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CONTACT US:
+91-9939586130
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www.aepstudycircle.com

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2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

# GRAVITATION

Every body in the Universe attracts every other body in the Universe. This force of attraction between any pair of bodies in the Universe is called *gravitational force*. It is a fundamental force and also is the weakest force in nature. Any body which has some mass (be it a very small mass like that of an electron or a very large mass like that of Sun or stars) exerts a gravitational force on any other body having some mass. Thus gravitational force is due to the mass of the interacting bodies. The law which governs the gravitational force between any pair of bodies in the Universe is called 'Newton's Universal law of gravitation, named after Sir Isaac Newton who discovered this law.

## UNIVERSAL LAW OF GRAVITATION

According to Newton's universal law of gravitation, *"every particle in the universe attracts every other particle in the universe with a force that is directly proportional to the product of the masses of those particles and inversely proportional to the square of the distance between those particles. This force acts along the line joining the two particles."*

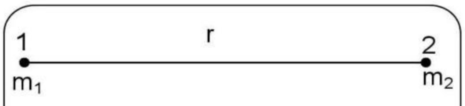



**Fig. 3.1**

Consider two particles 1 and 2 of mass '$m_1$' and '$m_2$' respectively, separated by a distance 'r'. The gravitational force of attraction between the two particles (F), as per Newton's universal law of gravitation is

$F \propto m_1 m_2$ and $F \propto \frac{1}{r^2}$

$\therefore \quad F \propto \frac{m_1 m_2}{r^2}$ or

$$F = \frac{Gm_1m_2}{r^2},$$

where 'G' is a constant of proportionality called *universal gravitational constant*. Particle 1 exerts a force on particle 2 $(\vec{F}_{21})$ and it is an attractive force directed towards 1 from 2. Similarly, particle 2 exerts a force on particle 1 $(\vec{F}_{12})$ and it is an attractive force directed towards 2 from 1.

Thus the gravitational force between 1 and 2 form an action-reaction pair.

$$\vec{F}_{12} = -\vec{F}_{21} \text{ ; } F = |\vec{F}_{12}| = |\vec{F}_{21}| \text{, then } F = \frac{Gm_1m_2}{r^2}$$

If $m_1 = m_2 = 1$ kg and $r = 1$ m, then

$$F = \frac{Gm_1m_2}{r^2} = \frac{G \times 1 \times 1}{1^2} = G$$

Thus the universal gravitational constant 'G' is numerically equal to the gravitational force between two particles of unit mass each, separated by unit distance. The value of 'G' was first experimentally established by Henry Cavendish. The SI unit of 'G' is N $m^2$ $kg^{-2}$ and its dimensional formula is $M^{-1}L^3T^{-2}$. The value of $G = 6.67 \times 10^{-11}$ N $m^2$ $kg^{-2}$.

This law is called *universal law of gravitation* because it holds good irrespective of the nature of the objects (like size, shape, mass etc.) and at all places and at all times. The value of G does not depend upon the mass of the particles, the distance between the particles or the medium separating them.

### *Characteristics of gravitational force*

(i) The gravitational force between any two bodies form an action-reaction pair. The force on each body due to the other body is of same magnitude but opposite in direction.
(ii) The gravitational force between any pair of bodies is always attractive in nature.
(iii) The gravitational force between a pair of bodies is independent of the presence or absence of any other body in their neighbourhood
(iv) The gravitational force between any pair of bodies is independent of the medium separating the bodies. Hence protecting a body (or shielding a body) from gravitational force is impossible.
(v) Gravitational force is a central force i.e., it acts along the line joining the two interacting particles.
(vi) Gravitational force is the weakest force in nature.

(vii) Gravitational force is negligibly small in case of light bodies but becomes quite significant in case of massive bodies like planets, satellites and stars.

(viii) Gravitational force is a long-range force i.e., it is effective even if the distance between the interacting particles is very large. For example, the gravitational force between Sun and planet Pluto exists even though the distance between them is large and is the cause for the motion of Pluto around the Sun.

(ix) Gravitational force is a conservative force. Hence potential energies are associated with gravitational forces

**Notes:**
Newton's law of Universal Gravitation is strictly applicable to particles or point masses. If the sizes of the bodies are very small compared to their distance of separation, such bodies can also be treated as particles. It can also be shown that a body having spherical symmetry of mass distribution can be treated as a particle, with mass concentrated at the centre of the sphere only for gravitational interaction at points outside the sphere. If the interacting bodies cannot be reduced to particles, integration method will have to be used for determining the gravitational force.

## CONCEPT STRANDS

### Concept Strand 1

Calculate the gravitational force between an electron (mass = $9.1 \times 10^{-31}$ kg) and a proton(mass = $1.67 \times 10^{-27}$ kg) separated by a distance of 1 m.

### *Solution*

Since the distance of separation (r = 1 m) is very large compared to the sizes of electron and proton, we can treat them as particles and apply the law of universal gravitation.

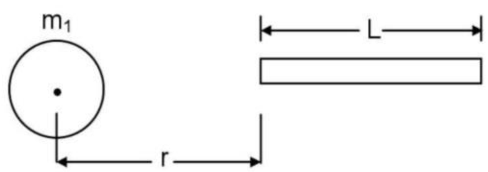
$$\therefore \quad F = \frac{GM_pM_e}{r^2} = \frac{6.67\times10^{-11}\times1.67\times10^{-27}\times9.1\times10^{-31}}{1^2}$$

$$= 1.01 \times 10^{-67}\ \text{N}$$

As this force is very small (in comparison to the electric force between an electron and a proton), gravitational force on small charged particles are usually neglected.

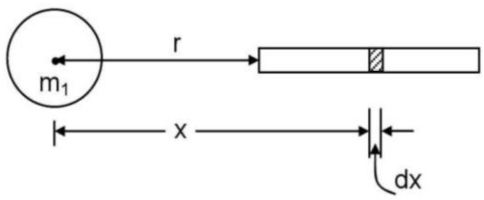
### Concept Strand 2

A uniform solid sphere of mass $m_1$ is separated from a uniform rod of length 'L' and mass $m_2$. Calculate the gravitational force exerted by the sphere on the rod.




### *Solution*

Due to spherical symmetry, the uniform sphere of mass $m_1$ can be considered as a particle of mass $m_1$ at the centre of the sphere (This is explained in later sections of this chapter). However, we cannot treat the uniform rod as a particle. Hence, consider an element of the rod, of length dx, at a distance x from the centre of the sphere. This elemental length of rod can be treated as a particle.




Mass of elemental rod, $dm = \left(\frac{m_2}{L}\right)dx$

Newton's law of universal gravitation can be applied between $m_1$ and dm

$\therefore$ Gravitational force on the elemental rod

$$dF = \frac{Gm_1dm}{x^2} = \frac{Gm_1m_2}{Lx^2}dx$$

$\therefore$ Total force on the rod,

$$F = \int_0^F dF = \int_{x=r}^{x=L+r} \frac{Gm_1m_2dx}{Lx^2}$$

$$= \frac{Gm_1m_2}{L}\int_{x=r}^{x=L+r}\frac{dx}{x^2}$$

$$= \frac{Gm_1m_2}{L}\int_{x=r}^{x=L+r}\frac{dx}{x^2} = \frac{Gm_1m_2}{L}\left(-\frac{1}{x}\right)_r^{L+r}$$

$$= -\frac{Gm_1m_2}{L}\left(\frac{1}{L+r}-\frac{1}{r}\right) = -\frac{Gm_1m_2}{L}\left[\frac{r-(L+r)}{r(L+r)}\right]$$

$$= \frac{-Gm_1m_2}{L}\times\frac{-L}{r(L+r)} = \frac{Gm_1m_2}{r(L+r)}$$

If $r >> L$, then $r + L \simeq r$

$$\therefore \quad F = \frac{Gm_1m_2}{r^2}$$

Hence if the sphere and rod are separated by a very large distance, much larger than the length of the rod, then the rod can also be treated as a particle.

### *Superposition principle of gravitational forces*

The gravitational force between any two particles is independent of the presence or absence of other particles. This gives rise to the superposition principle of gravitational forces. According to the superposition principle, the gravitational force on a particle of mass m, due to a distribution of particles of masses $m_1$, $m_2$.....,$m_n$ around it, is the vector sum of the gravitational forces exerted on m by each of the other particles $m_1$, $m_2$,.....$m_n$, the forces between each pair being independent of the other particles

**For example**

Consider a distribution of six particles ($m_1$, $m_2$, $m_3$, $m_4$, $m_5$ and $m_6$) around a particle of mass m as shown

$$\overline{F}_{01} = \frac{-Gmm_1}{|\overline{r}_1|^2}\hat{r}_1 = -\frac{Gmm_1}{|\overline{r}_1|^3}\overline{r}_1$$

$$\overline{F}_{02} = -\frac{Gmm_2}{|\overline{r}_2|^2}\hat{r}_2 = -\frac{Gmm_2}{|\overline{r}_2|^3}\overline{r}_2$$

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$$\overline{F}_{06} = -\frac{Gmm_6}{|\overline{r}_6|^2}\hat{r}_6 = \frac{-Gmm_6}{|\overline{r}_6|^3}\overline{r}_6$$

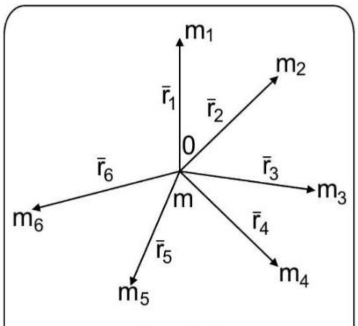



**Fig. 3.2**

$\overline{F}_G$ = gravitational force on m due to $m_1$, $m_2$,.....,$m_6$ as per superposition principle is

$$\overline{F}_a = \overline{F}_{01} + \overline{F}_{02} + \overline{F}_{03} + \overline{F}_{04} + \overline{F}_{05} + \overline{F}_{06}$$

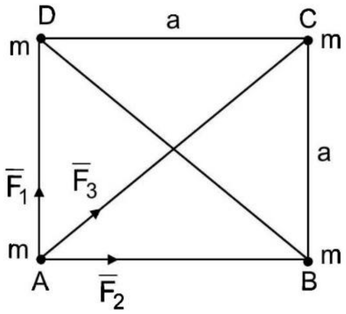
$$= -Gm\sum_{i=1}^{i=6}\frac{m_i}{|\overline{r}_i|^3}\overline{r}_i$$

## CONCEPT STRAND

### Concept Strand 3

Four identical particles, each of mass m, are placed at the corners of a square of side a. Calculate the net gravitational force on each particle.

### *Solution*




Let us consider a particle of mass 'm' placed at corner A of the square of side a

Gravitational force on A due to particle at D is $\overline{F}_1$ and

$$F_1 = |\overline{F}_1| = \frac{Gmm}{a^2} = \frac{Gm^2}{a^2}, \text{ along AD}$$

Gravitational force on A due to particle at B is $\overline{F}_2$ and

$$F_2 = |\overline{F}_2| = \frac{Gmm}{a^2} = \frac{Gm^2}{a^2}, \text{ along AB}$$

Gravitational force on A due to particle at C is $\overline{F}_3$ and

$$F_3 = |\overline{F}_3| = \frac{Gmm}{(\sqrt{2}\,a)^2} \quad (\because AC = \sqrt{2}\,a) = \frac{Gm^2}{2a^2}, \text{ along AC}$$

As per superposition principle, net gravitational force on A is $\overline{F} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$

$$= \overline{F}_R + \overline{F}_3 \qquad (\text{where } \overline{F}_R = \overline{F}_1 + \overline{F}_2)$$

$$F_R = \sqrt{F_1^2 + F_2^2} \qquad (\because \overline{F}_1 \text{ perpendicular to } \overline{F}_2)$$

$$|\overline{F}| = \frac{Gm^2}{a^2}\left[\sqrt{2} + \frac{1}{2}\right] = \frac{(2\sqrt{2}+1)Gm^2}{2a^2}, \text{ along AC}$$

The magnitude of the net gravitational force on the particles at B, C and D also will be of magnitude $\frac{(2\sqrt{2}+1)Gm^2}{2a^2}$ but their directions will be along BD (for particle at B), along CA (for particle at C) and along DB (for particle at D).

The gravitational force between larger bodies (other than particles) can be determined by using the superposition principle.

## GRAVITATIONAL FIELD

The presence of a particle/body (or a mass) modifies the space surrounding the particle/body. This modified space surrounding a particle or body (or anything having mass) is called the gravitational field of that particle or body. Any other mass brought inside this gravitational field will experience a gravitational force due to its interaction with this gravitational field. The field concept is very useful in dealing with non–contact forces (or action at a distance)

We know that the net gravitational force on a particle may be due to another single particle or due to a distribution of large number of particles. The advantage of the gravitational field concept is that it helps us to measure the net gravitational force on a mass, without bothering whether it is a single particle or a distribution of particles that exert this force on the concerned mass.

Every point in a gravitational field is characterized by two properties, out of which one is a vector quantity and the other is a scalar. The vector quantity is called Gravitational Field Intensity $\overline{E}$ and the scalar quantity is called Gravitational Potential V.

## GRAVITATIONAL FIELD INTENSITY ($\overline{E}$)

Gravitational field intensity $\overline{E}$ at a point in a gravitational field is a vector quantity, defined mathematically as $\overline{E} = \underset{\Delta m \to 0}{Lt} \frac{\overline{F}}{\Delta m}$, where $\Delta m$ is an infinitesimally small mass (but not zero) placed at the point, where it experiences a gravitational force $\overline{F}$. To get a practical measure of the gravitational field, it is defined as the gravitational force exerted on a unit mass placed at that point. The SI unit of gravitational field intensity is newton per kilogram (N $kg^{-1}$) and its dimensional formula is $M^0LT^{-2}$ (same as the dimensional formula of acceleration).

The gravitational field intensity ($\overline{E}$) is also known as *strength of the gravitational field* or simply *Gravitational Field*. If a particle of mass m is brought to a point, where the gravitational field is $\overline{E}$, the net gravitational force ($\overline{F}$) acting on that particle at that point is given by

$$\overline{F} = m\overline{E}$$

## CONCEPT STRAND

### Concept Strand 4

The gravitational field at a point is given by $\overline{E} = (6\hat{i} + 8\hat{j} + 10\hat{k})$ N kg$^{-1}$. Calculate the net gravitational force exerted on a particle of mass 5 kg placed at that point.

#### Solution

$$\overline{F} = m\overline{E}$$

$$= 5 \times \left[6\hat{i} + 8\hat{j} + 10\hat{k}\right] \text{ N}$$

$$= \left[30\hat{i} + 40\hat{j} + 50\hat{k}\right] \text{ N}$$

$\therefore$ Magnitude of the gravitational force $F = \left|\overline{F}\right|$

$$= \sqrt{30^2 + 40^2 + 50^2}$$

$$= \sqrt{900 + 1600 + 2500} = \sqrt{5000}$$

$$= 70.71 \text{ N}$$

This gravitational force acts in the same direction as the gravitational field intensity $(\overline{E})$ at that point.

**Note:**
In this problem, we do not know anything about the source/sources which exert this net gravitational force on the particle of mass 5 kg. Hence it is not correct to say that the gravitational force exerted by a field is in the direction of the source.

### Gravitational field intensity due to a particle or point mass

Consider a particle of mass M, placed at a point O. We want to determine the gravitational field intensity due to M, at a point P near it. OP is the position vector $\overline{r}$ (taking position O as the origin).

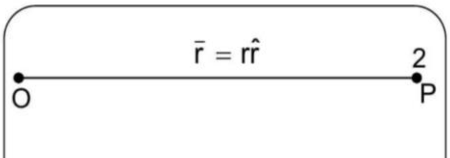



**Fig. 3.3**

If a point particle of mass m is placed at P, the gravitational force on it will be $\frac{GMm}{r^2}$ towards O (i.e., along PO)

$$\therefore \quad \overline{F} = -\frac{GMm}{r^2}\hat{r}$$

The gravitational field intensity at P at the location of the point mass m is given by

$$\overline{E} = \frac{\overline{F}}{m} = -\frac{GM}{r^2}\hat{r}$$

This is defined for all points, wherever the point mass is kept except at the location of the particle of mass M.

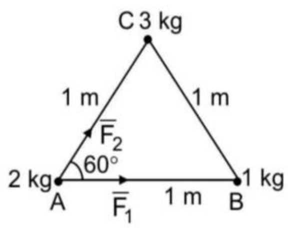
## CONCEPT STRAND

### Concept Strand 5

Three particles of masses 1 kg, 2 kg and 3 kg are placed at the corners of an equilateral triangle of side 1 m. Calculate the magnitude and direction of the gravitational field intensity at the location of the 2 kg mass. What is the net gravitational force on the 2 kg mass?

#### Solution

This problem can be solved by two methods




**Method 1**

Calculate the force $\overline{F}_1$ and $\overline{F}_2$ on the 2 kg mass due to 1 kg and 3 kg respectively and find out the resultant force $\overline{F} = \overline{F}_1 + \overline{F}_2$, which is the net force on 2 kg mass. Gravitational field at the location of 2 kg mass is given by $E = \frac{\overline{F}}{m} = \frac{\overline{F}}{2\text{ kg}}$.

**Method 2**

Let $\overline{E}_1$ be the gravitational field at the location of 2 kg mass due to 1 kg mass and $\overline{E}_2$ be the gravitational field at the same location due to 2 kg mass.

We have

$$\overline{E}_1 = \frac{-Gm_1}{r_1^2}\hat{r}_1 \ (m_1 = 1\text{ kg}, r_1 = 1\text{ m } \hat{r}_1 \text{ along BA})$$

$$\therefore \quad E_1 = \frac{G \times 1}{(1)^2} = G, \text{ along AB}$$

Similarly,

$$\overline{E}_2 = -\frac{Gm_2}{r_2^2}\hat{r}_2 \ (m_2 = 3\text{ kg}, r_2 = 1\text{ m}, \hat{r}_2 = \text{along CA})$$

$$\therefore \quad E_2 = \frac{G \times 3}{1^2} = 3G, \text{ along AC}$$

By superposition principle, the gravitational field at A(location of 2 kg mass) is given by

$$\overline{E} = \overline{E}_1 + \overline{E}_2$$

$$\therefore \quad E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

$$= \sqrt{G^2 + (3G)^2 + 2 \times G \times 3G \times \cos 60^\circ}$$

($\because \theta = 60^\circ$ between $\overline{E}_1$ and $\overline{E}_2$)

$$= \sqrt{G^2 + 9G^2 + 3G^2}$$

$$= \sqrt{13G^2} = \sqrt{13}\,G \ \text{ N kg}^{-1}$$

$$= \sqrt{13} \times 6.67 \times 10^{-11}\text{ N kg}^{-1}$$

$$= 24.05 \times 10^{-11}\text{ N kg}^{-1}$$

If $\overline{E}$ makes an angle of $\phi$ with AB,

$$\tan\phi = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta} \ \text{(From parallelogram law of vectors)}$$

$$= \frac{3G\sin 60^\circ}{G + 3G\cos 60^\circ} = \frac{3\sin 60^\circ}{1 + 3\cos 60^\circ}$$

$$= \frac{3\sqrt{3}}{5} = 1.039$$

$$\therefore \phi = \tan^{-1}(1.039) = 46.1^\circ$$

Hence the gravitational field at A (location of 2 kg mass) makes an angle of 46.1° in the anti-clockwise direction with AB.

The net gravitational force on 2 kg mass,

$$F = mE$$
$$= 2 \times 24.05 \times 10^{-11}$$
$$= 48.1 \times 10^{-11}\text{ N}$$

## *Gravitational field intensity due to a thin uniform ring at a point on its axis*

Consider a thin, uniform ring of mass M and radius a with centre at O. The gravitational field intensity $(\overline{E})$ due to this ring at a point P on the axis of the ring, distant 'r' from the centre of the ring is to be determined.

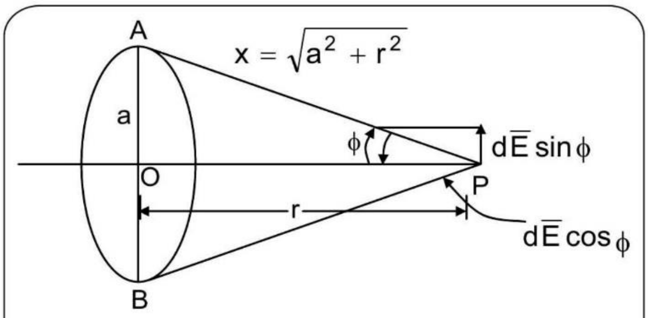



**Fig. 3.4**

Let $\lambda$ = mass per unit length of ring = $\frac{\text{Mass}}{\text{circumference}}$

$$= \frac{M}{2\pi a}$$

Consider an element of the ring, of length '$d\ell$' at A. Its mass $dm = \lambda d\ell = \frac{Md\ell}{2\pi a}$

The gravitational field intensity at P due to this elemental ring at A is $d\overline{E} = \frac{Gdm}{x^2}$, along PA

AP makes an angle $\phi$ with OP. Now $d\overline{E}$ can be resolved as $d\overline{E}\sin\phi$ perpendicular to the axis OP and $d\overline{E}\cos\phi$ along the axis as shown.

$d\overline{E}\sin\phi$ component gets cancelled by the field of a diametrically opposite element at B. Hence the effective component of all ring elements is only $d\overline{E}\cos\phi$

$$dE\cos\phi = \frac{Gdm}{x^2}\cos\phi$$

$$= \frac{GM}{x^2 \times 2\pi a}\cos\varphi\ d\ell \ (\because dm = \frac{Md\ell}{2\pi a})$$

$\therefore$ The resultant gravitational field at P due to all ring elements is $\overline{E} = \int dE\cos\phi$, along PO

$$\therefore \quad E = \int \frac{GM}{x^2 \times 2\pi a}\cos\phi\ d\ell$$

$$= \frac{GM}{2\pi a x^2}\cos\varphi \int_{\ell=0}^{\ell=2\pi a} d\ell = \frac{GM}{x^2}\cos\varphi$$

$$= \frac{GM}{x^2}.\frac{r}{x} \qquad (\because \cos\phi = \frac{r}{x})$$

$$= \frac{GMr}{x^3}$$

$$= \frac{GMr}{(a^2+r^2)^{3/2}} \qquad \left[\because x = (a^2+x^2)^{1/2}\right]$$

$$\therefore \quad E = \frac{GMr}{(a^2+r^2)^{3/2}}$$

**Notes:**

(i) At the centre of the ring (point O), r = 0
$\Rightarrow$ E = 0. Hence *the gravitational field at the centre of the thin, uniform ring is zero.*

(ii) If a << r, $E = \frac{GMr}{(r^2)^{3/2}} = \frac{GM}{r^2}$. So for points on the axis which are at a very large distance from the centre of the ring, the ring can be treated as a particle (or point mass).

(iii) The position where the gravitational field $\overline{E}$ becomes maximum or minimum is determined by putting $\frac{dE}{dr} = 0$.

$$E = \frac{GMr}{(a^2+r^2)^{3/2}}$$

$$\frac{dE}{dr} = 0 \Rightarrow r = \pm\frac{a}{\sqrt{2}}$$

At $+\frac{a}{\sqrt{2}}$, E is maximum negative (minimum) and at $-\frac{a}{\sqrt{2}}$, E is maximum positive (maximum).

The variation of $\overline{E}$, along the axis of the ring on either side of the ring is as shown below in Fig.3.4.

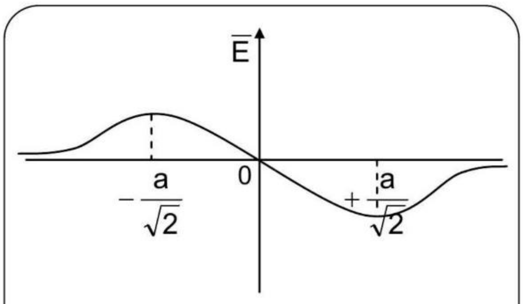



**Fig. 3.5**

### *Gravitational field intensity due to a uniform disc at a point on its axis*

Consider a uniform disc of mass M and radius a with centre at O. The point P is on the axis of the disc at a distance r from centre O.

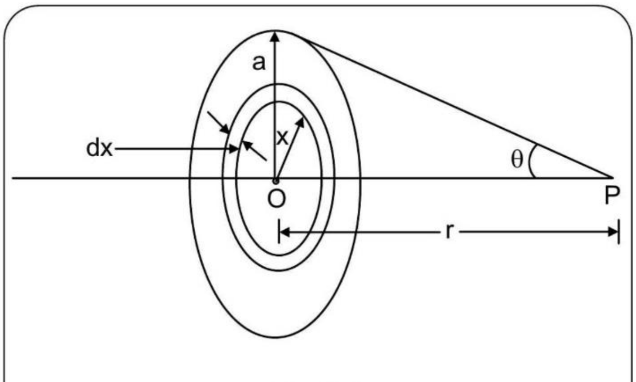



**Fig. 3.6**

The disc can be dividend into thin, uniform rings. Consider one such ring of radius x and width along the disc equal to dx.

Mass of ring $dm = \frac{\text{Mass of disc}}{\text{Surface Area of disc}} \times$ surface area of ring

$$= \left(\frac{M}{\pi a^2}\right)(2\pi x dx) = \frac{2M}{a^2}xdx$$

The gravitational field at P due to this elemental ring is

$$dE = \frac{G\,dm\,r}{(x^2+r^2)^{3/2}} \text{ along PO}$$

$$\therefore \quad dE = \frac{Gr}{(x^2+r^2)^{3/2}}.\frac{2M}{a^2}xdx = \frac{2GMr}{a^2}.\frac{x}{(x^2+r^2)^{3/2}}dx$$

$\therefore$ Gravitational field at P due to disc,

$$E = \int dE = \frac{2GMr}{a^2}\int_{x=0}^{x=a}\frac{x\,dx}{(x^2+r^2)^{3/2}}$$

$$= \frac{2GMr}{a^2}\left|-\frac{1}{\sqrt{x^2+r^2}}\right|_0^a = \frac{2GMr}{a^2}\left[\frac{1}{r}-\frac{1}{\sqrt{a^2+r^2}}\right]$$

$$= \frac{2GM}{a^2}\left[1-\frac{r}{\sqrt{a^2+r^2}}\right]$$

$$= \frac{2GM}{a^2}[1-\cos\theta], \text{ where } \cos\theta = \frac{r}{\sqrt{a^2+r^2}}$$

$$E = \frac{2GM}{a^2}[1-\cos\theta]$$

**Notes:**

(1) When P is very near to centre O, $\theta \approx 90°$, $\cos\theta = 0$
$\Rightarrow E = \frac{2GM}{a^2}$ (maximum value)
When P is far away from O, $\theta = 0°$, $\cos\theta = 1 \Rightarrow E = 0$

(2) If the disc is infinitely large, $\cos\theta = \cos 90° = 0$ and $E = \frac{2GM}{a^2}$ for all points on the axis.

Hence *the gravitational field due to an infinitely large disc along its axis is uniform (i.e., it is independent of the distance from the disc).*

## Gravitational field due to a thin, uniform shell (hollow sphere)

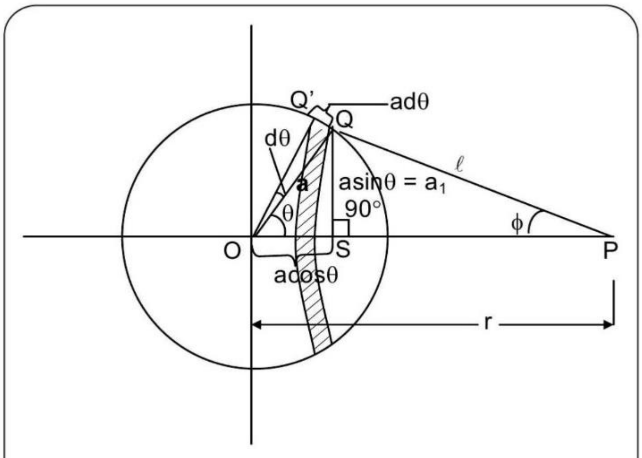



Fig. 3.7

Consider a thin, uniform spherical shell of mass M and radius a, with centre at point O. We want to calculate the gravitational field due to this shell at a point P, distant r from O.

Figure 3.7 shows a spherical shell. The shaded area represents a thin ring of radius $a_1 = a\sin\theta$ and width $Q'Q = ad\theta$

$PQ = \ell$ and angle OPQ is $\phi$

From $\Delta PSQ$, we have $PQ^2 = PS^2 + QS^2$

i.e., $\ell^2 = [r - OS]^2 + QS^2$
$= r^2 - 2r(OS) + (OS)^2 + QS^2$

$\Rightarrow \ell^2 = r^2 + a^2 - 2r(OS)$ $[\because QS^2 + OS^2 = OQ^2 = a^2]$
$= r^2 + a^2 - 2ra\cos\theta$ $(\because OS = a\cos\theta)$

$\therefore \ell^2 = a^2 + r^2 - 2ar\cos\theta$ —(i)

Area of shaded ring = circumference × width
$= 2\pi a_1 \times ad\theta$
$= 2\pi a\sin\theta . ad\theta$
$= 2\pi a^2 \sin\theta d\theta$

Mass of shaded ring, $dm = \frac{M}{\text{Area of shell}} \times$ area of ring

$$= \frac{M}{4\pi a^2}\times 2\pi a^2\sin\theta d\theta$$

$$= \frac{M}{2}\sin\theta d\theta \quad \text{— (ii)}$$

The gravitational field at P due to this ring is

$dE = \frac{Gdm}{\ell^2}\cos\phi$ ($\because \sin\phi$ components of the ring get cancelled for diametrically opposite points)

$$\therefore\ dE = \frac{GM}{2}\frac{\sin\theta d\theta\cos\phi}{\ell^2} \quad \text{— (iii)}$$

From $\Delta POQ$, we have $a^2 = \ell^2 + r^2 - 2\ell r\cos\phi$

$$\therefore\ \cos\phi = \frac{\ell^2 + r^2 - a^2}{2\ell r} \quad \text{— (iv)}$$

We have $\ell^2 = a^2 + r^2 - 2ar\cos\theta$ from (i)
Differentiating (i), we get

$$2\ell d\ell = 2ar\sin\theta d\theta$$

$$\therefore\ \sin\theta d\theta = \frac{\ell d\ell}{ar} \quad \text{— (v)}$$

Using values from (iv) and (v) is (iii), we get

$$dE = \frac{Gm}{4ar^2}\left[1 - \frac{(a^2-r^2)}{\ell^2}\right]d\ell$$

i.e., $$E = \int dE = \int_{\ell_1}^{\ell_2}\frac{Gm}{4ar^2}\left[1-\frac{(a^2-r^2)}{\ell^2}\right]d\ell$$

i.e., $E = \dfrac{Gm}{4ar^2}\left[\ell + \dfrac{(a^2 - r^2)}{\ell}\right]_{\ell_1}^{\ell_2}$

The following cases are of particular interest:

***(i) P outside the shell (r > a)***

In this case, value of $\ell$ varies from $\ell_1 = (r - a)$ to $\ell_2 = (r + a)$

$$\therefore \quad E = \frac{Gm}{4ar^2}\left[\ell + \frac{(a^2 - r^2)}{\ell}\right]_{r-a}^{r+a}$$

$$= \frac{Gm}{4ar^2}\left[(r + a) + (a - r) - \{(r - a) - (a + r)\}\right]$$

$$= \frac{Gm}{4ar^2}\left[2a + 2a\right] = \frac{Gm}{r^2}$$

$\therefore$ $E = \dfrac{Gm}{r^2}$ for all outside points. Hence, *for calculation of gravitational field at external points of a thin, uniform spherical shell, the shell can be treated as a point mass(particle) kept at the geometric centre of the shell.*

***(ii) P inside the shell (r < a)***

In this case, $\ell$ varies from $\ell_1 = (a - r)$ to $\ell_2 = (a + r)$

$$\therefore E = \frac{Gm}{4ar^2}\left[\ell + \frac{(a^2 - r^2)}{\ell}\right]_{a-r}^{a+r}$$

$$= \frac{Gm}{4ar^2}\left[(a + r) + (a - r) - \{(a - r) + (a + r)\}\right]$$

$\therefore$ $E = 0$ for inside points of shell. Hence, *the gravitational field inside a thin, uniform spherical shell due to the mass of the shell, is zero.*

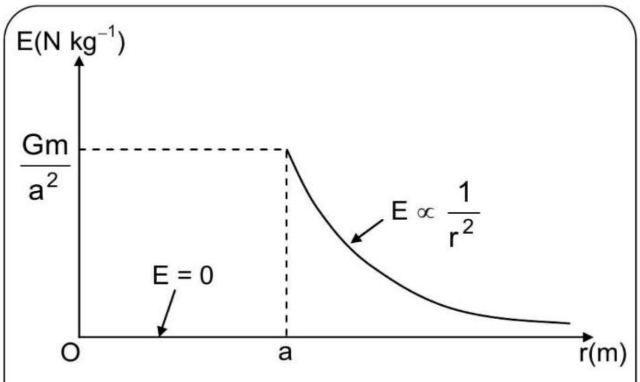



**Fig. 3.8**

Variation of gravitational field due to uniform, thin spherical shell of radius 'a'.

## *Gravitational field intensity due to a uniform solid sphere*

Consider a uniform solid sphere of mass M and radius 'a' with centre at point O. Let us evaluate its gravitational field intensity at a point P, distant 'r' from O.

***(i) P is outside the sphere (r > a)***

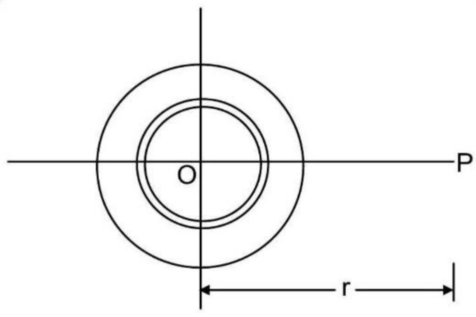



**Fig. 3.9**

The solid sphere can be divided into concentric uniform thin shells, each of mass dm

The gravitational field at P due to thin shell is $dE = \dfrac{Gdm}{r^2}$

$\therefore$ Total gravitational field at P due to solid sphere

$$E = \int dE = \int \frac{Gdm}{r^2} = \frac{GM}{r^2}$$

$$\therefore \quad E = \frac{GM}{r^2} \text{ for outside points}$$

Hence, *a uniform solid sphere can be treated as a point mass(particle) kept at its geometric centre for evaluation of gravitational field at all outside points of the solid sphere.*

***(ii) P is inside the sphere (r < a)***

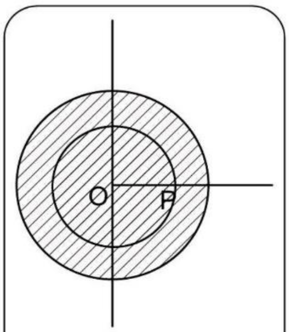



**Fig. 3.10**

In this case, we can treat the solid sphere as made of two parts, namely, (1) solid sphere of radius 'r' and (2) uniform spherical shell of inside radius r and outside radius a. The gravitational field at P is due to the superposition of the gravitational fields due to these two portions. We know that gravitational field at P due to the shell is zero (as P is in the inside of the shell). Hence gravitational field intensity at P is due to a solid sphere of radius r (instead of a).

Mass of reduced sphere, $M' = \frac{M}{\left(\frac{4}{3}\pi a^3\right)} \times \frac{4}{3}\pi r^3 = \frac{M}{a^3} r^3$

Gravitational field at P is $E = \frac{GM'}{r^2}$

$$= \frac{G}{r^2}\frac{M}{a^3}r^3 = \frac{GMr}{a^3}$$

$\therefore \quad E = \left(\frac{GM}{a^3}\right) r$ for inside points of solid sphere i.e., $E \propto r$ for inside points

Hence *the gravitational field intensity at inside points of a uniform solid sphere, is directly proportional to the distance of that point from the geometric centre of the solid sphere.*

Since r = 0 at the centre of the solid sphere, the gravitational field intensity at the centre of the solid sphere is zero.

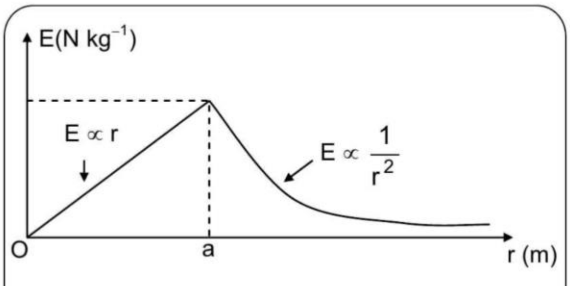


**Fig. 3.11**

Variation of gravitational field intensity due to a uniform solid sphere of radius 'a' is shown in Fig.3.11.

## GRAVITATIONAL POTENTIAL (V)

*The gravitational potential at a point is equal to the work done by an external force on a particle of unit mass in bringing it from infinity to its position in the gravitational field.* Gravitational potential is a scalar quantity. Its SI unit is joule per kilogramme($J\ kg^{-1}$) and its dimensional formula is $M^0L^2T^{-2}$. While bringing the unit mass from infinity to its position in the gravitational field, at every point in the path, the applied external force is equal and opposite to the gravitational force on the particle at those points i.e., the particle is brought slowly from infinity to its position so that its kinetic energy is zero at all positions. *Hence gravitational potential (V) can also be defined as the negative of the work done by the gravitational force as a particle of unit mass is brought from infinity to its position in the gravitational field.* If 'W' is the work done by an external force, in bringing a particle of mass 'm' from infinity to its position in the gravitational field, then the gravitational potential at that point (V) is given by

$$V = \frac{W}{m}$$

Also $W = -W_G$, where $W_G$ = work done by gravitational force in bringing the particle from infinity to its position and $W_G = \int_\infty^r \overline{F}_G.\overline{dr}$, where $\overline{F}_G$ = gravitational force on particle and $\overline{dr}$ = displacement of particle

$$\Rightarrow \quad W = -\int_\infty^r \overline{F}_G.\overline{dr} \qquad \therefore V = \frac{W}{m} = -\frac{W_G}{m} = \frac{-\int_\infty^r \overline{F}_G.\overline{dr}}{m}$$

$$= -\int_\infty^r \frac{\overline{F}_G}{m}.\overline{dr}$$

$$\therefore \qquad V = -\int_\infty^r \overline{E}.\overline{dr}$$

$$\left(\because \frac{\overline{F}_G}{m} = \overline{E}\right)$$

Conventionally the potential of a particle at infinity is taken as zero.

### Gravitational potential (V) at a distance r from a point mass (M)

Consider a particle of mass M placed at point O. We want to determine the gravitational potential at point P, distant r from O.

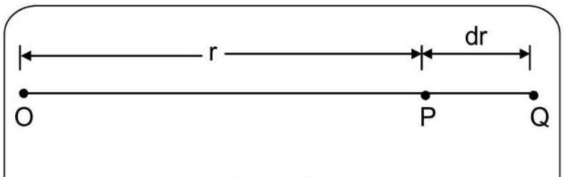


Fig. 3.12

The gravitational force acting on a particle of unit mass at P is the gravitational field intensity at P due to the mass M at O.

$$\therefore \quad F_G = E = \frac{GM}{r^2}\text{, along PO.}$$

If the unit mass is displaced from P through a small distance $\overline{dr}$ to Q, the small amount of work done by gravitational force

$$dW_G = \overline{F}_G.\overline{dr} = \overline{E}.\overline{dr}$$

$$= E\, dr \cos 180°$$

$(\because \overline{E}$ and $\overline{dr}$ are in opposite directions)

$$= -E\, dr = -\frac{GM}{r^2}dr$$

Work done by the gravitational force in transferring the unit mass from infinity to P is given by

$$W_G = \int_\infty^r dW_G = \int_\infty^r -\frac{GM}{r^2}dr$$

Gravitational potential at P is given by

$$V = -W_G = -\int_\infty^r -\frac{GM}{r^2}dr = -GM\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$= -\frac{GM}{r} + \frac{GM}{\infty} = -\frac{GM}{r}$$

$$V = -\frac{GM}{r}$$

Since gravitational potential at infinite distance is considered to be zero $\left(\text{i.e., } \frac{GM}{\infty} = 0\right)$, the gravitational potential comes out to be always negative

### *Relation between gravitational field intensity ($\overline{E}$) and gravitational potential (V)*

Since V is obtained by integration of $\overline{E}$, the converse, namely differentiation of V gives $\overline{E}$. Since V is in general $V \equiv V(x, y, z)$,

$$\overline{E} = -\overline{\nabla}V,$$

where

$$\overline{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\text{, i.e.,}$$

$$\overline{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

is the relation between gravitational field intensity $\overline{E}$ and gravitational potential V

When V depends on x alone,

$$E_x = -\frac{dV}{dx}\text{, where } E = |\overline{E}|$$

Similarly, $E_y = \frac{-dV}{dy}$ or $E_z = -\frac{dV}{dz}$

we can also write $E = -\frac{dV}{dr}$, when V is spherically symmetric

### *Gravitational potential due to a thin uniform ring along the axis of the ring*

Consider a thin, uniform ring of mass m and radius a with centre at O.

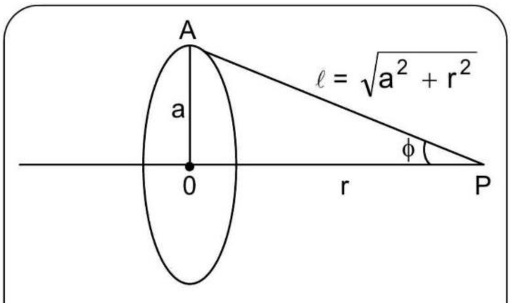


Fig. 3.13

An element of length $d\ell$ of the ring at A has a mass dm $= \frac{Md\ell}{2\pi a}$

The gravitational potential at P on the axis of the ring, distant r from the centre O, due to dm is given by $dV = \frac{-G\, dm}{\ell}$

$\therefore$ Total gravitational potential at P due to the ring

$$V = \int dV = \int_0^M -\frac{Gdm}{\ell} = -\frac{GM}{\ell}$$

$$\therefore \quad V = \frac{-GM}{(a^2 + r^2)^{1/2}}$$

At the centre of the ring, r = 0

$\therefore \quad V = -\dfrac{GM}{a}.$

If r >> a, $V = -\dfrac{GM}{r}$ *i.e., for distant points along the axis, the thin uniform ring behaves like a particle of mass m at its centre.*

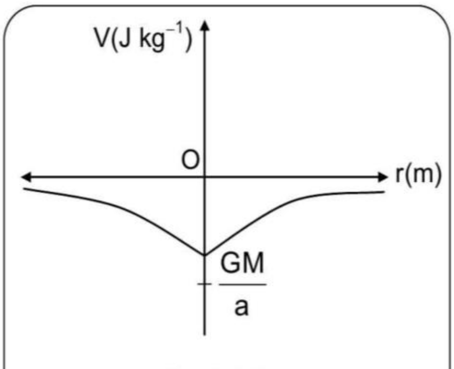



**Fig. 3.14**

Variation of gravitational potential along the axis of a thin, uniform ring of radius 'a' is given in Fig.3.14.

### *Gravitational potential due to a thin uniform spherical shell*

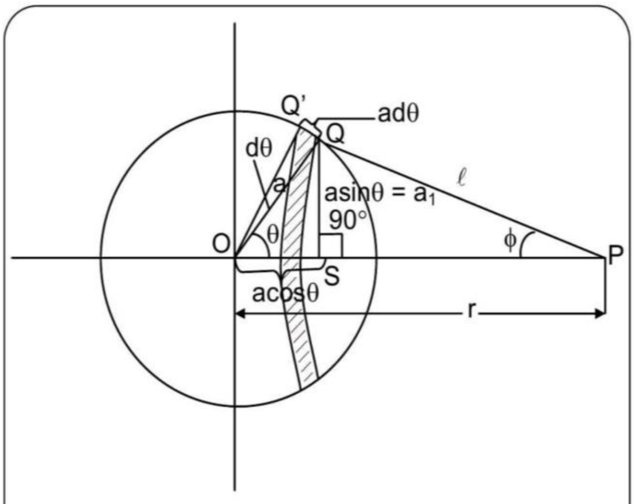



**Fig. 3.15**

Consider a thin, uniform shell of mass M and radius a with centre at point O. The mass of the element ring(shaded area) is $dm = \dfrac{M}{2}\sin\theta d\theta$ (already derived in the section on gravitational field)

Also $\ell^2 = a^2 + r^2 - 2ar\cos\theta$ (from Δ POQ in Fig. 3.15)

$\therefore \quad 2\ell d\ell = 2ar\sin\theta d\theta$

$\Rightarrow \quad \sin\theta d\theta = \dfrac{\ell d\ell}{ar}$

$\therefore \; dm = \dfrac{M\ell d\ell}{2ar}$ (This is also derived in the section on gravitational field)

The gravitational potential at P due to dm is given by

$$dV = -\frac{G\,dm}{\ell} \text{ (for ring)}$$

$$= -\frac{GM\ell d\ell}{2ar\ell}$$

i.e., $\quad dV = -\dfrac{GMd\ell}{2ar}$

As we vary θ from zero to π, the rings formed on the shell cover up the whole shell. The potential due to the shell is obtained by integrating dV within the limits θ = 0 to θ = π

***(i) P outside the shell (r > a)***

$\ell^2 = a^2 + r^2 - 2ar\cos\theta$

$\Rightarrow$ when θ = 0, ℓ = r – a and when θ = π, ℓ = r + a

$$\therefore \quad V = \int dV = \int_{\ell=(r-a)}^{\ell=(r+a)} -\frac{GM}{2ar}d\ell$$

$$= -\frac{GM}{2ar}\left|\ell\right|_{r-a}^{r+a}$$

$$= -\frac{GM}{2ar}\left[(r+a)-(r-a)\right] = -\frac{GM}{r}$$

$\therefore \quad V = -\dfrac{GM}{r}$ for all external points

Hence, *the thin uniform shell can be treated as a point mass, of same mass as the shell, placed at the centre of the shell for calculation of gravitational potential at all external points.*

***(ii) P inside the shell (r < a)***

In this case, when θ = 0, ℓ = a – r and when θ = π, ℓ = a + r

$$V = \int dV = \int_{a-r}^{a+r} -\frac{GM}{2ar}d\ell = -\frac{GM}{a}$$

$\therefore \quad V = -\dfrac{GM}{a}$ (inside the shell, V is independent of 'r')

Hence, *the gravitational potential due to a thin, uniform spherical shell is the same at all points inside it and also on its surface. i.e., the gravitational field is uniform.* Hence the interior of a thin, uniform spherical shell is an equipotential volume.

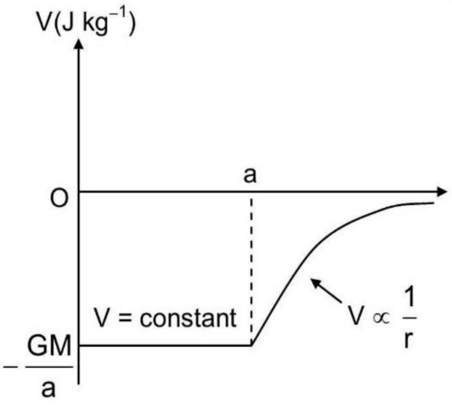


Fig. 3.16

Variation of gravitational potential due to a thin, uniform spherical shell of radius 'a' is shown in Fig. 3.16.

We know that E = 0, inside a spherical shell due to its mass alone. Also $E = -\frac{dV}{dr} \Rightarrow \frac{dV}{dr} = 0 \Rightarrow V$ = constant inside a thin, uniform spherical shell. *Thus, it is not necessary that if the gravitational field is zero at a point, the gravitational potential is zero at that point.*

## Gravitational potential due to a uniform solid sphere

***(i) P outside the sphere (r > a)***

Consider a uniform solid sphere of mass M and radius a, with centre at O.

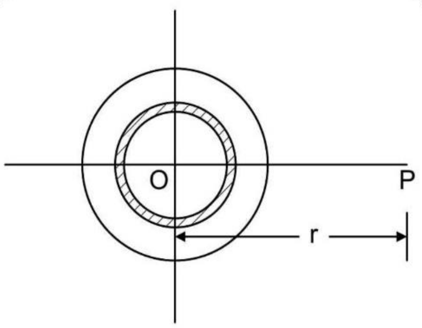


Fig. 3.17

The solid sphere can be divided into a large number of concentric uniform spherical shells, each of mass dm. The gravitational potential at P due to the shell of mass dm is

$$dV = -\frac{G\,dm}{r}$$

Total gravitational potential at P due to the entire sphere,

$$V = \int dV = \int_0^M -\frac{Gdm}{r} = -\frac{GM}{r}$$

$$\therefore \quad V = -\frac{GM}{r} \text{ for all external points}$$

Hence *a solid uniform sphere can be treated as a particle at its centre, having the same mass as the sphere, for calculation of gravitational potential at all external points.*

***(ii) P inside the sphere (r < a)***

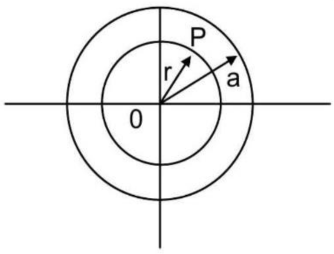


Fig. 3.18

The gravitational potential V at P is due to

(a) a uniform solid sphere of radius r and mass $M_S$ and

(b) a hollow sphere of outside radius a and inside radius r

$\therefore V = V_S + V_H$, where $V_S$ = potential due to solid sphere of mass $M_S$ and

$V_H$ = potential due to hollow sphere

$$M_S = \frac{M}{\left(\frac{4}{3}\pi a^3\right)} \times \frac{4}{3}\pi r^3 = \frac{Mr^3}{a^3}$$

$$\therefore \quad V_S = -\frac{GM_S}{r} = -\frac{GMr^3}{a^3 r} = \frac{-GMr^2}{a^3} \qquad \text{— (i)}$$

For calculation of $V_H$, we take an elemental shell of radius x and thickness dx($x_{min} = r$, $x_{max} = a$)

$$dM_H = \frac{M}{\left(\frac{4}{3}\pi a^3\right)} .4\pi x^2 dx = \frac{3Mx^2 dx}{a^3}$$

$$dV_H = -\frac{GdM_H}{x} = \frac{-3GMxdx}{a^3}$$

$$\therefore \quad V_H = \int dV_H = \int_{x=r}^{x=a} \frac{-3GMxdx}{a^3} = -\frac{3GM}{a^3}\left|\frac{x^2}{2}\right|_r^a$$

$$= -\frac{3GM}{2a^3}\left(a^2 - r^2\right)$$

$$\therefore \quad V = V_S + V_H = -\frac{GMr^2}{a^3} - \frac{3GM}{2a^3}\left(a^2 - r^2\right)$$

$$= -\frac{GM}{2a^3}[2r^2 + 3a^2 - 3r^2] = -\frac{GM}{2a^3}\left[3a^2 - r^2\right]$$

$$\therefore \quad V = -\frac{GM}{2a^3}\left(3a^2 - r^2\right)$$

for interior points. At the centre of the sphere, r = 0

$$\Rightarrow \quad V = -\frac{3}{2}\frac{GM}{a}$$

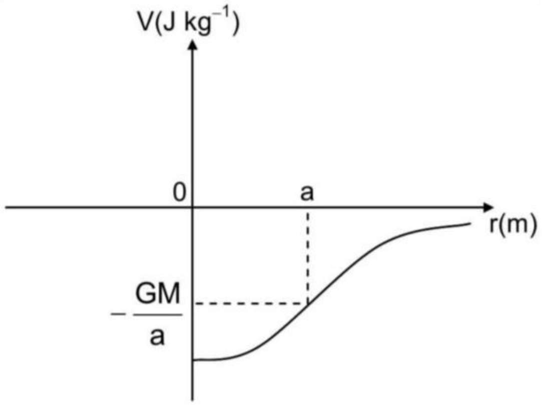



**Fig. 3.19**

Variation of gravitational potential due to a uniform solid sphere of radius 'a' is shown in Fig.3.19.

## GRAVITATIONAL POTENTIAL ENERGY

Since gravitational force is a conservative force, we can define gravitational potential energy associated with a system of particles, interacting through gravitational force.

We had earlier defined the gravitational potential $V = -\frac{W_G}{m}$, where $W_G$ = work done by gravitational force on a particle of mass m, in bringing it slowly from infinity to its position in the gravitational field.

Since $-W_G$ is the negative of the work done by a conservative force, it is equal to the change in potential energy between the final and initial positions($\because \Delta U = -W_G$, where $\Delta U$ equals the change in potential energy)

$$\therefore \quad V = \frac{\Delta U}{m} = \frac{U(r) - U(\infty)}{m}, \text{ where}$$

U(r) = gravitational potential energy of the particle at position r

U(∞) = gravitational potential energy at infinity (conventionally taken as zero)

$$V = \frac{U(r)}{m}$$

That is *gravitational potential at a point in a gravitational field is the gravitational potential energy of a unit mass placed at that point.*

$\therefore$ U = mV *is the gravitational potential energy of a particle of mass m, placed at a point in a gravitational field, where the gravitational potential is V.* Gravitational potential energy of a particle is zero at infinite distance or it is always negative.

## CONCEPT STRAND

### Concept Strand 6

What is the gravitational potential energy of a particle of mass 5 kg, when placed at a point where the gravitational potential is $-10^{-6}$ J kg$^{-1}$?

*Solution*

$$U = mV$$
$$= 5 \times (-10^{-6}) \text{ J}$$
$$= -5 \times 10^{-6} \text{ J}$$

## Gravitational potential difference

The gravitational potential difference between two points in a gravitational field is the change in potential energy of a particle of unit mass, when it is moved from one point to the other, against the gravitational force.

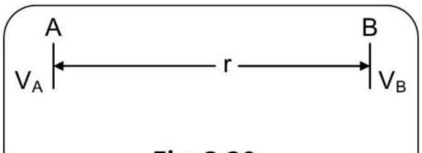



**Fig. 3.20**

Let A and B be two points, at gravitational potentials $V_A$ and $V_B$ respectively. If a particle of mass m is placed at A, its potential energy at A is $U(A) = mV_A$ — (i)

If this particle is moved very slowly from A to B, at B, its potential energy is $U(B) = mV_B$ — (ii)

$\therefore$ $\Delta U$ = change in potential energy

$= U(B) - U(A)$

$= mV_B - mV_A = m(V_B - V_A)$

Potential difference between A and B is given by

$$\Delta V = \frac{\Delta U}{m} = \frac{m(V_B - V_A)}{m} = V_B - V_A$$

If A is at infinity, $V_A = 0 \Rightarrow \Delta V = V_B$

*i.e., Gravitational potential at any point in a gravitational field is the change in potential energy of a particle of unit mass brought from infinity to that point in the gravitational field.*

## Gravitational potential energy of a system of two particles

The gravitational potential energy of a system of two particles is the negative of the work done by the gravitational force in assembling the system by bringing the particles from infinity to the desired configuration.

Consider two particles of masses $m_1$ and $m_2$, placed at A and B respectively, separated by a distance 'r'.

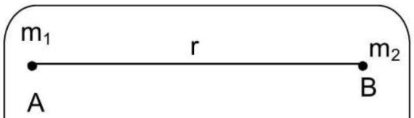



**Fig. 3.21**

If we consider that these particles were initially at infinite distance, initial potential energy of A = $m_1V_\infty = 0$ and initial potential energy of B = $m_2V_\infty = 0$ ($\because V_\infty = 0$)

$\therefore$ $U_i$ = Initial potential energy of the system = $m_1V_\infty + m_2V_\infty = 0$

If particle $m_1$ was brought from infinity to A, no work is done as there is no gravitational field(or force). But $m_1$ sets up a gravitational field all around A so that gravitational potential at B is $V_B = -\frac{Gm_1}{r}$

Potential energy of $m_2$ when it is brought to B is = $m_2V_B = -\frac{Gm_1m_2}{r}$

$\therefore$ Total potential energy of system of A and B

$$= -\frac{Gm_1m_2}{r} + 0 = \frac{-Gm_1m_2}{r}$$

$$\therefore \quad U = -\frac{Gm_1m_2}{r}$$

This is the potential energy of the two particle system. If we consider $m_2$ was brought to B first from infinity, it will produce a gravitational potential at A which is $V_A = -\frac{Gm_2}{r}$

The change in potential energy of $m_1$ when it is brought to A is $U = m_1V_A = -\frac{Gm_1m_2}{r}$

Hence it is immaterial how the particles were brought to their configuration. The final energy of the configuration is the same and is $U = \frac{-Gm_1m_2}{r}$

## Gravitational potential energy of three particle system

Let us consider that all the three particles were at infinite distances apart initially so that they do not exert any gravitational force on each other.

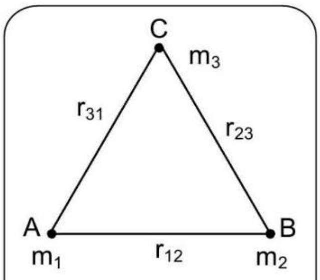



**Fig. 3.22**

If $m_1$ was brought first to A, no work is done as there is no gravitational force.

$\therefore$ $U_A = 0$ (single particle system where V = 0)

Now $m_1$ establishes a gravitational field all around it and the potential at B due to A is $V_{BA} = -\frac{Gm_1}{r_{12}}$

When particle $m_2$ is brought to B, the potential energy of the two particle system of $m_1$ and $m_2$ is $U_{AB} = m_2V_{BA} = -\frac{Gm_1m_2}{r_{12}}$

The gravitational potential at C due to A is $-\frac{Gm_1}{r_{31}}$ and at C due to B is $-\frac{Gm_2}{r_{23}}$

Hence the potential energy of 2 particle system B and C is $U_{BC} = -\frac{Gm_2m_3}{r_{23}}$

and the potential energy of 2 particle system C and A is $U_{CA} = -\frac{Gm_3m_1}{r_{31}}$

Hence the total gravitational potential energy of the 3 particles system is

$$U = U_{AB} + U_{BC} + U_{CA}$$

$$= -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_3m_1}{r_{31}}$$

i.e., $U = -G\left[\frac{m_1m_2}{r_{12}} + \frac{m_2m_3}{r_{23}} + \frac{m_3m_1}{r_{31}}\right]$, is the gravitational potential energy of a three particles system.

**Note:**

If there are n particles in a system, they form $\frac{n(n-1)}{2}$ pairs of particles. The total gravitational potential energy of the system is the sum of the gravitational potential energy of all such pairs.

## *'Self energy' of bodies*

The energy possessed by a body due to interaction of the particles inside the body, is called the self energy of the body.

For a single particle, there is no self energy. For other bodies, self energy of the body is the negative of the work done by the gravitational forces in assembling the body from infinity to their corresponding configuration to make the desired body.

## *Self energy of a thin, uniform hollow sphere*

Consider a thin, uniform hollow sphere of mass m and radius R. Its initial mass is zero and as particles of mass dm get added to it, its mass increases (becomes more positive) while potential energy decreases(becomes more negative).

When the mass of the shell is 'm', if a particle of mass dm is added to the shell, the potential energy of the system is

$$dU = -Vdm = \frac{Gm}{R}dm, \text{ where } V = \text{potential at surface}$$

$$= \frac{-Gm}{R}$$

$\therefore$ Total potential energy of the system(self energy)

$$U = \int_0^U dU = \int_{m=0}^{m=M} -\frac{Gm}{R}dm = -\frac{G}{R}\left|\frac{m^2}{2}\right|_0^m = \frac{-GM^2}{2R}$$

$$U = -\frac{GM^2}{2R}$$

is the self energy of a thin, uniform spherical shell of mass M and radius R.

## *Self energy of a uniform solid sphere*

Consider a uniform solid sphere of mass M and radius R. Initially, its mass is zero as no particle is in the system. As particles are brought from infinity, both mass and radius keep on increasing.

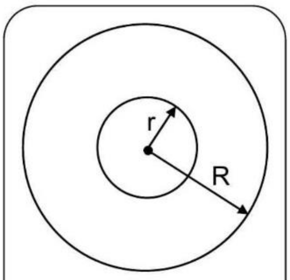



**Fig. 3. 23**

At an instant, when its radius is r, its mass is m, so that the potential at its surface is $dV = -\frac{Gm}{r}$

When a small particle of mass dm is added to the system, potential energy of the system is dV dm.

i.e., $$dU = -\frac{Gm}{r}dm \qquad \text{— (i)}$$

But m = volume × density

$$= \frac{4}{3}\pi r^3 \times \frac{M}{\frac{4}{3}\pi R^3} = \frac{Mr^3}{R^3}$$

$$\therefore \quad dm = \frac{3Mr^2}{R^3}dr \qquad \text{— (i)}$$

$$\therefore dU = -\frac{Gm}{r}dm = -\frac{GMr^3}{rR^3}.\frac{3Mr^2}{R^3}dr = -\frac{3GM^2}{R^6}.r^4dr$$

$$\therefore \quad U = \int dU = \int_{r=0}^{r=R} \frac{-3GM^2}{R^6}r^4dr$$

$$= \frac{-3GM^2}{R^6}\left|\frac{r^5}{5}\right|_0^R = \frac{-3GM^2}{5R^6} \times R^5$$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

is the self energy of the uniform solid sphere of mass M and radius R.

## CONCEPT STRAND

### Concept Strand 7

Considering Earth as a uniform solid sphere of radius 6400 km and mass $5.98 \times 10^{24}$ kg, calculate the minimum energy required to separate all particles of Earth to infinite distance of separation.

### *Solution*

$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}; M = 5.98 \times 10^{24} \text{ kg}$

$$U_i = \text{The self energy of Earth} = \frac{-3GM^2}{5R}$$

When all particles are at infinite distance, $U_f = 0$

$$\therefore \quad \Delta U = U_f - U_i = 0 + \frac{3GM^2}{5R}$$

$$= \frac{3 \times 6.67 \times 10^{-11} \times (5.98 \times 10^{24})^2}{5 \times (6.4 \times 10^6)}$$

$$= 22.36 \times 10^{31} \text{ J}$$

$$= 2.36 \times 10^{32} \text{ J}$$

Hence it is not easy to disintegrate Earth.

## ACCELERATION DUE TO GRAVITY

The force of attraction between two bodies produces acceleration of the bodies towards each other.

Consider the attraction between the Earth (mass M) and a body (mass m). The force of attraction F produces acceleration of both bodies given by

$$F = \frac{GMm}{R^2} \qquad \text{— (i)}$$

$$F = mg = Ma \qquad \text{— (ii)}$$

But, M being very large, a is unnoticeable. However, g, the acceleration due to gravity of the body of mass m is measurable. Equations (i) and (ii) yield

$$g = \frac{GM}{R^2}$$

The acceleration due to gravity of Earth on a particle is independent of its mass. The value of g at most places on Earth is about $9.8 \text{ m s}^{-2}$ and the value has been standardized as $9.8066 \text{ m s}^{-2} \approx 9.8 \text{ m s}^{-2}$.

Note that, in problems, unless otherwise mentioned, we take $g = 9.8 \text{ m s}^{-2}$.

## CONCEPT STRANDS

### Concept Strand 8

What is the acceleration of Earth towards a spherical body of radius 1 m and density same as that of the Earth and which is on the surface of the Earth?

#### Solution

$$\frac{GMm}{R^2} = Ma \Rightarrow a = \frac{Gm}{R^2} = \frac{m}{M}g$$

$$= \frac{1}{(6.4\times10^6)^3}g \approx 10^{-20}\ \text{m s}^{-2}\ \text{(negligibly small)}$$

### Concept Strand 9

If the acceleration due to gravity on a planet is $\frac{1}{4}$ of that on Earth and the radius of Earth is 20 times that of the planet, what is the density ratio of the planet to that of the Earth?

#### Solution

$$\frac{g_P}{g_e} = \frac{M_P R_e^2}{M_e R_P^2} \Rightarrow \frac{1}{4} = \frac{1}{20}\frac{\rho_P}{\rho_e} \Rightarrow \frac{\rho_P}{\rho_e} = 5$$

$$\left(\because M_P = \frac{4}{3}\pi R_P^3 \rho_P \ \text{ and } \ M_e = \frac{4}{3}\pi R_e^2 \rho_e\right)$$

### Concept Strand 10

What is the distance above Earth where a body will experience equal values of acceleration due to gravity due to the Earth and due to a planet with $\frac{1}{4}$ of the mass of the Earth, and at a distance D from centre of Earth?

#### Solution

Let D be the distance between the Earth and the planet. Then,

$$\frac{GM_e m}{(R+h)^2} = \frac{GM_P m}{[D-(R+h)]^2}$$

$$\Rightarrow \frac{[D-(R+h)]^2}{(R+h)^2} = \frac{M_P}{M_e} = \frac{1}{4}$$

$$\Rightarrow \frac{D-(R+h)}{R+h} = \frac{1}{2}$$

$$\Rightarrow \frac{D}{R+h} = \frac{3}{2}$$

$$\Rightarrow h = \frac{2}{3}D - R$$

### Concept Strand 11

If an outer shell of thickness 100 km disintegrates from the Earth, what will be the percentage change in the acceleration due to gravity on the surface of the Earth?

#### Solution

$$g' = G\frac{M'}{R'^2} = g\frac{R^2}{M}\frac{M'}{R'^2} = g\frac{R^2 R'^3}{R^3 R'^2} = g\frac{R'}{R}$$

$$\therefore \frac{g-g'}{g}\times100 = 1.5\%$$

## Variation of acceleration due to gravity

If acceleration due to gravity is considered to be the net effect of Earth on a body on or near it, it can vary due to the following:

### *(i) Effect of altitude*

The force of attraction experienced by a body at a height 'h' above the surface of the Earth is

$$F = \frac{GMm}{(R+h)^2} \quad \text{— (4)}$$

where R is the radius of the Earth. In the above formula we have assumed that all the mass of the Earth is concentrated at the centre. This assumption is valid so long as the force is measured outside the bodies in question. The acceleration due to gravity can be calculated by writing

$$mg' = \frac{GMm}{(R+h)^2}$$

where $g' = \frac{GM}{R^2}\frac{1}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(\frac{R}{R+h}\right)^2$

If h << R, as is usual for bodies near the surface of the Earth,

$$g' = g\left(1-\frac{2h}{R}\right)$$

If we treat Earth as a uniform solid sphere of mass M and radius R, the gravitational field of Earth at height 'h' above Earth,

$$E = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\cdot\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+h}\right)^2$$

which is the same expression as the effective acceleration due to gravity at height 'h'.

It should be understood that gravitational field intensity and acceleration due to gravity are two different physical quantities but their magnitudes and directions are same.

### (ii) Effect of depth below the surface of the Earth

The average density of Earth can be calculated from the knowledge of g and R

$$g = \frac{GM}{R^2} = \frac{GM}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi R$$

$$g = \rho_{av}\frac{4\pi GR}{3}$$

$$\rho_{av} = \frac{3g}{4\pi GR} = 5.483 \text{ g cm}^{-3}$$ (substituting standard values of g, G, and R)

The Earth consists of an outer crust (~40 km) of density ~3 g $cm^{-3}$, a mantle (~3000 km) of density varying from 3.5 to 5.5 g $cm^{-3}$ and an inner solid core, of density ~13 g $cm^{-3}$. However, in gravitational problems we assume that the density of Earth is a constant, and equal to $\rho_{av}$(~5.5 g $cm^{-3}$)

A body at a depth d below the surface of the Earth is subject to an attractive force due to the volume of Earth below it, that is, due to the mass $\frac{4}{3}\pi(R-d)^3\rho$, where ρ is the average density of the Earth. Hence,

$$mg' = G\frac{\frac{4}{3}\pi(R-d)^3\rho.m}{(R-d)^2}$$

leading to

$$g' = \frac{G}{R^3}\left(\frac{4}{3}\pi R^3\rho\right)(R-d) = \frac{GM}{R^2}\frac{(R-d)}{R}$$

$$g' = g\left(1-\frac{d}{R}\right)$$

At the centre of Earth, $d = R \Rightarrow g' = 0$

*It may be noted that g' = 0 at the centre of the Earth.*

Treating Earth as a uniform solid sphere, gravitational field at depth 'd' below surface of Earth = gravitational field at radius r = (R – d)

i.e., $$E = \frac{GMr}{R^3} = \frac{GM(R-d)}{R^2 . R}$$

$$= g\left[1-\frac{d}{R}\right]$$

which is the same expression as g' at depth d

### (iii) Apparent change in acceleration due to gravity due to Earth's rotation

Earth rotates about the N–S axis at an angular frequency $\omega = 7.3\times10^{-5}$ $rads^{-1}$Any point particle P on the surface is subject to a pseudo force.(because of rotation of Earth, a frame of reference attached to Earth is a non-inertial frame and hence the pseudo force)

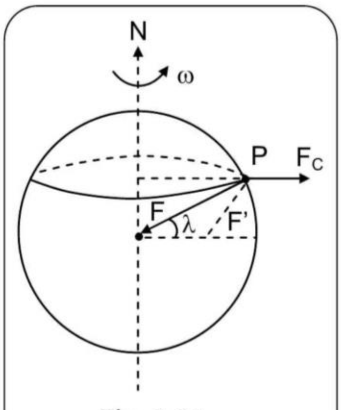



**Fig. 3.24**

$$F_c = m\,\omega^2 r = m\omega^2 R\cos\lambda$$

where, λ is the latitude at the point P. Therefore, the total force on the particle is the resultant F' of $F_c$ and the gravitational attraction F

$$F' = \sqrt{F^2 + F_c^2 + 2FF_c\cos(180^\circ - \lambda)}$$

$$= \sqrt{\left(\frac{GMm}{R^2}\right)^2 + (m\omega^2R\cos\lambda)^2 - 2\frac{GMm}{R^2}.m\omega^2R\cos^2\lambda}$$

$$= \frac{GMm}{R^2}\left[1+\left(\frac{\omega^2R^3\cos\lambda}{GM}\right)^2 - 2.\frac{\omega^2R^3\cos^2\lambda}{GM}\right]^{1/2}$$

The second term in the bracket << the first term and the last term. Expanding binomially and neglecting higher order terms

$$\therefore \quad F' = \frac{GMm}{R^2}\left[1-\frac{\omega^2R^3\cos^2\lambda}{GM}\right]$$

$$\therefore \quad mg' = \frac{GMm}{R^2}\left[1 - \frac{\omega^2 R^3 \cos^2\lambda}{GM}\right]$$

$$g' = g\left(1 - \frac{\omega^2 R \cos^2\lambda}{g}\right)$$

Note that, at the poles, $\lambda = 90°$ and $g' = g$ while at the equator, g' has the lowest value $g' = g(1 - \omega^2 R)$

### (iv) Effect of the shape of the Earth

In all the cases above, we have assumed that the Earth is perfectly spherical, but the shape is ellipsoidal, bulging at the equator and flattened at the poles. Thus

Equatorial radius > polar radius.

As $g \propto \frac{1}{R^2}$, it increases from the equator to the poles, being minimum at equator and maximum at the poles.

## CONCEPT STRAND

### Concept Strand 12

How much faster will the Earth have to rotate for a particle on the equator to fly off?

### Solution

Weight at the equator $W_e = m\, g_e = m(g - \omega^2 R)$

For $W_e = 0, g = \omega^2 R \Rightarrow \omega$

$= 1.237 \times 10^{-3}$ rad s$^{-1}$

The increase is $\frac{1.237 \times 10^{-3}}{7.275 \times 10^{-5}} \approx 17$

The Earth will have to rotate 17 times faster.

## ESCAPE VELOCITY

Escape velocity is the minimum velocity required for a particle to escape from the gravitational field of a celestial body such as a planet. The magnitude of escape velocity depends on the mass of the planet, and the distance from it. Consider a particle of mass m, at a distance r from a body of mass M. Then, the potential energy of the configuration is given by

$$PE = \frac{-GMm}{r}$$

This can be taken as the PE of the particle in the field of the massive body M.

For the particle to escape, its total energy should be zero.

i.e., $\quad KE + PE = 0$

$$\Rightarrow \quad KE = -PE = \frac{GMm}{r}$$

$$KE = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{r}}$$

For a particle on the surface of Earth, r = R, the radius of Earth.

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

With $g = 9.8$ m s$^{-2}$ and $R = 6.4 \times 10^6$ m, the escape velocity on the surface of the earth is $v_e = 11.2 \times 10^3$ ms$^{-1}$

Escape velocity does not depend on (i) the mass of the particle (ii) the angle of projection.

**Note:**

*If a body is launched from the surface of Earth with speed greater than $v_e$ (i.e., escape velocity), at infinite distance from Earth, only its PE becomes zero. Its kinetic energy will not be zero.*

## CONCEPT STRAND

### Concept Strand 13

What is the energy necessary for a particle of mass 1kg to escape from the gravity of the Earth?

*Solution*

$$W = \int dW = GMm\int_R^{\infty}\frac{dr}{r^2}$$

$$= GMm\left[\frac{-1}{r}\right]_R^{\infty} = \frac{GMm}{R}$$

$$= \frac{6.67\times10^{-11}\times5.98\times10^{24}\times1}{6.4\times10^{6}}$$

$$=6.23\times10^{7}\,J$$

## SATELLITES IN CIRCULAR ORBITS

Satellites are usually launched into circular orbits. Consider a satellite in an orbit of radius $r = R + h$ where R is the radius of the Earth and h is the height above the Earth at which the satellite is orbiting. The centripetal force necessary to keep the satellite in its orbit is provided by the gravitational attraction of the Earth directed towards the centre of the Earth. Thus we may write

$$\frac{m v_0^2}{r} = \frac{GMm}{r^2}$$

where $v_0$ is the orbital velocity.

$\therefore v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{gR^2}{R+h}}$. Hence $v_0$ is independent of mass of satellite.

$$v_0 \text{ near surface of Earth } = \sqrt{\frac{Gm}{R}} = \frac{v_e}{\sqrt{2}}\left(\because v_e = \sqrt{\frac{2Gm}{R}}\right)$$

$$v_0 = \frac{v_e}{\sqrt{2}}$$

Angular velocity ω is given by

$$\omega = \frac{v_0}{r} = \sqrt{\frac{gR^2}{(R+h)^3}} \qquad \omega = \frac{\sqrt{gR^2}}{r^{3/2}}$$

The time period T is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{gR^2}}r^{3/2} \quad T^2 \propto r^3$

### Geostationary satellites

Geostationary satellites have time period same as that of the Earth so that their position in the orbit is stationary with respect to the Earth. The radius of the geostationary orbit can be calculated as

$$r = R + h = \left[gR^2\left(\frac{T}{2\pi}\right)^2\right]^{1/3}$$

Substituting the values of g and R we get $h \approx 36000$ km.

*A geostationary satellite moves in the same direction as rotation of Earth (West to East), with a time period of 24 hour and its orbital plane passes through the equatorial plane of Earth.*

## CONCEPT STRAND

### Concept Strand 14

Satellite A is launched such that its time period is 3 times that of a geostationary satellite B. What is the height of A above the Earth?

*Solution*

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$\therefore \frac{T_1^2}{T_2^2} = 9 = \frac{r_1^3}{r_G^3}$

But $r_G = 6.625$ R for a geostationary satellite, where R is the radius of Earth.

$\therefore 9 = \frac{r_1^3}{(6.625R)^3} \Rightarrow h = 88196$ km where h is the height of satellite A above the surface of the Earth.

### *First and second cosmic velocities*

For satellites in orbits close to the Earth, equation for orbital speed $v_0$ can be written as

$v_0 = \sqrt{gR} = 7.92\ \text{km s}^{-1}$ where $v_0$ is called the first cosmic velocity.

The escape velocity from the surface of Earth is also known as the second cosmic velocity.

$$v_e = \sqrt{2gR} = \sqrt{2}v_0 = 11.2\ \text{km s}^{-1}$$

## CONCEPT STRAND

### Concept Strand 15

A planet revolving around the star in a circular orbit of radius R with a time period T is subject to a gravitational force proportional to $R^{-3}$. Calculate T as a function of R.

### *Solution*

$$\frac{mv^2}{R} = kR^{-3} \Rightarrow v = \sqrt{\frac{kR^{-2}}{m}}$$

$$\frac{v}{R} = \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{mR^2}{kR^{-2}}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}}R^2$$

### *Energy of a satellite*

Consider a satellite in a circular orbit of radius r around a planet of mass M.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Kinetic energy $= \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Potential energy $= \frac{-GMm}{r}$

$$TE = \text{Total energy} = PE + KE = \frac{-GMm}{2r}$$

$$\Rightarrow \quad E = -KE = \frac{PE}{2}$$

For satellites,

$$KE : PE : TE = 1 : -2 : -1$$

## Kepler's laws

### *First law (The law of orbits)*

"The path of a planet is an elliptical orbit around the sun with the sun at one of its foci".

### *Second law (The law of areas)*

"The radius vector, drawn from the sun to the planet sweeps out equal areas in equal intervals of time". Alternatively, "The areal velocity of a planet in its orbit is a constant".

Consider a small elemental area dA swept out by the planet;

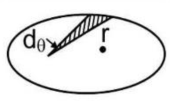



Fig. 3.25

$$dA = \frac{1}{2}r^2\,d\theta$$

$$\therefore \quad \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m}$$

where L is the angular momentum, a constant.

### Third law (The law of periods)

"The square of the time period of a planet around the sun is proportional to the cube of the semi major axis of its orbit". This has been proved for satellites in circular orbits in.

### Angular momentum of a satellite

If m is the mass of satellite, $v_0$ is its orbiting speed and r is the radius of the orbit, then

$$\bar{v}_0 \perp \bar{r}$$

Angular momentum of the satellite in its orbit is $\bar{L} = \bar{r} \times \bar{p}$

$$= \bar{r} \times m\bar{v}_0$$

$$= m\left(\bar{r} \times \bar{v}_0\right)$$

$$\therefore \quad L = mrv_0 \; (\because \sin\theta = \sin 90° = 1)$$

$$L = mr\sqrt{\frac{Gm}{r}} = \sqrt{m^2Gmr}$$

$$\therefore \quad L = mrv_0 = \sqrt{m^3Gr}$$

is the angular momentum of the satellite. Here, we have considered that there is no spin for the satellite. Since no external torque acts on the satellite during its orbital motion, its angular momentum is conserved. i.e. L = constant

## CONCEPT STRAND

### Concept Strand 16

A planet in its elliptical orbit has the farthest distance from the Sun($r_1$) equal to three times its nearest distance from the Sun($r_2$). Will the orbital speed of the planet be different at those points? Explain

### Solution

Let m be the mass of the planet. If $v_1$ and $v_2$ are the orbital speeds of the planet with respect to the Sun, at positions $r_1$ and $r_2$ respectively, then

Angular momentum of planet at position $\bar{r}_1$ is $\bar{L}_1 = m\left(\bar{r}_1 \times \bar{v}_1\right) \Rightarrow L_1 = mr_1v_1 \left(\because \bar{r}_1 \perp \bar{v}_1\right)$

Angular momentum of planet at position $\bar{r}_2$ is $\bar{L}_2 = m\left(\bar{r}_2 \times \bar{v}_2\right) \Rightarrow L_2 = mr_2v_2 \left(\because \bar{r}_2 \perp \bar{v}_2\right)$. Since angular momentum of the planet is conserved ($\because$ no torque acts on the planet), $L_1 = L_2 \Rightarrow mr_1v_1 = mr_2v_2$

$$\therefore \quad \frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{r_2}{3r_2} = \frac{1}{3}$$

$$\therefore \quad v_2 = 3v_1$$

Hence the orbital speed of planet in the nearest position to Sun is three times the orbital speed of planet in the farthest position.

### Nature of trajectory

Let a satellite be projected from certain height from Earth's surface with a velocity v along the x-direction as shown in Fig. 3.24.

(i) If v = 0, it will fall vertically down towards Earth.

(ii) If $0 < v < v_0$, where $v_0 = \sqrt{gR}$, then the projectile will fall back to Earth in a spiral path.

(iii) If $v = v_0$, where $v_0 = \sqrt{gR}$, the satellite will move in a circular orbit of radius R.

(iv) If $v_0 < v < v_e$, where $v_e = \sqrt{2gR}$, the satellite will move in an elliptical orbit.

(v) If $v = v_e$, the projectile will escape from Earth's gravity in a parabolic path.

(vi) If $v > v_e$, the projectile will escape from Earth's gravity in a hyperbolic path.

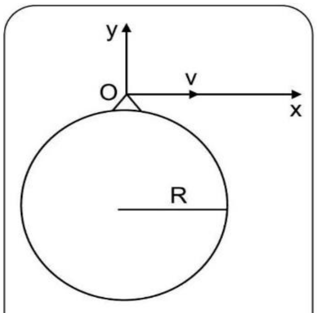


Fig. 3.26

### *Potential Energy at different positions*

M = mass of Earth, m = mass of body,

G = universal gravitational constant

R = radius of Earth, W = weight of body (mg)

g = acceleration due to gravity (on surface of Earth)

| | | | | |
|---|---|---|---|---|
| 1. | P.E of body on surface of Earth | $\frac{-GMm}{R}$ | $-mgR$ | $-WR$ |
| 2. | P.E at altitude h = R | $\frac{-GMm}{2R}$ | $-\frac{mgR}{2}$ | $\frac{-WR}{2}$ |
| 3. | P.E at finite altitude 'h' | $\frac{-GMm}{(R+h)}$ | $-\frac{mgR}{\left(1+\frac{h}{R}\right)}$ | $\frac{-WR}{\left(1+\frac{h}{R}\right)}$ |
| 4. | P.E at infinite altitude | Zero | Zero | Zero |
| 5. | Minimum work done to transfer a body from Earth's surface to an altitude h = R | $\frac{GMm}{2R}$ | $\frac{mgR}{2}$ | $\frac{WR}{2}$ |

## SUMMARY

| | |
|---|---|
| $F=\frac{Gm_1m_2}{r^2}$ | F → Gravitational force between two particles of masses $m_1$ and $m_2$.<br>G → Gravitational Constant<br>r → Distance between the two masses |
| $g=\frac{GM_e}{R_e^2}=\frac{4}{3}\pi GR_e\rho_e$ | g → Acceleration due to gravity on the surface of Earth<br>$R_e$ → Radius of Earth<br>$M_e$ → Mass of Earth<br>$\rho_e$ → Density of Earth |
| $g=\frac{GM}{R^2}$ | g → Acceleration due to gravity on the surface of a planet or satellite<br>M → Mass of the planet or satellite<br>R → radius of the planet or satellite |
| $g_\lambda = g-\omega^2R\cos^2\lambda$ | $g\lambda$ → Acceleration due to gravity at a given latitude $\lambda$<br>$\omega$ → Angular velocity of rotation of Earth |
| $g_e = g-\omega^2R_e$<br>$g_p = g$ | $g_e$ → Acceleration due to gravity at the equator<br>$g_p$ → Acceleration due to gravity at the poles |
| $g_h=\frac{GM}{(R_e+h)^2}=g\left(\frac{R_e}{R_e+h}\right)^2$ | $g_h$ → Acceleration due to gravity at a height h above the surface of Earth |
| If $\frac{g_h}{g}=n$, $h=R\left(\sqrt{n}-1\right)$ | g = acceleration due to gravity on the surface of Earth.<br>R = radius of Earth |
| $g_h=g\left(1-\frac{2h}{R_e}\right)$ | $g_h$ → Acceleration due to gravity at a height h above the surface of Earth ($h << R_e$) |

| | |
|---|---|
| $g_d = \frac{4}{3}\pi G\rho(R_e - d)$ | $g_d \rightarrow$ Acceleration due to gravity at a depth "d" from surface of earth.<br>$\rho \rightarrow$ Average density of Earth |
| $g_d = g\left(1 - \frac{d}{R_e}\right)$<br>If $\frac{g_d}{g} = n$, then $d = R_0\left[\frac{n-1}{n}\right]$<br>$g_{\text{centre of Earth}} = 0$ | $\rho \rightarrow$ density of Earth.<br>$R_e \rightarrow$ radius of Earth<br>d = depth below surface of Earth |
| $I = \frac{GM}{r^2}$ | $I \rightarrow$ Gravitational intensity at a distance 'r' from the centre of a particle of mass M |
| $V = -\frac{GM}{r}$ | $V \rightarrow$ Gravitational Potential at a distance 'r' from a particle of mass M |
| $P.E = -\frac{Gm_1m_2}{r}$ | $PE \rightarrow$ Potential Energy of a system of two particles of masses $m_1$ and $m_2$ separated by a distance 'r' |
| $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$ | $v_e \rightarrow$ Escape velocity from the surface of Earth |
| $v_o = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{r}}$ | $v_o \rightarrow$ Orbital velocity of a satellite which is orbiting in a circular path of radius 'r'(at a height h from the surface) |
| $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$ | $T \rightarrow$ Time period of revolution of a satellite which is revolving at a height 'h' |
| Energy of a satellite<br>$KE = \frac{1}{2}mv_0^2 = \frac{1}{2}\frac{GMm}{(R+h)}$<br>$PE = -\frac{GMm}{(R+h)} = -2K.E$<br>$TE = -\frac{1}{2}\frac{GMm}{(R+h)} = -K.E$<br>K.E:P.E:T.E =1:-2:-1 | $KE \rightarrow$ Kinetic Energy of a satellite<br>$PE \rightarrow$ Potential Energy of a satellite<br>$TE \rightarrow$ Total Energy of a satellite |
| $T = 2\pi\sqrt{\frac{R}{g}}$ | $T \rightarrow$ Time Period of a planet around the Sun<br>$g \rightarrow$ Acceleration due to gravity of Sun at the orbit |