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IN THIS CHAPTER ....

In this chapter, we study a useful counting technique in determining the number of different ways of arranging and selecting objects without actually listing them. Firstly, examine a principle which is most fundamental to the learning of these techniques.

Fundamental Principles of Counting [FPC]

Fundamental Principle of Multiplication

If an operation can be performed in m different ways, following which a second operation can be performed in n different ways, then the two operations in succession can be performed in $m \times n$ ways. This can be extended to any finite number of operations.

e.g. A hall has 12 gates. After entering into the hall the man come out through a different gate in 11 ways.

Hence, by the fundamental principle of multiplication, the total number of ways of man come out through different gates $= 12 \times 11 = 132$.

Fundamental Principle of Addition

If an operation can be performed in m different ways and another operation, which is independent of the first operation can be performed in n different ways. Then, either of the two operations can be performed in $(m + n)$ ways. This can be extended to any finite number of mutually exclusive operations.

e.g. There are 25 students in a class in which 15 boys and 10 girls.

The class teacher select either a boy or a girl for monitor of the class. Since, there are 15 ways to select a boy and there are 10 ways to select a girl.

Hence, by the fundamental principle of addition, the number of ways in which either a boy or a girl can be chosen as a monitor $= 10 + 15 = 25$ ways.

Example 1. *If $x < 4 < y$ and $x, y \in \{1, 2, 3, \dots, 10\}$, then find the number of ordered pairs (x, y).*

(a) 30 (b) 18 (c) 40 (d) 100

Sol. (b) We have, $x < 4 < y$,

where $x, y \in \{1, 2, 3, \dots, 10\}$

$\Rightarrow x = \{1, 2, 3\}$ and $y = \{5, 6, 7, 8, 9, 10\}$

Here, x have 3 options and y have 6 options.

$\therefore$ By multiplication rule,

Number of ordered pairs $= 3 \times 6 = 18$

Factorial Notation

The product of first n natural numbers is denoted by $n!$ and read as 'factorial n'.

Thus, $n! = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1$

e.g. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

and $4! = 4 \times 3! = 4 \times 3 \times 2! = 4 \times 3 \times 2 \times 1 = 24$

Properties of Factorial Notation

(i) $0! = 1! = 1$

(ii) Factorials of negative integers and fractions are not defined.

(iii) $n! = n(n-1)! = n(n-1)(n-2)!$

(iv) $\dfrac{n!}{r!} = n(n-1)(n-2)\dots(r+1)$

Exponent of Prime p in $n!$

Let n be a positive integer and p be a prime number. Then, last integer amongst $1, 2, 3, \dots, (n-1), n$ which is divisible by p is $\left[\dfrac{n}{p}\right]p$, where $\left[\dfrac{n}{p}\right]$ denotes the greatest integer less than or equal to $\dfrac{n}{p}$.

e.g. $\left[\dfrac{12}{5}\right] = 2, \left[\dfrac{15}{5}\right] = 3$ etc.

Let $E_p(n!)$ denotes the exponent of prime p in $n!$, then

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \dots + \left[\frac{n}{p^a}\right]$$

where a is a greatest positive integer such that $p^a \le n \le p^{a+1}$.

Example 2. *Let $E = \left[\dfrac{1}{3} + \dfrac{1}{50}\right] + \left[\dfrac{1}{3} + \dfrac{2}{50}\right] + \left[\dfrac{1}{3} + \dfrac{3}{50}\right] + \dots +$ upto 50 terms, then the exponent of 2 in $E!$ is*

(a) 17 (b) 25

(c) 15 (d) None of these

Sol. (c) Let $E = \left[\dfrac{1}{3} + \dfrac{x}{50}\right]$, where $x = 1, 2, \dots, 50$

For $1 \le x \le 33, \dfrac{1}{3} < \dfrac{1}{3} + \dfrac{x}{50} < 1$

$\therefore \left[\dfrac{1}{3} + \dfrac{x}{50}\right] = 0$, for $1 \le x \le 33$

For $34 \le x \le 50$, $1 < \dfrac{1}{3} + \dfrac{x}{50} < \dfrac{4}{3}$

$\Rightarrow \left[\dfrac{1}{3} + \dfrac{x}{50}\right] = 1$, for $34 \le x \le 50$

Thus, $E = 17$ and $p = 2$

$\because 2^4 < 17 < 2^5$

$\therefore a = 4$

Exponent of 2 in $(17)! = \left[\dfrac{17}{2}\right] + \left[\dfrac{17}{4}\right] + \left[\dfrac{17}{8}\right] + \left[\dfrac{17}{16}\right]$

$= 8 + 4 + 2 + 1 = 15$

Permutation

Each of different arrangements which can be made by taking some or all of a number of things is called a permutation.

e.g. Arrangements of objects taking 2 at a time from given 3 objects (a, b, c) are ab, bc, ca, cb, ac, ba, then total number of arrangements i.e. total number of permutation is 6.

Meaning of ^nP_r

Number of permutations of n distinct objects taking r at a time is denoted by ^nP_r.

$$^nP_r = \frac{n!}{(n-r)!}, \forall 0 \le r \le n$$

$$= n(n-1)(n-2)\dots(n-r+1), \forall n \in N \text{ and } r \in W.$$

Properties of ^nP_r

(i) The number of permutations of n distinct object taken all at a time is $^nP_n = n!$.

(ii) $^nP_0 = 1$, $^nP_1 = n$ and $^nP_{n-1} = n!$

(iii) $^nP_r = n \cdot {}^{n-1}P_{r-1} = r \cdot {}^{n-1}P_{r-1} + {}^{n-1}P_r$

(iv) $^{n-1}P_r = (n-r)\,{}^{n-1}P_{r-1}$

Important Result on Permutations

(i) The number of permutations of n things taken all at a time p are alike of one kind, q are alike of second kind, r are alike of third kind and remaining are distinct, is $\dfrac{n!}{p!\,q!\,r!}$.

(ii) The number of permutations of n different things, taken r at a time when each thing may be repeated any number of times is n^r.

(iii) Number of permutations under certain conditions

(a) Number of permutations of n different things taken r at a time when a particular thing is to be always included in each arrangement is $r \cdot {}^{n-1}P_{r-1}$.

(b) Number of permutations of n different things taken r at a time, when a particular thing is never taken in each arrangement is ${}^{n-1}P_r$.

(c) Number of permutations of n different things taken all at a time, when m specified things always come together is $m! \times (n-m+1)!$.

(d) Number of permutations of n different things, taken all at a time, when m specified things never come together is $n! - m! \times (n-m+1)!$.

(e) Number of permutations of n different things, taken r at a time when $p\,(p<r)$ particular things are to be always included in each arrangement is $p!\{r-(p-1)\}^{n-p}P_{r-p}$.

Example 3. *If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$, then the value of n is equal to*

(a) 3 (b) 4 (c) 5 (d) 6

Sol. (*b*) We have, ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$

$$\Rightarrow \frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)(2n) \times n!}{(n+2)(n+1)n \times 2n!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{3}{5}$$

$$\Rightarrow 10(2n+1) = 3(n+2)(n+1)$$

$$\Rightarrow 3n^2 - 11n - 4 = 0$$

$$\Rightarrow (n-4)(3n+1) = 0 \Rightarrow n = 4$$

Example 4. *Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is* **(JEE Main 2020)**

(a) $6!$ (b) $\frac{5}{2}(6!)$ (c) $\frac{1}{2}(6!)$ (d) 5^6

Sol. (*b*) To make 6-digit numbers from given digits 1, 3, 5, 7 and 9, we must repeat a digit and we can done the same in ${}^5C_1$ ways.

Now, the arrangement of these 6-digits in which two are identical is $\frac{6!}{2!}$.

So, required numbers of 6-digit numbers $= {}^5C_1 \frac{6!}{2!} = \frac{5}{2}(6!)$

Example 5. *Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?* **(JEE Main 2020)**

(a) $2!\,3!\,4!$ (b) $(3!)^3 \cdot (4!)$

(c) $(3!)^2 \cdot (4!)$ (d) $3!\,(4!)^3$

Sol. (*b*) Since two families has 3 members each and one family with four members. So that can be seated among themself, so same family members are not separated in $3!$, $3!$ and $4!$ respectively.

Now, the groups (means families) can arrange in $3!$ ways.

So, required number of ways is

$$3! \times 3! \times 4! \times 3! = (3!)^3 \cdot 4!$$

Circular Permutation

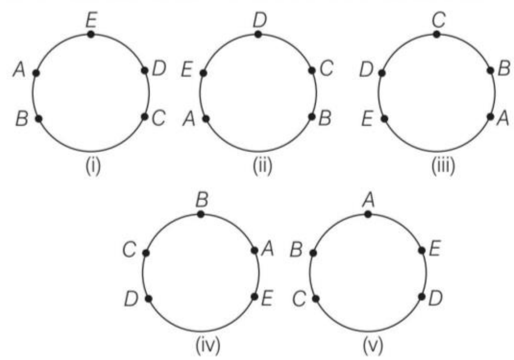
If objects are arranged along a closed curve, then permutation is known as circular permutation.

In other words, the permutation in a row has a beginnings and end but there is no beginning and end in circular permutation. So, we need to consider one object is fixed and the remaining objects are arranged in $(n-1)!$ ways.

e.g. Consider five persons A, B, C, D and E to be seated on the circumference of a circular table in order (which has no head). Now, shifting A, B, C, D and E one position in anti-clockwise direction, we will get arrangements as follows

we see that arrangements in all figures are same.

$\therefore$ The number of circular permutations of n different things taken all at a time is $\frac{{}^nP_n}{n} = (n-1)!$, if clockwise and anti-clockwise orders are taken as different.



In a circular permutation, if the position are given by number, then it is treated as a linear arrangement.

Important Results on Circular Permutation

(i) Number of circular permutations of n different things taken all at a time $= (n-1)!$. If clockwise and anti-clockwise orders are taken as different.

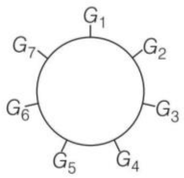
(ii) The number of circular permutations of n different things taken all at a time $= \frac{1}{2}(n-1)!$. If clockwise and anti-clockwise orders are taken as not different.

Example 6. *The number of ways in which 5 ladies and 7 gentlemen can be seated in a round table so that no two ladies sit together, is*

(a) $\frac{7}{2}(720)^2$ (b) $7(360)^2$

(c) $7(720)^2$ (d) 720

Sol. (a) First we fix the alternate positions of 7 gentlemen in a round table by 6! ways.



There are seven positions between the gentlemen in which 5 ladies can be seated in $^7P_5$ ways.

$\therefore$ Required number of ways

$$= 6! \times \frac{7!}{2!} = \frac{7}{2}(720)^2$$

Combination

Each of the different groups or selections which can be made by some or all of a number of given things without reference to the order of the things in each group is called a combination.

e.g. The groups made by taking 2 objects at a time from three objects (a, b, c) are ab, bc, ca. Then, the number of groups is 3 each of which is known as combination.

Note In permutation order of objects is important whereas in combination order of objects is not important.

Meaning of ^nC_r

The number of combinations of n different things taken r at a time is denoted by ^nC_r or $C(n, r)$ or $\binom{n}{r}$.

Then, $$^nC_r = \frac{n!}{r!(n-r)!} = \frac{^nP_r}{r!} \qquad (0 \le r \le n)$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots 2\cdot 1}$$

$n \in N$ and $r \in W$

If $r > n$, then $^nC_r = 0$

Properties of ^nC_r

(a) ^nC_r is a natural number.

(b) $^nC_0 = {^nC_n} = 1, {^nC_1} = n$

(c) $^nC_r = {^nC_{n-r}}$

(d) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

(e) $^nC_x = {^nC_y} \Leftrightarrow x = y$ or $x + y = n$

(f) $n \cdot {^{n-1}C_{r-1}} = (n-r+1)\, {^nC_{r-1}}$

(g) If n is even, then the greatest value of ^nC_r is $^nC_{n/2}$.

(h) If n is odd, then the greatest value of ^nC_r is $^nC_{\frac{n+1}{2}}$ or $^nC_{\frac{n-1}{2}}$.

(i) $^nC_r = \frac{n}{r} \cdot {^{n-1}C_{r-1}}$

(j) $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$

(k) $^nC_0 + {^nC_1} + {^nC_2} + \ldots + {^nC_n} = 2^n$

(l) $^nC_0 + {^nC_2} + {^nC_4} + \ldots = {^nC_1} + {^nC_3} + {^nC_5} + \ldots = 2^{n-1}$

(m) $^{2n+1}C_0 + {^{2n+1}C_1} + {^{2n+1}C_2} + \ldots + {^{2n+1}C_n} = 2^{2n}$

(n) $^nC_n + {^{n+1}C_n} + {^{n+2}C_n} + \ldots + {^{2n-1}C_n} = {^{2n}C_{n+1}}$

Important Results on Combinations

(i) The number of combinations of n different things, taken r at a time, where p particular things occur is $^{n-p}C_{r-p}$.

(ii) The number of combinations of n different things, taken r at a time, where p particular things never occur is $^{n-p}C_r$.

(iii) The total number of combinations of n different things taken one or more at a time or the number of ways of n different things selecting atleast one of them is

$^nC_1 + {^nC_2} + \ldots + {^nC_n} = 2^n - 1$

(iv) The number of combinations of n identical things taking r $(r \le n)$ at a time is 1.

(v) The number of ways of selecting r things out of n alike things is $(n+1)$, (where $r = 0, 1, 2, 3, \ldots, n$).

(vi) If out of $(p+q+r)$ things, p are alike of one kind, q are alike of second kind and rest are alike of third kind, then the total number of combinations is

$[(p+1)(q+1)(r+1)] - 1$

(vii) If out of $(p+q+r+t)$ things, p are alike of one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of combinations is $(p+1)(q+1)(r+1)2^t - 1$.

Example 7. *Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to* ***(JEE Mains 2021)***

(a) 5 (b) 2 (c) 4 (d) 6

Sol. (c) Total matches between boys of both team

$= {^7C_1} \times {^4C_1} = 28$

Total matches between girls of both team

$= {^nC_1}\, {^6C_1} = 6n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

Example 8. *A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with atleast 3 females, then* **(JEE Main 2019)**

(a) $m = n = 68$ (b) $m + n = 68$
(c) $m = n = 78$ (d) $n = m - 8$

Sol. (c) Since there are 8 males and 5 females. Out of these 13 members committee of 11 members is to be formed. According to the question, m = number of ways when there is at least 6 males

$$= ({}^8C_6 \times {}^5C_5) + ({}^8C_7 \times {}^5C_4) + ({}^8C_8 \times {}^5C_3)$$
$$= (28 \times 1) + (8 \times 5) + (1 \times 10) = 28 + 40 + 10 = 78$$

and n = number of ways when there is at least 3 females

$$= ({}^5C_3 \times {}^8C_8) + ({}^5C_4 \times {}^8C_7) + ({}^5C_5 \times {}^8C_6)$$
$$= 10 \times 1 + 5 \times 8 + 1 \times 28 = 78$$

So, $m = n = 78$

Division of Objects into Groups

Objects are Different

(i) The number of ways of dividing n different objects into 3 groups of p, q and r things $(p + q + r = n)$ is

(a) $\dfrac{n!}{p!\,q!\,r!}$; p, q and r are unequal.

(b) $\dfrac{n!}{p!\,2!\,(q!)^2}$; $q = r$

(c) $\dfrac{n!}{3!\,(p!)^3}$; $p = q = r$

(ii) The number of ways of dividing n different objects into r groups is

$$\frac{1}{r!}\left[r^n - \binom{r}{1}(r-1)^n + \binom{r}{2}(r-2)^n - \binom{r}{3}(r-3)^n + \dots\right]$$

(iii) The number of ways of dividing n different objects into r groups taking into account the order of the groups and also the order of objects in each group is

$$^{(n+r-1)}P_n = r(r+1)(r+2)\dots(r+n-1).$$

Objects are Identical

(i) The number of ways of dividing n identical objects among r persons such that each gets $1, 2, 3, \dots$ or k objects is the coefficient of x^{n-r} in the expansion of $(1 + x + x^2 + \dots + x^{k-1})^r$.

(ii) The number of ways of dividing n identical objects among r persons such that each one may get atmost n objects is $\binom{n+r-1}{r-1}$,

OR

The total number of ways of dividing n identical objects into r groups, if blank groups are allowed, is $^{n+r-1}C_{r-1}$.

(iii) The total number of ways of dividing n identical objects among r persons, each one of whom, receives atleast one item is $^{n-1}C_{r-1}$.

OR

The number of ways in which n identical things can be divided into r groups such that blank groups are not allowed, is $^{n-1}C_{r-1}$.

Example 9. *The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is* **(JEE Main 2019)**

(a) $2^{20} - 1$ (b) 2^{21}
(c) 2^{20} (d) $2^{20} + 1$

Sol. (c) Given that, out of 31 objects 10 are identical and remaining 21 are distinct, so in following ways, we can choose 10 objects.

0 identical + 10 distincts, number of ways $= 1 \times {}^{21}C_{10}$

1 identical + 9 distincts, number of ways $= 1 \times {}^{21}C_9$

2 identicals + 8 distincts, number of ways $= 1 \times {}^{21}C_8$

10 identicals + 0 distinct, number of ways $= 1 \times {}^{21}C_0$

So, total number of ways in which we can choose 10 objects is

$$^{21}C_{10} + {}^{21}C_9 + {}^{21}C_8 + \dots + {}^{21}C_0 = x \quad \text{(let)} \dots \text{(i)}$$

$$\Rightarrow {}^{21}C_{11} + {}^{21}C_{12} + {}^{21}C_{13} + \dots + {}^{21}C_{21} = x \quad \dots \text{(ii)}$$

$$[\because {}^nC_r = {}^nC_{n-r}]$$

On adding both Eqs. (i) and (ii), we get

$$2x = {}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{10} + {}^{21}C_{11} + {}^{21}C_{12} + \dots + {}^{21}C_{21}$$

$$\Rightarrow 2x = 2^{21}$$

$$\Rightarrow x = 2^{20}$$

Arrangement in Groups

(a) The number of ways in which n different things can be arranged into r different groups is $^{n+r-1}P_n$ or $n!\,{}^{n-1}C_{r-1}$.

(b) The number of ways in which n different things can be distributed into r different groups is

$$r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n - \dots + (-1)^{r-1} \cdot {}^rC_{r-1}$$

or coefficient of x^n in $n!(e^x - 1)^r$.
Here, blank groups are not allowed.

(c) The number of ways in which n identical things can be distributed into r different groups is $^{n+r-1}C_{r-1}$ or $^{n-1}C_{r-1}$, according as blank groups are or are not admissible.

Applications of Permutation and Combination

Functional Applications

(i) The number of all permutations (arrangements) of n different objects taken r at a time,

(a) when a particular object is to be always included in each arrangement is ${}^{n-1}C_{r-1} \times r!$

(b) when a particular object is never taken in each arrangement is ${}^{n-1}C_r \times r!$

(ii) If the sets A has m elements and B has n elements, then

(a) the number of functions from A to B is n^m.

(b) the number of one-one functions from A to B is ${}^nP_m$, $m \le n$.

(c) the number of onto functions from A to B is

$$n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \dots, m \le n.$$

(d) the number of increasing (decreasing) functions from A to B is $\binom{n}{m}$, $m \le n$.

(e) the number of non-decreasing (non-increasing) functions from A to B is

$$\binom{m+n-1}{m}, \ m \le n.$$

(f) the number of bijections from A to B is $n!$, if $m = n$.

(g) the number of bijections from A to A such that $f(x) \ne x, \forall x \in A$, is

$$m!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + \frac{(-1)^m}{m!}\right].$$

Example 10. *The number of functions f from $\{1,2,3, \dots, 20\}$ onto $\{1,2,3,\dots, 20\}$ such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4, is* **(JEE Main 2019)**

(a) $(15)! \times 6!$ (b) $5^6 \times 15$

(c) $5! \times 6!$ (d) $6^5 \times (15)!$

Sol. (a) According to given information,
we have if $k \in \{4, 8, 12, 16, 20\}$
Then, $f(k) \in \{3, 6, 9, 12, 15, 18\}$
[$\because$ codomain $(f) = \{1, 2, 3, \dots, 20\}$]
Now, we need to assign the value of $f(k)$ for $k \in \{4, 8, 12, 16, 20\}$ this can be done in ${}^6C_5 \cdot 5!$ ways $= 6 \cdot 5! = 6!$ and remaining 15 elements can be associated by 15! ways.
$\therefore$ Total number of onto functions $= \underline{|15} \cdot \underline{|6} = 15!6!$

Geometrical Applications

(i) Out of n non-concurrent and non-parallel straight lines, the number of point of intersection are ${}^nC_2$.

(ii) The number of straight lines passing through n points $= {}^nC_2$.

(iii) The number of straight lines passing through n points out of which m are collinear $= {}^nC_2 - {}^mC_2 + 1$.

(iv) In a polygon, the total number of diagonals out of n points (no three points are collinear) $= {}^nC_2 - n = \dfrac{n(n-3)}{2}$.

(v) Number of triangles formed by joining n points is ${}^nC_3$.

(vi) Number of triangles formed by joining n points out of which m are collinear are ${}^nC_3 - {}^mC_3$.

(vii) The number of parallelogram in two systems of parallel lines (when Ist set contains m parallel lines and 2nd set contains n parallel lines) $= {}^nC_2 \times {}^mC_2$

(viii) The number of rectangles of any size in a square of $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} n^2$.

Example 11. *There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points, then*

(a) $N > 190$ (b) $N \le 100$

(c) $100 < N \le 140$ (d) $140 < N \le 190$

Sol. (b) If out of n points, m are collinear, then
Number of triangles $= {}^nC_3 - {}^mC_3$
$\therefore$ Number of triangles $= {}^{10}C_3 - {}^6C_3 = 120 - 20 \Rightarrow N = 100$

Example 12. *Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is* **(JEE Main 2019)**

(a) 180 (b) 210 (c) 170 (d) 190

Sol. (c) It is given that, there are 20 pillars of the same height have been erected along the boundary of a circular stadium. Now, the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then total number of beams = number of diagonals of 20-sided polygon.
$\because$ ${}^{20}C_2$ is selection of any two vertices of 20-sided polygon which included the sides as well.
So, required number of total beams $= {}^{20}C_2 - 20$
[$\because$ the number of diagonals in a n-sided closed polygon $= {}^nC_2 - n$]

$$= \frac{20 \times 19}{2} - 20 = 190 - 20 = 170$$

Dearrangements

If n distinct objects are arranged in a row, then the number of ways in which they can be dearranged so that none of them occupies its original place is

$$n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots+(-1)^n\frac{1}{n!}\right\}$$

and it is denoted by $D(n)$.

If $r\ (0 \le r \le n)$ objects occupy the places assigned to them *i.e.*, their original places and none of the remaining $(n-r)$ objects occupies its original places, then the number of such ways is

$$D(n-r) = {}^nC_r \cdot D(n-r)$$
$$= {}^nC_r \cdot (n-r)! \left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+(-1)^{n-r}\frac{1}{(n-r)!}\right\}$$

Example 13. *Ajay writes letters to his five friends and addresses the corresponding. The number of ways can the letters be placed in the envelops so that atleast two of them are in the wrong envelopes are*

(a) 120 (b) 125 (c) 119 (d) 124

Sol. (c) Required number of ways $= \sum_{r=2}^{5} {}^5C_{5-r}\, D(r)$

$$= \sum_{r=2}^{5} \frac{5!}{r!(5-r)!} \cdot r! \left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+\frac{(-1)^r}{r!}\right\}$$
$$= \sum_{r=2}^{5} \frac{5!}{(5-r)!} \left\{\frac{1}{2!}-\frac{1}{3!}+\ldots+\frac{(-1)^r}{r!}\right\}$$
$$= \frac{5!}{3!}\left\{\frac{1}{2!}\right\} + \frac{5!}{2!}\left\{\frac{1}{2!}-\frac{1}{3!}\right\} + \frac{5!}{1!}\left\{\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right\} + \frac{5!}{0!}\left\{\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right\}$$
$$= 10 + 20 + (60-20+5) + (60-20+5-1)$$
$$= 10 + 20 + 45 + 44 = 119$$

Number of Integral Solutions of Linear Equations

Consider the equation

$$x_1 + x_2 + x_3 + x_4 + \ldots + x_r = n \qquad \ldots(i)$$

where $x_1, x_2, \ldots, x_r$ and n are non-negative integers

objects are to be divided into r groups where a group may contain any number of objects.

Therefore, total number of solutions of Eq. (i),

$$= \text{Coefficient of } x^n \text{ in } (x^0 + x^1 + \ldots + x^n)^r$$
$$= {}^{n+r-1}C_r \text{ or } {}^{n+r-1}C_{n-1}$$

Example 14. *The total number of 3-digit numbers, whose sum of digits is 10, is ....* ***(JEE Main 2020)***

(a) 54 (b) 56 (c) 52 (d) 50

Sol. (a) Let the digits of 3-digit numbers are x, y, z such that

$x+y+z=10$ and $x, y, z \in \{0, 1, 2, 3, \ldots, 9\}$, but $x \ne 0$

Now, let $x = t+1,\ t \in \{0, 1, 2, 3, \ldots, 8\}$

So, $t+1+y+z=10$

$\Rightarrow \quad t+y+z=9$

having non-negative integral solution

$$= {}^{9+3-1}C_{3-1}$$
$$= {}^{11}C_2 = 55$$

But, it include the case, when $t=9$

$\Rightarrow \quad x = 10,$

which is not possible, so required number of 3-digit numbers

$$= 55 - 1 = 54$$

Hence, answer is 54.

Practice Exercise

ROUND I Topically Divided Problems

Permutation

1. The exponent of 3 in 100! is
(a) 47 (b) 48 (c) 49 (d) 50

2. How many different non-digit numbers can be formed from the digits of the number 223355888 by rearrangement of the digits so that the odd digits occupy even places?
(a) 16 (b) 36 (c) 60 (d) 180

3. How many even numbers of 3 different digits can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition of digits is not allowed)?
(a) 224 (b) 280
(c) 324 (d) None of these

4. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is *(JEE Main 2021)*
(a) 77 (b) 42 (c) 35 (d) 82

5. The number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place is
(a) 1440 (b) 144 (c) 7! (d) ${}^4C_4 \times {}^3C_3$

6. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.
(a) 1440 (b) 1450
(c) 1460 (d) None of these

7. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is *(JEE Main 2019)*
(a) 306 (b) 310 (c) 360 (d) 288

8. The number of 6 digits numbers that can be formed using the digits 0, 1, 2,5, 7 and 9 which are divisible by 11 and no digit is repeated, is *(JEE Main 2019)*
(a) 60 (b) 72 (c) 48 (d) 36

9. The number of natural numbers less than 7000 which can be formed by using the digits 0, 1, 3, 7, 9 (repitition of digits allowed) is equal to *(JEE Main 2019)*
(a) 374 (b) 375 (c) 372 (d) 250

10. We are to form different words with the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1/m_2 is equal to
(a) 30 (b) 60 (c) 90 (d) 180

11. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is
(a) 324 (b) 341
(c) 359 (d) None of these

12. If the letters of the word MOTHER are written in all possible orders and these words are written out as in a dictionary, then the rank of the word MOTHER is
(a) 240 (b) 261
(c) 308 (d) 309

13. There are 10 persons named $P_1, P_2, P_3, \dots, P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.
(a) 4210 (b) 4200
(c) 4203 (d) 4205

14. If a denotes the number of permutations of $x+2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x-11$ things taken all at a time such that $a = 182\, bc$, then the value of x is
(a) 15 (b) 12
(c) 10 (d) 18

15. In a circus there are ten cages for accommodating ten animals. Out of these four cages are so small that five out of 10 animals cannot enter into them. In how many ways will it be possible to accommodate ten animals in these ten cages?
(a) 66400 (b) 86400
(c) 96400 (d) None of these

16. The total number of permutations of n (>1) different things taken not more than r at a time, when each thing may be repeated any number of times is
(a) $\frac{n(n^n-1)}{n-1}$ (b) $\frac{n^r-1}{n-1}$
(c) $\frac{n(n^r-1)}{n-1}$ (d) None of these

17. The number of ways in which 10 candidates $A_1, A_2, \dots, A_{10}$ can be ranked such that A_1 is always above A_{10} is
(a) 5! (b) 2(5!) (c) 10! (d) $\frac{1}{2}(10!)$

Circular Permutation

18. If eleven members of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is
(a) $10!\times 2$ (b) 10!
(c) $9!\times 2$ (d) None of these

19. The number of ways in which seven persons can be arranged at a round table, if two particular persons may not sit together is
(a) 480 (b) 120
(c) 80 (d) None of these

20. In how many ways can 15 members of a council sit along a circular table, when the Secretary is to sit on one side of the Chairman and the Deputy Secretary on the other side?
(a) $2\times 12!$ (b) 24
(c) $2\times 15!$ (d) None of these

21. 20 persons are invited for a party. In how many different ways can they and the host be seated at circular table, if the two particular persons are to be seated on either side of the host?
(a) 20 ! (b) 2·18 !
(c) 18 ! (d) None of these

22. In how many ways can 5 boys and 5 girls sit in a circle so that no two boys sit together?
(a) $5!\times 5!$ (b) $4!\times 5!$
(c) $\frac{5!\times 5!}{2}$ (d) None of these

Combination

23. How many numbers lying between 10 and 1000 can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition of digits is allowed) ?
(a) 1024 (b) 810
(c) 2346 (d) None of these

24. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is *(JEE Main 2021)*
(a) 560 (b) 1050 (c) 1625 (d) 575

25. How many 10-digit numbers can be written by using the digits 1 and 2 ?
(a) ${}^{10}C_1 + {}^{9}C_2$ (b) 2^{10}
(c) ${}^{10}C_2$ (d) 10!

26. The number of times the digits 3 will be written when listing the integers from 1 to 1000 is
(a) 269 (b) 300 (c) 271 (d) 302

27. The figures 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is
(a) 72 (b) 96 (c) 90 (d) 98

28. The value of ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$ is equal to
(a) ${}^{47}C_6$ (b) ${}^{52}C_5$
(c) ${}^{52}C_4$ (d) None of these

29. If ${}^{n}C_3 + {}^{n}C_4 > {}^{n+1}C_3$, then
(a) $n>6$ (b) $n>7$
(c) $n<6$ (d) None of these

30. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to *(JEE Main 2020)*
(a) ${}^{50}C_7 - {}^{30}C_7$ (b) ${}^{51}C_7 - {}^{30}C_7$
(c) ${}^{51}C_7 + {}^{30}C_7$ (d) ${}^{50}C_6 - {}^{30}C_6$

31. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is *(JEE Main 2020)*
(a) 3000 (b) 1500 (c) 2255 (d) 2250

32. Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the ith box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is *(JEE Main 2019)*
(a) 82 (b) 120 (c) 240 (d) 164

33. In how many ways can a student choose a program of 5 courses, if 9 courses are available and 2 specific courses are compulsory for every student?
(a) 34 (b) 36 (c) 35 (d) 37

34. Every body in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is
(a) 11 (b) 12 (c) 13 (d) 14

35. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is
(a) ${}^{16}C_{11}$ (b) ${}^{16}C_5$ (c) ${}^{16}C_9$ (d) ${}^{20}C_9$

36. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw?
(a) 64 (b) 45
(c) 46 (d) None of these

37. The number of ways in which we can choose a committee from four men and six women so that the committee includes atleast two men and exactly twice as many women as men is
(a) 94 (b) 126
(c) 128 (d) None of these

38. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done, is
(a) 216 (b) 240 (c) 600 (d) 3125

39. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which hall can be illuminated.
(a) $2^{10}-2$ (b) $2^{10}-1$
(c) $2^{10}+1$ (d) None of these

40. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.
(a) 111 (b) 112
(c) 113 (d) None of these

41. In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Find the number of ways in which the student can make the choice.
(a) 3 (b) 2 (c) 4 (d) 5

42. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to *(JEE Main 2019)*
(a) 28 (b) 27 (c) 25 (d) 24

43. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is *(JEE Main 2019)*
(a) 350 (b) 500 (c) 200 (d) 300

44. A pack of 52 cards are shuffled together. The number of ways in which a man can be dealt 26 cards so that he does not get two cards of the same suit and same denomination, is
(a) ${}^{52}C_{26}\cdot 2^{26}$ (b) ${}^{104}C_{26}$
(c) $2\cdot {}^{52}C_{26}$ (d) None of these

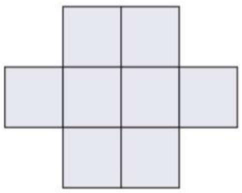
45. In how many ways can 21 English and 19 Hindi books be placed in a row so that no two Hindi books are together?
(a) 1540 (b) 1450 (c) 1504 (d) 1405

46. In a football championship, there were played 153 matches. Every team played one match with each other. The number of teams participating in the championship is
(a) 17 (b) 18 (c) 9 (d) 13

47. A father with 8 children takes them 3 at a time to the zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go the garden, is
(a) 336 (b) 112
(c) 56 (d) None of these

48. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then number of ways in which the car can be filled, is
(a) 10 (b) 20
(c) 30 (d) None of these

49. Six X's have to be placed in the square of the figure such that each row contains atleast one 'X'. In how many different ways can this be done?



(a) 28 (b) 27
(c) 26 (d) None of these

50. A question paper is divided into two parts A and B and each part contains 5 questions. The number of ways in which a candidate can answer 6 questions selecting atleast two questions from each part is
(a) 80 (b) 100
(c) 200 (d) None of these

51. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct, is
(a) 11 (b) 12 (c) 27 (d) 63

52. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is
(a) 6 (b) 11
(c) 13 (d) None of these

53. A lady gives a dinner party for six guests. The number of ways in which they may be selected from among ten friends, if two of the friends will not attend the party together, is
(a) 112 (b) 140
(c) 164 (d) None of these

54. A person is permitted to select atleast one and atmost n coins from a collection of $2n+1$ (distinct) coins. If the total number of ways in which he can select coins is 255, then n is equal to
(a) 4 (b) 8 (c) 16 (d) 32

55. In an steamer, there are stalls for 12 animals and there are horses, cows and calves (not less than 12 each) ready to be shipped in how many ways can the ship load be made?
(a) $3^{12}-1$ (b) 3^{12} (c) $(12)^3-1$ (d) $(12)^3$

56. In an examination of 9 papers a candidate has to pass in more papers, then the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful, is
(a) 255 (b) 256 (c) 193 (d) 319

Division of Objects into Groups

57. Given 5 different green dyes, four different blue dyes and three different red dyes, the number of combinations of dyes which can be chosen taking atleast one green and one blue dye is
(a) 3600 (b) 3720 (c) 3800 (d) 3600

58. There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one in each box, could be placed such that a ball does not go to box of its own colour, is
(a) 8 (b) 7
(c) 9 (d) None of these

59. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?
(a) 50 (b) 100
(c) 150 (d) 200

60. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth players just one card, is
(a) $\frac{52!}{(17!)^3}$ (b) $52!$
(c) $\frac{52!}{17!}$ (d) None of these

61. 18 mice were placed in two experimental groups and one control group with all group equally large. In how many ways can the mice be placed into three groups?
(a) $\frac{18!}{(6!)^2}$ (b) $\frac{18!}{(6!)^3}$
(c) $\frac{180}{(6!)^3}$ (d) None of these

62. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.
(a) 779 (b) 781
(c) 780 (d) 782

63. In how many ways can ₹16 be divided into 4 persons when none of them get less than ₹ 3?
(a) 70 (b) 35
(c) 64 (d) 192

64. A library has a copies of one book, b copies of each of two books, c copies of each of three books and single copies of d books. The total number of ways in which these books can be distributed, is
(a) $\frac{(a+b+c+d)!}{a!\,b!\,c!}$ (b) $\frac{(a+2b+3c+d)!}{a!\,(b!)^2(c!)^3}$
(c) $\frac{(a+2b+3c+d)}{a!\,b!\,c!}$ (d) None of these

65. Eleven books consisting of 5 Mathematics, 4 Physics and 2 Chemistry are placed on a shelf. The number of possible ways of arranging them on the assumption that the books of the same subject are all together, is
(a) 4! 2!
(b) 11!
(c) 5! 4! 3! 2!
(d) None of these

66. Three boys of class X, four boys of class XI and five boys of class XII sit in a row. The total number of ways in which these boys can sit so that all the boys of same class sit together is equal to
(a) $(3!)^2 (4!) (5!)$
(b) $(3!) (4!)^2 (5!)$
(c) $(3!) (4!) (5!)$
(d) $(3!) (4!) (5!)^2$

Applications of Permutation and Combination

67. The number of mappings (functions) from the set $A = \{1, 2, 3\}$ into the set $B = \{1, 2, 3, 4, 5, 6, 7\}$ such that $f(i) \le f(j)$, whenever $i < j$, is
(a) 84
(b) 90
(c) 88
(d) None of these

68. Let $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 4, 5\}$ that are onto and $f(x) \ne i$ is equal to
(a) 9
(b) 44
(c) 16
(d) None of these

69. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is
(a) 105 (b) 15 (c) 175 (d) 185

70. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
(a) 6 (b) 18 (c) 12 (d) 9

71. The maximum number of points of intersection of 6 circles is
(a) 25 (b) 24 (c) 50 (d) 30

72. If a polygon has 44 diagonals, then the number of its sides are
(a) 11
(b) 7
(c) 8
(d) None of these

73. The number of diagonals in a polygon of m sides is
(a) $\frac{1}{2!} m (m-5)$
(b) $\frac{1}{2!} m (m-1)$
(c) $\frac{1}{2!} m (m-3)$
(d) $\frac{1}{2!} m (m-2)$

74. The number of triangles that can be formed by 5 points in a line and 3 points on a parallel line is
(a) ${}^8C_3$
(b) ${}^8C_3 - {}^5C_3$
(c) ${}^8C_3 - {}^5C_3 - 1$
(d) None of these

75. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total numbers of m points are taken on I_1, n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points is
(a) ${}^{m+n+k}C_3$
(b) ${}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$
(c) ${}^mC_3 + {}^nC_3 + {}^kC_3$
(d) None of the above

76. Six points in a plane be joined in all possible ways by indefinite straight lines and if no two of them be coincident or parallel and no three pass through the same point (with the exception of the original 6 points). The number of distinct points or intersection is equal to
(a) 105
(b) 45
(c) 51
(d) None of these

77. The greatest possible number of points of intersection of 8 straight lines and 4 circles is
(a) 32 (b) 64 (c) 76 (d) 104

78. There are n distinct points on the circumference of a circle. The number of pentagons that can be formed with these points as vertices is equal to the number of possible triangles. Then, the value of n is
(a) 7 (b) 8 (c) 15 (d) 30

ROUND II Mixed Bag

Only One Correct Option

1. In a city no two persons have identical set of teeth and there is no person without a tooth. Also, no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, the maximum population of the city is
(a) 2^{32} (b) $(32)^2 - 1$ (c) $2^{32} - 1$ (d) 2^{32-1}

2. A rectangle with sides $2m-1$ and $2n-1$ divided into squares of unit length. The number of rectangle which can be formed with sides of odd length is
(a) m^2n^2
(b) $mn(m+1)(n+1)$
(c) 4^{m+n-1}
(d) None of the above

3. The lock of a safe consists of five discs each of which features the digits 0, 1, 2, ..., 9. The safe can be opened by dialing a special combination of the digits. The number of days sufficient enough to open the safe. If the work day lasts 13 h and 5 s are needed to dial one combination of digits is
(a) 9 (b) 10 (c) 11 (d) 12

4. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are
(a) 97 (b) 105 (c) 135 (d) 146

5. Let A be the set of 4-digit numbers $a_1\, a_2\, a_3\, a_4$, where $a_1 < a_2 < a_3 < a_4$, then $n(A)$ is equal to
(a) 84 (b) 126
(c) 210 (d) None of these

6. If the total number of m elements subsets of the set $A = \{a_1, a_2, a_3, \dots, a_n\}$ is λ times the number of 3 elements subsets containing a_4, then n is
(a) $(m-1)\lambda$ (b) $m\lambda$ (c) $(m+1)\lambda$ (d) 0

7. Sixteen men compete with one another in running, swimming and riding. How many prize lists could be made, if there were altogether 6 prizes of different values, one for running, 2 for swimming and 3 for riding?
(a) $16 \times 15 \times 14$ (b) $16^3 \times 15^2 \times 14$
(c) $16^3 \times 15 \times 14^2$ (d) $16^2 \times 15 \times 14$

8. The number of ways in which we can select four numbers from 1 to 30 so as to exclude every selection of four consecutive numbers is
(a) 27378 (b) 27405
(c) 27399 (d) None of these

9. The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(a) ${}^7P_2 2$ (b) ${}^7C_2 2^5$
(c) ${}^7C_2 5^2$ (d) None of these

10. If the difference of the number of arrangements of three things from a certain number of dissimilar things and the number of selections of the same number of things from them exceeds 100, then the least number of dissimilar things is
(a) 8 (b) 6 (c) 5 (d) 7

11. A person always prefers to eat 'parantha' and 'vegetable dish' in his meal. How many ways can he make his platter in a marriage party, if there are three types of paranthas, four types of 'vegetable dish', three types of 'salads' and two types of 'sauces'?
(a) 3360 (b) 4096
(c) 3000 (d) None of these

12. There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices on these points is
(a) $3p^2(p-1)+1$ (b) $3p^2(p-1)$
(c) $p^2(4p-3)$ (d) None of these

13. In how many different ways can the first 12 natural numbers be divided into three different groups such that numbers in each group are in AP?
(a) 1 (b) 5 (c) 6 (d) 4

14. Two packs of 52 cards are shuffled together. The number of ways in which a man can be dealt 20 cards, so that he does not get two cards of the same suit and same denomination is
(a) ${}^{56}C_{20} \times 2^{20}$ (b) ${}^{104}C_{20}$
(c) $2 \times {}^{52}C_{20}$ (d) None of these

15. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is *(JEE Main 2019)*
(a) 180 (b) 175 (c) 160 (d) 162

16. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is *(JEE Main 2019)*
(a) 12 (b) 11 (c) 9 (d) 7

17. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then, the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is *(JEE Main 2017)*
(a) 485 (b) 468
(c) 469 (d) 484

18. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is *(JEE Main 2016)*
(a) 46th (b) 59th
(c) 52nd (d) 58th

19. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
(a) 256 (b) 220 (c) 219 (d) 211

20. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is *(JEE Main 2021)*
(a) 26664 (b) 122664
(c) 122234 (d) 22264

21. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is
(a) 880 (b) 629 (c) 630 (d) 879

22. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty, is
(a) 5^2 (b) 3^5 (c) 2^5 (d) 5^3

23. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done, is
(a) 3 (b) 36 (c) 66 (d) 108

24. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then, the number of such arrangements is
(a) atleast 500 but less than 750
(b) atleast 750 but less than 1000
(c) atleast 1000
(d) less than 500

25. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent ?
(a) $7 \cdot {}^6C_4 \cdot {}^8C_4$ (b) $8 \cdot {}^6C_4 \cdot {}^7C_4$
(c) $6 \cdot 7 \cdot {}^8C_4$ (d) $6 \cdot 8 \cdot {}^7C_4$

26. The set $S = \{1, 2, 3, \dots, 12\}$ is to be partitioned into three sets A, B and C of equal size. Thus, $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is
(a) $12!/3!(4!)^3$ (b) $12!/3!(3!)^4$
(c) $12!/(4!)^3$ (d) $12!/(3!)^4$

Numerical Value Based Questions

27. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in dictionary, then the position of the word 'MOTHER' is............ *(JEE Main 2020)*

28. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is *(JEE Main 2020)*

29. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ....

30. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ....

31. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is .... *(JEE Main 2020)*

32. The number of permutations of the word 'AUROBIND' in which vowels appear in an alphabetical order is ....

33. 5 Indian and 5 American couples meet at party and shake hand. If no wife shake hand with her own husband and no Indian wife shakes hand with a male, then the number of handshakes that take place in the party is ....

34. Define a 'Good word' as a sequence of letters that consists only of the letters A, B and C and in which A never immediately followed by B, B is never immediately followed by A. If the number of n letter good words is 384, then the value of n is ....

35. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box could be placed such that a ball does not go to a box of its own colour, is ....

Answers

Round I

1. (c)	2. (c)	3. (a)	4. (a)	5. (b)	6. (a)	7. (b)	8. (a)	9. (a)	10. (a)
11. (a)	12. (d)	13. (b)	14. (b)	15. (b)	16. (c)	17. (d)	18. (c)	19. (a)	20. (a)
21. (b)	22. (b)	23. (b)	24. (c)	25. (b)	26. (b)	27. (c)	28. (c)	29. (a)	30. (b)
31. (d)	32. (b)	33. (c)	34. (b)	35. (c)	36. (a)	37. (a)	38. (a)	39. (b)	40. (b)
41. (a)	42. (c)	43. (d)	44. (a)	45. (a)	46. (b)	47. (c)	48. (b)	49. (c)	50. (c)
51. (d)	52. (c)	53. (b)	54. (a)	55. (b)	56. (b)	57. (b)	58. (c)	59. (c)	60. (a)
61. (b)	62. (c)	63. (b)	64. (b)	65. (c)	66. (a)	67. (a)	68. (b)	69. (d)	70. (b)
71. (d)	72. (a)	73. (c)	74. (c)	75. (b)	76. (c)	77. (d)	78. (b)		

Round II

1. (c)	2. (a)	3. (c)	4. (c)	5. (b)	6. (b)	7. (b)	8. (a)	9. (b)	10. (d)
11. (a)	12. (c)	13. (d)	14. (d)	15. (a)	16. (a)	17. (a)	18. (d)	19. (c)	20. (a)
21. (d)	22. (b)	23. (d)	24. (c)	25. (a)	26. (c)	27. (309)	28. (120)	29. (240)	30. (135)
31. (2454)	32. (1680)	33. (135)	34. (8)	35. (9)					

Solutions

Round I

1. Now, $100! = 1\cdot2\cdot3\cdot\ldots98\cdot99\cdot100$

$$= (1\cdot2\cdot4\cdot5\ldots98\cdot100)(3\cdot6\cdot9\ldots96\cdot99)$$
$$= K\cdot3^{33}(1\cdot2\cdot3\ldots32\cdot33)$$
$$[\because \text{let } K = 1\cdot2\cdot4\cdot5\ldots98\cdot100]$$
$$= [K(1\cdot2\cdot4\ldots31\cdot32)]\,3^{33}\cdot(3\cdot9\cdot12\ldots30\cdot33)$$
$$= K_1\cdot3^{33}\cdot3^{11}(1\cdot2\cdot3\ldots10\cdot11)$$
$$[\because \text{let } K(1\cdot2\cdot4\ldots31\cdot32) = K_1]$$
$$= K_1(1\cdot2\cdot4\ldots10\cdot11)3^{33}\cdot3^{11}(3\cdot6\cdot9\cdot12)$$
$$= K_2 3^{33}\cdot3^{11}\cdot3^4(1\cdot2\cdot3\cdot4)$$
$$= K_3\cdot3^{33}\cdot3^{11}\cdot3^4\cdot3 \quad [\because \text{let } K_2(1\cdot2\cdot3\cdot4) = K_3]$$
$$= K_3\cdot3^{49}$$

Hence, exponent of 3 is 49.

2. In a nine digits number, there are four even places for the four odd digits 3, 3, 5, 5.

$\therefore$ Required number of ways $= \dfrac{4!}{2!2!}\cdot\dfrac{5!}{2!3!} = 60$

3. The number will be even, if last digit is either 2, 4, 6 or 8 i.e. the last digit can be filled in 4 ways and remaining two digits can be filled in ${}^8P_2$ ways.

Hence, required number of numbers of three different digits $= {}^8P_2 \times 4 = 224$.

4. ***Case* I** 1, 1, 1, 1, 1, 2, 3

Number of ways $= \dfrac{7!}{5!} = 42$

***Case* II** 1, 1, 1, 1, 2, 2, 2

Number of ways $= \dfrac{7!}{4!3!} = 35$

Total number of ways = 42 + 35 = 77

5. In a word ARTICLE, vowels are A, E, I and consonants are C, L, R, T.

In a seven letter word, there are three even places in which three vowels are placed in 3! way. In rest of the four places, four consonants are placed in 4! ways.

$\therefore$ Required number of ways $= 3! \times 4! = 6 \times 24 = 144$

6. Two women occupy the chair from 1 to 4 in ${}^4P_2$ ways and 3 men occupy the remaining chairs in ${}^6P_3$ ways.

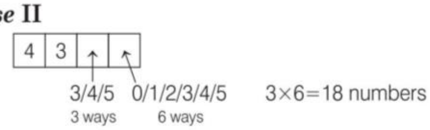
$\therefore$ Required number of ways $= {}^4P_2 \times {}^6P_3$

$= 12 \times 120 = 1440$

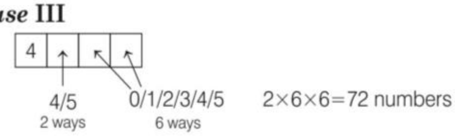
7. Following are the cases in which the 4-digit numbers strictly greater than 4321 can be formed using digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed)

***Case* I**

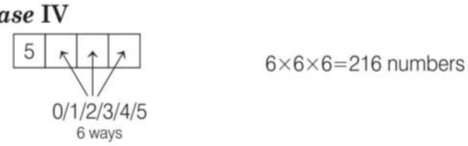
***Case* II**



***Case* III**



***Case* IV**



So, required total numbers $= 4 + 18 + 72 + 216 = 310$

8. Since, the sum of given digits

$$0+1+2+5+7+9=24$$

Let the six-digit number be *abcdef* and to be divisible by 11, so the difference of sum of odd placed digits and sum of even placed digits should be either 0 or a multiple of 11 means $|(a+c+e)-(b+d+f)|$ should be either 0 or a multiple of 11.

Hence, possible case is

$$a+c+e=12=b+d+f \text{ (only)}$$

Now, ***Case* I**

set $\{a, c, e\}=\{0,5,7\}$ and set $\{b, d, f\}=\{1,2,9\}$

So, number of 6-digits numbers $=(2\times 2!)\times(3!)=24$

[$\because a$ can be selected in ways only either 5 or 7]

***Case* II**

Set $\{a, c, e\}=\{1,2,9\}$ and set $\{b, d, f\}=\{0,5,7\}$

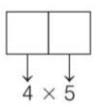
So, number of 6-digits numbers $=3!\times 3!=36$

So, total number of 6-digits numbers $=24+36=60$

9. Using the digits 0, 1, 3, 7, 9

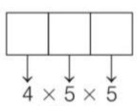
number of one digit natural numbers that can be formed = 4,

number of two digit natural numbers that can be formed = 20,

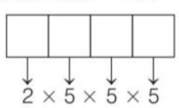


[$\because$ 0 can not come in Ist box]

number of three digit natural numbers that can be formed = 100



and number of four digit natural numbers less than 7000, that can be formed = 250



[$\because$ only 1 or 3 can come in Ist box]

$\therefore$ Total number of natural numbers formed

$$=4+20+100+250=374$$

10. In the word INTEGER, we have 5 letters other than 'I' and 'N' of which two are identical (E's). We can arrange these letters in $\frac{5!}{2!}$ ways. In any such arrangements, 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways.

So, required number of ways $=\frac{5!}{2!}\cdot {}^6P_2=m_1$.

Now, if word start with 'I' and end with 'R', then the remaining letters are 5.

So, total number of ways $=\frac{5!}{2!}=m_2$

$$\therefore \quad \frac{m_1}{m_2}=\frac{5!}{2!}\cdot\frac{6!}{4!}\cdot\frac{2!}{5!}=30$$

11. The number of words starting from A are $=5!=120$

The number of words starting from I are $=5!=120$

The number of words starting from KA are $=4!=24$

The number of words starting from KI are $=4!=24$

The number of words starting from KN are $=4!=24$

The number of words starting from KRA are $=3!=6$

The number of words starting from KRIA are $=2!=2$

The number of words starting from KRIN are $=2!=2$

The number of words starting from KRISA are $=1!=1$

The number of words starting from KRISNA are $=1!=1$

Hence, rank of the word KRISNA

$$=2(120)+3(24)+6+2(2)+2(1)=324$$

12. The number of words starting from E are $=5!=120$

The number of words starting from H are $=5!=120$

The number of words starting from ME are $=4!=24$

The number of words starting from MH are $=4!=24$

The number of words starting from MOE are $=3!=6$

The number of words starting from MOH are $=3!=6$

The number of words starting from MOR are $=3!=6$

The number of words starting from MOTE are $=2!=2$

The number of words starting from MOTHER are $=1!=1$

Hence, rank of the word MOTHER

$$=2(120)+2(24)+3(6)+2+1=309$$

13. In out of 10 persons, P_1 is always consider and P_4 and P_5 is not consider.

i.e. We have to select 4 persons out of 7 person and after that they arrange it.

$\therefore$ Required number of ways $={}^7C_4\times 5!=\frac{7\times 6\times 5}{3\times 2\times 1}\times 120$

$$=35\times 120=4200$$

14. **Hint** We have, $a={}^{x+2}P_{x+2}=(x+2)!$

and $b={}^xP_{11}=\frac{x!}{(x-11)!}$ and $c={}^{x-11}P_{x-11}=(x-11)!$

15. At first we have to a accommodate those 5 animals in cages which cannot enter in 4 small cages, therefore number of ways are ${}^6P_5$ and rest of the five animals arrange in 5! ways.

Total number of ways $=5!\times {}^6P_5=120\times 720=86400$

16. When we arrange one things at a time, the number of possible permutations is n. When we arrange them two at a time the number of possible permutations are $n\times n=n^2$ and so on. Thus, the total number of permutations are

$$n+n^2+\ldots+n^r=\frac{n(n^r-1)}{n-1} \qquad [\because n>1]$$

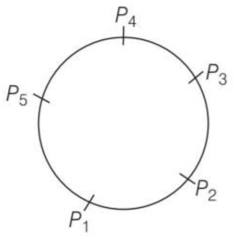
17. Two positions for A_1 and A_{10} can be selected in ${}^{10}C_2$ ways. Rest eight students can be ranked in 8! ways. Hence, total number of ways is ${}^{10}C_2 \times 8! = (1/2)(10!)$.

18. Since, out of eleven members two members sit together, then the number of arrangements $= 9! \times 2$

[$\because$ two numbers can be sit in two ways]

19. $\because$ Remaining 5 can be seated in 4! ways.

Now, on cross marked five places 2 person can sit in ${}^5P_2$ ways.



So, number of arrangements $= 4! \times \dfrac{5!}{3!}$

$= 24 \times 20 = 480$ ways

20. Since, total members are 15 but three special members constitute one member.

Therefore, required number of arrangements are $12! \times 2$, because, chairman remains between the two specified persons and the person can sit in two ways.

21. There are total $20 + 1 = 21$ persons. The two particular persons and the host be taken as one unit so that these remain $21 - 3 + 1 = 19$ persons be arranged in round table in 18! ways. But the two persons on either sides of the host can themselves be arranged in 2! ways.

$\therefore$ Required number of ways $= 2! \times 18! = 2 \cdot 18!$

22. First we fix the alternate position of girls and they arrange in 4! ways and in the five places five boys can be arranged in ${}^5P_5$ ways.

$\therefore$ Total number of ways $= 4! \times {}^5P_5 = 4! \times 5!$

23. ***Case* I** When number in two digits.

Total number of ways $= {}^9C_1 \times {}^9C_1 = 9 \times 9 = 81$

***Case* II** When number in three digits

Total number of ways $= {}^9C_1 \times {}^9C_1 \times {}^9C_1 = 9 \times 9 \times 9$

$= 729$

$\therefore$ Total number of ways $= 81 + 729 = 810$

24. $(2I, 4F) + (3I, 6F) + (4I, 8F)$

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

25. Each digit can be placed in 2 ways.

$\therefore$ Required number of ways $= 2^{10}$

26. Any number between 1 to 999 is a 3-digit number xyz where the digits x, y, z are any digits from 0 to 9.

Now, we first count the numbers in which 3 occurs once only. Since, 3 can occur at one place in ${}^3C_1$ ways, there are ${}^3C_1 \cdot (9 \times 9) = 3 \cdot 9^2$ such numbers.

Again, 3 can occur in exactly two places in ${}^3C_2 \cdot (9)$ such numbers.

Lastly ,3 can occur in all the three digits in one such number only 333.

$\therefore$ The number of times 3 occurs

$= 1 \times (3 \times 9^2) + 2 \times (3 \times 9) + 3 \times 1 = 300$

27. The number forms by the figure 4, 5, 6, 7, 8 which is greater than 56000 is in two cases.

***Case* I** Let the ten thousand digit place number be greater than 5.

The number of numbers $= {}^3C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 3 \times 4 \times 3 \times 2 \times 1 = 72$

***Case* II** Let the ten thousand digit number be 5 and thousand digit number be either 6 or greater than 6.

Then, the number of numbers $= {}^3C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 3 \times 3 \times 2 \times 1 = 18$

$\therefore$ Required number of ways $= 72 + 18 = 90$

28. $${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 = {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4) = {}^{52}C_4$$

29. $${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$$

$$\Rightarrow \quad {}^{n+1}C_4 > {}^{n+1}C_3 \qquad ({}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$$

$$\Rightarrow \quad \frac{{}^{n+1}C_4}{{}^{n+1}C_3} > 1 \Rightarrow \frac{n-2}{4} > 1 \Rightarrow n > 6$$

30. The value of

$$\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \ldots + {}^{30}C_6$$

$$\Rightarrow \sum_{r=0}^{20} {}^{50-r}C_6 + {}^{30}C_7 = {}^{30}C_7 + [{}^{30}C_6 + {}^{31}C_6 + {}^{32}C_6 + \ldots + {}^{50}C_6]$$

$$\Rightarrow \sum_{r=0}^{20} {}^{50-r}C_6 + {}^{30}C_7 = {}^{31}C_7 + {}^{31}C_6 + {}^{32}C_6 + \ldots + {}^{50}C_6$$

[as ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

Similarly, $\displaystyle\sum_{r=0}^{20} {}^{50-r}C_6 + {}^{30}C_7 = {}^{51}C_7$

$$\Rightarrow \sum_{r=0}^{20} {}^{50-r}C_6 = {}^{51}C_7 - {}^{30}C_7$$

Hence, option (b) is correct.

31. As each section has 5 questions, so number of ways to select 5 questions are

S_1	S_2	S_3
1	1	3
1	3	1
3	1	1

and

1	2	2
2	1	2
2	2	1

$\therefore$ Total number of selection of 5 questions

$= 3\times(^5C_1\times{}^5C_1\times{}^5C_3)+3\times(^5C_1\times{}^5C_2\times{}^5C_2)$

$= 3(5\times5\times10)+3(5\times10\times10) = 750+1500 = 2250$

32. Given there are three boxes, each containing 10 balls labelled 1, 2, 3, ... , 10.

Now, one ball is randomly drawn from each boxes, and n_i denote the label of the ball drawn from the ith box, $(i = 1, 2, 3)$.

Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is same as selection of 3 different numbers from numbers {1, 2, 3, ... , 10}

$= {}^{10}C_3 = 120$

33. Total number of available courses = 9
Out of these 5 courses have to be chosen. But it is given that 2 courses are compulsory for every student *i.e.*, you have to choose only 3 courses instead of 5, out of 7 instead of 9.

It can be done in $^7C_3$ ways $= \dfrac{7\times6\times5}{6} = 35$ ways

34. Let total number of persons be n.

Since, total number of hand shakes = 66

$\therefore \quad {}^nC_2 = 66 \Rightarrow \dfrac{n(n-1)}{2} = 66$

$\Rightarrow \quad n^2 - n - 132 = 0 \Rightarrow (n-12)(n+11) = 0$

$\Rightarrow \quad n = 12 \quad [\because n$ cannot be negative]

35. $\therefore$ Required number of ways $= {}^{22-4-2}C_{11-2} = {}^{16}C_9$

36. A selection of 3 balls so as to include atleast one black ball, can be made in the following 3 mutually exclusive ways

(i) The number of ways in which 1 black balls and 2 others are selected $= {}^3C_1\times{}^6C_2 = 3\times15 = 45$

(ii) The number of ways in which 2 black balls and 1 other are selected $= {}^3C_2\times{}^6C_1 = 3\times6 = 18$

(iii) The number of ways in which 3 black balls and no other are selected $= {}^3C_3 = 1$

$\therefore$ Total numbers of ways $= 45+18+1 = 64$

37. The number of ways in which we can choose a committee = Choose two men and four women

+ Choose three men and six women

$= {}^4C_2\times{}^6C_4 + {}^4C_3\times{}^6C_6$

$= 6\times15 + 4\times1 = 90+4 = 94$

38. Since, a five digit number is formed using digits {0, 1, 2, 3, 4 and 5} divisible by 3 *i.e.*, only possible when sum of digits is multiple of 3 which gives two cases.

***Case* I** {using digits 0, 1, 2, 4, 5}

Number of numbers $= {}^4C_1\times{}^4C_1\times{}^3C_1\times{}^2C_1\times{}^1C_1$

$= 4\times4\times3\times2\times1 = 96$

***Case* II** {using digits 1, 2, 3, 4, 5}

Number of numbers $= {}^5C_1\times{}^4C_1\times{}^3C_1\times{}^2C_1\times{}^1C_1$

$= 5\times4\times3\times2\times1 = 120$

$\therefore$ Total numbers formed $= 120+96 = 216$

39. Total number of ways

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + \dots + {}^{10}C_{10}$

$= 2^{10} - 1 \quad [\because {}^nC_0 + {}^nC_1 + {}^nc_2 + \dots = 2^n]$

40. Any number divisible by 5, if either 0 or 5 in unit place

6			0 or 5
Th	H	T	U

In unit place, the number of ways $= {}^2C_1 = 2$

In thousand place, number 6 is fixed. In ten and hundred place the number of ways of selection $= 8\times7$.

$\therefore$ Required number of ways $= 2\times8\times7 = 112$

41. Since, questions 1 and 2 are compulsory, so students has to select two question in out of three questions.

$\therefore$ Required number of ways $= {}^3C_2 = 3$

42. It is given that a group of students comprises of 5 boys and n girls.

The number of ways, in which a team of 3 students can be selected from this group such that each team consists of at least one boy and at least one girls, is

= (number of ways selecting one boy and 2 girls)

+ (number of ways selecting two boys and 1 girl)

$= (^5C_1\times{}^nC_2)\ (^5C_2\times{}^nC_1) = 1750$ [given]

$\Rightarrow \left(5\times\dfrac{n(n-1)}{2}\right) + \left(\dfrac{5\times4}{2}\times n\right) = 1750$

$\Rightarrow \quad n(n-1) + 4n = \dfrac{2}{5}\times1750$

$\Rightarrow \quad n^2 + 3n = 2\times350$

$\Rightarrow \quad n^2 + 3n - 700 = 0$

$\Rightarrow \quad n^2 + 28n - 25n - 700 = 0$

$\Rightarrow \quad n(n+28) - 25(n+28) = 0$

$\Rightarrow \quad n = 25 \quad [\because n \in N]$

43. Number of girls in the class = 5 and number of boys in the class = 7

Now, total ways of forming a team of 3 boys and 2 girls

$= {}^7C_3 \cdot {}^5C_2 = 350$

But, if two specific boys are in team, then number of ways $= {}^5C_1 \cdot {}^5C_2 = 50$

Required ways, i.e. the ways in which two specific boys are not in the same team $= 350 - 50 = 300$.

Alternate Method

Number of ways when A is selected and B is not

$= {}^5C_2 \cdot {}^5C_2 = 100$

Number of ways when B is selected and A is not

$= {}^5C_2 \cdot {}^5C_2 = 100$

Number of ways when both A and B are not selected

$= {}^5C_3 \cdot {}^5C_2 = 100$

$\therefore$ Required ways $= 100 + 100 + 100$

$= 300$

44. $\because$ 26 cards can be chosen out of 52 cards in ${}^{52}C_{26}$ ways. There are two ways in which each card can be dealt because a card can be either from the first pack or from the second.

$\therefore$ Total number of ways $= {}^{52}C_{26} \cdot 2^{26}$

45. First we fix the alternate position of English books.

Then, there are 22 vacant places for Hindi books.

Hence, total number of ways $= {}^{22}C_{19} = \frac{22!}{3!19!} = 1540$

46. Let there are n teams.

Each team play to every other team in ${}^nC_2$ ways

$\therefore \quad {}^nC_2 = 153$ (given)

$\Rightarrow \quad \frac{n!}{(n-2)!2!} = 153$

$\Rightarrow \quad n(n-1) = 306$

$\Rightarrow \quad n^2 - n - 306 = 0 \Rightarrow (n-18)(n+17) = 0$

$\Rightarrow \quad n = 18$ [$\because$ n is never negative]

47. The number of times he will go to the garden is same as the number of selecting 3 children from 8.

Therefore, the required number of ways $= {}^8C_3 = 56$

48. Since, 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ ways. Now, from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways.

Therefore, the required number of ways in which the car can be filled $= {}^5C_2 \times {}^2C_1 = 10 \times 2 = 20$

49. In all, we have 8 squares in which 6 'X' have to be placed and it can be done in ${}^8C_6 = 28$ ways.

But this includes the possibility that either the top or horizontal row does not have any 'X'. Since, we want each row must have atleast one 'X', these two possibilities are to be excluded.

Hence, required number of ways $= 28 - 2 = 26$

50. The number of ways that the candidate may select

(i) if 2 questions from A and 4 questions from B

$= {}^5C_2 \times {}^5C_4 = 50$

(ii) 3 questions from A and 3 questions from B

$= {}^5C_3 \times {}^5C_3 = 100$

(iii) 4 questions from A and 2 questions from B

$= {}^5C_4 \times {}^5C_2 = 50$

Hence, total number of ways $= 50 + 100 + 50 = 200$

51. Each question can be answered in 4 ways and all question can be answered correctly in only one way.

So, required number of ways $= ({}^4C_1)^3 - 1 = 4^3 - 1 = 63$

52. Let there be 'n' men participants.

Then, the number of games that the men play between themselves is $2 \cdot {}^nC_2$ and the number of games that the men played with the women is $2 \cdot (2n)$.

$\therefore \quad 2 \cdot {}^nC_2 - 2 \cdot 2n = 66$ (given)

$\Rightarrow \quad n(n-1) - 4n - 66 = 0 \Rightarrow n^2 - 5n - 66 = 0$

$\Rightarrow \quad (n+5)(n-11) = 0 \Rightarrow n = 11$

$\therefore$ Number of participants $= 11$ men $+ 2$ women $= 13$

53. There are two cases arise

***Case* I** They do not invite the particular friend

$= {}^8C_6 = 28$

***Case* II** They invite one particular friend

$= {}^8C_5 \times {}^2C_1 = 112$

$\therefore$ Required number of ways $= 28 + 112 = 140$

54. Since, the person is allowed to select atmost n coins out of $(2n+1)$ coins, therefore in order to select one, two, three, ..., n coins. Thus, if T is the total number of ways of selecting atleast one coin, then

$$T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 255 \quad \ldots\text{(i)}$$

Using the binomial theorem

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1} = 2^{2n+1}$$

$\Rightarrow \quad {}^{2n+1}C_0 + 2({}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n) + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow \quad 1 + 2(T) + 1 = 2^{2n+1}$

$\Rightarrow \quad 1 + T = \frac{2^{2n+1}}{2} = 2^{2n}$ [from Eq. (i)]

$\Rightarrow \quad 1 + 255 = 2^{2n}$

$\Rightarrow \quad 2^{2n} = 2^8 \Rightarrow n = 4$

55. First stall can be filled in 3 ways, second stall can be filled in 3 ways and so on.

$\therefore$ Number of ways of loading steamer

$= {}^3C_1 \times {}^3C_1 \times \ldots \times {}^3C_1$ (12 times)

$= 3 \times 3 \times \ldots . \times 3$ (12 times) $= 3^{12}$

56. $\because$ The candidate is unsuccessful, if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ Numbers of ways to be unsuccessful

$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$

$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$

$= \frac{1}{2}({}^9C_0 + {}^9C_1 + \ldots + {}^9C_9)$

$= \frac{1}{2}(2^9) = 2^8 = 256$

57. In each dye of chosen, there are two possibility either chose or reject it.

$\therefore$ The total number of ways in which atleast one green and one blue dye is chosen

$= (2^5 - 1)(2^4 - 1)2^3 = 31 \times 15 \times 8 = 3720$

58. The number of ways in which four different balls can be placed in four different boxes

$= {}^4C_1 + {}^3C_1 + {}^2C_1 + {}^1C_1 = 4 + 3 + 2 + 1 = 10$

$\therefore$ Required number of ways $= 10 - 1 = 9$

[since, only one way in which the same ball have a same box]

59. Let the boxes be marked as A, B, C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities.

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$$= A(1)\,B(1)\,C(3)$$
$$= {}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 = 5 \cdot 4 \cdot 1 = 20$$

Since, the box containing 3 balls could be any of the three boxes A, B, C. Hence, the required number of ways $20 \times 3 = 60$.

(ii) Any two box containing 2 balls each and 3rd containing 1 ball, the number of ways

$$= A(2)\,B(2)\,C(1) = {}^5C_2 \cdot {}^3C_2 \cdot {}^1C_1$$
$$= 10 \times 3 \times 1 = 30$$

Since, the box containing 1 ball could be any of the three boxes A, B, C.

Hence, the required number of ways $= 30 \times 3 = 90$.

Hence, total number of ways $= 60 + 90 = 150$.

60. For the first player, distribute the cards in ${}^{52}C_{17}$ ways. Now, out of 35 cards left 17 cards can be put for second player in ${}^{35}C_{17}$ ways. Similarly, for third player put them in ${}^{18}C_{17}$ ways. One card for the last player can be put in ${}^1C_1$ way. Therefore, the required number of ways for the proper distribution

$$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^1C_1$$
$$= \frac{52!}{35!\,17!} \times \frac{35!}{18!\,17!} \times \frac{18!}{17!\,1!} \times 1! = \frac{52!}{(17!)^3}$$

61. $$\frac{\text{Total arrangement}}{\text{Equally likely arrangement}} = \frac{18!}{6!\,6!\,6!} = \frac{18!}{(6!)^3}$$

62. Total number of ways

= (Attempt 3 from group I and 4 from group II)
+ (Attempt 4 from group I and 3 from group II)
+ (Attempt 5 from group I and 2 from group II)
+ (Attempt 2 from group I and 5 from group II)

$$= {}^6C_3 \times {}^6C_4 + {}^6C_4 \times {}^6C_3 + {}^6C_5 \times {}^6C_2 + {}^6C_2 \times {}^6C_5$$
$$= 2({}^6C_3 \times {}^6C_4) + 2({}^6C_5 \times {}^6C_2)$$
$$= 2(20 \times 15) + 2(6 \times 15)$$
$$= 600 + 180 = 780$$

63. Required number of ways

$$= \text{Coefficient of } x^{16} \text{ in } (x^3 + x^4 + x^5 + \ldots + x^{16})^4$$
$$= \text{Coefficient of } x^{16} \text{ in } x^{12}\,(1 + x + x^2 + \ldots + x^{12})^4$$
$$= \text{Coefficient of } x^4 \text{ in } (1 - x^{13})^4\,(1 - x)^{-4}$$
$$= \text{Coefficient of } x^4 \text{ in } (1 - 13x^5 + \ldots)$$
$$\times \left[1 + 4x + \ldots + \frac{(r+1)(r+2)(r+3)}{3!}x^r\right]$$
$$= \frac{(4+1)(4+2)(4+3)}{3!} = 35$$

64. Total number of books $= a + 2b + 3c + d$

$\therefore$ The total number of arrangements

$$= \frac{(a + 2b + 3c + d)!}{a!(b!)^2(c!)^3}$$

65. Since, the books consisting of 5 Mathematics, 4 Physics and 2 Chemistry can be put together of the same subject in $5!\,4!\,2!$ ways.

But these subject books can be arranged itself in 3! ways

$\therefore$ Required number of ways $= 5!\,4!\,3!\,2!$.

66. We can think of three packets. One consisting of three boys of class X, other consisting of four boys of class XI and last one consisting of five boys of class XII. These packets can be arranged in 3! ways and contents of these packets can be further arranged in 3!4! and 5! ways, respectively.

Hence, the total number of ways is $3! \times 3! \times 4! \times 5!$.

67. If the function is one one, then select any three from the set B in ${}^7C_3$ ways i.e. 35 ways.

If the function is many one, then there are two possibilities. All three corresponds to same element number of such functions $= {}^7C_1 = 7$ ways.

Two corresponds to same element. Select any two from the set B. The larger one corresponds to the larger and the smaller one corresponds to the smaller the third may corresponds to any two.

Number of such functions $= {}^7C_2 \times 2 = 42$

So, the required number of mappings $= 35 + 7 + 42 = 84$

68. Total number of functions

= Number of dearrangement of 5 objects

$$= 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44$$

69. We know, a triangle will be formed by taking three points at a time.

$\therefore$ Required number of triangles $= {}^{12}C_3 - {}^7C_3$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} - \frac{7 \times 6 \times 5}{3 \times 2} = 220 - 35 = 185$$

70. Total number of parallelogram formed

$$= {}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$$

71. Two circles intersect maximum at two distinct points. Now, two circles can be selected in ${}^6C_2$ ways.

$\therefore$ Total number of points of intersection are

$${}^6C_2 \times 2 = 30$$

72. Let n be the number of diagonals of a polygon.

Then, $${}^nC_2 - n = 44$$

$$\Rightarrow \quad \frac{n(n-1)}{2} - n = 44$$

$$\Rightarrow \quad n^2 - 3n - 88 = 0$$

$$\Rightarrow \quad n = -8 \text{ or } 11$$

$$\therefore \quad n = 11$$

73. Required number of diagonals $= {}^mC_2 - m$

$$= \frac{m(m-1)}{2!} - m$$

$$= \frac{m}{2!}(m-3)$$

74. The triangle will be formed by joining any three non-collinear points.

$\therefore$ Required number of ways $= {}^8C_3 - {}^5C_3 - {}^3C_3$

$$= {}^8C_3 - {}^5C_3 - 1$$

75. Total number of points are $m + n + k$, the triangles formed by these points $= {}^{m+n+k}C_3$

Joining of three points on the same line gives no triangle, the number of such triangles is

$${}^mC_3 + {}^nC_3 + {}^kC_3$$

$\therefore$ Required number of triangles

$$= {}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$$

76. Number of lines from 6 points $= {}^6C_2 = 15$

Points of intersection obtained from these lines

$$= {}^{15}C_2 = 105$$

Now, we find the number of times, the original 6 points come.

Consider one point say A_1. Joining A_1 to remaining 5 points, we get 5 lines and any two lines from these 5 lines gives A_1 as the point of intersection.

$\therefore A_1$ is common in ${}^5C_2 = 10$ times out of 105 points of intersections.

Similar is the case with other five points.

$\therefore$ 6 original points come $6 \times 10 = 60$ times in points of intersection.

Hence, the number of distinct points of intersection

$$= 105 - 60 + 6 = 51$$

77. The required number of points

$$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + ({}^8C_1 \times {}^4C_1) \times 2$$

$$= 28 + 12 + 32 \times 2 = 104$$

78. Since, there are n distinct points on a circle.

For making a pentagon it requires a five points.

According to given condition,

$${}^nC_5 = {}^nC_3 \Rightarrow n = 8$$

Round II

1. We have, 32 places for teeth. For each place, we have two choices either there is a tooth or there is no tooth. Therefore, the number of ways to fill up these places is 2^{32}. As there is no person without a tooth, the maximum population is $2^{32} - 1$.

2. For length, number of choices is

$$(2m-1) + (2m-3) + \ldots + 3 + 1 = m^2$$

Similarly, for breadth number of choices is

$$(2n-1) + (2n-3) + \ldots + 3 + 1 = n^2$$

Hence, required number of choices is m^2n^2.

3. Total time required = (total number of dials required to sure open the lock) $\times 5\,s$

$$= 10^5 \times 5\,s = \frac{500000}{60 \times 60 \times 13} \text{ days}$$

$$= 10.7 \text{ days}$$

Hence, 11 days are enough to open the safe.

4. Let n be the number of sides of the polygon.

$$n \cdot 160° = (n-2) \cdot 180° \Rightarrow 20° \cdot n = 360°$$

$\therefore \quad n = 18$

Then number of diagonals $= {}^{18}C_2 - 18 = 153 - 18 = 135$

5. Required number of ways $= {}^9C_4 = 126$

6. Total number of m elements subsets of $A = {}^nC_m$ ...(i)

and number of m elements subsets of A each containing the element $a_4 = {}^{n-1}C_{m-1}$

According to the question, ${}^nC_m = \lambda \cdot {}^{n-1}C_{m-1}$

$$\Rightarrow \frac{n}{m} \cdot {}^{n-1}C_{m-1} = \lambda \cdot {}^{n-1}C_{m-1}$$

$$\Rightarrow \lambda = \frac{n}{m} \Rightarrow n = m\lambda$$

7. Number of ways of giving one prize for running = 16

Number of ways of giving two prizes for swimming

$$= 16 \times 15$$

Number of ways of giving three prizes for riding

$$= 16 \times 15 \times 14$$

$\therefore$ Required ways of giving prizes

$$= 16 \times 16 \times 15 \times 16 \times 15 \times 14$$

$$= 16^3 \times 15^2 \times 14$$

8. The number of ways of selecting four numbers from 1 to 30 without any restriction is ${}^{30}C_4$. The number of ways of selecting four consecutive [*i.e.*, (1, 2, 3, 4), (2, 3, 4, 5), ..., (27, 28, 29, 30)] number is 27.

Hence, the number of ways of selecting four integers which excludes consecutive four selections is

$${}^{30}C_4 - 27 = \frac{30 \times 29 \times 28 \times 27}{24} - 27 = 27378$$

9. Other than 2, remaining five places can be filled by 1 and 3 for each place. The number of ways for five places is $2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

For 2, selecting 2 places out of 7 is ${}^7C_2$.

Hence, for required number of ways is ${}^7C_2 \times 2^5$.

10. Hint ${}^nP_3 - {}^nC_3 > 100$

$$\Rightarrow \frac{n!}{(n-3)!} - \frac{n!}{3!(n-3)!} > 100$$

11. The number of ways he can select atleast one parantha is $2^3 - 1 = 7$. The number of ways he can select atleast one vegetable dish is $2^4 - 1 = 15$. The number of ways be can select zero or more items from salads and sauces is 2^5.

Hence, the total number of ways is $7 \times 15 \times 32 = 3360$

12. Select any three points from total $3p$ points, which can be done in ${}^{3p}C_3$ ways. But this also includes selection of three collinear points.

Now, three collinear points from each straight line can be selected in ${}^{p}C_3$ ways.

Then, the number of triangles is

$${}^{3p}C_3 - 3 \cdot {}^{p}C_3 = p^2(4p-3)$$

13. No group of four numbers from the first 12 natural numbers can have the common difference 4.

If one group including 1 is selected with the common difference 1, then the other two group can have the common difference 1 or 2.

If one group including 1 is selected with the common difference 2, then one of the other two group can have the common difference 2 and the remaining group will have common difference 1.

If one group including 1 is selected with the common difference 3, then the other two groups can have the common difference 3.

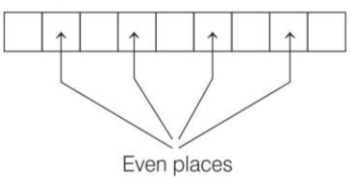
Therefore, the required number of ways is $2+1+1=4$.

14. 20 cards can be chosen out of 52 cards in ${}^{52}C_{20}$ ways. There are two ways in which each card can be dealt, because a card can be either from the first pack or from the second.

Hence, the total number of ways is ${}^{52}C_{20} \times 2^{20}$.

15. Given digits are 1, 1, 2, 2, 2, 2, 3, 4, 4.

According to the question, odd numbers 1, 1, 3 should occur at even places only.



$\therefore$ The number of ways to arrange odd numbers at even places are ${}^4C_3 \times \frac{3!}{2!}$

and the number of ways to arrange remaining even numbers are $\frac{6!}{4!2!}$.

So, total number of 9-digit numbers, that can be formed using the given digits are

$${}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{4!2!} = 4 \times 3 \times 15 = 180$$

16. Since, there are m-men and 2-women and each participant plays two games with every other participant.

$\therefore$ Number of games played by the men between themselves $= 2 \times {}^mC_2$

and the number of games played between the men and the women $= 2 \times {}^mC_1 \times {}^2C_1$

Now, according to the question,

$$2\,{}^mC_2 = 2\,{}^mC_1\,{}^2C_1 + 84$$

$$\Rightarrow \quad \frac{m!}{2!(m-2)!} = m \times 2 + 42$$

$$\Rightarrow \quad m(m-1) = 4m + 84$$

$$\Rightarrow \quad m^2 - m = 4m + 84$$

$$\Rightarrow \quad m^2 - 5m - 84 = 0$$

$$\Rightarrow \quad m^2 - 12m + 7m - 84 = 0$$

$$\Rightarrow \quad m(m-12) + 7(m-12) = 0$$

$$\Rightarrow \quad m = 12 \qquad [\because m > 0]$$

17. Given, X has 7 friends, 4 of them are ladies and 3 are men while Y has 7 friends, 3 of them are ladies and 4 are men.

$\therefore$ Total number of required ways

$$= {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3 + {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 + {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 + {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0$$

$$= 1 + 144 + 324 + 16 = 485$$

18. Clearly, number of words start with A $= \frac{4!}{2!} = 12$

Number of words start with L $= 4! = 24$

Number of words start with M $= \frac{4!}{2!} = 12$

Number of words start with SA $= \frac{3!}{2!} = 3$

Number of words start with SL $= 3! = 6$

Note that, next word will be "SMALL".

Hence, the position of word "SMALL" is 58th.

19. Given, $n(A) = 2, n(B) = 4 \therefore n(A \times B) = 8$

The number of subsets of $A \times B$ having 3 or more elements

$$= {}^8C_3 + {}^8C_4 + \ldots + {}^8C_8$$

$$= ({}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + \ldots + {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$$

$$\therefore \quad 2^n = {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n$$

$$= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$$

$$= 256 - 1 - 8 - 28 = 219$$

20. Digits are 1, 2, 2, 3

Total distinct numbers $= \frac{4!}{2!} = 12$

Total numbers when 1 at unit place is 3.

2 at unit place is 63 at unit place is 3.

So, sum $= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

$= (1111) \times 24$

$= 26664$

21. The number of ways to choose zero or more white balls

$= (10+1)$

[$\because$ all white balls are mutually identical]

Number of ways to choose zero or more green balls

$= (9+1)$

[$\because$ all green balls are mutually identical]

Number of ways to choose zero or more black balls
$= (7+1)$

[$\because$ all black balls are mutually identical]

Hence, number of ways to choose zero or more balls of any colour $= (10+1)(9+1)(7+1)$

Also, number of ways to choose a total of zero balls = 1

Hence, the number of ways to choose atleast one ball (irrespective of any colour)

$= (10+1)(9+1)(7+1) - 1 = 880 - 1 = 879$

22. The number of different ordered pairs (Y, Z) such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z = \phi$.

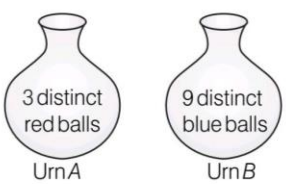
Since, $Y \subseteq X, Z \subseteq X$, hence we can only use the elements of X to construct sets Y and Z.

$n(Y)$	Number of ways to make Y	Number of ways to make Z such that $Y \cap Z = \phi$
0	${}^5C_0$	2^5
1	${}^5C_1$	2^4
2	${}^5C_2$	2^3
3	${}^5C_3$	2^2
4	${}^5C_4$	2^1
5	${}^5C_5$	2^0

Hence, total number of ways to construct sets Y and Z such that $Y \cap Z = \phi$ is

$${}^5C_0 \times 2^5 + {}^5C_1 \times 2^{5-1} + \ldots + {}^5C_5 \times 2^{5-5} = (2+1)^5 = 3^5$$

23.



The number of ways in which two balls from urn A and two balls from urn B can be selected

$= {}^3C_2 \times {}^9C_2 = 3 \times 36 = 108$

24. The number of ways in which 4 novels can be selected

$= {}^6C_4 = 15$

The number of ways in which 1 dictionary can be selected $= {}^3C_1 = 3$

4 novels can be arranged in 4! ways.

$\therefore$ The total number of ways $= 15 \times 4! \times 3 = 15 \times 24 \times 3$

$= 1080$

25. Given word is MISSISSIPPI.

Here, I = 4 times, S = 4 times,
P = 2 times, M = 1 time

_M_I_I_I_I_P_P_

Required number of words $= {}^8C_4 \times \dfrac{7!}{4!2!} = {}^8C_4 \times \dfrac{7 \times 6!}{4!2!}$

$= 7 \cdot {}^8C_4 \cdot {}^6C_4$

26. $\therefore$ Required number of ways $= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4$

$= \dfrac{12!}{8!4!} \times \dfrac{8!}{4!4!} \times 1 = \dfrac{12!}{(4!)^3}$

27. Given word in MOTHER, now alphabetical order of letters is EHMORT, so number of words start with letter.

E ------ is 5! H ------ is 5!
M E ------ is 4! M H ------ is 4!
M O E ------ is 3! M O H ------ is 3!
M O R ------ is 3! M O T E ------ is 2!
M O T H E R is 1

So, position of the word 'MOTHER' is

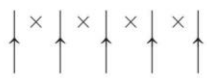
$5! + 5! + 4! + 4! + 3! + 3! + 3! + 2! + 1$

$= 120 + 120 + 24 + 24 + 6 + 6 + 6 + 2 + 1 = 309$

28. Given word is LETTER, having vowels E, E and consonants L, T, T, R.

Now, the number of ways to arrange the consonants are $\dfrac{4!}{2!} = 12$.

Now, we have five place to put vowels E, E.



So, number of ways to arrange vowels is, ${}^5C_2 = 10$

So, number of required words $= 12 \times 10 = 120$.

29. The given word is 'SYLLABUS' having letters SS, LL, ABUY.

Now, number of ways to select two like letters are ${}^2C_1$.

And number of ways to select two distinct letters are ${}^5C_2$.

And number of ways to permutate the 2 like letters and 2 distinct letters is $\dfrac{4!}{2!}$.

So, number of required words $= {}^2C_1 \times {}^5C_2 \times \dfrac{4!}{2!}$

$= 2 \times 10 \times 12$

$= 240$

30. Number of ways to select four questions from six questions $= {}^6C_4$

And number of ways to answer these questions correctly $= 1 \times 1 \times 1 \times 1 = 1$

And number of ways to answer remain two questions wrongly $= 3 \times 3 = 9$

$\therefore$ Required number of ways $= {}^6C_4 \times 1 \times 9$

$= \dfrac{6!}{2!4!} \times 9$

$= \dfrac{6 \times 5}{2} \times 9 = 135$

31. Given word is 'EXAMINATION' having letters (AA), (II), (NN), EXMOT, we have to form 4 letter words, then following cases are possible

(I) 2 same, 2 same and number of words are

$$^3C_2 \times \frac{4!}{2!2!} = 18$$

(II) 2 same, 2 different and number of words are

$$^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 3 \times \frac{7 \times 6}{2} \times \frac{4 \times 3 \times 2}{2}$$

$$= 21 \times 36 = 756$$

(III) All are different and number of words are

$$^8C_4 \times 4! = \frac{8 \times 7 \times 6 \times 5}{4!} 4! = 56 \times 30 = 1680$$

So, total number of 4 letter words are

$18 + 756 + 1680 = 2454$

Hence, answer is 2454.

32. Let the first locate vowels in alphabetical order at any 4 places out of 8 and this can be done in $^8C_4$ ways.

Now, we left with 4 letters R, B, N, D and 4 places that can be filled in 4! ways.

$\therefore$ Number of permutation of the word 'AUROBIND' in which vowels appear in an alphabetical order $= {}^8C_4 \times 4!$

$$= \frac{8!}{4! \times 4!} \times 4! = \frac{8!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$$

33. Given, 5 Indian and 5 American couples meet at party and shake hand.

$\therefore$ Total number of possible hand shakes $= {}^{20}C_2$

Number of hand shakes, when wife shake hand with her own husband = 10 (5 Indian and 5 American)

Number of hand shakes when Indian wife shakes hand with a male $= {}^5C_1 \times {}^{10}C_1 = 50$ (it include the case where the Indian wife shake hand with her own husband)

$\therefore$ Total number of hand shakes that take place in the party $= {}^{20}C_2 - 5 - 50 = 135$

$$= 15 \times 9 = 135$$

34. There are 3 choices for the first of n-letter and two choices for each subsequent letters.

Hence, using fundamental principle, number of good words $= 3 \cdot 2^{n-1} = 384$

$$= 2^{n-1} = 128 = 2^7$$

$\therefore \quad n = 8$

35. Number of dearrangements in such problems is given by

$$n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^n \frac{1}{n!}\right\}$$

Hence, the required number of dearrangements is

$$4!\left\{\frac{1}{2!} - \frac{1}{3!} - \frac{1}{4}\right\} = 12 - 4 + 1 = 9$$