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MATHEMATICS CBSE

CIRCLES

SET-03

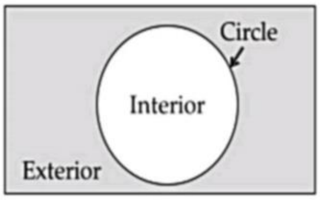
Syllabus

- *Through examples, arrive at definition of circle and related concepts– radius, circumference, diameter, chord, arc, secant, sector, segment, subtended angle.*
- *(Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse.*
- *(Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and conversely, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*
- *(Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre (or their respective centres) and conversely.*
- *(Prove) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*
- *(Motivate) Angles in the same segment of a circle are equal.*
- *(Motivate) The sum of either of the pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.*

Revision Notes

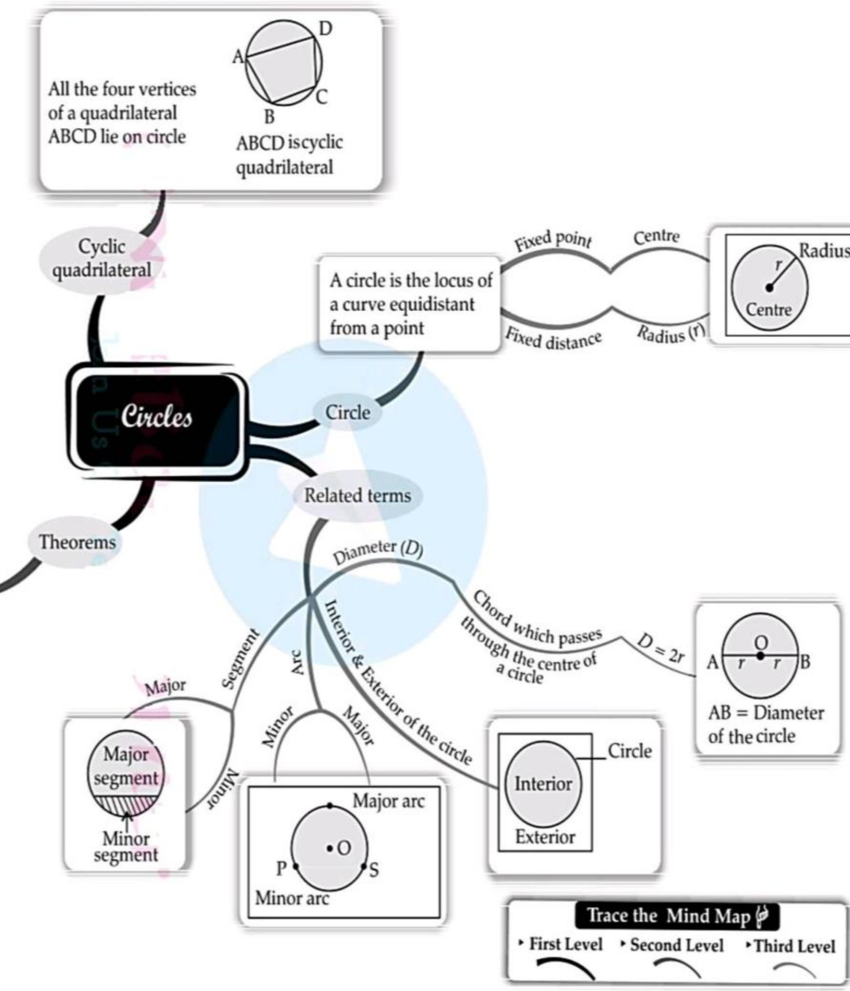
Basic Properties

- A circle is a collection (set) of all those points in a plane, each one of which is at a constant distance from a fixed point in the plane.
- The fixed point is called the centre and the constant distance is called the radius of the circle.
- All the points lying inside a circle are called its interior points and all those points which lie outside the circle are called its exterior points.
- The collection (set) of all interior points of a circle is called the interior of the circle while the collection of all exterior points of a circle is called the exterior of the circle.



- In a circle, equal chords subtend equal angles at the centre.

Statement	Figure
1. Equal chords of a circle subtend equal angles at the centre.	$AB = CD$ then $\angle AOB = \angle COD$
2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.	$\angle AOB = \angle COD$ then $AB = CD$
3. The perpendicular from the centre of a circle to a chord bisects the chord.	$OM \perp AB$ then $AM = MB$
4. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.	If $AM = MB$ then $OM \perp AB$
5. Equal chords of a circle are equidistant from the centre.	$AB = CD$ then $OL = OM$
6. Chords equidistant from the centre of a circle are equal in length.	If $OL = OM$ then $AB = CD$
7. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.	$\angle POQ = 2\angle PAQ$
8. Angles in the same segment of a circle are equal.	$\angle RPS = \angle RQS$
9. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.	$\angle P + \angle R = 180°$ $\angle Q + \angle S = 180°$
10. If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is cyclic.	If $\angle P + \angle R = 180°$ $\angle Q + \angle S = 180°$ then PQRS is a cyclic quadrilateral



- The chords corresponding to congruent arcs are equal.
- If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
- If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent.
- An infinite number of circles can be drawn through a given point P.
- An infinite number of circles can be drawn through the two given points.
- Only one circle can be drawn through the three given non-collinear point.
- Perpendicular bisectors of two chords of a circle intersect each other at the centre of the circle.
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

Cyclic Quadrilaterals

- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are con-cyclic, *i.e.*, lie on the same circle.
- If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.
- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- **Concentric Circles :** Circles with a common centre are called concentric circles.
- The degree measure of a semi-circle is 180°.
- The degree measure of a circle is 360°.
- The degree measure of a major arc is $(360° - \theta)$, where θ is the degree measure of the corresponding minor arc.
- Area of a circle $= \pi r^2$ sq. units

(A) Objective Questions — 1 Mark Each

Stand Alone MCQs

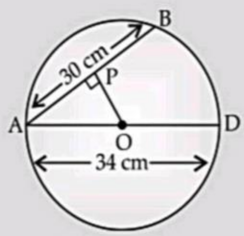
Q. 1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is

(A) 17 cm (B) 15 cm
(C) 4 cm (D) 8 cm

A [NCERT Exemp.]

Ans. Option (D) is correct.

Explanation:



Draw $OP \perp AB$

We know that the perpendicular from the centre of the circle bisects the chord.

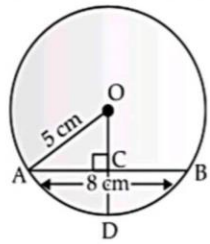
Therefore, $AP = \frac{1}{2} \times AB = \frac{1}{2} \times 30 = 15$ cm

Radius $= OA = \frac{1}{2} \times 34 = 17$ cm

In right ΔOPA, we have

$$OP = \sqrt{OA^2 - AP^2}$$
$$= \sqrt{17^2 - 15^2}$$
$$= \sqrt{289 - 225}$$
$$= \sqrt{64} = 8 \text{ cm}$$

Q. 2. In the given figure, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to



(A) 2 cm (B) 3 cm
(C) 4 cm (D) 5 cm

A [NCERT Exemp.]

Ans. Option (A) is correct.

Explanation: The perpendicular drawn from the centre to a chord bisect the chord,

$$AC=\frac{1}{2}\times AB=\frac{1}{2}\times 8=4 \text{ cm}$$

$$OC=\sqrt{OA^2-AC^2}$$
$$=\sqrt{5^2-4^2}$$
$$=\sqrt{25-16}$$
$$=\sqrt{9}=3$$

OC = 3 cm

Now, CD = OD – OC

= 5 cm – 3 cm = 2 cm

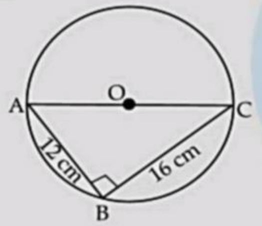
Q. 3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to *BC*, then the radius of the circle passing through the points A, B and C is :

(A) 6 cm (B) 8 cm
(C) 10 cm (D) 12 cm

A [NCERT Exemp.]

Ans. Option (C) is correct.

Explanation:



AB is perpendicular to BC, therefore, ABC is right triangle.

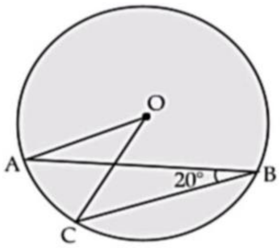
In right triangle ABC, we have

$$AC=\sqrt{AB^2+BC^2}$$
$$=\sqrt{12^2+16^2}$$
$$=\sqrt{144+256}$$
$$=\sqrt{400}=20$$

Therefore, AC = 20 cm (Diameter of circle)

$$\text{Radius}=\frac{1}{2}\times 20=10 \text{ cm}$$

Q. 4. In the given figure, if $\angle ABC$ = 20°, then $\angle AOC$ is equal to :



(A) 20° (B) 40°
(C) 60° (D) 10°

A [NCERT Exemp.]

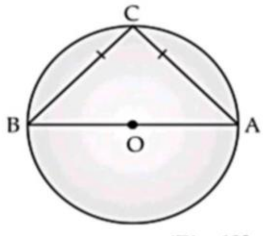
Ans. Option (B) is correct.

Explanation: Arc AC of a circle subtends AOC at the centre O and ABC at a point B on the remaining part of the circle.

Since, the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle AOC = 2\angle ABC$$
$$\angle AOC = 2\times 20^\circ = 40^\circ$$

Q. 5. In the given figure, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to :



(A) 30° (B) 60°
(C) 90° (D) 45°

A [NCERT Exemp.]

Ans. Option (D) is correct.

Explanation: As AOB is a diameter of the circle,

$\angle C=90^\circ$ [Angles in a semi-circle is 90°]

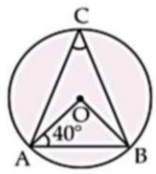
Now, AC = BC

$\angle A=\angle B$ [Angles opposite to equal sides of triangle are equal]

Using angle-sum property of a triangle, we have

$$\angle A+\angle B+\angle C=180^\circ$$
$$\angle A+\angle A+90^\circ=180^\circ$$
$$2\angle A=180^\circ-90^\circ$$
$$\angle A=\frac{90^\circ}{2}=45^\circ$$

Q. 6. In the given figure, if $\angle OAB$ = 40°, then $\angle ACB$ is equal to :



(A) 50° (B) 40°
(C) 60° (D) 70°

A [NCERT Exemp.]

Ans. Option (A) is correct.

Explanation: In ΔOAB

OA = OB [Radii of circle]

[Angles opposite to equal sides of a triangle are equal.]

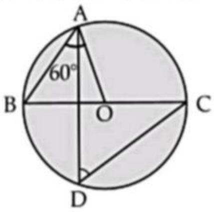
Therefore,

$$\angle AOB=180^\circ-(40^\circ+40^\circ)=100^\circ$$

Since, the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle ACB=\frac{1}{2}\angle AOB=\frac{1}{2}\times 100^\circ=50^\circ$$

Q. 7. In the given figure, BC is a diameter of the circle and ∠BAO = 60°. Then ∠ADC is equal to :



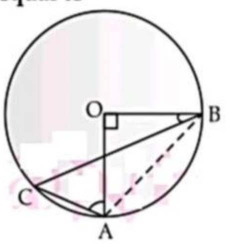
(A) 30° (B) 45°
(C) 60° (D) 120°

A [NCERT Exemp.]

Ans. Option (C) is correct.

> *Explanation:* In ΔOAB,
> OA = OB [Radii of the same circle]
> ∠ABO = ∠BAO
> [Angles opposite to equal sides are equal]
> ∠ABO = ∠BAO = 60° [Given]
> Now, ∠ADC = ∠ABC = 60°
> [Angles in the same segment of a circle are equal.]
> Therefore, ∠ADC = 60°

Q. 8. In the given figure, ∠AOB = 90° and ∠ABC = 30°, then ∠CAO is equal to



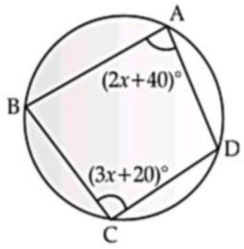
(A) 30° (B) 45°
(C) 90° (D) 60°

A [NCERT Exemp.]

Ans. Option (D) is correct.

> *Explanation:*
> In ΔOAB, we have
> $OA = OB$ [Radii of the same circle]
> $\angle OAB = \angle OBA$
> In triangle OAB, we have
> $\angle OAB + \angle OBA + \angle AOB = 180°$
> [Sum of angles of a triangle is 180°]
> $2\angle OAB = 180° - \angle AOB$
> $= 180° - 90°$
> $\angle OAB = \frac{1}{2} \times 90° = 45°$
> Also, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 90° = 45°$
> Now, in ΔCAB we have
> $\angle CAB = 180° - (\angle ABC + \angle ACB)$
> $= 180° - (30° + 45°) = 105°$
> Now, $\angle CAO = \angle CAB - \angle OAB$
> $\angle CAO = 105° - 45° = 60°$

Q. 10. In the given figure, ABCD is a cyclic quadrilateral in which ∠A = (2x + 40°) and ∠C = (3x + 20)°, then the value of *x* is:

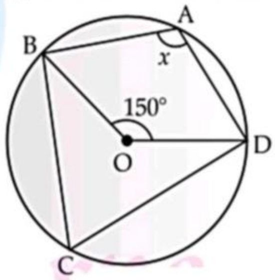


(A) 20° (B) 40°
(C) 24° (D) 48°

Ans. Option (C) is correct.

> *Explanation:* We know that the sum of opposite angles of a cyclic quadrilateral are 180°.
> ∴ ∠A + ∠C = 180°
> ⇒ 2x + 40 + 3x + 20 = 180°
> ⇒ 5x = 180° – 60° = 120°
> ⇒ x = 24°

Q. 11. In the given figure, O is the centre of a circle and ∠BOD = 150°, then the value of *x* is:

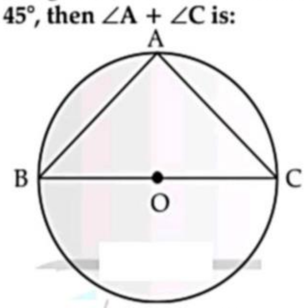


(A) 105° (B) 115°
(C) 100° (D) 110°

Ans. Option (A) is correct.

> *Explanation:*
> ∠BOD = 150° (Given)
> ∴ Reflex ∠BOD = 360° – 150° = 210°
> So, $x = \frac{1}{2} \times \text{Reflex } \angle BOD$
> $= \frac{1}{2} \times 210° = 105°$

Q. 12. In the given figure, BOC is a diameter of a circle and ∠B = 45°, then ∠A + ∠C is:



(A) 45° (B) 60°
(C) 90° (D) 135°

Ans. Option (D) is correct.

Explanation: Since an angle in a semi-circle is a right angle,

$$\angle A = 90^\circ$$

and $\angle B = 45^\circ$

Then, $\angle A + \angle B + \angle C = 180^\circ$

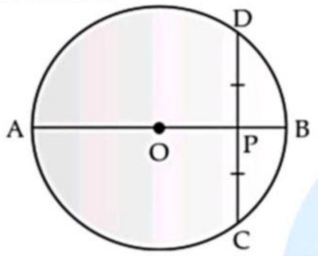
(Angle sum property of a triangle)

$\therefore \quad \angle A + \angle C = 180^\circ - \angle B$

$= 180^\circ - 45^\circ$

$= 135^\circ$

Q. 13. In the given figure, O is the centre of a circle and diameter AB bisects the chord CD at a point P such that CP = PD = 10 cm and PB = 6 cm, then the radius of the circle is:



(A) 12.3 cm (B) 11.3 cm

(C) 13 cm (D) 10.3 cm

Ans. Option (B) is correct.

Explanation: Let the radius of the circle be r cm, then OC = OD = OB = r cm, OP = $(r-6)$ cm

But CP = PD = 10 cm (given)

Since AB bisects the chord CD at a point P, then $\angle OPD = 90^\circ$.

$\therefore$ In ΔOPD, $OD^2 = OP^2 + PD^2$

(Using Pythagoras theorem)

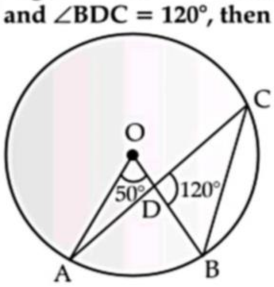
$r^2 = (r-6)^2 + (10)^2$

$\Rightarrow \quad r^2 = r^2 + 36 - 12r + 100$

$\Rightarrow \quad 12r = 136$

$\Rightarrow \quad r = 11.3$ cm

Q. 14. In the given figure, O is the centre of a circle, ∠AOB = 50° and ∠BDC = 120°, then ∠OBC is:



(A) 45° (B) 60°

(C) 35° (D) 90°

Ans. Option (C) is correct.

Explanation:

$\because \quad \angle AOB = 50^\circ$ (Given)

$\therefore \quad \angle DCB = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 50^\circ$

$= 25^\circ$

Now, in ΔDBC,

$\angle BDC + \angle DCB + \angle DBC$

$= 180^\circ$

(Angle sum property)

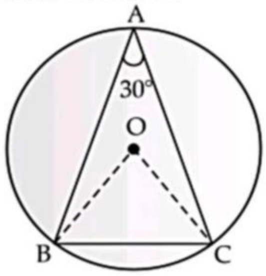
$\Rightarrow 120^\circ + 25^\circ + \angle DBC = 180^\circ$

$\Rightarrow \quad \angle DBC = 180^\circ - (120^\circ + 25^\circ)$

$= 180^\circ - 145^\circ$

$= 35^\circ$

Q. 15. In the given figure, O is the centre of a circle and ∠BAC = 30°, then ∠OBC is:



(A) 60° (B) 45°

(C) 40° (D) 70°

Ans. Option (A) is correct.

Explanation:

Here, $\angle BAC = 30^\circ$ (Given)

Then, $\angle BOC = 2 \times \angle BAC$

$= 2 \times 30^\circ = 60^\circ$

$= 2 \times 30^\circ = 60^\circ$

Let $\angle OBC$ be x then $\angle OCB$ also be x

$\because \quad OB = OC = \text{Radius}$

Now, in ΔOBC,

$\angle BOC + \angle OBC + \angle OCB = 180^\circ$

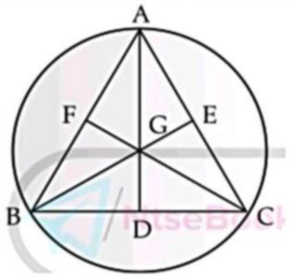
$\Rightarrow \quad 60^\circ + x + x = 180^\circ$

$\Rightarrow \quad x = 60^\circ$

Case-based MCQs

I. Read the following text and answer the following questions on the basis of the same:

A circular park of radius 20 m is situated in a colony. Three boys Anant, Bibhav and Chandra are sitting at equal distances on its boundary, each having a toy telephone in his hands to talk to each other.



Here, A, B and C be the positions of Anant, Bibhav and Chandra and also let D, E and F are the medians of ΔABC and G be its centroid. Given answer the following questions.

Q. 1. ΔABC is a/an .... triangle.

(A) Equilateral (B) Right
(C) Isosceles (D) Scalene

Ans. Option (A) is correct.

> *Explanation:* Here, $\widehat{AB} = \widehat{BC} = \widehat{CA}$
> $\Rightarrow$ Chord AB = Chord BC = Chord CA
> $\therefore$ ABC is an equilateral triangle.

Q. 2. The length of GA is:

(A) 10 m (B) 12 m
(C) 20 m (D) 15 m

Ans. Option (C) is correct.

> *Explanation:* In an equilateral triangle, the centroid coincides with its circumcentre
> $\therefore$ GA = GB = GC = 20 m.

Q. 3. The length of GD is:

(A) 12 m (B) 10 m
(C) 20 m (D) 15 m

Ans. Option (B) is correct.

> *Explanation:* Since, the centroid of a triangle divides a median in the ratio 2:1, then
> $$\frac{GA}{GD} = \frac{2}{1}$$
> $$\Rightarrow \quad \frac{20}{GD} = \frac{2}{1}$$
> $$\Rightarrow \quad GD = 10 \text{ m}$$

Q. 4. The length of BD is:

(A) $20\sqrt{2}$ m (B) $10\sqrt{2}$ m
(C) $20\sqrt{3}$ m (D) $10\sqrt{3}$ m

Ans. Option (D) is correct.

> *Explanation:* In right ΔBDG,
> $$(BG)^2 = (BD)^2 + (GD)^2$$
> $$\Rightarrow \quad (20)^2 = (BD)^2 + (10)^2$$
> $$\Rightarrow \quad BD = \sqrt{400 - 100}$$
> $$= \sqrt{300}$$
> $$= 10\sqrt{3} \text{ m}$$

Q. 5. The length of each telephone string is:

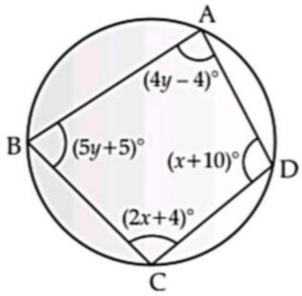
(A) $10\sqrt{3}$ m (B) $20\sqrt{3}$ m
(C) $12\sqrt{3}$ m (D) $15\sqrt{3}$ m

Ans. Option (B) is correct.

> *Explanation:* $BC = 2 \times BD$
> $$= 2 \times 10\sqrt{3}$$
> $$= 20\sqrt{3} \text{ m.}$$

II. Read the following text and answer the questions given below:

Four boys are playing with a ball in a circular park. The positions of each boy is represented by A, B, C and D in the following diagram.



Give answer the following questions:

Q. 1. The values of x and y are:

(A) $x = 40$ and $y = 25$
(B) $x = 25$ and $y = 40$
(C) $x = 50$ and $y = 15$
(D) $x = 30$ and $y = 35$

Ans. Option (A) is correct.

> *Explanation:* Since, the opposite angles of a cyclic quadrilateral are supplementary.
> $$\therefore \quad \angle A + \angle C = 180°$$
> $$\Rightarrow \quad 4y - 4 + 2x + 4 = 180°$$
> $$\Rightarrow \quad x + 2y = 90° \quad ...(i)$$
> and $$\angle B + \angle D = 180°$$
> $$\Rightarrow \quad 5y + 5 + x + 10 = 180°$$
> $$\Rightarrow \quad x + 5y = 165° \quad ...(ii)$$
> On solving, we get $x = 40$ and $y = 25$

Q. 2. The value of $\angle A$ is:

(A) 50° (B) 96°
(C) 84° (D) 60°

Ans. Option (B) is correct.

> *Explanation:*
> $$\angle A = 4y - 4$$
> $$= 4 \times 25 - 4 = 96°$$

Q. 3. The value of $\angle C$ is:

(A) 84° (B) 50°
(C) 60° (D) 130°

Ans. Option (A) is correct.

> *Explanation:*
> $$\because \quad \angle A + \angle C = 180° \quad \text{[From eq. (i)]}$$
> $$\Rightarrow \quad 96° + \angle C = 180°$$
> $$\Rightarrow \quad \angle C = 180° - 96° = 84°$$

Q. 4. The value of $\angle B$ is:

(A) 100° (B) 60°
(C) 120° (D) 130°

Ans. Option (D) is correct.

> *Explanation:*
> $$\angle B = (5y + 5)°$$
> $$= (5 \times 25 + 5)° = 130°$$

Q. 5. The sum of ∠A and ∠B is:

(A) 186° **(B)** 225°

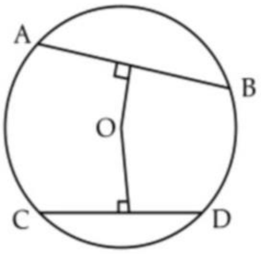
(C) 226° **(D)** 230°

Ans. Option (C) is correct.

> *Explanation:* From questions (ii) and (iv),
> $$\angle A + \angle B = 96^\circ + 130^\circ = 226^\circ$$

III. Read the following text and answer the questions given below:

Rohan draws a circle of radius 10 cm with the help of compass and scale. He also draws two chords, AB and CD in such a way that AB and CD are 6 cm and 8 cm from the centre O. Now, he has some doubts that are given below. Help him out by answering these questions:



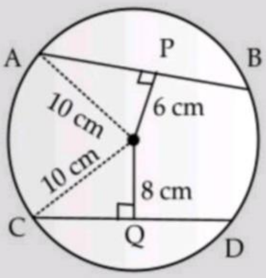
Q. 1. What is the length of AB ?

(A) 12 cm **(B)** 11 cm

(C) 16 cm **(D)** 8 cm

Ans. Option (C) is correct.

> *Explanation:* Join OA & OC.
>
> Since perpendicular from the centre bisects the chord.



> Thus $AP = BP = \frac{1}{2}AB$
>
> and $CQ = QD = \frac{1}{2}CD$
>
> ∴ $AP^2 = OA^2 - OP^2$ (By Pythagoras theorem)
>
> $= 10^2 - 6^2 = 64$
>
> ∴ $AP = 8$ cm
>
> *i.e.,* $AB = 2\,AP = 2 \times 8 = 16$ cm

Q. 2. What is the length of CD ?

(A) 10 cm **(B)** 12 cm

(C) 16 cm **(D)** 21 cm

Ans. Option (B) is correct.

> *Explanation:*
>
> $CQ^2 = OC^2 - OQ^2$ (By Pythagoras theorem)
>
> $= (10)^2 - (8)^2$
>
> $= 100 - 64$
>
> $= 36$ cm
>
> ∴ $CQ = 6$ cm
>
> Now, $CD = 2 \times CQ$
>
> $= 2 \times 6 = 12$ cm

Q. 3. A circle divides the plane, on which it lies, in _________ parts

(A) 1 **(B)** 2

(C) 3 **(D)** 4

Ans. Option (B) is correct.

Q. 4. A quadrilateral is called cyclic if all the four vertices of it lie on a _________

(A) Circle **(B)** Quadrilateral

(C) Pentagon **(D)** Triangle

Ans. Option (A) is correct.

Q. 5. Which statement is not true ?

(A) Equal chords of a circle subtend equal angles at the centre

(B) The perpendicular from the centre of a circle to a chord bisects the chord.

(C) Angles in the same segment of a circle are equal.

(D) The sum of either pair of opposite angles of a cyclic quadrilateral is 90°.

Ans. Option (D) is correct.

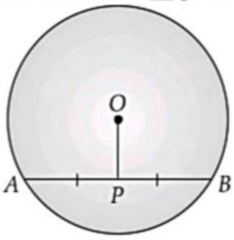
> *Explanation:* The sum of either pair of opposite angles of a cyclic quadrilateral is 90°. This statement is false as sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

(B) SUBJECTIVE QUESTIONS

Very Short Answer Type Questions

(1 Mark Each)

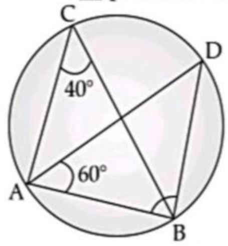
Q. 1. In the given figure, O is the centre of the circle and PA = PB. Find ∠OPA. R [Board Term II, 2013]



Sol. Given, $PA = PB$; $OP \perp AB$

Hence, $\angle OPA = 90°$

Q. 2. In the given figure, A, B, C and D are the points on a circle such that ∠ACB = 40° and ∠DAB = 60°, then find the measure of ∠DBA. U [Board Term II, 2012, Set-01]



Sol. $\angle ACB = \angle ADB$

[Angles in the same segment]

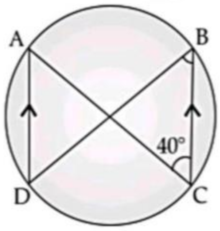
∴ $\angle ADB = 40°$

Now, in ΔADB,

$\angle ADB + \angle DBA + \angle BAD = 180°$

or, $40° + \angle DBA + 60° = 180°$ or, $\angle DBA = 80°$

Q. 3. In the given figure, AD || BC and ∠BCA = 40°. Find the measure of ∠DBC. R [Board Term II, 2012]



Sol. $\angle BDA = \angle BCA = 40°$

[Angles in the same segment]

Now, since AD || BC,

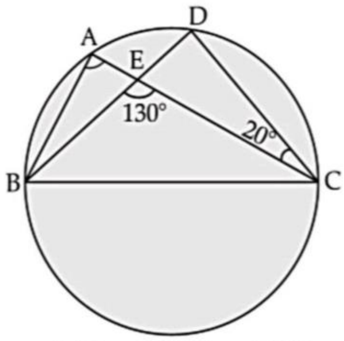
$\angle DBC = \angle BDA$

[Alternate interior angles]

∴ $\angle DBC = 40°$

AI **Q. 4. In the given figure, A, B, C and D are four points of a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.**

[Board Term II, 2012, KVS 2017, 2019, NCERT]



Sol. $\angle BEC = \angle EDC + \angle DCE$

(Exterior angle property)

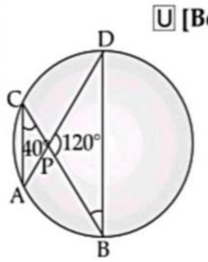
$130° = \angle EDC + 20°$

$\angle EDC = 110° = \angle BDC$

$\angle BAC = \angle BDC = 110°$

(Angles in the same segment)

AI **Q. 5. In the given figure, ∠ACP = 40° and ∠BPD = 120°, then Find ∠CBD.** U [Board Term II, 2012]



Sol. $\angle ADB = \angle ACB = 40°$

[∵ Angles in the same segment are equal]

Now, in ΔDPB

$\angle DPB + \angle DBP + \angle PDB = 180°$

[Angle sum property]

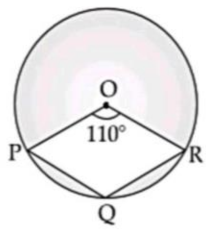
or, $120° + \angle DBP + 40° = 180°$

or, $\angle DBP = 180 - (120° + 40°)$

or, $\angle DBP = 20°$

∴ $\angle CBD = \angle PBD = 20°$

AI **Q. 6. In the given figure, if ∠POR is 110°, then find the value of ∠PQR.** R [Board Term II, 2012]



Sol. Reflex angle POR $= 360° - 110°$

$= 250°$

∴ By degree measure theorem,

$\angle PQR = \frac{1}{2}$(reflex angle POR)

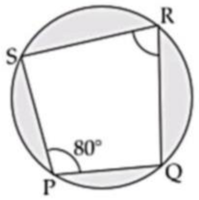
$= \frac{1}{2}(250°) = 125°$

Q. 7. What is sum of the opposite angles of a cyclic quadrilateral? R [Board Term II, 2011]

Sol. Sum of opposite angles of a cyclic quadrilateral is 180°.

Q. 8. In the given figure, quadrilateral PQRS is cyclic. If ∠P = 80°, then ∠R is equal to ....

R [Board Term I, 2011]



Sol. Since, quadrilateral PQRS is cyclic

∴ ∠P + ∠R = 180°

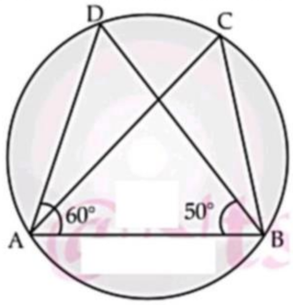
or, 80° + ∠R = 180°

or, ∠R = 100°

Short Answer Type Questions-I

(2 Marks Each)

AI **Q. 1. In the figure, if ∠DAB = 60°, ∠ABD = 50°, then find ∠ACB.** U [Board Term II, KVS 2016]



Sol. In ΔADB,

By angle sum property

∠ABD + ∠ADB + ∠BAD = 180°

∴ 50° + ∠ADB + 60° = 180°

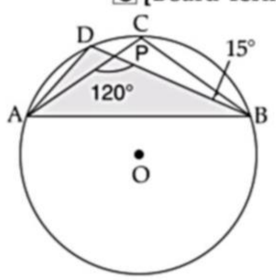
∴ ∠ADB = 180° – (50° + 60°)

= 70°

∴ ∠ACB = ∠ADB = 70°

(∵ angles in the same segment of a circle are equal)

AI **Q. 2. In the given figure, O is the centre of the circle and chord AC and BD intersect at P such that ∠APB = 120° and ∠PBC = 15°, find the value of ∠ADB.** U [Board Term II, KVS 2014]



∠PCB + ∠PBC = ∠APB

(exterior angle of a Δ is equal to the sum of two opposite angles) 1

∠PCB + 15° = 120°

∴ ∠PCB = 105°

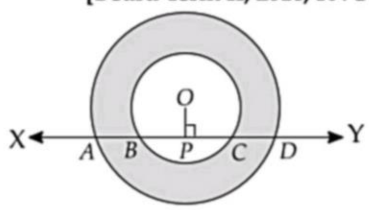
or, ∠ACB = 105°

or, ∠ADB = ∠ACB = 105°

[Angle in same segment] 1

Q. 3. If a line intersects two concentric circles with common centre O, at A, B, C and D. Prove that AB = CD. A [NCERT]

[Board Term II, 2016; KVS 2014; 2012]



Sol. Draw OP perpendicular to XY from the centre to a chord bisecting it.

OP ⊥ to chord BC.

or, BP = PC ...(i)

Similarly, AP = PD ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

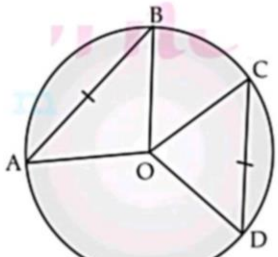
AP – BP = PD – PC

or, AB = CD **Hence Proved**

Q. 4. Prove that "equal chords of a circle subtend equal angles at the centres."

A [Board Term II, KVS 2016, 2012, NCERT]

Sol. Given, AB and CD are the chords of a circle with centre at O such that AB = CD



To Prove : ∠AOB = ∠COD

Proof : In ΔAOB and ΔCOD,

AO = CO (radii of same circle)

AB = CD (given)

BO = DO (radii of same circle)

ΔAOB ≅ ΔCOD (*SSS*)

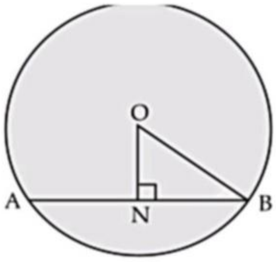
∴ ∠AOB = ∠COD (c.p.c.t.)

Hence Proved.

Q. 5. A chord of length 10 cm is at a distance of 12 cm from the centre of a circle. Find the radius of the circle. U [Board Term II, 2016]

Sol. Given, AB = 10 cm

ON = 12 cm



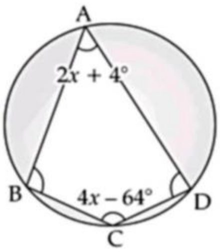
Also, ON ⊥ AB
and AN = BN
(∵ Perpendicular drawn from the centre of the circle to chord of circle bisects the chord)
In ΔONB,

$$OB^2 = ON^2 + NB^2$$

(By pythagoras theorem)

∴ $OB^2 = 12^2 + 5^2$ (∵ BN = 5 cm)
$= 144 + 25 = 169$
∴ OB = 13 cm
Hence, the radius of the circle is 13 cm.

Q. 6. In the given figure, find the value of x.
R [Board Term II, 2012]



Sol. In a cyclic quadrilateral,

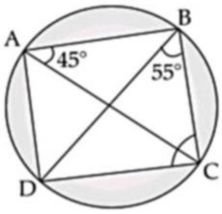
$$\angle A + \angle C = 180^\circ$$

(opposite angles of cyclic quadrilateral are supplementary)

or, $2x + 4^\circ + 4x - 64^\circ = 180^\circ$
or, $6x - 60^\circ = 180^\circ$
or, $6x = 180^\circ + 60^\circ = 240^\circ$
or, $x = \frac{240^\circ}{6}$
∴ $x = 40^\circ$

Q. 7. ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠DBC = 55° and ∠BAC = 45°, find ∠BCD. **R [Board Term II, 2012]**

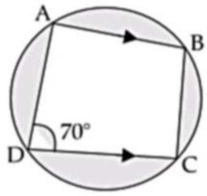
Sol.



∠BAC = ∠BDC = 45°,
(angles in the same segment)
In ΔDBC,
∠DBC + ∠BCD + ∠CDB = 180°
[Angle sum property]
or, 55° + ∠BCD + 45° = 180°
or, ∠BCD = 80°

AI Q. 8. ABCD is a cyclic quadrilateral in which AB || CD. If ∠D = 70°, find all the remaining angles.
R [Board Term II, 2012]

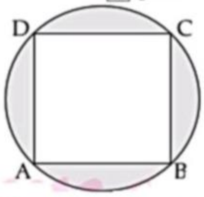
Sol.



Since, sum of the opposite pairs of angles in a cyclic quadrilateral is 180°.
Hence, ∠B + ∠D = 180°
or, ∠B = 180° – 70° = 110°
Again, AB || CD and AD is its transversal, so
∠A + ∠D = 180°
(Co-interior angles)
∠A + 70° = 180°
or, ∠A = 180° – 70° = 110°
and ∠A + ∠C = 180°
or, 110° + ∠C = 180°
∴ ∠C = 180° – 110 = 70°

Q. 9. Prove that a cyclic parallelogram is a rectangle.
A [NCERT],[KVS - 2014]

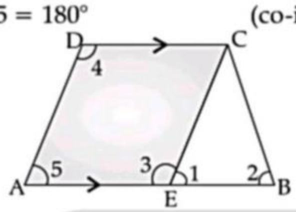
Sol.



ABCD is a parallelogram and opposite angles of it are equal.
∴ ∠B = ∠D
But, ∠B + ∠D = 180°
(Opp. angles of a cyclic quadrilateral)
or, 2∠D = 180°
or, ∠D = 90°
We know that, if one angle of a parallelogram is 90°, then it is a rectangle.
or, ABCD is a rectangle.

Q. 10. If non-parallel sides of a trapezium are equal. Prove that the trapezium is cyclic. **A [NCERT]**

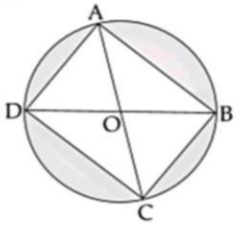
Sol. Draw CE || AD.
AECD is a parallelogram
CE = DA = CB
or, ∠1 = ∠2
∠4 + ∠5 = 180° (co-interior angles)



But, ∠5 = ∠1 (corresponding angles)
= ∠2
∴ ∠4 + ∠2 = 180°
or, ABCD is a cyclic trapezium.

Q. 11. If diagonals of a cyclic quadrilateral are diameters of the circle through the opposite vertices of the quadrilateral, prove that the quadrilateral is a rectangle. R **[Board Term II 2017]**

Sol. ABCD is a cyclic quadrilateral in which AC and BD are diameters



Since AC is diameter

$\therefore \quad \angle ABC = \angle ADC = 90°$ (angles of semicircle)

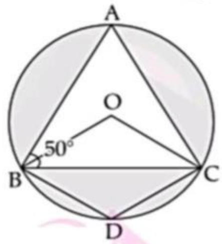
BD is the diameter

$\therefore \quad \angle BAD = \angle BCD = 90°$ (angles of semicircle)

Since all angles of quadrilateral ABCD are 90°.

$\therefore$ ABCD is a rectangle **Hence Proved.**

AI **Q. 12. In the given figure, O is the centre of the circle and BA = AC. If ∠ABC = 50°, find ∠BOC and ∠BDC.** A **[Board Term II, 2017]**



Sol. $AB = AC$ (Given)

$\therefore \quad \angle ABC = \angle ACB = 50°$ (isosceles Δ property)

By angle sum property of a triangle

$\angle BAC = 180° - \angle ABC - \angle ACB$

$= 180° - 50° - 50° = 80°$

$\therefore \quad \angle BOC = 2\angle BAC$ (angle at the centre is twice the angle at the circumference)

$= 2 \times 80° = 160°$

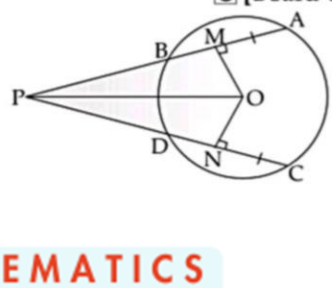
$\angle BDC + \angle BAC = 180°$ (Opp. angles of cyclic quadrilateral)

$\therefore \quad \angle BDC = 180° - 80° = 100°$

Short Answer Type Questions-II

(3 Marks Each)

Q. 1. In the given figure, AB and CD are two chords of a circle with centre O such that MP = NP. If OM ⊥ AB and ON ⊥ DC, show that AB = CD. U **[Board Term II, 2014]**



Sol. Construction : Join OP.

Proof : In ΔOMP and ΔONP,

$\angle OMP = \angle ONP = 90°$ (given)

$OP = OP$ (common)

$MP = NP$ (given)

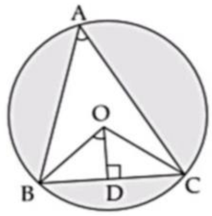
$\therefore \quad \Delta OMP \cong \Delta ONP$ (RHS)

$\therefore \quad OM = ON$ (c.p.c.t.)

$\therefore \quad AB = CD$ **Hence Proved**

(chords equidistant from the centre are equal)

Q. 2. If O is the circumcentre of a ΔABC and OD ⊥ BC, then prove that ∠BOD = ∠BAC. U **[Board Term II, 2013]**



Sol. Given : $OD \perp BC$

Proof : In ΔOBD and ΔOCD,

$OB = OC$ (radii)

$OD = OD$ (common)

$\angle ODB = \angle ODC$ (each 90°)

$\Delta OBD \cong \angle OCD$ (RHS rule)

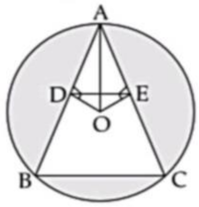
$\angle BOD = \angle COD$ (c.p.c.t.)

and $\angle BOC = 2\angle BOD$

But $\angle BOC = 2\angle BAC$ (angle of centre is twice the angle at the circumference)

or, $\angle BOD = \angle BAC$

Q. 3. In the given figure, AB and AC are two chords of circle whose centre is O. If OD ⊥ AB, OE ⊥ AC and AO bisects ∠DAE, prove that ΔADE is an isosceles triangle and ∠ABC = ∠ACB.



U **[Board Term II, 2016, 2012]**

Sol. In ΔAOD and ΔAOE,

$\angle OAD = \angle OAE$ (AO is bisector)

$\angle ADO = \angle AEO = 90°$ (Given)

$AO = AO$ (common)

$\therefore \quad \Delta ADO \cong \Delta AEO$ (By AAS)

$\therefore \quad AD = AE$ (c.p.c.t.)

i.e., ΔADE is an isosceles Δ.

Also $OD = OE$ (c.p.c.t.)

$\therefore \quad AB = AC$

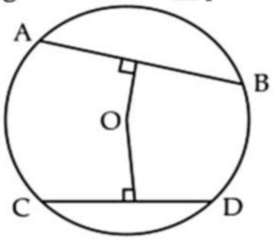
(chords equidistant from centre are equal)

$\angle ABC = \angle ACB$

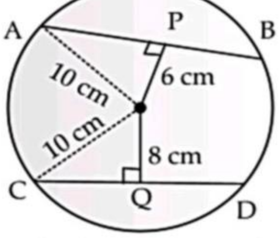
(isosceles Δ prop. for ΔABC) **Hence Proved**

[CBSE Marking Scheme, 2016]

Q. 4. In the given figure, AB and CD are two chords of a circle with centre O at a distance of 6 cm and 8 cm from O. If the radius of the circle is 10 cm, find the length of chords. U **[Board Term II, 2014]**



Sol. Join OA and OC.



Since, perpendicular from centre bisects the chord,

$\therefore$ $AP = BP = \frac{1}{2}AB$

and $CQ = QD = \frac{1}{2}CD$

In ΔOAP, by Pythagoras theorem,

$$AP^2 = OA^2 - OP^2$$
$$= 10^2 - 6^2 = 64$$

$\therefore$ $AP = 8$ cm or, $AB = 16$ cm

In ΔOQC,

$$CQ^2 = OC^2 - OQ^2$$
$$= 10^2 - 8^2$$

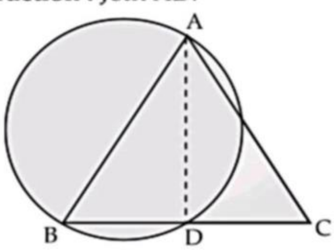
or, $CQ = 6$ cm or, $CD = 12$ cm

Q. 5. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter, bisects the third side. A **[Board Term II, 2016]**

Sol. Given, ΔABC is an isosceles triangle with $AB = AC$. A circle is drawn taking AB as the diameter which intersects the side BC at D.

To Prove that : $BD = DC$

Construction : Join AD.



Proof : $\angle ADB = 90°$

(angle in semi-circle is 90°)

$\angle ADB + \angle ADC = 180°$ (linear-pair)

or, $\angle ADC = 90°$

In ΔABD and ΔACD,

$AB = AC$ (given)

$\angle ADB = \angle ADC$ (proved)

$AD = AD$ (common)

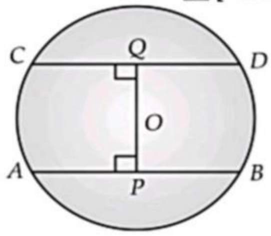
$\therefore$ $\Delta ABD \cong \Delta ACD$ [RHS Rule]

$\therefore$ $BD = DC$ (c.p.c.t)

Hence Proved.

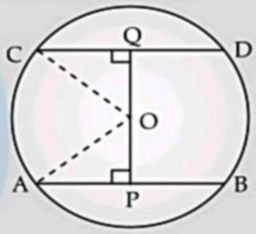
Q. 6. In the given figure, AB and CD are two parallel chords of a circle with centre O and radius 5 cm such that AB = 8 cm and CD = 6 cm. If OP is perpendicular to AB and OQ is perpendicular to CD, determine the length of PQ.

U **[Board Term II, 2016]**



Sol. Construction : Join *OA* and *OC*.

Since, perpendicular from centre of the circle to the chord bisects the chord.



$\therefore$ $AP = PB = \frac{1}{2}AB = 4$ cm

and, $CQ = QD = \frac{1}{2}CD = 3$ cm 1

In ΔOAP,

$OP^2 = OA^2 - AP^2$

(Pythagoras theorem)

or, $OP^2 = 5^2 - 4^2$

$= 25 - 16$

$= 9$

$\therefore$ $OP = 3$ cm ½

In ΔOCQ,

$OQ^2 = OC^2 - CQ^2$

(Pythagoras theorem)

$= 5^2 - 3^2$

$= 25 - 9$

$= 16$

$\therefore$ $OQ = 4$ cm ½

$\therefore$ $PQ = OP + OQ$

$= 3 + 4$

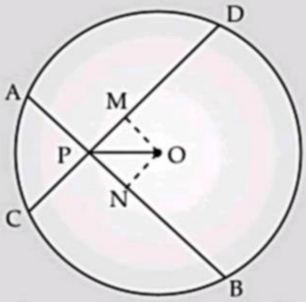
$= 7$ cm 1

[CBSE Marking Scheme, 2016]

AI **Q. 7. If two equal chords of a circle intersect within a circle, prove that the line segment joining the point of intersection to the centre makes equal angles with the chords.**

U **[NCERT][Board Term II, 2016]**

Sol. Let AB and CD be two equal chords intersecting at P.



1

Let O be the centre of the circle. Draw OM ⊥ CD and ON ⊥ AB.

In ΔOMP and ΔONP,

∠OMP = ∠ONP = 90° (By construction)

OM = ON

(equal chords are equidistant from the centre)

OP = OP (common)

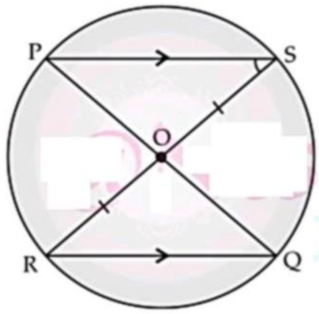
∴ ΔOPM ≅ ΔOPN (R.H.S.)

∴ ∠OPM = ∠OPN (c.p.c.t) **2**

Hence Proved

[CBSE Marking Scheme, 2016]

Q. 8. In the given figure, a diameter PQ of a circle bisects the chord RS at the point O. If PS is parallel to RQ, prove that RS is also a diameter of the circle.



A **[Board Term II, 2016]**

Sol. Given, PS || RQ

∴ ∠PSO = ∠QRO (alternate angles)

and OS = RO (given)

Also, ∠POS = ∠QOR (vertically opposite angles)

ΔPSO ≅ ΔQRO (ASA congruency rule)

OP = OQ (c.p.c.t.)

or, O is the mid-point of PQ.

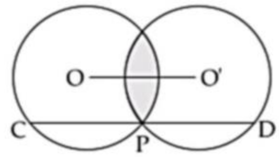
or, O is the centre of the circle.

Now, since RS passes through O, it means that RS passes through the centre of the circle.

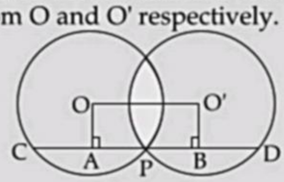
∴ RS is a diameter of the circle. **Hence Proved. 3**

Q. 9. Two circles whose centres are O and O' intersect at P. Through P, a line parallel to OO', intersecting the circles at C and D is drawn as shown. Prove that CD = 2 OO'

A **[Board Term II, 2015, 2014, NCERT Exemp.]**



Sol. Construction : Draw OA and O'B perpendicular to CD from O and O' respectively.



Proof : OA ⊥ CD

∴ OA bisects the chord CP (perpendicular from the centre to the chord bisects the chord)

∴ $AP = \frac{1}{2}CP$

or, CP = 2AP ...(i) **1**

Similarly, O'B ⊥ PD

∴ $BP = \frac{1}{2}PD$

or, PD = 2BP ...(ii)

CD = CP + DP

= 2AP + 2BP [from (i) and (ii)]

= 2(AP + BP) ...(iii) **1**

In quadrilateral ABO'O,

OA = O'B (two lines ⊥ to same line CD)

AB || OO' (Given)

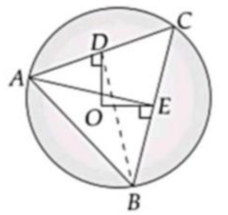
∴ ABO'O is || gm and AB || OO'.

(opp. sides of parallelogram are equal)

∴ CD = 2AB = 2OO' **1**

[CBSE Marking Scheme, 2015]

Q. 10. In the given figure, O is the centre of the circle, OD ⊥ AC, OE ⊥ BC and OD = OE. Show that ΔDBA ≅ ΔEAB.



U **[Board Term II 2017]**

Sol. OD = OE (given)

∴ AC = BC

(chords equidistant from the centre are equal)

In ΔACE and ΔBCD,

AC = BC (Prove above)

CE = CD $\left(\frac{1}{2}BC = \frac{1}{2}AC\right)$

∠C = ∠C (Common angle)

So, ΔACE ≅ ΔBCD (SAS Rule)

i.e., AE = BD (c.p.c.t) ....(i)

In ΔDBA and ΔEAB,

BD = AE (From (i))

$DA = EB \quad \left(\frac{1}{2}AC = \frac{1}{2}BC\right)$

$AB = AB$ (Common)

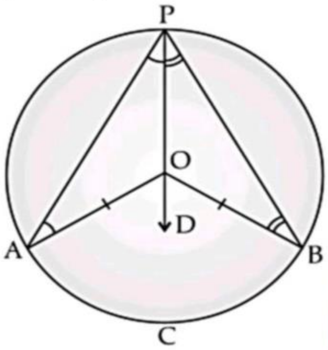
$\therefore \quad \Delta DBA \cong \Delta EAB$ (By *SSS*)

Hence Proved

Q. 11. Prove that 'The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.'

[Board Term-II, 2012, KVS-2016, NCERT]

Sol. Given : A circle having centre O. Arc ACB makes $\angle APB$ and $\angle AOB$ at circumference and centre of circle respectively.



To Prove : $\angle AOB = 2\angle APB$

Construction : Join PO and entered to the point D

Proof : Here, $\angle AOD = \angle OAP + \angle APO$ (Exterior angle property)

or, $\angle AOD = \angle APO + \angle APO$ $(OA = OP \therefore \angle OAP = \angle APO)$

or, $\angle AOD = 2\angle APO$ ...(i)

Similarly, $\angle DOB = \angle OPB + \angle OBP$ (Exterior angle property)

or, $\angle DOB = \angle OPB + \angle OPB$ $(OP = OB \therefore \angle OPB = \angle OBP)$

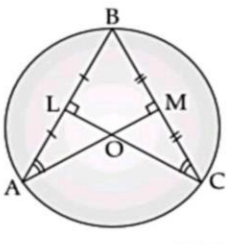
or, $\angle DOB = 2\angle OPB$ ...(ii)

Now, $\angle AOD + \angle DOB = 2\angle APO + 2\angle OPB$ [From (i) and (ii)]

$\angle AOB = 2(\angle APO + \angle OPB)$

$\angle AOB = 2\angle APB$ **Hence Proved.**

Q. 12. In the given figure, O is the centre of the circle and L and M are the mid-points of AB and CB respectively. If $\angle OAB = \angle OCB$, prove that BL = BM.



U [Board Term II 2017]

Sol. OL is a line from the centre to the mid-point of chord AB.

$\therefore$ OL is perpendicular to AB.

i.e., $\angle ALO = 90°$

OM is a line from the centre to the mid-point of chord BC.

$\therefore$ OM is perpendicular to BC.

i.e., $\angle CMO = 90°$

In ΔALO and ΔCMO,

$\angle ALO = \angle CMO = 90°$ (proved above)

$\angle LAO = \angle MCO$ (given)

$\therefore \quad AO = CO$ (radius)

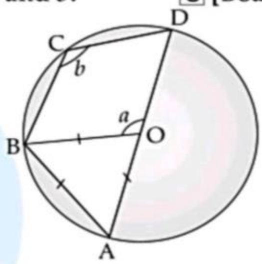
$\therefore \quad \Delta ALO \cong \Delta CMO$ (By AAS)

$\therefore \quad AL = CM$ (c.p.c.t)

$\therefore \quad BL = BM$

(L and M are mid-point of AB and CB respectively)

Q. 13. In the given figure, AB is a chord equal to the radius of the given circle with centre O. Find the values of *a* and *b*. **U [Board Term II, 2014]**



Sol. $OB = OA$ (radii of circle)

$OA = OB = AB$ (given)

$\therefore$ ΔOAB is an equilateral triangle.

$\therefore \quad \angle AOB = 60°$ **1**

(angles of equilateral Δ are 60° each)

$\therefore \quad a + \angle AOB = 180°$ (linear pair)

$\therefore \quad a + 60° = 180°$

$\therefore \quad a = 120°$

Reflex angle $BOD = 2\angle BCD$ **1**

(angle subtended by an arc at the centre is twice at the circumference)

$360° - a = 2b$

$360° - 120° = 2b$

$2b = 240°$

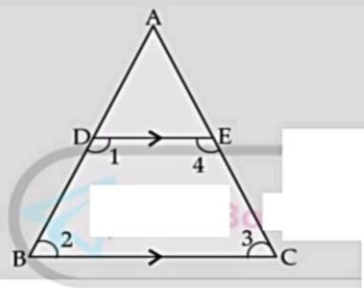
$b = 120°$ **1**

[CBSE Marking Scheme, 2014]

Q. 14. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

U [Board Term II , 2012, NCERT Exemp.]

Sol.



1

Given, DE ∥ BC

AB = AC

or, $\angle 2 = \angle 3$ ...(i)

(Angles opposite to equal sides to are always equal)

Since, DE is parallel to BC, the consecutive interior angles as supplementary

$\angle 1 + \angle 2 = 180°$ ...(ii)

and $\angle 3 + \angle 4 = 180°$ ...(iii)

$\therefore$ $\angle 1 + \angle 3 = 180°$ [By (i) and (ii)]

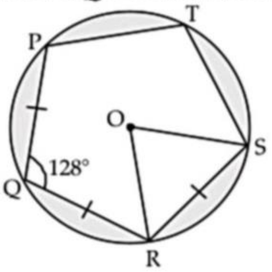
$\angle 2 + \angle 4 = 180°$ [By (i) and (iii)]

But these are opposite angles of a quadrilateral.

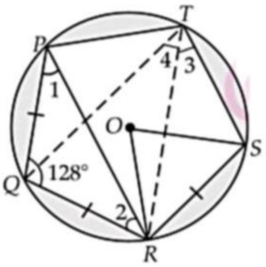
$\therefore$ BCED is a cyclic quadrilateral.

[CBSE Marking Scheme 2012]

AI Q. 15. In the given figure, PQ = QR = RS and ∠PQR = 128°. Find ∠PTQ, ∠PTS and ∠ROS. U **[NCERT]**



Sol. Given, PQ = QR = RS, ∠PQR = 128°



$$\angle 1 = \angle 2 = \frac{(180° - 128°)}{2}$$

$$= \frac{52°}{2} = 26°$$

$\therefore \angle PTQ = \angle QRP = 26°$

(angle in same segment)

$\angle PTS = 26° + \angle 4 + \angle 3$

$= 26° + 26° + 26°$

$= 78°$

$\angle ROS = 2\angle RTS$

(angle subtended by an arc at the centre is twice at circumference)

$= 2 \times 26° = 52°$

Long Answer Type Questions

(4 Marks Each)

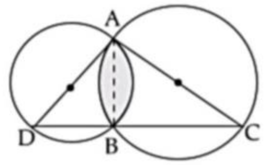
Q. 1. Two circles intersect at two points A and B. AD and AC are the diameters of the two circles. Prove that D, B and C are collinear.

R **[Board Term II, 2012, KVS 2016]**

OR

Two circles intersect at two points A and B. AD and AC are diameters of the two circles. Prove that B lies on the line segment DC. [NCERT]

Sol. Construction : Join AB



Proof : $\angle ABD = 90°$ (angle in a semi-circle)

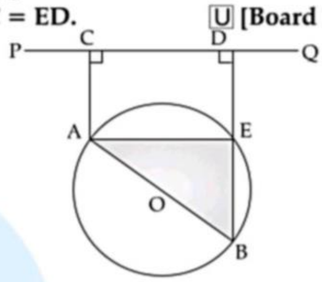
$\angle ABC = 90°$ (angle in a semi-circle)

$\angle ABD + \angle ABC = 180°$

$\therefore$ DBC is a line

$\therefore$ D, B and C are collinear. **Hence Proved.**

Q. 2. In the given figure, AB is a diameter of the circle with centre O. If AC and BD are perpendicular on a line PQ and BD meets the circle at E, then prove that AC = ED. U **[Board Term II, 2013]**



Sol. Proof : $\angle AEB = 90°$ (angle in semi-circle)

$\angle AEB + \angle AED = 180°$ (linear pair)

$\therefore$ $\angle AED = 90°$

$\angle EAC + \angle ACD + \angle CDE + \angle AED = 360°$

(sum of angles of a quad.)

or, $\angle EAC + 90° + 90° + 90° = 360°$

or, $\angle EAC = 360° - 270°$

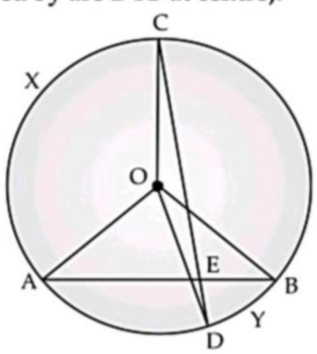
$= 90°$

Hence, each angle of quadrilateral is 90°.

$\therefore$ EACD is a rectangle

$\therefore$ AC = ED. **Hence Proved. 1**

Q. 3. In the given figure, AB and CD are two chords of a circle, with centre O, intersecting each other at point E, prove that ∠AEC is equal to $\frac{1}{2}$ (angle subtended by arc CXA at the centre + angle subtended by arc DYB at centre).



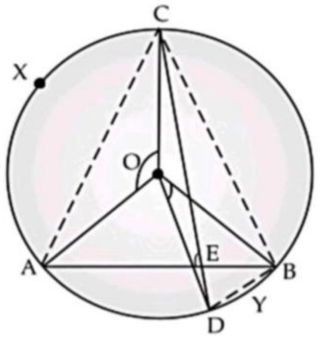
A **[Board Term II, 2016, NCERT Exemplar]**

Sol. Given : AB and CD are two chords of circle with centre O, which intersects at E.

To Prove that : $\angle AEC = \frac{1}{2}(\angle COA + \angle DOB)$

Construction : Join AC, BC and BD.

Proof : AC is a chord.



$\therefore \quad \angle AOC = 2\angle ABC$ ...(i)

(angle subtended by an arc at the centre is twice the angle of remaining circle)

Similarly, $\angle DOB = 2\angle DCB$ ...(ii)

(angle subtended by an arc at the centre is twice the angle of remaining circle)

Adding (i) and (ii), we get

$\angle AOC + \angle DOB = 2(\angle ABC + \angle DCB)$ ...(iii)

In ΔCEB,

$\angle AEC = \angle ECB + \angle CBE$

(exterior angle is sum of two opposite interior angles)

$\angle AEC = \angle DCB + \angle ABC$ ...(iv)

From (iii) and (iv), we get

$\angle AOC + \angle DOB = 2\angle AEC$

Hence, $\angle AEC = \frac{1}{2}(\angle COA + \angle DOB)$

Hence Proved.

AI Q. 4. PQ and RS are two parallel chords of a circle whose centre is O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between PQ and RS when they lie,

(i) On the same side of centre O.

(ii) On the opposite sides of centre O.

A **[Board Term II, 2015]**

Sol. Given, OP = OR = 10 cm (radii of same circle)

PQ = 16 cm,

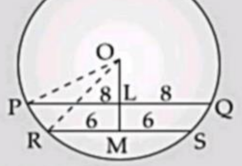
RS = 12 cm

Draw OL ⊥ PQ and OM ⊥ RS. 1

Since, perpendicular from the centre to the chord bisects the chord.

$\therefore \quad PL = LQ = \frac{1}{2}PQ = 8$ cm

$RM = MS = \frac{1}{2}RS = 6$ cm



In right triangle OLP,

$OP^2 = OL^2 + PL^2$

(By Pythagoras theorem)

$100 = OL^2 + 64$

$OL = \sqrt{100-64} = \sqrt{36}$ 1

OL = 6 cm

In right triangle OMR,

$OR^2 = OM^2 + RM^2$

(By Pythagoras theorem)

$100 = OM^2 + 36$

OM = 8 cm

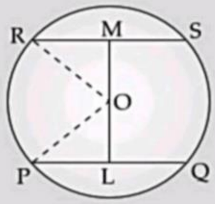
(i) If PQ and RS lie on same side of centre O.

Distance between PQ and RS

= LM = OM – OL

= 8 – 6 = 2 cm 1

(ii) If PQ and RS lie on opposite sides of centre O



Distance between PQ and RS

= LM

= OL + OM

= 6 + 8 cm

= 14 cm 1

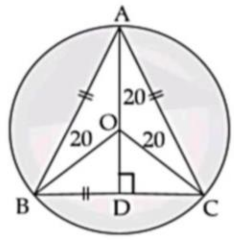
Q. 5. A circular park of radius 20 m is situated in a village. Three girls Rita, Sita and Gita are sitting at equal distance on its boundary each having a toy telephone in their hands to talk to each other. Find the length of the string of each phone. (There is no slack in the string). A

OR

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and Dayd are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

[NCERT]

Sol.



Here, A, B and C are the three points where three girls are sitting.

ΔABC is an equilateral triangle.

In an equilateral triangle, the circumcentre is the point of intersection of median.

$\therefore$ O divides AD in the ratio 2 : 1.

Hence, if AO = 20 m

then, OD = 10 m

Also, median is same as the altitude for an equilateral triangle.

In right ΔODC,

$OC^2 = OD^2 + DC^2$

or, $20^2 = 10^2 + DC^2$

or, $DC^2 = 400 - 100 = 300$

or, $DC = 10\sqrt{3}$ m

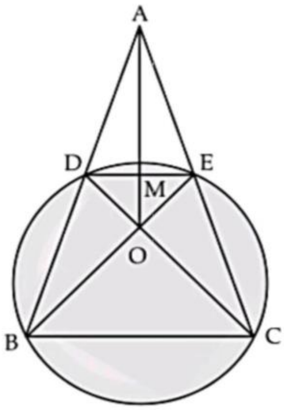
or, $BC = 2DC$

$= 20\sqrt{3}$ m

Length of the string of each phone = $20\sqrt{3}$ m

Q. 6. D and E are respectively the points on equal sides AB and AC of an isosceles triangle ABC such that B, C, E and D are con-cyclic, as shown in the given figure, if O is the point of intersection of CD and BE, prove that AO is the bisector of the line segment DE.

A [Board Term II, 2016]



Sol. As DECB is concyclic quadrilateral then

$\angle EDB + \angle ECB = 180°$ and $\angle DBC + \angle DEC = 180°$

Also, $\angle EDB + \angle ADE = 180°$ and $\angle AED + \angle DEC = 180°$ (linear pairs)

Hence, we can say that, $\angle ADE = \angle ECB = \angle ACB$ and $\angle AED = \angle DBC = \angle ACB$.

As triangle is isosceles, so we can say $\angle ADE = \angle ACB = \angle AED = \angle ABC$

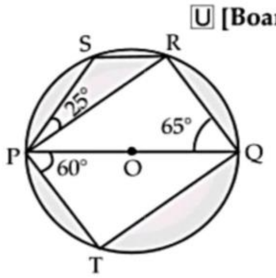
So, DE is parallel to BC and AD = AE and DB = EC and DECB is an isosceles trapezium. So, DC and EB will be equal and if they intersect at O, AO will be the median of the triangle ABC and triangle ADE as well.

Hence, AO is the bisector of the line segment DE.

Q. 7. In the given figure, PQ is the diameter of the circle. If $\angle PQR = 65°$, $\angle QPT = 60°$, then find the measure of :

(i) $\angle QPR$ (ii) $\angle PRS$ (iii) $\angle PSR$ (iv) $\angle PQT$.

U [Board Term II, 2012]



Sol. (i) $\angle QRP = 90°$ (Angle in the semi-circle)

(ii) $\therefore$ $\angle QPR = 25°$ (By angle sum property)

$\angle QPS = \angle QPR + \angle RPS = 50°$

$\angle QRS = 180° - 50° = 130°$

(PQRS is a cyclic quad.)

$\angle PRS = 130° - \angle QRP$

$= 130° - 90° = 40°$

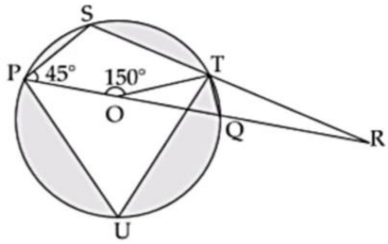
(iii) $\angle PSR = 180° - 65° = 115°$

(PQRS is a cyclic quad.)

(iv) $\angle PTQ = 90°$

$\therefore$ $\angle PQT = 180° - (90° + 60°) = 30°$

AI Q. 8. In the given figure $\angle SPQ = 45°$, $\angle POT = 150°$ and O is the centre of circle. Find the measures of $\angle RQT$, $\angle RTQ$ and $\angle PUT$. **U [Board Term II, 2015]**



Sol. In the given figure

$\angle POT + \text{reflex } \angle POT = 360°$

$150° + \text{reflex } \angle POT = 360°$

$\text{reflex } \angle POT = 210°$ **1**

$\text{reflex } \angle POT = 2\angle PST$

(angle subtended by arc at the centre is twice at circumference)

$210° = 2\angle PST$

$\therefore$ $\angle PST = 105°$ **1**

$\angle PQT + \angle PST = 180°$

(opposite angles of cyclic quadrilateral are supplementary)

$\angle PQT = 180° - 105°$

$= 75°$

$\angle RQT + \angle PQT = 180°$ (linear pair)

$\angle RQT = 180° - 75°$

$= 105°$

$\angle RTQ = \angle SPQ$

(Exterior angle of a cyclic quadrilateral is equal to interior opposite angle)

$= 45°$ **1**

$\angle PUT = \frac{1}{2}\angle POT$

(angle subtended by an arc at the centre is twice the angle at the remaining circle)

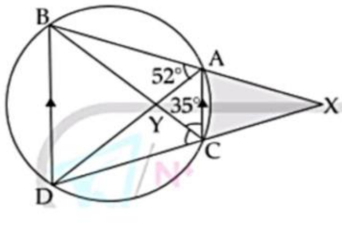
$= \frac{1}{2} \times 150°$

$= \frac{1}{2} \times 150°$

$= 75°$ **1**

[CBSE Marking Scheme, 2015]

Q. 9. In the given figure, ABDC is a cyclic quadrilateral in which AC || BD.



(i) If ∠BAD = 52°, ∠BCA = 35°. Find ∠ACX.

(ii) Prove that ∠CBD = ∠ADB. Also prove that, DY = BY.

(iii) Prove that, ΔXBD is an isosceles triangle

(iv) Prove that, XA = XC A [Board Term II, 2014]

Sol. (i) ∠BCD = ∠BAD

= 52°

(angles in the same segment)

∠BCD + ∠BCA + ∠ACX = 180°

(angle on the straight line)

or, 52° + 35° + ∠ACX = 180°

or, ∠ACX = 180° – 87°

= 93°

or, ∠ACX = 93°

(ii) ∠CBD = ∠DAC

(angles in the same segment) ...(i)

∠DAC = ∠ADB

(alternate angle) ...(ii)

From (i) and (ii),

∠CBD = ∠ADB ...(iii)

In ΔYBD,

∴ ∠YBD = ∠YDB [from (iii)]

DY = BY

(sides opposite to equal angles in a Δ are equal)

(iii) ∠ABD = ∠ACX

(exterior angle of a cyc. quad. is equal to interior opp. angles) ....(iv)

∠BDC = ∠ACX

(corresponding angles) ...(v)

From (iv) and (v),

∠ABD = ∠BDC ...(vi)

∴ XB = XD

∴ ΔXBD is an isosceles triangle

(iv) ∠BDC = ∠ACX

(corresponding angles)

∠ABD = ∠XAC

(corresponding angles)

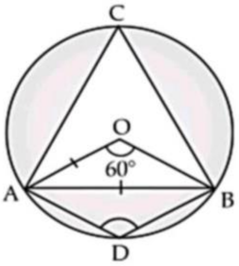
∴ ∠ACX = ∠XAC [from (vi)]

∴ XA = XC

Q. 10. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc. U [Board Term II, 2012]

Sol. According to the question,

OA = AB = OB 1



∴ ΔOAB is an equilateral triangle

∠AOB = 60°

$\angle ACB = \frac{1}{2}\angle AOB$

(angle subtended by an arc at the circumference is half of the angle at the centre of circle)

$\angle ACB = \frac{1}{2} \times 60°$

∠ACB = 30°

∠ACB + ∠ADB = 180° (opposite angles of cyclic quadrilateral are supplementary)

or, ∠ADB = 180° – ∠ACB

∠ADB = 180° – 30° = 150°

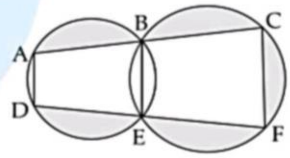
Commonly made Error

- Students fail to identify which angle is subtended by the chord from which arc.

Answering Tip

- A chord has its both points on the circle.

AI **Q. 11. In the given figure, B and E are points on line segments AC and DF respectively. Prove that AD ∥ CF.** U [Board Term II 2012]



Sol. Join B and E.

ADEB is a cyclic quadrilateral

∴ ∠ADE = ∠EBC,

(ext. angle of a cyclic quad. = int. opp. angle) ...(i)

Similarly, FEBC is a cyclic quadrilateral,

∴ ∠EBC + ∠CFE = 180° ...(ii)

(opp. angles of quad. are supp.)

From (i) and (ii),

∠ADE + ∠CFE = 180°

But these are co-interior angles for the lines AD and CF.

Since, co-interior angles are supplementary.

∴ AD ∥ CF **Hence Proved.**

Q. 12. ABCD is a cyclic quadrilateral whose diagonals intersect at E. If ∠DBC = 70°, ∠BAC = 30°, find ∠BCD. Further, if AB = BC, find ∠ECD. A [Board Term II, 2012]

Sol. Proof : ∠BAC = ∠BDC = 30°

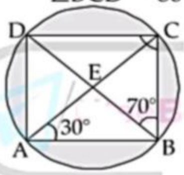
(angles in the same segment)

In ΔDBC,

∠BDC + ∠DBC + ∠BCD = 180° (A.S.P.)

30° + 70° + ∠BCD = 180°

∠BCD = 80°



Now, $AB = BC$

$\therefore$ $\angle BAC = \angle BCA$

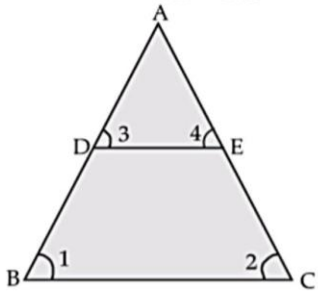
(angles opp. to equal sides)

$\therefore$ $\angle BCA = 30°$

or, $\angle ECD = 80° - 30° = 50°$

Q. 13. D and E are points on equal sides AB and AC of isosceles ΔABC such that AD = AE. Prove that the points B, C, E and D are con-cyclic. U **[NCERT]**

Sol. Given : $AB = AC$



To prove : $\angle DBC + \angle CED = 180°$

Proof : $AB = AC$ or, $\angle 1 = \angle 2$

$AD = AE$ or, $\angle 3 = \angle 4$ ...(i)

$\angle A + \angle 1 + \angle 2 = 180° = \angle A + \angle 3 + \angle 4$

$2\angle 1 = 2\angle 3$ ...(ii)

By eqn. (i) and (ii),

$\therefore$ $\angle 1 = \angle 3 = \angle 2 = \angle 4$

or, $DE \parallel BC$

(Corresponding angle)

$\therefore$ $\angle 1 + \angle BDE = 180°$

($\angle BDE = \angle CED$, as $\angle 3 = \angle 4$)

$\angle 1 + \angle CED = 180°$

$\therefore$ Points B, C, E, D are con-cyclic. **Proved.**