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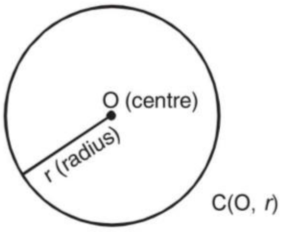
CIRCLES

Syllabus Reference

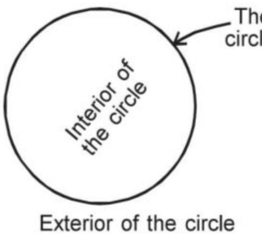
SOME DEFINITIONS

Circle: A circle is a collection of all those points in a plane, which are at a constant distance from a fixed point in the plane.



The fixed point (say O) is called the centre of the circle and the constant distance is called the radius of the circle. A circle with centre 'O' and radius 'r' is usually denoted by C(O, r).

Circular Region: The circle and its interior, is called the circular region.



POSITION OF A POINT (P) WITH RESPECT TO A CIRCLE

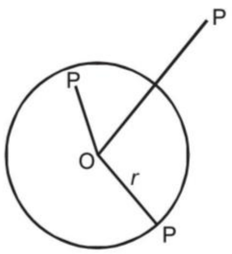
A point P is said to lie inside, outside or on the circle C(O, r) according as:

(*i*) $OP < r$

(*ii*) $OP > r$ or

(*iii*) $OP = r$

All the points lying inside a circle are called its interior points and all those points which lie outside the circle are called its exterior points.



Circumference of a circle: The distance covered around the circle or the perimeter of a circle is called its circumference.

Chord of a circle: A line segment joining any two points on a circle is called the chord of the circle.

Diameter of a circle: A chord passing through the centre of the circle is called a diameter of the circle.

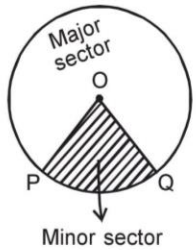
Semicircle: A diameter divides a circle into two equal parts and each equal part is called a semicircle.

Arc of a circle: A continuous piece of a circle is called an arc of the circle.

Segment of a circle: Any chord divides the circular region into two parts, each is called a segment of the circle.

Concentric circles: Circles having the same centres are called concentric circles.

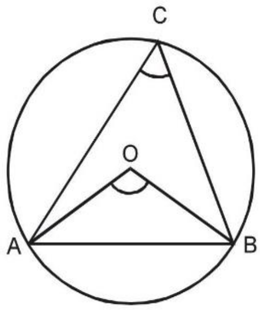
Sector of a circle: The part of the plane region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.



ANGLE SUBTENDED BY A CHORD AT A POINT

Let AB be the chord of a circle C(O, r) with centre 'O' and radius 'r'.

Take any point C on the circle. Join CA, CB, OA and OB. Now, $\angle AOB$ is the angle subtended by the chord AB at the centre of the circle and $\angle ACB$ is the angle subtended by the chord AB at the point C on the circle.



THEOREM 1. ***Equal chords of a circle subtend equal angles at the centre.***

THEOREM 2. ***(Converse of theorem 1)***

If the angles subtended by the chords of a circle at the centre are equal, then these chords are equal.

CONGRUENT CIRCLES

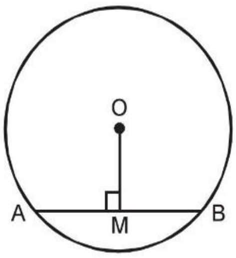
Two circles are said to be congruent if and only if either of them can be superposed on the other so as to cover it exactly.

Or

Two circles are said to be congruent to each other if and only if they have equal radii.

PERPENDICULAR DRAWN FROM THE CENTRE TO THE CHORD

Consider a circle C(O, *r*) with centre O and radius '*r*'. Draw OM ⊥ AB, where AB is the chord of the circle.



THEOREM 3. ***The perpendicular from the centre to a chord bisects the chord.***

THEOREM 4. ***(Converse of theorem 3)***

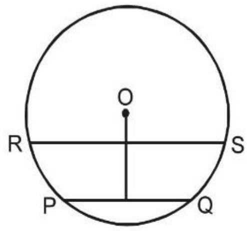
The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

THEOREM 5. ***There is one and only one circle passing through three non-collinear points.***

EQUAL CHORDS AND THEIR DISTANCES FROM THE CENTRE

A circle can have infinitely many chords. We shall see that longer chord is nearer the centre than the smaller chord.

Distance of the centre from the diameter [*i.e.*, the longest chord of the circle] is zero, because the centre lies on the diameter.



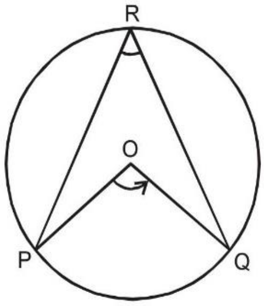
THEOREM 6. ***Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).***

THEOREM 7. ***(Converse of above theorem)***

Chords equidistant from the centre of a circle are equal in length.

ANGLE SUBTENDED BY AN ARC OF A CIRCLE

Consider a circle C(O, *r*) with centre O and radius *r*. Take an arc PQ. ∠POQ is the angle subtended at the centre of the circle by an arc PQ and ∠PRQ is the angle subtended at the circumference other than arc PQ or on the remaining part of the circle.



THEOREM 8. ***The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.***

COROLLARY: ***Angle in a semi-circle is a right angle.***

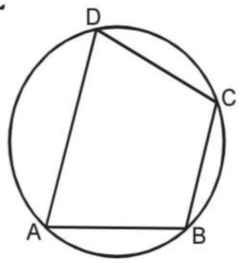
THEOREM 9. ***Angles in the same segment of a circle are equal.***

THEOREM 10. ***(Converse of theorem 9)***

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e., they are concyclic).

CYCLIC QUADRILATERAL

A quadrilateral ABCD is called a cyclic quadrilateral, if all the four vertices A, B, C and D lie on a circle. The four points A, B, C and D are concyclic.



THEOREM 11. ***The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.***

Or

The opposite angles of a cyclic quadrilateral are supplementary.

THEOREM 12. ***(Converse of theorem 11)***
If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

THEOREM 13. ***If one side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.***

CONGRUENT CIRCLES

Two circles are said to be congruent to each other if and only if they have equal radii.

Congruent arcs of a circle, subtend equal angles at the centre.

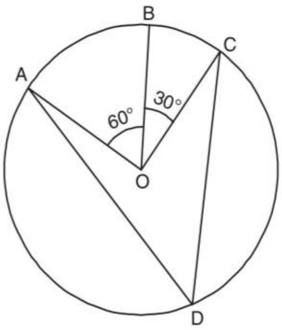
NCERT & BOARD QUESTIONS CORNER
(Remembering & Understanding Based Questions)

Very Short Answer Type Questions

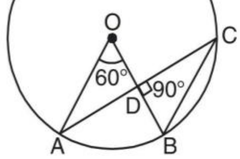
1. ***A, B and C are three points on a circle with centre O, such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is another point on the circle other than arc ABC. Find $\angle ADC$.***

Sol. Since angle subtended at the centre by an arc is double the angle subtended at the remaining part of the circle.

$$\therefore \angle ADC = \frac{1}{2}\angle AOC$$
$$= \frac{1}{2} \times 90°$$
$$= 45°$$



2. ***In the given figure, O is the centre of the circle, $\angle AOB = 60°$ and $\angle CDB = 90°$. Find $\angle OBC$.***



Sol. Since angle subtended at the centre by an arc is double the angle subtended at the remaining part of the circle.

$$\therefore \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60° = 30°$$

Now, in ΔCBD, by using angle sum property, we have

$\angle CBD + \angle BDC + \angle DCB = 180°$

$\angle CBO + 90° + \angle ACB = 180°$

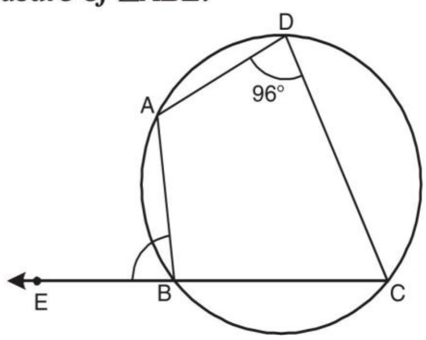
[$\because \angle CBO = \angle CBD$ and $\angle ACB = \angle DCB$ the same $\angle s$]

$\angle CBO + 90° + 30° = 180°$

$\angle CBO = 180° - 90° - 30° = 60°$

or $\angle OBC = 60°$

3. ***In the given figure, ABCD is a cyclic quadrilateral. If $\angle ADC = 96°$, then find the measure of $\angle ABE$.***



Sol. Here, ABCD is a cyclic quadrilateral

$\therefore \quad \angle ABC + \angle ADC = 180^\circ$

$\angle ABC + 96^\circ = 180^\circ$

$\angle ABC = 180^\circ - 96^\circ = 84^\circ$

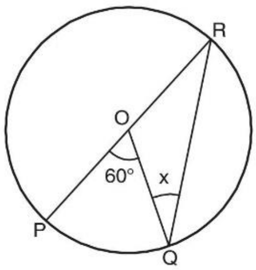
Also, $\angle ABE + \angle ABC = 180^\circ$ [a linear pair]

$\angle ABE + 84^\circ = 180^\circ$

$\angle ABE = 180^\circ - 84^\circ = 96^\circ$

Hence, $\angle ABE = 96^\circ$

4. ***If O is the centre of the circle, find the value of x.***



Sol. Here, ∠PRQ and ∠POQ are angles subtended by an arc PQ at the remaining part and at the centre of a circle with centre O, respectively.

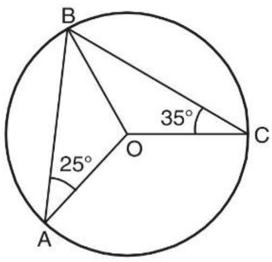
$\therefore \quad \angle PRQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 60^\circ = 30^\circ$

Also, in ΔOQR, OQ = OR = radii of same circle

$\Rightarrow \quad \angle PRQ = x$

$\Rightarrow \quad x = 30^\circ$

5. ***In the given figure, O is the centre of circle. If ∠OAB=25° and ∠OCB = 35°, then find the measure of ∠AOC.***



Sol. ∵ In ΔOAB, OA = OB

$\Rightarrow \quad \angle OAB = \angle OBA = 25^\circ$ ...(*i*)

In ΔOCB, OC = OB

$\Rightarrow \quad \angle OCB = \angle OBC = 35^\circ$ ...(*ii*)

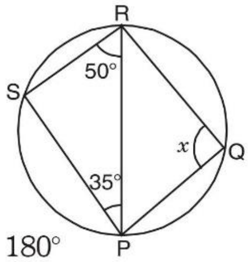
Adding (*i*) and (*ii*), we have

$\angle ABC = 25^\circ + 35^\circ = 60^\circ$

Now, $\angle AOC = 2\angle ABC$

$= 2 \times 60^\circ = 120^\circ$

6. ***In the figure, PQRS is a cyclic quadrilateral. Find the value of x.***



Sol. In ΔPRS, by using angle sum property, we have

$\angle PSR + \angle SRP + \angle RPS = 180^\circ$

$\angle PSR + 50^\circ + 35^\circ = 180^\circ$

$\angle PSR = 180^\circ - 85^\circ = 95^\circ$

Since PQRS is a cyclic quadrilateral

$\therefore \quad \angle PSR + \angle PQR = 180^\circ$

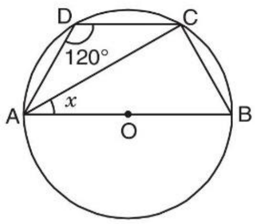
[∵ opp ∠s of a cyclic quad. are supplementary]

$95^\circ + x = 180^\circ$

$x = 180^\circ - 95^\circ$

$x = 85^\circ$

7. ***In the figure, O is the centre of a circle passing through points A, B, C and D and ∠ADC = 120°. Find the value of x.***



Sol. Since ABCD is a cyclic quadrilateral

$\therefore \angle ADC + \angle ABC = 180^\circ$

[∵ opp. ∠s of a cyclic quad. are supplementary]

$120^\circ + \angle ABC = 180^\circ$

$\angle ABC = 180^\circ - 120^\circ = 60^\circ$

Now, $\angle ACB = 90^\circ$ [angle in a semicircle]

In rt. ∠ed ΔACB, ∠ACB = 90°

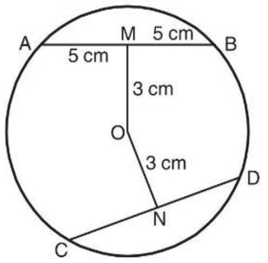
$\therefore \angle CAB + \angle ABC = 90^\circ$

$x + 60^\circ = 90^\circ$

$x = 90^\circ - 60^\circ$

$x = 30^\circ$

8. ***In the given figure, O is the centre of the circle. AB and CD are two chords, such that OM ⊥ AB and ON ⊥ CD. If OM = ON = 3 cm and AM = BM = 5 cm, then find the length of the chord CD.***



Sol. Here, OM ⊥ AB,

ON ⊥ CD and OM = ON = 3 cm

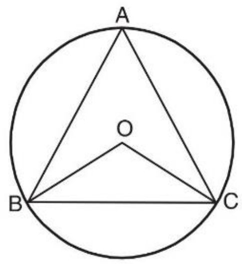
Since equidistant chords are equal in length

∴ CD = 10 cm

9. ***An equilateral triangle ABC is inscribed in a circle with centre O. Find the measure of ∠BOC.***

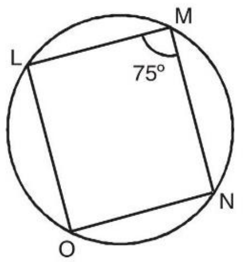
Sol. Since ΔABC is an equilateral

∴ $\angle A = \angle B = \angle C = 60^\circ$



$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$
$$= 90^\circ + \frac{1}{2} \times 60^\circ$$
$$= 120^\circ$$

10. ***In the given figure, ∠M is 75°, then find ∠O.***



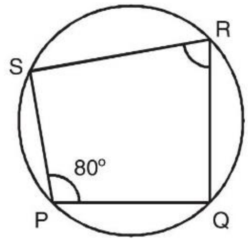
Sol. ∵ Sum of opposite angles of a cyclic quadrilateral is 180°

∴ $\angle O + \angle M = 180^\circ$

$\angle O + 75^\circ = 180^\circ$

⇒ $\angle O = 105^\circ$

11. ***In the figure, quadrilateral PQRS is cyclic. If ∠P = 80°, then find ∠R.***



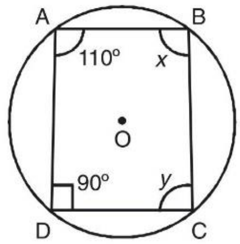
Sol. ∵ Sum of opposite angles of a cyclic quadrilateral is 180°

∴ $\angle P + \angle R = 180^\circ$

⇒ $80^\circ + \angle R = 180^\circ$

⇒ $\angle R = 100^\circ$

12. ***ABCD is a cyclic quadrilateral as shown in the figure, then find the value of (x + y).***

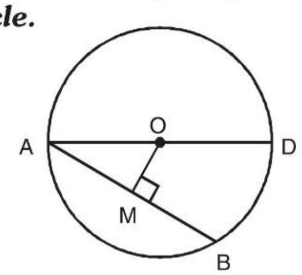


Sol. ∵ $x + y + 110^\circ + 90^\circ = 360^\circ$

$x + y + 200^\circ = 360^\circ$

⇒ $x + y = 160^\circ$

13. ***AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, then find the distance of AB from the centre of the circle.***



Sol. ∵ The perpendicular drawn from centre to the chord bisects it.

∴ $AM = \frac{1}{2}AB = \frac{1}{2} \times 30 \text{ cm} = 15 \text{ cm}$

Also, $OA = \frac{1}{2}AD = \frac{1}{2} \times 34 \text{ cm}$

$= 17 \text{ cm}$

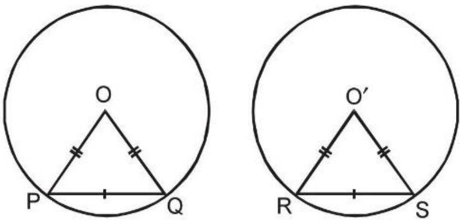
In rt. ΔOAM, we have

$$OA^2 = OM^2 + AM^2$$
$$17^2 = OM^2 + 15^2$$
$$\Rightarrow \quad 289 = OM^2 + 225$$
$$\Rightarrow \quad OM^2 = 289 - 225$$
$$\Rightarrow \quad OM^2 = 64$$
$$\Rightarrow \quad OM = \sqrt{64} = 8\,\text{cm}$$

14. *Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.*

Sol. ***Given:*** Two congruent circles C (O, r) and C(O′, r), such that chord PQ = chord RS.

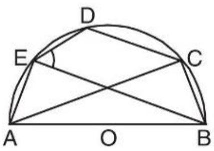
To Prove: $\angle POQ = \angle RO'S$



Proof: In ΔPOQ and $\Delta RO'S$

$$PO = RO' = r$$
$$QO = SO' = r$$
$$PQ = RS \quad \text{[given]}$$
$$\Rightarrow \quad \Delta POQ \cong \Delta RO'S$$
[by SSS congruence axiom]
$$\Rightarrow \quad \angle POQ = \angle RO'S \quad \text{[c.p.c.t.]}$$

15. *In the given figure, ABCDE is a pentagon. Whose vertices lie on the semicircle with centre O. Find the sum of ∠ACD and ∠DEB.*



Sol. Here, $\angle AEB = 90^\circ$

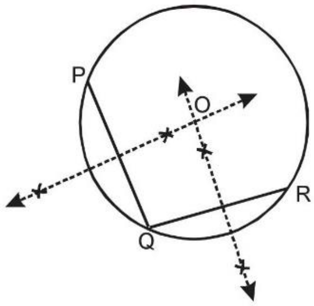
[∵ ∠ in a semicircle is right angle]

Now, ACDE is a cyclic quadrilateral and ∠AED, ∠ACD is a pair of opposite angles

$$\therefore \quad \angle AED + \angle ACD = 180^\circ$$
$$\angle AEB + \angle DEB + \angle ACD = 180^\circ$$
$$90^\circ + \angle DEB + \angle ACD = 180^\circ$$
$$\angle DEB + \angle ACD = 180^\circ - 90^\circ$$
$$\angle DEB + \angle ACD = 90^\circ$$

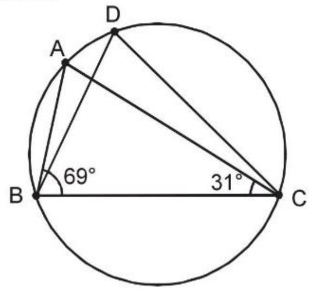
16. *Suppose you are given a circle. Give method to find its centre.*

Sol. Take any three distinct points P, Q and R on the given circle. Join PQ and QR.



Draw the perpendicular bisectors of PQ and RQ. The two perpendicular bisectors intersect at O. Thus, O is the centre of the given circle.

17. *In the given figure, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.*



Sol. In ΔABC, by angle sum property, we have

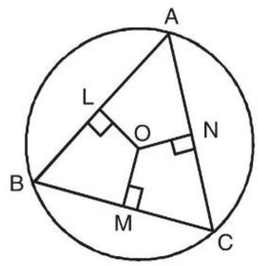
$$\angle BAC + \angle ABC + \angle BCA = 180^\circ$$
$$\Rightarrow \quad \angle BAC + 69^\circ + 31^\circ = 180^\circ$$
$$\Rightarrow \quad \angle BAC = 180^\circ - 69^\circ - 31^\circ = 80^\circ$$

Also, $\angle BDC = \angle BAC$

[angles in the same segment of a circle]

$$\Rightarrow \quad \angle BDC = 80^\circ$$

18. *In the figure, O is the centre of the circle, OM ⊥ BC, OL ⊥ AB, ON ⊥ AC and OM = ON = OL.*



Is ΔABC equilateral? Give reasons.

Sol. OL ⊥ AB, OM ⊥ BC and ON ⊥ AC

and OM = ON = OL

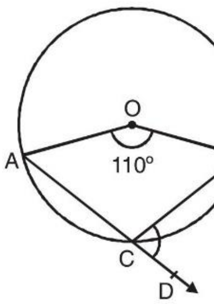
[Perpendicular distance of chords from the centre of a circle]

∴ AB = BC = AC

[Chords equidistant from the centre of a circle are equal]

Hence, ΔABC is an equilateral triangle.

19. ***If O is centre of circle shown in figure and ∠AOB = 110°, then find ∠BCD.***



Sol. $\angle AOB = 110^\circ$

$\Rightarrow$ Reflex $\angle AOB = 360^\circ - 110^\circ = 250^\circ$

Reflex $\angle AOB = 2\angle ACB$

$250^\circ = 2\angle ACB$

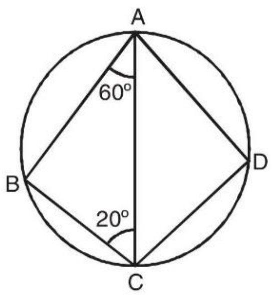
$\Rightarrow \angle ACB = \frac{250^\circ}{2} = 125^\circ$

$\angle ACB + \angle BCD = 180^\circ$ [linear pair]

$125^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 125^\circ = 55^\circ$

20. ***In the figure, if ∠BAC = 60°, ∠ACB = 20°, find ∠ADC.***



Sol. In ΔABC, we have

$\angle ABC = 180^\circ - (\angle BAC + \angle BCA)$ [angle sum property]

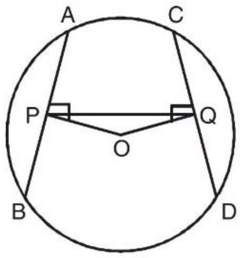
$= 180^\circ - 80^\circ = 100^\circ$

∵ ABCD is a cyclic quadrilateral,

∴ $\angle ABC + \angle ADC = 180^\circ$

$\angle ADC = 180^\circ - 100^\circ = 80^\circ$

21. ***In the figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠POQ = 150°, find ∠APQ.***



Sol. As AB = CD

So, OP = OQ

[equal chords are equidistant from the centre]

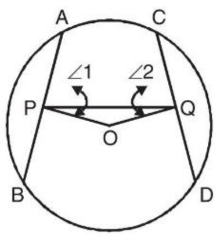
$\angle 1 = \angle 2$

[angles opp. to equal sides are equal]

$\angle 1 + \angle 2 + \angle POQ = 180^\circ$

$2\angle 1 = 180^\circ - 150^\circ = 30^\circ$

∴ $\angle 1 = 15^\circ$



Since APB is a line segment

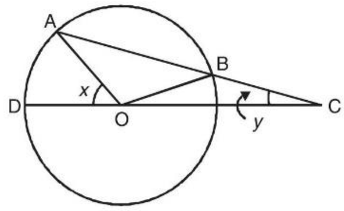
$\angle APQ + \angle 1 = 90^\circ$

$\angle APQ = 90^\circ - \angle 1$

$= 90^\circ - 15^\circ = 75^\circ$

∴ $\angle APQ = 75^\circ$

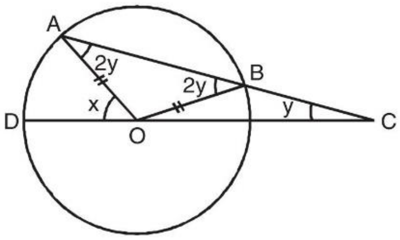
22. ***In the figure, chord AB of circle with centre O, is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If ∠ACD = y and ∠AOD = x, show that x = 3y.***



Sol. In ΔOBC, OB = BC

$\Rightarrow \quad \angle BOC = \angle BCO = y$

...[angles opp. to equal sides are equal]



$\angle OBA$ is the exterior angle of ΔBOC

So, $\quad \angle ABO = 2y$

...[ext. angle is equal to the sum of int. opp. angles]

Similarly, $\angle AOD$ is the exterior angle of ΔAOC

$\therefore \quad x = 2y + y = 3y$

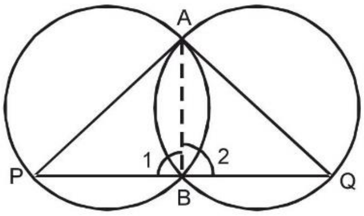
23. ***If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.***

Sol. AP is a diameter

$\therefore \quad \angle 1 = 90°$ ...(*i*)

[$\because$ angle in a semicircle is right angle]

Similarly, $\quad \angle 2 = 90°$ ...(*ii*)



Adding (*i*) and (*ii*), we have

$\angle 1 + \angle 2 = 90° + 90°$

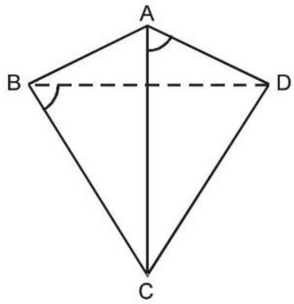
$\angle PBQ = 180°$

$\Rightarrow$ P, B and Q are collinear.

Hence, B lies on the side PQ of the ΔAPQ.

24. ***ABC and ADC are two right triangles with common hypotenuse AC. Prove that: $\angle CAD = \angle CBD$.***

Sol. Since ΔABC and ΔADC are two right triangles with common hypotenuse AC.



$\therefore \quad \angle ABC = \angle ADC = 90°$

Also, $\quad \angle ABC + \angle ADC = 90° + 90°$

$= 180°$

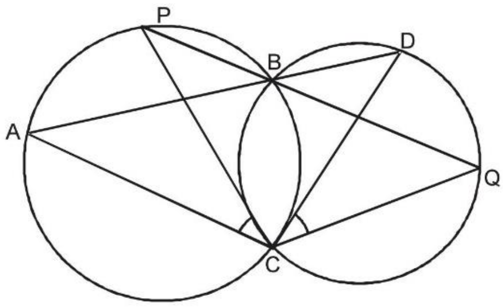
$\Rightarrow$ ABCD is a cyclic quadrilateral.

[$\because$ opposite angles are supplementary]

$\Rightarrow \quad \angle CAD = \angle CBD$

[$\because$ angles in the same segment]

25. ***Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that: $\angle ACP = \angle QCD$.***



Sol. $\quad \angle ACP = \angle ABP$ ...(*i*)

[angles in the same segment]

$\angle QCD = \angle QBD$ ...(*ii*)

[angles in the same segment]

Also, $\quad \angle ABP = \angle QBD$ ...(*iii*)

[vertically opposite angles]

$\therefore$ From (*i*), (*ii*) and (*iii*), we have

$\angle ACP = \angle QCD$

26. ***In a cyclic quadrilateral PQRS, if $\angle P - \angle R = 50°$, then find the measure of $\angle P$ and $\angle R$.***

Sol. PQRS is cyclic quad., we have

$\angle P + \angle R = 180°$

[opp. angles are supplementary]

$\angle P - \angle R = 50°$ [given]

$\Rightarrow \quad 2\angle P = 230°$

$\angle P = \frac{230°}{2} = 115°$

$\angle P = 115°$

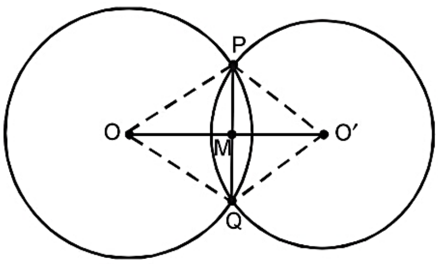
$\angle R = 115° - 50° = 65°$

$\angle R = 65°$

Short Answer Type-II Questions

27. ***If two circles intersect in two points, prove that the line through their centres is the perpendicular bisector of the common chord.***

Sol. ***Given:*** Two circles C(O, *r*) and C(O′, *s*) intersect at P and Q.



To Prove: OO′ is perpendicular bisector of the chord PQ.

Const.: Join OP, OQ, O′P and O′Q

Proof: In ΔOPO′ and ΔOQO′

OP = OQ [radii of same circle]

O′P = O′Q [radii of same circle]

OO′ = OO′ [common]

$\Rightarrow$ ΔOPO′ ≅ ΔOQO′ [by SSS congruence axiom]

$\Rightarrow$ ∠POM = ∠QOM [c.p.c.t.]

Now, in ΔPOM and ΔQOM

OP = OQ [radii of same circle]

∠POM = ∠QOM [proved above]

OM = OM [common]

$\Rightarrow$ ΔPOM ≅ ΔQOM [by SAS congruence axiom]

PM = QM and

∠PMO = ∠QMO [c.p.c.t.]

Also, ∠PMO + ∠QMO = 180° [a linear pair]

$\Rightarrow$ ∠PMO = ∠QMO = 90°

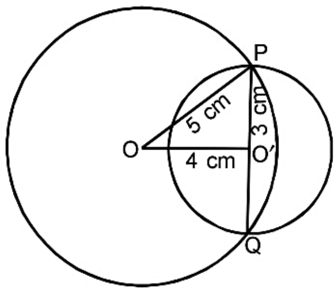
Hence, OO′ is the perpendicular bisector of the chord PQ.

28. ***Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.***

Sol. Let C(O, 5 cm) and C(O′, 3 cm) be the two circles intersecting each other in P and Q. Let PQ be the common chord.

Now, OP = 5 cm, O′P = 3 cm

and OO′ = 4 cm



$\therefore$ $(O'P)^2 + (OO')^2 = (OP)^2$

i.e., $(3)^2 + (4)^2 = (5)^2$

Then, by Pythagoras Theorem, we have

∠OO′P = 90°

or OO′ ⊥ O′P

Also, we know that the line joining the centres is the perpendicular bisector of the common chord.

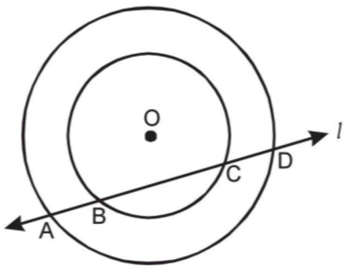
$\therefore \quad O'P = \frac{1}{2} PQ$

$\Rightarrow \quad PQ = 2 \times O'P$

$= 2 \times 3 = 6$ cm

Hence, the length of the common chord is 6 cm.

29. ***If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see fig.).***

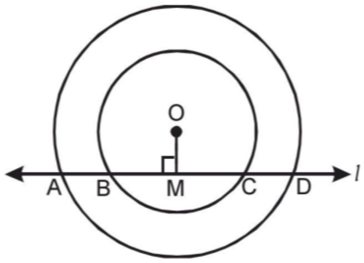


Sol. Draw $OM \perp l$

Since perpendicular from the centre of a circle to a chord of the circle bisects the chord.

$\therefore \quad BM = CM \quad ...(i)$

and $\quad AM = DM \quad ...(ii)$



Subtracting (*i*) from (*ii*), we have

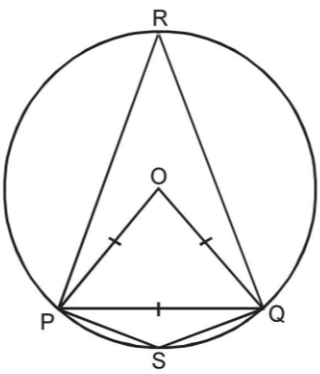
$$AM - BM = DM - CM$$

$$AB = CD$$

30. ***A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.***

Sol. Let PQ be the chord whose length is equal to the radius of the circle. Let R and S be the points in major arc and minor arc PQ respectively.

$\therefore \quad OP = PQ = OQ = r$



$\Rightarrow \quad \Delta OPQ$ is an equilateral triangle.

$\Rightarrow \quad \angle POQ = 60°$

Now, $\angle POQ$ and $\angle PRQ$ are the angles subtended by arc PQ at the centre of the circle and in the remaining part of the circle.

$$\therefore \quad \angle PRQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 60°$$

$$= 30°$$

Also, $\angle PSQ + \angle PRQ = 180°$

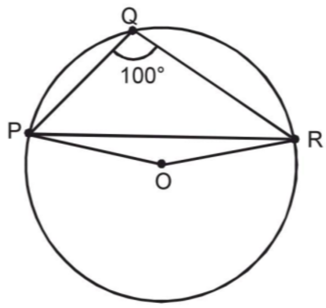
[$\because$ opposite angles of a cyclic quadrilateral]

$\Rightarrow \quad \angle PSQ + 30° = 180°$

$\Rightarrow \quad \angle PSQ = 180° - 30° = 150°$

Hence, the required angles are 150° and 30°.

31. ***In the given figure, $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.***



Sol. Since angle subtended at the centre by an arc of a circle is twice the angle subtended in the remaining part of the circle.

$\therefore \quad$ Reflex $\angle POR = 2\angle PQR$

$= 2 \times 100° = 200°$

$\Rightarrow \quad \angle POR = 360° - 200°$

$= 160° \quad ...(i)$

Also, OP = OR = radii of the same circle

$\Rightarrow \quad \angle OPR = \angle ORP \quad ...(ii)$

[$\because$ angles opposite to equal sides of a triangle]

Now, $\angle OPR + \angle ORP + \angle POR = 180°$

$\angle OPR + \angle OPR + 160° = 180°$

[using (*i*) and (*ii*)]

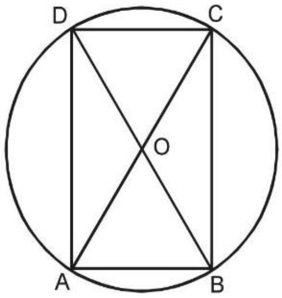
$\Rightarrow \quad 2\angle OPR = 180° - 160° = 20°$

$$\Rightarrow \quad \angle OPR = \frac{1}{2} \times 20° = 10°$$

Hence, the required $\angle OPR = 10°$.

32. ***If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.***

Sol. ABCD is a cyclic quadrilateral. AC and BD are the diagonals of the quadrilateral ABCD.



Since AC is a diameter [given]

∴ $\angle ABC = \angle ADC = 90°$ [angles in a semicircle]

Similarly, BD is a diameter [given]

∴ $\angle BCD = \angle BAD = 90°$

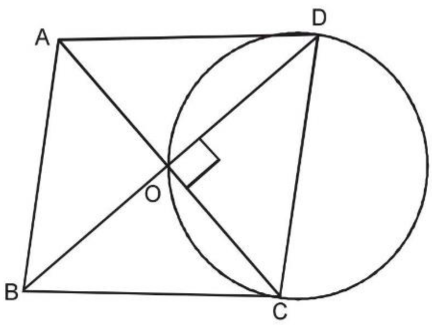
Now, in cyclic quadrilateral ABCD, we have

$\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90°$

Hence, ABCD is a rectangle.

33. *Prove that the circle drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.*

Sol. *Given:* A circle C(O, *r*) is drawn taking the side DC of a rhombus ABCD as its diameter.



To Prove: Circle passes through the point of intersection O of the diagonals AC and BD.

Proof: Since diagonals of a rhombus bisect each other at right angles.

∴ $\angle COD = 90°$

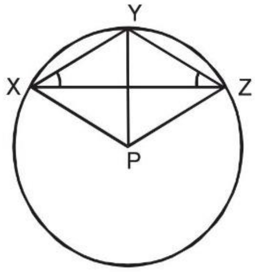
⇒ ΔCOD is a right triangle with hypotenuse CD.

Circle drawn with CD as diameter passes through the vertex O of right ΔCOD.

Similarly, circles described on AB, BC, DA as diameters will pass through O, the point of intersection of its diagonals.

34. *In the given figure, P is the centre of the circle. Prove that:*

$\angle XPZ = 2(\angle XZY + \angle YXZ)$



Sol. Arc XY subtends $\angle XPY$ at the centre P and $\angle XZY$ in the remaining part of the circle.

∴ $\angle XPY = 2(\angle XZY)$ ...(*i*)

Similarly, arc YZ subtends $\angle YPZ$ at the centre P and $\angle YXZ$ in the remaining part of the circle.

∴ $\angle YPZ = 2(\angle YXZ)$ ...(*ii*)

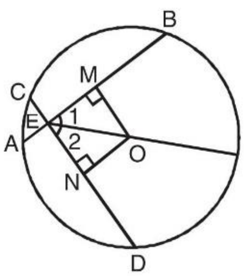
Adding (*i*) and (*ii*), we have

$\angle XPY + \angle YPZ = 2(\angle XZY + \angle YXZ)$

$\angle XPZ = 2(\angle XZY + \angle YXZ)$

Long Answer Type Questions

35. *If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.*



Sol. *Given:* In a circle with centre O chords AB and CD intersect at E such that

$\angle 1 = \angle 2$

To Prove: AB = CD

Construction: OM ⊥ AB and ON ⊥ CD

Proof: In ΔOME and ΔONE, we have

$\angle 1 = \angle 2$ [given]

$\angle OME = \angle ONE = 90°$

OE = OE [common]

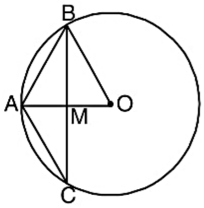
∴ By AAS congruence rule,

$\Delta OME \cong \Delta ONE$

∴ OM = ON [c.p.c.t.]

Hence, AB = CD [chords equidistant from the centre are equal]

36. ***In a circle of radius 5 cm, AB and AC are two chords such that AB = AC = 6 cm, as shown in the figure. Find the length of the chord BC.***



Sol. Here, OA = OB = 5 cm [radii]

AB = AC = 6 cm

∴ B and C are equidistant from A.

∴ AO is the perpendicular bisector of chord BC and it intersect BC in M.

Now, in rt. ∠ed ΔAMB, ∠M = 90°

∴ By using Pythagoras Theorem, we have

$$BM^2 = AB^2 - AM^2$$
$$= 36 - AM^2 \quad ...(i)$$

Also, in rt. ∠ed ΔBMO, ∠M = 90°

∴ By using Pythagoras Theorem, we have

$$BM^2 = BO^2 - MO^2$$
$$= 25 - (AO - AM)^2 \quad ...(ii)$$

From (*i*) and (*ii*), we obtain

$25 - (AO - AM)^2 = 36 - AM^2$

$25 - AO^2 - AM^2 + 2AO \times AM = 36 - AM^2$

$25 - 25 + 2 \times 5 \times AM = 36$

$10\ AM = 36$

$AM = 3.6$ cm

From (*i*), we have

$$BM^2 = 36 - (3.6)^2$$
$$= 36 - 12.96$$
$$= 23.04$$

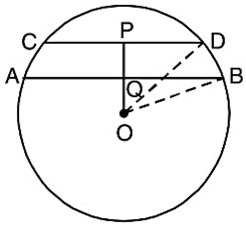
⇒ $BM = \sqrt{23.04} = 4.8$ cm

Thus, $BC = 2 \times BM$

$= 2 \times 4.8 = 9.6$ cm

Hence, the length of the chord BC is 9.6 cm.

37. ***In the figure, AB and CD are two chords of a circle with centre O such that AB = 16 cm, CD = 12 cm and AB || CD. If OP ⊥ CD and if the distance between AB and O is 6 cm, find the radius of the circle and the distance of CD from centre O.***



Sol. Here, AB||CD and AB = 16 cm, CD = 12 cm, OP ⊥ CD and OP ⊥ AB

Since perpendicular from the centre of a circle to a chord of a circle bisects the chord, we have

$$AQ = BQ = \frac{1}{2} \times 16 = 8 \text{ cm}$$

and $$CP = DP = \frac{1}{2} \times 12 = 6 \text{ cm}$$

Also, OQ = 6 cm

Let PQ = x cm ⇒ OP = $(x + 6)$ cm

Join OB and OD

Now, in rt. ∠ed ΔOQB, by using Pythagoras Theorem, we have

$$OB^2 = OQ^2 + BQ^2$$
$$= 6^2 + 8^2$$
$$= 36 + 64 = 100$$
$$OB = 10 \text{ cm}$$

Again, in rt. ∠ed ΔOPD, by using Pythagoras Theorem, we have

$OP^2 + DP^2 = OD^2$

$(x + 6)^2 + 6^2 = 10^2$

$(x + 6)^2 = 100 - 36 = 64$

$(x + 6)^2 = (8)^2$

⇒ $x + 6 = \pm 8$

Either $x + 6 = 8$ or $x + 6 = -8$

⇒ $x = 2$ or $x = -14$

Rejecting negative value, which is not possible

⇒ $x = 2$

∴ OP = 2 + 6 = 8 cm

Hence, the radius of the circle is 10 cm and the distance of CD from centre O is 8 cm.

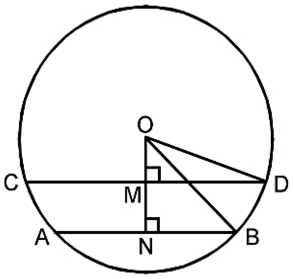
38. ***The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?***

Sol. Here, AB || CD, AB = 6 cm, CD = 8 cm and ON = 4 cm

OM ⊥ CD, ON ⊥ AB

Since the perpendicular from the centre of a circle to the chord of the circle, bisects the chord.

$\therefore \quad AN = BN = \frac{1}{2}AB = \frac{1}{2} \times 6 = 3$ cm



and $CM = DM = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4$ cm

Let OM be x cm

Now, in rt. ΔONB, ∠N = 90°

∴ By Pythagoras Theorem, we have

$OB^2 = ON^2 + NB^2$

$\Rightarrow \quad OB^2 = (4)^2 + (3)^2$

$\Rightarrow \quad OB^2 = 16 + 9 = 25$

$\Rightarrow \quad OB = \sqrt{25} = 5$ cm

Again, in rt. ΔOMD, ∠M = 90°

∴ By Pythagoras Theorem, we have

$OM^2 = OD^2 - MD^2$

$OM^2 = (5)^2 - (4)^2$

$OM^2 = 25 - 16$

$OM^2 = 9$

$OM = \sqrt{9} = 3$ cm

Hence, the distance of the other chord (CD) from the centre is 3 cm.

39. ***Bisectors of angles A, B and C of triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the ΔDEF are*** $90^\circ - \frac{\angle A}{2}$, $90^\circ - \frac{\angle B}{2}$ ***and*** $90^\circ - \frac{\angle C}{2}$ ***respectively.***

Sol. Let $\angle BAD = x$, $\angle ABE = y$

and $\angle ACF = z$, then

$\angle CAD = x$, $\angle CBE = y$

and $\angle BCF = z$

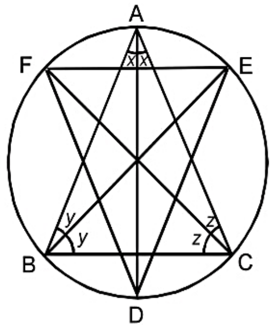
[AD, BE and CF is bisector of ∠A, ∠B and ∠C]

In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad 2x + 2y + 2z = 180^\circ$

or $\quad x + y + z = 90^\circ \quad$ ...(*i*)



Now, $\quad \angle ADE = \angle ABE$

and $\quad \angle ADF = \angle ACF$

[angles in the same segment of a circle]

$\Rightarrow \quad \angle ADE = y$ and $\angle ADF = z$

$\Rightarrow \quad \angle ADE + \angle ADF = y + z$

or $\quad \angle D = y + z \quad$ ...(*ii*)

From (*i*) and (*ii*), we have

$x + \angle D = 90^\circ$

$\Rightarrow \quad \angle D = 90^\circ - x$

or $\quad \angle D = 90^\circ - \frac{\angle A}{2} \quad \left[\because x = \frac{\angle A}{2}\right]$

Similarly, $\quad \angle E = 90^\circ - \frac{\angle B}{2}$

and $\quad \angle F = 90^\circ - \frac{\angle C}{2}$

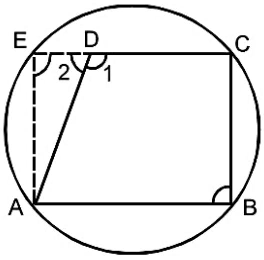
40. ***ABCD is a parallelogram. The circle through A, B and C intersects (produce if necessary) at E. Prove that: AE = AD.***

Sol. ***Given:*** ABCD is a parallelogram. Circle through A, B and C intersects CD produced in E.

To Prove: AE = AD

Proof: ABCE is a cyclic quadrilateral.

$\therefore \quad \angle B + \angle E = 180^\circ \quad$ ...(*i*)



ABCD is a parallelogram.

$\therefore$ $\angle B = \angle 1$ ...(*ii*)

Also, $\angle 1 + \angle 2 = 180°$ [a linear pair]

or $\angle B + \angle 2 = 180°$ ...(*iii*) [using (*ii*)]

Now, from (*i*) and (*iii*), we have

$\angle B + \angle E = \angle B + \angle 2$

$\Rightarrow$ $\angle E = \angle 2$

In ΔADE, we have

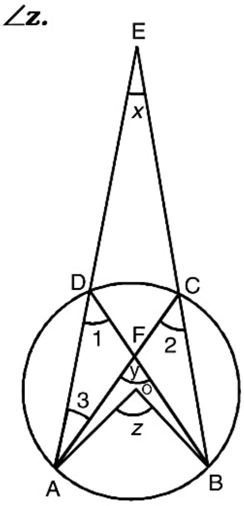
$\angle E = \angle 2$

$\Rightarrow$ $AD = AE$

[sides opposite to equal angles of a Δ]

41. ***In the given figure, O is the centre of the circle. Prove that:***

$\angle x + \angle y = \angle z.$



Sol. $\angle 1 = \angle 2$

[angles in the same segment]

Also, $\angle AOB = 2\angle ADB$

$\angle z = 2\angle 1 = \angle 1 + \angle 1$

$\angle z = \angle 1 + \angle 2$ ...(*i*)

In ΔADF,

ext. $\angle ACB = \angle y = \angle 1 + \angle 3$

$\Rightarrow$ $\angle y = \angle z - \angle 2 + \angle 3$ [using (*i*)] ...(*ii*)

Again, in ΔAEC,

ext. $\angle 2 = \angle x + \angle 3$ ...(*iii*)

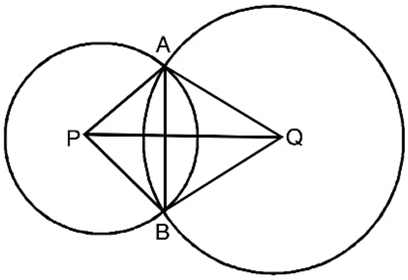
From (*ii*) and (*iii*), we have

$\angle y = \angle z - \angle x - \angle 3 + \angle 3$

$\angle y = \angle z - \angle x$

or $\angle z = \angle x + \angle y$

42. ***Prove that line joining the centres of two intersecting circles subtends equal angles at the two points of intersection of circles.***



Sol. ***Given:*** P and Q are centre of two intersecting circles intersects at A and B.

To Prove: $\angle PAQ = \angle PBQ$

Construction: Join P and Q

Proof: In ΔPAQ and ΔPBQ, we have

$PQ = PQ$ [common side]

$AP = BP$ [radii of a circle]

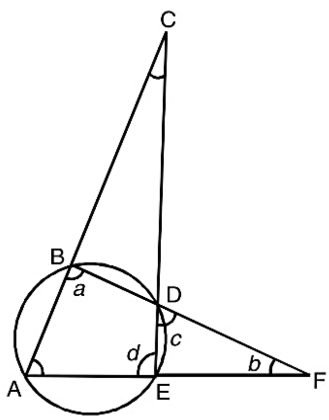
$AQ = BQ$ [radii of a circle]

$\therefore$ By SSS congruence rule, we have

$\Delta PAQ \cong \Delta PBQ$

$\Rightarrow$ $\angle PAQ = \angle PBQ$ [c.p.c.t.]

43. ***In the given figure, find the value of a, b, c and d. Given $\angle BCD = 43°$ and $\angle BAE = 62°$.***



Sol. In ΔACE,

$\angle ACE + \angle EAC + \angle AEC = 180^\circ$

$43^\circ + 62^\circ + d = 180^\circ$

$d = 180^\circ - (43^\circ + 62^\circ)$

$= 180^\circ - 105^\circ = 75^\circ$

$a + d = 180^\circ$

[opp. angles of cyclic quad. are supplementary]

$a + 75^\circ = 180^\circ$

$a = 180^\circ - 75^\circ = 105^\circ$

In ΔABF,

$\angle BAF + \angle ABF + \angle AFB = 180^\circ$

$62^\circ + 105^\circ + b = 180^\circ$

$b = 180^\circ - (62^\circ + 105^\circ)$

$b = 180^\circ - 167^\circ = 13^\circ$

$\angle DEF = 180^\circ - 75^\circ = 105^\circ$

In ΔDEF,

$\angle DEF + \angle EFD + \angle EDF = 180^\circ$

$105^\circ + 13^\circ + c = 180^\circ$

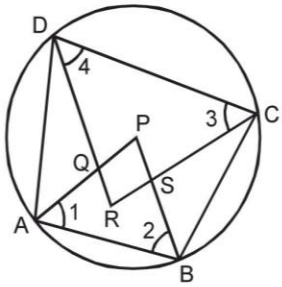
$118^\circ + c = 180^\circ$

$c = 180^\circ - 118^\circ = 62^\circ$

$\therefore a = 105^\circ, b = 13^\circ, c = 62^\circ, d = 75^\circ$

44. *Show that the quadrilateral formed by angle bisectors of a cyclic quadrilateral, is also cyclic.*

Sol. ***Given:*** A cyclic quadrilateral ABCD, in which AP, BP, CR and DR are the angle bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively, such that a quadri-lateral. PQRS is formed.



To Prove: PQRS is a cyclic quadrilateral.

Proof: Since ABCD is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180^\circ$ and

$\angle B + \angle D = 180^\circ$ ...(*i*)

Also, AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$, respectively.

$\therefore \quad \angle 1 = \frac{1}{2}\angle A, \angle 2 = \frac{1}{2}\angle B,$

$\angle 3 = \frac{1}{2}\angle C$ and $\angle 4 = \frac{1}{2}\angle D$

From (*i*), we have

$\frac{1}{2}\angle A + \frac{1}{2}\angle C = \frac{1}{2}(\angle A + \angle C)$

$= \frac{1}{2} \times 180^\circ = 90^\circ$

$\frac{1}{2}\angle B + \frac{1}{2}\angle D = \frac{1}{2}(\angle B + \angle D)$

$= \frac{1}{2} \times 180^\circ = 90^\circ$

or $\quad \angle 1 + \angle 3 = 90^\circ$

and $\quad \angle 2 + \angle 4 = 90^\circ$ ...(*ii*)

Now, in ΔAPB, by angles sum property of a Δ

$\angle 1 + \angle 2 + \angle P = 180^\circ$ ...(*iii*)

Again, in ΔCRD, by angles sum property of a Δ

$\angle 3 + \angle 4 + \angle R = 180^\circ$ ...(*iv*)

Adding (*iii*) and (*iv*), we have

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle P + \angle R$

$= 180^\circ + 180^\circ$

$\Rightarrow \quad 90^\circ + 90^\circ + \angle P + \angle R = 360^\circ$ [using (*ii*)]

$\Rightarrow \quad \angle P + \angle R = 360^\circ - 180^\circ = 180^\circ$

i.e., the sum of one pair of the opposite angles of quadrilateral PQRS is 180°.

Hence, the quadrilateral PQRS is a cyclic quadrilateral.

45. *If O is the circumcircle of a ΔABC and $OD \perp BC$, prove that: $\angle BOD = \angle A$.*

Sol. Join OC.

Now, in ΔBDO and ΔCDO, we have

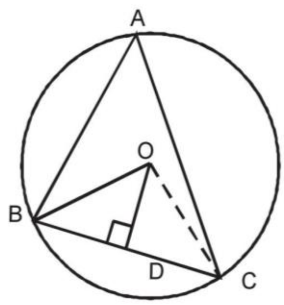
$OB = OC$ [radii of same circle]

$\angle BDO = \angle CDO = 90^\circ$

$OD = OD$ [common]

$\Rightarrow \quad \Delta BDO \cong \Delta CDO$

[by RHS congruence axiom]



$\Rightarrow \quad \angle BOD = \angle COD$ [c.p.c.t.]

$\Rightarrow \quad 2\angle BOD = 2\angle COD = \angle BOC$ ...(*i*)

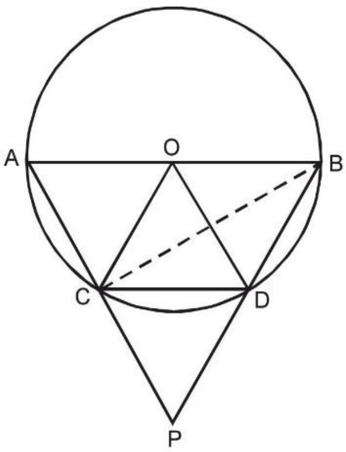
Now, an arc BC subtends $\angle BOC$ at the centre O of the circle and $\angle BAC$ in the remaining part of the circle.

$\therefore \quad \angle BAC = \frac{1}{2}\angle BOC$

$\Rightarrow \quad \angle A = \frac{1}{2} \times 2\angle BOD$ [using (*i*)]

$\Rightarrow \quad \angle A = \angle BOD$

46. ***In the given figure, AB is a diameter of a circle C(O, r). Chord CD is equal to the radius OC. If AC and BD when produced intersect at P, prove that: ∠APB is constant.***



Sol. Join BC and OD.

In ΔOCD, we have

$OC = OD = CD = \text{radius}$ [given]

$\Rightarrow$ ΔOCD is an equilateral triangle.

$\Rightarrow \quad \angle COD = 60°$

Now, arc CD subtends ∠COD at the centre of the circle with centre O and ∠CBD in the remaining part of the circle.

$\therefore \quad \angle CBD = \frac{1}{2}\angle COD = \frac{1}{2} \times 60° = 30°$

$\angle ACB = 90°$ [angle in a semicircle]

Also, ACP is a straight line

$\therefore \quad \angle BCP = 180° - \angle ACB$

$= 180° - 90° = 90°$

Thus, in ΔBCP, by angles sum property of a triangle, we have

$\angle BCP + \angle CPB + \angle CBP = 180°$

$\Rightarrow \quad 90° + \angle CPB + 30° = 180°$

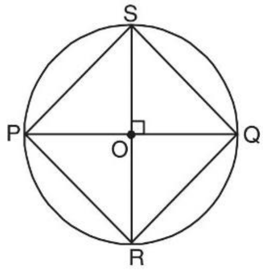
[$\because \angle CPB = \angle CBD = 30°$]

$\Rightarrow \quad \angle CPB = 180° - 90° - 30°$

$\Rightarrow \quad \angle CPB = 60° = \text{Constant}$

$\Rightarrow \quad \angle APB = \text{Constant}$

47. ***Two diameters of a circle intersect each other at right angles. Prove that the quadrilateral formed by joining their end-points is a square.***



Sol. Let PQ and RS be two perpendicular diameters of a circle with centre O. We have to prove that the quadrilateral PRQS is a square.

In ΔPOS and ΔQOS, we have

$OP = OQ$ [radii of same circle]

$\angle POS = \angle QOS$ [each = 90°]

$OS = OS$ [common]

∴ By SAS congruence rule, we have

$\Delta POS \cong \Delta QOS$

$\Rightarrow \quad PS = QS$ [c.p.c.t.]

Also, $\angle PSQ = 90°$

[∵ ∠ in a semicircle is right angle]

Similarly, we have

$QS = QR$ and $\angle SQR = 90°$

and $RQ = RP$ and $\angle QRP = 90°$

Now, in quadrilateral PRQS, we have

$PS = QS = QR = RP$

and $\angle PSQ = \angle SQR = \angle QRP = \angle RPS = 90°$

Hence, PRQS is a square.

APPLICATION BASED QUESTIONS (Solved)

1. ***Prove that two circles cannot intersect at more than two points.***

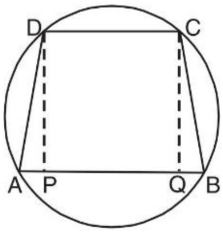
Sol. Let there be two circles which intersect at three points say at A, B and C. Clearly, A, B and C are not collinear. We know that through three non-collinear points A, B and C one and only one circle can pass. Therefore, there cannot be two circles passing through A, B and C. In other words, the two circles cannot intersect at more than two points.

2. ***If two non-parallel sides of a trapezium are equal, then it is cyclic.***

Sol. ***Given:*** A trapezium ABCD in which AB ∥ DC and AD = BC.

To Prove: ABCD is a cyclic trapezium.

Construction: Draw DP ⊥ AB, CQ ⊥ AB.



Proof: In rt. ∠ed ΔAPD and ΔBQC

Hyp. AD = Hyp. BC [given]

DP = CQ

[distance between two parallel lines is equal]

∠P = ∠Q [right angle]

By using RHS congruence rule, we have

ΔAPD ≅ ΔBQC

⇒ ∠A = ∠B and ∠ADP = ∠BCQ [c.p.c.t.]

Now, ∠ADP = ∠BCQ

⇒ 90° + ∠ADP = 90° + ∠BCQ

⇒ ∠ADC = ∠BCD [∵ ∠PDC = ∠QCD = 90°]

or ∠D = ∠C

Also, ∠A + ∠B + ∠C + ∠D = 360°

[∠sum property of the quadrilateral]

∠A + ∠A + ∠C + ∠C = 360°

2(∠A + ∠C) = 360°

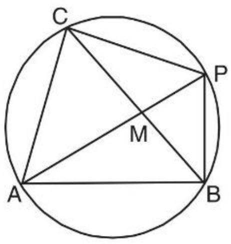
∠A + ∠C = 180°

Similarly, ∠B + ∠D = 180°

Hence, ABCD is a cyclic trapezium.

3. ***In any triangle, if the angle bisector of ∠A and the perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle.***

Sol. Let angle bisector of ∠A and perpendicular bisector of side BC intersect at a point P. Since PM is perpendicular bisector of BC.



∴ BP = CP

⇒ ∠BCP = ∠CBP

Also, ∠CBP = ∠CAP

[∵ ∠s in the same segment are equal]

Now, in ΔBCP, we have

∠BPC = 180° – ∠CBP – ∠BCP

= 180° – ∠CBP – ∠CBP [∵ ∠BCP = ∠CBP]

= 180° – 2∠CBP ...(*i*)

Again, ∠BAC = 2∠PAC

= 2∠CBP ...(*ii*)

[∵ ∠PAC = ∠CBP]

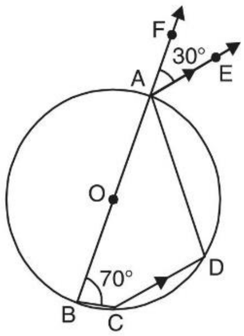
Adding (*i*) and (*ii*), we obtain

∠BPC + ∠BAC = 180°

Thus, ABPC is a cyclic quadrilateral *i.e.*, Points A, B, P and C are concyclic.

Hence, P lies on the circumcircle of ΔABC.

4. ***In the given figure, ABCD is a cyclic quadrilateral, in which AE is drawn parallel to CD and BA is produced to F. If ∠ABC = 70° and ∠FAE = 30°, find ∠BCD.***



Sol. Since opposite angles of a cyclic quadrilateral are supplementary

∴ ∠ABC + ∠ADC = 180°

⇒ 70° + ∠ADC = 180°

⇒ ∠ADC = 180° – 70° = 110°

Now, ∠DAE = ∠ADC

[alt. angles as AE || DC]

∴ ∠DAE = 110°

Again, ∠BAD = 180° – ∠DAE – ∠EAF

[∠s in a st. line]

= 180° – 110° – 30°

∠BAD = 40°

Now, ∠BCD + ∠BAD = 180°

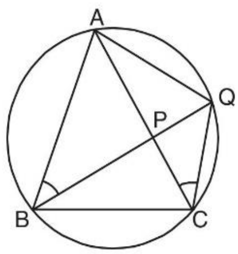
[opp. ∠s of a cyclic quadrilateral]

⇒ ∠BCD + 40° = 180°

⇒ ∠BCD = 180° – 40°

∠BCD = 140°

5. ***In the given figure, ABC is an isosceles triangle with AB = AC and P is a point on side AC. Through C a line is drawn to intersect BP produced in Q, such that ∠ABQ = ∠ACQ. Prove that***



$$\angle AQC = 90° + \frac{1}{2}\angle A.$$

Sol. Here, through C a line is drawn to intersect BP produced in Q, such that $\angle ABQ = \angle ACQ$

But, they are angles of the same segment

$\therefore$ A, B, C and Q are concyclic

Thus, ABCQ is a cyclic quadrilateral

$\Rightarrow \quad \angle ABC + \angle AQC = 180^\circ \quad ...(i)$

[opp. ∠s of a cyclic quadrilateral]

Also, in ΔABC, AB = AC

$\Rightarrow \quad \angle ACB = \angle ABC \quad ...(ii)$

[∠s opp. to equal sides of a Δ]

Again, $\angle A + \angle ABC + \angle ACB = 180^\circ$

[∠ sum property of a Δ]

$\angle A + \angle ABC + \angle ABC = 180^\circ$ [using (ii)]

$\angle A + 2\angle ABC = 180^\circ$

$\angle ABC = 90^\circ - \frac{1}{2}\angle A \quad ...(iii)$

From (i) and (iii), we have

$90^\circ - \frac{1}{2}\angle A + \angle AQC = 180^\circ$

$\Rightarrow \quad \angle AQC = 180^\circ - 90^\circ + \frac{1}{2}\angle A$

$= 90^\circ + \frac{1}{2}\angle A.$

ANALYZING, EVALUATING & CREATING TYPE QUESTIONS (Solved)

1. *Live and let live ! Keeping in mind the campaign slogan, three students Priyanka, Sania and David are protesting against killing innocent animals for commercial purposes in a circular park of radius 20 m. They are standing at equal distances on its boundary by holding banners in their hands.*

(i) Find the distance between each of them.

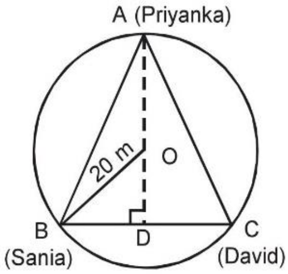
(ii) Which mathematical concept is used in it?

Sol. (i) Let us assume that A, B and C be the positions of Priyanka, Sania and David respectively on the boundary of circular park with centre O.

Draw AD ⊥ BC.

Since the centre of the circle coincides with the centroid of the equilateral ΔABC.

$\therefore$ Radius of circumscribed circle $= \frac{2}{3}$ AD



$\Rightarrow \quad 20 = \frac{2}{3} AD$

$\Rightarrow \quad AD = 20 \times \frac{3}{2}$

$\Rightarrow \quad AD = 30$ m

Now, AD ⊥ BC and let AB = BC = CA = x m

$\Rightarrow \quad BD = CD = \frac{1}{2}BC = \frac{x}{2}$

In rt. ΔBDA, ∠D = 90°

$\therefore$ By Pythagoras Theorem, we have

$AB^2 = BD^2 + AD^2$

$x^2 = \left(\frac{x}{2}\right)^2 + (30)^2$

$x^2 - \frac{x^2}{4} = 900$

$\frac{3}{4}x^2 = 900$

$x^2 = 900 \times \frac{4}{3}$

$x^2 = 1200$

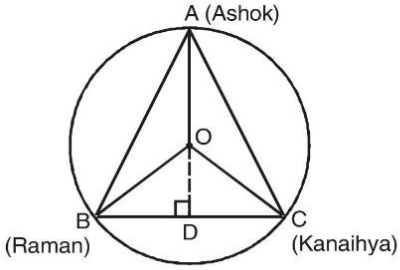
$x = \sqrt{1200} = 20\sqrt{3}$

Hence, distance between each of them is $20\sqrt{3}$ m.

(ii) Properties of circle, equilateral triangle and Pythagoras theorem.

2. ***A circular park of radius 10 m is situated in a colony. Three students Ashok, Raman and Kanaihya are standing at equal distances on its circumference each having a toy telephone in his hands to talk each other about Honesty, Peace and Discipline. Find the length of the string of each phone.***

Sol. Let us assume, A, B and C be the position of three students Ashok, Raman and Kanaihya respectively on the circumference of the circular park with centre O and radius 10 m. Since the centre of circle coincides with the centroid of the equilateral ΔABC.



$\therefore$ Radius of circumscribed circle $= \frac{2}{3}AD$

$\Rightarrow \quad 10 = \frac{2}{3}AD$

$\Rightarrow \quad AD = 15$ m

Now, $AD \perp BC$ and let $AB = BC = CA = x$

$\Rightarrow \quad BD = CD = \frac{1}{2}BC = \frac{x}{2}$

In rt. $\angle$ed ΔBDA, $\angle D = 90°$

$$AB^2 = BD^2 + AD^2$$

$$x^2 = \frac{x^2}{4} + 225$$

$$x^2 - \frac{x^2}{4} = 225$$

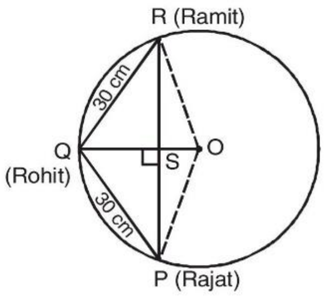
$\Rightarrow \quad x^2 = 225 \times \frac{4}{3} = 300$

$\Rightarrow \quad x = 10\sqrt{3}$ m

Thus, the length of each string is $10\sqrt{3}$ m.

3. ***Three scouts Rajat, Rohit and Ramit in the cultural show holded three stringed balloons with a message 'Stop Child Labour'. Keeping themselves on the boundary of a circle of radius 25 cm, each scout holded the string tightly. Find the distance between Rajat and Ramit, when distance between Rajat and Rohit and Rohit and Ramit is 30 cm.***

Sol. Let P, Q and R be the position of Rajat, Rohit and Ramit respectively.



Let O be the centre of circle. Points P and R are equidistant from Q.

Now, OQ is the perpendicular bisector of PR.

Let OQ bisect PR at S

In rt. ΔPSQ, $\angle S = 90°$

$$PS^2 = PQ^2 - SQ^2$$

$\Rightarrow \quad PS^2 = 900 - SQ^2 \qquad ...(i)$

Also, in rt. ΔOSP, we have

$$PS^2 = OP^2 - OS^2$$

$\Rightarrow \quad = 625 - (OQ - SQ)^2$

$\Rightarrow \quad 900 - SQ^2 = 625 - (25 - SQ)^2$

$= 625 - 625 - SQ^2 + 50\,SQ$

$\Rightarrow \quad 50\,SQ = 900 \Rightarrow SQ = 18$ cm $\quad ...(ii)$

From (*i*) and (*ii*), we have

$$PS^2 = 900 - (18)^2 = 576$$

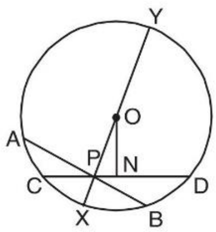
$\Rightarrow \quad PS = 24$ cm

$\Rightarrow \quad PR = 2 \times 24 = 48$ cm

Thus, distance between Rajat and Ramit is 48 cm.

4. ***Prove that among all the chords of a circle passing through a given point inside the circle that one is smallest which is perpendicular to the diameter passing through the point.***

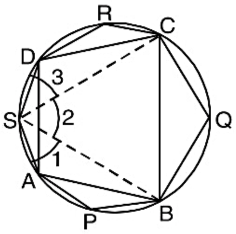
Sol. Let P be the given point inside a circle with centre O. Draw the chord AB which is perpendicular to the diameter XY through P. Let CD be any other chord through P. Draw ON perpendicular to CD from O. Then ΔONP is a right triangle (see fig.). Therefore, its hypotenuse OP is larger than ON. We know that the chord nearer to the centre is larger



than the chord which is farther to the centre. Therefore, CD > AB. In other words, AB is the smallest of all chords passing through P.

5. *Prove that the sum of the angles in the four segments of a circle exterior to a cyclic quadrilateral is equal to six right angles.*

Sol. ***Given:*** A cyclic quadrilateral ABCD and ∠P, ∠Q, ∠R and ∠S are four angles in the four external segments.



To Prove: ∠P + ∠Q + ∠R + ∠S = 6 right angles.

Const.: Join SB and SC.

Proof: Since sum of opposite angles of a cyclic quadrilateral is 180°

∴ In cyclic quad. SAPB, we have

$$\angle 1 + \angle P = 180° \quad ...(i)$$

Similarly, in cyclic quad. SBQC, we have

$$\angle 2 + \angle Q = 180° \quad ...(ii)$$

and in cyclic quad. SCRD, we have

$$\angle 3 + \angle R = 180° \quad ...(iii)$$

Adding (*i*), (*ii*) and (*iii*), we obtain

$$\angle 1 + \angle 2 + \angle 3 + \angle Q + \angle R + \angle P = 180° + 180° + 180°$$

$$\angle S + \angle Q + \angle R + \angle P = 540°$$

or ∠P + ∠Q + ∠R + ∠S = 6 × 90° = 6 right angles.

6. *Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half of the difference of the angles subtended by the chords AC and DE at the centre.*

Sol. Here, we have to prove that

$$\angle ABC = \frac{1}{2}(\angle AOC - \angle DOE),$$

where O is the centre of the circle.

Consider ΔOAD and ΔOCE

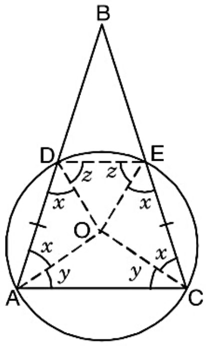
OA = OC
OD = OE } [radii of same circle]

AD = CE [given]

By using SSS congruence rule, we have

ΔOAD ≅ ΔOCE

⇒ ∠AOD = ∠COE



Also, ΔOAD and ΔOCE are isosceles triangles

∴ ∠OAD = ∠ODA = ∠OCE = ∠OEC = x (say)

In ΔOAC, we have OA = OC = r

⇒ ∠OAC = ∠OCA = y (say) [∠s opp. to equal sides of a Δ]

∴ ∠AOC = 180° – 2y ...(*i*)

In ΔODE, we have OD = OE = r

⇒ ∠ODE = ∠OED = z (say)

∴ ∠DOE = 180° – 2z ...(*ii*)

Since ACED is a cyclic quadrilateral

∴ ∠ACE + ∠ADE = 180°

$$\Rightarrow x + y + x + z = 180°$$

$$\Rightarrow 2x + y + z = 180° \quad ...(iii)$$

In ΔBDE, we have

∠BDE + ∠BED + ∠DBE = 180°

$$\Rightarrow 180° – (x + z) + 180° – (x + z) + \angle DBE = 180°$$

$$\Rightarrow \angle DBE = 2x + 2z – 180° = 180° – y + z – 180° = z – y \quad ...(iv)$$

[using (*iii*)]

From (*i*) and (*ii*), we have

$$\angle AOC – \angle DOE = 180° – 2y – 180° + 2z = 2(z – y)$$

$$\frac{1}{2}(\angle AOC – \angle DOE) = z – y \quad ...(v)$$

From (*iv*) and (*v*), we have

$$\angle DBE = \frac{1}{2}(\angle AOC – \angle DOE)$$

or

$$\angle ABC = \frac{1}{2}(\angle AOC – \angle DOE).$$