AEP STUDY CIRCLE

SINCE 2001...

IIT-NEET-CBSE

ACCENTS EDUCATIONAL PROMOTERS

24 YEARS OF LEGACY

XI IIT-NEET

PHYSICS WPE

SET-01

ONLINE-OFFLINE LEARNING ACADEMY

YOUR GATEWAY TO EXCELLENCE IN

IIT-JEE, NEET AND CBSE EXAMS

Work-Energy Power

IIT-JEE

NEET

CBSE

CONTACT US:

+91-9939586130

+91-9955930311

www.aepstudycircle.com

aepstudycircle@gmail.com

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

The topic of Work, Energy, and Power is a critical part of IIT-JEE-NEET Physics. To help students prepare effectively, here is an overview of the Top 150 Important Questions related to this chapter. These questions are designed to cover a wide range of concepts, problem-solving techniques, and applications frequently tested in competitive exams.

Work

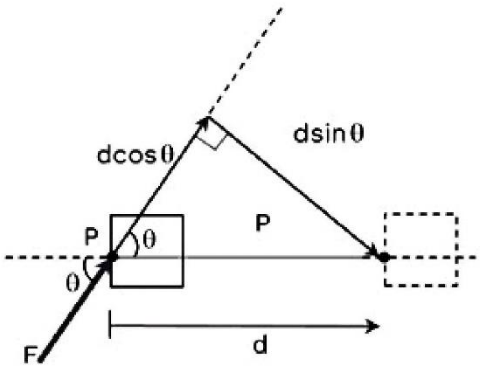
This collection of 150 Important Questions ensures comprehensive coverage of the topic, enabling students to tackle even the most challenging problems in the exam confidently.

Work is defined as dot product of force and displacement vectors.
It is a scalar quantity.

Work, $W = \vec{F}.\vec{s}$, $\vec{F}$ = Force vector, $\vec{s}$ = Displacement vector

$= F.s.\cos\theta$ θ = Angle between $\vec{F}$ and $\vec{s}$

Whenever work is done, energy is converted from one form to other form of energy.
In other words, work done by a force (W)= Force (F) × Displacement of the point of application of force in the direction of force.



$W = F.(d\cos\theta)$

Q. **The displacement x of a particle of mass 1.0 kg, moving along x-axis under the action of a constant force is given as $t = \sqrt{x} - 3$ (in SI units), where t is time. What will be the work done by the force in the first 10 second is**
(A) 640 J (B) 676 J (C) 320 J (D) None of these

Sol. Given : $t = \sqrt{x} - 3$ or $x = (t+3)^2 = t^2 + 6t + 9$
At $t = 0$, $x = 0^2 + 6(0) + 9 = 9$m
At $t = 10$, $x = 10^2 + 6(10) + 9 = 169$m
Displacement, $s = 169 - 9 = 160$ m

Velocity, $v = \frac{dx}{dt} = 2t + 6$

Acceleration, $a = \frac{dv}{dt} = 2$

Force, $F = ma = 1 \times 2 = 2N$
$\therefore$ Work done in 10 s, $W = F.s = 2 \times 160 = 320$ J

Q. **Displacement x (in meters) of a body of mass 1 kg as a function of time t, on a horizontal smooth surface is given as $x = 2t^2$. The work done in the first one second by the external force is**

(A) 1 J **(B) 2 J** **(C) 4 J** **(D) 8 J**

Sol. Given : m = 1 kg, $x = 2t^2$

$\therefore \quad \frac{dx}{dt} = 4t \quad$ or $\quad \frac{d^2x}{dt^2} = 4$

$\Rightarrow \quad$ Acceleration, $a = \frac{d^2x}{dt^2} = 4\,ms^{-2}$

Work done = Force × Displacement

W = m a. s

$W = 1 \times 4 \times 2t^2 = 8t^2$ J

At t = 1 s, W = 8 J

Q. **A 2 kg mass kept on a horizontal table is moved in the horizontal direction by a distance of 50 cm. What is the work done by normal reaction on the mass?**

(A) 10 J **(B) 0 J** **(C) 100 erg** **(D) 100 J**

Sol. The direction of displacement of block and the the direction of normal reaction are perpendicular to each other. So, the work done by normal reaction on the block will be zero..

Q. **A force of 50 N is acting on a body at an angle θ with the horizontal. If 150 J of work is done by displacing it through 6 m, then θ is**

(A) 60° **(B) 30°** **(C) 0°** **(D) 45°**

Sol. Work done, $W = Fs\cos\theta$ or $\cos\theta = \frac{W}{Fs}$

Here, F = 50 N, s = 6 m, W = 150 J

$\therefore \quad \cos\theta = \frac{150}{50 \times 6} = \frac{150}{300} = \frac{1}{2} \quad$ or $\quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

Q. **A coolie pushes a box through a distance of 20 m on a railway platform. If the coolie exerts a force of 20 kg wt in a direction inclined at 60° with ground, then work done by him is**

(A) 1960 J **(B) 196 J** **(C) 1.96 J** **(D) 196 kJ**

Sol. The work done by the coolie is

$W = F\,s\cos\theta$

Here, $F = 20\text{ kgwt} = 20 \times 9.8\text{ N} = 196\text{ N}$

$s = 20\text{ m},\ \theta = 60°$

$\therefore$ $W = 196 \times 20 \times \cos 60° = 1960\text{ J}$

Q. **A body moves from a position $\vec{r}_1 = (2\hat{i} - 3\hat{j} - 4\hat{k})\,m$ to a position $\vec{r}_2 = (3\hat{i} - 4\hat{j} + 5\hat{k})\,m$ under the influence of a constant force $\vec{F} = (4\hat{i} + \hat{j} + 6\hat{k})\,N$. The work done by the force is**

(A) 57 J **(B) 58 J** **(C) 59 J** **(D) 60 J**

Sol. Here, $\vec{r}_1 = 2\hat{i} - 3\hat{j} - 4\hat{k},\ \vec{r}_2 = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{F} = 4\hat{i} + \hat{j} + 6\hat{k}$

$\therefore$ $\vec{r} = \vec{r}_2 - \vec{r}_1 = (3\hat{i} - 4\hat{j} + 5\hat{k}) - (2\hat{i} - 3\hat{j} - 4\hat{k}) = \hat{i} - \hat{j} + 9\hat{k}$

$\therefore$ Work done, $W = \vec{F} \cdot \vec{r}$

$= (4\hat{i} + \hat{j} + 6\hat{k}) \cdot (\hat{i} - \hat{j} + 9\hat{k}) = 4 - 1 + 54 = 57\,J$

Q. **A particle acted upon by constant forces $(4\hat{i} + \hat{j} - 3\hat{k})\,N$ and $(3\hat{i} + \hat{j} - \hat{k})\,N$ is displaced from the point $(\hat{i} + 2\hat{j} + 3\hat{k})\,m$ to point $(5\hat{i} + 4\hat{j} + \hat{k})\,m$. The total work done by the forces in SI unit is**

(A) 20
(B) 40
(C) 50
(D) 30
(E) 35

Sol. Here, $\vec{F}_1 = 4\hat{i} + \hat{j} - 3\hat{k}$, $\vec{F}_2 = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{r}_2 = 5\hat{i} + 4\hat{j} + \hat{k}$

Displacement,

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 4\hat{i} + 2\hat{j} - 2\hat{k}$$

Total force, $\vec{F} = \vec{F}_1 + \vec{F}_2$

$$= (4\hat{i} + \hat{j} - 3\hat{k}) + (3\hat{i} + \hat{j} - \hat{k}) = 7\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\therefore \quad W = \vec{F} \cdot \vec{r} = (7\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (4\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= 28 + 4 + 8 = 40 \text{ J}$$

Q. **A 5 N force acts on a 15 kg body initially at rest. The work done by the force in the third second of its motion is (in joules) approximately equal to**
(A) 9 **(B) 15** **(C) 4** **(D) 20**

Sol. Here, $u = 0$, $m = 15$ kg, $F = 5$ N

$\therefore \quad F = ma \quad \text{or} \quad 5 = 15 \times a \quad \text{or} \quad a = \frac{1}{3} \text{ms}^{-2}$

Distance travelled by the body in n^{th} second is

$$S_n = u + \frac{a}{2}(2n - 1)$$

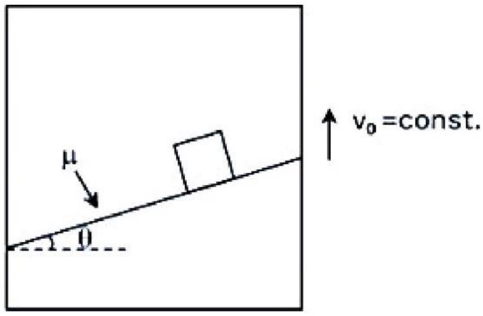
Hence, distance travelled by the body in 3^{rd} second is

$$S_3 = 0 + \frac{1/3}{2}(2 \times 3 - 1) = \frac{5}{6} \text{m}$$

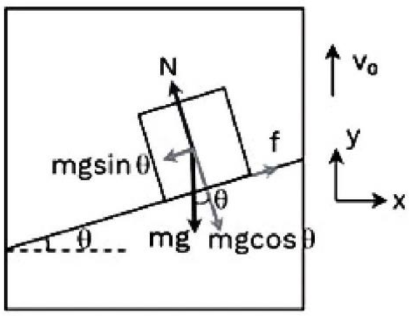
$\therefore$ Work done by the force in 3^{rd} second is

$$W = F \times S_3 = 5 \times \frac{5}{6} = \frac{25}{6} \text{J} = 4.16 \text{ J} \approx 4\text{J}$$

Q. **Consider that an inclined plane is formed inside an elevator moving upwards with constant velocity. The block is not slipping on the inclined plane. Find work done by friction, by gravity and by normal reaction in time t_0.**



Sol.



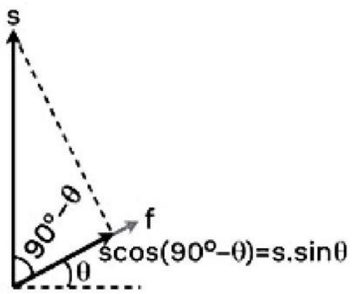
In time t_0, Displacement, $\vec{s} = v_0 t_0 \hat{j}$

As the block is not slipping,

So, $f = mg\sin\theta$

$N = mg\cos\theta$

For work done by friction:



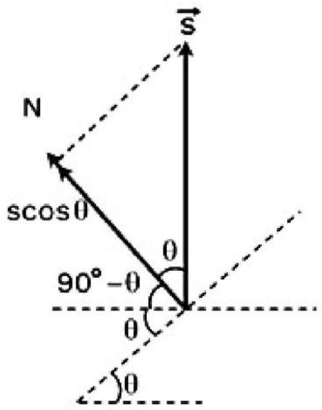
$$W_f = f.s.\sin\theta = (mg\sin\theta)(v_0 t_0)\sin\theta$$

$$W_f = mgv_0 t_0 \sin^2\theta$$

For work done by gravity:

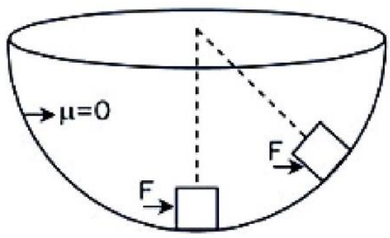
$$W_g = mg.s.\cos 180°$$
$$= -mgs$$
$$= -mgv_0t_0$$

For work done by normal reaction:

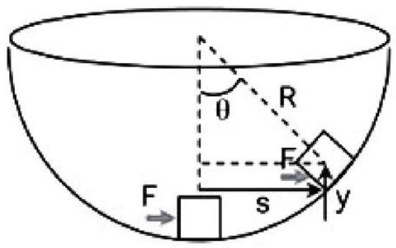


$$W_N = N.s.\cos\theta$$
$$= mg\cos\theta.v_0t_0.\cos\theta$$
$$W_N = mgv_0t_0\cos^2\theta$$

Q. **A constant force F displaces a block of mass m inside a smooth hemisphere as shown. Find work done by force F, and work done by gravity.**



Sol. From adjacent figure, $\sin\theta = \frac{s}{R}$



Displacement in direction of force F, $s = R.\sin\theta$

Work done by force F, $W_f = F.s.\cos 0° = FR\sin\theta$

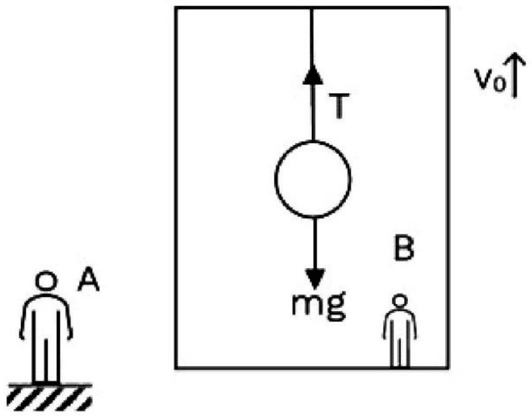
Now, $y = R - R\cos\theta$

Work done by gravity, $W_g = mg.y.\cos 180°$

$$= -mg.\ y$$
$$= -mgR(1-\cos\theta)$$

Work is a frame dependent quantity. Value of work done by a force can change on changing the reference frame from which motion is being observed.

Consider a lift moving upwards with constant velocity v_0. A ball of mass m is hanging by a string as shown.



As per observer A, displacement of ball in time t, $s = v_0t$ ...(upwards)

Work done by mg as per the ground observer = – (mg. s) = – mgv_0t

As per observer B, displacement of ball in time t is zero.

So, work done by mg = 0

Thus, we can say that work is frame dependent.

Work done by Variable Force

If force F is a function of x, then work done by force is given by,

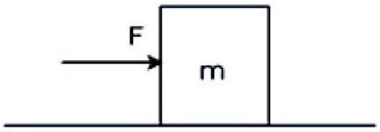
$$W = \int F.dx = \int_{x_1}^{x_2} f(x).dx$$

Q. If a force F = 2x² displaces a block from x = 0 to x = 4, then what is work done?

Sol. Work done by the force, $W_F = \int F.dx = \int_0^4 2x^2.dx$

$$= 2.\left[\frac{x^3}{3}\right]_0^4 = 2 \times \frac{(4^3 - 0^3)}{3} = \frac{128}{3} J$$

Q. If a force F = 2t displaces a block of mass m = 2kg as shown. Find work done by force in a time interval of 3 seconds starting from zero.



Sol. Acceleration, $a = \frac{F}{m} = \frac{2t}{2} = t$

$$a = \frac{dv}{dt} = t$$

$$\int_0^v dv = \int_0^t t.dt$$

$$v = \frac{t^2}{2}$$

Now, $v = \frac{dx}{dt} = \frac{t^2}{2} \Rightarrow dx = \frac{t^2}{2}.dt$

Work done by force,

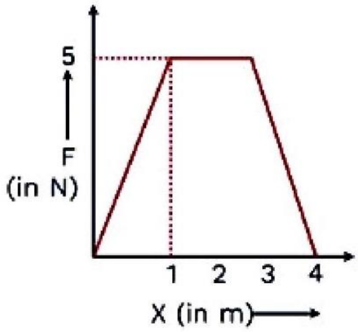
$$W_f = \int F.dx$$

$$= \int_0^3 2t.\left(\frac{t^2}{2}\right).dt$$

$$= \int_0^3 t^3.dt = \left[\frac{t^4}{4}\right]_0^3$$

$$= \frac{3^4 - 0^4}{4} = \frac{81}{4} J$$

Q. **The variation of force with displacement for a body moving along a straight line is shown in figure. Calculate the work done by the force on the particle in 1st metre of the path?**

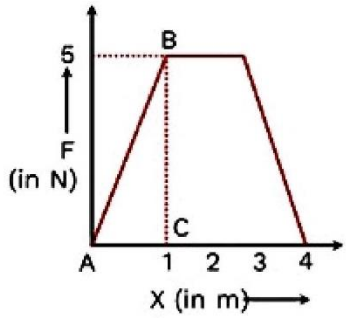


(A) 5 J **(B) 10 J** **(C) 15 J** **(D) 2.5 J**

Sol. Work done, W = Area under curve of F–x graph from x = 0 m to x = 1 m

= Area of triangle ABC

$= \frac{1}{2} \times 5 \times 1 = 2.5$ J



Q. **A force of (5 + 3x) N acting on a body of mass 20 kg along the x-axis displaces it from x = 2 m to x = 6 m. The work done by the force is**

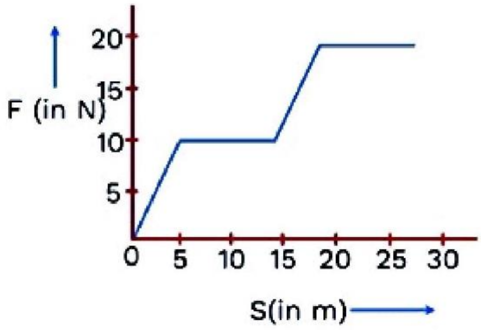
(A) 20 J **(B) 48 J** **(C) 68 J** **(D) 86 J**

Sol. Force, F = (5 + 3x) N

Work done, $W = \int_{x_1}^{x_2} F\,dx = \int_2^6 (5 + 3x)dx$

$= \left[5x + \frac{3}{2}x^2\right]_2^6 = 68$ J

Q. **The work done by a force acting on a body is shown in the graph drawn below. What is the work done in covering an initial distance of 20 m is**



(A) 225 J **(B) 200 J** **(C) 400 J** **(D) 175 J**

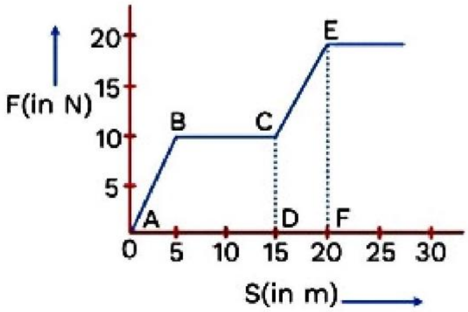
Sol. Work done,

W = Area under F-S graph from S = 0 to S = 20 m

= Area of trapezium ABCD + Area of trapezium CEFD

$= \left(\frac{1}{2} \times (10+15) \times 10\right) + \left(\frac{1}{2}(10+20) \times 5\right)$

= 125 + 75 = 200 J



Q. **A force $\vec{F} = -(y\hat{i} + x\hat{j})$ acts on a particle moving in the x-y plane. The particle starts from origin and it is moved along positive x-axis to the point (2a, 0) and after that it is moved parallel to the y-axis to the point (2a, 2a). Find the value of total work done on the particle is**

(A) $-4a^2$ **(B) $-2a^2$** **(C) $4a^2$** **(D) $2a^2$**

Sol. Given, $\vec{F} = -(y\hat{i} + x\hat{j})$

$$\vec{\nabla} \times \vec{F} = 0$$

$\Rightarrow$ Force is conservative and work done is path independent.

From graph, x = y ...(i)

Net work done by the applied force

$$W = \int \vec{F} \cdot \vec{dr} = \int -\left(y\hat{i} + x\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$$

$$= -\int ydx - \int xdy = -\int_0^{2a} xdx - \int_0^{2a} ydy \qquad \text{...(Using (i))}$$

$$= -\left[\frac{x^2}{2}\right]_0^{2a} - \left[\frac{y^2}{2}\right]_0^{2a} = -\left(\frac{4a^2}{2} - 0\right) - \left(\frac{4a^2}{2} - 0\right) = -4a^2$$

Q. **A rubber band exerts a force $F = ax + bx^2$ where a and b are constants, when it is stretched by a distance x. What is the work done in stretching the un-stretched rubber-band by L?**

(A) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$ **(B)** $aL^2 + bL^3$ **(C)** $\frac{1}{2}\left(aL^2 + bL^3\right)$ **(D)** $\frac{aL^2}{2} + \frac{bL^3}{3}$

Sol. Restoring force, $F = ax + bx^2$

Work done in stretching the rubber-band by a small amount dx is given by,

$$dW = F.dx$$

Net work done in stretching the rubber-band by L is

$$W = \int dW = \int_0^L Fdx$$

or $$W = \int_0^L \left(ax + bx^2\right)dx = \left[a\frac{x^2}{2} + b\frac{x^3}{3}\right]_0^L \text{ or } W = \frac{aL^2}{2} + \frac{bL^3}{3}$$

Q. **Find the value of work done on a particle of mass m by a force which is given by, $\vec{F} = K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K is a constant of appropriate dimensions), if the particle is being moved from the point (a, 0) to the point (0, a) along a circular path of having radius "a" about the origin in the x–y plane.**

(A) $\frac{2K\pi}{a}$ **(B)** $\frac{K\pi}{a}$ **(C)** $\frac{K\pi}{2a}$ **(D)** 0

Sol.

$$\vec{F} = K\left[\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{i} + \frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{j}\right]$$

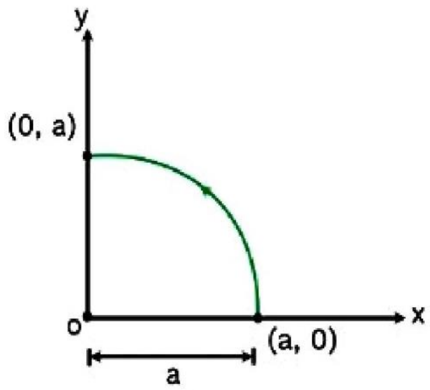
Equation of circular path will be

$$x^2 + y^2 = a^2$$

$$\vec{F} = K\left[\frac{x}{(a^2)^{\frac{3}{2}}}\hat{i} + \frac{y}{(a^2)^{\frac{3}{2}}}\hat{j}\right]$$

so,

$$\vec{F} = \frac{K}{a^3}\left(x\hat{i} + y\hat{j}\right)$$



$$dW = \vec{F}\cdot d\vec{s} = \vec{F}\cdot(dx\hat{i} + dy\hat{j})$$

$$W = \int dW = \frac{K}{a^3}\int_a^0 x\cdot dx + \frac{K}{a^3}\int_0^a y\cdot dy$$

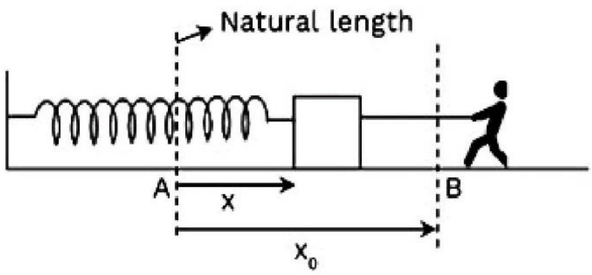
$$W = \frac{K}{a^3}\left[\frac{x^2}{2}\right]_a^0 + \frac{K}{a^3}\left[\frac{y^2}{2}\right]_0^a$$

$$= \frac{K}{a^3}\left[0 - \left(\frac{a^2}{2}\right)\right] + \frac{K}{a^3}\left[\frac{a^2}{2} - 0\right]$$

$$\boxed{W = -\frac{K}{2a} + \frac{K}{2a} = 0}$$

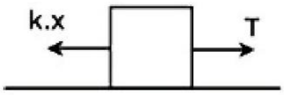
Work done by Spring Force

Let's consider that a block is attached to a spring of spring constant k. A man is pulling the block very slowly as shown.



At any instant, if elongation in the spring is x, then spring force tries to pull the block towards itself.

On drawing FBD of the block,



As the block is being pulled slowly, so it does not have kinetic energy at any instant and hence net force on the block will be balanced.

So, T = k.x

For total displacement (x_0) from A to B,

Work done by spring,

$$W_s = \int_0^{x_0} (kx).dx.\cos 180^\circ$$ ...(as the angle between spring force and displacement is 180°)

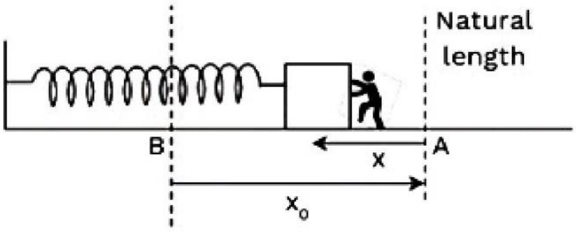
$$\text{So, } W_s = -k\left[\frac{x^2}{2}\right]_0^{x_0} = \frac{-kx_0^2}{2}$$

Work done by man,

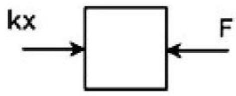
$$W_m = \int_0^{x_0} T.dx.\cos 0^\circ$$

$$= \int_0^{x_0} kx.dx.1 = k\left[\frac{x^2}{2}\right]_0^{x_0} = \frac{kx_0^2}{2}$$

Similarly, if a man is compressing a spring as shown,



Then, free body diagram of the block at any instant can be drawn as,



Then,

Work done by spring, $W_{spring} = \int_0^{x_0} (kx)(dx)\cos 180^\circ = -\frac{kx_0^2}{2}$

Work done by man, $W_{man} = \int_0^{x_0} F.dx.\cos 0^\circ = \int_0^{x_0} kx.dx = \frac{kx_0^2}{2}$

Q. **A spring with force constant k is initially stretched by x_1. If it is further stretched by x_2, then the increase in its potential energy is**

Sol. Initial potential energy, $U_1 = \frac{1}{2}kx_1^2$

Find potential energy, $U_2 = \frac{1}{2}k(x_1 + x_2)^2$

$= \frac{1}{2}k(x_1^2 + x_2^2 + 2x_1 x_2)$

Increase in potential energy, $\Delta U = U_2 - U_1$

$= \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + \frac{1}{2}(2k\,x_1x_2) - \frac{1}{2}kx_1^2$

$= \frac{1}{2}kx_2[x_2 + 2x_1]$

Q. **When two springs A and B with force constants k_A and k_B are stretched by the same force, then the respective ratio of the work done on them is**

Sol. We know that, when we stretch a spring of force constant k by a distance x, then work done is, $W = \frac{1}{2}kx^2$

But force F = kx or $x = \frac{F}{k}$

$\therefore \quad W = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}\frac{F^2}{k}$

As both springs A and B are stretched by the same force F, so

$W_A = \frac{1}{2}\frac{F^2}{k_A}$ and $W_B = \frac{1}{2}\frac{F^2}{k_B}$

Required ratio is $\frac{W_A}{W_B} = \frac{k_B}{k_A}$

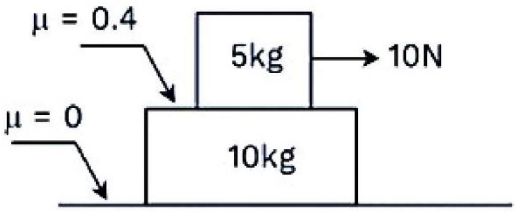
Q. **Two blocks of same mass m are connected to a spring of spring constant k. If both are given velocity v in opposite directions, then the maximum elongation of the spring is**

Sol. $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

or $\quad mv^2 = \frac{1}{2}kx^2$ or $kx^2 = 2mv^2$ $\quad$ or $\quad x = \sqrt{\frac{2mv^2}{k}}$

Work done by Friction

Q. **Find the work done by friction on the block of mass 5kg in 2s for the given system.**



Sol. Let's assume that both blocks move together, then,

$$a = \frac{10}{5+10} = \frac{2}{3} m/s^2$$

For 10kg block,

$$a_{max} = \frac{f_{max}}{10} = \frac{\mu \times 5 \times g}{10} = \frac{0.4 \times 5 \times 10}{10} = 2m/s^2$$

As $a < a_{max}$, so both blocks can move together.

Friction force between blocks, $f = 10 \times a = 10 \times \frac{2}{3} = \frac{20}{3} N$

For 5kg block,

In time t = 2s,

Displacement, $s = \frac{1}{2} a t^2 = \frac{1}{2} \times \frac{2}{3} \times 2^2 = \frac{4}{3} m$

So, Work done by friction on 5kg block,

$$W_f = f.s.\cos 180^\circ$$

$$= -fs = -\frac{20}{3} \times \frac{4}{3} = -\frac{80}{9} Nm$$

Work Energy Theorem

It states that work done by all forces on a body is equal to the change in its kinetic energy.

$$\sum W_{(\text{all forces})} = \Delta K.E.$$

$$W_{(\text{all forces})} = K_{\text{final}} - K_{\text{initial}}$$

Q. A ball is thrown vertically upwards with initial speed 20m/s. Mass of ball is 2kg. If maximum height attained is 18m, find work done by air drag.

Sol. By Work energy theorem,

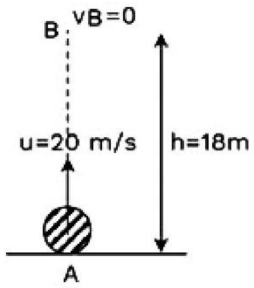
$$W_{mg} + W_{drag} = \Delta KE$$

$$(mg \times 18 \times \cos 180^\circ) + W_{drag} = K_B - K_A$$

$$(-2 \times 10 \times 18) + W_{drag} = \left(\frac{1}{2} \times 2 \times 0^2\right) - \left(\frac{1}{2} \times 2 \times 20^2\right)$$

$$-360 + W_{drag} = -400$$

$$W_{drag} = -40 J$$



–ve sign of W_{drag} shows that drag force is acting in downward direction.

Q. The position of a body having mass of 4 g, acted upon by a constant force varies with time as,
$x = 4t^2 + t$. where x is in metre and t in second.
What is the work done on the body by the force during first 2 seconds?

Sol. Here, $m = 4\text{ g} = 4 \times 10^{-3}$ kg,
Position, $x = 4t^2 + t$

$\therefore$ Velocity, $v = \frac{dx}{dt} = 8t + 1$

At t = 0 s, $v_0 = 8(0) + 1 = 1\ ms^{-1}$
At t = 2 s, $v_2 = 8(2) + 1 = 17\ ms^{-1}$
By work energy theorem,

$$W = \frac{1}{2}m(v_2^2 - v_0^2) = \frac{1}{2}(4 \times 10^{-3})(17^2 - 1^2)$$

$$W = \frac{1}{2}(4 \times 10^{-3})(288) = 576 \times 10^{-3} J$$

$$W = 0.576 J = 576 mJ$$

Q. **A block of mass 3 kg starts from rest and slides down a curved path in the shape of a quarter-circle of radius 2 m and reaches the bottom of the path with a 4 m/s speed. If g = 10 m/s², the amount of work done against friction is**

Sol. Here, mass of the block,

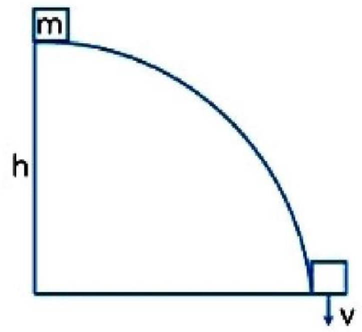
$m = 3$ kg

Initial speed of the block,

$u = 0$ (as it starts from rest)

Final speed of the block,

$v = 4$ m/s



Height, h (in this case, the radius of quarter circle) = 2 m

The change in kinetic energy of the block is

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 - 0 = 24\text{J}$$

The work done by the gravitational force is

$W_g = mgh = (3 \text{ kg}) (10 \text{ m/s2}) (2 \text{ m}) = 60$ J

If W_f is the work done by the friction, then according to work energy theorem,

$W_g + W_f = \Delta K$

Or $W_f = \Delta K - W_g = 24 \text{ J} - 60 \text{ J} = -36$ J

As work done against friction is equal and opposite to work done by the friction,

∴ The amount of work done against friction is 36 J

Q. **A body possessing kinetic energy T moving on a rough horizontal surface is stopped in a distance y. The frictional force exerted on the body is**

(A) Ty **(B)** $\frac{\sqrt{T}}{y}$ **(C)** $\frac{T}{y}$ **(D)** $\frac{T}{\sqrt{y}}$

Sol. According to work-energy theorem, $T = f\,y$

where f is the frictional force exerted on the body

or Frictional force, $f = \frac{T}{y}$

Q. **A block of mass 5 kg is resting on a smooth surface. What should be the angle at which a force of 20 N should be applied so that the body acquires a kinetic energy of 40 J after moving 4 m?**

(A) 30° **(B) 45°** **(C) 60°** **(D) 120°**

Sol. According to work-energy theorem

W = Change in kinetic energy

$$FS\cos\theta = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Substituting the given values, we get

$$20 \times 4 \times \cos\theta = 40 - 0 \qquad (\because u = 0)$$

or $$\cos\theta = \frac{40}{80} = \frac{1}{2} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Q. **A force F = 6t, where t is time, acts on a particle having mass = 1 kg. If the particle starts from rest, then the work done on the body during the first 1 sec will be**

(A) 4.5 J **(B) 22 J** **(C) 9 J** **(D) 18 J**

Sol. We have been given, F = 6t

or $$m\frac{dv}{dt} = 6t$$

Rearranging and integrating both sides

$$\Rightarrow \int_0^v dv = 6\int_0^1 t\,dt \qquad (\therefore m = 1 \text{ kg})$$

$$\Rightarrow v = 6\left[\frac{t^2}{2}\right]_0^1 \Rightarrow v = \frac{6}{2} = 3 ms^{-1}$$

Work done by the force during the first 1 s is given by the change in the kinetic energy of the object.

$$W = \Delta K.E. = \frac{1}{2}mv^2 \quad \Rightarrow \quad W = \frac{1}{2} \times 1 \times (3)^2 = 4.5J$$

Q. **A particle is moving along a circle having radius r with constant tangential acceleration. If the velocity of the particle is v at the end of second revolution after the revolution has started, then the tangential acceleration is**

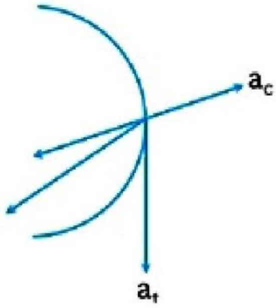
(A) $\frac{v^2}{8\pi r}$ **(B)** $\frac{v^2}{6\pi r}$ **(C)** $\frac{v^2}{4\pi r}$ **(D)** $\frac{v^2}{2\pi r}$

Sol. According to work-energy theorem,

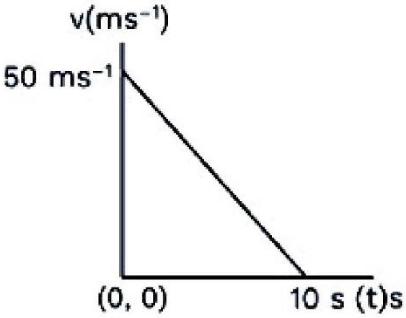
$$K_f - K_i = W_{\text{tangential force}} + W_{\text{centripetal force}}$$

$$\therefore \quad \frac{1}{2}mv^2 - 0 = F_t \times s + 0$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = ma_t \times (2 \times 2\pi r) \Rightarrow a_t = \frac{v^2}{8\pi r}$$



Q. **Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is**



Sol. Here, m = 10 kg, t = 2s

$u = 50\ ms^{-1}$ at t = 0 s

$$a = \frac{\Delta v}{\Delta t} = \frac{50-0}{0-10} = -5ms^{-2}$$

Speed of the body at t = 2 s,

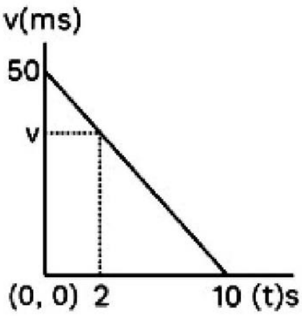
$v = u + at = 50 + (-5) \times 2 = 40\ ms^{-1}$

Using work energy theorem,

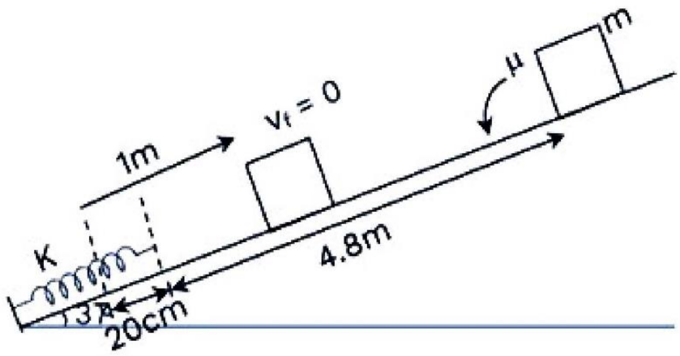
Work done on the body = Change in kinetic energy of the body

$$w = \frac{1}{2} \times 10[40^2 - 50^2]$$

$$= 5 \times (40 - 50)(40 + 50) = -4500\ J$$



Q. **Find K and μ when the block stops after 1m after rebounding from spring. Take m = 2kg.**



Sol.

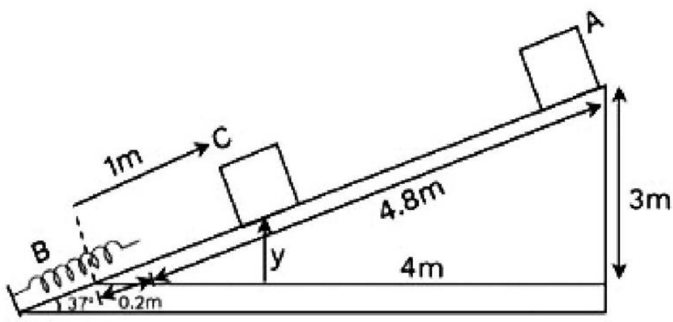
$V_A = 0$

$V_B = 0$

$v_C = 0$

$$\sin 37^\circ = \frac{y}{(1\text{ m})}$$

$$y = \frac{3}{5}\text{m}$$



By work energy theorem for journey A – B,

$$W_{mg} + W_f + W_{spring} + W_N = K_B - K_A$$

$$mg(3) + (\mu mg\cos 37^\circ)(-5) + \left(-\frac{1}{2}K(0.2)^2\right) + 0 = (0-0)$$

$$(2\times 10\times 3) + \left[\mu\times 2\times 10\times\frac{4}{5}\times(-5)\right] - \frac{K}{2}\times(0.2)^2 = 0$$

$$60 = 80\mu + 0.02K$$

$$3000 = 4000\mu + K \qquad \text{...(1)}$$

By Work energy theorem for journey B – C;

$$W_{mg} + W_f + W_{spring} + W_N = K_C - K_B$$

$$mg(-y) + (\mu mg\cos 37^\circ)(-1) + \frac{1}{2}K(0.2)^2 + 0 = (0-0)$$

$$\left[2\times 10\times\left(-\frac{3}{5}\right)\right] + \left[\mu\times 2\times 10\times\frac{4}{5}\times(-1)\right] + \frac{1}{2}K(0.2\times 0.2) = 0$$

$$-12 - 16\mu + 0.02K = 0$$

$$0.02K - 16\mu = 12$$

$$K - 800\mu = 600$$

$$-600 = 800\mu - K \quad \text{...(2)}$$

Adding (1) and (2);

$$2400 = 4800\mu$$

$$\mu = 0.5$$

$$K = 800(0.5) + 600 = 1000\,N/m$$

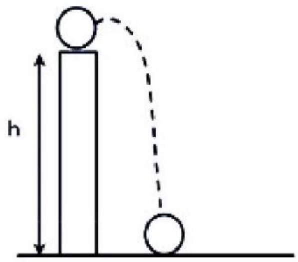
Potential Energy

The capacity to do work is known as potential energy.
For a system, change in potential energy = Negative of work done by the system.

$$\Delta PE = -\int F.dx$$

$$\boxed{\Delta PE = -W_{system} = U_f - U_i}$$

Q. **A man puts a ball from ground and keeps it on a table at height h. What are changes in various potential energies?**



Sol. It is to be assumed that the man moves the ball from the top very slowly so that the ball does not get any kinetic energy at any instant.
If F is force applied by man, then at any instant, F=mg
Work done by gravity, W_g = mg × h × cos180° = −(m g h)
Work done by man, W_m = F × h × cos0° = +(m g h)

$(\Delta U)_g$ = Change in gravitational potential energy = $-W_g$

$$\Delta U_g = -[-mgh]$$

$$\boxed{\Delta U_g = mgh}$$

Change in potential energy of man, $(\Delta U)_{man} = -W_m$

$$\boxed{\Delta U_{man} = -mgh}$$

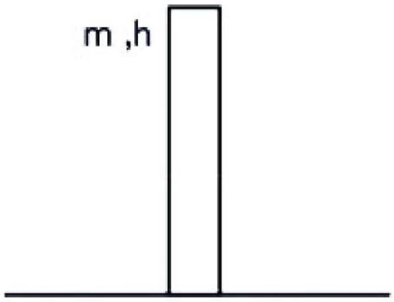
Potential energy at a point is not defined. Change in potential energy is found by previous explanation. So, to find potential energy at a point, we need to take a reference point where potential energy value can be assumed to be zero.

The choice of the reference point is to be taken as per convenience.

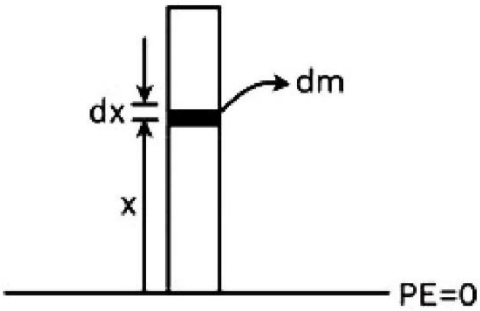
Then, $U_P - U_{reference} = -W_{system}$

$$U_P - 0 = -W_{system}$$

Q. **A uniform rod having mass m and length h is kept on ground as shown. Find gravitational potential energy of the rod.**



Sol. Assuming ground level as reference point (U = 0),



Consider an elemental mass dm of length dx at height x,

Then, potential energy of the elemental mass, $dU = (dm)g.x$

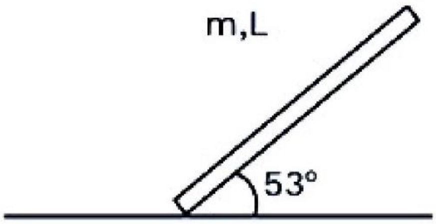
$$= \left(\frac{m}{h}\right).dx.g.x$$

Total potential energy, $U = \int dU = \frac{mg}{h}\int_0^h x.dx$

$$= \frac{mg}{h}.\left[\frac{x^2}{2}\right]_0^h = \frac{mg}{h}\left[\frac{h^2-0}{2}\right]$$

$$U = \frac{mgh}{2}$$

Q. **A rod of length L is kept on ground as shown. If m is mass of rod, then find its potential energy.**



Sol. Consider an elemental mass dm of length dx as shown in the figure.

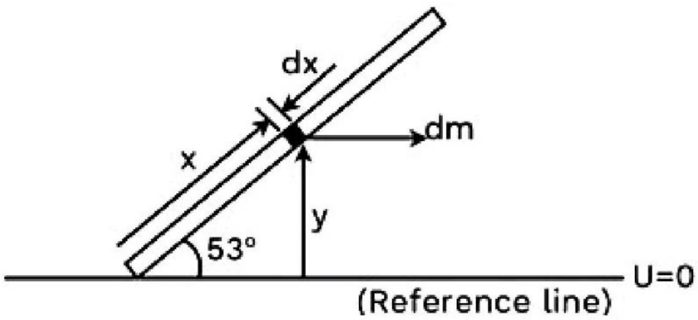
For elemental mass, $dm = \left(\frac{m}{L}\right).dx$

Also, $\sin 53^\circ = \frac{y}{x}$

Potential energy of the elemental mass dm,

$$dU = (dm).g.y$$

$$= \left(\frac{m}{L}\right)dx.g.x\sin 53^\circ$$



Total potential energy,

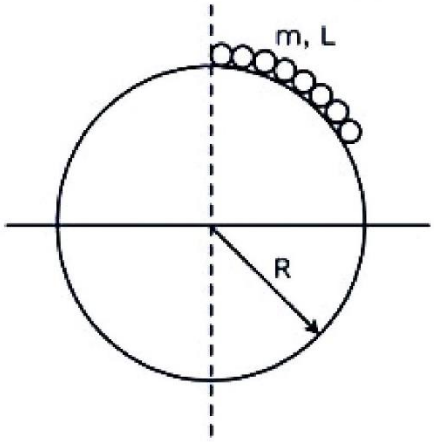
$$U = \int dU = \frac{mg\sin 53^\circ}{L}\int_0^L x.dx$$

$$= \frac{mg}{L}.\sin 53^\circ\left[\frac{x^2}{2}\right]_0^L$$

$$= \frac{mgL}{2}.\sin 53^\circ = \frac{mgL}{2}\times\frac{4}{5}$$

$$U = \frac{2}{5}mgL$$

Q. **Find potential energy of the chain in given arrangement.**



Sol. Consider an elemental mass dm of length R.dθ as shown.

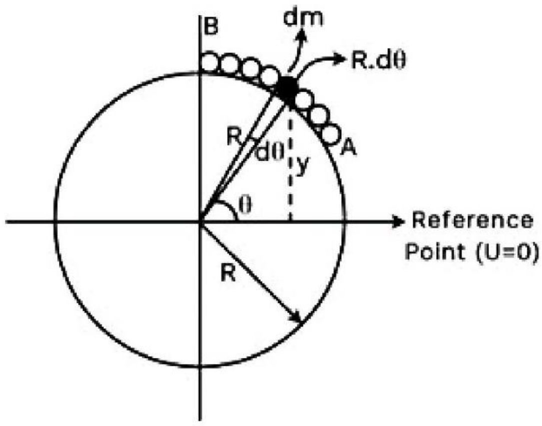
Then, $dm = \frac{m}{L}.R.d\theta$

and, $y = R\sin\theta$

So, $dU = dm.g.y$

$$= \frac{m}{L}.R.d\theta.g.R\sin\theta$$

$$dU = \frac{mgR^2}{L}\sin\theta.d\theta$$

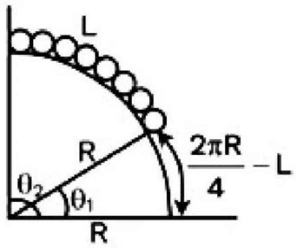


Total potential energy of chain, $U = \int dU = \frac{mgR^2}{L}\int_{\theta_1}^{\theta_2}\sin\theta.d\theta$

From adjacent figure,

$$\theta_1 = \frac{\left(\frac{2\pi R}{4} - L\right)}{R}$$

$$\theta_1 = \left(\frac{\pi}{2} - \frac{L}{R}\right)$$



and, $\theta_2 = \frac{\pi}{2}$

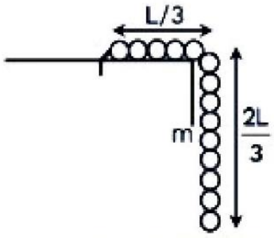
So, $U = \frac{mgR^2}{L}[-\cos\theta]_{\theta_1}^{\theta_2}$

$$= \frac{mgR^2}{L}(\cos\theta_1 - \cos\theta_2)$$

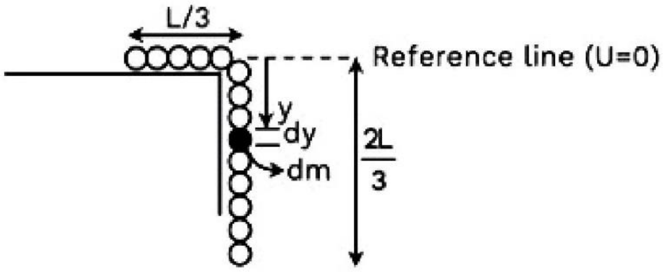
$$= \frac{mgR^2}{L}\left[\cos\left(\frac{\pi}{2} - \frac{L}{R}\right) - \cos\frac{\pi}{2}\right]$$

$$U = \frac{mgR^2}{L}\sin\frac{L}{R}$$

Q. **Find the potential energy of the chain shown in figure.**



Sol. For all mass elements on length $\frac{L}{3}$, U will be zero as they all lie on reference line.



Now, for a small element below the reference line,

$$dm = \left(\frac{m}{L}\right)dy$$

Potential energy of the elemental mass, $dU = dm.g.(-y)$

$$dU = -\frac{m}{L}g.y.dy$$

Total potential energy, $U = \int dU = -\frac{mg}{L}\int_0^{\frac{2L}{3}} y.dy$

$$= -\frac{mg}{L}\left[\frac{y^2}{2}\right]_0^{\frac{2L}{3}}$$

$$= -\frac{mg}{2L}\left[\frac{4L^2}{9}\right] = -\frac{2}{9}mgL$$

Conservative Forces and Potential Energy

The forces which are internal to a system can be of 2 types:

a. Conservative forces, like gravity; and
b. Dissipative forces like friction.

Internal forces are a result of the natural dynamics of the system as opposed to external forces which are applied by an external source.

We know that the work done by a force F on a body for small displacement is given as,

$dW = F{\cdot}dr.$

If the work done by an internal force F, when the particle moves from a general position r1 to another position r2, can be expressed as the difference in a scalar function of r between the two ends of the trajectory,

$$W_{12} = \int_{r_1}^{r_2} F \cdot dr = -\left[U(r_2) - U(r_1)\right] = U_1 - U_2,$$

then, it is said that the force is a conservative force.

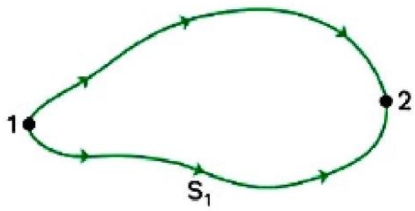
In the above expression, the scalar function U(r) is called the potential energy. It is clear that the potential energy satisfies $dU = -F{\cdot}dr$ (the minus sign is included for convenience).

There are two main consequences that follow from the existence of a potential function:

(i) the work done by a conservative force between points r_1 and r_2 is independent of the path. This follows from (1) since W_{12} only depends on the initial and final potentials U_1 and U_2 (and not on how we go from r_1 to r_2), and

(ii) the work done by potential forces is recoverable.

Consider the work done in going from point r_1 to point r_2, W_{12}. If we go, now, from point r_2 to r_1, we have that $W_{21} = -W_{12}$ since the total work $W_{12} + W_{21} = (U_1 - U_2) + (U_2 - U_1) = 0$



In one dimension any force which is only a function of position is conservative. That is, if we have a force, F(x), which is only a function of position, then F(x)dx is always a perfect differential. This means that we can define a potential function as

$$U(x) = -\int_{x_0}^{x} F(x)dx$$

where x_0 is arbitrary.

In two and three dimensions, we would, in principle, expect that any force which depends only on position, F(r), to be conservative. However, it turns out that, in general, this is not sufficient. In multiple dimensions, the condition for a force field to be conservative is that it can be expressed as the gradient of a potential function. That is,

$F_c = -\nabla U.$

This result follows from the gradient theorem, which is after called the fundamental theorem of calculus, which states that the integral

$$-\int_{r_1}^{r_2} \nabla U \cdot dr = -(U_2 - U_1)$$

is independent of the path between r_1 and r_2. Therefore, the work done by conservative forces depends only upon the end points r_2 and r_1 rather than the details of the path taken between them.

$$\int_{r_1}^{r_2} F_C \cdot dr = -\int_{r_1}^{r_2} \nabla U \cdot dr = -(U_2 - U_1)$$

In the general case, we will deal with internal forces that are a combination of conservative and non-conservative forces.

$$F = F_C + F_{NC} = -\nabla U + F_{NC}$$

The Gradient Operator, ∇

The gradient operator, ∇ (called "del"), in cartesian coordinates is defined as

$$\nabla(\ ) \equiv \frac{\partial(\ )}{\partial x}\hat{i} + \frac{\partial(\ )}{\partial y}\hat{j} + \frac{\partial(\ )}{\partial z}\hat{k}$$

When operating on a scalar function U (x, y, z), the result ∇U is a vector, called the gradient of U. The components of ∇U are the derivatives of U along each of the coordinate directions,

$$\nabla U \equiv \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$

If we consider a particle moving due to conservative forces with potential energy U (x, y, z), as the particle moves from point $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ to point $\vec{r} + d\vec{r} = (x + dx)\hat{i} + (y + dy)\hat{j} + (z + dz)\hat{k}$, The potential energy changes by $dU = \left[U(x + dx, y + dy, z + dz) - U(x, y, z)\right]$.

For small increments dx, dy and dz, and dU, can be expressed, using Taylor series expansions, as

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz = \nabla U \cdot dr$$

where $d\vec{r} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$

This equation expresses the fundamental property of the gradient. The gradient allows us to find the change in a function induced by a change in its variables.

Conservation of Energy

When all the forces which are doing work on the system are conservative, then the work is given by the equation,

$W_{12} = \int_{r_1}^{r_2} F \cdot dr = -\left[U(r_2) - U(r_1)\right] = U_1 - U_2$, and the principle of work and energy,

$$K_1 + W_{12} = K_2,$$

which reduces to,

$K_1 + U_1 = K_2 + U_2$

or more generally, since the points r_1 and r_2 are arbitrary,

$E = K + U = \text{constant}.$

The above equation shows that the total energy remains constant, and during the motion only exchanges between kinetic and potential energy take place.

In the general case, however, we will have a combination of conservative, F_C, and non-conservative, F_{NC}, forces. In this case, the work done by the conservative forces will be calculated using the corresponding potential function, i.e., $W_{12}^C = U_1 - U_2$, and the work done by the non-conservative forces will be path dependent and will need to be calculated using the work integral. Thus, in the general case, we will have,

$$K_1 + U_1 + \int_{r_1}^{r_2} F_{NC} \cdot dr = K_2 + U_2$$

The work done by non-conservative forces which oppose the motion is negative. Therefore, the sum of $K_2 + U_2$ will be less that $K_1 + U_1$.

Examples of Conservative Forces

Gravity near the earth's surface

On a "flat earth", the specific gravity g points down (along the -z axis), so F = – mgk.
Call U = 0 on the surface z = 0, and then

$$U(z) = -\int_0^z (-mg)\,dz,$$

$$U(z) = mgz.$$

For the motion of a projectile, the total energy is then

$$E = \frac{1}{2}mv^2 + mgz = \text{constant}.$$

Since v_x and v_y remain constant, we also have $\frac{1}{2}mv_z^2 + mgz = \text{constant}$.

Gravity

In a central gravity field,

$$F = -G\frac{Mm}{r^2} = -\nabla\left(-G\frac{Mm}{r}\right),$$

and so, taking $U(r \to \infty) = 0$,

$$U = -G\frac{Mm}{r}.$$

where G is the universal gravitational constant.

Spring Force

For small displacement, the force supported by a spring is F = −kx. The elastic potential energy of the spring is the work done on it to deform it by an amount x. Thus, we have

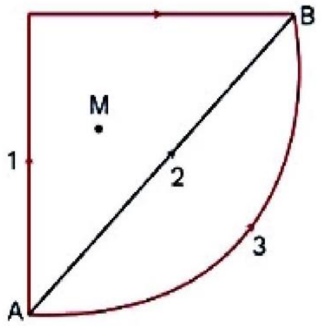
$$U = -\int_0^x -kx\,dx = \frac{1}{2}kx^2.$$

If the deformation, either tensile or compressive, increases from x_1 to x_2 during the motion, then the change in potential energy of the spring is the difference between its final and initial values, or,

$$\Delta U = \frac{1}{2}k\left(x_2^2 - x_1^2\right).$$

In general, if for a force, $\vec{\nabla} \times \vec{F}$ is equal to zero, it is said to be a conservative force.

Q. **If the work done in moving a particle from A to B along three different paths 1, 2, and 3 are W_1, W_2 and W_3 respectively (as shown) in the gravitational field of a point mass M, then the relation between W_1, W_2 and W_3 is**



(A) $W_1 > W_2 > W_3$

(B) $W_1 = W_2 = W_3$

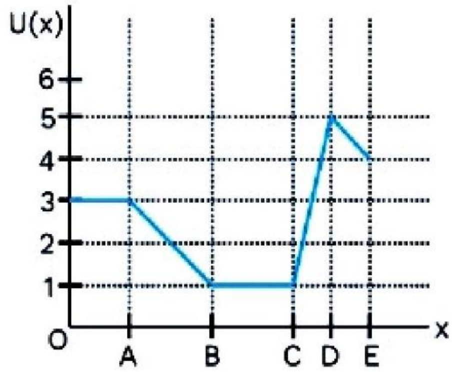
(C) $W_1 < W_2 < W_3$

(D) $W_2 > W_1 > W_3$

Sol. Gravitational force is a conservative force, so work done by it on a particle is independent of path followed.
Work done by a conservative force depends only on the initial and final positions of the particle.

∴ $W_1 = W_2 = W_3$

Q. The figure shows the potential energy function U(x) for a system in which a particle is in one-dimensional motion. Arrange regions AB, BC, CD and DE according to the magnitude of the force on the particle in decreasing order.



Sol. $\because \quad F = -\frac{dU}{dx}$

Magnitude of the slope is greatest in the region CD, then in region AB, then in DE and is zero in region BC.

$\therefore \quad CD > AB > DE > BC$

Q. The general form of potential energy curve for atoms or molecules can be represented by the following equation $U(R) = \frac{A}{R^n} - \frac{B}{R^m}$. Here, R is the interatomic or molecular distance, A and B are coefficients, n and m are the exponents. In the above equation

(A) First term represents the attractive part of the potential.

(B) Second term represents the attractive part of the potential.

(C) Both terms represent the attractive part of the potential.

(D) Second term represents the repulsive part of the potential.

Sol. $U(R) = \frac{A}{R^n} - \frac{B}{R^m}$

The first term shows the repulsive part of the potential function and the second term represents the attractive part of the potential function.

Q. **In a diatomic molecule, the potential energy function for the force between 2 atoms is given by, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a, b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U_{(x=\infty)} - U_{\text{at equilibrium}}]$, then D is**

Sol. Here,

$$U = \frac{a}{x^{12}} - \frac{b}{x^6}$$

∴ Force,

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right)$$

$$= -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$$

At equilibrium, F = 0

$$\therefore \quad \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0 \quad \text{or} \quad x^6 = \frac{2a}{b}$$

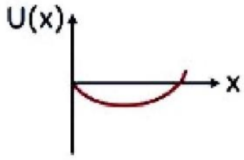
$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

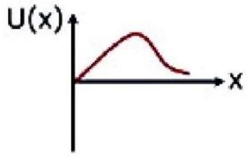
$$U_{(x=\infty)} = 0$$

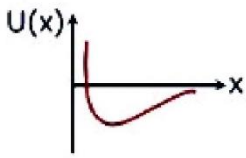
Now, we can find the dissociation energy as,

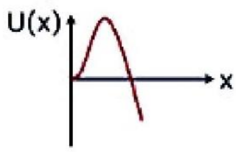
$$D = \left[U_{(x=\infty)} - U_{\text{at equilibrium}}\right] = \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$$

Q. **The force applied on a particle which is constrained to move along x-axis varies with x-coordinate as**
$F(x) = -kx + ax^3$. Here k and a are positive constants.
For $x \geq 0$, the functional form of the potential energy U(x) of the particle is

(A) 

(B) 

(C) 

(D) 

Sol. $F(x) = -kx + ax^3$

Relation between F and U is given by

$$F = -\frac{dU}{dx}$$

So, $$U = \int dU = -\int F.dx$$

$$U = -\int(-kx + ax^3)dx$$

$$U = -\left[-\frac{kx^2}{2} + \frac{ax^4}{4}\right]$$

$$\boxed{U = \frac{kx^2}{2} - \frac{ax^4}{4}}$$

U becomes zero when

$$\frac{kx^2}{2} = \frac{ax^4}{4}$$

$$x^2 = \frac{2k}{a}$$

$$x = \pm\sqrt{\frac{2k}{a}}$$

For $x \geq 0$, U is zero at,

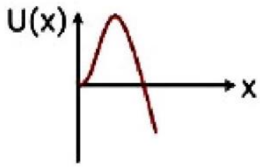
$$x = \sqrt{\frac{2k}{a}}.$$

Also, at $x = 0$, $U = 0$

For $x < \sqrt{\frac{2k}{a}}$, U is +ve

For $x > \sqrt{\frac{2k}{a}}$, U is –ve

Force is zero at x=0, this shows that the slope of U-x graph is zero at x=0.
So, U–x graph can be drawn as



Q. **Find the equilibrium position if the potential energy of a conservative system is given by $V(x) = (x^3 - 3x)$ joule, where x is measured in metre.**

Sol. Here, $V(x) = (x^2 - 3x)$ J

For a conservative field, force,

$$F = -\frac{dV}{dx}$$

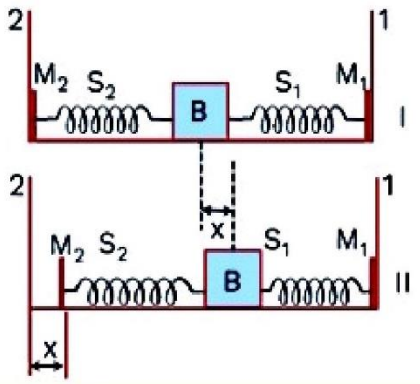
$$\therefore \quad F = -\frac{d}{dx}\left(x^2 - 3x\right) = 3 - 2x$$

At equilibrium position, F = 0

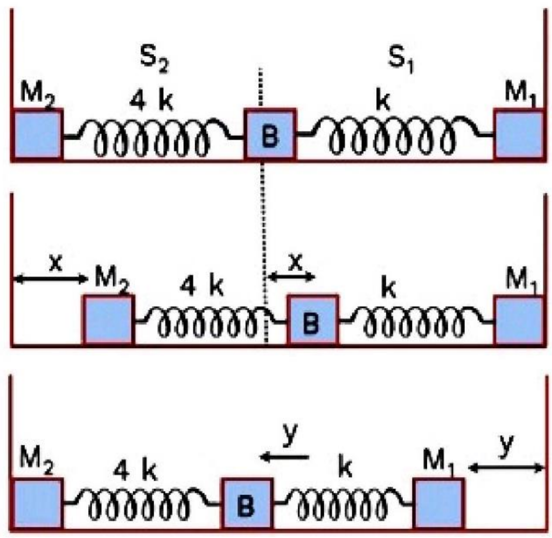
$\therefore \quad -2x + 3 = 0$ or

$$x = \frac{3}{2}\text{m} = 1.5\,\text{m}$$

Q. **A block (B) is attached to two upstretched spring S_1 and S_2 with spring constants k and 4k, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The spring and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is**



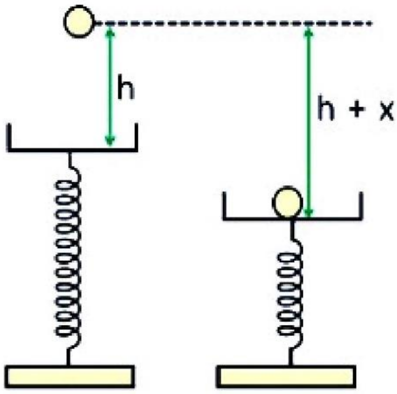
Sol. When the block B is displaced to the right, the spring S_2 will have no tension, it will be in the natural length. And when the block B is displaced to the left, the spring S_1 will have no tension, it will be in its natural length. Whenever a spring pulls a massless support, the spring will be in natural length.



At maximum compression, velocity of B will be zero.
By energy conservation,

$$\frac{1}{2}(4k)\,y^2 = \frac{1}{2}kx^2 \quad \text{or} \quad \frac{y}{x} = \frac{1}{2}$$

Q. **A ball having mass m is dropped from a height h on a platform fixed at the top of a vertical spring, as shown in figure. The platform moves down by a distance x. Then the spring constant is**



(A) $\frac{mg}{(h+x)}$

(B) $\frac{mg}{(h+2x)}$

(C) $\frac{2mg(h+x)}{x^2}$

(D) $\frac{mg}{(2h+x)}$

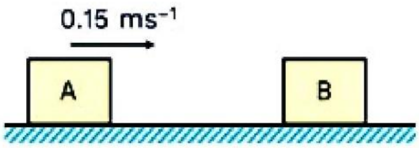
Sol. Loss in potential energy of the ball = mg(h + x)

Gain in elastic potential energy of the spring $= \frac{1}{2}kx^2$

As per law of conservation of mechanical energy,

$$mg(h+x) = \frac{1}{2}kx^2 \quad \text{or} \quad k = \frac{2mg(h+x)}{x^2}$$

Q. **Two rectangular blocks A and B of masses 2kg and 3kg respectively are connected by a spring of spring constant 10.8 N m^{-1} and are placed on a frictionless horizontal surface. The block A was given an initial velocity of 0.15 m s^{-1} in the direction shown in the figure. The maximum compression of the spring during the motion is**



Sol. At maximum compression of spring, both blocks have same speed.
By conservation of momentum,

$$2(0.15) = (2+3)V$$

$$V = \frac{2 \times 0.15}{5} = 2 \times 0.03 = 0.06 \text{ m/s}$$

By energy conservation,

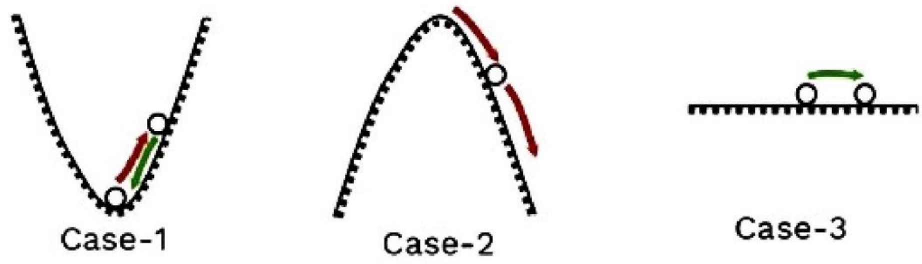
$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}(2)(0.15)^2 - \frac{1}{2}(2+3)(0.06)^2$$

$$10.8\ x_{max}^2 = 0.045 - 0.009$$

$$x_{max} = \sqrt{\frac{1}{300}} = 0.05 \text{ m}$$

Equilibrium

At equilibrium condition, net force on a body = 0.
There are 3 kinds of equilibrium
(a) Stable equilibrium
(b) Unstable equilibrium
(c) Neutral equilibrium
Let's say 3 balls are kept as shown in equilibrium.



If ball is slightly displaced in case 1, the ball will come back due to presence of a restoring components of mg. So, the placement of ball is such that the ball tries to restore its initial equilibrium condition. This is called "Stable equilibrium".

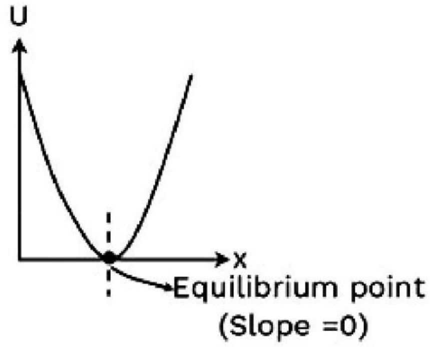
If ball is slightly displaced in case 2, the ball will fall further. There is no restoring force. Such an equilibrium condition is "unstable equilibrium.

If ball is slightly displaced in case 3, it stays at equilibrium even at displaced position. There is no restoring force, nor is any disturbing force. It is called "neutral equilibrium".

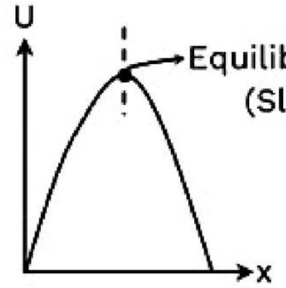
Relation between Force and Potential Energy
Force (F) and potential energy (U) are related as

$$F = -\frac{dU}{dx}$$

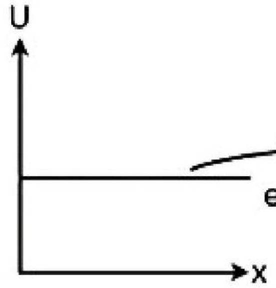
(Negative of slope of U – x graph is equal to force).
U – x graph for stable equilibrium can be drawn as-



U-x graph for unstable equilibrium can be drawn as-



U-x graph for neutral equilibrium will be-



When body passes a point of stable equilibrium, slope of U-x graph increases with x.
When body passes a point of unstable equilibrium, slope of U-x graph decreases.

$\therefore$ For stable equilibrium,

$$\frac{d}{dx}(\text{Slope}) = +ve$$

$$\frac{d}{dx}\left[\frac{dU}{dx}\right] = +ve \text{ , i.e. } \frac{d^2U}{dx^2} = +ve$$

For unstable equilibrium,

$$\frac{d}{dx}(\text{Slope}) = -ve$$

$$\frac{d}{dx}\left[\frac{dU}{dx}\right] = -ve \text{ , i.e. } \frac{d^2U}{dx^2} = -ve$$

Power

Rate of doing work is called power.

Power is delivered by a force. SI unit of power is watt (W).

Power, $P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F}.\vec{s})$

$$P = \vec{F}.\frac{d\vec{s}}{dt}$$

When force $\vec{F}$ is constant,

$$P = \vec{F}.\vec{v} \;....\left(\because \frac{d\vec{s}}{dt} = \vec{v}\right)$$

Above formulae give instantaneous power.

For a time interval, average power can be defined as,

Average power, $P = \frac{W_{\Delta t}}{\Delta t}$

Also, $P = \vec{F}.\vec{v} = m\vec{a}.\vec{v}$

Putting $\vec{a} = \frac{d\vec{v}}{dt}$

Power, $P = m\vec{v}.\frac{d\vec{v}}{dt}$

$\Rightarrow \int P.dt = \int mv.dv$

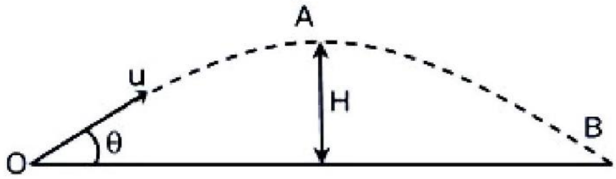
Also, $\vec{a} = \vec{v}.\frac{d\vec{v}}{dx}$

So, $P = mv.\frac{dv}{dx}.v$

$\int P.dx = \int m.v^2.dv$

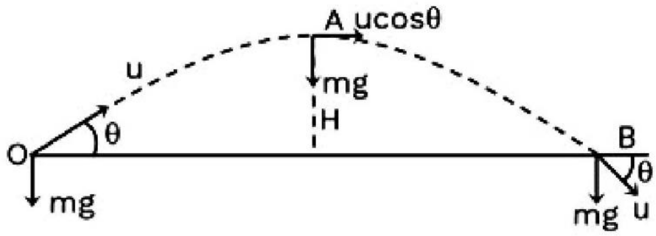
These are the equations related to power, displacement and velocity.

Q. **For given projectile motion, find power delivered between O–A and instantaneous power at maximum height and at O.**



Sol. For journey O – A

Average power, $P_{avg} = \frac{\Delta W}{\Delta t}$



$$= \frac{(-mg)H}{\left(\frac{T}{2}\right)}$$

$$= \frac{-2mgH}{T}$$

$$P_{avg} = \frac{-2mgH}{\left(\frac{2u\sin\theta}{g}\right)} = -\frac{mg^2H}{u\sin\theta}$$

At point A, instantaneous power, $P_{ins.} = P_A = mg\,.\,u\cos\theta\,.\,\cos 90^\circ = 0$

At O, instantaneous power, $P_{ins.} = P_o = mg.\ u.\ \cos(90 + \theta) = -m\,g\,u\sin\theta$

At B, instantaneous power, $P_{ins.} = P_B = mg.\ u.\ \cos(90 - \theta) = \quad m\,g\,u\sin\theta$

Q. **2000 stones each of mass 5kg each a loaded on a truck at a height 12m by a crane in time 10 minutes. Find power delivered.**

Sol. Average power, $P_{avg} = \frac{W}{\Delta t} = \frac{n[mgh]}{\Delta t}$

$$= \frac{(2000 \times 5 \times 10 \times 12)\,J}{(10 \times 60)\,s} = 2000\,W$$

Commercial unit of power is “horse power (h.p.)”.
1 h.p. = 746 W

So, Power of crane = 2000W = $\frac{2000}{746}$hp = 2.68hp

Q. **Find the power of a tube well motor if it pumps water of 10kg/s at a height of 12m at speed 5m/s.**

Sol. Power of motor, P = K.E. /s + P.E./s

$= \frac{1}{2} \times 10 \times 5^2 + (10 \times g \times h)$

$= \left(\frac{1}{2} \times 10 \times 25\right) + (10 \times 10 \times 12)$

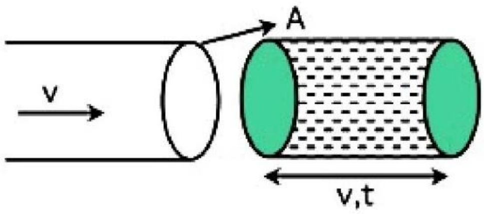
$= 125 + 1200$

$= 1325\,J/s$ = 1325 Watt

Q. **To increase the flow output of a tube well n times, what should be the increase in power of the motor?**

Sol. Power, P = F.v

In 1s, mass of water coming out, $\frac{dm}{dt} = \rho Av$



If flow output is increased n times,

Then, $\left(\frac{dm}{dt}\right)' = (\rho Av') \Rightarrow \rho Av' = n\rho Av \Rightarrow v' = nv$

Now, $F = \frac{dp}{dt} = \frac{d(mv)}{dt}$, p = momentum

$F = m.\frac{dv}{dt} + v.\frac{dm}{dt}$

As v is constant, so $\frac{dv}{dt} = 0$

Then, $F = v.\frac{dm}{dt}$

So, new force, $F' = v'\left(\frac{dm}{dt}\right)' = (nv)n.\left(\frac{dm}{dt}\right)$

$\Rightarrow \quad F' = n^2\left[v.\dfrac{dm}{dt}\right] = n^2F$

New power, $P' = F'.v' = n^2F \times nv$

$P' = n^3P$

Q. **A particle is moving along x-axis under the action of a force, F which varies with its position (x) as $F \propto \dfrac{1}{\sqrt[4]{x}}$. How the power of this applied force varies with x.**

Sol. Force, $F \propto x^{-\frac{1}{4}}$

Then, acceleration, $a \propto x^{-\frac{1}{4}}$

$a = kx^{-\frac{1}{4}}$ (where k is a proportionality constant)

Then, $a = \dfrac{dv}{dt} = \dfrac{dx}{dt}\dfrac{dv}{dx} = kx^{-\frac{1}{4}}$

$v\dfrac{dv}{dx} = kx^{-\frac{1}{4}}$

$v.dv = kx^{-\frac{1}{4}}.dx$

$\int v.dv = \int kx^{-\frac{1}{4}}.dx$

$\dfrac{v^2}{2} = k\dfrac{x^{\frac{3}{4}}}{\left(\frac{3}{4}\right)}$, then, $v \propto x^{\frac{3}{8}}$

Now, power, $P = Fv$

$P \propto \dfrac{x^{\frac{3}{8}}}{x^{\frac{1}{4}}}$, i.e., $\boxed{P \propto x^{\frac{1}{8}}}$

Q. **Provided a racing car does not lose traction, the time taken by it to race from rest through a distance depends mainly on engine's power P. Derive the distance S in terms of t and P, assuming that power P is constant.**

Sol.

$$\text{Power, } P = mav = mv.\frac{dv}{dt}$$

$$\int_0^v v.dv = \frac{P}{m}\int_0^t dt$$

$$\text{Then, } \frac{v^2}{2} = \frac{Pt}{m}$$

$$v = \sqrt{\frac{2Pt}{m}}$$

$$\text{Also, } v = \frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}\text{, then, } \int_0^S dx = \sqrt{\frac{2P}{m}}\int_0^t t^{\frac{1}{2}}.dt$$

$$S = \sqrt{\frac{2P}{m}}\left(\frac{2}{3}t^{\frac{3}{2}}\right) = \sqrt{\frac{8P}{9m}}\left(t^{\frac{3}{2}}\right)$$