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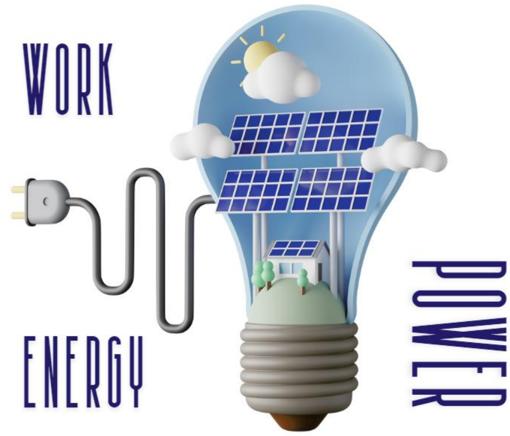


WORK POWER ENERGY

REVIEW OF BASIC CONCEPTS

1. Work Done by a Force

(a) Work done by a constant force

When a constant force **F** acting on a body produces a displacement **S**, then the work done by the force is given by

$$W = \mathbf{F} \cdot \mathbf{S} = FS\cos\theta$$

where θ is the angle between the force vector **F** and the displacement vector **S** [see Fig. 4.1]. F and S are the magnitudes of **F** and **S** respectively.

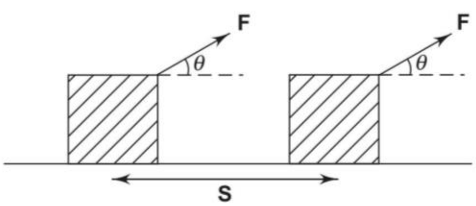


Fig. 4.1

(i) If θ is acute, $\cos\theta$ is positive. Hence work is positive for acute θ. In this case the force increases the speed of the body.

(ii) If $\theta = 90°$, $W = 0$, i.e. if the force is perpendicular to displacement work done by the force is zero.

(iii) If θ is obtuse, W is negative. In this case the force decreases the speed of the body.

(iv) If $\theta = 0$, i.e. force **F** is in the same direction as displacement **S**, then $W = FS$.

(v) If $\theta = 180°$, force **F** is opposite to **S** (example frictional force), $W = -FS$. Work done by frictional and viscous force is always negative.

(b) Work done by a variable force

Suppose a force **F** is not constant but depends on the position vector **r** of the body, then the work done by the force **F** in moving the body from a position r_1 to a position r_2 is given by

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot \mathbf{dr}$$

EXAMPLE 1 A block of mass $m = 5$ kg slides down from the top of an inclined plane of inclination $\theta = 30°$ with the horizontal. The coefficient of sliding friction between the block and the plane is 0.25. The length of the plane is 2 m. Find the work done by the (a) gravitational force, (b) frictional force and (c) normal reaction if the block slides to the the bottom of the plane.

SOLUTION Refer to Fig. 4.2.

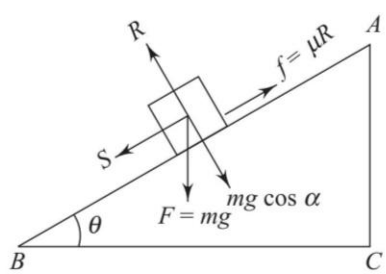


Fig. 4.2

Displacement $S = AB = 2$ m from A to B.

(a) Angle between **F** and **S** is $(90° - \theta) = 90° - 30° = 60°$

$\therefore$ Work done by gravitational force is

$$W_1 = FS\cos 60° = mgS\cos 60°$$

$$= 5 \times 9.8 \times 2 \times \frac{1}{2} = 49 \text{ J}$$

(b) Work done by frictional force is

$$W_2 = fS\cos 180°$$

$$= -fS = -\mu RS$$

$= -\mu mg(\cos\theta)S$

$= -0.25 \times 5 \times 9.8 \times \cos 30° \times 2$

$= -21.2$ J

(c) Since the normal reaction R is perpendicular to displacement S, work done by normal reaction is

$$W_3 = RS\cos 90° = 0$$

EXAMPLE 2 A block of mass $m = 2$ kg is raised vertically upwards by means of a massless string through a distance of $S = 4$ m with a constant acceleration $a = 2.2$ ms^{-2}. Find the work done by (a) tension and (b) gravity. Also find the net work done on the block.

SOLUTION

(a) From the free body diagram (Fig. 4.3)

$T - mg = ma$

$\Rightarrow T = m(a+g)$

$= 2 \times (2.2 + 9.8)$

$= 24$ N

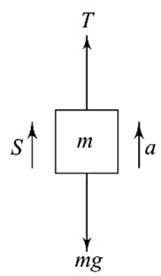


Fig. 4.3

$\therefore$ Work done by tension is ($\because$ T and S are in the same direction)

$W_1 = TS\cos 0°$

$= 24 \times 4 \times 1 = 96$ J

(b) Since the gravitational force mg and displacement S are in opposite directions, work done by gravity is

$$W_2 = mgS\cos 180°$$
$$= -2 \times 9.8 \times 4 = -78.4 \text{ J}$$

(c) Net work done $W = W_1 + W_2 = 96 - 78.4 = 17.6$ J

EXAMPLE 3 A block of mass $m = 2$ kg is suspended by a light string from the ceiling of a lift. The lift starts moving down with an acceleration $a = 1.8$ ms^{-2}. Find the work done by the tension in the string during the first 5 seconds.

SOLUTION Tension $T = m(g - a) = 2 \times (9.8 - 1.8) = 16$ N

Distance moved in $t = 5$ s is

$$S = \frac{1}{2}at^2 = \frac{1}{2} \times 1.8 \times (5)^2 = 22.5 \text{ m}$$

Since the tension and displacement are in opposite directions, the work done by tension is

$$W = TS\cos 180° = -TS$$
$$= -16 \times 22.5 = -360 \text{ J}$$

EXAMPLE 4 A constant force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ newton displaces a body from position $\mathbf{r}_1 = (4\hat{\mathbf{i}} - 5\hat{\mathbf{j}})$ metre to $\mathbf{r}_2 = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ metre. Find the work done by the force.

SOLUTION

Displacement $\mathbf{S} = \mathbf{r}_2 - \mathbf{r}_1$

$$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) - (4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) = -3\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$$

$$\therefore \quad W = \mathbf{F}\cdot\mathbf{S} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(-3\hat{\mathbf{i}} + 8\hat{\mathbf{j}})$$
$$= -6 + 24 = 18 \text{ J}$$
$$[\because \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 1 \text{ and } \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = 0]$$

EXAMPLE 5 A body of mass $m = 0.5$ kg travels in a straight line with a velocity $v = 5x^{3/2}$ where v is in ms^{-1} and x is in metre. Find the work done in displacing the body from $x = 0$ to $x = 2$ m.

SOLUTION

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(5x^{3/2})$

$$= 5 \times \frac{3}{2}x^{1/2}\frac{dx}{dt}$$
$$= \frac{15}{2}x^{1/2} \times (5x^{3/2}) \qquad \left[\because \frac{dx}{dt} = v\right]$$
$$= \frac{75}{2}x^2$$

$\therefore$ Work done $W = \displaystyle\int_{x=0}^{x=2} F\,dx = \int_0^2 ma\,dx$

$$= 0.5 \times \frac{75}{2}\int_0^2 x^2 dx$$
$$= 0.5 \times \frac{75}{2} \times \left|\frac{x^3}{3}\right|_0^2$$
$$= \frac{0.5 \times 75}{2 \times 3}(8 - 0)$$
$$= 50 \text{ J}$$

EXAMPLE 6 The displacement x of a particle of mass m moving in a straight line varies with time t as $x = kt^{3/2}$ under the action of a force F where k is a constant. The work done by the force is proportional to

(a) $\sqrt{t}$ (b) t

(c) $t^{3/2}$ (d) t^2

SOLUTION $x = kt^{3/2}$

Velocity $v = \dfrac{dx}{dt} = \dfrac{3}{2}kt^{1/2}$ (i)

Acceleration $a = \dfrac{dv}{dt} = \dfrac{3}{2}k \times \dfrac{1}{2} \times t^{-1/2} = \dfrac{3k}{4}t^{-1/2}$

Force $F = ma = \dfrac{3km}{4}t^{-1/2}$ (ii)

Work done $W = \int F dx$ (iii)

From Eq (i) $dx = \frac{3}{2} k\, t^{\frac{1}{2}}\, dt$ (iv)

Using (ii) and (iv) in (iii) we have

$$W = \int \frac{3km}{4} t^{-1/2} \times \frac{3}{2} k t^{1/2}\, dt$$

$$= \frac{9k^2 m}{8} \int dt = \frac{9k^2 m}{8} t$$

Hence $W \propto t$.

EXAMPLE 7 The force F on a particle of mass m moving in a straight line varies with its velocity v as $F = \frac{k}{v}$ where k is a constant. The work done by the force in time t is

(a) $\left(\frac{kt}{mv}\right)^2$ (b) $\frac{kt}{v}$

(c) $\frac{kt^2}{m}$ (d) kt

SOLUTION $F = \frac{k}{v} \Rightarrow Fv = k \Rightarrow F\frac{dx}{dt} = k$

$\therefore$ $F dx = k\, dt$

Work done is $W = \int F\, dx = k \int_0^t dt = kt.$

So the correct choice is (d).

EXAMPLE 8 A uniform chain of mass M and Length L has a part l of its length hanging over the edge of the table. If the friction between the chain and the edge is neglected, the work done to pull the length l on the table is

(a) $\frac{Mgl^2}{L}$ (b) $\frac{Mgl^2}{2L}$

(c) $\frac{MgL^2}{l}$ (d) $\frac{MgL^2}{2l}$

SOLUTION Mass per unit length $= \frac{M}{L}$. Mass of part of length l is $m = \frac{Ml}{L}$. Force (weight) of this part is

$$F = mg = \frac{Mlg}{L}$$

Work done $W = \int_0^l F dl = \frac{Mg}{L} \int_0^l l dl = \frac{Mgl^2}{2L}$

2. Energy

Energy can be defined as the capacity or ability to do work and is measured by the amount of work a body can do. So, energy is measured in the same units as work, namely, joule. Like work, energy is a scalar quantity.

Energy can exist in various forms, such as heat energy, electrical energy, sound energy, light energy, chemical energy, nuclear energy, mechanical energy, etc. We will be mainly concerned with mechanical energy. Mechanical energy is of two types, *kinetic* and *potential.*

Kinetic Energy: Energy due to Motion A moving object can do work on another object when it strikes it. In other words, an object in motion has the ability to do work and, by definition, has energy. *The energy possessed by a body by virtue of its motion is called kinetic energy.*

An initially motionless body can move and acquire a velocity only if a force acts on it. The work done by the force in causing the body to move measures the kinetic energy (written as KE) of the moving body, i.e.

$$\text{KE} = W$$

The kinetic energy of a body of mass m, moving with a velocity v is given by

$$\text{KE} = \frac{1}{2} mv^2$$

This relation holds even if the force is variable, i.e. if the force varies both in magnitude and direction.

Work-Energy Principle Suppose a body of mass m moves with an initial velocity u. A force F acts on it, as a result of which it acquires a final velocity v. The work done by the force is given by

$$W = \int_u^v F dx = m \int_u^v a dx = m \int_u^v \frac{dv}{dt} dx$$

or $$W = m \int_u^v v\, dv = \frac{1}{2} m\left(v^2 - u^2\right) \quad \left(\because \frac{dx}{dt} = v\right)$$

$$= \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

= final KE – initial KE

= change in KE

Thus, *the work done by a force in displacing a body measures the change in its kinetic energy.* This is the work-energy principle.

Thus, when a force does work on a body, its kinetic energy increases; the increase in kinetic energy being equal to the amount of work done. The converse of this is also true. When the kinetic energy of a body is decreased by a retarding force, the decrease is equal to the work done by the body against the retarding force. Thus kinetic energy and work are equivalent quantities and are, therefore, measured in the same units, namely, joule.

Potential Energy: Energy due to Position or Configuration An object can have energy not only by virtue of its motion, but also because of its position or configuration. *The energy possessed by a body owing to its position or configuration is called potential energy.* For example, a wound watch spring has potential energy on account of its wound state or configuration of the coils. As the spring unwinds, it does work to move the hands of the watch. Thus, a wound spring has the *potentiality* to do work.

Gravitational Potential Energy An object held at a position above the surface of the earth has potential energy by virtue of its position. When it falls from that position, it can do work. The potential energy of an object held above the earth is called *gravitational potential energy*. To calculate the energy stored in a body which has been lifted above the earth's surface against the gravitational force, we have to calculate the amount of work done in carrying it there.

Consider a body of mass m. It is lifted vertically to a height h above the earth by applying a force F vertically upward. The force F must be just enough to overcome the gravitational attraction, i.e.

$$F = mg$$

where g is the acceleration due to gravity at that place. For bodies not too far above the surface of the earth, the value of g is practically constant. Hence the work done by a constant force F in displacing a body by a height h can be calculated by the product $F \times h = mgh$. Thus gravitational potential energy of a body of mass m at a height h above the surface of the earth is mgh. The gravitational potential energy on the surface of the earth is taken to be zero.

$$\text{Gravitational PE} = mgh \quad \text{or} \quad U = mgh$$

Potential Energy of a Spring. Consider a perfectly elastic spring. One end of the spring is fixed to a rigid wall and other end is fixed to a block which is placed on a frictionless horizontal surface as shown in Fig. 4.4. We assume that the mass of the spring is negligible compared to the mass of the block.

If we stretch the spring by a distance x, the spring will exert a force on us during stretching. This force is due to the reaction of the spring and is called the *restoring force* which is proportional to the displacement x and acts in a direction opposite to the displacement, i.e.

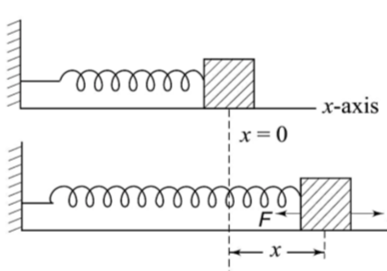


Fig. 4.4

$$F \propto -x$$

or $$F = -kx$$

where k is the force constant of the spring. The negative sign indicates that the force acts in a direction opposite to displacement.

To stretch a spring by a displacement x, we must exert a force F' on it, equal but opposite to the force F exerted by the spring on us. Therefore, the applied force is

$$F' = -F = kx$$

Notice that F' is a variable force as it depends on x. Therefore, the work done by the applied force in stretching the spring through a distance x is given by

$$W = \int_0^x F'\,dx$$

$$= \int_0^x (kx)dx = k\int_0^x x\,dx$$

$$= k\left|\frac{x^2}{2}\right|_0^x = \frac{1}{2}kx^2$$

It is evident that the work done in compressing the spring by an amount x is also given by $W = \frac{1}{2}kx^2$. The work done in stretching or compressing a spring is stored in it in the form of potential energy which is due to the changed configuration of the coils of the spring. Hence the potential energy of a massless elastic spring of force constant k when it is stretched or compressed by an amount x is given by

$$U = W = \frac{1}{2}kx^2$$

3. Conservative and Non-Conservative Forces

(a) Conservative force

A force is conservative if

(i) the work done by it on a body in moving it from one position to another depends only on the initial and final positions of the body and not on the path followed by it between the two positions.

or

(ii) the net work done by the force on a body that moves through any closed path is zero.

The above two conditions are equivalent. Examples of conservative forces are gravitational force, electrostatic force and spring force.

(b) Non-conservative force

A force is non-conservative if

(i) The work done by it on a body in moving it from one position to another depends on the path followed by the body between the two positions.

or

(ii) The work done by the force on a body that moves through a closed path is non zero.

Examples of non-conservative forces are frictional and viscous forces.

4. Conservative Force and Potential Energy

For a conservative force F that depends upon position r, there is a potential energy function U which also depends on r. When a conservative force does positive work, the potential of the system decreases, i.e.

Work done = decrease in potential energy

$$\text{or} \quad F dx = -dU$$

$$\text{or} \quad F = -\frac{dU}{dx}$$

Hence *the negative derivative of the potential energy function with respect to position gives the conservative force acting on the system.*

5. Principle of Conservation of Energy

The total energy of an isolated system remains constant; it may change from one form to another.

EXAMPLE 9 An elastic spring of negligible mass has a force constant $k = 4\ \text{Nm}^{-1}$. One end of the spring is fixed to the wall and the other end touches a block of mass $m = 250$ g placed on a horizontal surface. The spring is compressed by an amount $x = 5$ cm as shown in Fig. 4.5. The coefficient of friction between the block and the horizontal surface is $\mu = 0.2$. If the system is released, the find the speed of the block when it leaves the spring.

SOLUTION Loss in P.E. of spring + work done against friction = gain in K.E.

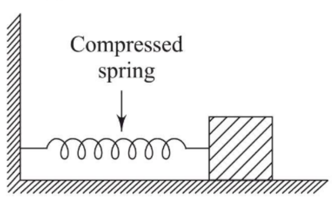


Fig. 4.5

$$\text{or} \quad \frac{1}{2}kx^2 + \mu mgx = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}\times 4\times(0.05)^2 + 0.2\times 0.250\times 10\times 0.05 = \frac{1}{2}\times 0.250\times v^2$$

which gives $v = 0.5\ \text{ms}^{-1}$

EXAMPLE 10 A bullet moving with a speed of 100 ms^{-1} travels a distance of 2 cm in a plank of wood before coming to rest. How much distance will the same bullet travel in the same plank before coming to rest if it were moving with a speed of 200 ms^{-1}?

SOLUTION Since the bullet and the plank are the same, the resistive force F exerted on the bullet is the same in the two cases. The kinetic energy is spent in doing work against friction. Hence

$$\frac{1}{2}mv_1^2 = Fx_1$$

$$\text{and} \quad \frac{1}{2}mv_2^2 = Fx_2$$

These equations give

$$x_2 = x_1 \times \frac{v_2^2}{v_1^2} = 2\text{ cm}\times\left(\frac{200}{100}\right)^2 = 8\text{ cm}$$

EXAMPLE 11 A bullet moving with initial kinetic energy K enters a block of wood at A. It loses 1/4 its initial kinetic energy after travelling a distance $AB = 3$ cm in the block. How much further distance will it penetrate before coming to rest? Assume that the resistance offered by the block is constant.

SOLUTION Refer to Fig. 4.6.

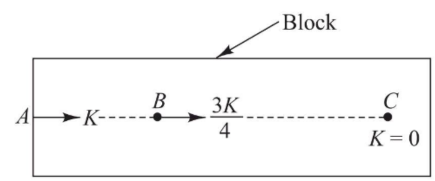


Fig. 4.6

Let F be the constant resistance force. For distance AB, we have change in kinetic energy = work done, i.e.

$$K - \frac{3K}{4} = F\times AB$$

$$\Rightarrow \quad F = \frac{K}{4\,AB}$$

Let us suppose that the bullet comes to rest at C. For distance BC, we have

$$\frac{3K}{4} - 0 = F\times BC$$

$$\Rightarrow \quad BC = \frac{3K}{4F} = \frac{3K}{4}\times\frac{4AB}{K} = 3AB$$

$$\Rightarrow \quad BC = 3\times 3\text{ cm} = 9\text{ cm}$$

EXAMPLE 12 A body of mass $m = 250$ g tied to a string is whirled in a vertical circle of radius $r = 1$ m. Find the minimum speed the body must have to complete the circle when the body is (a) at the top of the circle and (b) at the bottom of the circle. Also find the tension in the string when the body is at the bottom of the circle. Take $g = 10\ ms^{-2}$.

SOLUTION

(a) When, the body is at top A of the circle, the net force towards centre O is $TA + mg$. Hence [see Fig. 4.7]

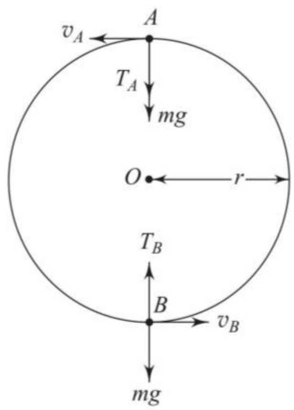


Fig. 4.7

$$T_A + mg = \frac{mv_A^2}{r}$$

Where v_A = velocity of the body at top A

and T_A = tension in the string when the body is at A.

For v_A to be minimum, $T_A \simeq 0$ which gives

$$v_A = \sqrt{gr} = \sqrt{10 \times 1} = 3.16\ ms^{-}$$

(b) Let v_B be the minimum velocity when the body is at bottom B of the circle so that it can complete the circle, then from conservation of energy, increase in kinetic energy as the body moves from A to B = decrease in gravitational potential energy. Hence

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = mg \times AB = mg \times 2r$$

$$\Rightarrow \quad v_B^2 = 4gr + v_A^2 = 4gr + gr \quad (\because v_A = \sqrt{gr})$$

$$\therefore \quad v_B = \sqrt{5gr} = \sqrt{5 \times 10 \times 1} = 7.07\ ms^{-1}$$

(c) Net force towards centre O when the body is at B is $(T_B - mg)$. Hence

$$T_B - mg = \frac{mv_B^2}{r}$$

$$\Rightarrow \quad T_B = \frac{mv_B^2}{r} + mg$$

$$= 5\ mg + mg \quad (\because v_B = \sqrt{5gr})$$

$$= 6\ mg$$

$$= 6 \times 0.250 \times 10 = 15\ \text{N}$$

EXAMPLE 13 A small block of mass m, starts from rest at A and slides on a frictionless track which ends in a circular loop of radius r. If $h = 6r$, find the speed of the block when it reaches C as shown in Fig. 4.8(a). What is the force exerted on the block by the track when it is at C? Also find the minimum height h so that the block is able to complete the circle.

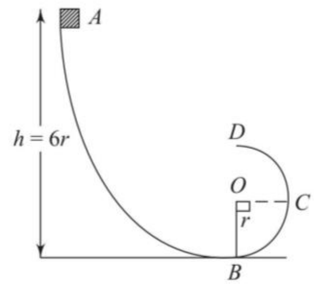


Fig. 4.8(a)

SOLUTION Let v be the speed of the block when it reaches C. From conservation of energy, gain in K.E. = loss in P.E., i.e.

$$\frac{1}{2}mv^2 - 0 = mgh - mg \times OB = mgh - mgr$$

$$\frac{1}{2}v^2 = g \times 6r - gr = 5gr$$

$$\Rightarrow \quad v = \sqrt{10gr}$$

When the block is at C, the track exerts a normal reaction N on the block. Since the block is moving in a circular path, the necessary centripetal force for circular motion is provided by the normal reaction (Fig. 4.8(b)).

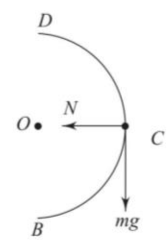


Fig. 4.8(b)

$$\therefore \quad N = \frac{mv^2}{r} = \frac{m \times 10gr}{r} = 10\ mg$$

Thus the track exerts a force on the block equal to 10 times the weight of the block.

To complete the circle, the minimum speed at D must be $v_{min} = \sqrt{5gr}$. Hence

$$mgh_{min} = \frac{1}{2}mv_{min}^2 \Rightarrow h_{min} = 2.5r.$$

EXAMPLE 14 The force between two point charges q_1 and q_2 separated by a distance r is $F = \frac{kq_1q_2}{r^2}$

where k is a constant. Find the potential energy of the system of charges.

SOLUTION $F = -\dfrac{dU}{dr} \Rightarrow dU = -F dr$. Integrating

$$U = -\int_0^r F dr = -kq_1q_2 \int_0^r r^{-2} dr$$

$$\Rightarrow \quad U = \frac{kq_1q_2}{r}$$

EXAMPLE 15 One end of an elastic spring of natural length L and spring constant k is fixed to a wall and the other end is attached to a block of mass m lying on a horizontal frictionless table (Fig. 4.9). The block is moved to a position A so that the spring is compressed to half its natural length and then released. What is the velocity of the block when it reaches position B which is at a distance $\frac{3L}{4}$ from the wall.

(a) $\dfrac{L}{4}\sqrt{\dfrac{k}{m}}$ (b) $\dfrac{L}{2}\sqrt{\dfrac{2k}{m}}$

(c) $\dfrac{L}{4}\sqrt{\dfrac{3k}{m}}$ (d) $\dfrac{L}{2}\sqrt{\dfrac{k}{m}}$

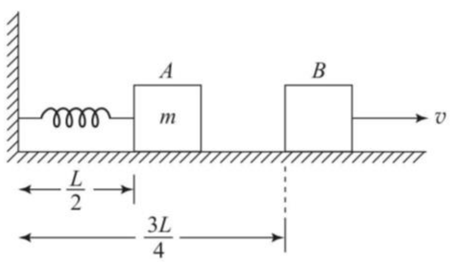


Fig. 4.9

SOLUTION At position A, the compression of the spring is $\frac{L}{2}$. At position B, the compression is $L - \frac{3L}{4} = \frac{L}{4}$. Therefore, loss of potential energy as the block moves from A to B is given by

$$\frac{1}{2}k\left(\frac{L}{2}\right)^2 - \frac{1}{2}k\left(\frac{L}{4}\right)^2 = \frac{3kL^2}{32}$$

From conservation of energy, loss in P.E. = gain in K.E., i.e.

$$\frac{3kL^2}{32} = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{L}{4}\sqrt{\frac{3k}{m}}$$

EXAMPLE 16 In Ex. 15 above, the speed of the block when it leaves the spring is

(a) $\dfrac{L}{2}\sqrt{\dfrac{k}{m}}$ (b) $L\sqrt{\dfrac{k}{m}}$

(c) $\dfrac{3L}{2}\sqrt{\dfrac{k}{m}}$ (d) $2L\sqrt{\dfrac{k}{m}}$

SOLUTION The block will leave the spring when it acquires its natural length L when its P.E. is zero. Hence the loss of P.E = $\frac{1}{2}k\left(\frac{L}{2}\right)^2$. If v is the velocity when the block leaves the spring, then its K.E. = $\frac{1}{2}mv^2$. Hence

$$\frac{1}{2}k\left(\frac{L}{2}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{L}{2}\sqrt{\frac{k}{m}}$$

EXAMPLE 17 A uniform chain of mass M and length L has a part $L/3$ hanging over the edge of a table. If the friction between the table and the chain is neglected, the kinetic energy of the chain as it completely falls off the edge of the table is

(a) $\dfrac{2MgL}{3}$ (b) $\dfrac{4MgL}{9}$

(c) $\dfrac{3}{2}MgL$ (d) MgL

SOLUTION Mass per unit length of the chain is $\frac{M}{L}$. Consider a small element of length dx at a distance x below the edge of the table. The mass of this element is $m = \left(\frac{M}{L}\right) dx$. The potential energy of the hanging part of the chain is

$$U = \int_0^{L/3} \frac{Mg}{L} x\, dx$$

$$= \frac{Mg}{L}\left|\frac{x^2}{2}\right|_0^{L/3} = \frac{MgL}{18}$$

Let us take the potential energy to be zero when the chain is on the table. Therefore, the total initial P.E. of the chain is

$$U_i = 0 + U = \frac{MgL}{18}$$

If the full chain (of length L) were to fall, the P.E. would be

$$U_f = \int_0^L \frac{Mg}{L} x\, dx = \frac{MgL}{2}$$

$$\therefore \text{ Loss in P.E.} = U_f - U_i = \frac{MgL}{2} - \frac{MgL}{18} = \frac{4}{9} MgL$$

From law of conservation of energy,

Gain in K.E. = Loss in P.E. = $\frac{4}{9}$ MgL

Since the initial K.E. is zero, the K.E. as the chain slips off the edge = $\frac{4}{9}MgL$

5. Power

The rate of doing work is called power, i.e.

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

The faster a given amount of work is done, the greater is the power of the agent that does the work.

In the SI system, the unit of power is the watt (symbol W). *Power is said to be 1 watt when 1 joule of work is done in 1 second,* i.e.

$$1 \text{ W} = 1 \text{ Js}^{-1}$$

Since the watt is a small unit for the measurement of power, larger units, namely kilowatt (kW) and megawatt (MW) are often used.

$$1 \text{ kW} = 1000 \text{ W} = 10^3 \text{ W}$$

$$1 \text{ MW} = 1{,}000{,}000 \text{ W} = 10^6 \text{ W}$$

The power of an agent can also be expressed in terms of the force applied and the velocity of the object on which the force is applied. Now, power P is given by

$$P = \frac{W}{t} = \frac{\mathbf{F}\cdot\mathbf{S}}{t}$$

$$= \mathbf{F}\cdot\mathbf{v} \;(\because \frac{\mathbf{S}}{t} = \text{rate of change of displacement} = \mathbf{v})$$

Power is a scalar quantity as it is the ratio of two scalars W and t, or a scalar product of two vectors **F** and **v**.

EXAMPLE 18 An engine pulls a car of mass 1000 kg on a level road at a constant velocity of 5 ms^{-1}. If the frictional force is 500 N, what power does the engine generate? What extra power must the engine supply to maintain the same speed up an inclined plane having a gradient of 1 in 10?

SOLUTION Since the car moves at a constant velocity, its acceleration is zero. Hence the engine has to do work only to overcome the frictional force f.

$\therefore$ $\quad$ Power = $f \times v = 500 \times 5 = 2500$ W = 2.5 kW

For an inclined plane having a gradient of 1 in 10, $\sin\theta = \frac{1}{10}$. To maintain the same speed up the inclined plane, the engine has to do extra work against the force $mg \sin\theta$. Therefore,

$$\text{Extra power} = mg\sin\theta \times v$$

$$= 1000 \times 9.8 \times \frac{1}{10} \times 5 = 4900 \text{ W}$$

$$= 4.9 \text{ kW}$$

EXAMPLE 19 An electric pump on the ground floor of a building takes 10 minutes to fill a tank of volume 2000 litre with water. If the tank is 40 m above the ground and the efficiency of the pump is 40%, how much electric power is consumed by the pump in filling the tank? Take $g = 10 \text{ ms}^{-2}$.

SOLUTION Volume of tank V = 2000 litre

$$= 2000 \times 10^{-3} \text{ m}^3 = 2 \text{ m}^3$$

$$\text{Mass of water } m = \rho V = 1000 \times 2 = 2 \times 10^3 \text{ kg}$$

Work done to lift this mass to a height h = 40 m is

$$W = mgh = 2 \times 10^3 \times 10 \times 40 = 8 \times 10^5 \text{ J}$$

$$\text{Power needed} = \frac{W}{t} = \frac{8\times10^5}{10\times60} = \frac{4}{3}\times10^3 \text{ W}$$

If P is the total power consumed, the useful power available = 40% if P = 0.4 P. Hence

$$0.4\,P = \frac{4}{3}\times10^3 \quad\Rightarrow\quad P = 3.33\times10^3 \text{ W}$$

$$= 3.33 \text{ kW}$$

EXAMPLE 20 A constant power P is supplied to a car of mass m = 3000 kg. The velocity of the car increases from $u = 2 \text{ ms}^{-1}$ to $v = 5 \text{ ms}^{-1}$ when the car travels a distance x = 117 m. Find the value of P. Neglect friction.

SOLUTION $P = Fv = mav \quad\Rightarrow\quad a = \frac{P}{mv}$

Now $\quad a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{v\,dv}{dx}.$

$$\therefore \quad v\frac{dv}{dx} = \frac{P}{mv}$$

$$\Rightarrow \quad v^2 dv = \frac{P}{m}dx$$

$$\therefore \quad \int_u^v v^2 dv = \frac{P}{m}\int_0^x dx$$

$$\Rightarrow \quad \frac{1}{3}(v^3 - u^3) = \frac{Px}{m}$$

$$\Rightarrow \quad P = \frac{m(v^3-u^3)}{3x}$$

$$= \frac{3000\times[(5)^3-(2)^3]}{3\times117}$$

$$= 1000 \text{ W} = 1 \text{ kW}$$

EXAMPLE 21 A car of mass m starts from rest at time $t = 0$ and is driven on a straight horizontal road by the engine which exerts a constant force F. If friction is negligible, the car acquires kinetic energy E at time t and develops a power P. Which of the following is/are correct ?

(a) $E \propto t$ (b) $E \propto t^2$
(c) $P \propto t$ (d) $P \propto t^2$

SOLUTION Since F is constant, acceleration $a = \frac{F}{m}$ is constant. At time t, the velocity of the car is

$$v = u + at = 0 + at = at = \frac{Ft}{m}.$$

$\therefore$ Kinetic energy E at time

$$t = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Ft}{m}\right)^2 = \left(\frac{F^2}{m}\right)t^2.$$

Since F is constant, $E \propto t^2$.

Power P at time $t = Fv = F \times \frac{Ft}{m} = \left(\frac{F^2}{m}\right)t$

Thus $P \propto t$. So the correct choices are (b) and (c).

6. Collisions

Elastic Collisions: *If there is no change of kinetic energy during a collision it is called an elastic collision.* The collision between subatomic particles is generally elastic. The collision between two steel or glass balls is nearly elastic.

Inelastic Collisions: *If there is a loss of kinetic energy during a collision, it is called an inelastic collision.* Since there is always some loss of kinetic energy in any collision, collisions are generally inelastic. If the loss is negligibly small, the collision is very nearly elastic. Perfectly elastic collisions are not possible. If two bodies stick together, after colliding, the collision is perfectly inelastic, e.g. a bullet striking a block of wood and being embedded in it. The loss of kinetic energy usually results in heat or sound energy.

In may be remembered that *the total momentum remains conserved in both elastic and inelastic collisions.* Further, since the interacting forces become effectively zero after the collision, *the potential energy remains the same both before and after the collision.*

One-dimensional or Head-on Collision

Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same straight line (with $u_1 > u_2$) colliding with each other. Let v_1 and v_2 be their respective velocities after the collision. If velocities u_1, u_2, v_1 and v_2 are all along the same straight line, the collision is known as one-dimensional or head-on collision (Fig. 4.10)

From the law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

If momentum along positive x-axis is taken to be positive, the momentum along the negative x-axis is taken to be negative.

Two-dimensional or Oblique Collision

If the velocities of the colliding bodies are not along the same straight line, the collision is known as two-dimensional or oblique collision (Fig. 4.11)

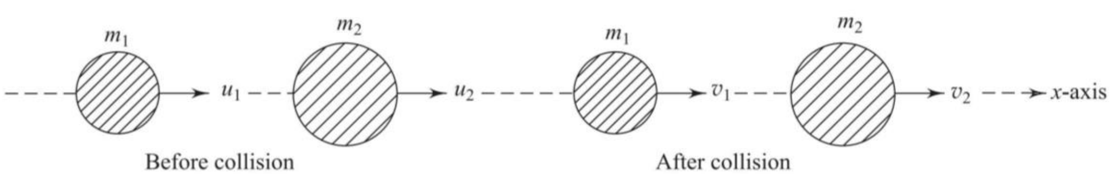


Fig. 4.10

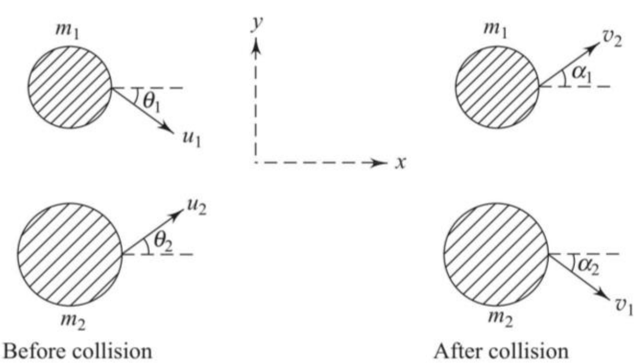


Fig. 4.11

In this case, we apply the law of conservation of momentum separately for x and y components of momenta. The components of momentum along the positive x-axis and positive y-axis are taken to be positive and components of momentum along negative x-axis and negative y-axis are taken to be negative.

Momentum conservation of x-components gives

$$m_1u_1 \cos \theta_1 + m_2u_2 \cos \theta_2 = m_1v_1 \cos \alpha_1 + m_2v_2 \cos \alpha_2$$

Momentum conservation of y-components gives

$$-m_1u_1 \sin \theta_1 + m_2u_2 \sin \theta_2 = m_1v_1 \sin \alpha_1 - m_2v_2 \sin \alpha_2$$

Cofficient of Restitution

Newton proved experimentally that, when two bodies collide, the ratio of the relative velocity after collision to the relative velocity before collision is constant for the two bodies. This constant is known as *coefficient of restitution* and is denoted by letter e.

$$e = \frac{\text{velocity of separation after collision}}{\text{velocity of approach before collision}}$$

or $$e = -\frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}$$

(i) For a perfectly elastice collision, $e = 1$.

(ii) For a perfectly in inelastic collision, $e = 0$, because the two bodies stick together and hence $\mathbf{v}_2 = \mathbf{v}_1$.

(iii) Perfectly elastic or perfectly inelastic collisions do not occur in nature. Hence, for any collision, e lies between 0 and 1.

(iv) For a head-on collision (Fig. 4.10)

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = -\frac{v_2 - v_1}{u_2 - u_1}$$

(v) For an oblique collision (Fig. 4.11)

Velocity of approach $= u_1 \cos \theta_1 - u_2 \cos \theta_2$

Velocity of separation $= v_2 \cos \alpha_2 - v_1 \cos \alpha_1$

$$\therefore \quad e = \frac{v_2 \cos\alpha_2 - v_1 \cos\alpha_1}{u_1 \cos\theta_1 - u_2 \cos\theta_2}$$

Velocities after Head-on Elastic Collision

Refer to Fig. 4.10 again. From the law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{1}$$

The coefficient of restitution is defined as

$$e = \frac{v_1 - v_2}{u_2 - u_1}$$

$$\Rightarrow \quad v_1 - v_2 = e\,(u_2 - u_1) \tag{2}$$

Eliminating v_2 from (1) and (2), we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{m_2\,(1+e)}{m_1 + m_2}\right)u_2 \tag{3}$$

Using (3) in (2), we get

$$v_2 = \left(\frac{m_1\,(1+e)}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 \tag{4}$$

Perfectly Elastic Collision

For perfectly elastic collision, $e = 1$. Putting $e = 1$ in Eqs. (3) and (4) we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \tag{5}$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \tag{6}$$

Special cases

(i) *If both bodies have the same mass, then*

$$m_1 = m_2 = m$$

In this case,

$$v_1 = u_2$$

and $$v_2 = u_1$$

This means that *in a one-dimensional elastic collision between two bodies of equal mass, the bodies merely exchange their velocities after the collision.*

(ii) *If one of the bodies, say m_2, is initially at rest*, then

$$u_2 = 0$$

In this case,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

and $$v_2 = \left(\frac{2m_1u_1}{m_1 + m_2}\right)$$

If, in addition, $m_1 = m_2 = m$, these equations give

$$v_1 = 0 \quad v_2 = u_1$$

Thus, *if a body suffers a one-dimensional elastic collision with another body of the same mass at rest, the first body is stopped dead, but the second begins in move with the velocity of the first.*

However, if the body at rest, namely B, is much more massive than the colliding body A, i.e. $m_2 \gg m_1$, such that m_1 is negligibly small, then

$$v_1 = -u_1$$

and $$v_2 \to 0$$

Thus, *if a very light body suffers an elastic collision with a very heavy body at rest, the velocity of the lighter body is reversed on collision, while the heavier body remains practically at rest.* A common example of this type of collision is the dropping of a hard steel ball on a hard concrete floor. The ball rebounds and regains its original height from where it was dropped while the much more massive ground remains at rest.

Finally, if the body at rest is much lighter than the colliding body, i.e. if $m_2 \ll m_1$, we have

$$v_1 \simeq u_1 \quad v_2 \simeq 2u_1$$

i.e. *the velocity of the massive body remains practically unchanged on collision with the lighter body at rest and the lighter body acquires nearly twice the initial velocity of the massive body.*

(iii) *Kinetic energy delivered by incident body to a stationary body in perfectly elastic head-on collision.*

Kinetic energy of m_1 before collision is $K_i = \frac{1}{2}m_1u_1^2$

and after collision is $K_f = \frac{1}{2}m_1v_1^2$. Therefore

$$\frac{K_i}{K_f} = \frac{u_1^2}{v_1^2}$$

or $$\frac{K_i - K_f}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

The fractional decrease in kinetic energy of m_1 is

$$\frac{\Delta K}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

If $u_2 = 0$, $\frac{v_1}{u_1} = \frac{m_1 - m_2}{m_1 + m_2}$. Therefore,

$$\frac{\Delta K}{K_i} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

> **Note**
> The fraction of kinetic energy lost by mass m_1 is maximum if $m_1 = m_2$ and minimum if $m_2 \to \infty$.

(iv) *Change in kinetic energy of a system in a perfectly inelastic head-on collision.*

In a perfectly inelastic collision, the two stick together after the collision. Hence $v_1 = v_2$ and $e = 0$.

Putting $e = 0$ in Eqs. (1) and (2), we get

$$v_1 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2}{m_1 + m_2}\right)u_2$$

and $$v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2}{m_1 + m_2}\right)u_2$$

when mass m_2 is stationary, $u_2 = 0$. Then

$$v_1 = \left(\frac{m_1}{m_1 + m_2}\right)u_1 \qquad (7)$$

and $$v_2 = \left(\frac{m_1}{m_1 + m_2}\right)u_1$$

Notice that $v_1 = v_2 = v$ (say)

Total K.E. of the system before collision is

$$K_i = \frac{1}{2}m_1u_1^2$$

and after the collision is

$$K_f = \frac{1}{2}(m_1 + m_2)v^2$$

$\therefore$ Loss in K.E. of the system is

$$K_i - K_f = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2 \qquad (8)$$

From Eq. (7)

$\frac{v}{u_1} = \left(\frac{m_1}{m_1 + m_2}\right)$. Using this in Eq. (8)

we get

$$K_i - K_f = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

In general, if $u_2 \neq 0$, we have

$$K_i - K_f = \left(\frac{m_1m_2}{2(m_1 + m_2)}\right)(u_1 - u_2)^2 \qquad (9)$$

Oblique Impact on a Fixed Horizontal Plane

Consider a body of mass m moving with a velocity u making an angle α with the normal ON to a fixed horizontal floor as shown in Fig. 4.12. After collision with the horizontal plane, the body is deflected with a velocity v making an angle β with the normal. Since the horizontal plane is fixed, it remains at rest. Hence the impact takes place along the normal. The normal component of u is $u\cos\alpha$ along $-y$ direction and the normal component of v is $v\cos\beta$ along the $+y$ direction. Now

$$e = -\frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$e = -\frac{(v\cos\beta)\hat{\mathbf{j}}}{(u\cos\alpha)(-\hat{\mathbf{j}})} = \frac{v\cos\beta}{u\cos\alpha}$$

$\Rightarrow \quad v\cos\beta = eu\cos\alpha \quad (1)$

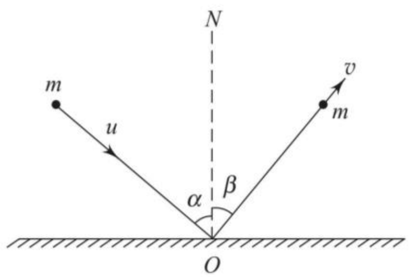


Fig. 4.12

Since the impulsive force acts along the normal, the momentum along the normal is not conservved. Since the component of the impulsive force along the horizontal is zero, the momentum along the horizontal is conserved. Hence

$u\sin\alpha = v\sin\beta \quad (2)$

From Eqs. (1) and (2), we get

$v = (e^2\cos^2\alpha + \sin^2\alpha)^{1/2}\,u \quad (3)$

and $\tan\beta = \dfrac{\tan\alpha}{e} \quad (4)$

For a perfectly elastic collision, $e = 1$ and Eqs. (3) and (4) give

$v = u$

and $\beta = \alpha$

i.e. for a perfectly elastic collision, the body rebounds from a fixed surface with the same speed and at the same angle on the other side of the normal.

Direct Impact on a Fixed Plane

If the body falling normally on a fixed plane rebounds after impact, then, in this case $\alpha = \beta = 0$. Using this in Eqs. (3) we get

$v = eu$

EXAMPLE 22 A body is dropped from rest from a height h = 5.0 m. After rebounding twice from a horizontal floor, to what height will it rise if the coefficient of restitution is 0.8?

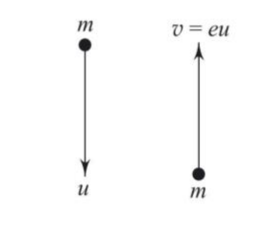


Fig. 4.13

SOLUTION Speed of the body just before first impact with floor = $\sqrt{2gh}$

Speed just after first impact = $e\sqrt{2gh}$. This is also the speed just before the second impact. Therefore, speed just after second impact = $e^2\sqrt{2gh}$. This is the initial speed for the upward motion of the body after the second impact, i.e. $u = e^2\sqrt{2gh}$. Therefore, height attained after two impacts is

$$h_2 = \frac{u^2}{2g} = \frac{1}{2g}\left(e^2\sqrt{2gh}\right)^2 = e^4 h$$

$$= (0.8)^4 \times 5 = 2.05 \text{ m}$$

Note

(1) Height attained after n impacts is $h_n = e^{2n}h$

(2) Speed of rebound after nth impact is $v_n = e^n\sqrt{2gh}$

(3) Total distance travelled before the body comes to rest = $h\left(\dfrac{1+e^2}{1-e^2}\right)$.

EXAMPLE 23 A steel ball of mass m moving with velocity u_1 undergoes a perfectly elastic head-on collision with another identical steel ball moving with velocity u_2. Show that, after the collision, they merely exchange their velocities.

SOLUTION Refer to Fig. 4.10 again. From conservation of momentum,

$mu_1 + mu_2 = mv_1 + mv_2$

$\Rightarrow \quad u_1 + u_2 = v_1 + v_2 \quad$ (i)

From the definition of coefficient of restitution,

$v_2 - v_1 = e\,(u_1 - u_2)$

For a perfectly elastic collision, $e = 1$. Hence

$v_2 - v_1 = u_1 - u_2 \quad$ (ii)

From (i) and (ii) we get

$v_1 = u_2$ and $v_2 = u_1$

EXAMPLE 24 A steel ball of mass m moving with a velocity u undergoes a perfectly elastic oblique collision with another indentical steel ball initially at rest. Show that, after the collision, they move at right angles to each other.

SOLUTION Refer to Fig. 4.14

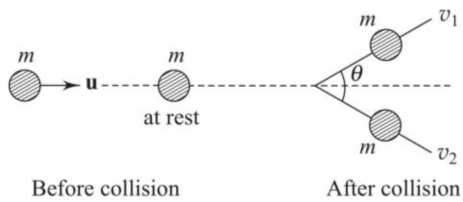


Fig. 4.14

From conservation of momentum,

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$\Rightarrow \quad \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2 \qquad \text{(i)}$$

Taking the scalar product of $\boldsymbol{u}$ with itself, we have

$$\mathbf{u}\cdot\mathbf{u} = (\mathbf{v}_1+\mathbf{v}_2)\cdot(\mathbf{v}_1+\mathbf{v}_2)$$

$$\Rightarrow \quad u^2 = v_1^2 + 2\mathbf{v}_1\cdot\mathbf{v}_2 + v_2^2 \qquad \text{(ii)}$$

Since kinetic energy is also conserved in an elastic collision, we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad u^2 = v_1^2 + v_2^2 \qquad \text{(iii)}$$

Using (iii) in (ii), we get

$$2\,\mathbf{v}_1\cdot\mathbf{v}_2 = 0$$

$$\Rightarrow \quad \mathbf{v}_1\cdot\mathbf{v}_2 = 0$$

$$\Rightarrow \quad v_1 v_2 \cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = 0 \quad \Rightarrow \quad \theta = 90°$$

EXAMPLE 25 Two steel balls of the same mass m moving in opposite directions with the same speed u collide head-on. If the collision is perfectly elastic, predict the result of the collision.

SOLUTION $m_1 = m_2 = m$, $u_1 = u$ and $u_2 = -u$. Let v_1 and v_2 be their velocities after collision.

Total momentum before collision $= m_1u_1 + m_2u_2 = m(u - u) = 0$

Total momentum after collision $= mv_1 + mv_2 = m(v_1 + v_2)$

From conservation of momentum,

$$0 = m(v_1 + v_2) \Rightarrow v_2 = -v_1$$

Since $e = 1$, we have

$$v_2 - v_1 = u_1 - u_2 = u - (-u) = 2u$$

Putting $v_1 = -v_2$, we get $v_2 = u$. Also $v_1 = -u$. Thus, after the collision, the two balls move in opposite directions with equal speeds, each equal to u but their directions are reversed.

EXAMPLE 26 A ball mass 2 kg moving with a velocity of 8 ms^{-1} collides head-on with another ball of mass 4 kg moving with a velocity of 2 ms^{-1} moving in the same direction. The collision is elastic and the coefficient restitution is $e = 0.5$.

(a) Find the velocities of the balls after the collision.

(b) Calculate the loss of kinetic energy due to collision.

SOLUTION Refer to Fig 4.10 again.

(a) Given $m_1 = 2$ kg, $m_2 = 4$ kg, $u_1 = 8$ ms^{-1}, $u_2 = 2$ ms^{-1} and $e = 0.5$

From conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad 2\times 8 + 4\times 2 = 2v_1 + 4v_2$$

$$\Rightarrow \quad 12 = v_1 + 2v_2 \qquad \text{(i)}$$

Since $e = 0.5$, we have

$$v_2 - v_1 = e(u_1 - u_2) = 0.5\times(8-2) = 3 \qquad \text{(ii)}$$

Eliminating v_2 from (i) and (ii) we get $v_1 = 2$ ms^{-1}. Using this in (i) or (ii), we get $v_2 = 5$ ms^{-1}

(b) Kinetic energy before collision is

$$K_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= \frac{1}{2}\times 2\times(8)^2 + \frac{1}{2}\times 4\times(2)^2 = 72\text{ J}$$

Kinetic energy after collision is

$$K_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\times 2\times(2)^2 + \frac{1}{2}\times 4\times(5)^2 = 54\text{ J}$$

$\therefore$ Loss of K.E. $= K_i - K_f = 72 - 54 = 18$ J

EXAMPLE 27 In Example 27, what is the loss of kinetic energy if the ball of mass 4 kg is moving towards the mass of mass 2 kg, their speeds being the same?

SOLUTION In this case $u_2 = -2$ ms^{-1}. Equations (i) and (ii) become

$$4 = v_1 + 2v_2 \qquad \text{(iii)}$$

and

$$v_2 - v_1 = 5 \qquad \text{(iv)}$$

Equations (iii) and (iv) give $v_1 = -2$ ms^{-1} and $v_2 = 3$ ms^{-1}

$$K_i = \frac{1}{2}\times 2\,(8)^2 + \frac{1}{2}\times 4\times(-2)^2 = 72\text{ J}$$

$$K_f = \frac{1}{2}\times 2\times(-2)^2 + \frac{1}{2}\times 4\times(3)^2 = 22\text{ J}$$

$\therefore$ Loss of K.E $= 72 - 22 = 50$ J

EXAMPLE 28 Two blocks B and C of masses 1 kg and 2 kg respectively are connected by a massless elastic spring of spring constant 150 Nm^{-1} and placed on a horizontal frictionless surface as shown in Fig. 4.15. A third block A of mass 1 kg moves with a velocity of

3 ms^{-1} along the line joining B and C and collides with B. If the collision is perfectly elastic and the natural length of the spring is 80 cm, find the minimum separation between blocks B and C.

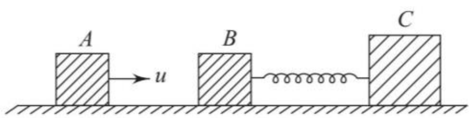


Fig. 4.15

SOLUTION Given $m_A = m_B = 1$ kg, $m_C = 2$ kg and $u = 3\ ms^{-1}$. Block A will collide with block B. Since they have equal masses and the collision is perfectly elastic, A will come to rest and B will move to the right with a velocity u. Block B will compress the spring. Hence block C will accelarate and block B will retard until both B and C move with the same velocity. Let this common velocity be v. Since no external force acts, the momentum of B and C is conserved, i.e.

$$m_B u = (m_B + m_C)v$$

$$\Rightarrow \quad 1 \times 3 = (1+2)v \quad \Rightarrow \quad v = 1\ ms^{-1}$$

If x is the maximum compression, then from the principle of conservation of energy,

$$\frac{1}{2}m_A u^2 = \frac{1}{2}(m_B + m_C)v^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}\times 1 \times (3)^2 = \frac{1}{2}\times(1+2)\times(1)^2 + \frac{1}{2}\times 150 \times x^2$$

which gives $x = 0.2$ m $= 20$ cm

$\therefore$ Minimum separation between B and C = 80 cm – 20 cm = 60 cm

EXAMPLE 29 A block of $m_1 = m$ is moving on a frictionless horizontal surface with velocity $u_1 = 2u$ towards another block of mass $m_2 = 3m$ moving on the same surface with velocity $u_2 = u$ in the same direction. A massless spring of force constant k is attached to m_2 as shown in Fig. 4.16. When block m_1 collides with the spring, show that the maximum compression of the spring is given by $x = \frac{u}{2}\sqrt{\frac{3m}{k}}$.

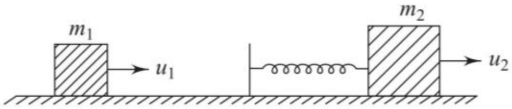


Fig. 4.16

SOLUTION When block m_1 collides with spring, it begins to get compressed. As a result m_2 gains speed. The compression of the spring is maximum at the instant when the relative velocity of m_1 with respect to m_2 is zero, i.e. when both m_1 and m_2 have equal velocities. Let v be the common velocity of the blocks. From conservation of momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad 2mu + 3mu = (m + 3m)v$$

$$\Rightarrow \quad v = \frac{5u}{4}$$

From the law of conservation of energy

Loss in K.E. = gain in P.E of spring

If x is the maximum compression, then

$$\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}\times m \times (2u)^2 + \frac{1}{2}\times 3m \times u^2 - \frac{1}{2}(m+3m)\times\left(\frac{5u}{4}\right)^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{3}{4}mu^2 = kx^2 \Rightarrow x = \frac{u}{2}\sqrt{\frac{3m}{k}}$$

7. Useful Formulae and Tips

1. A body of mass m is dropped from a height h. Due to the friction of air, it will hit the ground with a speed less than $\sqrt{2gh}$. If v is the speed with which it hits the ground, the work done by friction is

$$W_f = \frac{1}{2}mv^2 - mgh = \frac{1}{2}m(v^2 - 2gh)$$

If friction is absent, $W_f = 0$, then $v = \sqrt{2gh}$.

2. Two block A and B of masses m_1 and m_2 are released from the same height at the same time. Block A slides along an inclined plane of inclination θ and block B falls vertically downwards (Fig. 4.17)

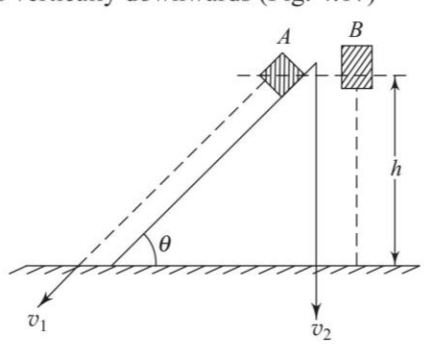


Fig. 4.17

If the inclined plane is frictionless, gain in KE = loss in PE, i.e.

$$\frac{1}{2}m_1v_1^2 = m_1gh \quad \Rightarrow \quad v_1 = \sqrt{2gh}$$

If the air friction is neglected.

$$\frac{1}{2}m_2v_2^2 = m_2gh \quad \Rightarrow \quad v_2 = \sqrt{2gh}$$

Thus both block will hit the ground with the same speed independent of the mass. But the times taken to reach the ground will be different.

For block A, $t_1 = \sqrt{\frac{2h}{g\sin^2\theta}}$

For block B, $t_2 = \sqrt{\frac{2h}{g}}$

3. If a block of mass m in contact with a spring compressed by a distance x is released, the block will leave the spring with a velocity v determined from

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

which gives $v = \sqrt{\frac{k}{m}}\,x$, where k is the spring constant.

4. If a block of mass m moving with speed u comes in contract with a relaxed spring of spring constant k, its velocity v when the spring is compressed by an amount x is obtained from.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

which gives $v = \left(\frac{k\,x^2}{m} - u^2\right)^{1/2}$

5. If two springs of spring constants k_1 and k_2 are stretched by the same force F, then $F = k_1x_1 = k_2\,x_2$. Potential energy stored is

$$U_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}Fx_1$$

and $$U_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}Fx_2$$

which give $\frac{U_1}{U_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$

6. If the two springs are stretched by the same amount x, then $F_1 = k_1x$ and $F_2 = k_2x$.

$U_1 = \frac{1}{2}k_1x^2$ and $U_2 = \frac{1}{2}k_2x^2$.

$$\frac{U_1}{U_2} = \frac{k_1}{k_2} = \frac{F_1}{F_2}$$

7. A chain has a length L and mass M. A part L/n is hanging at the edge of the table. The length of the chain lying on the table is $(L - L/n)$. Then work done against gravity to pull the the hanging part on the table $= \frac{MgL}{2n^2}$

8. If a body of mass m moving with velocity v is stoped in a distance x by a retarding force F, then

$$\frac{1}{2}mv^2 = Fx$$

(a) If two bodies of masses m_1 and m_2 moving with the same velocity are subjected to the same retarding force, the ratio of the stopping distance is

$$\frac{x_1}{x_2} = \frac{m_1}{m_2}$$

(b) If the two bodies are moving with equal kinetic energy and are stopped by the same retarding force, then

$$x_1 = x_2$$

(c) If the two bodies are moving with equal linear momentum and are stopped by the same force, then.

$$\frac{p^2}{2m} = Fx$$

and $$\frac{x_1}{x_2} = \frac{m_2}{m_1}$$

1 SECTION Multiple Choice Questions with One Correct Choice

(Level A)

1. A raindrop of radius r falls from a certain height h above the ground. The work done by the gravitational force is proportional to
   (a) r (b) r^2
   (c) r^3 (d) r^4

2. A pendulum has a length l. Its bob is pulled aside from its equilibrium position through an angle α and then released. The speed of the bob when it passes through the equilibrium position is given by
   (a) $\sqrt{2gl}$ (b) $\sqrt{2gl\cos\alpha}$
   (c) $\sqrt{2gl(1-\cos\alpha)}$ (d) $\sqrt{2gl(1-\sin\alpha)}$

3. A small ball is pushed from a height h along a smooth hemispherical bowl. With what speed should the ball be pushed so that it just reaches the top of the opposite end of the bowl? The height of the top of the bowl is R.
   (a) $\sqrt{2gh}$ (b) $\sqrt{2gR}$
   (c) $\sqrt{2g(R+h)}$ (d) $\sqrt{2g(R-h)}$

4. Two particles of masses m and $4m$ have linear momenta in the ratio of 2 : 1. What is the ratio of their kinetic energies?
   (a) $\sqrt{2}$ (b) 2
   (c) 4 (d) 16

5. Two particles of masses m and $4m$ have kinetic energies in the ratio of 2:1. What is the ratio of their linear momenta?
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
   (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

6. A moving bullet hits a solid target resting on a frictionless surface and gets embedded in it. What is conserved in this process?
   (a) momentum and kinetic energy
   (b) kinetic energy alone
   (c) momentum alone
   (d) neither momentum nor kinetic energy

7. A bullet is fired from a rifle which recoils after firing. The ratio of the kinetic energy of the rifle to that of the bullet is
   (a) zero (b) one
   (c) less than one (d) more than one

8. A bullet is fired at a plank of wood with a speed of 200 ms^{-1}. After passing through the plank, its speed reduces to 180 ms^{-1}. Another bullet, of the same mass and size but moving with a speed of 100 ms^{-1} is fired at the same plank. What would be the speed of this bullet after passing through the plank? Assume that the resistance offered by the plank is the same for both the bullets?
   (a) 48 ms^{-1} (b) 49 ms^{-1}
   (c) 50 ms^{-1} (d) 51 ms^{-1}

9. A particle of mass m has half the kinetic energy of another particle of mass $m/2$. If the speed of the heavier particle is increased by 2 ms^{-1}, its new kinetic energy equals the original kinetic energy of the lighter particle. The ratio of the original speeds of the lighter and heavier particles is
   (a) 1 : 1 (b) 1 : 2
   (c) 1 : 3 (d) 1 : 4

10. In Q. 9, what is the original speed of the heavier particle?
   (a) $2(1+\sqrt{2})$ ms^{-1}
   (b) $2(1-\sqrt{2})$ ms^{-1}
   (c) $(2\sqrt{2}+1)$ ms^{-1}
   (d) $(2\sqrt{2}-1)$ ms^{-1}

11. Two identical cylindrical vessels, with their bases at the same level, each contains a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the vessels are interconnected?
   (a) $A\rho g\,(h_1-h_2)^2$ (b) $A\rho g\,(h_1+h_2)^2$
   (c) $A\rho g\left(\frac{h_1-h_2}{2}\right)^2$ (d) $A\rho g\left(\frac{h_1+h_2}{2}\right)^2$

12. The bob of a pendulum is released from a horizontal position A as shown in Fig. 4.18. The length of the

pendulum is 2 m. If 10% of the initial energy of the bob is dissipated as heat due to the friction of air, what would be the speed of the bob when it reaches the lowermost point B? Take $g = 10\ \text{ms}^{-2}$.

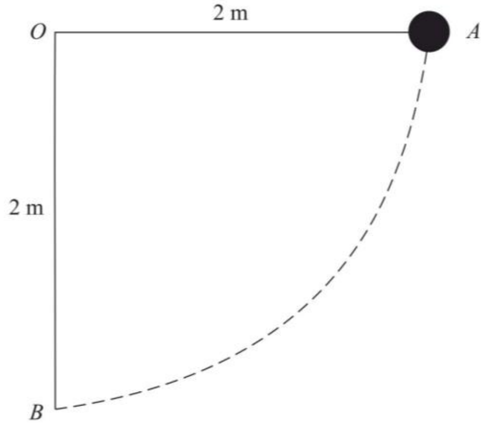


Fig. 4.18

(a) $3\ \text{ms}^{-1}$ (b) $4\ \text{ms}^{-1}$
(c) $5\ \text{ms}^{-1}$ (d) $6\ \text{ms}^{-1}$

13. A wooden block of mass 0.9 kg is suspended from the ceiling of a room by thin wires. A bullet of mass 0.1 kg moving horizontally with a speed of $10\ \text{ms}^{-1}$ strikes the block and sticks to it. What is the height to which the block rises? Take $g = 10\ \text{ms}^{-2}$.
(a) 2.5 m (b) 5.0 m
(c) 7.5 m (d) 10.0 m

14. In Q. 13, what is the loss in kinetic energy of the system due to impact?
(a) 450 J (b) 400 J
(c) 350 J (d) 300 J

15. Two identical balls marked 2 and 3, in contact with each other and at rest on a horizontal frictionless table, are hit head-on by another identical ball marked 1 moving initially with a speed v as shown in Fig. 4.19. What is observed, if the collision is elastic?

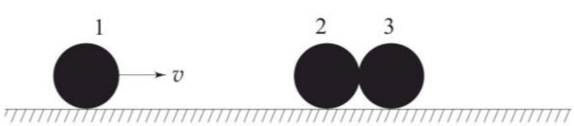


Fig. 4.19

(a) Ball 1 comes to rest and balls 2 and 3 roll out with speed $\frac{v}{2}$ each.
(b) Balls 1 and 2 come to rest and ball 3 rolls out with speed v.
(c) Balls 1, 2 and 3 roll out with speed $\frac{v}{3}$ each.
(d) Balls 1, 2 and 3 come to rest.

16. The bob A of a pendulum released from a height h hits head–on another bob B of the same mass of an identical pendulum initially at rest. What is the result of this collision? Assume the collision to be elastic (see Fig. 4.20).

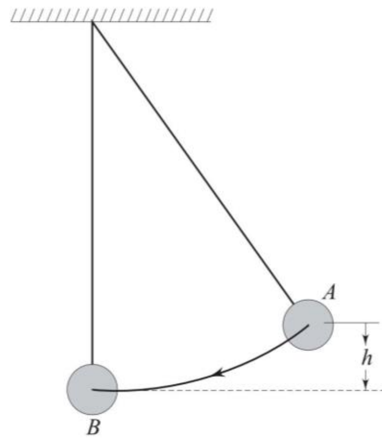


Fig. 4.20

(a) Bob A comes to rest at B and bob B moves to the left attaining a maximum height h.
(b) Bobs A and B both move to the left, each attaining a maximum height $\frac{h}{2}$.
(c) Bob B moves to the left and bob A moves to the right, each attaining a maximum height $\frac{h}{2}$.
(d) Both bobs come to rest.

(Level B)

17. A steel ball falls from a height h on a floor for which the coefficient of restitution is e. The height attained by the ball after two rebounds is
(a) eh (b) e^2h
(c) e^3h (d) e^4h

18. Two balls marked 1 and 2 of the same mass m and a third ball marked 3 of mass M are arranged over a smooth horizontal surface as shown in Fig. 4.21. Ball 1 moves with a velocity v_1 towards balls 2 and 3. All collisions are assumed to be elastic. If $M < m$, the number of collisions between the balls will be
(a) one (b) two
(c) three (d) four

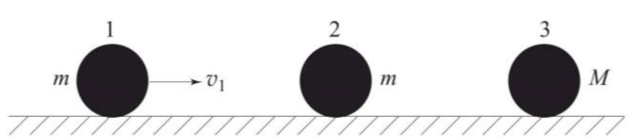


Fig. 4.21

19. In Q. 18, if $M > m$, the number of collisions between the balls will be
(a) one (b) two
(c) three (d) four

20. The distance x moved by a body of mass 0.5 kg by a force varies with time t as
$$x = 3t^2 + 4t + 5$$
where x is expressed in metre and t in second. What is the work done by the force in the first 2 seconds?
(a) 25 J (b) 50 J
(c) 75 J (d) 100 J

21. A body of mass m is dropped from a height h above the ground. The velocity v of the body when it has lost half its initial potential energy is given by
(a) $v = \sqrt{gh}$ (b) $v = \sqrt{2gh}$
(c) $v = \sqrt{\frac{gh}{2}}$ (d) $v = 2\sqrt{gh}$

22. A body of mass m is thrown vertically upwards with a velocity v. The height h at which the kinetic energy of the body is half its initial value is given by
(a) $h = \frac{v^2}{g}$ (b) $h = \frac{v^2}{2g}$
(c) $h = \frac{v^2}{3g}$ (d) $h = \frac{v^2}{4g}$

23. A car of mass m moving at a speed v is stopped in a distance x by the friction between the tyres and the road. If the kinetic energy of the car is doubled, its stopping distance will be
(a) $8x$ (b) $4x$
(c) $2x$ (d) x

24. A body of mass m is dropped from a certain height. It has a velocity v when it is at a height h above the ground. Which of the following will remain constant during the free fall?
(a) $v^2 + 2gh$ (b) $v^2 - 2gh$
(c) $v + \sqrt{\frac{gh}{2}}$ (d) $v - 2\sqrt{gh}$

25. A body of mass m moving with a speed v suffers a perfectly inelastic collision with another body of M at rest. The speed of the composite body will be
(a) $\left(\frac{m+M}{m}\right)v$ (b) $\left(\frac{m}{m+M}\right)v$
(c) $\left(\frac{M}{m+M}\right)v$ (d) $\left(\frac{m+M}{M}\right)v$

26. In Q. 25, the ratio of the final kinetic energy of the system to the initial kinetic energy is
(a) $\frac{m}{m+M}$ (b) $\frac{M}{m+M}$
(c) $\frac{m+M}{m}$ (d) $\frac{m+M}{M}$

27. A ball of mass m moving horizontally at a speed v collides with the bob of a simple pendulum at rest. The mass of the bob is also m. If the collision is perfectly inelastic, the height to which the two balls rise after the collision will be given by
(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$
(c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

28. In Q. 27, the ratio of the kinetic energy of the system immediately after the collision to that before the collision will be
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

29. In Q.27, if the collision is perfectly elastic, the bob of the pendulum will rise to a height of
(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$
(c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

30. A body of mass m_1 moving at a constant speed undergoes an elastic collision with a body of mass m_2 initially at rest. The ratio of the kinetic energy of mass m_1 after the collision to that before the collision is
(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ (b) $\left(\frac{m_1 + m_2}{m_1 - m_2}\right)^2$
(c) $\left(\frac{2m_1}{m_1 + m_2}\right)^2$ (d) $\left(\frac{2m_2}{m_1 + m_2}\right)^2$

31. In Q. 30, the ratio of the kinetic energies of masses m_2 and m_1 after the collision is

(a) $\frac{m_1 m_2}{(m_1 - m_2)^2}$ (b) $\frac{\sqrt{2} m_1 m_2}{(m_1 - m_2)^2}$

(c) $\frac{2m_1 m_2}{(m_1 - m_2)^2}$ (d) $\frac{4m_1 m_2}{(m_1 - m_2)^2}$

32. A ball P of mass 2 kg undergoes an elastic collision with another ball Q at rest. After collision, ball P continues to move in its original direction with a speed one-fourth of its original speed. What is the mass of ball Q?
(a) 0.9 kg (b) 1.2 kg
(c) 1.5 kg (d) 1.8 kg

33. A ball of mass m moving with a velocity v undergoes an oblique elastic collision with another ball of the same mass m but at rest. After the collision, if the two balls move with the same speeds, the angle between their directions of motion will be
(a) 30° (b) 60°
(c) 90° (d) 120°

34. In Q. 33, if the two balls move with different speeds after the collision, the angle between their directions of motion will be
(a) less than 90° (b) more than 90°
(c) exactly 90° (d) exactly 180°

35. A box is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
(a) $t^{1/2}$ (b) $t^{3/4}$
(c) $t^{3/2}$ (d) t^2

36. A bullet, incident normally on a wooden plank, loses one-tenth of its speed in passing through the plank. The least number of such planks required to stop the bullet is
(a) 5 (b) 6
(c) 7 (d) 8

37. A bullet is fired normally on an immovable wooden plank. It loses 25% of its momentum in penetrating a thickness of 3.5 cm. The total thickness penetrated by the bullet is
(a) 8 cm (b) 10 cm
(c) 12 cm (d) 14 cm

38. A bullet is fired normally on an immovable wooden plank. It loses 25% of its kinetic energy in penetrating a thickness x of the plank. What is the total thickness penetrated by the bullet?
(a) $2x$ (b) $4x$
(c) $6x$ (d) $8x$

39. A uniform rod of mass m and length l is made to stand vertically on one end. The potential energy of the rod in this position is

(a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$

(c) $\frac{mgl}{2}$ (d) mgl

40. If the rod in Q. 39 is held inclined at an angle of 60° with the vertical, what will be the potential energy of the rod in this position?

(a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$

(c) $\frac{mgl}{2}$ (d) mgl

41. A body, having kinetic energy k, moving on a rough horizontal surface, is stopped in a distance x. The force of friction exerted on the body is

(a) $\frac{k}{x}$ (b) $\frac{\sqrt{k}}{x}$

(c) $\frac{k}{\sqrt{x}}$ (d) kx

42. A body of mass m, having momentum p, is moving on a rough horizontal surface. If it is stopped in a distance x, the coefficient of friction between the body and the surface is given by

(a) $\mu = \frac{p^2}{2gm^2x}$ (b) $\mu = \frac{p^2}{2mgx}$

(c) $\mu = \frac{p}{2mgx}$ (d) $\mu = \frac{p}{2gm^2x}$

43. A uniform chain of mass M and length L is held on a horizontal frictionless table with $\frac{1}{n}$ th of its length hanging over the edge of the table. The work done is pulling the chain up on the table is

(a) $\frac{Mgl}{n}$ (b) $\frac{Mgl}{2n}$

(c) $\frac{Mgl}{n^2}$ (d) $\frac{Mgl}{2n^2}$

44. A body of mass $m = 1$ kg is dropped from a height $h = 40$ cm on a horizontal platform fixed to one end of an elastic spring, the other being fixed to a base, as shown in Fig. 4.22. As a result the spring

is compressed by an amount $x = 10$ cm. What is the force constant of the spring. Take $g = 10\ ms^{-2}$.

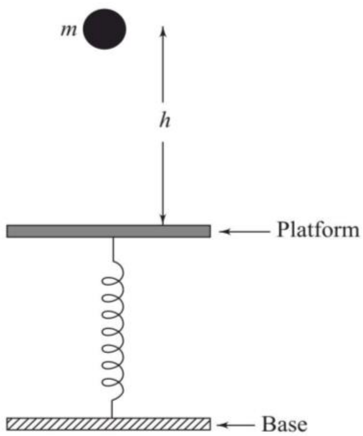


Fig. 4.22

(a) 600 Nm^{-1} (b) 800 Nm^{-1}
(c) 1000 Nm^{-1} (d) 1200 Nm^{-1}

45. A block of wood of mass M is suspended by means of a thread. A bullet of mass m is fired horizontally into the block with a velocity v. As a result of the impact, the bullet is embedded in the block. The block will rise to vertical height given by

(a) $\frac{1}{2g}\left(\frac{mv}{M+m}\right)^2$ (b) $\frac{1}{2g}\left(\frac{mv}{M-m}\right)^2$

(c) $\frac{1}{2g}\frac{mv^2}{(M+m)}$ (d) $\frac{1}{2g}\frac{mv^2}{(M-m)}$

46. A moving particle of mass m makes a head-on collision with a particle of mass $2m$ initially at rest. If the collision is perfectly elastic, the percentage loss of energy of the colliding particle is

(a) 50% (b) 66.7%
(c) 88.9% (d) 100%

47. A body of mass m moving with a velocity v in the x-direction collides with a body of mass M moving with a velocity V in the y-direction. They stick together during collision. Then

(a) the magnitude of the momentum of the composite body is $\sqrt{(mv^2)+(MV)^2}$
(b) the composite body moves in a direction making a angle $\theta = \tan^{-1}\left(\frac{MV}{mv}\right)$ with the x-axis.
(c) the loss of kinetic energy as a result of collision is $\frac{1}{2}\frac{Mm}{(M+m)}(V^2+v^2)$
(d) all the above choices are correct.

48. A body of mass 2 kg moving with a velocity $(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ ms^{-1} collides with another body of mass 3 kg moving with a velocity $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ ms^{-1}. If they stick together, the velocity in ms^{-1} of the composite body is

(a) $\frac{1}{5}(8\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ (b) $\frac{1}{5}(-4\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})$

(c) $\frac{1}{5}(8\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})$ (d) $\frac{1}{5}(-4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}})$

49. A body of mass 2 kg has an initial velocity $\mathbf{v}_i = (\hat{\mathbf{i}}+\hat{\mathbf{j}})$ ms^{-1}. After collision with another body its velocity becomes $\mathbf{v}_f = (5\hat{\mathbf{i}}+6\hat{\mathbf{j}}+\hat{\mathbf{k}})$ ms^{-1}. If the impact time is 0.02 s, the average force of impact on the body (in newton) is

(a) $50\ (4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}})$ (b) $50\ (4\hat{\mathbf{i}}-5\hat{\mathbf{j}}-\hat{\mathbf{k}})$

(c) $100\ (4\hat{\mathbf{i}}+5\hat{\mathbf{j}}-\hat{\mathbf{k}})$ (d) $100\ (4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}})$

50. A body falls from a height h on a horizontal surface and rebounds. Then it falls again, and again rebounds and so on. If the restitution coefficient is $\frac{1}{3}$, the total distance covered by the body before it comes to rest is

(a) $\frac{h}{4}$ (b) $\frac{5h}{4}$

(c) $2h$ (d) $3h$

51. In Q. 50 above, the total time taken by the body to come to rest is

(a) $\sqrt{\frac{2h}{g}}$ (b) $2\sqrt{\frac{2h}{g}}$

(c) $3\sqrt{\frac{2h}{g}}$ (d) $4\sqrt{\frac{2h}{g}}$

52. A body P strikes another body Q of mass that is p times that of body P and moving with a velocity that is $\frac{1}{q}$ of the velocity of body P. If body P comes to rest, the coefficient of restitution is

(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{q(p-1)}$

(c) $\frac{p-q}{q(p-1)}$ (d) $\frac{p+q}{q(p-1)}$

53. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. Sphere A is projected along the groove and at the end of time T impinges on sphere B. If e is the coefficient of restitution, the second impact will occur after a time equal to

(a) T (b) eT
(c) $\frac{2T}{e}$ (d) $2\,eT$

54. Two masses M and m (with $M > m$) are connected by means of a pulley as shown in Fig. 4.23. The system is released. At the instant when mass M has fallen through a distance h, the velocity of mass m will be

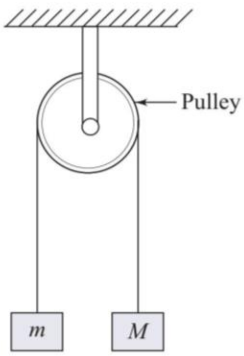


Fig. 4.23

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{2gh\,M}{m}}$
(c) $\sqrt{\frac{2gh\,(M-m)}{(M+m)}}$ (d) $\sqrt{\frac{2gh\,(M+m)}{(M-m)}}$

55. A mass m, lying on a horizontal frictionless surface is connected to mass M as shown in Fig. 4.24. The system is now released. The velocity of mass m when mass M as descended a distance h is

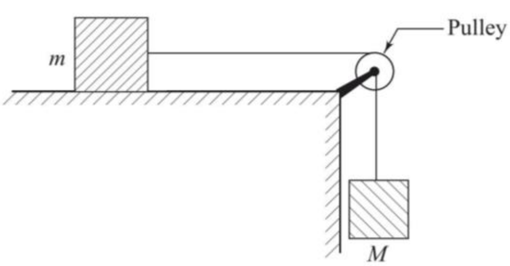


Fig. 4.24

(a) $\sqrt{\frac{2\,Mgh}{m}}$ (b) $\sqrt{\frac{2\,mgh}{M}}$
(c) $\sqrt{\frac{2\,Mgh}{(M+m)}}$ (d) $\sqrt{2gh}$

56. An escalator is moving downwards with a uniform speed u. A man of mass m is running upwards on it at a uniform speed v. If the height of the escalator is h, the work done by the man in going up the escalator is

(a) zero (b) mgh
(c) $\frac{mghu}{(v-u)}$ (d) $\frac{mgh\,v}{(v-u)}$

57. A uniform rope of length L is lying in a lump on a frictionless table. A small part of the rope is held hanging through a hole in the table just below the lump. The system is then released. The speed of the end of the rope as it leaves the hole is

(a) $\sqrt{2gL}$ (b) $2\sqrt{\frac{gL}{3}}$
(c) $\sqrt{\frac{2\,gL}{3}}$ (d) zero

58. The potential energy (in joule) of a body of mass 2 kg moving in the $x-y$ plane is given by

$$U = 6x + 8y$$

where the position coordinates x and y are measured in metre. If the body is at rest at point (6 m, 4 m) at time $t = 0$, it will cross the y-axis at time t equal to

(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

59. In Q. 58 above, the speed of the body when it crosses the y-axis is

(a) zero (b) 5 ms^{-1}
(c) 10 ms^{-1} (d) 20 ms^{-1}

60. A bullet of mass m is fired horizontally with a velocity v on a wooden block of mass M suspended from a support and gets embedded in it. The kinetic energy of the bullet + block system is

(a) $\frac{1}{2}\,mv^2$ (b) $\frac{1}{2}\,(M+m)v^2$
(c) $\frac{Mmv^2}{2(M+m)}$ (d) $\frac{m^2v^2}{2(M+m)}$

61. A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(a) $\frac{16}{25}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{9}{2}$

62. An isolated particle of mass m is moving in a horizontal plane $(x - y)$, along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at

(a) $y = -5$ cm (b) $y = +20$ cm
(c) $y = +5$ cm (d) $y = -20$ cm

63. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is (see Fig. 4.25)

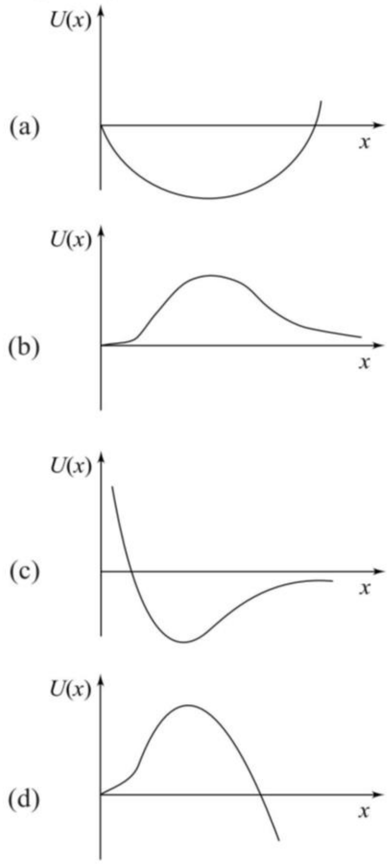


Fig. 4.25

64. A body of mass 6 kg is acted upon by a force which causes a displacement in it given by $x = \frac{t^2}{4}$ metre where t is the time in second. The work done by the force is 2 seconds is

(a) 12 J (b) 9 J
(c) 6 J (d) 3 J

65. A ladder 2.5 m long and of weight 150 N has its centre of mass 1 m from its bottom. A weight of 40 N is attached to the top end. The work required to raise the ladder from the horizontal position to the vertical position is

(a) 190 J (b) 250 J
(c) 285 J (d) 475 J

66. A body of mass 5 kg rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10 m by a horizontal force of 25 N. The kinetic energy acquired by it is (take $g = 10$ ms^{-2})

(a) 200 J (b) 150 J
(c) 100 J (d) 50 J

67. A body is moving up an inclined plane of angle θ with an initial kinetic energy E. The coefficient of friction between the plane and the body is μ. The work done against friction before the body comes to rest is:

(a) $\frac{\mu \cos\theta}{E\cos\theta + \sin\theta}$ (b) $\mu E \cos\theta$

(c) $\frac{\mu E \cos\theta}{\mu\cos\theta - \sin\theta}$ (d) $\frac{\mu E\cos\theta}{\mu\cos\theta + \sin\theta}$

68. A particle falls from a height h on a fixed horizontal plate and rebounds. If e is the coefficient of restitution, the total distance travelled by the particle before it stops rebounding is

(a) $\frac{h(1+e^2)}{(1-e^2)}$ (b) $\frac{h(1-e^2)}{(1+e^2)}$

(c) $\frac{h(1-e^2)}{2(1+e^2)}$ (d) $\frac{h(1+e^2)}{2(1-e^2)}$

69. A force F = $(3x + 4)$ newton displaces an object of mass 10 kg from $x = 0$ to $x = 4$m. If the speed of the object at $x = 0$ is 2ms^{-1}, its speed at $x = 4$m will be

(a) 3ms^{-1} (b) $\sqrt{3}$ ms^{-1}
(c) $2\sqrt{3}$ ms^{-1} (d) 4ms^{-1}

70. The potential energy U of a particle of mass m=3kg moving in the x-direction is given by

$$U = 3(x - 1) - (x - 3)^3$$

when U is in joule and x is in metre. Which of the graphs shown in Fig. 4.26 best represents the variation of the acceleration A of the particle with position x ?

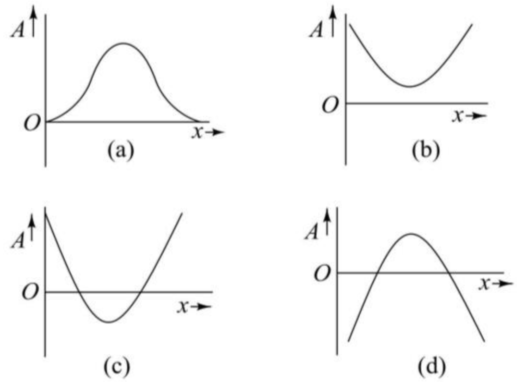


Fig. 4.26

71. A block of wood of mass M is moving at a speed V and a bullet of mass m is moving towards it wil a velocity v as shown in Fig. 4.27.

Fig. 4.27

If the bullet gets embedded in the block after impact, the value of v so that the block + bullet system is stopped dead after impact must be

(a) $\dfrac{MV}{M+m}$ (b) $\dfrac{mV}{M+m}$

(c) $\dfrac{MV}{M}$ (d) $\dfrac{mV}{M}$

72. A steel ball of mass m tied to a light string of length L is released from rest when the string is horizontal. The tension in the string when the ball is at a height $\dfrac{L}{2}$ above the lowest position (see fig. 4.28) is

(a) $\dfrac{1}{2}mg$ (b) $\sqrt{2}\,mg$

(c) $\dfrac{3}{2}mg$ (d) $2\,mg$

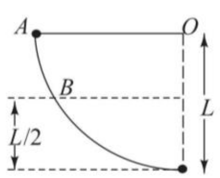


Fig. 4.28

73. A steel ball of mass m tied to a light string of length L in released from rest when the string is horizontal. It strikes a steel block of mass $M = 3m$ which is initially at rest at the bottom as shown in Fig.4.29. If the collision is elastic, to what height h will the ball rise after the collision ?

(a) $\dfrac{2L}{3}$ (b) $\dfrac{L}{2}$

(c) $\dfrac{L}{3}$ (d) $\dfrac{L}{4}$

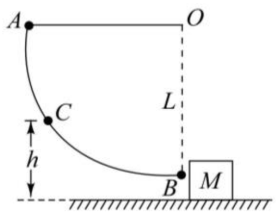


Fig. 4.29

74. When a force acts on a body of mass m, its position x varies with time t as

$$x = \frac{kt^3}{3}$$

where k is a constant. The work done by the force in the first 2 seconds is

(a) $2\,mk^2$ (b) $4\,mk^2$

(c) $8\,mk^2$ (d) $16\,mk^2$

75. A pump of power P is used to pump water in a certain pipe at a certain rate. To pump twice as much water through the same pipe in the same time, the power of the pump must be increased to

(a) $2P$ (b) $4P$

(b) $8P$ (d) $16P$

76. A force acts on a car initially at rest. The engine of the car drives it with a constant power P . If the car acquires a velocity v when it has travelled a distance x, then v is proportional to

(a) $x^{1/3}$ (b) $x^{2/3}$

(c) $x^{3/2}$ (d) x^3

77. When a force acts on a body of mass m, its position x varies with time t as

$$x = at^4 + bt + c$$

where a, b and c are constants. The work done by the force during the first second of the motion is

(a) $ma\,(2a + c)$ (b) $2ma\,(a + 2b)$

(c) $3ma\,(2a + 3b)$ (d) $4ma\,(2a + b)$

78. The kinetic energy K of a particle moving in a circle of radius r is given by

$$K = \frac{kx^2}{2}$$

where k is a constant and x is the distance moved along the arc. The net force on the particle is

(a) kx (b) $\frac{kx^2}{r}$

(c) $kx\left(1+\frac{x^2}{r^2}\right)^{1/2}$ (d) $kx\left(1-\frac{x^2}{r^2}\right)^{1/2}$

79. A block begins to move on a rough horizontal surface with an initial velocity u. If it loses $\frac{1}{4}$ of its initial kinetic energy in time t, the coefficient of kinetic friction between the block and the surface is

(a) $\frac{u}{gt}$ (b) $\frac{\sqrt{3}u}{2gt}$

(c) $\frac{u}{2gt}\left(\sqrt{3}-1\right)$ (d) $\frac{u}{2gt}\left(2-\sqrt{3}\right)$

80. Ball A moving with momentum p undergoes a one dimensinal collision with a stationary ball B of the same mass. During the collision ball B imparts an impulse I to ball A. The coefficient of restitution is

(a) $\frac{2I}{p}$ (b) $\frac{2I}{p}-1$

(c) $\frac{I}{p}+1$ (d) $\frac{2I}{p}+1$

81. A hydrogen filled balloon of mass M has a rope of length l of negligible mass attached to it with a man of mass m at the other end of the rope. The whole system is in equilibruim in mid-air. The man climbs up the rope and reaches the balloon. As a result the balloon descends by a height h. The work done by the man in reaching the balloon is

(a) $(M+m)gh$ (b) $(M-m)\,g\,(l-h)$

(c) Mgh (d) mgl

82. A ball of mass m moving with a velocity v_1 strikes normally a massive rock of mass M ($>> m$) moving towards the ball with a velocity v_2. If the collision is perfectly elastic, the magnitude of the velocity of the ball after impact is

(a) $v_1 + v_2$ (b) $v_1 - v_2$

(c) $v_1 + 2v_2$ (d) $v_1 - 2v_2$

Answers

(Level A)

1. (c)	2. (c)	3. (d)	4. (d)
5. (a)	6. (c)	7. (c)	8. (b)
9. (b)	10. (a)	11. (c)	12. (d)
13. (b)	14. (a)	15. (b)	16. (a)

(Level B)

17. (d)	18. (b)	19. (c)	20. (c)
21. (a)	22. (d)	23. (c)	24. (a)
25. (b)	26. (a)	27. (d)	28. (b)
29. (b)	30. (a)	31. (d)	32. (b)
33. (c)	34. (c)	35. (c)	36. (b)
37. (a)	38. (b)	39. (c)	40. (a)
41. (a)	42. (a)	43. (d)	44. (c)
45. (a)	46. (c)	47. (d)	48. (a)
49. (d)	50. (b)	51. (b)	52. (d)
53. (c)	54. (c)	55. (c)	56. (d)
57. (b)	58. (b)	59. (c)	60. (d)
61. (b)	62. (a)	63. (d)	64. (d)
65. (b)	66. (b)	67. (d)	68. (a)
69. (c)	70. (c)	71. (c)	72. (c)
73. (d)	74. (c)	75. (c)	76. (a)
77. (d)	78. (c)	79. (d)	80. (b)
81. (d)	82. (c)		

Solutions

(Level A)

1. Mass of the drop m = volume × density of water = $\frac{4\pi}{3}r^3\rho$, where ρ is the density of water. Work done by gravitational force is

$$W = mgh = \frac{4\pi}{3}r^3\rho gh$$

Thus $W \propto r^3$. Hence the correct choice is (c).

2. As shown in Fig. 4.26, the height attained by the bob when the string subtends an angle α with the vertical is

$$h = l - l\cos\alpha = l\,(1-\cos\alpha)$$

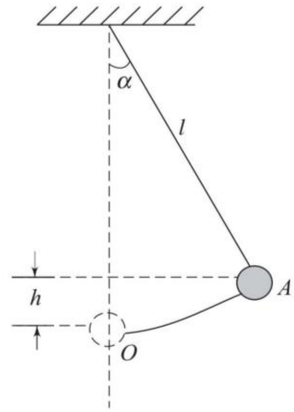


Fig. 4.26

Its potential energy at the highest point $A = mgh$, where m is the mass of the bob. Let v be the speed of the bob when it passes through O. Its kinetic energy at $O = \frac{1}{2} mv^2$. From the principle of conservation of energy, we have

$$\frac{1}{2} mv^2 = mgh$$

or $$v = \sqrt{2gh} = \sqrt{2gl(1-\cos\alpha)}$$

Hence the correct choice is (c).

3. From the principle of conservation of energy, we have

$$\frac{1}{2} mv^2 + mgh = 0 + mgR$$

which gives $v = \sqrt{2g(R-h)}$. Hence the correct choice is (d).

4. Given $p_1 = m_1v_1 = 2p$ and $p_2 = m_2v_2 = p$, so that

$$\frac{m_1v_1}{m_2v_2} = 2$$

The ratio of their kinetic energies is

$$\frac{(\text{KE})_1}{(\text{KE})_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{m_1^2v_1^2}{m_2^2v_2^2}\cdot\frac{m_2}{m_1}$$

But $m_2 = 4\,m_1$ and $\frac{m_1v_1}{m_2v_2} = 2$. Therefore,

$$\frac{(\text{KE})_1}{(\text{KE})_2} = (2)^2 \times 4 = 16$$

Hence the correct choice is (d).

5. Given $(\text{KE})_1 = \frac{1}{2} m_1v_1^2 = 2K$

and $(\text{KE})_2 = \frac{1}{2} m_2v_2^2 = K$, so that

$$\frac{m_1v_1^2}{m_2v_2^2} = 2$$

or $$\frac{m_1^2v_1^2}{m_2^2v_2^2} = \frac{2m_1}{m_2}$$

or $$\frac{p_1}{p_2} = \sqrt{\frac{2m_1}{m_2}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}} \quad (\because m_2 = 4\,m_1)$$

Hence the correct choice is (a)

6. The correct choice is (c).

7. Let M be the mass of the rifle and m that of the bullet and let V and v be their respective speeds. From the principle of conservation of momentum, we have $MV = mv$. The ratio of their kinetic energies is

$$\frac{K_r}{K_b} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}mv^2} = \frac{MV^2}{mv^2} = \frac{(MV)^2}{(mv)^2}\cdot\frac{m}{M} = \frac{m}{M}$$

Since $m < M$, $K_r < K_b$. Hence the correct choice is (c).

8. Let m be the mass of each bullet. Since the resistance offered by the plank is the same for the two bullets, the amount of work done by the plank is the same for the two bullets. From work–energy principle, the decrease in the kinetic energy is the same for the two bullets.

Decrease in KE of first bullet $= \frac{1}{2} mu_1^2 - \frac{1}{2} mv_1^2$

$$= \frac{1}{2} m(200)^2 - \frac{1}{2} m(180)^2 \quad \text{(i)}$$

If v_2 is the speed of the second bullet after passing through the plank, then

Decrease in KE of second bullet $= \frac{1}{2} mu_2^2 - \frac{1}{2} mv_2^2$

$$= \frac{1}{2} m(100)^2 - \frac{1}{2} mv_2^2 \quad \text{(ii)}$$

Equating (i) and (ii) we have,

$$\frac{1}{2} m(200)^2 - \frac{1}{2} m(180)^2 = \frac{1}{2} m(100)^2 - \frac{1}{2} mv_2^2$$

which gives $v_2^2 = 2400$ or $v = 49$ ms $^{-1}$

Hence the correct choice is (b).

9. Let v and v' be the original speeds of the heavier and the lighter particles respectively. We then have

$$\frac{1}{2} mv^2 = \frac{1}{2} \times \left\{\frac{1}{2}\left(\frac{m}{2}\right)v'^2\right\}$$

$\therefore$ $$v^2 = \frac{1}{4} v'^2$$

or $$v' = 2v$$

Hence the correct choice is (b).

10. When the heavier particle is speeded up by 2.0 m s^{-1}, its kinetic energy becomes $\frac{1}{2} m(v + 2)^2$. Since this equals the original kinetic energy of the lighter particle, we have

$$\frac{1}{2}\,m(v+2)^2 = \frac{1}{2}\,(m/2)(4v^2)$$

$$v^2 + 4 + 4v = 2v^2$$

or $$v^2 - 4v - 4 = 0$$

$$v = \frac{4 \pm \sqrt{16+16}}{2} = \frac{4 \pm 2\sqrt{8}}{2} = 2 \pm 2\sqrt{2}$$

The positive root is $v = 2 + 2\sqrt{2} = 2(1 + \sqrt{2})$. Hence the correct choice is (a).

11. The work done by gravity equals the change in the potential energy of the system after the vessels are interconnected. We may regard the liquid in each vessel as equivalent to a point mass kept at their respective centres of mass. Remembering that the mass of the liquid is given by $(Ah\rho)$ and that the PE of a mass at a height h in earth's gravity is mgh, we have

$$\text{Total PE at start} = (\text{A}h_1\rho)g\,\frac{h_1}{2} + (\text{A}h_2\rho)g\,\frac{h_2}{2}$$

$$= \frac{A\rho g}{2}\,(h_1^2 + h_2^2)$$

After the vessels are connected, the height of liquid in each vessel is $(h_1 + h_2)/2$.

Hence PE after connection

$$= \left\{A\rho\left(\frac{h_1+h_2}{2}\right)g\left(\frac{h_1+h_2}{2}\right)\right\}$$

$$= \frac{A\rho g}{4}\,(h_1+h_2)^2$$

$$\text{Change in PE} = \frac{A\rho g}{4}\,\{(h_1+h_2)^2 - 2(h_1^2+h_2^2)\}$$

$$= -\frac{A\rho g}{4}\,(h_1-h_2)^2$$

$$= -A\rho g\left(\frac{h_1-h_2}{2}\right)^2$$

This must be equal to the work done 'by' gravity on the liquid. Thus the work done 'by' gravity is

$$A\rho g\left(\frac{h_1-h_2}{2}\right)^2$$

Hence the correct choice is (c).

12. PE at $A = mgh$. Since 10% of this energy is lost,

KE at point $B = mgh \times \frac{90}{100} = 0.9\,mgh$. Therefore,

$$\frac{1}{2}\,mv^2 = 0.9\,mgh$$

or $$v^2 = 1.8\,gh = 1.8 \times 10 \times 2 = 36$$

which gives $v = 6\ \text{ms}^{-1}$. Hence the correct choice is (d).

13. Mass of block $(M) = 0.9$ kg, mass of bullet $(m) = 0.1$ kg, initial velocity of bullet $(u) = 100\ \text{ms}^{-1}$ and initial velocity of block $(U) = 0$.

$\therefore$ Momentum of bullet and block before impact $= mu + MU = mu$. Let v be the velocity of the bullet + block after impact. Then, the momentum after impact $= (m + M)v$. From the principle of conservation of momentum, we have

$$mu = (m+M)v$$

or $$v = \frac{mu}{(M+m)} = \frac{0.1 \times 100}{(0.9+0.1)} = 10\ \text{ms}^{-1}$$

$$\text{KE of bullet + block} = \frac{1}{2}\,(m+M)v^2 \qquad \text{(i)}$$

Let the bullet + block system rise to a height h. At this height,

$$\text{PE of bullet + block} = (m+M)\,gh \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$h = \frac{v^2}{2g} = \frac{(10)^2}{2 \times 10} = 5\ \text{m}$$

Hence the correct choice is (b).

14. Initial KE $= \frac{1}{2}\,mu^2$

Final KE $= \frac{1}{2}\,(m+M)v^2$

$$\therefore \quad \text{Loss in KE} = \frac{1}{2}\,mu^2 - \frac{1}{2}\,(m+M)v^2$$

$$= \frac{1}{2} \times 0.1 \times (100)^2 - \frac{1}{2}\,(0.1+0.9) \times (10)^2$$

$$= 450\ \text{J}$$

Hence the correct choice is (a).

15. The system consists of three identical balls marked 1, 2 and 3. Let m be the mass of each ball. Before the collision,

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= \frac{1}{2}\,mv^2 + 0 + 0 = \frac{1}{2}\,mv^2$$

Case (a) After the collision,

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= 0 + \frac{1}{2}\,m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\,m\left(\frac{v}{2}\right)^2$$

$$= \frac{1}{4}\ mv^2$$

Case (b)

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= 0 + 0 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

Case (c)

KE of the system = KE of 1 + KE of 2 + KE of 3

$$= \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}m\left(\frac{v}{3}\right)^2$$

$$= \frac{1}{6}\ mv^2$$

Case (d)

KE of the system = 0

Now, in an elastic collision, the kinetic energy of the system remains unchanged. Hence choice (b) is the only possible result of the collision.

16. Suppose the bob A acquires a velocity v on reaching the bob B. In a head-on elastic collision between two bodies of the same mass when one of them is at rest, the velocities are exchanged after the collision. Hence the bob A will come to rest at the lowermost position (occupied by B before collision) and the bob B will move to the left attaining a maximum height h. Hence the correct choice is (a).

(Level B)

17. The velocity attained after a fall through a height h is given by

$$v^2 = 2gh$$

Thus $h \propto v^2$. The velocity after first rebound is ev. Therefore, the height attained after first rebound = e^2h. Velocity after second rebound is e^2v. Hence the height attained after second rebound is e^4h. Thus the correct choice is (d).

18. The first collision will be between balls 1 and 2. Since both have the same mass, after the collision ball 1 will come to rest and ball 2 will move with speed v_1. This ball will collide with the stationary ball 3. After this second collision, let v_2 and v_3 be the speeds of balls 2 and 3 respectively. Since the collisions are elastic, v_2 and v_3 are given by

$$v_2 = \left(\frac{m-M}{m+M}\right) v_1 \qquad \text{(i)}$$

and $$v_3 = \left(\frac{2m}{m+M}\right) v_1 \qquad \text{(ii)}$$

If $M < m$, it follows from (i) and (ii) that $v_2 < v_3$ and both have the same direction. Therefore, ball 2 cannot collide with ball 3 again. Hence there are only two collisions. Thus, the correct choice is (b).

19. If $M > m$, we have from Eq. (i)

$$v_2 = -\left(\frac{M-m}{M+m}\right) v_1$$

The negative sign indicates that, after the second collision, ball 2 will move in opposite direction towards the ball 1 which is at rest after the first collision. Therefore, ball 2 will make another collision with ball 1.

Hence, in this case, there are three collisions in all between the balls. Thus the correct choice is (c).

20. Velocity $(v) = \frac{dx}{dt} = \frac{d}{dt}(3t^2 + 4t + 5) = 6t + 4.$

Acceleration is $a = \frac{dv}{dt} = \frac{d}{dt}(6t + 4) = 6\ \text{ms}^{-2}$.

Therefore, applied force is $F = ma = 0.5 \times 6 = 3$ N. Now $t = 2$s, the distance moved is $x = 3 \times (2)^2 + 4 \times 2 + 5 = 25$ m $\quad (\because u = 4\text{ms}^{-1}$ and $x = 5$ m at $t = 0)$

$\therefore$ Work done $W = Fx = 3 \times 25 = 75$ J. Hence the correct choice is (c).

21. Initial PE = mgh. Now, gain in KE = loss in PE. Thus

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\ mgh$$

or $$v = \sqrt{gh}$$

Hence the correct choice is (a).

22. Initial KE = $\frac{1}{2}\ mv^2$. Now, gain in PE = loss in KE. Thus

$$mgh = \frac{1}{4}\ mv^2$$

or $$h = \frac{v^2}{4g}$$

Hence the correct choice is (d).

23. If a is the deceleration due to the force of friction f, then

$$2ax = v^2$$

or $$\frac{1}{2}\ mv^2 = max$$

or $$\text{KE} = fx \qquad (\because f = ma)$$

Thus if KE is doubled, x is also doubled. Hence the correct choice is (c).

24. The total energy = KE + PE remains constant during the free fall, i.e.

$$mgh + \frac{1}{2}mv^2 = \text{constant}$$

or $$gh + \frac{v^2}{2} = \text{constant}$$

Hence the correct choice is (a).

25. Initial momentum of the system = mv, since body of mass M is at rest. After the inelastic collision, the bodies stick together and the mass of the composite body is $(m + M)$. If V is the speed of the composite body, its momentum will be $(m + M)V$. From the principle of conservation of momentum, we have

$$mv = (m + M)V$$

or $$V = \left(\frac{m}{m+M}\right)v$$

Hence the correct choice is (b).

26. Initial KE = $\frac{1}{2}mv^2$. Final KE = $\frac{1}{2}(m + M)V^2$. Therefore,

$$\frac{\text{Final KE}}{\text{Initial KE}} = \left(\frac{m+M}{m}\right)\frac{V^2}{v^2} = \frac{m}{m+M}$$

Hence the correct choice is (a).

27. In a perfectly inelastic collision, two bodies stick together. After the collision, the speed of the ball and the bob (sticking together) is $v' = v/2$. The height to which they will rise is given by

$$v' = \sqrt{2gh'}$$

or $$h' = \frac{v'^2}{2g} = \frac{v^2}{8g}$$

Hence the correct choice is (d).

28. Mass of the ball and the bob sticking together is $m' = 2m$. KE after collision $= \frac{1}{2}m'v'^2 = \frac{1}{2} \times 2m \times \left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$. KE before collision $= \frac{1}{2}mv^2$. Therefore, their ratio is 1:2. Hence the correct choice is (b).

29. In an elastic collision between two bodies of the same mass with one of them initially at rest, the moving body is brought to rest and the other moves with the same speed. Thus the ball will come to rest and the bob of the pendulum acquires a speed v. At this speed, it will rise to height h given by $h = v^2/2g$. Hence the correct choice is (b).

30. Let u_1 be the speed of mass m_1 before the collision. Here $u_2 = 0$. Therefore, the speeds of masses m_1 and m_2 after the collision respectively are

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

$\therefore$ KE of m_1 after collision $= \frac{1}{2}m_1v_1^2$

$= \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$. KE of m_1 before collision $= \frac{1}{2}m_1u_1^2$. The ratio of the two is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$.

Hence the correct choice is (a).

31. KE of m_2 after collision $= \frac{1}{2}m_2v_2^2$

$= \frac{1}{2}m_2\left(\frac{2m_1}{m_1 + m_2}\right)^2 u_1^2$.

KE of m_1 after collision $= \frac{1}{2}m_1v_1^2$

$= \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$. Dividing the two, we find that the correct choice is (d).

32. Since the collision is elastic, both momentum and kinetic energy are conserved. If m is mass of ball Q and v' its speed after the collision, the law of conservation of momentum gives

$$2v = 2 \times \frac{v}{4} + mv'$$

where v is the original speed of ball P. Thus

$$mv' = \frac{3v}{2} \text{ or } v' = \frac{3v}{2m} \quad \text{(i)}$$

The law of conservation of energy gives

$$\frac{1}{2} \times 2 \times v^2 = \frac{1}{2} \times 2 \times \left(\frac{v}{4}\right)^2 + \frac{1}{2}mv'^2$$

or $$mv'^2 = \frac{15v^2}{8} \quad \text{(ii)}$$

Using (i) in (ii) we get $m = 1.2$ kg. Hence the correct choice is (b).

33. Refer to Fig. 4.27.

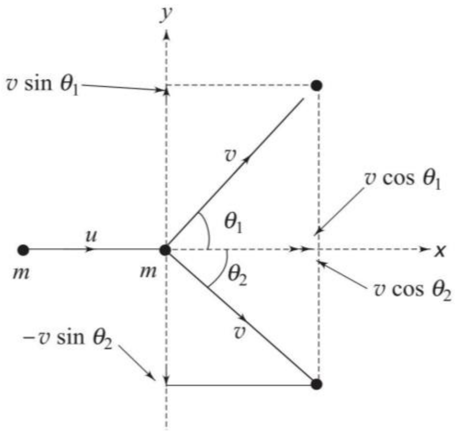


Fig. 4.27

Taking the components of the velocities along x and y–axes and using the law of conservation of momentum for x and y components we have

$$mu = mv \cos \theta_1 + mv \cos \theta_2 \quad \text{(i)}$$

and $$0 = mv \sin \theta_1 - mv \sin \theta_2 \quad \text{(ii)}$$

From (ii) we get $\sin \theta_1 = \sin \theta_2$ or $\theta_1 = \theta_2$. Using $\theta_1 = \theta_2 = \theta$ in (i) we have

$$mu = 2\, mv \cos \theta$$

or $$\cos \theta = \frac{u}{2v} \quad \text{(iii)}$$

Since the collision is elastic, kinetic energy is also conserved, i.e.

$$\frac{1}{2}\, mu^2 = \frac{1}{2}\, mv^2 + \frac{1}{2}\, mv^2$$

or $$u^2 = 2v^2 \text{ or } u = \sqrt{2}\, v \quad \text{(iv)}$$

Using (iv) and (iii) we have $\cos \theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$.

Thus $\theta_1 + \theta_2 = 2\theta = 90°$. Hence the correct choice is (c).

34. Refer to Fig. 4.27 again. Let $\mathbf{v}_1$ be the velocity of the ball moving along direction θ_1 and $\mathbf{v}_2$ be the velocity of the ball moving along θ_2.

From momentum conservation,

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$\Rightarrow \quad \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2$$

The magnitude of **u** is

$$u = \left[v_1^2 + v_2^2 + 2v_1v_2 \cos(\theta_1 + \theta_2)\right]^{1/2} \quad \text{(i)}$$

Since kinetic energy is also conversed,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad u^2 = v_1^2 + v_2^2 \quad \text{(ii)}$$

Using (ii) in (i) we have

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos(\theta_1 + \theta_2)$$

$$\Rightarrow \quad \cos(\theta_1 + \theta_2) = 0 \quad \Rightarrow \quad \theta_1 + \theta_2 = 90°$$

35. Power $P = Fv = mav = m\,\frac{dv}{dt}\cdot v$ or $v\,dv = \frac{P}{m}dt$.

Integrating, we have

$$\int v dv = \frac{P}{m}\int dt \qquad (\because P = \text{constant})$$

or $$\frac{v^2}{2} = \frac{Pt}{m}$$

or $$v = \sqrt{\frac{2P}{m}}\, t^{1/2}$$

or $$\frac{dx}{dt} = \sqrt{\frac{2P}{m}}\, t^{1/2}$$

or $$dx = \sqrt{\frac{2P}{m}}\, t^{1/2}\, dt$$

Integrating again, we have

$$\int d\,x = \sqrt{\frac{2P}{m}}\int t^{1/2} dt$$

or $$x = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}$$

i.e. $x \propto t^{3/2}$. Hence the correct choice is (c).

36. Let v be the speed of the bullet incident on the first plank. Its speed after it passes the plank $= \frac{9v}{10}$. If x is the thickness of the plank, the deceleration a due to the resistance of the plank is given by

$$2ax = v^2 - \left(\frac{9v}{10}\right)^2 = \frac{19v^2}{100} \quad \text{(i)}$$

Suppose the bullet is stopped after passing through n such planks. Then the distance covered by the bullet is $s = nx$. Thus, we have

$$v^2 - 0 = 2as = 2anx$$

or $$n = \frac{v^2}{2ax} = \frac{v^2 \times 100}{19v^2} \text{ [use Eq. (i) above]}$$

$$= \frac{100}{19} = 5.26$$

Thus the minimum number of planks required is 6. Hence the correct choice is (b).

37. Let u cms^{-1} be the speed of the bullet. Since the mass of the bullet remains unchanged, its speed becomes $v = \frac{3u}{4}$ cms^{-1} after it penetrates a distance $x = 3.5$ cm. The retardation a due to the resistance of the wooden is given by

$$u^2 - v^2 = 2ax$$

or $$u^2 - \left(\frac{3u}{4}\right)^2 = 2a \times 3.5$$

which gives $a = \frac{u^2}{16}$ cms^{-2}. The bullet will come to rest when its velocity $v' = 0$. If x' is the thickness penetrated by the bullet, then

$$u^2 - v'^2 = 2ax'$$

or $x' = \frac{u^2}{2a}$. But $a = \frac{u^2}{16}$ cms^{-2}. Therefore

$$x' = \frac{u^2 \times 16}{2u^2} = 8 \text{ cm}$$

Hence the correct choice is (a).

38. Since the wood offers a constant deceleration and hence a constant retardation force, the bullet will lose the remaining 75% of its kinetic energy after penetrating a further distance of $3x$. Therefore, the total distance penetrated by the bullet before it comes to rest = $x + 3x = 4x$. Hence the correct choice is (b).

39. The potential energy in the vertical position = work done in raising it from horizontal position to vertical position. In doing so, the mid–point of the rod is raised through a height $h = l/2$. Since the entire mass of the rod can be assumed to be concentrated at the mid–point (centre of mass), the work done = mgh = $mgl/2$. Hence the correct choice is (c).

40. Refer to Fig. 4.28. $AD = AB = l$. In the inclined position, let the centre of mass C of the rod be at a height h above the ground, so that $AC = l/2$. In triangle ACE, we have

$$h = AC \sin 30° = \frac{l}{2} \sin 30° = \frac{l}{4}$$

$$\therefore PE = mgh = \frac{mgl}{4}$$, which is choice (a).

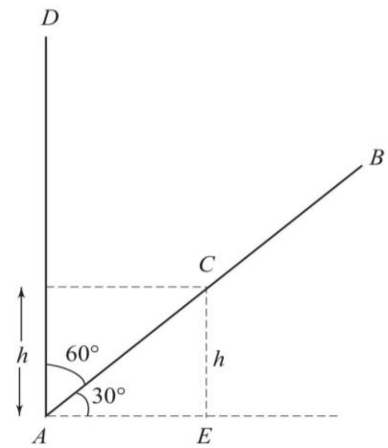


Fig. 4.28

41. Let f be the force of friction and m be the mass of the body. The retardation $a = f/m$. If v is the initial speed of the body, then

$$2ax = v^2$$

or $$max = \frac{1}{2} mv^2 = k$$

But $ma = f$. Therefore $fx = k$ or $f = k/x$. Hence the correct choice is (a).

42. Force of friction = μmg. Therefore, retardation $a = \mu mg/m = \mu g$. Also $2ax = v^2$ or $2am^2x = m^2v^2$. But $p = mv$. Therefore,

$$2\,am^2x = p^2$$

But $a = \mu g$. Therefore, $2\,\mu g\,m^2x = p^2$ or $\mu = \frac{p^2}{2gm^2x}$. Hence the correct choice is (a).

43. The mass per unit length of the chain $m = \frac{M}{L}$. The mass of the hanging portion of the chain is $m' = \frac{mL}{n}$. This mass can be assumed to be concentrated at the centre of the hanging portion of the chain which is a distance of $x = \frac{L}{2n}$ from the edge of the table. Therefore, the work done in pulling the hanging portion of the chain on to the table top is

$$W = m'gx = \frac{mL}{n} \times g \times \frac{L}{2n} = \frac{mgL^2}{2n^2} = \frac{MgL}{2n^2}$$

Hence the correct choice is (d).

44. Since the platform is depressed by an amount x, the total work done on the spring is $mg\,(h + x)$. This work is stored in the spring in the form of potential energy $\frac{1}{2}\,kx^2$. Equating the two, we have

$$\frac{1}{2}kx^2 = mg(h+x)$$

or $$k = \frac{2m\,g(h+x)}{x^2}$$

Given, $h = 0.4$ m, $x = 0.1$ m, $m = 1$ kg and $g = 10$ ms^{-2}. Substituting these values, we get $k = 1000$ Nm^{-1}. Hence the correct choice is (c).

45. Let V be the velocity of the block with the bullet embedded in it at the time of impact. Then from the principle of conservation of momentum, we have

$$mv = (M+m)V$$

or $$V = \frac{mv}{(M+m)} \qquad \text{(i)}$$

If the block, with the bullet embedded in it, rises to a vertical height h, then from the principle of conservation of energy, we have

$$\frac{1}{2}(M+m)V^2 = (M+m)gh$$

or $$V = \sqrt{2gh} \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$\sqrt{2gh} = \frac{mv}{(M+m)}$$

Squaring this equation, we find that h is given correctly by choice (a).

46. Percentage loss of energy $= \frac{4mM}{(M+m)^2} \times 100$

$$= \frac{4m \times 2m}{(2m+m)^2} \times 100$$

$$= \frac{800}{9} = 88.9\%$$

Hence the correct choice is (c).

47. Refer to Fig. 4.29. Here $\mathbf{p} = mv$ and $\mathbf{P} = M\mathbf{V}$. The resultant of $\mathbf{p}$ and $\mathbf{P}$ is

$$p_r = \sqrt{p^2 + P^2} = \sqrt{(mv)^2 + (MV)^2}$$

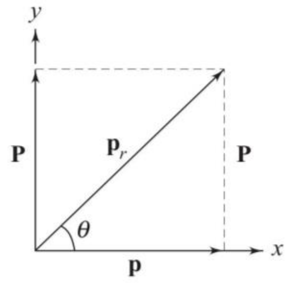


Fig. 4.29

which is choice (a). The angle which the resultant momentum p_r subtends with the x-axis is given by

$$\tan\theta = \frac{P}{p} = \frac{MV}{mv}$$, which is choice (b).

$$\text{Loss of KE} = \left(\frac{1}{2}mv^2 + \frac{1}{2}MV^2\right) - \frac{1}{2}\left[\frac{m^2v^2 + M^2V^2}{(M+m)}\right]$$

$$= \frac{1}{2}\frac{Mm}{(M+m)}(V^2 + v^2)$$, which is choice (c).

Hence the correct choice is (d).

48. $\mathbf{p}_1 = 2\text{ kg}\left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\text{ ms}^{-1} = \left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\right)\text{ kg ms}^{-1}$

$\mathbf{p}_2 = 3\text{ kg}\left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\text{ ms}^{-1} = \left(6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)\text{ kg ms}^{-1}$

Resultant momentum is

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$$
$$= \left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\right) + \left(6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)$$
$$= \left(8\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\text{ kg ms}^{-1}$$

Total mass $(m) = 2 + 3 = 5$ kg. Therefore, the velocity of composite body is

$$\mathbf{v} = \frac{\mathbf{P}}{m} = \frac{1}{5}\left(8\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\text{ ms}^{-1}$$

Hence the correct choice is (a).

49. Change in momentum $= m(\mathbf{v}_f - \mathbf{v}_i) = 2\text{ kg}\left(5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\text{ms}^{-1} - 2\text{ kg}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)\text{ ms}^{-1} = 2\left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\text{ kg ms}^{-1}$.

$$\text{Average force} = \frac{\text{change in momentum}}{\text{time of impact}}$$

$$= \frac{2\left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}{0.02\text{ s}} = 100\left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$$

Hence the correct choice is (d).

50. Total distance $= h + 2e^2h + 2e^4h + \ldots\ldots$

$$= h + 2e^2h(1 + e^2 + \ldots\ldots)$$

$$= h + \frac{2e^2h}{1-e^2} = h\left(\frac{1+e^2}{1-e^2}\right)$$

$$= h\left[\frac{1+\left(\frac{1}{3}\right)^3}{1-\left(\frac{1}{3}\right)^2}\right] = \frac{5h}{4}$$

Hence the correct choice is (b).

51. Total time $= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + \cdots$

$$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h}{g}}\,(e + e^2 + \cdots)$$

$$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h}{g}}\,\frac{e}{(1-e)} = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

$$= \sqrt{\frac{2h}{g}}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) = 2\sqrt{\frac{2h}{g}}$$

Hence the correct choice is (b).

52. Given $m_Q = p\, m_P$ and $v_Q = v_P/q$. From the principle of conservation of momentum, we have (since body P comes to rest after collision)

$$m_P\, v_P + m_Q\, v_Q = m_Q\, v$$

where v is the velocity of body Q after collision. Thus

$$m_P\, v_P + p\, m_P\, \frac{v_P}{q} = p\, m_P v.$$

which gives $\quad \frac{v}{v_P} = \frac{p+q}{pq} \qquad$ (i)

Now, the coefficient of restitution is given by

$$e = \frac{v}{v_p - v_Q} = \frac{v}{v_p - \frac{v_p}{q}}$$

which gives $\quad \frac{v}{v_P} = \frac{e}{q}\,(q-1) \qquad$ (ii)

Equating (i) and (ii), we get $e = \frac{p+q}{q\,(p-1)}$ which is choice (d).

53. Refer to Fig. 4.30. If sphere A is projected with velocity v, the time taken by it to strike B is equal to $\frac{\pi r}{v} = T$ or $\pi r = Tv$. Now, the coefficient of restitution is given by

$$e = \frac{v_B - v_A}{v}$$

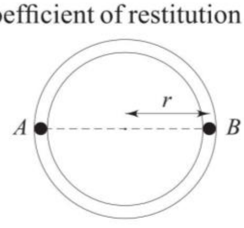


Fig. 4.30

where v_A and v_B are the velocities of A and B after the collision. Thus, $v_B - v_A = ev$. The spheres travel with this relative velocity. It is clear that one will overtake the other after travelling a distance $= 2\pi r$.

$$\therefore \text{ Time taken} = \frac{2\pi r}{v_B - v_A} = \frac{2\pi r}{ev} = \frac{2Tv}{ev} = \frac{2T}{e}$$

(since $\pi r = Tv$).

Hence the correct choice is (c).

54. If mass m falss through a distance h, mass m rises up through the same distance h. Let v be the common velocity of the masses when this happens. Now, loss in PE = gain in KE, i.e.

$$Mgh - mgh = \frac{1}{2}\,(M+m)\,v^2$$

which gives $v = \sqrt{\frac{2gh(M-m)}{(M+m)}}$, which is choice (c).

55. When M has descended a distance h, loss of PE = Mgh. If v is the common velocity of the masses, gain in KE $= \frac{1}{2}\,(M+m)\,v^2$. Hence

$$\frac{1}{2}\,(M+m)\,v^2 = Mgh$$

or $\qquad v = \sqrt{\frac{2Mgh}{(M+m)}}$, which is choice (c).

56. Relative speed of man with respect to escalator $= (v - u)$.

$\therefore$ Actual displacement of man per second $= (v - u)$. Hence, the actual displacement of man in going up the escalator of height h is $\frac{v\,h}{(v-u)}$. Therefore,

Work done $= mg \times \frac{v\,h}{(v-u)}$, which is choice (d)

57. Let the mass of the rope be M. If a small part of length x hangs through the hole, it weight

$$= \frac{Mx}{L}\,(g)$$

and its acceleration is $a = \frac{Mx}{L}\,\frac{g}{M} = \frac{xg}{L}$

$\therefore$ Force on the remaining part of length $(L - x)$ = mass of part of length $(L - x) \times$ acceleration (a)

$$= \frac{M}{L}\,(L-x) \times \frac{xg}{L} = \frac{Mg}{L^2}\,(Lx - x^2)$$

If the rope falls through a distance dx, the work done by gravity is

$$dW = \frac{Mg}{L^2}\,(Lx - x^2)\,dx$$

$\therefore$ Total work done is

$$W = \int_0^L dW = \frac{Mg}{L^2}\int_0^L \left(Lx - x^2\right) dx$$

$$= \frac{Mg}{L^2}\left|\frac{Lx^2}{2} - \frac{x^3}{3}\right|_0^L = \frac{MgL}{6}$$

Since the centre of mass of the rope falls through a distance $\frac{L}{2}$, decrease in PE = $\frac{MgL}{2}$. From the principle of conservation of energy, we have

Decrease in PE – Work done (W) = Increase in KE

or $$\frac{MgL}{2} - \frac{MgL}{6} = \frac{1}{2}Mv^2$$

which gives $v = \sqrt{\frac{4gL}{3}}$, which is choice (b).

58. Given $U = 6x + 8y$ joule and mass $m = 2$ kg. Force along x-axis is

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\,(6x + 8y) = -6 \text{ newton}$$

Force along y-axis is

$$F_y = -\frac{dU}{dy} = -\frac{d}{dy}\,(6x + 8y) = -8 \text{ newton}$$

Therefore, the x and y components of acceleration are

$$a_x = \frac{F_x}{m} = \frac{-6}{2} = -3 \text{ ms}^{-2}$$

and $$a_y = \frac{F_y}{m} = \frac{-8}{2} = -4 \text{ ms}^{-2}$$

$\therefore$ Resultant acceleration

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-3)^2 + (-4)^2} = 5 \text{ ms}^{-2}$$

The x and y coordinates of the body at time t are

$$x = x_0 + \frac{1}{2}a_x\,t^2 = 6 - \frac{1}{2}\times 3 \times t^2$$

$$= \left(6 - \frac{3}{2}t^2\right) \text{ metre}$$

and $$y = y_0 + \frac{1}{2}a_y\,t^2 = 4 - \frac{1}{2}\times 4 \times t^2$$

$$= (4 - 2t^2) \text{ metre}$$

The body will cross the y-axis when $x = 0$, i.e. at time t given by $\left(6 - \frac{3}{2}t^2\right) = 0$ or $t = 2$ s. Hence the correct choice is (b).

59. $v_x = a_x t = -3 \text{ ms}^{-2} \times 2 \text{ s} = -6 \text{ ms}^{-1}$

$v_y = a_y t = -4 \text{ ms}^{-2} \times 2 \text{ s} = -8 \text{ ms}^{-1}$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-6)^2 + (-8)^2} = 10 \text{ ms}^{-1}$$

Hence the correct choice is (c).

60. Initial momentum (p) = momentum of bullet + momentum of block = mv + 0 = mv. From the principle of conservation of momentum, final momentum of bullet + block system of mass (M + m) = Initial momentum p. Now

$$\text{KE} = \frac{p^2}{2\times(M \times m)} = \frac{m^2v^2}{2(M+m)}$$

Hence the correct choice is (d).

61. A ball dropped from a height h_1 on reaching the planet's surface will have a velocity given by

$$v_1 = \sqrt{2gh_1}$$

Let v_2 be the velocity with which the ball bounces. It will attain a height h_2 given by

$$v_2 = \sqrt{2gh_2}$$

$$\therefore \quad \frac{v_2}{v_1} =$$

or $$1 - \frac{v_2}{v_1} = 1 - 0.6 \text{ or } \frac{v_1 - v_2}{v_1} = 0.4 = \frac{2}{5}$$

Hence the correct choice is (b).

62. Let m' and M be the masses of the lighter and heavier fragments respectively. Since the particle is moving along the x-axis, the y-component of momentum will be zero immediately after and before explosion, i.e.

$$m'\,v_y + MV_y = 0$$

where v_y and V_y are the velocities of the lighter and heavier fragments respectively immediately after explosion. Thus

$$V_y = -\left(\frac{m'}{M}\right)v_y = -\left(\frac{m/4}{3m/4}\right)v_y = -\frac{1}{3}v_y$$

Since y = + 15 cm, the direction of v_y is along the positive y-axis and that of V_y will be along the negative y-axis. An instant later (say, at time t), it is given that

$$y = 15 \text{ cm} = v_y\,t$$

$$\therefore \quad Y = V_y\,t = -\frac{1}{3}\,v_y\,t = -\frac{1}{3}\,y$$

$$= -\frac{1}{3} \times 15 \text{ cm} = -5 \text{ cm}$$

Hence the correct choice is (a).

63. The potential energy of the particle is given by

$$U = -\int F dx = -\int (-kx + ax^3)\, dx$$

or $$U = k\frac{x^2}{2} - a\frac{x^4}{4} = \frac{x^2}{4}(2k - ax^2) \quad \text{(i)}$$

From Eq. (i) it follows that $U = 0$ at two values of x which are $x = 0$ and $x = \sqrt{2k/a}$. Hence graphs (b) and (c) are not possible. Also U is maximum or minimum at a value of x given by $\frac{dU}{dx} = 0$, i.e.

$$0 = \frac{d}{dx}\left(\frac{kx^2}{2} - \frac{ax^4}{4}\right)$$

$$= kx - ax^3 = x(k - ax^2)$$

or $$x = \sqrt{k/a}.$$

At this value of x, U is maximum if $\frac{d^2U}{dx^2} < 0$,

Now $\frac{d^2U}{dx^2} = \frac{d}{dx}(kx - ax^3) = k - 3ax^2$.

At $x = \sqrt{k/a}$,

$$\frac{d^2U}{dx^2} = k - 3a\frac{k}{a} = k - 3k = -2k,$$

which is negative.

Hence U is maximum at $x = \sqrt{k/a}$.

Hence graph (a) is also not possible. Also U is negative for $x > \sqrt{2k/a}$. Therefore, the correct graph is (d).

64. The velocity of the body at time t is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^2}{4}\right) = \frac{t}{2}$$

$\therefore$ At $t = 0$, $v = u = 0$ and at $t = 2$ s, $v = 1$ ms^{-1}, Now, work done = increase in KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 6 \times (1)^2 = 3 \text{ J}$$

65. Work done = increase in potential energy in (i) raising the weight 150 N of the ladder through a height 1 m and (ii) raising a weight 40 N through 2.5 m

$$= 150 \text{ N} \times 1 \text{ m} + 40 \text{ N} \times 2.5 \text{ m}$$

$$= 250 \text{ Nm} = 250 \text{ J}$$

Hence the correct choice is (b).

66. Friction force = $\mu\, mg = 0.2 \times 5 \times 10 = 10$ N. Effective force F = applied force – frictional force = 25 – 10 = 15 N. Kinetic energy = work done by force F in pulling the body through a distance S (= 10 m) = 15 × 10 = 150 J, which is choice (b).

67. The retardation is given by (see Fig. 4.31)

$$a = g(\mu \cos\theta + \sin\theta) \quad \text{(i)}$$

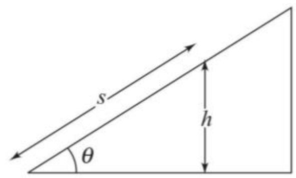


Fig. 4.31

Let u be the initial velocity of the body. If it is stopped after moving a distance s up the plane, then

$$u^2 = 2as$$

$\therefore$ $$\text{Kinetic energy} = E = \frac{1}{2}mu^2$$

$$= \frac{1}{2}m \times 2as = mas \quad \text{(ii)}$$

Now, work done against friction is

$$W = \text{gain in PE} = mgh$$

It is clear from the figure that $h = s \sin\theta$. Therefore,

$$W = mgs \sin\theta \quad \text{(iii)}$$

From (i), we have

$$g = \frac{a}{(\mu\cos\theta + \sin\theta)} \quad \text{(iv)}$$

Also $$\mu = \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{or} \quad \sin\theta = \mu\cos\theta \quad \text{(v)}$$

Using (iv) and (v) in (iii), we have

$$W = \frac{mas(\mu\cos\theta)}{(\mu\cos\theta + \sin\theta)} \quad \text{(vi)}$$

Using (ii) in (vi), we get

$$W = \frac{\mu E \cos\theta}{(\mu\cos\theta + \sin\theta)}$$, which is choice (d).

68. The total distance travelled is

$$S = h + 2e^2 h + 2e^4 h + 2e^6 h + \ldots$$

$$= h + 2h(e^2 + e^4 + e^6 + \ldots)$$

$$= h + 2h\left(\frac{e^2}{1 - e^2}\right)$$

$$= h\left[1+\frac{2e^2}{1-e^2}\right]=\frac{h\left(1+e^2\right)}{\left(1-e^2\right)}$$

Hence the correct choice is (a).

69. Work done by the force is

$$W = \int f dx \int_0^4 (3x+4)dx$$

$$= \left|\frac{3x^2}{2}+4x\right|_0^4 = 40 \text{ J}$$

From workenergy principle, work done = change in kinetic energy.

$$W = \Delta k = \frac{1}{2}m\,(v^2-u^2)$$

$$40 = \frac{1}{2}\times 10\times (v^2-2^2)$$

$\Rightarrow \quad v = 2\sqrt{3}\ \text{ms}^{-1}$, which is choice (c)

70. $$F = -\frac{dU}{dx} = -\frac{d}{dx}\left[3(x-1)-(x-3)^3\right]$$

$$= -3+3\,(x-3)^2$$

$\therefore$ Acceleration is

$$A = \frac{F}{m} = \frac{-3+3\,(x-3)^2}{3}$$

$$= (x-3)^2-1 \qquad (1)$$

From Eq. (1), it follows that $A = 0$ when $(x-3)^2-1=0$ which gives $x = 2\text{m}$ and $x = 4\text{m}$. Also A is minimum $= -1\ \text{ms}^{-2}$ at $x = 3\text{m}$. Hence correct graphs is (c).

71. Let $+mv$ be the momentom of the bullet before impact. Then momentum of the block $= -MV$. The total momentum of the system before impact $= (mv-MV)$. Since the block + bullet system is brought to rest after impact, its momentum after impact is zero. From conservation of momentum,

$mv - MV = 0 \Rightarrow v = \frac{MV}{m}$, which is choice (c).

72. Refer to Fig 4.32.

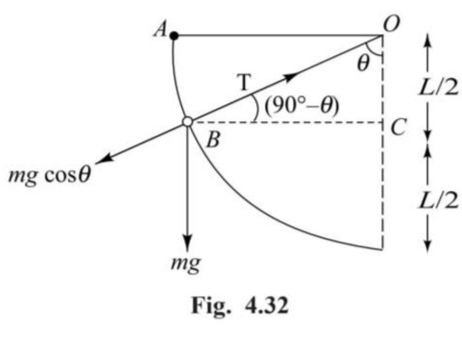


Fig. 4.32

The equation of motion when the ball is at position B is

$$\frac{mv^2}{L} = T - mg\cos\theta \qquad (1)$$

Where v is the speed of the ball at point B. From conservation of energy,

Total energy at A = total energy at B

$$mgL + 0 = \frac{1}{2}mv^2 + mg\left(\frac{L}{2}\right)$$

$$\Rightarrow \quad mv^2 = mgL \qquad (2)$$

Using (2) in (1), we have

$$mg = T - mg\cos\theta$$

$$\Rightarrow \quad T = mg\,(1+\cos\theta)$$

In triangle OBC

$$\sin(90^0-\theta) = \frac{OC}{OB} = \frac{L/2}{L} = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\therefore \quad T = mg\left(1+\frac{1}{2}\right) = \frac{3}{2}\,mg$$

So the correct choice is (c).

73. Let v be the speed of the ball just before impact. From conservation of energy,

total energy at A = total energy at B

$$mg\,L + 0 = 0 + \frac{1}{2}mu^2$$

$$\Rightarrow \quad u^2 = 2gL$$

Since the collision is elastic, the ball rebounds after the impact with a speed v given by

$$v = \left(\frac{M-m}{M+m}\right)u$$

$$= \left(\frac{3m-m}{3m+m}\right)u = \frac{u}{2}$$

If the ball to a height h, then from the conservation of energy,

total energy at C = total energy at B

$$mg\,h + 0 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{u^2}{8g} \qquad \left(\because v = \frac{u}{2}\right)$$

$$= \frac{2gL}{8g} \qquad (\because u^2 = 2gL)$$

$$= \frac{L}{4}$$

So that correct choice is (d).

74. Given $x = \frac{kt^3}{3}$

$$v = \frac{dx}{dt} = kt^2$$

$$a = \frac{dv}{dt} = 2kt$$

$$F = ma = 2mkt$$

$\therefore$ Work done in interval $t = 0$ to $t = 2$s *is*

$$W = \int F dx$$

$$= 2mk\int_0^2 t dx$$

$$= 2mk\int_0^2 t\frac{dx}{dt}dt$$

$$= 2mk\int_0^2 tv\,dt$$

$$= 2mk^2\int_0^2 t^3\,dt \qquad (\because \quad v = 2kt)$$

$$= 8mk^2$$

So the correct choice is (c)

75. If A is the cross-sectional area of the pipe, the volume of water flowing a small distance Δx in time $\Delta t = A\Delta x$. The Volume flowing per second $= \frac{A\Delta x}{\Delta t} = Av$, Where v is the speed of water. Mass of water flowing per second $= \rho Av$ where ρ is the density of water. Therefore, increase in kinetic energy per second (which is power)

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2}\rho Av \times v^2$$

or $$P = \frac{1}{2}\rho Av^3$$

In order to doube the mass of water flowing per second, v must be doubled. Since $P \propto v^3$, power P must be increased $(2)^3 = 8$ times. So the correct choice is (c).

76. $P = Fv = mav = m\frac{dv}{dt} \times v$

$$\Rightarrow \quad v\frac{dv}{dt} = \frac{P}{m}$$

$$\Rightarrow \quad v\frac{dv}{dx} \times \frac{dx}{dt} = \frac{P}{m}$$

$$\Rightarrow \quad v^2\frac{dv}{dx} = \frac{P}{m} \qquad \left(\because \frac{dx}{dt} = v\right)$$

$$\Rightarrow \quad v^2\,dv = \frac{P}{m}dx$$

Integrating, we have

$$\int_o^v v^2 dv = \frac{P}{m}\int_o^x dx \qquad (\because P = \text{constant})$$

$$\Rightarrow \quad \frac{v^3}{3} = \frac{Px}{m}$$

$$\Rightarrow \quad v = \left(\frac{3Px}{m}\right)^{1/3}$$

So the correct choice is (a).

77. Given $x = at^4 + bt + c$.

$$v = \frac{dx}{dt} = 4\,at^3 + \text{b}$$

$$\frac{dv}{dt} = 12\,at^2$$

$$P = \frac{dW}{dt}$$

$$\Rightarrow \quad Fv = \frac{dW}{dt}$$

$$\Rightarrow \quad m\frac{dv}{dt}\,v = \frac{dW}{dt} \qquad \left(\because \quad acc. = \frac{dv}{dt}\right)$$

or $$\frac{dW}{dt} = m \times 12at^2 \times (4\,at^3 + b)$$

$$= 48\,ma^2\,t^5 + 12\,mabt^2$$

$$\Rightarrow \quad dW = 48\,ma^2\,t^5\,dt + 12\,mabt^2\,dt$$

Integrating, we get

$$W = 48ma^2\int_0^1 t^5 dt + 12mab\int_0^1 k^2 dt$$

$$= 8\,ma^2 + 4\,mab$$

$$w = 4ma\,(b + 2a), \text{ which is choice (d).}$$

ALTERNATIVE METHOD

From work-energy principle,

work done = Change in K.E.

= Final K.E. – Initial K.E.

$$= \frac{1}{2} mv^2 \text{ at } t = 1s - \frac{1}{2} mv^2 at\ t = 0$$

$$= \frac{1}{2} m(4a + b)^2 - \frac{1}{2} mb^2$$

$$= \frac{1}{2}m\left[(4a+b)^2 - b^2\right]$$

$$= 4ma(2a + b)$$

78. Given $\frac{mv^2}{2} = \frac{kx^2}{2}$

$$\Rightarrow \quad mv^2 = kx^2 \qquad (1)$$

Differentiating w.r.t. time t,

$$2mv\frac{dv}{dt} = 2kx\frac{dx}{dt} = 2\ kx\ v$$

$$\Rightarrow \quad m\frac{dv}{dt} = kx$$

$$\Rightarrow \quad m\ a_t = kx$$

where a_t is tangential acceleration. Therefore, tangential force is

$$F_t = kx$$

Radial (centripetal) force is

$$F_r = \frac{mv^2}{r} = \frac{kx^2}{r} \qquad \text{[use Eq. (1)]}$$

$$\therefore \quad \text{Net force} = \sqrt{F_t^2 + F_r^2}$$

$$= \left(k^2x^2 + \frac{k^2x^4}{r^2}\right)^{1/2}$$

$$= kx\left(1 + \frac{x^2}{r^2}\right)^{1/2}$$

So the correct choice is (c).

79. $K_i = \frac{1}{2} mu^2$. Let v be the velocity of the block when it has lost $\frac{1}{4}$ th of its kinetic energy, then its new K.E. is $3K_i / 4$. Therefore,

$$\frac{1}{2} mu^2 = \frac{3}{8} mv^2 \Rightarrow v = \frac{\sqrt{3}u}{2}$$

Frictional force is $f = \mu mg$. Therefore, retardation $a = -\frac{f}{m} = -\mu g$. From $v = u + at$. we get

$$\frac{\sqrt{3}u}{2} = u - \mu gt$$

$$\Rightarrow \quad \mu = \frac{u}{2gt}\left(2 - \sqrt{3}\right), \text{ which is choice (d).}$$

80. Let m be the mass of each ball. Let u_1 and u_2 be the velocities A and B before collision and v_1 and v_2 are collision. Then

$$u_1 = \frac{p}{m} \text{ and } u_2 = 0 \text{ (given)}$$

Impulse = change in momentum. It is given that

I = change in momentum of B.

$= mv_2 - 0$

$$\Rightarrow \quad v_2 = \frac{I}{m}$$

Therefore,

$$v_1 = \frac{p - I}{m}$$

Now $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$= \frac{\left(\frac{I}{m} - \frac{p-I}{m}\right)}{\left(\frac{p}{m} - 0\right)} = \frac{2I}{p} - 1$$

So the correct choice is (b).

81. Net height gained by the man = $(l - h)$. Therefore, gain in potential energy of man = $mg(l - h)$. The weight mg of the man pulls the balloon down by a height h. Hence work done by this force on the balloon = mgh. This is equal to the gain in potential energy of the balloon. Therefore, the work done by man in reaching the balloon is

W = increase in P.E. of man + increase in P.E. of balloon = $mg\ (l - h) + mgh = mgl$. So the correct choice is (d).

82. For a perfectly elastic one-dimensional collision, the velocity of the ball after the colision is

$$v_1 = \left(\frac{m - M}{m + M}\right)u_1 + \frac{2M(-u_2)}{m + M}$$

Since $M >> m$,

$$v_1 = -u_1 - 2u_2 = -(u_1 + 2u_2).$$

The magnitude of v_1 is

$$|\,v_1\,| = u_1 + 2u_2$$

2 SECTION Multiple Choice Questions Based on Passage

So the correct choice is (c).

Questions 1 to 2 are based on the following passage.

Passage I

A light rod of length L having a body of mass M attached to its end hangs vertically. It is turned through 90° so that it is horizontal and then released.

1. The centripetal acceleration when the rod makes an angle θ with the vertical is
   (a) $g \cos \theta$ (b) $2g \cos \theta$
   (c) $g \sin \theta$ (d) $2g \sin \theta$
2. The tension in the rod when it makes an angle θ with the vertical is
   (a) $Mg \cos \theta$ (b) $2\, Mg \cos \theta$
   (c) $3\, Mg \cos \theta$ (d) zero

Solutions

1. The rod is released from the horizontal position OA. Let OB be the position of the rod when it makes an angle θ with the vertical. (Fig. 4.33).

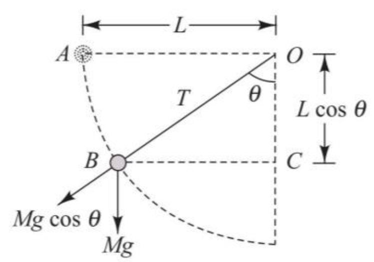


Fig. 4.33

The loss of PE when the body falls from A to $B = Mg \times OC = MgL \cos \theta$. If v is the velocity of the body at B, then

$$\frac{1}{2} Mv^2 = MgL \cos \theta \text{ or } v^2 = 2gL \cos \theta \quad (1)$$

$$\text{centripetal acceleration} = \frac{v^2}{L} = \frac{2gL\cos\theta}{L} = 2g \cos \theta,$$

which is choice (b).

2. The centripetal force when the body is at B is

$$F_c = \frac{Mv^2}{L}$$

Thus, we have

$$T - Mg \cos \theta = \frac{Mv^2}{L} \quad (2)$$

Using (1) in (2), we get

$$T - Mg \cos \theta = \frac{M}{L} \times 2gL \cos \theta = 2\, Mg \cos \theta$$

or $T = 3\, Mg \cos \theta$

Thus the correct choice is (c).

Questions 3 to 5 are based on the following passage.

Passage II

A small roller coaster starts at point A with a speed u on a curved track as shown in Fig. 4.34. The friction between the roller coaster and the track is negligible and it always remains in contact with the track.

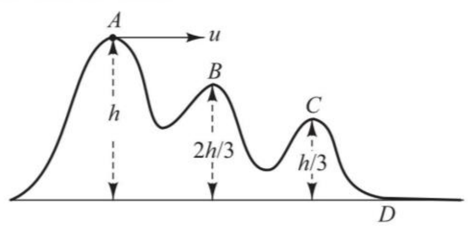


Fig. 4.34

3. The speed of the roller coaster at point B on the track will be
   (a) $(u^2 + gh)^{1/2}$ (b) $\left(u^2 + \frac{2gh}{3}\right)^{1/2}$
   (c) $(u^2 + 2gh)^{1/2}$ (d) $\left(u^2 + \frac{3gh}{2}\right)^{1/2}$
4. The speed of the roller coaster at point C on the track will be
   (a) $\left(u^2 + \frac{gh}{3}\right)^{1/2}$ (b) $\left(u^2 + \frac{2gh}{3}\right)^{1/2}$
   (c) $\left(u^2 + \frac{4gh}{3}\right)^{1/2}$ (d) $(u^2 + 2gh)^{1/2}$

5. The speed of the roller coaster at point D on the track will be
(a) $(u^2 + gh)^{1/2}$ (b) $(u^2 + 2gh)^{1/2}$
(c) $(u^2 + 3gh)^{1/2}$ (d) $(u^2 + 4gh)^{1/2}$

Solutions

3. Total energy at A = KE + PE = $\frac{1}{2}mu^2 + mgh$. If v_b is the speed at point B, the total energy at $B = \frac{1}{2}mv_b^2 + mg(2h/3)$. From the principle of conservation of energy, we have
$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv_b^2 + \frac{2mgh}{3}$ which gives
$v_b = \left(u^2 + \frac{2gh}{3}\right)^{1/2}$, which is choice (b).
4. Similarly, the speed at point C is given by
$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv_c^2 + \frac{mgh}{3}$ which gives
$v_c = \left(u^2 + \frac{4gh}{3}\right)^{1/2}$, which is choice (c).
5. At point D, the energy is entirely kinetic. If the speed of the roller coaster at point D is v_d, then we have
$\frac{1}{2}mv_d^2 = mgh + \frac{1}{2}mu^2$
or $v_d = (u^2 + 2gh)^{1/2}$, which is choice (b).

Questions 6 to 8 are based on the following passage.

Passage III

The displacement x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metre and t is in second.

6. The displacement of the particle when its velocity is zero is
(a) zero (b) 1 m
(c) 2 m (d) 3 m
7. The acceleration of the particle
(a) increases with time
(b) decreases with time
(c) increases with time up to $t = 3$ s and then decreases with time.
(d) remains constant at 2 ms^{-2}.
8. The work done by the force in first 6 s is
(a) 1 J (b) 3 J
(c) 6 J (d) zero

Solutions

6. Given $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$ or
$$x = (t-3)^2 \quad (1)$$
Differentiating (1) with respect to t, we get
$$\frac{dx}{dt} = 2(t-3)$$
or $$v = 2(t-3) \quad (2)$$
From (2) it follows that $v = 0$ at $t = 3$ s. Using $t = 3$ s in (1), we get $x = 0$. Thus, the displacement of the particle is zero when its velocity is zero. Thus, the correct choice is (a).
7. From Eq. (2), we have
$$a = \frac{dv}{dt} = \frac{d}{dt}[2(t-3)] = 2 \text{ ms}^{-2}.$$
Hence the correct choice is (d).
8. From Eq. (2), the initial velocity, i.e., velocity at $t = 0$ is
$$v_0 = 2(0-3) = -6 \text{ ms}^{-1}$$
Final velocity, i.e., velocity at $t = 6$ s is
$$v = 2(6-3) = 6 \text{ ms}^{-1}$$
Work done = final KE – initial KE
$$= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v^2 - v_0^2)$$
$$= \frac{1}{2}m[(6)^2 - (-6)^2] = 0,$$ which is choice (d).

Questions 9 to 12 are based on the following passage.

Passage IV

The kinetic energy of a particle moving along a circle of radius R depends on distance (s) as $K = as^2$ where a is a constant.

9. The centripetal force is given by
(a) $\frac{as^2}{2R}$ (b) $\frac{as^2}{R}$
(c) $\frac{2as^2}{R}$ (d) $\frac{4as^2}{R}$

10. The speed of the particle around the circle is

(a) $2s\left(\frac{a}{m}\right)^{1/2}$ (b) $s\left(\frac{a}{m}\right)^{1/2}$

(c) $s\left(\frac{2a}{m}\right)^{1/2}$ (d) $s\left(\frac{a}{2m}\right)^{1/2}$

11. The tangential force acting on the particle is

(a) mas (b) $2mas$

(c) as (d) $2as$

12. The net force acting on the particle is

(a) $2as\left(1+\frac{s}{R}\right)$ (b) $as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

(c) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (d) zero

Solutions

9. Given KE (K) = $\frac{1}{2}mv^2 = as^2$. Therefore, the centripetal force is

$$f_c = \frac{mv^2}{R} = \frac{2\times\left(\frac{1}{2}mv^2\right)}{R} = \frac{2as^2}{R},$$

which is choice (c).

10. The speed v of the particle around the circle is given by

$$\frac{1}{2}mv^2 = as^2 \quad \text{or} \quad v = s\left(\frac{2a}{m}\right)^{1/2}$$

Hence the correct choice is (c).

11. The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left[s\left(\frac{2a}{m}\right)^{1/2}\right] = \left(\frac{2a}{m}\right)^{1/2}\frac{ds}{dt}$$

But $\frac{ds}{dt} = v = s\left(\frac{2a}{m}\right)^{1/2}$.

Therefore,

$$a_t = \left(\frac{2a}{m}\right)^{1/2}\times s\left(\frac{2a}{m}\right)^{1/2} = \frac{2as}{m}$$

$\therefore$ Tangential force is $f_t = ma_t$

$$= m\times\frac{2as}{m} = 2as, \text{ which is choice (d).}$$

12. Net force acting on the particle is

$$f = (f_c^2 + f_t^2)^{1/2}$$

$$= \left[\left(\frac{2as^2}{R}\right)^2 + (2as)^2\right]^{1/2}$$

$$= 2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$$

Thus the correct choice is (c).

Questions 13 to 15 are based on the following passage.

Passage V

A conical pendulum consists of a string of length L fixed at one end carrying a body of mass m at the other end. The mass is revolved in a circle in the horizontal plane about a vertical axis passing through the fixed end of the string. The angular frequency of revolution of the body is ω. The string makes an angle θ with the vertical axis.

13. The tension in the string is

(a) $\frac{m\omega^2}{L}$ (b) $\frac{L\omega^2}{m}$

(c) $m\omega^2 L$ (d) $m\omega L^2$

14. The angle of inclination of the string with the vertical is given by

(a) $\cos\theta = \frac{g}{\omega^2 L}$ (b) $\sin\theta = \frac{g}{\omega^2 L}$

(c) $\cos\theta = \frac{\omega^2 L}{g}$ (d) $\sin\theta = \frac{\omega^2 L}{g}$

15. The linear speed of the body is

(a) ωL (b) $\omega L\sin\theta$

(c) $\omega L\cos\theta$ (d) $\omega L\tan\theta$

Solutions

13. Let T be tension in the string. Figure 4.35 shows the forces acting on the system. Tension T can be resolved into two mutually perpendicular components. The horizontal component $T\sin\theta$ provides the centripetal force for circular motion and the vertical component $T\cos\theta$ balances the weight mg.

Thus

$$T\cos\theta = mg \quad (1)$$

and $$T\sin\theta = \frac{mv^2}{r} = m\omega^2 r$$

But $r = L\sin\theta$. Therefore,

$$T\sin\theta = m\omega^2 L\sin\theta \text{ or } T = m\omega^2 L \quad (2)$$

Hence the correct choice is (c).

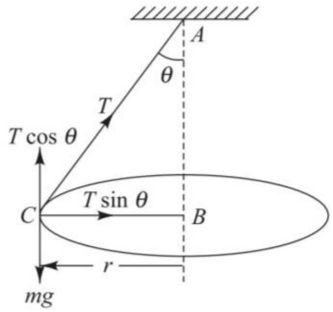


Fig. 4.35

14. From (1), we have $\cos\theta = \dfrac{mg}{T}$ (3)

Using (2) in (3), we get

$$\cos\theta = \frac{mg}{m\omega^2 L} = \frac{g}{\omega^2 L}$$, which is choice (a).

15. Linear velocity is $v = \omega r = \omega L \sin\theta$, which is choice (b).

3 SECTION Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

A simple pendulum of length l is displaced from its mean position O to position A so that the string makes an angle θ_1 with the vertical and then released. If air resistances is neglected, the speed of the bob when the string makes an angle θ_2 with the vertical is $v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$.

Statement-2

The total momentum of a system is conserved if no external force acts on it.

2. **Statement-1**

A uniform rod of mass m and length l is held at an angle θ with the vertical. The potential energy of the rod in this position is $\frac{1}{2}\,mg\,l\cos\theta$.

Statement-2

The entire mass of the rod can be assumed to be concentrated at its centre of mass.

3. **Statement-1**

A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement-2

The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

4. **Statement-1**

A man carrying a bucket of water and walking on a rough level road with a uniform velocity does no work while carrying the bucket.

Statement-2

The work done on a body by a force **F** in giving it a displacement **S** is defined as

$$W = \mathbf{F}\cdot\mathbf{S} = FS\cos\theta$$

where θ is the angle between vectors **F** and **S**.

5. **Statement-1**

A crane P lifts a car up to a certain height in 1 min. Another crane Q lifts the same car up to the same height in 2 min. Then crane P consumes two times more fuel than crane Q.

Statement-2
Crane P supplies two times more power than crane Q.

6. **Statement-1**
Two inclined frictionless tracks of different inclinations θ_1 and θ_2 meet at A from where two blocks P and Q of different masses m_1 and m_2 are allowed to slide down from rest, one on each track as shown in Fig. 4.36. Then blocks P and Q will reach the bottom with the same speed.

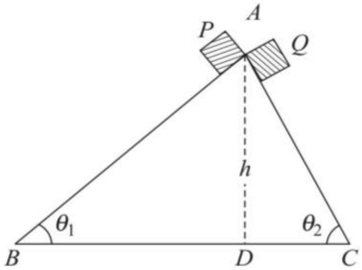


Fig. 4.36

Statement-2
Blocks P and Q have equal accelerations down their respective tracks.

7. **Statement-1**
In Q.6 above, block P will take a longer time to reach the bottom than block Q.
Statement-2
Block Q has a greater acceleration down the track than block P.

8. **Statement-1**
Comets move around the sun in highly elliptical orbits. The work done by the gravitational force of the sun on a comet over a complete orbit is zero.
Statement-2
The gravitational force is conservative.

9. **Statement-1**
The total energy of a system is always conserved irrespective of whether external forces act on the system.
Statement-2
If external forces act on a system, the total momentum and energy will increase.

10. **Statement-1**
The rate of change of the total linear momentum of a system consisting of many particles is proportional to the vector sum of all the internal forces due to inter-particle interactions.
Statement-2
The internal forces can change the kinetic energy of the system of particles but not the linear momentum of the system.

11. **Statement-1**
An elastic spring of force constant k is stretched by a small length x. The work done in extending the spring by a further length x is $2\ kx^2$.
Statement-2
The work done in extending an elastic spring by a length x is proportional to x^2.

12. **Statement-1**
Two identical balls B and C lie on a horizontal smooth straight groove so that they are touching. A third identical ball A moves at a speed v along the groove and collides with B (see Fig. 4.37). If the collisions are perfectly elastic, then after the collision, balls A and B will come to rest and ball C moves with velocity v to the right.

Fig. 4.37

Statement-2
In an elastic collision, linear momentum and kinetic energy are both conserved.

13. **Statement-1**
Two bodies A and B of masses m and $2\ m$ respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with a velocity u and collides elastically with A as shown in Fig. 4.38. At a certain instant t_0 after the collision, it is found that the velocities of A and B are the same $= u/3$.

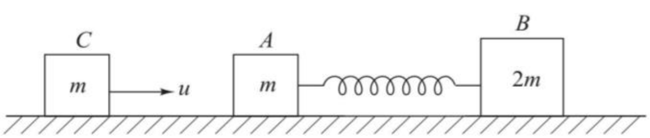


Fig. 4.38

Statement-2
In an elastic collision, the kinetic energy of the system is conserved.

14. **Statement-1**
In an inelastic collision between two bodies, the total energy does not change after the collision but the kinetic energy of the system decreases.
Statement-2
The loss of kinetic energy appears as heat in the system.

15. **Statement-1**
In a collision between two bodies, each body exerts an equal and opposite force on the other at each instant of time during the collision.

Statement-2
The total energy of the system is conserved.

16. **Statement-1**
The term 'collision' between two bodies does not necessarily mean that the two bodies actually strike against each other.
Statement-2
In physics, a collision is said to take place if the one body influences the motion of the other.

17. **Statement-1**
In an inelastic collision, the two colliding bodies stick to each other after the collision and move with a common velocity.
Statement-2
There is a loss of total kinetic energy in an inelastic collision.

18. **Statement-1**
In a collision between two bodies, the linear momentum of each body remains constant.
Statement-2
If no external force acts, the total linear momentum of a system is conserved.

19. **Statement-1**
In an elastic collision between two bodies, the energy of each body is conserved.
Statement-2
The total energy of an isolated system is conserved.

20. **Statement-1**
The total energy of a system is always conserved irrespective of whether external forces act on the system.
Statement-2
The total energy of an isolated system is always conserved.

21. **Statement-1**
A body P of mass M moving with speed u collides head-on and elastically with a body Q of m initially at rest. If $m \ll M$, body Q will have a maximum speed equal to $2u$ after the collision.
Statement-2
In an elastic collision, the momentum and kinetic energy are both conserved.

Solutions

1. The correct choice is (b). It is clear from Fig. 4.39 that $PQ = l\cos\theta_1$ and $PR = l\cos\theta_2$. Therefore, $h_1 = l - l\cos\theta_1 = l(1 - \cos\theta_1)$ and $h_2 = l(1 - \cos\theta_2)$.

Let m be the mass of the bob and v be its speed when it reaches position B. Then, from the principle of conservation of energy, K.E. at B = loss of P.E. as the bob moves from A to B.

Hence

$$\frac{1}{2}mv^2 = mgh_1 - mgh_2$$

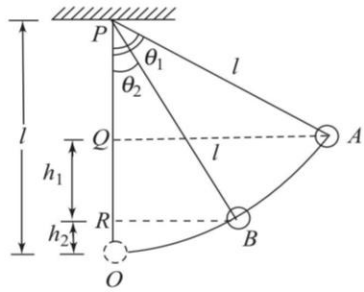


Fig. 4.36

$$= mg[l(1-\cos\theta_1) - l(1-\cos\theta_2)]$$
$$= mg\,l\,(\cos\theta_2 - \cos\theta_1)$$
$$\Rightarrow \quad v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$$

2. The correct choice is (a). Let C be the centre of mass of the rod AB so that $AC = l/2$. Let h be the height of C above the ground. In triangle ACD, we have $CD = AC\sin(90° - \theta)$ (see Fig. 4.40).
Or $h = \frac{l}{2}\cos\theta$. Since the entire mass of the rod can be assumed to be concentrated at the centre of mass, therefore, potential energy = work done to raise the rod from horizontal position on the ground to the position shown in the figure $= mgh = \frac{l}{2}mgl\cos\theta$.

3. Statement-1 is true. The decrease in mechanical energy is smaller when the block is made to go up on the inclined surface because some part of the kinetic energy is converted into gravitational potential energy. Statement-2 is false. The coefficient of friction does not depend on the angle of inclination of the plane. Hence the correct choice is (c).

4. The correct choice is (a). Since the velocity is uniform, the man exerts no net force on the bucket in the direction of motion. The only force he exerts on the bucket is against gravity (to overcome) the weight mg of the bucket) and this force is perpendicular to the displacement (i.e. $\theta = 90°$). Hence $W = FS\cos 90° = 0$.

5. The two cranes do the same amount of work = mgh. Hence they consume the same amount of fuel. Crane P does the same amount of work in half the time. Hence crane P supplies two times more power than crane Q. Thus the correct choice is (d).

6. The acceleration of blocks P and Q respectively are

$$a_1 = \frac{m_1 g \sin\theta_1}{m_1} = g \sin\theta_1$$

and $$a_2 = \frac{m_2 g \sin\theta_2}{m_2} = g \sin\theta_2$$

Since $\theta_2 > \theta_1$; $a_2 > a_1$. The potential energy of block P at $A = m_1gh$. When it reaches the bottom B, its kinetic energy is $\frac{1}{2} m_1 v_1^2$ where v_1 is its speed when it reaches B. Now P.E at A = K.E. at B. Hence

$$m_1gh = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gh}.$$

Similarly $$m_2gh = \frac{1}{2} m_2 v_2^2 \Rightarrow v_2 = \sqrt{2gh} = v_1.$$

Hence the correct choice is (c).

7. The correct choice is (a). If t_1 and t_2 are the times taken by P and Q to reach the bottom, then

$$v_1 = u_1 + a_1t_1 = a_1t_1 \qquad (\because u_1 = 0)$$

and $$v_2 = u_2 + a_2t_2 = a_2t_2 \qquad (\because u_2 = 0)$$

Now $v_1 = v_2$. Hence $a_1t_1 = a_2t_2$.

Thus $$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

Since $a_2 > a_1$; $t_1 > t_2$.

8. The correct choice is (a). For a conservation force, the work done in moving a body from one point to another does not depend on the nature of the path and the work done over a closed path is zero, irrespective of the nature of the path.

9. Statement-1 is false; the total energy of an isolated system is conserved. Statement-2 is true. Hence the correct choice is (d).

10. Statement-1 is false and Statement-2 is true. The rate of change of momentum is proportional to the net external force acting on the system. Hence the correct choice is (d).

11. The correct choice is (d). Potential energy stored in the spring when it is extended by x is $U_1 = \frac{1}{2}kx^2$

Potential energy stored in the spring when it is further extended by x is

$$U_2 = \frac{1}{2}k(x + x)^2 = 2kx^2$$

$\therefore$ Work done = gain in potential energy = $U_2 - U_1$ = $2\,kx^2 - \frac{1}{2}kx^2 = \frac{3}{2}\,kx^2$

12. The correct choice is (a). Linear momentum will be conserved if A comes to rest and B and C move to the right with a velocity $v/2$ each or A, B and C all move to the right with velocity $v/3$ each. It is easy to see that in these two cases, the kinetic energy is not conserved. Hence the only result of the collision is the one given in Statement-1.

13. The correct choice is (b). Let C collide with A at $t = 0$. Since the collision in elastic and A and C have equal masses, C will come to rest and A will move to the right with velocity u and at this instant the spring is uncompressed and B is at rest. Hence at $t = 0$, the momentum of the system = mu. When A moves to the right, it compresses the spring and as a result B beings to move to the right. Let v be the common velocity of A and B at time t_0. From the principle of conservation of linear momentum, we have

Momentum of C before collision = momentum of A after collision + momentum of B after collision or $mu = mv + (2m)\,v \Rightarrow v = \frac{u}{3}$.

14. The correct choice is (a). The total energy (which includes all forms of energy) is conserved in any process.

15. The correct choice is (b). Statement-1 follows from Newton's third law of motion.

16. The correct choice is (a).

17. The correct choice is (d). The two colliding body need not get stuck after an inelastic collision.

18. The correct choice is (d). Since the velocities of the two bodies change due to collision, the linear momentum of each body will change but the total linear momentum of the system of two bodies is conserved.

19. The correct choice is (d). Due to change in velocity, the energy of each body changes on collision but the total energy of the system of two bodies is conserved.

20. The correct choice is (d). If an external force acts on a system, it is accelerated which will increase the total energy.

21. The correct choice is (a). If v and V are the velocities of Q and P after the collision, then from conservation of momentum and kinetic energy, we have

$$Mu = mv + MV \;\Rightarrow\; M(u - V) = mv \quad (1)$$

$$\frac{1}{2}Mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

$$\Rightarrow \quad M(u - V)(u + V) = mv^2 \quad (2)$$

From Equations (1), (2), we get

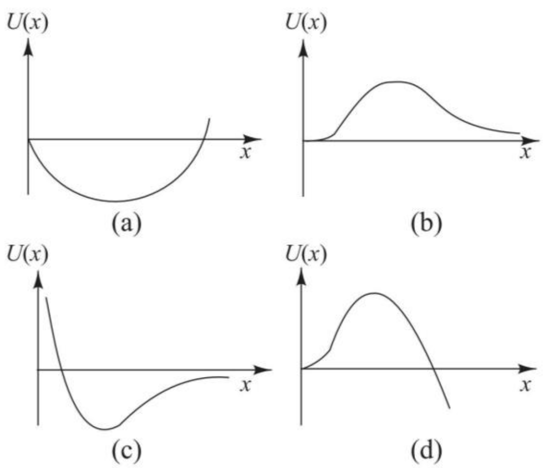
$$v = \frac{2Mu}{(M + m)} = \frac{2u}{\left(1 + \frac{m}{M}\right)}$$

If $M \gg m$, then v is maximum equal to $2u$ (since $\frac{m}{M} \rightarrow$ zero).

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. A spring of force consttant 800 Nm^{-1} has an extension of 5 cm, The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J
(c) 32 J (d) 24 J **[2002]**

2. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle



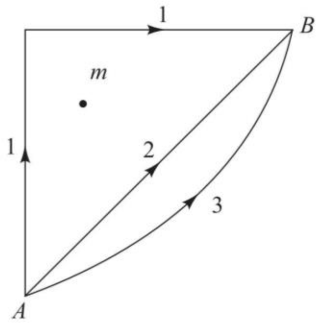
[2002]

3. Consider the following statements
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.
(a) A does not imply B and B does not imply A.
(b) A implies B but B does not imply B.
(c) A does not imply B but B implies A.
(d) A implies B and B implies A. **[2003]**

4. A spring of spring constant 5×10^3 Nm^{-1} is stretched initially by 5 cm from the unstretched position. Then the work required to stretch by another 5 cm is
(a) 12.50 J (b) 18.75 J
(c) 25.00 J (d) 6.25 J **[2003]**

5. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$ **[2003]**

6. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in the figure) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.
(a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$
(a) $W_1 < W_3 < W_2$ (b) $W_1 < W_2 < W_3$
[2003]



7. A particle move in a straight line with retardation proportional to its displacement. It loss of kinetic energy for any displacement x is proportional to
(a) x^2 (b) e^x
(c) x (d) $\log_e x$ **[2004]**

8. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? Take $g = 10$ ms^{-2}.
(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J **[2004]**

9. A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ newton is applied to a particle which displaces it from the origin to a point $\mathbf{r} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}})$ metre. The work done on the particle is
(a) –7J (b) +7J
(c) +10 J (d) +13 J **[2004]**

10. A body of mass m accelerates uniformly from rest to velocity v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$

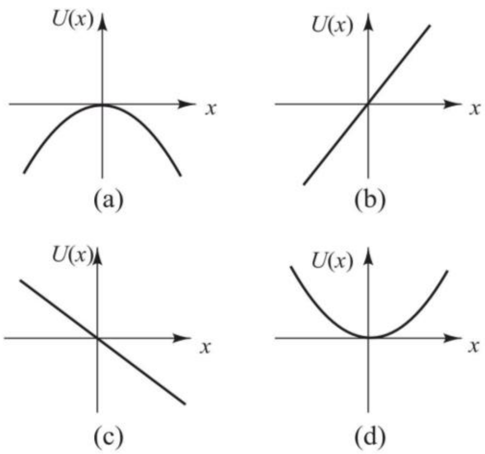
(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$ **[2004]**

11. A wire fixed at the upper end stretches by length l by applying force F. The work done is stretching is

(a) $\frac{1}{2}Fl$ (b) Fl

(c) $2\,Fl$ (d) $2\sqrt{2}\,Fl$ **[2004]**

12. A particle at the origin is under the influence of a force $F = kx$, where k is a positive constant. If the potential energy U is zero at $x = 0$, the variation of potential energy with the coordinate x is represented by



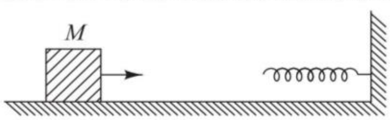
[2004]

13. A ball of mass 20 kg is stationary at the top of the hill of height 100 m. It rolls down the smooth surface of the hill to the ground and then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is (take $g = 10\ ms^{-2}$)

(a) $40\ ms^{-1}$ (b) $20\ ms^{-1}$

(c) $10\ ms^{-1}$ (d) $10\sqrt{30}\ ms^{-1}$ **[2005]**

14. A block of mass M moving on a frictionless horizontal surface collides with spring of spring constant k and compresses it by length L. The maximum momentum of the block after the collision is



(a) $L\sqrt{Mk}$ (b) $\frac{KL^2}{2M}$

(c) zero (d) $\frac{ML^2}{k}$ **[2005]**

15. A mass m moves with velocity v and collides with another identical mass initially at rest. After the collision the first mass moves with a velocity $v/\sqrt{3}$ in a direction perpendicular to the initial direction of motion. If the collision is inelastic, the speed of the second mass after collision is

(a) v (b) $\sqrt{3}v$

(c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{\sqrt{3}}$ **[2005]**

16. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15\ Nm^{-1}$. What will be its initial acceleration if it is released from a point 20 cm away from the origin?

(a) $10\ ms^{-2}$ (b) $5\ ms^{-2}$

(c) $15\ ms^{-2}$ (d) $3\ ms^{-2}$ [....]

17. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is

(a) $\frac{Mg}{\sqrt{2}}$ (b) $Mg(\sqrt{2}-1)$

(c) $Mg(\sqrt{2}+1)$ (d) $Mg\sqrt{2}$ **[2006]**

18. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is (take $g = 10\ ms^{-2}$)

(a) 1.25 J (b) 0.5 J

(c) –0.5 J (d) –1.25 J **[2006]**

19. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is $4\ ms^{-1}$. The kinetic energy of the other mass is

(a) 192 J (b) 96 J

(c) 144 J (d) 288 J **[2006]**

20. The potential energy of 1 kg particle free to move along the x-axis is given by

$$v(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\text{joule}$$

The total mechanical energy of the particle is 2 J. Then the maximum speed (in m/s) is

(a) $\frac{1}{\sqrt{2}}$ (b) 2

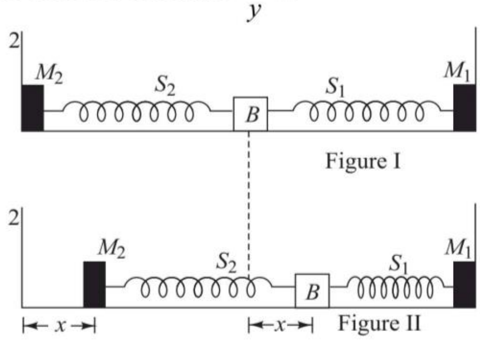
(c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{2}$ **[2006]**

21. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by
(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm **[2007]**

22. An athlete in the Olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range
(a) 20,000 J – 50,000 J (b) 2,000 J – 5,000 J
(c) 200 J – 500 J (d) 2×10^5 J – 3×10^5 J **[2008]**

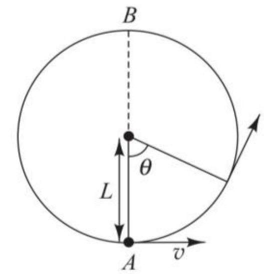
23. A block of mass 0.5 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is
(a) 0.34 J (b) 0.16 J
(c) 1.00 J (d) 0.67 J **[2008]**

24. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively (see Figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (Figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacement x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{x}{y}$ is



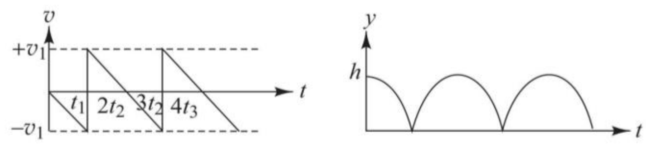
(a) 4 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$ **[2008]**

25. A bob of mass m is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A satisfies

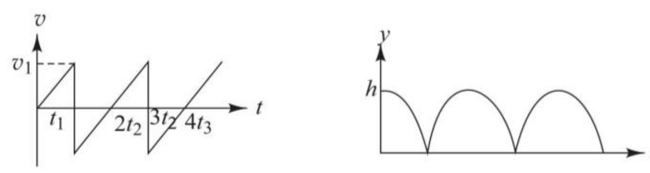


(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$
(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$ **[2008]**

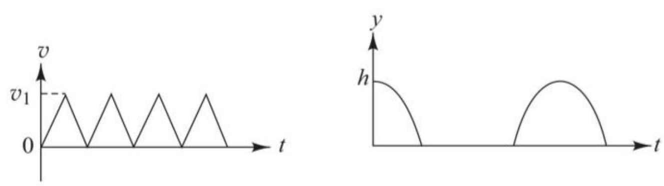
26. Consider a rubber ball falling freely from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be:



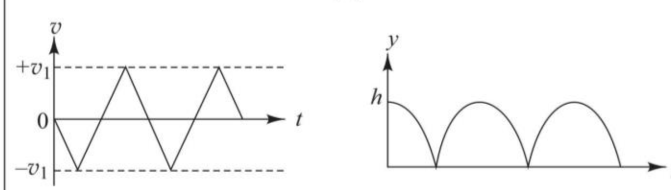
(a)



(b)



(c)



(d)

[2009]

27. **Statement-1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2: Principle of conservation of momentum holds true for al kinds of collisions.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

(b) Statement-1 is ture, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is false, Statement-2 is true.

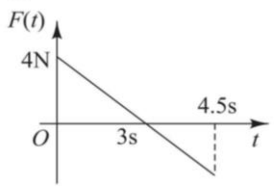
(d) Statement-1 is true, Statement-2 is false. **[2010]**

28. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecules is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$

(c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$ **[2010]**

29. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J

(c) 5.06 J (d) 14.06 J **[2010]**

30. This question has two **statements** 1 and 2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two spring S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on S_1 than that on S_2.

Statement 1 : If stretched by the same amount, work done on S_1 will be more than that on S_2.

Statement 2 : k_1 is less than k_2.

(a) Statement 1 is false, statement 2 is true.

(b) Statement 1 is true, statement 2 is false.

(c) Statement 1 is true, statement 2 is true and statement 2 is the correct explanation for statement 1.

(d) Statement 1 is true, Statement 2 is true but statement 2 is not the correct explanation of statement 1. **[2012]**

31. The question has Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement-II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(b) Statement-I is true, Statement-II is false.

(c) Statement-I is false, Statement-II is true.

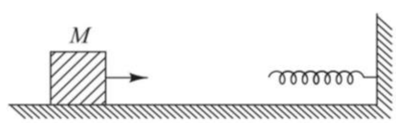
(d) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I. **[2003]**

32. The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{\mathbf{i}} + \frac{y}{(x^2+y^2)^{3/2}}\hat{\boldsymbol{j}}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x–y plane is:

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$

(c) $\frac{K\pi}{2a}$ (d) 0 **[2013]**

33. A block of mass 4 kg moving on a frictionless horizontal surface collides with spring of force constant $4\ \text{Nm}^{-1}$ and compresses it by length 50 cm. The maximum momentum (in kg ms^{-1}) of the block after the collision is



(a) 2 (b) 4

(c) 1 (d) 0.5 **[2013]**

Answers

1. (b)	2. (d)	3. (c)	4. (b)
5. (b)	6. (b)	7. (a)	8. (b)

9. (b)	10. (b)	11. (a)	12. (a)
13. (a)	14. (a)	15. (c)	16. (a)
17. (b)	18. (d)	19. (d)	20. (c)
21. (a)	22. (b)	23. (d)	24. (c)
25. (d)	26. (a)	27. (a)	28. (d)
29. (c)	30. (a)	31. (c)	32. (d)
33. (a)			

Solutions

1. Here $x_1 = 5$ cm = 0.05 m and $x_2 = 15$ cm = 0.15 m

$$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}\times 800\times[0.15)^2 - (0.05)^2]$$
$$= 8\ \mathbf{J}$$

2. Refer to the solution of Q.65 on page 4.32 of this chapter.

3. The kinetic energy of a system of particles is zero only if the velocity of every particles is zero. Linear momentum of a system of particles is zero if the vector sum of linear momenta of individual particles is zero which is possible even if velocities are non-zero. Hence statement B implies statement A but does not imply B.

4. Here $x_1 = 5$ cm = 0.05 m and $x_2 = 10$ cm = 0.1 m

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$
$$= \frac{1}{2}\times 5\times 10^3\ [(0.1)^2 - (0.5)^2] = 18.75\ \text{J}$$

5. $P = Fv = mav = m\frac{dv}{dt}\ v = m\frac{dv}{dx}.\frac{dx}{dt}\ v = mv^2\frac{dv}{dx}$

$$\therefore \quad v^2 dv = \frac{P}{m}dx$$

Integrating (assuming that the body starts from rest), we have

$$\int_0^v v^2 dv = \frac{P}{m}\int_0^x dx \qquad (\because P = \text{constant})$$

$$\Rightarrow \quad \frac{v^3}{3} = \frac{Px}{m}$$

$$\Rightarrow v = \left(\frac{3P}{m}\right)^{1/3} = x^{1/3} = k\ x^{1/3} \text{ where } k = \left(\frac{3P}{m}\right)^{1/3} =$$
constant

$$\Rightarrow \quad \frac{dx}{dt} = k\,x^{1/3}$$

$$\Rightarrow \quad x^{-1/3}\,dx = k\,dt$$

Integrating $\int_0^x x^{-1/3}dx = k\int_0^t dt$

$$\Rightarrow \quad \frac{x^{2/3}}{2/3} = kt \Rightarrow x = \left(\frac{2k}{3}\right)^{3/2} \times t^{3/2}$$

Hence $\quad x \propto t^{3/2}$

6. Gravitational force is conservative. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. Hence the correct choice is (b).

7. $a = -kx$ where k is a positive constant

or $\quad \frac{dv}{dt} = -kx \Rightarrow \frac{dv}{dx}.\frac{dx}{dt} = -kx$

or $\quad vdv = -kxdx \qquad \left(\because v = \frac{dx}{dt}\right)$

Integrating,

$$\int_u^v vdv = -k\int_0^x xdx$$

$$\frac{1}{2}(v^2 - u^2) = \frac{kx^2}{2}$$

where u = initial velocity and v = final velocity. If m is the mass of the particle, then

Initial K.E. is $K_i = \frac{1}{2}mu^2$

Final K.E. is $K_f = \frac{1}{2}mv^2$

Loss in K.E. = $k_i - k_f = m\,(u^2 - v^2)$

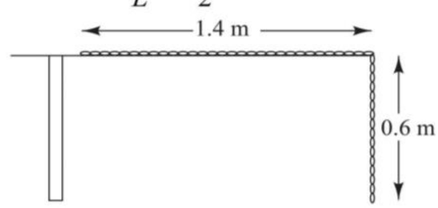
Using (i) in (ii)

Loss in K.E. = $\frac{km}{2}x^2$

Thus loss in K.E. $\propto x^2$

8. Mass per unit length of the chain is

$$\frac{M}{L} = \frac{4}{2} = 2\ \text{kgm}^{-1}$$



Mass of hanging portion is

$$m = 2\times 0.6 = 1.2\ \text{kg}$$

This mass can be assumed to be concentrated at the centre of the hanging portion of the chain which is at a distance of $x = 0.3$ from the edge of the table.

$\therefore$ Work done = $mgx = 1.2\times 10\times 0.3 = 3.6$ J

9. $\mathbf{W} = \mathbf{F}.\mathbf{r}$

$= (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}).(2\hat{\mathbf{i}} - \hat{\mathbf{j}})$

$= 10 - 3 = 7$ J

$(\because \hat{\mathbf{i}}.\hat{\mathbf{i}} = \hat{\mathbf{j}}.\hat{\mathbf{j}} = 1 \text{ and } \hat{\mathbf{i}}.\hat{\mathbf{j}} = \hat{\mathbf{k}}.\hat{\mathbf{j}} = 0)$

10. If a is the uniform acceleration. then

$$v_1 = 0 + at_1$$

$$\Rightarrow \quad a = \frac{v_1}{t_1}$$

At an instant of time t, the velocity v of the body is

$$v = 0 + at = \frac{v_1 t}{t_1}$$

Instantaneous power $P = Fv = mav$

Using (i) and (ii), we have

$$P = m \times \frac{v_1}{t_1} \times \frac{v_1 t}{t_1} = \frac{m v_1^2 t}{t_1^2}$$

> **Note** Choice (a) has dimensions of momentum and choice (d) has dimensions of energy. Choice (b) is the only one which has dimensions of power (ML^2T^{-3})

11. $W = \frac{1}{2}kx^2 = \frac{1}{2}kl^2 = \frac{1}{2}(kl) \times l$

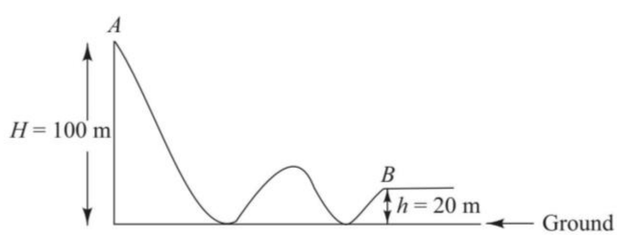
$$= \frac{1}{2}F \times l \qquad (\because F = kx = kl)$$

12. Potential energy function is

$$U(x) = -\int_0^x F\, dx = -k\int_0^x x\, dx = \frac{1}{2}kx^2$$

The value of $U(x)$ is always negative for both positive and negative values of x. Thus the variation of potential energy with x is an inverted parabola as shown in choice (a).

13. Refer to the following figure.



Velocity at $A = 0$. Let v be the velocity at B. From the law of conservation of energy, loss of P.E. = gain in K.E. i.e.

$$mg(H - h) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 10(100 - 20) = \frac{1}{2}v^2 \quad \Rightarrow \quad v = 40 \text{ ms}^{-1}$$

14. The momentum is maximum when kinetic energy is maximum. This happens when the entire potential energy of the compressed spring is transferred to the block, i.e. when

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

here v is the maximum velocity imparted to the block. Thus

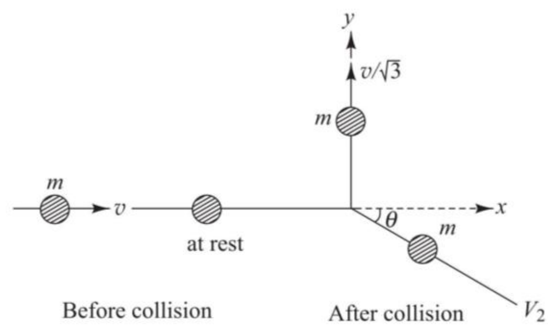
$$Mv^2 = kL^2$$

$$\Rightarrow \quad M^2v^2 = MkL^2$$

$$\Rightarrow \quad p^2 = MkL^2$$

$$\Rightarrow \quad p = L\sqrt{Mk}$$

15. Refer to the following figure.



Conservation of x and y components of momentum gives

$$mv = mv_2\cos\theta \Rightarrow v = v_2\cos\theta \qquad \text{(i)}$$

and $m\frac{v}{\sqrt{3}} = -mv_2\sin\theta \;\Rightarrow\; \frac{v}{\sqrt{3}} = -v_2\sin\theta \qquad$ (ii)

Squarring (i) and (ii) and adding we get

$$v_2 + \frac{v^2}{3} = v_2^2$$

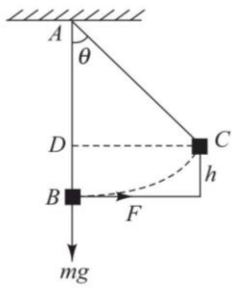
$$\Rightarrow \quad v_2 = \frac{2v}{\sqrt{3}}$$

> **Note** This question is wrong as it violates the law of conservation of energy. If we calculate the total initial kinetic energy and final kinetic energy, we get $k_i = \frac{1}{2}mv^2$ and $k_f = \frac{1}{2}mv^2 \times \frac{5}{3}$, i.e. $k_f > k_i$ which is not possible.

16. $F = -kx = -15 \times 0.2 = -3$ N. Therefore

$$|a| = \frac{|F|}{m} = \frac{3}{0.3} = 10 \text{ ms}^{-2}$$

17. Let l be the length of the string. The mass is moved from B to C by a horizontal force F until $\theta = 45°$. The work done against the force of gravity is



$$W_g = Mgh = MgDB$$
$$= Mg\,(AB - AD)$$
$$= Mg\,(l - l\cos\theta)$$
$$= Mg\,l\,(1 - \cos\theta)$$

This work is done against the force of gravity. Hence, by sign convention, it is negative. Thus

$$W_g = -Mg\,l\,(1 - \cos\theta)$$

The work done by the applied horizontal force **F** is

$$F_a = F \times \text{horizontal distance moved}$$
$$= F \times CD = F\,l\sin\theta$$

∴ Total work done is

$$W = W_g + W_a$$
$$= -Mg\,l\,(1 - \cos\theta) + F\,l\sin\theta$$

Since the mass is at rest at positions B and C, the change in kinetic energy is zero. From work energy principle, work done = change in kinetic energy, i.e.

$$-Mg\,l\,(1 - \cos\theta) + Fl\,\text{Sin}\,\theta = 0$$

or $$F = \frac{Mg(1-\cos\theta)}{\sin\theta} = \frac{Mg(1-\cos 45°)}{\sin 45°}$$

$$= \frac{Mg\left(1 - \frac{1}{\sqrt{2}}\right)}{1/\sqrt{2}} = Mg(\sqrt{2} - 1)$$

18. The vertical height to which the particle rises is

$$h = \frac{u^2}{2g} = \frac{(5)^2}{2 \times 10} = 1.25 \text{ m}$$

Work done $W_g = mgh = (100 \times 10^{-3}) \times 10 \times 1.25 = 1.25$ J

Since the work W_g is done against the force of gravity, it is negative. Hence the correct choice is (d), i.e. $W_g = -1.25$ J.

19. Let v be the speed of the 4 kg mass just after the explosion. Since the bomb was at rest, its momentum is zero. From the conservation of momentum, we have

$$4 \times v - 12 \times 4 = 0 \quad \Rightarrow \quad v = 12 \text{ ms}^{-1}$$

$$\therefore \quad \text{K.E. of 4 kg mass} = \frac{1}{2} \times 4 \times (12)^2 = 288 \text{ J}$$

20. $$V = \frac{x^4}{4} - \frac{x^2}{2}$$

$$\therefore \quad \frac{dV}{dx} = x^3 - x$$

V is maximum or minimum if $\frac{dV}{dx} = 0$, i.e. if $x^3 - x = 0$ or $x(x^2 - 1) = 0$ which gives $x = 0$, 1 and –1. Now

$$\frac{d^2V}{dx^2} = 3x^2 - 1$$

V is maximum if $\frac{d^2V}{dx^2}$ is negative and V is minimum if $\frac{d^2V}{dx^2}$ is positive. For $x = 0$, $\frac{d^2V}{dx^2} = -1$.

Hence for $x = 0$, V is maximum. For $x = \pm 1$, $\frac{d^2V}{dx^2} = 2$.

Hence for $x = \pm 1$, V is minimum, Now

$$V = \frac{x^4}{4} - \frac{x^2}{2}$$

Therefore, the minimum potential energy is (put $x = \pm 1$)

$$V_{min} = \frac{(\pm 1)^4}{4} - \frac{(\pm 1)^2}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \text{ joule}$$

Maximum kinetic energy is $K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} v_{max}^2$

($\because\ m = 1$ kg).

Given, total energy $E = 2$ joule. From $E = V + K$, we have

$$2 = \frac{1}{4} + \frac{1}{2} v_{max}^2$$

which gives $$v_{max} = \frac{3}{\sqrt{2}} \text{ ms}^{-1}.$$

21. Let x be the compression of the spring. The kinetic energy of the block $\left(= \frac{1}{2} mv^2\right)$ is used up in

(i) imparting potential energy $\left(= \frac{1}{2} k\,x^2\right)$ to the spring and

(ii) doing work ($= fx$) against friction. Thus

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2 + fx$$

$$\frac{1}{2} \times 2 \times (4)^2 = \frac{1}{2} \times 10{,}000 \times x^2 + 15x$$

$$\Rightarrow \quad 5000x^2 + 15x - 16 = 0$$

The two roots are $x = 0.055$ m and –0.058 m. Since The negative value of x is not permissible,
$x = 0.055$ m = 5.5 cm

22. Average speed of athlete $v = \dfrac{100\,\text{m}}{10\,\text{s}} = 10\ \text{ms}^{-1}$. Now kinetic energy $K = \dfrac{1}{2}mv^2 \Rightarrow m = \dfrac{2K}{v^2}$, where m is the mass of the athlete. Thus

$$m = \frac{2K}{(10)^2} = \frac{K}{50}$$

Choice (a), (c) and (d) give absurd values of m. Choice (b) gives m in the range 40 kg to 100 kg. So the correct choice is (b).

23. Let v be the velocity of the composite body. From conservation of momentum, we have

$$0.5 \times 2 = (0.5 + 1.0)v \Rightarrow v = \frac{2}{3}\ \text{ms}^{-1}$$

Initial K.E. is $K_i = \dfrac{1}{2} \times 0.5 \times (2)^2 = 1.0$ J

Final K.E. is $K_f = \dfrac{1}{2}(1.0 + 0.5) \times \left(\dfrac{2}{3}\right)^2 \text{J} = \dfrac{1}{3}\text{J}$

$\therefore$ Loss of energy $= 1.0 - \dfrac{1}{3} = 0.67$ J

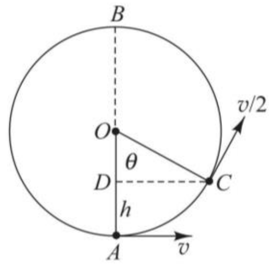
24. Potential energy stored in spring S_1 when the block B is moved through a distance x is $U_1 = \dfrac{1}{2}k_1x^2 = \dfrac{1}{2}kx^2$. When the block is released, it moves to the left, compressing the spring S_2 through a distance y. The potential energy stored in spring S_2 when its compression is y is $U_2 = \dfrac{1}{2}k_2y^2 = \dfrac{1}{2}(4\,k)y^2 = 2ky^2$. Since y is maximum compression of spring S_2, from conservation of energy, we have $U_1 = U_2$, i.e.

$$\frac{1}{2}kx^2 = 2\,k\,y^2$$

which gives $\dfrac{y}{x} = \dfrac{1}{2}$.

25. Refer to the figure. Here $OA = OB = OC = L$ and $OD = OC \cos\theta = L\cos\theta$. Therefore

$h = OA - OD = L - L\cos\theta$



or $h = L(1 - \cos\theta)$. From conservation of energy, total energy at A = total energy at C, i.e.

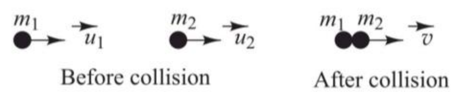
$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgL(1-\cos\theta)$$

$$v^2 = \frac{8gL}{3}(1-\cos\theta) \quad (1)$$

The minimum velocity the bob must have at A so as to reach B is $v = \sqrt{5gL}$. Putting this in Eq. (1), we get $\cos\theta = -\dfrac{7}{8}$. Therefore θ lies between $\dfrac{3\pi}{4}$ and π.

26. While falling the velocity of the ball is negative as it is directed downwards. After the collision, the velocity is reversed and is positive as it is now directed upwards. Hence the correct choice is (a).

27. The principle of conservation of mementum holds for bothelasticaswellasinelasticcollisions.Inacompletely inelastic collision, the two particles stick together.



$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)\vec{v}$$

$$\Rightarrow \quad \vec{v} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2} = \frac{\vec{P}_1 + \vec{P}_2}{m_1 + m_2}$$

$\because$ K.E. after collision $= \dfrac{1}{2}(m_1 + m_2)v^2$, which can be zero if $\vec{v} = 0$ which is possible if

$$\vec{P}_1 = -\vec{P}_2$$

This is not possible for two particles moving in the same direction. Hence the particles do not lose all their kinetic energy. So the correct choice is (a).

28. $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$ (1)

$\therefore \quad U(x = \infty) = 0$

Force $\quad F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{a}{x^{12}} - \dfrac{b}{x^6}\right)$

$$\Rightarrow \quad F = -\left(-\frac{12a}{x^{13}} + \frac{6b}{x^7}\right) \quad (2)$$

At equilibrium $F = 0$. Putting $F = 0$ in Eq. (2) we get

$$x^6 = \frac{2a}{b}$$

Putting $x^6 = \dfrac{2a}{b}$ in Eq. (1), we get

$$U \text{ at equilibrium} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\frac{2a}{b}} = -\frac{b^2}{4a}$$

$\therefore \quad D = [U \text{ at } x = \infty - U \text{ at equilibrium}]$

$$= 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$$, which is choice (d).

29. Slope of graph $= -\frac{4\text{ N}}{3\text{ s}} = -\frac{4}{3}\text{ Ns}^{-1}$. Therefore,

$F = -\frac{4}{3}t + 4$. Now change in momentum $= \int F\,dt$

$$\Rightarrow \quad m\,v - 0 = \int_0^{4.5s}\left(-\frac{4}{3}t + 4\right)dt$$

or $$2v = \left|-\frac{2t^2}{3} + 4t\right|_0^{4.5s} = 4.5$$

Which gives $v = 2.25\text{ ms}^{-1}$. Therefore,

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times(2.25)^2$$

$\simeq 5.06$ J, which is choice (c).

30. $F = k_1x_1 = k_2x_2$

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2k_1}(k_1x_1)^2 = \frac{F^2}{2k_1}$$

Similarly $$W_2 = \frac{F^2}{2k_2}$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{k_2}{k_1}$$

It is given that $W_1 > W_2$. Hence $k_2 > k_1$. So Statement 2 is true.

If the springs are stretched by the same amount x, then $W_1 = \frac{1}{2}k_1x^2$ and $W_2 = \frac{1}{2}k_2x^2$. Hence

$$\frac{W_1}{W_2} = \frac{k_1}{k_2}$$

Since $k_1 < k_2$; $W_1 < W_2$, so statement 1 is false. The correct choice is (a).

31. The maximum loss of kinetic energy occurs when the collision is perfectly inelastic, i.e. when the particles get stuck together after the collision. So Statement-II is correct.

From conservation of momentum,

$$mv + 0 = (m+M)\,V$$

$$\Rightarrow \quad V = \frac{mv}{(m+M)}$$

Here V is the velocity of the composite body.

Total K.E. before collision is

$$k_i = \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

Total K.E. after collision is

$$K_f = \frac{1}{2}(m+M)V^2$$

$$= \frac{1}{2}(m+M)\times\left(\frac{mv}{m+M}\right)^2$$

$$= \frac{1}{2}\frac{m^2v^2}{(m+M)}$$

$\therefore$ Loss of $\quad \text{K.E.} = K_i - K_f$

$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{(m+M)}$$

$$= \frac{1}{2}mv^2\left[1 - \frac{m}{(m+M)}\right]$$

$$= \frac{M}{(m+M)}\times\frac{1}{2}mv^2$$

$\therefore \quad f = \frac{M}{(m+M)}$. So Statement-I is false.

32. $W = \int \vec{F}.\vec{dr}$

$$= \int F_x dx + \int F_y\,dy$$

$$= K\int_{(a,0)}^{(0,a)}\frac{xdx}{(x^2+y^2)^{3/2}} + K\int_{(a,0)}^{(0,a)}\frac{ydy}{(x^2+y^2)^{3/2}}$$

$$= -\frac{K}{2}\left|(x^2+y^2)^{-1/2}\right|_{(a,0)}^{(0,a)} - \frac{K}{2}\left|(x^2+y^2)^{-1/2}\right|_{(a,0)}^{(0,a)}$$

$= -0 - 0$

$= 0$

33. The momentum is maximum when kinetic energy is maximum. This happens when the entire potential energy of the compressed spring is transferred to the block, i.e. when

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

here v is the maximum velocity imparted to the block and L is the length of the spring. Thus

$$Mv^2 = kL^2$$

$$\Rightarrow \quad M^2v^2 = MkL^2$$

$$\Rightarrow \quad p^2 = MkL^2$$

$$\Rightarrow \quad p = L\sqrt{Mk}$$

$$= 0.5 + \sqrt{4\times 4}$$

$$= 2\text{ kg ms}^{-1}$$