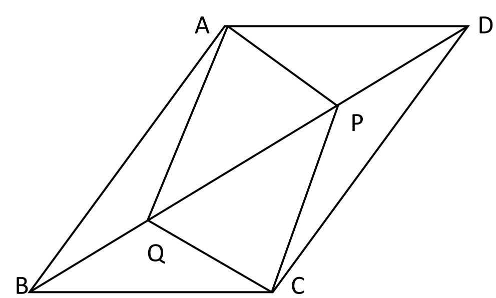
(i) ΔAPD ≅ ΔCQB

(ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram.



Sol. **Given:** In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ

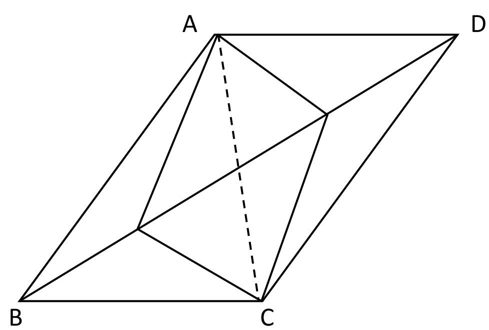
To prove: (i) ΔAPD ≅ ΔCQB

(ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram.



Construction: Join AC to intersect BD at O.

Proof: (i) In ΔAPD and ΔCQB,

AP = CQ [From (ii)]

PD = QB [Given]

AD = CB [Opposite sides of ∥gm ABCD]

∴ ΔAPD ≅ ΔCQB [SSS Rule]

(ii) ∵ APCQ is a ∥gm [Proved in (i) above]

∴ AP = CQ

(iii) In ΔAQB and ΔCPD,

AQ = CP [From (iii)]

QB = CD [Given]

AB = CD [Opp. sides of ∥gm ABCD]

∴ ΔAQB ≅ ΔCPD [SSS Rule]

(iv) ∵ APCQ is a ∥gm [Proved in (i) above]

∴ AQ = CP [Opp. sides of a ∥gm is equal]

(v) ∵ The diagonals of a parallelogram bisect each other.

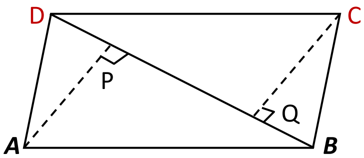
∴ OB = OD

∴ OB – BQ = OD – DP [∵ BQ = DP (given)]

Also, OA = OC ... (2) [∵ Diagonals of a ∥ gm bisects each other]

In view of (1) and (2), APCQ is a parallelogram.

Q. 10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively (See figure) Show that:
(i) ΔAPB ≅ ΔCQD
(ii) AP = CQ



Sol. Given: ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.

To prove: (i) ΔAPB ≅ ΔCQD

(ii) AP = CQ

Proof: (i) In ΔAPB and ΔCQD

AB = CD [Opp. sides of ∥gm ABCD]

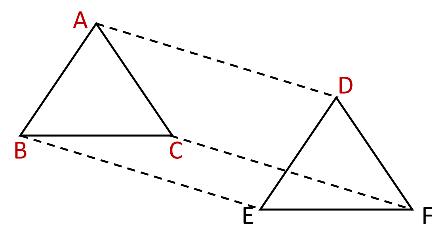
∠ABP = ∠CDQ [∵ AB ∥ DC and transversal BD intersects them]

∠APB = ∠CQD [Each = 90°]

∴ ΔAPB ≅ ∠CQD [AAS Rule]

(ii) ∵ ΔAPB ≅ ΔCQD [Proved above in (i)] ∴ AP = CQ

Q. 11. In ΔABC and ΔDEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that:
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD ∥ CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) ΔABC ≅ ΔDEF



Sol. **Given:** In ΔABC and ΔDEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively.

To prove: (i) quadrilateral ABED is a parallelogram

(ii) Quadrilateral BECF is a parallelogram

(iii) AD ∥ CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF.

Proof: (i) In quadrilateral ABED,

AB = DE and AB ∥ DE [Given]

∴ quadrilateral ABED is a parallelogram. | ∵ A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length

(ii) In quadrilateral BECF,

BC = EF and BC ∥ EF [Given]

∴ quadrilateral BECF is a parallelogram. | ∵ A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length

(iii) ∵ ABED is a parallelogram [Proved in (i)]

∴ AD ∥ BE and AD = BE ... (1) [∵ Opp. sides of ∥gm are parallel and equal]

∵ BEFC is a parallelogram [Proved in (ii)]

From (1) and (2), we obtains AD ∥ CF and AD = CF

(iv) In quadrilateral BCFD,

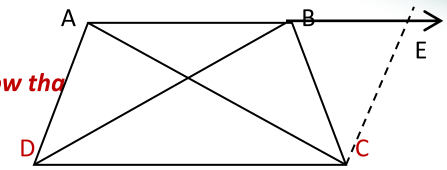
AD ∥ CF and AD = CF [From (iii)

∴ Quadrilateral ACFD is a parallelogram. | ∵ A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length

(v) ∵ ACFD is a parallelogram

∴ AC ∥ DF and AC = DF [In a parallelogram opposite sides are parallel and of equal length]

(vi) In Δ ABC and ΔDEF, AB = DE [∵ ABED is a parallelogram]
BC = EF [∵ BEFC is a parallelogram]
AC = DF [proved in (v)]
∴ ΔABC ≅ ΔDEF [SSS Rule]



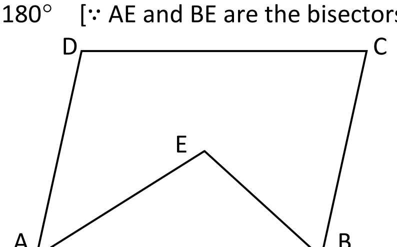
Q. 12. ABCD is a trapezium in which AB ∥ CD and AD = BC (see figure). Show tha
(i) ∠A = ∠B (ii) ∠C = ∠D
(iii) ΔABC ≅ ΔBAD
(iv) diagonal AC = diagonal BD
[Hint: Extend AB and draw a line through C parallel to DA intersecting AB product at E]

Sol. **Given:** ABCD is a trapezium in which AB ∥ CD and AD = BC
To prove: (i) ∠A = ∠B (ii) ∠C = ∠D
(iii) ΔABC ≅ ΔBAD (iv) diagonal AC = diagonal BD
Construction: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.
Proof: (i) AB ∥ Cd [Given]
and AD ∥ EC [By construction]
∴ AECD is a parallelogram [A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]
∴ AD = EC [Opp. sides of a ∥gm are equal]
But AD = BC [Given]
∴ EC = BC
∴ ∠CBE = ∠CEB ... (1) [Angles of opposite to equal sides of a triangle are equal]
∠B + ∠CBE = 180° ... (2) [Linear Pair Axiom]
∵ AD ∥ EC [By construction]
And transversal AE intersects them
∴ ∠A + ∠CEB = 180° ... (3) [The sum of consecutive interior angles on the same side of the transversal is 180°]
From (2) and (3),
∠B + ∠CBE = ∠A + ∠CEB [From (1)]
But ∠CBE = ∠CEB
∴ ∠B = ∠A or ∠A = ∠B
(ii) ∵ AB ∥ CD
∴ ∠A + ∠D = 180° [The sum of consecutive interior angles on the same side of the transversal is 180°]
and ∠B + ∠C = 180° ∴ ∠A + ∠D = ∠B + ∠C
But ∠A = ∠B [Proved in (i)]
∴ ∠D = ∠C or ∠C = ∠D
(iii) In Δ ABC and ΔBAD, AB = BA [Common]
BC = AD [given]
∠ABC = ∠BAD [SAS Rule]
∴ ΔABC ≅ ΔBAD [From (iii) above]
(iv) ∵ ΔABC ≅ ΔBAD [c.p.c.t.]
∴ AC = BD

ADDITIONAL EXAMPLES

Q. 1. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.

Sol. **Given:** ABCD is a parallelogram. The angle bisector AE and BE of adjacent angles ∠A and B meet at E.
To prove: ∠AEB = 90°
Proof: ∵ AD ∥ BC [Opp. sides of ∥gm]
and transversal AB intersects them
∴ ∠DAB + ∠CBA = 180° [∵ sum of consecutive interior angles on the same side of a transversal is 180°]
=> 2 ∠EAB + 2 ∠EBA = 180° [∵ AE and BE are the bisectors of ∠DAB and ∠CBA respectively]



=> ∠EAB + ∠EBA = 90° ... (1)

In ΔEAB,

∠EAB + ∠EBA + ∠AEB = 180° [∵ The sum of the three angles of a triangle is 180°]

=> 90° + ∠AEB = 180° [From (1)]

=> ∠AEB = 90°

Q. 2. AB and CD are two parallel lines and a transversal l intersects AB at X and CD at Y. Prove that the bisectors of the interior angles from a rectangle.

Sol. **Given:** AB and CD are two parallel lines and a transversal l intersects AB at X and CD at Y.

To prove: The bisectors of the interior angles from a rectangle.

Proof: AB ∥ CD

and EF intersects them

∴ ∠BXY = ∠CYX [Alternate ∠s] => ½ ∠BXY = ½ ∠CYX [Halves of equals are equal]

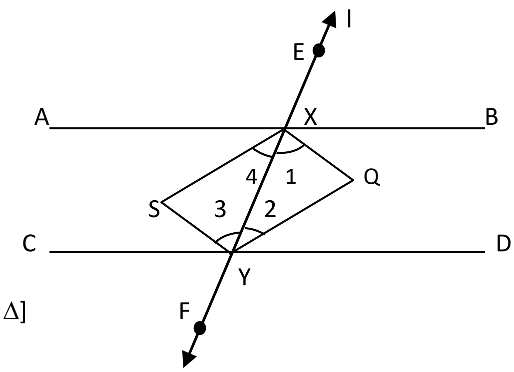
=> ∠1 = ∠3

But these angles form a pair of equal alternate angles for lines XQ and SY and a transversal XY.

∴ XQ ∥ SY ... (1)

Similarly, we can prove that

SX ∥ YQ ... (2)



In view of (1) and (2),

SYQY is a parallelogram

[∵ A quadrilateral is a parallelogram if both pairs of its opposite sides are parallel]

Now, ∠BXY + ∠DYX = 180° [Consecutive interior ∠s]

=> ½ ∠BXY + ½ ∠DYX = 90°

=> ∠1 + ∠2 = 90°

But ∠1 + ∠2 + ∠XQY = 180° [Angle sum property of a Δ]

=> 90° + ∠XQY = 180°

=> ∠XQY = 90°

=> ∠YSX = 90° [Opposite ∠s of ∥gm are equal]

and ∠SXQ = 90° [∵ Consecutive interior angles are supplementary]

Now, ∠SXQ = 90°

=> ∠SYQ = 90° [Opposite ∠s of a ∥gm are equal]

Thus, each angle of the parallelogram SYQX is 90°. Hence parallelogram SYQX is a rectangle.

Q. 3. Which of the following statements are true (T) and which are false (F)?

(i) In a parallelogram, the diagonals are equal.

(ii) In a parallelogram, the diagonals bisect each other.

(iii) In a parallelogram, the diagonals intersect at right angles.

(iv) In any quadrilaterals, if a pair of opposite sides is equal, it is a parallelogram.

(v) If all angles of a quadrilateral are equal, it is a parallelogram.

(vi) If all sides of a quadrilateral are equal, it is a parallelogram.

(vii) If three sides of a quadrilateral are equal, it is a parallelogram.

(viii) If three angles of a quadrilateral are equal, it is a parallelogram.

Sol. (i) False (F) (ii) True (T) (iii) False (F) (iv) False (F) (v) True (T) (vi) True (T)

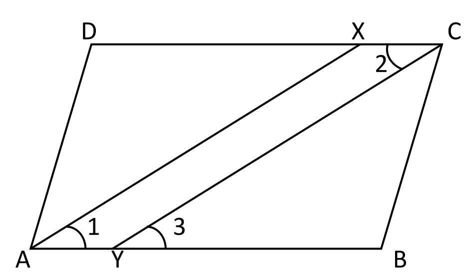
(vii) False (F) (viii) False (F)

Q. 4. ABCD is a parallelogram and line segments AX, CY bisect the angles A and C respectively. Show that AX ∥ CY.

Sol. **Given:** ABCD is a parallelogram and line segments AX, CY bisect the angles A and C respectively.

To prove: AX ∥ CY.

Proof: ∵ ABCD is a parallelogram.



∴ ∠A = ∠C [Opposite ∠s]

=> ½ ∠A = ½ ∠C [∵ Halves of equals are equal]

=> ∠1 = ∠2 ... (1)

[∵ AX is the bisector of ∠A and CY is the bisector of ∠C]

Now, AB∥ DC and CY intersects them

∴ ∠2 = ∠3 ... (2) [Alternate interior ∠s]

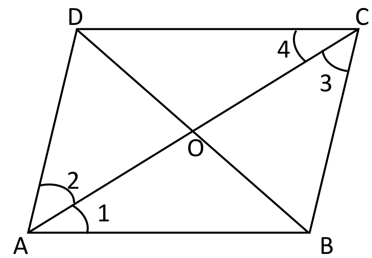
From (1) and (2), we get

∠1 = ∠3

But these are corresponding angles ∴ AX ∥ CY

Q. 5. *If a diagonal of a parallelogram bisects one of the angles of the parallelogram, it also bisects the second angle and then the two diagonals are perpendicular to each other.*

Sol. **Given:** ABCD is a ∥gm. Diagonal AC bisects ∠A.



To prove: (i) AC bisects ∠C

(ii) AC ⊥ BD.

Proof: (i) ∵ AB ∥ DC and AC intersects them

∴ ∠1 = ∠4 [Alternate Interior ∠s]

Similarly, ∠2 = ∠3 [Alternate interior ∠s]

But ∠1 = ∠2

∴ ∠3 = ∠4

=> AC bisects ∠C.

(ii) In ΔADC,

∠2 = ∠4

∴ AD = CD [Sides opposite to equal angles of a triangle are equal]

In ΔAOD and ΔCOD,

OA = OC [∵ Diagonals of a ∥gm bisect each other

OD = OD [Common side]

AD = CD [Proved above]

∴ ΔAOD ≅ ΔCOD [SSS Axiom]

∴ ∠AOD = ∠COD [c.p.c.t.]

But ∠AOD + ∠COD = 180° [Linear pair Axiom]

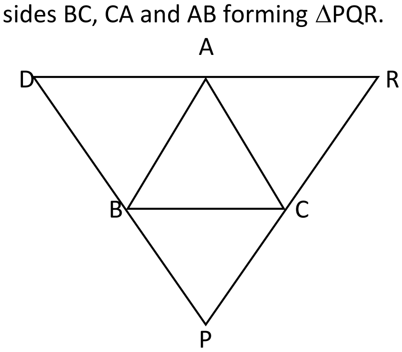
=> 2 ∠AOD = 180° [∵ ∠AOD = ∠COD] => ∠AOD = 90°

=> AC ⊥ BD

Q. 6. *Given ΔABC, lines are drawn through A, B and C parallel respectively to the sides BC, CA and AB forming ΔPQR. Show that BC = ½ QR.*

Sol. **Given:** ΔABC, lines are drawn through A, B and C parallel respectively to the sides BC, CA and AB forming ΔPQR.

To prove: BC = ½ QR



Proof: ∵ AQ ∥ CB and AC ∥ QB

∴ AQBC is a parallelogram

∴ BC = QA ... (1) [Opposite sides of a ∥gm]

∵ AR ∥ BC and AB ∥ RC

∴ ARCB is a parallelogram

∴ BC = AR ... (2) [Opposite sides of a ∥gm]

From (1) and (2),

QA = AR = ½ QR ... (3)

From (1) and (3), BC = ½ QR

Q. 7.*In the given figure, ABCD is a parallelogram and X, Y are the mid-points of the sides AB and DC respectively. Show that quadrilateral AXCY is a parallelogram*.

Sol. **Given:** ABCD is a parallelogram and X, Y are the mid-points of the sides AB and DC respectively.

To prove: Quadrilateral AXCY is a parallelogram.

Proof: AB = DC [Opposite sides of a parallelogram are equal]

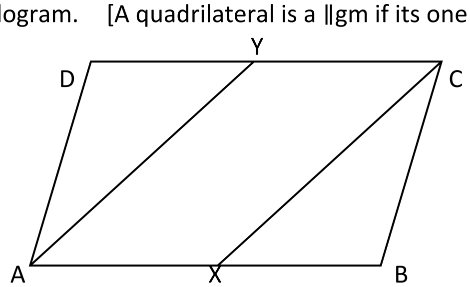
=> ½ AB = ½ DC [Halves of equals are equal]

=> AX = YC ... (1) [∵ X and Y are the mid-points of the sides AB and DC respectively]

∵ AB ∥ DC ... (2)

In view of (1) and (2),

AXCY is a parallelogram. [A quadrilateral is a ∥gm if its one pair of opposite sides are parallel and equal]



TEST YOUR KNOWLEDGE

Q. 1. Prove that the angle bisectors of a parallelogram form a rectangle.

Q. 2. ABCD is a parallelogram and P and Q are the mid-points of the opposite sides AB and CD respectively. If AQ intersects DP at S and BQ intersects CP at R, prove that PSQR is a parallelogram.

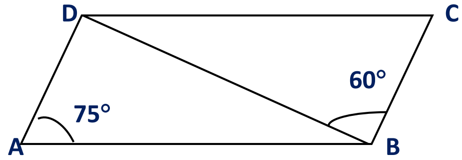
Q. 3. In a parallelogram ABCD, two points P and Q are taken on its diagonal BD such that DP = BQ. Prove that APCQ is a parallelogram.

Q. 4. Find all the other three angles of a parallelogram, if one of its angles is given to be (i) 70° (ii) 135° (iii) 95°

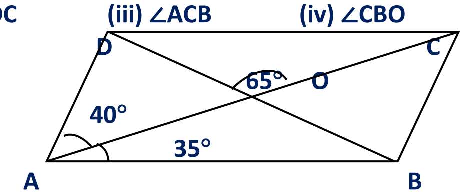
Q. 5. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Q. 6. Find the measures of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

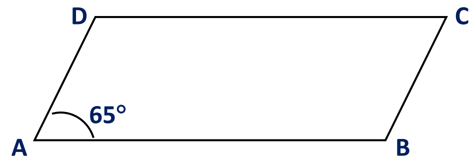
Q. 7. In the following figure, ABCD is a parallelogram in which ∠DAB = 75° and ∠DBC = 60°. Find ∠CDB and ∠ADB.



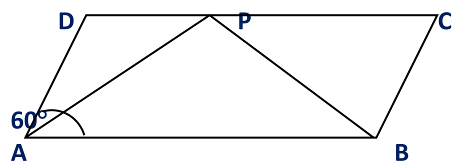
Q. 8. In the following figure, ABCD is a parallelogram ∠DAO = 40°, ∠BAO = 35° and ∠COD = 65°. Find
(i) ∠ABO (ii) ∠ODC (iii) ∠ACB (iv) ∠CBO



Q. 9. In the following figure, ABCD is a parallelogram in which ∠A = 65°. Find ∠B, ∠C and ∠D.



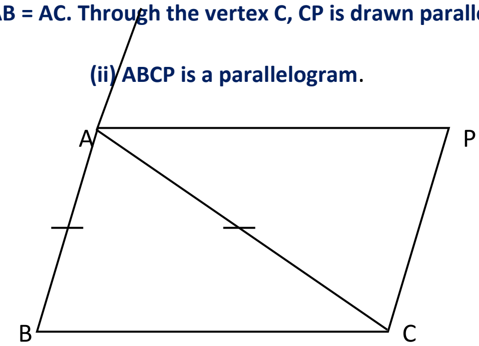
Q. 10. In the following figure, ABCD is a parallelogram in which ∠A = 60°. If the bisectors of ∠A and ∠B meet at P, prove that ∠APB = 90°. Also, prove that AD = DP, PC = BC and DC = 2AD.



Q. 11. In the following figure, ABCD is a parallelogram. O is mid-point of the side DC. OA and OB are joined if ∠AOB = 90°, then prove that AB

Q. 12. The sides AB and CD of a parallelogram ABCD are produced to E and F respectively such that BE = AB and DF = AD. Prove that ECF is a straight line.

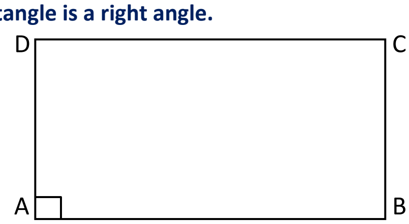
Q. 13. In an isosceles triangle ABC, AB = AC. Through the vertex C, CP is drawn parallel to BA meeting the bisector AP of the exterior angle CAB at P. Prove that:
(i) ∠PAC = ∠BCA (ii) ABCP is a parallelogram.



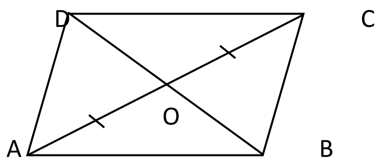
Q. 14. IF AD, BC are the equal sides of an isosceles trapezium ABCD, prove that ∠A = ∠B.

Q. 15. Prove that the opposite angles of an isosceles trapezium are supplementary.

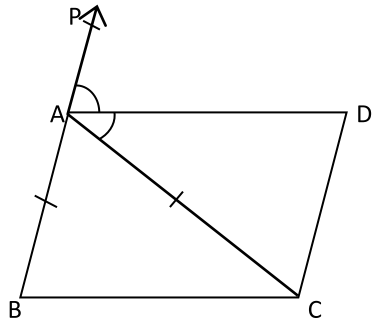
Q. 16. Show that each angle of a rectangle is a right angle.



Q. 17. Show that the diagonals of a rhombus are perpendicular to each other.



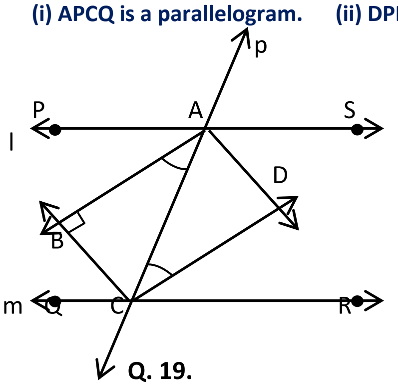
Q. 18. ABC is an isosceles triangle isn which AB = AC. AD bisects exterior angle ∠PAC and CD ∥ AB (see figure). Show that:
(i) ∠DAC = ∠BCA and (ii) ABCD is a parallelogram



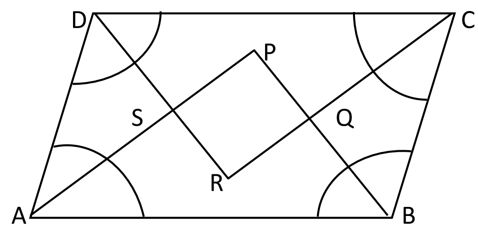
Q. 19. Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Q. 20. Show that the bisectors of angles of a parallelogram form a rectangle.

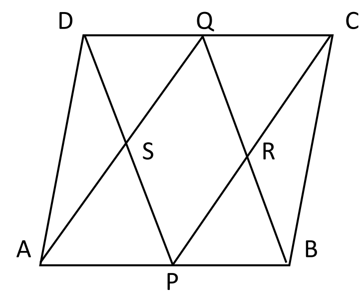
Q. 21. ABCD is a parallelogram in which P and Q are mid-points of opposite sides AB and CD (see fig.). If AQ intersects DP at S and BQ intersects CP at R. Show that:
(i) APCQ is a parallelogram. (ii) DPBQ is a parallelogram. (iii) PSQR is a parallelogram.



Q. 19.



Q. 20.



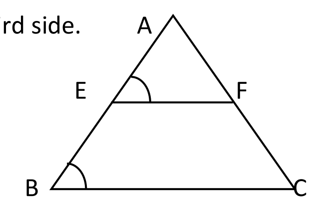
Q. 21.

Answers

4. (i) 110°, 70°, 110° (ii) 45°, 135°, 45° (iii) 85°, 95°, 85°
5. 108°, 72°, 108°, 72° 6. 68°, 112°, 68°, 112°
7. 45°, 60° 8. (i) 80°, (ii) 80° (iii) 40° (iv) 25° 9. 115°, 65°, 115°

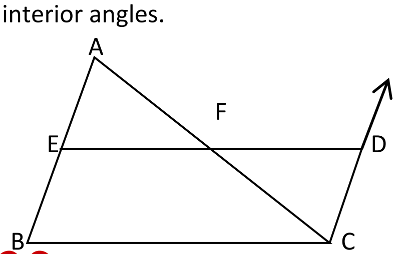
1. The mid-point Theorem

The line segment joining the mid-points of two sides of a triangle is parallel to the third side.



Given: ABC is a triangle in which mid-point E of AB and mid-point F of AC are joined.
To prove: EF ∥ BC
Construction: Extend EF to D such that EF = FD. Join CD.
Proof: In ∆AEF and ∆CDF,

AF = CF [∵ F is the mid-point of AC
EF = DF [By construction]
∠AFE = ∠CFD [Vertically Opposite Angles]
∴ ∆AEF ≅ ∆CDF [SAS Rule] ∴ ∠EAF = ∠DCF
But these are equal alternate interior angles.
∴ EF ∥ BC



2. Converse of the Mid-point Theorem

Theorem: The line drawn through the mid-point of the one side of a triangle, parallel to another side bisects the third side.

Verification by activity

In activity 1, we also observe that

EF = ½ BC

Verification by exhaustion

Given: ABC is a triangle in which E is the mid-point of AB. EF ∥ BC.

To Prove: AF = FC

Construction: Draw a line FG ∥ AB to intersect BC at G.

Proof: EF ∥ BG [Given]

EB ∥ FG [By construction]

∴ EFGB is a parallelogram [A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

∴ EB = FG ... (1)

∵ E is the mid-point of AB ∴ AE = EB ... (2)

From (1) and (2),

FG = AE ... (3)

∵ FG ∥ AB and transversal AC intersects them

∴ ∠GFC = ∠EAF ... (4) [Corresponding angles]

∵ EF ∥ BC and transversal AC intersects them

∴ ∠GCF = ∠EFA ... (5) [Corresponding angles]

In ΔFGC and Δ AEF,

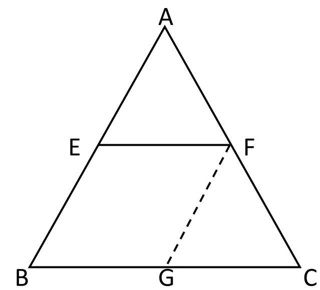
FG = AE [From (3)]

∠GFC = ∠EAF [From (4)]

∠CGF = EFA [From (5)]

∴ ΔFGC ≅ ΔAEF [AAS Rule]

∴ FC = AF [c.p.c.t.]



Alter: Given: ABC is a triangle in which E is the mid-point of AB. EF ∥ BC.

To prove: AF = CF

Construction: Through C drawn CM ∥ BA and extend EF to intersect CM at D.

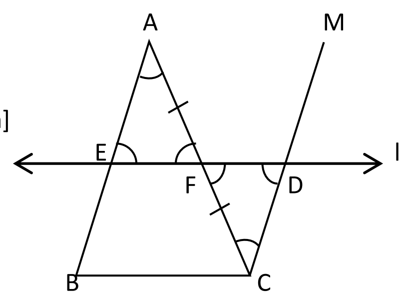
Proof: In ΔAEF and ΔCDF,

∠AFE = ∠CFD [Vertically Opposite angles]

∠FAE = ∠FCD [∵ EA ∥ CD and transversal AC intersects them]

AE = CD | AE = EB (∵ E is the mid-point of AB)
∵ EF ∥ BC and AB ∥ MC
∴ Quadrilateral EBCD is a ∥gm
∴ EB = CD
∴ AE = CD

∴ ΔAEF ≅ ΔCDF [AAS Rule] ∴ AF = CF



ILLUSTRATIVE EXAMPLES (Exercise – 2)

Q. 1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see figure). AC is a diagonal. Show that: (i) SR ∥ AC and SR = ½ AC (ii) PQ = SR (iii) PQRS is a parallelogram.

Sol. **Given:** ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA, AC is a diagonal.

To prove: (i) SR ∥ AC and SR = ½ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Proof: (i) In ΔDAC,

∵ S is the mid-point of DA and R is mid-point of DC

∴ SR ∥ AC and SR = ½ AC [Mid-point theorem]

(ii) In ΔBAC,

∵ P is the mid-point of AB and Q is the mid-point of BC

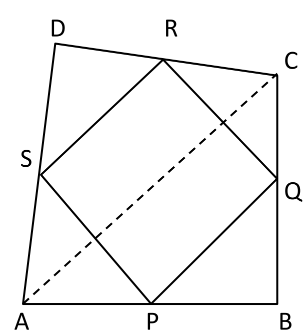
∴ PQ ∥ AC and PQ = ½ AC [Mid-point theorem]

But from (i), SR = ½ AC

∴ PQ = SR

(iii) PQ ∥ SR [From (ii)]

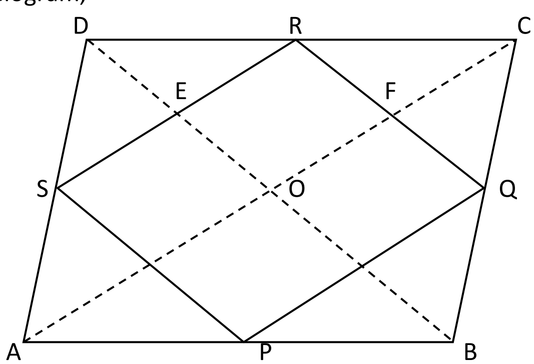
SR ∥ AC [From (i)]



∴ PQ ∥ SR [Two lines parallel to the same line are parallel to each other]
Also, PQ = SR [From (ii)]
∴ PQRS is a parallelogram [A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

Q. 2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

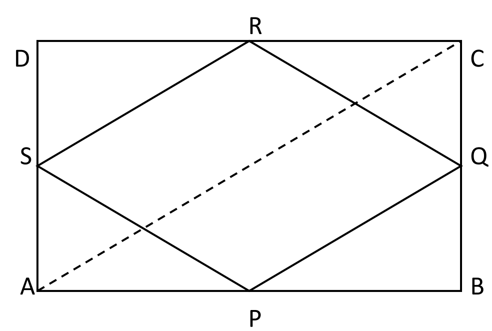
Sol. **Given:** □ABCD is a rhombus. P, Q, R, S are the mid-points of AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined.
To prove: □ABCD is a rectangle.
Construction: Join AC and BD
Proof: In triangles RDS and PBQ.
DS = QB [Halves of opposite sides of ∥gm ABCD which are equal]
DR = PB [Halves of opposite sides of ∥gm ABCD which are equal]
∠SDR = ∠QBP [Opposite ∠s of ∥gm ABCD which are equal]
∴ ∠SDR = ∠QBP [SAS Axiom]
∴ SR = PQ [c.p.c.t.]
In triangles RCQ and PAS,
RC = AP [Halves of opposite sides of ∥gm ABCD which are equal]
CQ = As [Halves of opposite of ∥gm ABCD which are equal]
∠RCQ = ∠PAS [Opposite ∠s of ∥gm ABCD which are equal]
∴ ΔRCQ ≅ ΔPAS [SAS Axiom]
∴ RQ = SP [c.p.c.t.]
∴ In □ PQRS, SR = PQ and RQ = SP
∴ □ PQRS is a parallelogram,



In ΔCDB, ∵ R and Q are the mid-point of DC and CB respectively.
∴ RQ ∥ DB => RF ∥ EO
Similarly, RE ∥ FO
∴ OFRE is a ∥gm ∴∠R = ∠EOF = 90° [∵ Opposite ∠s of a ∥gm are equal and diagonals of a rhombus intersect at 90°]
Thus □ PQRS is a rectangle.

Q. 3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. **Given:** ABCD is a rectangle. P, Q, R and S are the mid-point of AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.
To prove: Quadrilateral PQRS is a rhombus.
Construction: Join AC.
Proof: In ΔABC,
∵ P and Q are the mid-points of AB and BC respectively
∴ PQ ∥ AC and PQ = ½ AC ... (1)
In ΔADC,
∵ S and R are the mid-points of AD and DC respectively
∴ SR ∥ AC and SR = ½ AC ... (2)
From (1) and (2),
PQ ∥ SR and PQ = SR
∴ Quadrilateral PQRS is a parallelogram ... (3)
In rectangle ABCD,
AD = BC [Opposite sides]
=> ½ AD = ½ BC [Halves of equals are equal]



=> AS = BQ

In ΔAPS and ΔBPQ,

AP = BP [∵ P is the mid-point of AB]

AS = BQ [Proved above]

∠PAS = ∠PBQ [Each = 90°]

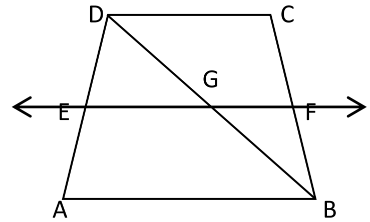
∴ ΔAPS ≅ ΔBPQ [SAS Axiom]

∴ PS = PQ ... (4) [c.p.c.t.] In view of (3) and (4), PQRS is a rhombus.

Q. 4. ABCD is a trapezium in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC.

Sol. **Given:** ABCD is a trapezium in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F.

To prove: F is the mid-point of BC.



Proof: Let DB intersect EF at G.

In ΔDAB,

∵ E is mid-point of DA and EG ∥ AB

∴ G is the mid-point of DB [By converse of mid-point theorem]

Again, in ΔBDC,

∵ G is the mid-point of BD is GF ∥ AB ∥ DC

∴ F is the mid-point of BC. [By converse of mid-point theorem]

Q. 5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. (see figure). Show that the line segments AF and EC trisect the diagonal BD.

Sol. **Given:** In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively

To prove: Line segments AF and EC trisect the diagonal BD.

Proof: AB ∥ DC [Opposite sides of ∥ gm ABCD]

∴ AE ∥ FC ... (1)

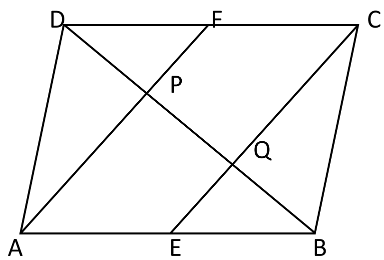
∵ AB = DC [Opposite sides of ∥gm ABCD]

∴ ½ AB = ½ DC [Halves of equals are equal]

=> AE = CF ... (2)

In view of (1) and (2), AECF is a parallelogram[A quadrilateral is a parallelogram if a pair of opposite sides of parallel and is of equal length]

∴ EC ∥ AF ... (3) [Opposite sides of ∥ gm AECF]



In ΔDBC,

∵ f is the mid-point of DC

and FP ∥ CQ [∵ EC ∥ AF]

∵ P is the mid-point of DQ [By converse of mid-point theorem]

=> DP = PQ ... (4)

Similarly, in ΔBAP,

BQ = PQ ... (5)

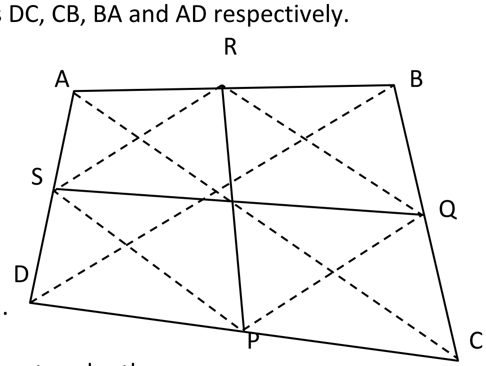
From (4) and (5), we obtain

DP = PQ = BQ => Line segment AF and EC trisect the diagonal BD.

Q. 6. Show that the line segment joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. **Given:** ABCD is a quadrilateral. P, Q, R and S are the mid-points of the sides DC, CB, BA and AD respectively.

To prove: PR and QS bisect each other.



Construction: Join PQ, QR, RS, SP, AC and BD.

Proof: In ΔABC,

∵ R and Q are the mid-points of AB and BC respectively.

∴ RQ ∥ AC and RQ = ½ AC

Similarly, we can show that

PS ∥ AC and PS = ½ AC

∴ RQ ∥ PS and RQ = PS

Thus, a pair of opposite sides of a quadrilateral PQRS are parallel and equal.

∴ ▭PQRS is a parallelogram.

Since the diagonals of a parallelogram bisect each other. ∴ PR and QS bisect each other.

Q. 7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that: (i) D is the mid-point of AC (ii) MD ⊥ AC (iii) CM = MA = ½ AB.

Sol. **Given:** ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that:

(i) D is the mid-point of AC (ii) MD ⊥ AC (iii) CM = MA = ½ AB.

Proof: (i) In ∆ ACB,

∵ M is the mid-point of AB and MD ∥ BC

∴ D is the mid-point of AC. [By converse of mid-point theorem]

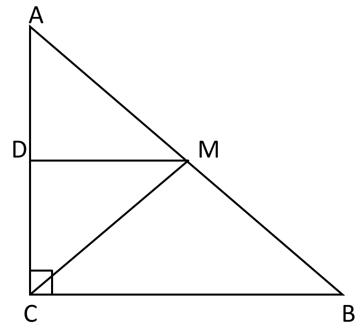
(ii) ∵ MD ∥ BC and AC intersects them

∴ ∠ADM = ∠ACB [Corresponding angles]

But ∠ACB = 90° [Given]

(iii) Now, ∠ADM + ∠CDM = 180° [Linear pair Axiom]

∴ ∠ADM = ∠CDM = 90°



In ∆ADM and ∆CDM,

AD = CD

∠ADM = ∠CDM [∵ D is the mid-point of AC]

DM = DM [Each = 90°]

∴ ∆ADM ≅ ∆CDM [Common]

∴ ∆ADM ≅ ∆CDM [SAS Rule]

∴ MA = MC [c.p.c.t.]

But M is the mid-point of AB ∴ MA = MB = ½ AB ∴ MA = MC = ½ AB=> CM = MA = ½ AB

ADDITIONAL EXAMPLES

Q. 1. Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square.

Sol. **Given:** ABCD is a square. P, Q, R and S are the mid-points of the consecutive sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove: □PQRS is a square.

Construction: Join AC and BD

Proof: RQ ∥ AC and RQ = ½ AC

SP ∥ AC and SP = ½ AC ∴ RQ = SP and RQ ∥ SP

Similarly, SR = PQ and SR ∥ PQ

∴ PQRS is a parallelogram

∵ RQ ∥ AC

∴ RE ∥ HO

∵ SR ∥ PQ

∴ HR ∥ OE

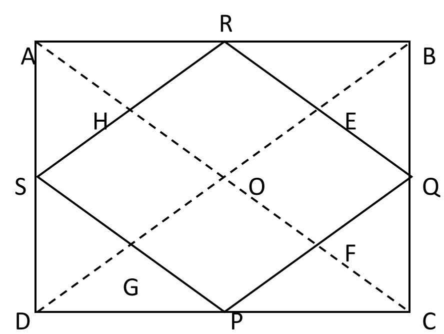
∴ OERH is a parallelogram

∴ ∠R = ∠HOE [Opposite ∠s of a ∥gm]

But ∠HOE = 90°

∴ ∠R = 90°

∴ □ PQRS is a rectangle.



But AC = BD ∴ PQ = QR = RS = SP ∴ □PQRS is a square.

Q. 2. Prove that the figure formed by joining the mid-points of the consecutive sides of a quadrilateral is parallelogram.

Sol. **Given:** ABCD is a quadrilateral. P, Q, R and S are the mid-points of the consecutive sides AB, BC, CD and DA respectively.

To prove: □PQRS is a parallelogram.

Construction: Join BD

Proof: In ∆CBD,

∵ Q is the mid-point of BC and R is the mid-point of CD.

∴ QR ∥ BD and QR = ½ BD ... (1)

In ∆ABD,

∵ P is the mid-point of AB is S is the mid-point of AD.

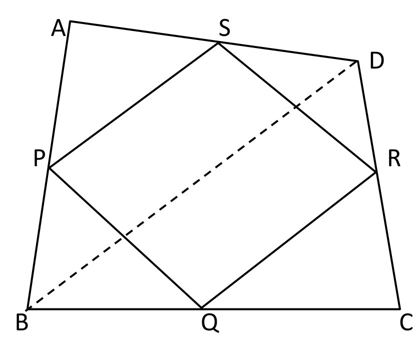
∴ PS ∥ BD and PS = ½ BD ... (2)

From (1) and (2),

QR = PS and QR ∥ PS

Thus, a pair of opposite sides of □PQRS are parallel and equal.

∴ □PQRS is a parallelogram.

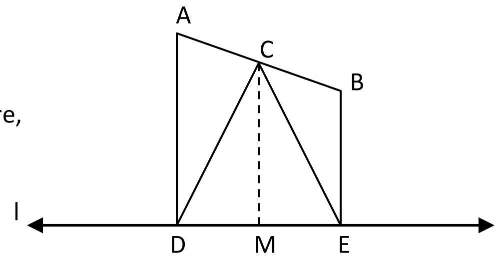


Q. 3. Points A and B are on the same side of a line l. AD and BE are perpendicular to l, meeting at D and E respectively. C is the mid-point of AB. Prove that CD = CE.

Sol. **Given:** Points A and B are on the same of a line l. AD and BE are perpendicular to l, meeting at D and E respectively. C is the mid-point of AB.

To prove: CD = CE

Construction: Draw CM $\perp$ l.



Proof: AD $\perp$ l and BE $\perp$ l.

$\therefore$ AD $\parallel$ BE.

Transversals AB and DE intersect the parallel lines AD and BE. Therefore,

$\because$ AC = CB [Given]

$\therefore$ DM = ME

In Δs CDM and CEM,

DM = EM [Prove above]

$\angle$CMD = $\angle$CME [Each = 90°]

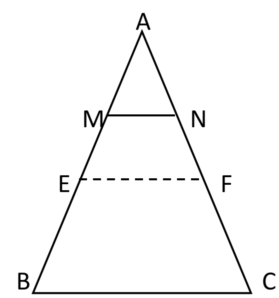
CM = CM [Common side]

$\therefore$ ΔCDM $\cong$ ΔCEM [SAS Axiom] $\therefore$ CD = CE [c.p.c.t.]

Q. 4. In triangle ABC, Points M and N on sides AB and AC respectively are taken so that AM = ¼ AB and AN = ¼ AC. Prove that MN = ¼ BC.

Sol. **Given:** In triangle ABC, points M and N on the sides AB and AC respectively are taken so that AM = ¼ AB and AN = ¼ AC.

To prove: MN = ¼ BC.



Construction: Join EF where E and F are the middle points of AB and AC respectively.

Proof: $\because$ E is the mid-point of AB and F is the mid-point of AC.

$\therefore$ EF $\parallel$ BC and EF = ½ BC ... (1)

Now, AE = ½ AB and AM = ¼ AB

$\therefore$ AM = ½ AE

Similarly, AN = ½ AF

=> M and N are the mid-points of AE and AF respectively.

$\therefore$ MN $\parallel$ EF and MN = ½ EF = ½ [½ BC] [From (1)]

= ¼ BC

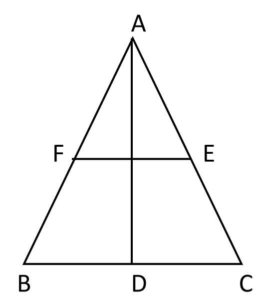
Q. 5. In figure, Δ ABC is isosceles with AB = AC. D, E and F are respectively the mid-points of sides BC, AC and AB. Show that line segment AD is perpendicular to the line segment EF and is bisected by it.

Sol. **Given:** ΔABC is isosceles with AB = AC. D, E and F are the mid-points of the sides BC, AC and AB respectively.

To prove: AD $\perp$ EF and AD is bisected by EF.

Construction: Join DE and DF.

Proof: In Δ ABC,



$\because$ D and E are the mid-point of BC and AC respectively

$\therefore$ DE $\parallel$ AB and DE = ½ AB ... (1)

In ΔABC,

$\because$ D and F are the mid-points of BC and BA respectively.

$\because$ DF $\parallel$ CA and DF = ½ CA ... (2)

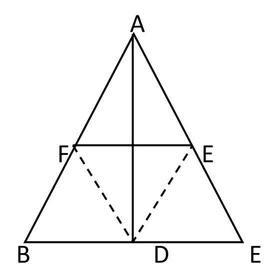
$\because$ AB = AC [Given]

$\therefore$ DE = DF ... (3) [From (1) and (2)]

Again, AB = AC

=> ½ AB = ½ AC [Halves of equals are equal]

=> AF = AE ... (4)



Also, DE = ½ AB = AF ... (5) [$\because$ F is the mid-point of AB]

and DE = ½ AC = AE ... (6) [$\because$ E is the mid-point of AC]

From (3), (4), (5) and (6), we have

DF = FA = AE = ED

=> AFDE is a rhombus.

$\because$ Diagonals of a rhombus bisect each other at 90°.

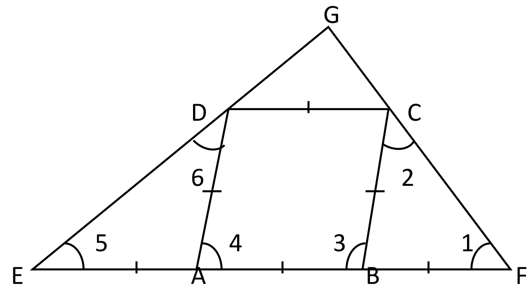
$\therefore$ AD $\perp$ EF and AD is bisected by EF.

Q. 6. ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF. Prove that ED and FC are perpendicular to each other.

Sol. **Given:** ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF.

To prove: ED ⊥ FC.

Proof:



AB = BF [By construction]

AB = BC [∵ ABCD is a rhombus]

∴ BC = BF

∴ ∠1 = ∠2 ... (1) [Angles opposite equal sides of a Δ are equal]

In ΔBCF,

Ext. ∠3 = ∠1 + ∠2 = ∠1 + ∠1 [From (1)]

= 2 ∠1 ... (2)

AB = AE [By construction]

AB = AD [∵ ABCD is a rhombus]

∴ AD = AE [∵ ABCD is a rhombus]

∴ ∠5 = ∠6 ... (3) [Angles opposite equal sides of a Δ are equal]

In ΔADE,

Ext. ∠4 = ∠5 + ∠6 = ∠5 + ∠5 [From (3)]

= 2 ∠5 ... (4)

∵ AD ∥ BC and transversal AB intersects them

∴ ∠3 + ∠4 = 180° [∵ Consecutive interior angles on the same side of a transversal are supplementary]

=> 2∠1 + 2∠5 = 180° [From (2) and (4)]

=> ∠1 + ∠5 = 90° ... (5)

In ΔGEF,

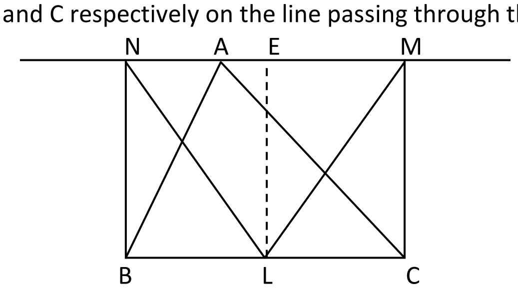
∠1 + ∠5 + ∠EGF = 180° [∵ The sum of the three angles of Δ is 180°]

=> 90° + ∠EGF = 180° [From (5)]

=> EG ⊥ GF => ED ⊥ FC.

Q. 7. ***BM and CN are perpendicular to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM = LN.***

Sol. **Given:** In ΔABC, BM and CN are the perpendiculars from B and C respectively on the line passing through the vertex A. L is the mid-point of BC.



To prove: LM = LN

Construction: Draw LE ⊥ MN

Proof: ∵ BM ∥ LE ∥ CN and BC cuts them such that

BL = LC ... (1)

∴ ME = EN ... (2) [∵ MN cuts them]

In Δ LEM and LEN,

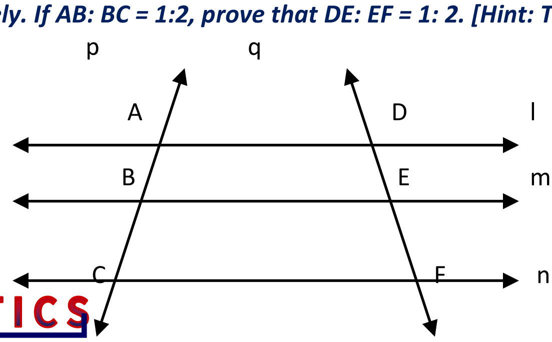
ME = EN [By (2)]

∠MEL = ∠NEL [Each = 90°]

LE = LE [Common side]

∴ ΔLEM ≅ ΔLEN [SAS Axiom] ∴ LM = LN [c.p.c.t.]

Q. 8. ***In figure, three parallel lines l, m and n are intersected by a transversal p at points A, B and C respectively and transversal q at D, E and F respectively. If AB: BC = 1:2, prove that DE: EF = 1: 2. [Hint: Through the mid-point of BC draw a line parallel to n.]***

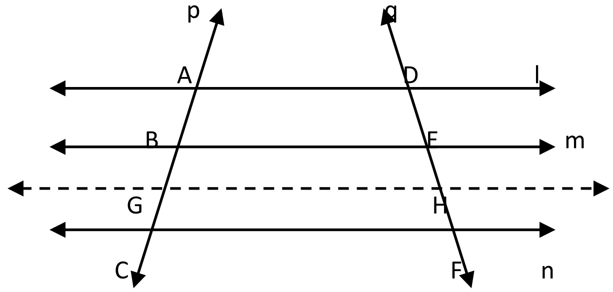


Sol. **Given:** Three parallel lines l, m and n are intersected by a transversal p at points A, B and C respectively and transversal q at D. E and F respectively. AB: BC = 1 : 2

To prove: DE : EF = 1 : 2

Construction: Through the mid-point G of BC, draw a line parallel to n.

Proof: BG = GC



∴ EH = HF

∴ AB : BC = 1 : 2 and G is the mid-point of BC.

∴ AB = BG = GC ∴ DE = EH = HF => DE : EF = 1 : 2

Q. 9. Fill in the blanks:

(i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is ....

(ii) The triangle formed by joining the mid-points of the sides of a right-triangle is ....

(iii) The figure formed by joining the mid-points of the consecutive sides of a quadrilateral is ....

(iv) If a line is divided by three parallel lines into two-segments of lengths in the ratio 1:3, another line will be divided by these parallel lines into two-segments of lengths in the ratio ....

Sol. (i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is an isosceles triangle.

(ii) The triangle formed by joining the mid-points of the sides of a right-triangle is a right triangle.

(iii) The figure formed by joining the mid-points of the consecutive sides of a quadrilateral is a parallelogram.

(iv) If a line is divided by three lines into two segments of lengths in the ratio 1:3, another line will be divided by these parallel lines into two segments of length in the ratio 1:3.

Q. 10. In the following figure, D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that ∆DEF is also an equilateral triangle.

Sol. **Given:** D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC

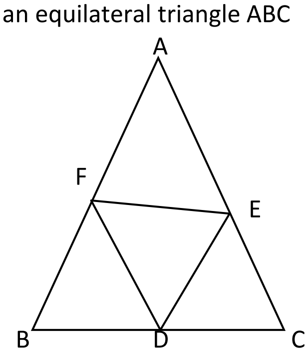
To prove: ∆DEF is also an equilateral triangle.

Proof: ∵ E is the mid-point of AB and F is the mid-point of AC.

∴ EF ∥ BC and EF = ½ BC

Similarly, ED ∥ AB and ED = ½ AB.

DE ∥ AC and DF = ½ AC



∵ ∆ABC is an equilateral triangle

∴ AB = BC = CA

=> $\frac{AB}{2} = \frac{BC}{2} = \frac{CA}{2}$

=> ED = EF = DF

=> ∆DEF is an equilateral triangle.

Q. 11. M and N divide the side AB of a ∆ABC into three equal parts. Line segments MP and NQ are both parallel to BC, and meet AC in P and Q respectively. Prove that P, Q divide AC into three equal parts.

Sol. ∵ MP ∥ BC and NQ ∥ BC

∴ MP ∥ NQ ... (1)

Therefore, ∵ AM = MN [Given]

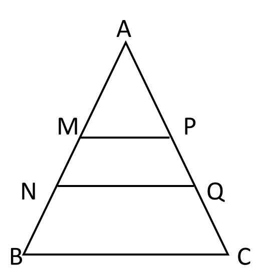
∴ AP = PQ ... (2)

∵ MP ∥ BC and MN = NM ∴ PQ = QC ... (3)

From (2) and (3)

AP = PQ = QC

=> P and Q divide AC into three equal parts.

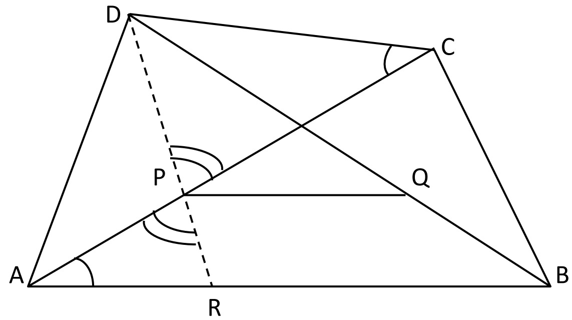


Q. 12. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half of the difference of these sides.

Sol. **Given:** ABCD is a trapezium. P and Q are the mid-points of the diagonals AC and BD respectively.

To prove: (i) PQ ∥ AB or DC

(ii) PQ = ½ (AB – DC)



Construction: Join DP and Produce DP to meet AB in R.

Proof: In ∆s APR and CPD,

∠PAR = ∠PCD [Alternate ∠s ∵ AB ∥ DC and AC intersects them

∠APR = ∠CPD [Vertically Opposite Angles]

AP = CP [Given]

∴ ∆APR ≅ ∆CPD [ASA Axiom]

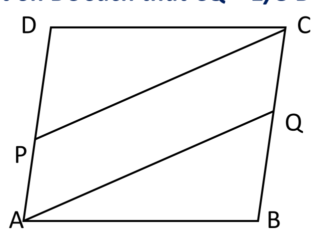
∴ PR = PD [c.p.c.t.] and AR = CD [c.p.c.t.]

In ∆DRB, ∵ P and Q are the mid-points of DR and BD respectively.

∴ PQ ∥ RB or AB or DC

and PQ = ½ RB = ½ (AB – AR) = ½ (AB – DC). [∵ AR = DC]

Q. 13. ABCD is a parallelogram. P is a point on AD such that AP = 1/3 AD. Q is a point on BC such that CQ = 1/3 BC. Prove that AQCP is a parallelogram.



Sol. AD = BC [Opposite sides of ∥ gm]

=> $\frac{1}{3}$ AD = $\frac{1}{3}$ BC

=> AP = CQ ... (1)

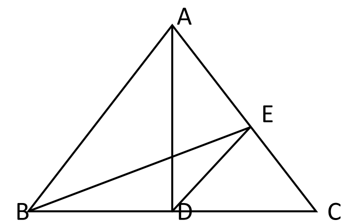
∵ AD ∥ BC [Opposite sides of ∥gm]

∴ AP ∥ CQ ... (2)

Thus, AQCP is a quadrilateral whose one pair of opposite sides AP and CQ are parallel and equal.

∴ AQCP is a parallelogram.

Q. 14. In the following figure, AD is a median and DE ∥ AB. Prove that BE is a median.



Sol. In ∆ ABC,

∵ AD is a median

∴ D is the mid-point of BC

DE is a line through D (mid-point of BC) and parallel to AB.

∴ E is the mid-point of AC.

Hence BE is the median.

Q. 15. In the following figure, triangle ABC is right-angled at B. Given that AB = 9 cm, AC 15 cm and D, E are the mid-points of AB and AC respectively. Calculate:

(i) The length of BC

(ii) The area of of ∆ADE.

Sol. (i) In right angled triangle ABC,

$BC^2 = AC^2 - AB^2 = (15)^2 - (9)^2 = 225 - 81 = 144$

=> $BC = \sqrt{144} = 12$ cm

(ii) ∵ D and E are the mid-points of AB and AC respectively.

∴ DE ∥ BC

and DE = ½ BC = ½ (12) = 6 cm

AD = BD = ½ AB = ½ (9) = $\frac{9}{2}$ cm

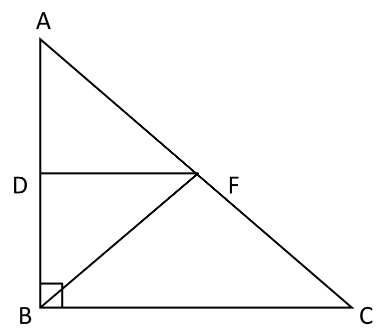
∵ DE ∥ BC and AB intersect them

∴ ∠ADE = ∠ABC = 90° [Corresponding ∠s]

=> ∆ADE is a right-angled triangle.

∴ Area of ∆ADE

$= \frac{(AD)\,(DE)}{2} = \frac{9}{2} \cdot \frac{6}{2} = \frac{27}{2} = 13.5 \text{ cm}^2$



TEST YOUR KNOWLEDGE

Q. 1. The diagonal of a quadrilateral ABCD are perpendicular to each other. Show that the quadrilateral formed by joining the mid-points of its sides is a rectangle.

Q. 2. E, F are respectively, the mid-points of non-parallel sides of a trapezium ABCD. Prove that:
(i) EF ∥ AB and (ii) EF = ½ (AB + CD)

Q. 3. ABC is a triangle; AD is a median and E is the mid-point of AD. BE is joined and produced to intersect AC in a point F. Prove that AF = 1/3 AC.

Q. 4. ABCD is a parallelogram and AB and CD are bisected at R and S respectively. Show that BRDS is a parallelogram.

Q. 5. If two points P and Q are taken in the equal sides AB and AC of an isosceles triangle ABC such that BP = CQ, prove that PQ ∥ BC.

Q. 6. Prove that the straight lines joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Q. 7. Prove that the straight lines joining the mid-points of the opposite sides of a parallelogram are parallel to the other pair of parallel sides.

Q. 8. In ΔABC, E is the mid-point of the median AD. BE is produced to meet AC in F. Prove that AF : FC = 1 : 2.

Q. 9. Prove that the median bisecting the hypotenuse of a right-angled triangle half of the hypotenuse.

Q. 10. Prove that the straight line joining the mid-points of the parallel sides of an isosceles trapezium is perpendicular to those very sides.

Q. 11. If the bisectors of angles B and C of a parallelogram ABCD meet in any point on side AD, then prove that BC = 2 AB.

Q. 12. In any ΔABC, BP and CQ are perpendicular on any line through A and M is the mid-point of the side BC. Show that MP = MQ.

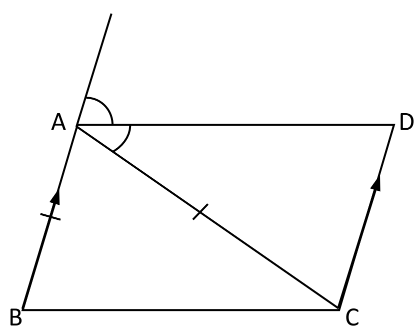
Q. 13. Let ABC be an isosceles triangle in which AB = AC. If D, E, F be the mid-points of the sides BC, CA and AB respectively, show that the segments AD and EF bisect each other at right angles.

Q. 14. ABCD is a rhombus. EABF is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angles.

Q. 15. ABCD is square. A is joined to a point P on BC and D is joined to a point Q on AB. If AP = DQ, prove that AP and DQ are perpendicular to each other.

Q. 16. ABCD is a parallelogram. AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that
BF = BC

Q. 17. In an isosceles triangle ABC, AB = AC. Through the vertex C, CP is drawn parallel to BA meeting the bisector AP of the exterior ∠CAP at P. Prove that:
(i) ∠PAC = ∠BCA
(ii) ABCP is a parallelogram



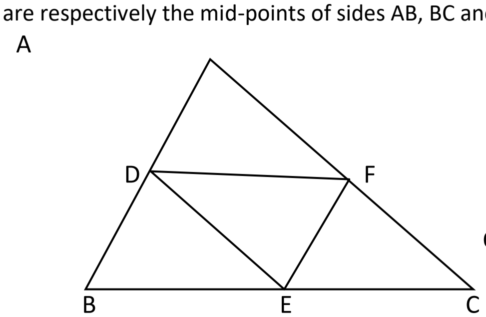
Q. 18. If two parallelograms PQAD and PQBC are on the opposite sides of PQ, prove that ABCD is a parallelogram.

Q. 19. P is the mid-point of the side AB of a parallelogram ABCD. A line through B parallel to PD meets DC in Q and AD produced at R. Prove that.
(i) AR = 2BC (ii) BR = 2BQ

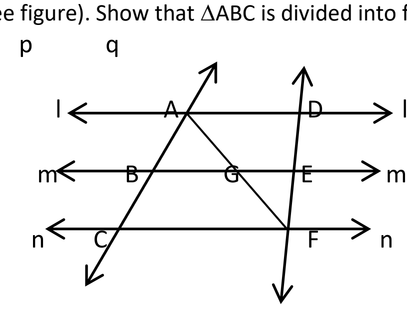
Q. 20. ABCD is a parallelogram. E is the mid-point of BC. DE and AB are produced to meet at F. Prove that AF = 2AB.

Q. 21. In ΔABC, D, E and F are respectively the mid-points of sides AB, BC and CA (see figure). Show that ΔABC is divided into four congruent triangles.

Q. 21.



Q. 22.



Q. 22. l, m and n are three parallel lines intersected by transversals p and q such that l, m and n cut off equal intercepts AB and BC on p (see figure). Show that l, m and n cut off equal intercepts DE and EF on q also.

MISCELLANEOUS EXERCISE

Q. 1. Prove that a straight line joining the mid-points of the opposite sides of a parallelogram is parallel to the other pair of opposite sides.

Q. 2. ABCD is a parallelogram. Points P and Q are taken on AB and CD respectively such that AP = CQ. Prove that AC and PQ bisect each other.

Q. 3. Prove that any straight line drawn through the point of intersection of the diagonals of a parallelogram terminated both way by the sides bisects the parallelogram and is itself bisected at the point.

Q. 4. In parallelogram ABCD, E and F are the mid-points of the opposite sides AD and BC respectively. Show that BE and DF trisect the diagonal AC.

Q. 5. Two points A and B lie on the same side of a line XY. AD $\perp$ XY and BE $\perp$ XY meet in D and E respectively. If C is the mid-point of AB, prove that CD = CE

Q. 6. Let ABCD be a trapezium in which AB ‖ DC and let E be the mid-point of AD. A line through E parallel to AB meets BC at G. Show that G is the mid-point of BC.

Q. 7. ΔABC is a triangle in which AB = 2AC. BA is a produced to D and ext. $\angle$CAD is bisected by AE cutting BC produced in E. Prove that C is the mid-point of BE.

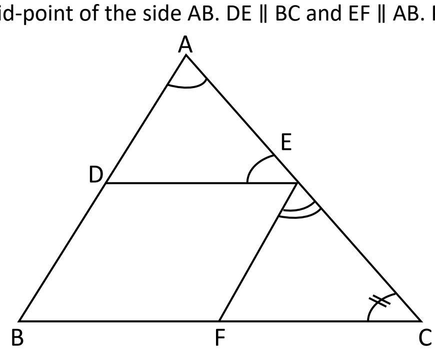
Q. 8. AB and CD are two non-intersecting straight line. Prove that the sum of the perpendiculars from A and B on CD is equal twice the perpendicular from the middle point of AB on CD.

Q. 9. Prove that the straight-line joining are mid-points of a pair of opposite sides of a quadrilateral to the midpoints of a pair of opposite sides of a quadrilateral to the midpoints of the diagonals from a parallelogram.

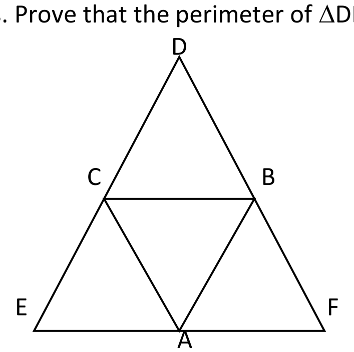
Q. 10. D, E, F are the mid-points of the sides BC, CA and AB of a triangle ABC. FG is drawn parallel to BE, meeting DE produced in G. Prove that the sides of the triangle CFG are equal to the medians of the triangle ABC.

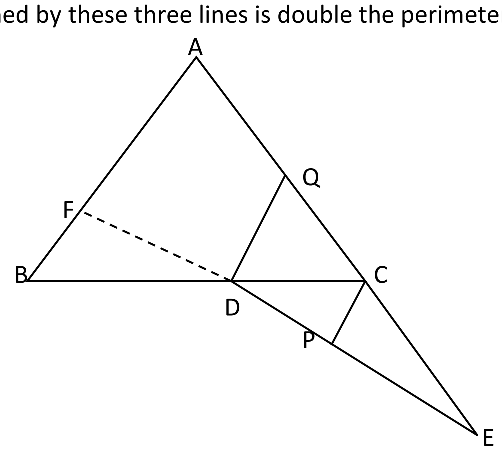
Q. 11. If through the middle part of the base of a triangle, a straight line is drawn parallel to one of the sides, prove that its intercepts on the internal and external bisectors of the vertical angle is equal to the other side.

Q. 12. In the following figure, D is the mid-point of the side AB. DE ‖ BC and EF ‖ AB. Prove that DE = ½ BC and CE = AE.



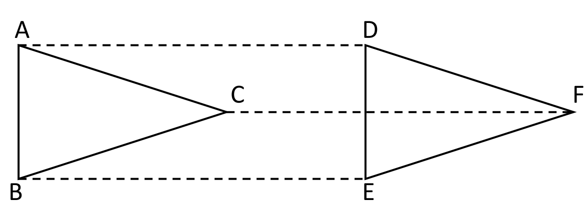
Q. 13. In the above fig. ABC is a triangle. Through the vertices A, B and C, three lines are drawn respectively parallel to their opposite sides. Prove that the perimeter of ΔDEF formed by these three lines is double the perimeter of the given ΔABC.

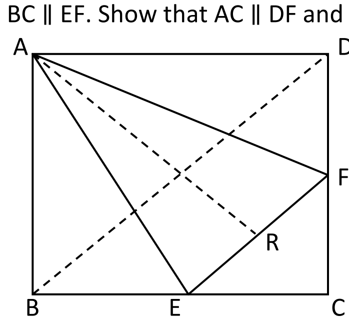




Q. 14. In the above figure, the side AD of a ΔABC is produced to E such that CE = ½ AC. If D is the mid-point of BC and ED produced meets AB at F and CP, DQ are drawn parallel to BA, prove that FD = 1/3 EF.

Q. 15. Let ABC and DEF be the two triangles such that AB ‖ DE; AB = DE; BC ‖ EF. Show that AC ‖ DF and AC = DF.

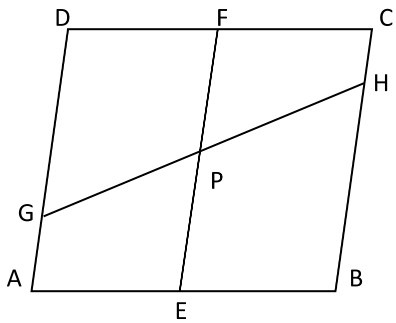


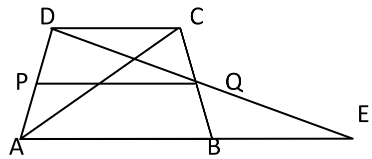


Q. 16. ABCD is a square and EF is parallel to BD. If R is the mid-point of EF, prove that:
(i) BE = DF (ii) AR bisects ∠BAD
(iii) If AR is produced it will pass through C.

Q. 17. ABCD is a parallelogram, E and F are the mid-points of AB and CD respectively. GH is any line that intersects AD, EF and BC at G, P and H respectively. Prove that GP = PH.

Q. 18. In the above figure, ABCD is a trapezium in which AB ∥ DC and P, Q are the mid-points of AD and BC respectively. DQ and AB when produced meet at E. Prove that:
(i) DQ = QE (ii) PR ∥ AB (iii) AR = RC





SUMMARY

Q. 1. Sum of the angles of a quadrilateral is 360°.

Q. 2. A diagonal of a parallelogram divides it into two congruent triangles.

Q. 3. In a parallelogram,
(i) Opposite sides are equal, (ii) Opposite angles are equal
(iii) diagonals bisect each other.

Q. 4. A quadrilateral is parallelogram, if
(i) Opposite sides are equal (ii) opposite angles are equal
(iii) diagonals bisect each other (iv) a pair of opposite sides is equal and parallel.

Q. 5. Diagonals of a rectangle bisect each other and are equal and vice-versa.

Q. 6. Diagonals of a rhombus bisect each other at right angles and vice-versa.

Q. 7. Diagonals of a square bisect each other at right angles and are equal, and vice-versa.

Q. 8. The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

Q. 9. A line through the mid-point of a side of a triangle parallel to another side bisects the third side.

Q. 10. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral, in order, is a parallelogram.

... **END.**