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XI IIT-JEE

PERMUTATION COMBINATION

MATHEMATICS

YOUR GATEWAY TO EXCELLENCE IN IIT-JEE, NEET AND CBSE EXAMS

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02 PERMUTATION COMBINATION

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1. FUNDAMENTAL PRINCIPLE OF COUNTING (counting without actual counting):

If an event A can occur in 'm' different ways and another event B can occur in 'n' different ways, then the total number of different ways of -

(a) simultaneous occurrence of both events in a definite order is $m \times n$. This can be extended to any number of events (known as multiplication principle).

(b) happening exactly one of the events is $m + n$ (known as addition principle).

Example : There are 15 IITs in India and let each IIT has 10 branches, then the IITJEE topper can select the IIT and branch in $15 \times 10 = 150$ number of ways.

Example : There are 15 IITs & 20 NITs in India, then a student who cleared both IITJEE & AIEEE exams can select an institute in $(15 + 20) = 35$ number of ways.

Illustration 1 : A college offers 6 courses in the morning and 4 in the evening. The possible number of choices with the student if he wants to study one course in the morning and one in the evening is-

(A) 24 (B) 2 (C) 12 (D) 10

Solution : The student has 6 choices from the morning courses out of which he can select one course in 6 ways.

For the evening course, he has 4 choices out of which he can select one in 4 ways.

Hence the total number of ways $6 \times 4 = 24$. **Ans.(A)**

Illustration 2 : A college offers 6 courses in the morning and 4 in the evening. The number of ways a student can select exactly one course, either in the morning or in the evening-

(A) 6 (B) 4 (C) 10 (D) 24

Solution : The student has 6 choices from the morning courses out of which he can select one course in 6 ways.

For the evening course, he has 4 choices out of which he can select one in 4 ways.

Hence the total number of ways $6 + 4 = 10$. **Ans. (C)**

Do yourself - 1 :

(i) There are 3 ways to go from A to B, 2 ways to go from B to C and 1 way to go from A to C. In how many ways can a person travel from A to C ?

(ii) There are 2 red balls and 3 green balls. All balls are identical except colour. In how many ways can a person select two balls ?

Greatest Integer

For every real number x, there exist an unique integer k such that $k \le x < k + 1$. Then k is called integral part or greatest integer or floor of x.

It is usually denoted by $\lfloor x \rfloor$ or [x]

Here are some example

x	–2.1	$-\sqrt{2}$	3	$\sqrt{5}$	π	2	100	$-\sqrt{70}$
$\lfloor x \rfloor$	–3	–2	3	2	3	2	100	–9

Note :

$x - 1 < \lfloor x \rfloor \le x$

$\lfloor x \rfloor = x \Leftrightarrow x$ is integer

$\lfloor x \rfloor = n \Leftrightarrow x \in [n, n + 1),\ n \in I$

For $n \in I$, $\lfloor x + n \rfloor = n + \lfloor x \rfloor$

$$\lfloor x \rfloor + \lfloor -x \rfloor = \begin{cases} 0, & x \in I \\ -1, & x \notin I \end{cases}$$

Graph of y = [x]

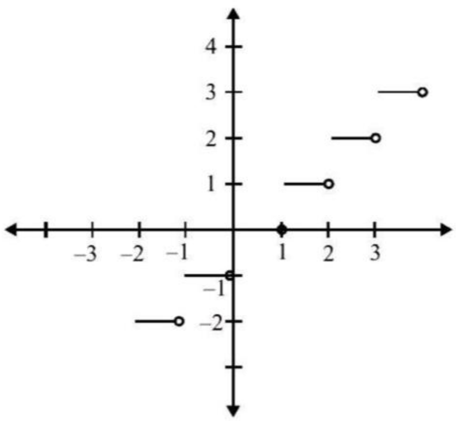


Illustration 3 : Solve following

(i) $4[x] - 8 = 0$ (ii) $3\left[-\frac{x}{3}\right] + 9 = 0$

(iii) $[|x|] = 2$ (iv) $|[x]| = 2$ (v) $\left[\frac{5+x}{2}\right] + \left[\frac{3+x}{2}\right] = -9$

Here [.] denotes greatest integer function.

Solution :

(i) $4[x] = 8 \Rightarrow [x] = 2 \Rightarrow x \in [2,3)$

(ii) $3\left[-\frac{x}{3}\right] + 9 = 0 \Rightarrow \left[-\frac{x}{3}\right] = -3 \Rightarrow -3 \le -\frac{x}{3} < -2 \Rightarrow 9 \ge x > 6 \Rightarrow x \in (6, 9]$

(iii) $[|x|] = 2 \Rightarrow 2 \le |x| < 3 \Rightarrow x \in (-3, -2] \cup [2, 3]$

(iv) $|[x]| = 2 \Rightarrow [x] = -2$ or $2 \Rightarrow x \in [-2, -1) \cup [2, 3)$

(v) $\left[\frac{5+x}{2}\right] + \left[\frac{3+x}{2}\right] = -9 \Rightarrow \left[2 + \frac{1+x}{2}\right] + \left[1 + \frac{1+x}{2}\right] = -9$

$\Rightarrow 2 + \left[\frac{1+x}{2}\right] + 1 + \left[\frac{1+x}{2}\right] = -9 \Rightarrow 2\left[\frac{1+x}{2}\right] = -12 \Rightarrow \left[\frac{1+x}{2}\right] = -6$

$\Rightarrow -6 \le \frac{x+1}{2} < -5 \Rightarrow -12 \le x + 1 < -10 \Rightarrow x \in [-13, -11)$

2. FACTORIAL NOTATION :

(i) A Useful Notation : n! (factorial n) = n.(n – 1).(n – 2).........3. 2. 1; n! = n. (n – 1)! where $n \in N$

(ii) 0! = 1! = 1

(iii) Factorials of negative integers are not defined.

(iv) n! is also denoted by $\underline{|n}$

(v) $(2n)! = 2^n.n!\,[1.\,3.\,5.\,7........(2n-1)]$

(vi) Prime factorisation of n! : Let p be a prime number and n be a positive integer, then exponent of p in n! is denoted by E_p (n!) and is given by

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ... + \left[\frac{n}{p^k}\right]$$

where, $p^k \le n < p^{k+1}$ and [x] denotes the integral part of x.

If we isolate the power of each prime contained in any number n, then n can be written as $n = 2^{\alpha_1}.3^{\alpha_2}.5^{\alpha_3}.7^{\alpha_4}....$, where α_i are whole numbers.

Illustration 4 : Find the exponent of 6 in 50!

Solution:

$$E_2(50!) = \left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right] + \left[\frac{50}{64}\right] \text{ (where [ ] denotes integral part)}$$

$E_2(50!) = 25 + 12 + 6 + 3 + 1 + 0 = 47$

$$E_3(50!) = \left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right] + \left[\frac{50}{81}\right]$$

$E_3(50!) = 16 + 5 + 1 + 0 = 22$

$\Rightarrow$ 50! can be written as $50! = 2^{47}.\,3^{22}$.........

Therefore exponent of 6 in 50! = 22 **Ans.**

3. PERMUTATION & COMBINATION :

(a) **Permutation :** Each of the arrangements in a definite order which can be made by taking some or all of the things at a time is called a PERMUTATION. In permutation, order of appearance of things is taken into account; when the order is changed, a different permutation is obtained.

Generally, it involves the problems of arrangements (standing in a line, seated in a row), problems on digit, problems on letters from a word etc.

${}^nP_r$ denotes the number of permutations of n **different** things, taken r at a time ($n \in N$, $r \in W$, $r \le n$)

$${}^nP_r = n\,(n-1)\,(n-2)\,.............\,(n-r+1) = \frac{n!}{(n-r)!}$$

Note :

(i) ${}^nP_n = n!$, ${}^nP_0 = 1$, ${}^nP_1 = n$

(ii) Number of arrangements of n **distinct** things taken all at a time = n!

(iii) ${}^nP_r$ is also denoted by A_r^n or P(n,r).

(b) Combination :

Each of the groups or selections which can be made by taking some or all of the things without considering the order of the things in each group is called a COMBINATION.

Generally, involves the problem of selections, choosing, distributed groups formation, committee formation, geometrical problems etc.

^nC_r denotes the number of combinations of n different things taken r at a time ($n \in N, r \in W, r \le n$)

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Note :

(i) ^nC_r is also denoted by $\binom{n}{r}$ or C (n, r).

(ii) $^nP_r = {^nC_r}.\ r!$

Illustration 5 : If *a* denotes the number of permutations of (x + 2) things taken all at a time, *b* the number of permutations of x things taken 11 at a time and *c* the number of permutations of (x – 11) things taken all at a time such that *a = 182 bc,* then the value of x is

(A) 15 (B) 12 (C) 10 (D) 18

Solution :

$$^{x+2}P_{x+2} = a \Rightarrow a = (x+2)!$$

$$^{x}P_{11} = b \Rightarrow b = \frac{x!}{(x-11)!}$$

and $^{x-11}P_{x-11} = c \Rightarrow c = (x-11)!$

$\because a = 182bc$

$$(x+2)! = 182\frac{x!}{(x-11)!}(x-11)! \Rightarrow (x+2)(x+1) = 182 = 14\times 13$$

$\therefore x+1 = 13 \quad \Rightarrow x = 12$ **Ans. (B)**

Illustration 6 : A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be drawn so that there are atleast two balls of each colour ?

Solution : The selections of 6 balls, consisting of atleast two balls of each colour from 5 red and 6 white balls, can be made in the following ways

Red balls (5)	White balls (6)	Number of ways
2	4	$^5C_2 \times {^6C_4} = 150$
3	3	$^5C_3 \times {^6C_3} = 200$
4	2	$^5C_4 \times {^6C_2} = 75$

Therefore total number of ways = 425 **Ans.**

Illustration 7 : How many 4 letter words can be formed from the letters of the word 'ANSWER' ? How many of these words start with a vowel ?

Solution : Number of ways of arranging 4 different letters from 6 different letters are $^{6}C_{4}4! = \frac{6!}{2!} = 360$.

There are two vowels (A & E) in the word 'ANSWER'.

Total number of 4 letter words starting with A : A _ _ _ = $^{5}C_{3}3! = \frac{5!}{2!} = 60$

Total number of 4 letter words starting with E : E _ _ _ = $^{5}C_{3}3! = \frac{5!}{2!} = 60$

$\therefore$ Total number of 4 letter words starting with a vowel = 60 + 60 = 120. **Ans.**

Illustration 8 : If all the letters of the word 'RAPID' are arranged in all possible manner as they are in a dictionary, then find the rank of the word 'RAPID'.

Solution : First of all, arrange all letters of given word alphabetically : 'ADIPR'

Total number of words starting with A _ _ _ _ = 4! = 24
Total number of words starting with D _ _ _ _ = 4! = 24
Total number of words starting with I _ _ _ _ = 4! = 24
Total number of words starting with P _ _ _ _ = 4! = 24
Total number of words starting with RAD _ _ = 2! = 2
Total number of words starting with RAI _ _ = 2! = 2
Total number of words starting with RAPD _ = 1
Total number of words starting with RAPI _ = 1

$\therefore$ Rank of the word RAPID = 24 + 24 + 24 + 24 + 2 + 2 + 1 + 1 = 102 **Ans.**

Do yourself -2 :

(i) Find the exponent of 10 in $^{75}C_{25}$.

(ii) If $^{10}P_r = 5040$, then find the value of r.

(iii) Find the number of ways of selecting 4 even numbers from the set of first 100 natural numbers.

(iv) If all letters of the word 'RANK' are arranged in all possible manner as they are in a dictionary, then find the rank of the word 'RANK'.

(v) How many words can be formed using all letters of the word 'LEARN' ? In how many of these words vowels are together ?

(vi) **Sketch the graph of**

(a) $y = [2x]$ (b) $y = \left[\frac{x}{3}\right]$ (c) $y = [-x]$

Here [.] denotes greatest integer function.

(vii) **Solve following**

(a) $\left[\frac{x}{3}\right] + 2 = 0$ (b) $3\left[\frac{x}{3}\right] - 2 = 0$ (c) $\left[\frac{|x|}{3}\right] = 10$

(d) $\left|\left[-\frac{x}{3}\right]\right| = 4$ (e) $\left[\frac{-x-1}{2}\right] + \left[\frac{5-x}{2}\right] = 3$

4. PROPERTIES OF ^nP_r and ^nC_r :

(a) The number of permutation of n different objects taken r at a time, when p particular objects are always to be included is $r!.^{n-p}C_{r-p}$ $(p \le r \le n)$

(b) The number of permutations of n different objects taken r at a time, when repetition is allowed any number of times is n^r.

(c) Following properties of ^nC_r should be remembered :

(i) $^nC_r = {^nC_{n-r}}$; $^nC_0 = {^nC_n} = 1$

(ii) $^nC_x = {^nC_y} \Rightarrow x = y$ or $x + y = n$

(iii) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

(iv) $^nC_0 + {^nC_1} + {^nC_2} + \dots\dots\dots + {^nC_n} = 2^n$

(v) $^nC_r = \frac{n}{r}\,{^{n-1}C_{r-1}}$

(vi) ^nC_r is maximum when $r = \frac{n}{2}$ if n is even & $r = \frac{n-1}{2}$ or $r = \frac{n+1}{2}$, if n is odd.

(d) The number of combinations of n different things taking r at a time,

(i) when p particular things are always to be included = $^{n-p}C_{r-p}$

(ii) when p particular things are always to be excluded = $^{n-p}C_r$

(iii) when p particular things are always to be included and q particular things are to be excluded $= {^{n-p-q}C_{r-p}}$

Illustration 9 : There are 6 pockets in the coat of a person. In how many ways can he put 4 pens in these pockets?

(A) 360 (B) 1296 (C) 4096 (D) none of these

Solution : First pen can be put in 6 ways.

Similarly each of second, third and fourth pen can be put in 6 ways.

Hence total number of ways = $6 \times 6 \times 6 \times 6 = 1296$ **Ans.(B)**

Illustration 10 : A delegation of four students is to be selected from a total of 12 students. In how many ways can the delegation be selected, if-

(a) all the students are equally willing ?

(b) two particular students have to be included in the delegation ?

(c) two particular students do not wish to be together in the delegation ?

(d) two particular students wish to be included together only ?

(e) two particular students refuse to be together and two other particular students wish to be together only in the delegation ?

Solution :

(a) Formation of delegation means selection of 4 out of 12.
Hence the number of ways = $^{12}C_4 = 495$.

(b) If two particular students are already selected. Here we need to select only 2 out of the remaining 10. Hence the number of ways = $^{10}C_2 = 45$.

(c) The number of ways in which both are selected = 45. Hence the number of ways in which the two are not included together = 495 – 45 = 450

(d) There are two possible cases

(i) Either both are selected. In this case, the number of ways in which the selection can be made = 45.

(ii) Or both are not selected. In this case all the four students are selected from the remaining ten students. This can be done in $^{10}C_4 = 210$ ways.

Hence the total number of ways of selection = 45 + 210 = 255

(e) We assume that students A and B wish to be selected together and students C and D do not wish to be together. Now there are following 6 cases.

(i) (A, B, C) selected, (D) not selected

(ii) (A, B, D) selected, (C) not selected

(iii) (A, B) selected, (C, D) not selected

(iv) (C) selected, (A, B, D) not selected

(v) (D) selected, (A, B, C) not selected

(vi) A, B, C, D not selected

For (i) the number of ways of selection = $^8C_1 = 8$

For (ii) the number of ways of selection = $^8C_1 = 8$

For (iii) the number of ways of selection = $^8C_2 = 28$

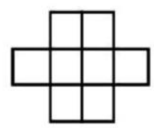
For (iv) the number of ways of selection = $^8C_3 = 56$

For (v) the number of ways of selection = $^8C_3 = 56$

For (vi) the number of ways of selection = $^8C_4 = 70$

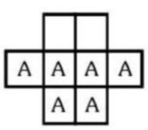
Hence total number of ways = 8 + 8 + 28 + 56 + 56 + 70 = 226. **Ans.**

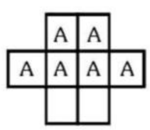
Illustration 11: In the given figure of squares, 6 A's should be written in such a manner that every row contains at least one 'A'. In how many number of ways is it possible ?



(A) 24 (B) 25 (C) 26 (D) 27

Solution : There are 8 squares and 6 'A' in given figure. First we can put 6 'A' in these 8 squares by $^8C_6$ number of ways.

(I) 

(II) 

According to question, atleast one 'A' should be included in each row. So after subtracting these two cases, number of ways are = $(^8C_6 - 2) = 28 - 2 = 26$. **Ans. (C)**

Illustration 12: There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices at these points is :

(A) $3p^2(p-1)+1$ (B) $3p^2(p-1)$ (C) $p^2(4p-3)$ (D) none of these

Solution : The number of triangles with vertices on different lines = $^pC_1 \times {}^pC_1 \times {}^pC_1 = p^3$

The number of triangles with two vertices on one line and the third vertex on any one of the other two lines = $^3C_1\,\{^pC_2 \times {}^{2p}C_1\} = 6p.\dfrac{p(p-1)}{2}$

So, the required number of triangles = $p^3 + 3p^2(p-1) = p^2(4p-3)$ **Ans. (C)**

Illustration 13: There are 10 points in a row. In how many ways can 4 points be selected such that no two of them are consecutive ?

Solution : Total number of remaining non-selected points = 6

. . . . . .

Total number of gaps made by these 6 points = 6 + 1 = 7

If we select 4 gaps out of these 7 gaps and put 4 points in selected gaps then the new points will represent 4 points such that no two of them are consecutive.

x . . x . x . . x .

Total number of ways of selecting 4 gaps out of 7 gaps = ${}^{7}C_4$ **Ans.**

In general, total number of ways of selection of r points out of n points in a row such that no two of them are consecutive : ${}^{n-r+1}C_r$

Do yourself-3 :

(i) Find the number of ways of selecting 5 members from a committee of 5 men & 2 women such that all women are always included.

(ii) Out of first 20 natural numbers, 3 numbers are selected such that there is exactly one even number. How many different selections can be made ?

(iii) How many four letter words can be made from the letters of the word 'PROBLEM'. How many of these start as well as end with a vowel ?

5. FORMATION OF GROUPS :

(a) (i) The number of ways in which (m + n) different things can be divided into two groups such that one of them contains m things and other has n things, is $\frac{(m+n)!}{m!\ n!}$ $(m \neq n)$.

(ii) If m = n, it means the groups are equal & in this case the number of divisions is $\frac{(2n)!}{n!\ n!\ 2!}$.

As in any one way it is possible to interchange the two groups without obtaining a new distribution.

(iii) If 2n things are to be divided equally between two persons then the number of ways :

$$\frac{(2n)!}{n!\ n!\ (2!)} \times 2!.$$

(b) (i) Number of ways in which (m + n + p) different things can be divided into three groups containing m, n & p things respectively is : $\frac{(m+n+p)!}{m!\ n!\ p!}$, $m \neq n \neq p$.

(ii) If m = n = p then the number of groups = $\frac{(3n)!}{n!\ n!\ n!\ 3!}$.

(iii) If 3n things are to be divided equally among three people then the number of ways in which it can be done is $\frac{(3n)!}{(n!)^3}$.

(c) In general, the number of ways of dividing n distinct objects into ℓ groups containing p objects each and m groups containing q objects each is equal to $\dfrac{n!}{(p!)^{\ell}(q!)^{m}\,\ell!m!}$

Here $\ell p + mq = n$

Illustration 14 : In how many ways can 15 students be divided into 3 groups of 5 students each such that 2 particular students are always together ? Also find the number of ways if these groups are to be sent to three different colleges.

Solution : Here first we seperate those two particular students and make 3 groups of 5,5 and 3 of the remaining 13 so that these two particular students always go with the group of 3 students.

$\therefore$ Number of ways $=\dfrac{13!}{5!5!3!}\cdot\dfrac{1}{2!}$.

Now if these groups are to be sent to three different colleges, total number of

ways $=\dfrac{13!}{5!5!3!}\cdot\dfrac{1}{2!}\cdot 3!$ **Ans.**

Illustration 15 : Find the number of ways of dividing 52 cards among 4 players equally such that each gets exactly one Ace.

Solution : Total number of ways of dividing 48 cards (Excluding 4Aces) in 4 groups $=\dfrac{48!}{(12!)^4 4!}$

Now, distribute exactly one Ace to each group of 12 cards. Total number of ways

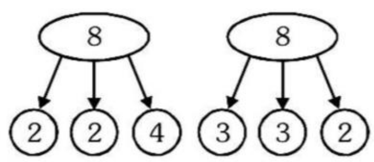
$=\dfrac{48!}{(12!)^4 4!}\times 4!$

Now, distribute these groups of cards among four players

$=\dfrac{48!}{(12!)^4 4!}\times 4!4! = \dfrac{48!}{(12!)^4}\times 4!$ **Ans.**

Illustration 16 : In how many ways can 8 different books be distributed among 3 students if each receives at least 2 books ?

Solution : If each receives at least two books, then the division trees would be as shown below :



(i) (ii)

The number of ways of division for tree in figure (i) is $\left[\dfrac{8!}{(2!)^2 4!2!}\right]$.

The number of ways of division for tree in figure (ii) is $\left[\dfrac{8!}{(3!)^2 2!2!}\right]$.

The total number of ways of distribution of these groups among 3 students

is $\left[\frac{8!}{(2!)^2 4!2!}+\frac{8!}{(3!)^2 2!2!}\right]\times 3!$. **Ans.**

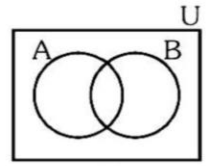
Do yourself-4 :

(i) Find the number of ways in which 16 constables can be assigned to patrol 8 villages, 2 for each.

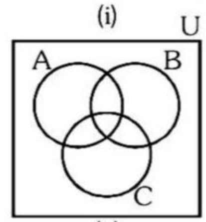
(ii) In how many ways can 6 different books be distributed among 3 students such that none gets equal number of books and each gets atleast one book ?

(iii) n different toys are to be distributed among n children. Find the number of ways in which these toys can be distributed so that exactly one child gets no toy.

6. PRINCIPLE OF INCLUSION AND EXCLUSION :



(i)



(ii)

In the Venn's diagram (i), we get

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A' \cap B') = n(U) - n(A \cup B)$

In the Venn's diagram (ii), we get

$n(A \cup B \cup C)$

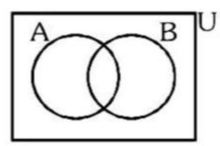
$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$

In general, we have $n(A_1 \cup A_2 \cup .... \cup A_n)$

$= \sum n(A_i) - \sum_{i \neq j} n(A_i \cap A_j) + \sum_{i \neq j \neq k} n(A_i \cap A_j \cap A_k) + ... + (-1)^n \sum n(A_1 \cap A_2 \cap ... \cap A_n)$

Illustration 17 : Find the number of permutations of letters a,b,c,d,e,f,g taken all at a time if neither 'beg' nor 'cad' pattern appear.



Solution : The total number of permutations without any restrictions; n(U) = 7!

(b e g) a c d f

Let A be the set of all possible permutations in which 'beg' pattern always appears : n(A) = 5!

(c a d) b e f g

Let B be the set of all possible permutations in which 'cad' pattern always appears : n(B) = 5!

(c a d) (b e g) f

$n(A \cap B)$: Number of all possible permutations when both 'beg' and 'cad' patterns appear.

$n(A \cap B) = 3!$.

Therefore, the total number of permutations in which 'beg' and 'cad' patterns do not appear

$n(A' \cap B') = n(U) - n(A \cap B) = n(U) - n(A) - n(B) + n(A \cap B)$

$= 7! - 5! - 5! + 3!$. **Ans.**

Do yourself-5 :

(i) Find the number of n digit numbers formed using first 5 natural numbers, which contain the digits 2 & 4 essentially.

7. PERMUTATIONS OF ALIKE OBJECTS :

Case-I : Taken all at a time -

The number of permutations of n things taken all at a time : when p of them are similar of one type, q of them are similar of second type, r of them are similar of third type and the remaining $n-(p+q+r)$ are all different is : $\frac{n!}{p!\ q!\ r!}$.

Illustration 18 : In how many ways the letters of the word "ARRANGE" can be arranged without altering the relative position of vowels & consonants.

Solution : The consonants in their positions can be arranged in $\frac{4!}{2!}=12$ ways.

The vowels in their positions can be arranged in $\frac{3!}{2!}=3$ ways

$\therefore$ Total number of arrangements $= 12 \times 3 = 36$ **Ans.**

Illustration 19 : How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places?

(A) 17 (B) 18 (C) 19 (D) 20

Solution : There are 4 odd digits (1, 1, 3, 3) and 4 odd places (first, third, fifth and seventh). At these places the odd digits can be arranged in $\frac{4!}{2!2!}=6$ ways

Then at the remaining 3 places, the remaining three digits (2, 2, 4) can be arranged in $\frac{3!}{2!}=3$ ways

$\therefore$ The required number of numbers $= 6 \times 3 = 18$. **Ans. (B)**

Illustration 20 :
(a) How many permutations can be made by using all the letters of the word HINDUSTAN ?
(b) How many of these permutations begin and end with a vowel ?
(c) In how many of these permutations, all the vowels come together ?
(d) In how many of these permutations, none of the vowels come together ?
(e) In how many of these permutations, do the vowels and the consonants occupy the same relative positions as in HINDUSTAN ?

Solution :
(a) The total number of permutations = Arrangements of nine letters taken all at a time $=\frac{9!}{2!}=181440$.

(b) We have 3 vowels and 6 consonants, in which 2 consonants are alike. The first place can be filled in 3 ways and the last in 2 ways. The rest of the places can be filled in $\frac{7!}{2!}$ ways.

Hence the total number of permutations $= 3 \times 2 \times \frac{7!}{2!} = 15120$.

(c) Assume the vowels (I, U, A) as a single letter. The letters (IUA), H, D, S, T, N, N can be arranged in $\frac{7!}{2!}$ ways. Also IUA can be arranged among themselves in $3! = 6$ ways.

Hence the total number of permutations $= \frac{7!}{2!} \times 6 = 15120$.

(d) Let us divide the task into two parts. In the first, we arrange the 6 consonants as shown below in $\frac{6!}{2!}$ ways.

$\times C \times C \times C \times C \times C \times C \times$ (Here C stands for a consonant and $\times$ stands for a gap between two consonants)

Now 3 vowels can be placed in 7 places (gaps between the consonants) in $^7C_3.3! = 210$ ways.

Hence the total number of permutations $= \frac{6!}{2!} \times 210 = 75600$.

(e) In this case, the vowels can be arranged among themselves in $3! = 6$ ways.

Also, the consonants can be arranged among themselves in $\frac{6!}{2!}$ ways.

Hence the total number of permutations $= \frac{6!}{2!} \times 6 = 2160$. **Ans.**

Illustration 21 : If all the letters of the word 'PROPER' are arranged in all possible manner as they are in a dictionary, then find the rank of the word 'PROPER' .

Solution : First of all, arrange all letters of given word alphabetically : EOPPRR

Total number of words starting with-

E _ _ _ _ _ $= \frac{5!}{2!2!} = 30$

O _ _ _ _ _ $= \frac{5!}{2!2!} = 30$

PE _ _ _ _ $= \frac{4!}{2!} = 12$

PO _ _ _ _ $= \frac{4!}{2!} = 12$

PP _ _ _ _ $= \frac{4!}{2!} = 12$

PRE _ _ _ $= 3! = 6$

PROE _ _ $= 2! = 2$

PROPER $= 1 = 1$

Rank of the word PROPER = 105 **Ans.**

Case-II : Taken some at a time

Illustration 22 : Find the total number of 4 letter words formed using four letters from the word "PARALLELOPIPED'.

Solution : Given letters are PPP, LLL, AA, EE, R, O, I, D.

Cases	No.of ways of selection	No.of ways of arrangements	Total
All distinct	${}^{8}C_{4}$	${}^{8}C_{4}\times 4!$	1680
2 alike, 2 distinct	${}^{4}C_{1}\times {}^{7}C_{2}$	${}^{4}C_{1}\times {}^{7}C_{2}\times \frac{4!}{2!}$	1008
2 alike, 2 other alike	${}^{4}C_{2}$	${}^{4}C_{2}\times \frac{4!}{2!2!}$	36
3 alike, 1 distinct	${}^{2}C_{1}\times {}^{7}C_{1}$	${}^{2}C_{1}\times {}^{7}C_{1}\times \frac{4!}{3!}$	56
		Total	2780

Ans.

Illustration 23 : Find the number of all 6 digit numbers such that all the digits of each number are selected from the set $\{1,2,3,4,5\}$ and any digit that appears in the number appears at least twice.

Solution :

Cases	No.of ways of selection	No.of ways of arrangements	Total
All alike	${}^{5}C_{1}$	${}^{5}C_{1}\times 1$	5
4 alike + 2 other alike	${}^{5}C_{2}\times 2!$	${}^{5}C_{2}\times 2\times \frac{6!}{2!4!}$	300
3 alike + 3 other alike	${}^{5}C_{2}$	${}^{5}C_{2}\times \frac{6!}{3!3!}$	200
2 alike + 2 other alike +2 other alike	${}^{5}C_{3}$	${}^{5}C_{3}\times \frac{6!}{2!2!2!}$	900
		Total	1405

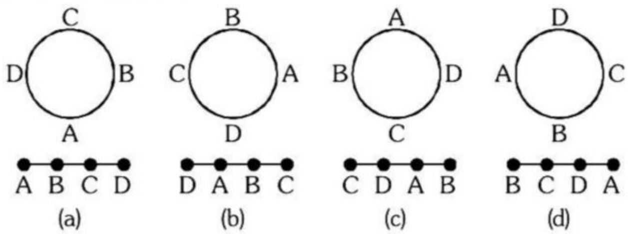
Ans.

Do yourself-6 :

(i) In how many ways can the letters of the word 'ALLEN' be arranged ? Also find its rank if all these words are arranged as they are in dictionary.

(ii) How many numbers greater than 1000 can be formed from the digits 1, 1, 2, 2, 3 ?

8. CIRCULAR PERMUTATION :



Let us consider that persons A,B,C,D are sitting around a round table. If all of them (A,B,C,D) are shifted by one place in anticlockwise order, then we will get Fig.(b) from Fig.(a). Now, if we shift A,B,C,D in anticlockwise order, we will get Fig.(c). Again, if we shift them, we will get Fig.(d) and in the next time, Fig.(a).

Thus, we see that if 4 persons are sitting at a round table, they can be shifted four times and the four different arrangements, thus obtained will be the same, because anticlockwise order of A,B,C,D does not change.

But if A,B,C,D are sitting in a row and they are shifted in such an order that the last occupies the place of first, then the four arrangements will be different.

Thus, if there are 4 things, then for each circular arrangement number of linear arrangements is 4.

Similarly, if n different things are arranged along a circle, for each circular arrangement number of linear arrangements is n.

Therefore, the number of linear arrangements of n different things is n × (number of circular arrangements of n different things). Hence, the number of circular arrangements of n different things is -

1/n × (number of linear arrangements of n different things) $= \frac{n!}{n} = (n-1)!$

Therefore note that :

(i) The number of circular permutations of n different things taken all at a time is : $(n-1)!$.

If clockwise & anti-clockwise circular permutations are considered to be same, then it is : $\frac{(n-1)!}{2}$.

(ii) The number of circular permutations of n different things taking r at a time distinguishing clockwise & anticlockwise arrangements is : $\frac{{}^nP_r}{r}$

Illustration 24 : In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls are together?

(A) $5! \times 5!$ (B) $5! \times 4!$ (C) $\frac{1}{2}(5!)^2$ (D) $\frac{1}{2}(5! \times 4!)$

Solution : Leaving one seat vacant between two boys, 5 boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls sit in 5! ways. Hence the required number of ways = $4! \times 5!$

Ans. (B)

Illustration 25 : The number of ways in which 7 girls can stand in a circle so that they do not have same neighbours in any two arrangements ?

(A) 720 (B) 380 (C) 360 (D) none of these

Solution : Seven girls can stand in a circle by $\frac{(7-1)!}{2!}$ number of ways, because there is no difference in anticlockwise and clockwise order of their standing in a circle.

$\therefore \quad \frac{(7-1)!}{2!} = 360$ **Ans. (C)**

Illustration 26 : The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is

(A) $9! \times 10!$ (B) $5(9!)^2$ (C) $(9!)^2$ (D) none of these

Solution : Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ The required number of ways $= \frac{1}{2} \times 9! \times 10! = 5\,(9!)^2$ **Ans. (B)**

Illustration 27 : A person invites a group of 10 friends at dinner. They sit
(i) 5 on one round table and 5 on other round table,
(ii) 4 on one round table and 6 on other round table.
Find the number of ways in each case in which he can arrange the guests.

Solution : (i) The number of ways of selection of 5 friends for first table is $^{10}C_5$. Remaining 5 friends are left for second table.
The total number of permutations of 5 guests at a round table is 4!. Hence, the total number of arrangements is $^{10}C_5 \times 4! \times 4! = \frac{10!4!4!}{5!5!} = \frac{10!}{25}$
(ii) The number of ways of selection of 6 guests is $^{10}C_6$.
The number of ways of permutations of 6 guests on round table is 5!. The number of permutations of 4 guests on round table is 3!
Therefore, total number of arrangements is : $^{10}C_6 5! \times 3! = \frac{(10)!}{6!4!}5!3! = \frac{(10)!}{24}$ **Ans. (B)**

Do yourself-7 :

(i) In how many ways can 3 men and 3 women be seated around a round table such that all men are always together ?

(ii) Find the number of ways in which 10 different diamonds can be arranged to make a necklace.

(iii) Find the number of ways in which 6 persons out of 5 men & 5 women can be seated at a round table such that 2 men are never together.

(iv) In how many ways can 8 persons be seated on two round tables of capacity 5 & 3.

9. TOTAL NUMBER OF COMBINATIONS :

(a) Given n different objects , the number of ways of selecting atleast one of them is, $^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combinations of n distinct things.

(b) (i) Total number of ways in which it is possible to make a selection by taking some or all out of p + q + r +......things, where p are alike of one kind, q alike of a second kind, r alike of third kind & so on is given by : $(p + 1)(q + 1)(r + 1).........-1$.

(ii) The total number of ways of selecting one or more things from p identical things of one kind, q identical things of second kind, r identical things of third kind and n different things is given by :
$(p + 1)(q + 1)(r + 1)2^n - 1$.

Illustration 28 : A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q so that $P \cap Q = \phi$ is :-

(A) $2^{2n} - {}^{2n}C_n$ (B) 2^n (C) $2^n - 1$ (D) 3^n

Solution : Let $A = \{a_1, a_2, a_3, ... a_n\}$. For $a_i \in A$, we have the following choices :

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

Out of these only (ii), (iii) and (iv) imply $a_i \notin P \cap Q$. Therefore, the number of ways in which none of $a_1, a_2, ...a_n$ belong to $P \cap Q$ is 3^n. **Ans. (D)**

Illustration 29 : There are 3 books of mathematics, 4 of science and 5 of english. How many different collections can be made such that each collection consists of-

(i) one book of each subject ?

(ii) at least one book of each subject ?

(iii) at least one book of english ?

Solution : (i) ${}^3C_1 \times {}^4C_1 \times {}^5C_1 = 60$

(ii) $(2^3-1)(2^4-1)(2^5-1) = 7 \times 15 \times 31 = 3255$

(iii) $(2^5-1)(2^3)(2^4) = 31 \times 128 = 3968$ **Ans.**

Illustration 30 : Find the number of groups that can be made from 5 red balls, 3 green balls and 4 black balls, if at least one ball of all colours is always to be included. Given that all balls are identical except colours.

Solution : After selecting one ball of each colour, we have to find total number of combinations that can be made from 4 red. 2 green and 3 black balls. These will be $(4+1)(2+1)(3+1)$ $= 60$ **Ans.**

Do yourself-8 :

(i) There are p copies each of n different books. Find the number of ways in which atleast one book can be selected ?

(ii) There are 10 questions in an examination. In how many ways can a candidate answer the questions, if he attempts atleast one question.

10. DIVISORS :

Let $N = p^a. q^b. r^c$ .... where p, q, r......... are distinct primes & a, b, c....... are natural numbers then :

(a) The total numbers of divisors of N including 1 & N is $= (a+1)(b+1)(c+1)$.......

(b) The sum of these divisors is

$$= (p^0 + p^1 + p^2 + ... + p^a)(q^0 + q^1 + q^2 + ... + q^b)(r^0 + r^1 + r^2 + ... + r^c)...$$

(c) Number of ways in which N can be resolved as a product of two factor is =

$$\frac{1}{2}(a+1)(b+1)(c+1)...... \text{ if N is not a perfect square}$$

$$\frac{1}{2}[(a+1)(b+1)(c+1)......+1] \text{ if N is a perfect square}$$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

Note :

(i) Every natural number except 1 has atleast 2 divisors. If it has exactly two divisors then it is called a prime. System of prime numbers begin with 2. All primes except 2 are odd.

(ii) A number having more than 2 divisors is called composite. 2 is the only even number which is not composite.

(iii) Two natural numbers are said to be relatively prime or coprime if their HCF is one. For two natural numbers to be relatively prime, it is not necessary that one or both should be prime. It is possible that they both are composite but still coprime, eg. 4 and 25.

(iv) 1 is neither prime nor composite however it is co-prime with every other natural number.

(v) Two prime numbers are said to be twin prime numbers if their non-negative difference is 2 (e.g.5 & 7, 19 & 17 etc).

(vi) All divisors except 1 and the number itself are called proper divisors.

Illustration 31: Find the number of proper divisors of the number 38808. Also find the sum of these divisors.

Solution : **(i)** The number $38808 = 2^3 . 3^2 . 7^2 . 11$

Hence the total number of divisors (excluding 1 and itself i.e.38808)

$= (3 + 1)(2 + 1)(2 + 1)(1 + 1) - 2 = 70$

(ii) The sum of these divisors

$=(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2)(7^0 + 7^1 + 7^2)(11^0 + 11^1) - 1 - 38808$

$= (15)(13)(57)(12) - 1 - 38808 = 133380 - 1 - 38808 = 94571.$ **Ans.**

Illustration 32: In how many ways the number 18900 can be split in two factors which are relative prime (or coprime) ?

Solution: Here $N = 18900 = 2^2 . 3^3 . 5^2 . 7^1$

Number of different prime factors in $18900 = n = 4$

Hence number of ways in which 18900 can be resolved into two factors which are relative prime (or coprime) $= 2^{4-1} = 2^3 = 8$. **Ans.**

Illustration 33: Find the total number of proper factors of the number 35700. Also find

(i) sum of all these factors,

(ii) sum of the odd proper divisors,

(iii) the number of proper divisors divisible by 10 and the sum of these divisors.

Solution: $35700 = 5^2 \times 2^2 \times 3^1 \times 7^1 \times 17^1$

The total number of factors is equal to the total number of selections from (5,5), (2,2), (3), (7) and (17), which is given by $3 \times 3 \times 2 \times 2 \times 2 = 72$.

These include 1 and 35700. Therefore, the number of proper divisors (excluding 1 and 35700) is $72 - 2 = 70$

(i) Sum of all these factors (proper) is :

$(5^0 + 5^1 + 5^2)(2^0 + 2^1 + 2^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 1 - 35700$

$= 31 \times 7 \times 4 \times 8 \times 18 - 1 - 35700 = 89291$

(ii) The sum of odd proper divisors is :

$(5^0 + 5^1 + 5^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 1$

$= 31 \times 4 \times 8 \times 18 - 1 = 17856 - 1 = 17855$

(iii) The number of proper divisors divisible by 10 is equal to number of selections from (5,5), (2,2), (3), (7), (17) consisting of at least one 5 and at least one 2 and 35700 is to be excluded and is given by $2 \times 2 \times 2 \times 2 \times 2 - 1 = 31$.

Sum of these divisors is :

$(5^1 + 5^2)(2^1 + 2^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 35700$

$= 30 \times 6 \times 4 \times 8 \times 18 - 35700 = 67980$ **Ans.**

Do yourself-9 :

(i) Find the number of ways in which the number 94864 can be resolved as a product of two factors.

(ii) Find the number of different sets of solution of xy = 1440.

11. TOTAL DISTRIBUTION :

(a) **Distribution of distinct objects :** Number of ways in which n distinct things can be distributed to p persons if there is no restriction to the number of things received by them is given by : p^n

(b) **Distribution of alike objects :** Number of ways to distribute n alike things among p persons so that each may get none, one or more thing(s) is given by ${}^{n+p-1}C_{p-1}$.

Illustration 34: In how many ways can 5 different mangoes, 4 different oranges & 3 different apples be distributed among 3 children such that each gets alteast one mango ?

Solution : 5 different mangoes can be distributed by following ways among 3 children such that each gets atleast 1 :

3 1 1

2 2 1

Total number of ways : $\left(\dfrac{5!}{3!1!1!2!} + \dfrac{5!}{2!2!2!}\right) \times 3!$

Now, the number of ways of distributing remaining fruits (i.e. 4 oranges + 3 apples) among 3 children = 3^7 (as each fruit has 3 options).

$\therefore$ Total number of ways $= \left(\dfrac{5!}{3!2!} + \dfrac{5!}{(2!)^3}\right) \times 3! \times 3^7$ **Ans.**

Illustration 35: In how many ways can 12 identical apples be distributed among four children if each gets atleast 1 apple and not more than 4 apples.

Solution : Let x,y,z & w be the number of apples given to the children.

$\Rightarrow x + y + z + w = 12$

Giving one-one apple to each

Now, $x + y + z + w = 8$ ....(i)

Here, $0 \le x \le 3,\ 0 \le y \le 3,\ 0 \le z \le 3,\ 0 \le w \le 3$

$x = 3 - t_1, y = 3 - t_2, z = 3 - t_3, w = 3 - t_4$.
Putting value of x, y, z, w in equation (i)
Put $12 - 8 = t_1 + t_2 + t_3 + t_4$
$\Rightarrow t_1 + t_2 + t_3 + t_4 = 4$
(Here max. value that t_1, t_2, t_3 & t_4 can attain is 3, so we have to remove those cases when any of t_i getting value 4)
$= {}^7C_3 -$ (all cases when atleast one is 4)
$= {}^7C_3 - 4 = 35 - 4 = 31$ **Ans.**

Illustration 36: Find the number of non negative integral solutions of the inequation $x + y + z \le 20$.

Solution : Let w be any number ($0 \le w \le 20$), then we can write the equation as :
$x + y + z + w = 20$ (here $x, y, z, w \ge 0$)
Total ways $= {}^{23}C_3$ **Ans.**

Illustration 37: Find the number of integral solutions of $x + y + z + w < 25$, where $x > -2, y > 1, z \ge 2, w \ge 0$.

Solution : Given $x + y + z + w < 25$
$x + y + z + w + v = 25$ ....(i)
Let $x = -1 + t_1, y = 2 + t_2, z = 2 + t_3, w = t_4, v = 1 + t_5$ where $(t_1, t_2, t_3, t_4 \ge 0)$
Putting value of x, y, z, w, v in equation (i)
$\Rightarrow t_1 + t_2 + t_3 + t_4 + t_5 = 21$.
Number of solutions $= {}^{25}C_4$ **Ans.**

Illustration 38: Find the number of positive integral solutions of the inequation $x + y + z \ge 150$, where $0 < x \le 60, 0 < y \le 60, 0 < z \le 60$.

Solution : Let $x = 60 - t_1, y = 60 - t_2, z = 60 - t_3$ (where $0 \le t_1 \le 59, 0 \le t_2 \le 59, 0 \le t_3 \le 59$)
Given $x + y + z \ge 150$
or $x + y + z - w = 150$ (where $0 \le w \le 147$) ....(i)
Putting values of x, y, z in equation (i)
$60 - t_1 + 60 - t_2 + 60 - t_3 - w = 150$
$30 = t_1 + t_2 + t_3 + w$
Total solutions $= {}^{33}C_3$ **Ans.**

Illustration 39: Find the number of positive integral solutions of $xy = 12$

Solution : $xy = 12$
$xy = 2^2 \times 3^1$
(i) 3 has 2 ways either 3 can go to x or y
(ii) 2^2 can be distributed between x & y as distributing 2 identical things between 2 persons
(where each person can get 0, 1 or 2 things). Let two person be ℓ_1 & ℓ_2
$\Rightarrow \ell_1 + \ell_2 = 2$
$\Rightarrow {}^{2+1}C_1 = {}^3C_1 = 3$
So total ways $= 2 \times 3 = 6$.

Alternatively :
$xy = 12 = 2^2 \times 3^1$
$x = 2^{a_1}3^{a_2}$ $\quad 0 \le a_1 \le 2$

$0 \le a_2 \le 1$

$y = 2^{b_1}3^{b_2}$ $\quad 0 \le b_1 \le 2$

$0 \le b_2 \le 1$

$2^{a_1+b_1}3^{a_2+b_2} = 2^2 3^1$

$\Rightarrow a_1 + b_1 = 2 \rightarrow {}^3C_1$ ways

$a_2 + b_2 = 1 \rightarrow {}^2C_1$ ways

Number of solutions $= {}^3C_1 \times {}^2C_1 = 3 \times 2 = 6$ **Ans.**

Illustration 40 : Find the number of solutions of the equation $xyz = 360$ when **(i)** $x,y,z \in N$ **(ii)** $x,y,z \in I$

Solution : **(i)** $xyz = 360 = 2^3 \times 3^2 \times 5$ $(x,y,z \in N)$

$x = 2^{a_1}3^{a_2}5^{a_3}$ (where $0 \le a_1 \le 3, 0 \le a_2 \le 2, 0 \le a_3 \le 1$)

$y = 2^{b_1}3^{b_2}5^{b_3}$ (where $0 \le b_1 \le 3, 0 \le b_2 \le 2, 0 \le b_3 \le 1$)

$z = 2^{c_1}3^{c_2}5^{c_3}$ (where $0 \le c_1 \le 3, 0 \le c_2 \le 2, 0 \le c_3 \le 1$)

$\Rightarrow 2^{a_1}3^{a_2}5^{a_3}.2^{b_1}3^{b_2}5^{b_3}.2^{c_1}3^{c_2}5^{c_3} = 2^3 \times 3^2 \times 5^1$

$\Rightarrow 2^{a_1+b_1+c_1}.3^{a_2+b_2+c_2}.5^{a_3+b_3+c_3} = 2^3 \times 3^3 \times 5^1$

$\Rightarrow a_1 + b_1 + c_1 = 3 \rightarrow {}^5C_2 = 10$

$a_2 + b_2 + c_2 = 2 \rightarrow {}^4C_2 = 6$

$a_3 + b_3 + c_3 = 1 \rightarrow {}^3C_2 = 3$

Total solutions $= 10 \times 6 \times 3 = 180$.

(ii) If $x,y,z \in I$ then, (a) all positive (b) 1 positive and 2 negative.

Total number of ways $= 180 + {}^3C_2 \times 180 = 720$ **Ans.**

Do yourself -10 :

(i) In how many ways can 12 identical apples be distributed among 4 boys. (a) If each boy receives any number of apples. (b) If each boy receives atleast 2 apples.

(ii) Find the number of non-negative integral solutions of the equation $x + y + z = 10$.

(iii) Find the number of integral solutions of $x + y + z = 20$, if $x \ge -4$, $y \ge 1$, $z \ge 2$

12. DEARRANGEMENT :

There are n letters and n corresponding envelopes. The number of ways in which letters can be placed in the envelopes (one letter in each envelope) so that no letter is placed in correct envelope is

$$n!\left[1 - \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots + \frac{(-1)^n}{n!}\right]$$

Proof : n letters are denoted by 1,2,3,........,n. Let A_i denote the set of distribution of letters in envelopes (one letter in each envelope) so that the i^{th} letter is placed in the corresponding envelope. Then,

$n(A_i) = 1 \times (n-1)!$ [since the remaining n–1 letters can be placed in n –1 envelops in (n–1)! ways]

Then, $n(A_i \cap A_j)$ represents the number of ways where letters i and j can be placed in their corresponding envelopes. Then,

$n(A_i \cap A_j) = 1 \times 1 \times (n-2)!$

Also $n(A_i \cap A_j \cap A_k) = 1 \times 1 \times 1 \times (n-3)!$

Hence, the required number is

$n(A_1' \cup A_2' \cup ..... \cup A_n') = n! - n(A_1 \cup A_2 \cup .... \cup A_n)$

$$= n! - \left[\sum n(A_i) - \sum n(A_i \cap A_j) + \sum n(A_i \cap A_j \cap A_k) + ... + (-1)^n \sum n(A_i \cap A_2 ..... \cap A_n)\right]$$

$$= n! - [{}^nC_1(n-1)! - {}^nC_2(n-2)! + {}^nC_3(n-3)! + ... + (-1)^{n-1} \times {}^nC_n 1]$$

$$= n! - \left[\frac{n!}{1!(n-1)!}(n-1)! - \frac{n!}{2!(n-2)!}(n-2)! + ... + (-1)^{n-1}\right] = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} + ... + \frac{(-1)^n}{n!}\right]$$

Illustration 41: A person writes letters to six friends and addresses the corresponding envelopes. In how many ways can the letters be placed in the envelopes so that

(i) all the letters are in the wrong envelopes.

(ii) at least two of them are in the wrong envelopes.

Solution : (i) The number of ways is which all letters be placed in wrong envelopes

$$= 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right) = 720\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720}\right)$$

$= 360 - 120 + 30 - 6 + 1 = 265.$

(i) The number of ways in which at least two of them in the wrong envelopes

$$= {}^6C_4 \,.\, 2!\left(1 - \frac{1}{1!} + \frac{1}{2!}\right) + {}^6C_3 \,.\, 3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right) + {}^6C_2 \,.\, 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

$$+ {}^6C_1 . 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) + {}^6C_0\, 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)$$

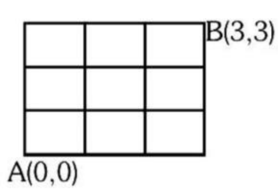
$= 15 + 40 + 135 + 264 + 265 = 719.$ **Ans.**

Do yourself - 11 :

(i) There are four balls of different colours and four boxes of colours same as those of the balls. Find the number of ways in which the balls, one in each box, could be placed in such a way that a ball does not go to box of its own colour.

<u>Miscellaneous Illustrations :</u>

Illustration 42: In how many ways can a person go from point A to point B if he can travel only to the right or upward along the lines (Grid Problem) ?



Solution : To reach the point B from point A, a person has to travel along 3 horizontal and 3 vertical strips. Therefore, we have to arrange 3H and 3V in a row. Total number of ways = $\frac{6!}{3!3!} = 20$ ways **Ans.**

Illustration 43: Find sum of all numbers formed using the digits 2,4,6,8 taken all at a time and no digit being repeated.

Solution : All possible numbers = 4! = 24

If 2 occupies the unit's place then total numbers = 6

Hence, 2 comes at unit's place 6 times.

Sum of all the digits occuring at unit's place

$= 6 \times (2 + 4 + 6 + 8)$

Same summation will occur for ten's, hundred's & thousand's place. Hence required sum

$= 6 \times (2 + 4 + 6 + 8) \times (1 + 10 + 100 + 1000) = 133320$ **Ans.**

Illustration 44: Find the sum of all the numbers greater than 1000 using the digits 0,1,2,2.

Solution :

(i) When 1 is at thousand's place, total numbers formed will be $= \frac{3!}{2!} = 3$

(ii) When 2 is at thousand's place, total numbers formed will be = 3! = 6

(iii) When 1 is at hundred's, ten's or unit's place then total numbers formed will be-

Thousand's place is fixed i.e. only the digit 2 will come here, remaining two places can be filled in 2! ways.

So total numbers = 2!

(iv) When 2 is at hundred's, ten's or unit's place then total numbers formed will be-

Thousand's place has 2 options and other two places can be filled in 2 ways.

So total numbers = $2 \times 2 = 4$

$\text{Sum} = 10^3(1 \times 3 + 2 \times 6) + 10^2(1 \times 2 + 2 \times 4) + 10^1(1 \times 2 + 2 \times 4) + (1 \times 2 + 2 \times 4)$

$= 15 \times 10^3 + 10^3 + 10^2 + 10$

$= 16110$ **Ans.**

Illustration 45 : Find the number of positive integral solutions of $x + y + z = 20$, if $x \neq y \neq z$.

Solution : $x \geq 1$

$y = x + t_1 \qquad t_1 \geq 1$

$z = y + t_2 \qquad t_2 \geq 1$

$x + x + t_1 + x + t_1 + t_2 = 20$

$3x + 2t_1 + t_2 = 20$

(i) $x = 1$ $\quad 2t_1 + t_2 = 17$

$t_1 = 1,2 \,.........\, 8 \Rightarrow 8$ ways

(ii) $x = 2$ $\quad 2t_1 + t_2 = 14$

$t_1 = 1,2 \,.........\, 6 \Rightarrow 6$ ways

(iii) $x = 3$ $\quad 2t_1 + t_2 = 11$

$t_1 = 1,2 \,.........\, 5 \Rightarrow 5$ ways

(vi) $x = 4$ $\quad 2t_1 + t_2 = 8$

$t_1 = 1,2,3 \Rightarrow 3$ ways

(v) $x = 5$ $\quad 2t_1 + t_2 = 5$

$t_1 = 1, 2 \Rightarrow 2$ ways

Total = 8 + 6 + 5 + 3 + 2 = 24

But each solution can be arranged by 3! ways.

So total solutions = 24 × 3! = 144. **Ans.**

Illustration 46: A regular polygon of 15 sides is constructed. In how many ways can a triangle be formed using the vertices of the polygon such that no side of triangle is same as that of polygon ?

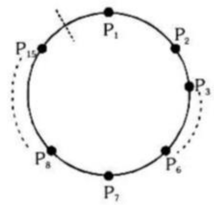
Solution : Select one point out of 15 point, therefore total number of ways = ${}^{15}C_1$

Suppose we select point P_1. Now we have to choose 2 more point which are not consecutive.

since we can not select P_2 & P_{15}.

Total points left are 12.

Now we have to select 2 points out of 12 points which are not consecutive



Total ways = ${}^{12-2+1}C_2 = {}^{11}C_2$

Every select triangle will be repeated 3 times.

So total number of ways $= \dfrac{{}^{15}C_1 \times {}^{11}C_2}{3} = 275$

Alternative :

First of all let us cut the polygon between points P_1 & P_{15}. Now there are 15 points on a straight line and we have to select 3 points out of these, such that the selected points are not consecutive.

x O y O z O w

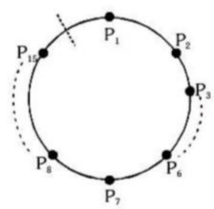
Here bubbles represents the selected points,

x represents the number of points before first selected point,

y represents the number of points between Ist & IInd selected point,

z represents the number of points between IInd & IIIrd selected point

and w represents the number of points after IIIrd selected point.



$x + y + z + w = 15 - 3 = 12$

here $x \geq 0$, $y \geq 1$, $z \geq 1$, $w \geq 0$

Put $y = 1 + y'$ & $z = 1 + z'$ $(y' \geq 0, z' \geq 0)$

$\Rightarrow x + y' + z' + w = 10$

Total number of ways = ${}^{13}C_3$

These selections include the cases when both the points P_1 & P_{15} are selected. We have to remove those cases. Here a represents number of points between P_1 & 3rd selected point & b represents number of points between 3rd selected point and P_{15}

$\Rightarrow a + b = 15 - 3 = 12 \quad (a \geq 1, b \geq 1)$

put $a = 1 + t_1$ & $b = 1 + t_2$

$t_1 + t_2 = 10$

Total number of ways = ${}^{11}C_1 = 11$

Therefore required number of ways = ${}^{13}C_3 - {}^{11}C_1 = 286 - 11 = 275$ **Ans.**

Illustration 47: Find the number of ways in which three numbers can be selected from the set $\{5^1, 5^2, 5^3, ...5^{11}\}$ so that they form a G.P.

Solution : Any three selected numbers which are in G.P. have their powers in A.P.

Set of powers is = $\{1,2,.........6,7,.....11\}$

By selecting any two numbers from $\{1,3,5,7,9,11\}$, the middle number is automatically fixed. Total number of ways = ${}^6C_2$

Now select any two numbers from $\{2,4,6,8,10\}$ and again middle number is automatically fixed. Total number of ways = ${}^5C_2$

$\therefore$ Total number of ways are = ${}^6C_2 + {}^5C_2 = 15 + 10 = 25$ **Ans.**

EXERCISE (O-1)

ONLY ONE CORRECT :

1. The number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number is

(A) 672 (B) 640 (C) 512 (D) none

PC0001

2. How many of the 900 three digit numbers have at least one even digit?

(A) 775 (B) 875 (C) 450 (D) 750

PC0002

3. Number of natural numbers between 100 and 1000 such that at least one of their digits is 7, is

(A) 225 (B) 243 (C) 252 (D) none

PC0003

4. Number of 4 digit numbers of the form N = abcd which satisfy following three conditions :

(i) $4000 \le N < 6000$ (ii) N is multiple of 5 (iii) $3 \le b < c \le 6$

is equal to

(A) 12 (B) 18 (C) 24 (D) 48

PC0004

5. Consider the five points comprising of the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

(A) 4 (B) 6 (C) 8 (D) 10

PC0005

6. The number of ways in which 5 different books can be distributed among 10 people if each person can get at most one book is :

(A) 252 (B) 10^5 (C) 5^{10} (D) ${}^{10}C_5.5!$

PC0006

7. A student has to answer 10 out of 13 questions in an examination . The number of ways in which he can answer if he must answer atleast 3 of the first five questions is :

(A) 276 (B) 267 (C) 80 (D) 1200

PC0007

8. A question paper on mathematics consists of twelve questions divided into three parts A, B and C, each containing four questions. In how many ways can an examinee answer five questions, selecting atleast one from each part.

(A) 624 (B) 208 (C) 1248 (D) 2304

PC0008

9. 5 Indian & 5 American couples meet at a party & shake hands . If no wife shakes hands with her own husband & no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is :

(A) 95 (B) 110 (C) 135 (D) 150

PC0009

10. The kindergarten teacher has 25 kids in her class . She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by :

(A) ${}^{25}C_5 - {}^{24}C_5$ (B) ${}^{24}C_5$ (C) ${}^{24}C_4$ (D) none

PC0010

11. Number of cyphers at the end of ${}^{2002}C_{1001}$ is

(A) 0 (B) 1 (C) 2 (D) 200

PC0011

12. Three vertices of a convex n sided polygon are selected. If the number of triangles that can be constructed such that none of the sides of the triangle is also the side of the polygon is 30, then the polygon is a

(A) Heptagon (B) Octagon (C) Nonagon (D) Decagon

PC0012

13. There are 12 books on Algebra and Calculus in our library , the books of the same subject being different. If the number of selections each of which consists of 3 books on each topic is greatest then the number of books of Algebra and Calculus in the library are respectively:

(A) 3 and 9 (B) 4 and 8 (C) 5 and 7 (D) 6 and 6

PC0013

14. Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word):

(A) 210 (B) 462 (C) 151200 (D) 332640

PC0014

15. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :

(A) 144 (B) 72 (C) 288 (D) 720

PC0015

16. If the letters of the word "VARUN" are written in all possible ways and then are arranged as in a dictionary, then the rank of the word VARUN is :

(A) 98 (B) 99 (C) 100 (D) 101

PC0016

17. A new flag is to be designed with six vertical strips using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent strips have the same colour is -

(A) 12×81 (B) 16×192 (C) 20×125 (D) 24×216

PC0017

18. Number of 5 digit numbers which are divisible by 5 and each number containing the digit 5, digits being all different is equal to k(4!), the value of k is

(A) 84 (B) 168 (C) 188 (D) 208

PC0018

19. A 5 digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 & 5 without repetition. The total number of ways this can be done is :

(A) 3125 (B) 600 (C) 240 (D) 216

PC0019

20. The number of n digit numbers which consists of the digits 1 & 2 only if each digit is to be used atleast once, is equal to 510 then n is equal to

(A) 7 (B) 8 (C) 9 (D) 10

PC0020

21. If m denotes the number of 5 digit numbers if each successive digits are in their descending order of magnitude and n is the corresponding figure, when the digits are in their ascending order of magnitude then $(m - n)$ has the value

(A) ${}^{10}C_4$ (B) ${}^{9}C_5$ (C) ${}^{10}C_3$ (D) ${}^{9}C_3$

PC0021

22. A rack has 5 different pairs of shoes. The number of ways in which 4 shoes can be chosen from it , so that there will be no complete pair is :

(A) 1920 (B) 200 (C) 110 (D) 80

PC0022

23. Number of ways in which 8 people can be arranged in a line if A and B must be next each other and C must be somewhere behind D, is equal to

(A) 10080 (B) 5040 (C) 5050 (D) 10100

PC0023

24. An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the fifth digit is either a 4 or 5 while has no memorising of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum number of distinct trials he has to try to make sure that he dials the correct telephone number, is

(A) 360 (B) 240 (C) 216 (D) none

PC0024

25. Number of 5 digit numbers divisible by 25 that can be formed using only the digits 1, 2, 3, 4, 5 & 0 taken five at a time is

(A) 2 (B) 32 (C) 42 (D) 52

PC0025

26. Let P_n denotes the number of ways of selecting 3 people out of 'n' sitting in a row, if no two of them are consecutive and Q_n is the corresponding figure when they are in a circle. If $P_n - Q_n = 6$, then 'n' is equal to :

(A) 8 (B) 9 (C) 10 (D) 12

PC0026

27. Number of 7 digit numbers the sum of whose digits is 61 is :

(A) 12 (B) 24 (C) 28 (D) none

PC0027

28. In a unique hockey series between India & Pakistan, they decide to play on till a team wins 5 matches. The number of ways in which the series can be won by India, if no match ends in a draw is :

(A) 126 (B) 252 (C) 225 (D) none

PC0028

29. There are 100 different books in a shelf. Number of ways in which 3 books can be selected so that no two of which are neighbours is

(A) ${}^{100}C_3 - 98$ (B) ${}^{97}C_3$ (C) ${}^{96}C_3$ (D) ${}^{98}C_3$

PC0029

30. Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting in a row, if no two of them are consecutive. If, $P_{n+1} - P_n = 15$ then the value of 'n' is :

(A) 7 (B) 8 (C) 9 (D) 10

PC0030

MATCH THE COLUMN :

31.

	Column-I		Column-II
(A)	Number of increasing permutations of *m* symbols are there from the *n* set numbers $\{a_1, a_2, \ldots, a_n\}$ where the order among the numbers is given by $a_1 < a_2 < a_3 < \ldots a_{n-1} < a_n$ is **PC0031**	(P)	n^m
(B)	There are *m* men and *n* monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys **PC0032**	(Q)	${}^mC_n$
(C)	Number of ways in which *n* red balls and $(m-1)$ green balls can be arranged in a line, so that no two red balls are together, is (balls of the same colour are alike) **PC0033**	(R)	${}^nC_m$
(D)	Number of ways in which '*m*' different toys can be distributed in '*n*' children if every child may receive any number of toys, is **PC0034**	(S)	m^n

32. Number of permutations of 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time, such that the digit

1 appearing somewhere to the left of 2

3 appearing to the left of 4 and

5 somewhere to the left of 6, is

(*e.g.* 815723946 would be one such permutation)

(A) $9 \cdot 7!$ (B) $8!$ (C) $5! \cdot 4!$ (D) $8! \cdot 4!$

PC0035

33. Seven different coins are to be divided amongst three persons . If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is

(A) 420 (B) 630 (C) 710 (D) none

PC0036

34. Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more, is :

(A) $\frac{(5!)^2}{8}$ (B) $\frac{9!}{2}$ (C) $\frac{9!}{3!\,(2!)^3}$ (D) none

PC0037

35. A gentleman invites a party of $m + n\ (m \neq n)$ friends to a dinner & places m at one table T_1 and n at another table T_2 , the table being round . If not all people shall have the same neighbour in any two arrangement, then the number of ways in which he can arrange the guests, is

(A) $\frac{(m+n)!}{4\ mn}$ (B) $\frac{1}{2}\frac{(m+n)!}{mn}$ (C) $2\frac{(m+n)!}{mn}$ (D) none

PC0038

36. A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. If internal arrangement inside the car does not matter then the number of ways in which they can travel, is

(A) 91 (B) 182 (C) 126 (D) 3920

PC0039

37. Let m denote the number of ways in which 4 different books are distributed among 10 persons, each receiving none or one only and let n denote the number of ways of distribution if the books are all alike. Then :

(A) $m = 4n$ (B) $n = 4m$ (C) $m = 24n$ (D) none

PC0040

38. There are (p + q) different books on different topics in Mathematics. $(p \neq q)$

If L = The number of ways in which these books are distributed between two students X and Y such that X get p books and Y gets q books.

M = The number of ways in which these books are distributed between two students X and Y such that one of them gets p books and another gets q books.

N = The number of ways in which these books are divided into two groups of p books and q books then,

(A) L = M = N (B) L = 2M = 2N (C) 2L = M = 2N (D) L = M = 2N

PC0041

39. Number of ways in which 7 green bottles and 8 blue bottles can be arranged in a row if exactly 1 pair of green bottles is side by side, is (Assume all bottles to be alike except for the colour).

(A) 84 (B) 360 (C) 504 (D) none

PC0042

40. There are 10 red balls of different shades & 9 green balls of identical shades. Then the number of arranging them in a row so that no two green balls are together is

(A) $(10\,!)\,.\,{}^{11}P_9$ (B) $(10\,!)\,.\,{}^{11}C_9$

(C) 10 ! (D) 10 ! 9 !

PC0043

41. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is :

(A) $6\,(7\,! - 4\,!)$ (B) $7\,(6\,! - 4\,!)$ (C) $8\,! - 5\,!$ (D) none

PC0044

42. Number of ways in which 5 A's and 6 B's can be arranged in a row which reads the same backwards and forwards, is

(A) 6 (B) 8 (C) 10 (D) 12

PC0045

43. Number of ways in which four different toys and five indistinguishable marbles can be distributed between Amar, Akbar and Anthony, if each child receives atleast one toy and one marble, is

(A) 42 (B) 100 (C) 150 (D) 216

PC0046

44. There are counters available in x different colours. The counters are all alike except for the colour. The total number of arrangements consisting of y counters, assuming sufficient number of counters of each colour, if no arrangement consists of all counters of the same colour is :

(A) $x^y - x$ (B) $x^y - y$ (C) $y^x - x$ (D) $y^x - y$

PC0047

45. The number of ways in which 8 distinguishable apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is $K \cdot {}^7P_3$ where K has the value equal to

(A) 14 (B) 66 (C) 44 (D) 22

PC0048

46. There are six periods in each working day of a school. Number of ways in which 5 subjects can be arranged if each subject is allotted at least one period and no period remains vacant is

(A) 210 (B) 1800 (C) 360 (D) 3600

PC0049

47. Number of positive integral solutions satisfying the equation $(x_1 + x_2 + x_3)(y_1 + y_2) = 77$, is

(A) 150 (B) 270 (C) 420 (D) 1024

PC0050

48. There are counters available in 3 different colours (atleast four of each colour). Counters are all alike except for the colour. If 'm' denotes the number of arrangements of four counters if no arrangement consists of counters of same colour and 'n' denotes the corresponding figure when every arrangement consists of counters of each colour, then :

(A) $m = 2n$ (B) $6m = 13n$ (C) $3m = 5n$ (D) $5m = 3n$

PC0051

49. One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world, and 30 read Business Today. Five students read all the three magazines. How many read exactly two magazines?

(A) 50 (B) 10 (C) 95 (D) 65

PC0052

50. A person writes letters to his 5 friends and addresses the corresponding envelopes. Number of ways in which the letters can be placed in the envelope, so that atleast two of them are in the wrong envelopes,is,

(A) 1 (B) 2 (C) 118 (D) 119

PC0053

EXERCISE (O-2)

ONLY ONE CORRECT :

1. A committee of 5 is to be chosen from a group of 9 people. Number of ways in which it can be formed if two particular persons either serve together or not at all and two other particular persons refuse to serve with each other, is

(A) 41 (B) 36 (C) 47 (D) 76

PC0054

2. There are m points on a straight line AB & n points on the line AC none of them being the point A. Triangles are formed with these points as vertices, when

(i) A is excluded (ii) A is included. The ratio of number of triangles in the two cases is:

(A) $\frac{m+n-2}{m+n}$ (B) $\frac{m+n-2}{m+n-1}$ (C) $\frac{m+n-2}{m+n+2}$ (D) $\frac{m(n-1)}{(m+1)(n+1)}$

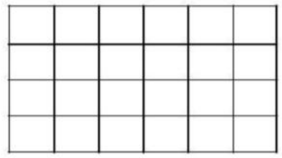
PC0055

3. There are 10 straight lines in a plane, such that no 3 are concurrent and no 2 are parallel to each other. If points of intersection of above lines are joined, then maximum number of lines thus formed are (including old lines) -

(A) 610 (B) 620 (C) 630 (D) 640

PC0056

4. Number of rectangles in the grid shown which are not squares is



(A) 160 (B) 162 (C) 170 (D) 185

PC0057

5. Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle. Then the maximum number of circles that can be drawn so that each contains atleast three of the given points is :

(A) 216 (B) 156 (C) 172 (D) none

PC0058

6. The number of ways of choosing a committee of 2 women & 3 men from 5 women & 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member & Mr. B can only serve, if Miss C is the member of the committee, is

(A) 60 (B) 84 (C) 124 (D) none

PC0059

7. Product of all the even divisors of $N = 1000$, is

(A) $32 \cdot 10^2$ (B) $64 \cdot 2^{14}$ (C) $64 \cdot 10^{18}$ (D) $128 \cdot 10^6$

PC0060

8. Two classrooms A and B having capacity of 25 and (n–25) seats respectively. A_n denotes the number of possible seating arrangements of room 'A', when 'n' students are to be seated in these rooms, starting from room 'A' which is to be filled up full to its capacity. If $A_n - A_{n-1} = 25!\,({}^{49}C_{25})$ then 'n' equals -

(A) 50 (B) 48 (C) 49 (D) 51

PC0061

MORE THAN ONE ARE CORRECT :

9. Lines $y = x + i$ & $y = -x + j$ are drawn in $x - y$ plane such that $i \in \{1,2,3,4\}$ & $j \in \{1,2,3,4,5,6\}$. If m represents the total number of squares formed by the lines and n represents the total number of triangles formed by the given lines & x-axis, then correct option/s is/are-

(A) $m + n = 50$ (B) $m - n = 2$ (C) $m + n = 48$ (D) $m - n = 4$

PC0062

10. The combinatorial coefficient C(n, r) is equal to

(A) number of possible subsets of r members from a set of n distinct members.

(B) number of possible binary messages of length n with exactly r 1's.

(C) number of non decreasing 2-D paths from the lattice point (0, 0) to (r, n).

(D) number of ways of selecting r things out of n different things when a particular thing is always included plus the number of ways of selecting 'r' things out of n, when a particular thing is always excluded.

PC0063

11. There are 10 questions, each question is either True or False. Number of different sequences of incorrect answers is also equal to

(A) Number of ways in which a normal coin tossed 10 times would fall in a definite order if both Heads and Tails are present.

(B) Number of ways in which a multiple choice question containing 10 alternatives with one or more than one correct alternatives, can be answered.

(C) Number of ways in which it is possible to draw a sum of money with 10 coins of different denominations taken some or all at a time.

(D) Number of different selections of 10 indistinguishable things taken some or all at a time.

PC0064

12. Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3, ... n is :

(A) $\left(\frac{n-1}{2}\right)^2$ if n is even

(B) $\frac{n(n-2)}{4}$ if n is odd

(C) $\frac{(n-1)^2}{4}$ if n is odd

(D) $\frac{n(n-2)}{4}$ if n is even

PC0065

13. The combinatorial coefficient ${}^{n-1}C_p$ denotes

(A) the number of ways in which *n* things of which p are alike and rest different can be arranged in a circle.

(B) the number of ways in which *p* different things can be selected out of *n* different thing if a particular thing is always excluded.

(C) number of ways in which *n* alike balls can be distributed in *p* different boxes so that no box remains empty and each box can hold any number of balls.

(D) the number of ways in which (n – 2) white balls and p black balls can be arranged in a line if black balls are separated, balls are all alike except for the colour.

PC0066

14. In a certain strange language, words are written with letters from the following six-letter alphabet : A, G, K, N, R, U. Each word consists of six letters and none of the letters repeat. Each combination of these six letters is a word in this language. The word "KANGUR" remains in the dictionary at,

(A) 248^{th} (B) 247^{th} (C) 246^{th} (D) 253^{rd}

PC0067

15. Six people are going to sit in a row on a bench. A and B are adjacent, C does not want to sit adjacent to D. E and F can sit anywhere. Number of ways in which these six people can be seated, is

(A) 200 (B) 144 (C) 120 (D) 56

PC0068

16. Six married couple are sitting in a room. Find the number of ways in which 4 people can be selected so that :

(A) they do not form a couple

(B) they form exactly one couple

(C) they form at least one couple

(D) they form atmost one couple

PC0069

17. The number of three digit numbers having only two consecutive digits identical is :
(A) 153 (B) 162 (C) 180 (D) 161

PC0070

18. Number of 3 digit numbers in which the digit at hundredth's place is greater than the other two digit is
(A) 285 (B) 281 (C) 240 (D) 204

PC0071

19. All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is :
(A) 5 (B) 325 (C) 345 (D) 365

PC0072

Paragraph for Question Nos. 20 to 22

16 players $P_1, P_2, P_3, \ldots\ldots P_{16}$ take part in a tennis tournament. Lower suffix player is better than any higher suffix player. These players are to be divided into 4 groups each comprising of 4 players and the best from each group is selected for semifinals.

20. Number of ways in which 16 players can be divided into four equal groups, is

(A) $\frac{35}{27}\prod_{r=1}^{8}(2r-1)$ (B) $\frac{35}{24}\prod_{r=1}^{8}(2r-1)$ (C) $\frac{35}{52}\prod_{r=1}^{8}(2r-1)$ (D) $\frac{35}{6}\prod_{r=1}^{8}(2r-1)$

PC0073

21. Number of ways in which they can be divided into 4 equal groups if the players P_1, P_2, P_3 and P_4 are in different groups, is :

(A) $\frac{(11)!}{36}$ (B) $\frac{(11)!}{72}$ (C) $\frac{(11)!}{108}$ (D) $\frac{(11)!}{216}$

PC0073

22. Number of ways in which these 16 players can be divided into four equal groups, such that when the best player is selected from each group, P_6 is one among them, is (k) $\frac{12!}{(4!)^3}$. The value of k is :
(A) 36 (B) 24 (C) 18 (D) 20

PC0073

23. Consider the word W = MISSISSIPPI

(a) If N denotes the number of different selections of 5 letters from the word W = MISSISSIPPI then N belongs to the set
(A) {15, 16, 17, 18, 19} (B) {20, 21, 22, 23, 24}
(C) {25, 26, 27, 28, 29} (D) {30, 31, 32, 33, 34}

(b) Number of ways in which the letters of the word W can be arranged if atleast one vowel is separated from rest of the vowels

(A) $\frac{8!\cdot 161}{4!\cdot 4!\cdot 2!}$ (B) $\frac{8!\cdot 161}{4\cdot 4!\cdot 2!}$ (C) $\frac{8!\cdot 161}{4!\cdot 2!}$ (D) $\frac{8!}{4!\cdot 2!}\cdot\frac{165}{4!}$

(c) If the number of arrangements of the letters of the word W if all the S's and P's are separated is (K)$\left(\frac{10!}{4!\cdot 4!}\right)$, then K equals -

(A) $\frac{6}{5}$ (B) 1 (C) $\frac{4}{3}$ (D) $\frac{3}{2}$

PC0074

24. The maximum number of permutations of 2n letters in which there are only a's & b's, taken all at a time is given by :

(A) ${}^{2n}C_n$

(B) $\frac{2}{1}\cdot\frac{6}{2}\cdot\frac{10}{3}\ldots\ldots\frac{4n-6}{n-1}\cdot\frac{4n-2}{n}$

(C) $\frac{n+1}{1}\cdot\frac{n+2}{2}\cdot\frac{n+3}{3}\cdot\frac{n+4}{4}\ldots\ldots\frac{2n-1}{n-1}\cdot\frac{2n}{n}$

(D) $\frac{2^n\cdot[1.3.5\ldots\ldots(2n-3)(2n-1)]}{n!}$

PC0075

25. Number of ways in which the letters of the word 'B U L B U L' can be arranged in a line in a definite order is also equal to the

(A) number of ways in which 2 alike Apples and 4 alike Mangoes can be distributed in 3 children so that each child receives any number of fruits.

(B) Number of ways in which 6 different books can be tied up into 3 bundles, if each bundle is to have equal number of books.

(C) coefficient of $x^2y^2z^2$ in the expansion of $(x + y + z)^6$.

(D) number of ways in which 6 different prizes can be distributed equally in three children.

PC0076

26. Which of the following statements are correct?

(A) Number of words that can be formed with 6 only of the letters of the word "CENTRIFUGAL" if each word must contain all the vowels is $3 \cdot 7!$

(B) There are 15 balls of which some are white and the rest black. If the number of ways in which the balls can be arranged in a row, is maximum then the number of white balls must be equal to 7 or 8. Assume balls of the same colour to be alike.

(C) There are 12 things, 4 alike of one kind, 5 alike and of another kind and the rest are all different. The total number of combinations is 240.

(D) Number of selections that can be made of 6 letters from the word "COMMITTEE" is 35.

PC0077

MATCH THE COLUMN:

27. **Column-I** | **Column-II**

(A) Four different movies are running in a town. Ten students go to watch these four movies. The number of ways in which every movie is watched by atleast one student, is (Assume each way differs only by number of students watching a movie) — (P) 11

PC0078

(Q) 36

(B) Consider 8 vertices of a regular octagon and its centre. If T denotes the number of triangles and S denotes the number of straight lines that can be formed with these 9 points then the value of (T – S) equals

PC0079

(C) In an examination, 5 children were found to have their mobiles in their pocket. The Invigilator fired them and took their mobiles in his possession. Towards the end of the test, Invigilator randomly returned their mobiles. The number of ways in which at most two children did not get their own mobiles is

(R) 52

(S) 60

PC0080

(D) The product of the digits of 3214 is 24. The number of 4 digit natural numbers such that the product of their digits is 12, is

PC0081

(E) The number of ways in which a mixed double tennis game can be arranged from amongst 5 married couple if no husband & wife plays in the same game, is

(T) 84

PC0082

28. A guardian with 6 wards wishes everyone of them to study either Law or Medicine or Engineering. Number of ways in which he can make up his mind with regard to the education of his wards if every one of them be fit for any of the branches to study, and atleast one child is to be sent in each discipline is:

(A) 120 (B) 216 (C) 729 (D) 540

PC0083

EXERCISE (S-1)

1. Four visitors A, B, C & D arrive at a town which has 5 hotels. In how many ways can they disperse themselves among 5 hotels, if 4 hotels are used to accommodate them.

PC0084

2. There are 6 roads between A & B and 4 roads between B & C.

(i) In how many ways can one drive from A to C by way of B ?

(ii) In how many ways can one drive from A to C and back to A, passing through B on both trips ?

(iii) In how many ways can one drive the circular trip described in (ii) without using the same road more than once.

PC0085

3. (i) Find the number of four letter word that can be formed from the letters of the word HISTORY. (each letter to be used atmost once)

(ii) How many of them contain only consonants?

(iii) How many of them begin & end in a consonant?

(iv) How many of them begin with a vowel?

(v) How many contain the letters Y?

(vi) How many begin with T & end in a vowel?

(vii) How many begin with T & also contain S?

(viii) How many contain both vowels?

PC0086

4. If repetitions are not permitted

(i) How many 3 digit numbers can be formed from the six digits 2, 3, 5, 6, 7 & 9 ?

(ii) How many of these are less than 400 ?

(iii) How many are even ?

(iv) How many are odd ?

(v) How many are multiples of 5 ?

PC0087

5. How many two digit numbers are there in which the tens digit and the units digit are different and odd ?

PC0088

6. Every telephone number consists of 7 digits. How many telephone numbers are there which do not include any other digits but 2 , 3 , 5 & 7 ?

PC0089

7. (a) In how many ways can four passengers be accommodated in three railway carriages, if each carriage can accommodate any number of passengers.

PC0090

(b) In how many ways four persons can be accommodated in 3 different chairs if each person can occupy only one chair.

PC0091

8. How many odd numbers of five distinct digits can be formed with the digits 0,1,2,3,4 ?

PC0092

9. Number of ways in which 7 different colours in a rainbow can be arranged if green is always in the middle.

PC0093

10. Find the number of ways in which the letters of the word "MIRACLE" can be arranged if vowels always occupy the odd places.

PC0094

11. A letter lock consists of three rings each marked with 10 different letters. Find the number of ways in which it is possible to make an unsuccessful attempts to open the lock.

PC0095

12. (i) Prove that : ${}^{n}P_{r} = {}^{n-1}P_{r} + r.\ {}^{n-1}P_{r-1}$

PC0096

(ii) If ${}^{20}C_{r+2} = {}^{20}C_{2r-3}$, find ${}^{12}C_{r}$.

PC0097

(iii) Prove that : ${}^{n-1}C_{3} + {}^{n-1}C_{4} > {}^{n}C_{3}$ if $n > 7$.

PC0098

(iv) Find r if ${}^{15}C_{3r} = {}^{15}C_{r+3}$.

PC0099

13. Find the number of ways in which two squares can be selected from an 8 by 8 chess board of size 1×1 so that they are not in the same row and in the same column.

PC0100

14. There are 10 seats in a double decker bus, 6 in the lower deck and 4 on the upper deck. Ten passengers board the bus, of them 3 refuse to go to the upper deck and 2 insist on going up. The number of ways in which the passengers can be accommodated is ______. (Assume all seats to be duly numbered)

PC0101

15. In a certain algebraical exercise book there are 4 examples on arithmetical progressions, 5 examples on permutation-combination and 6 examples on binomial theorem. Number of ways a teacher can select for his pupils atleast one but not more than 2 examples from each of these sets, is _______.

PC0102

16. In how many ways can a team of 6 horses be selected out of a stud of 16, so that there shall always be 3 out of $A\,B\,C\,A'\,B'\,C'$, but never $A\,A'$, $B\,B'$ or $C\,C'$ together.

PC0103

17. There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. Find the number of participants & the total numbers of games played in the tournament.

PC0104

18. Each of 3 committees has 1 vacancy which is to be filled from a group of 6 people. Find the number of ways the 3 vacancies can be filled if ;

(i) Each person can serve on atmost 1 committee.

(ii) There is no restriction on the number of committees on which a person can serve.

(iii) Each person can serve on atmost 2 committees.

PC0105

19. Find the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, \ldots\ldots. 3^{100}, 3^{101}\}$ so that they form a G.P.

PC0106

20. Find the number of ways in which letters of the word VALEDICTORY be arranged so that the vowels may never be separated.

PC0107

21. An examination paper consists of 12 questions divided into parts A & B.
Part-A contains 7 questions & Part-B contains 5 questions. A candidate is required to attempt 8 questions selecting atleast 3 from each part. In how many maximum ways can the candidate select the questions ?

PC0108

22. 5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separate from the first 2.

PC0109

23. During a draw of lottery, tickets bearing numbers 1, 2, 3,......, 40, 6 tickets are drawn out & then arranged in the descending order of their numbers. In how many ways, it is possible to have 4^{th} ticket bearing number 25.

PC0110

24. Find the number of distinct natural numbers upto a maximum of 4 digits and divisible by 5, which can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit not occuring more than once in each number.

PC0111

25. A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied.

PC0112

26. Find the number of permutations of the word "AUROBIND" in which vowels appear in an alphabetical order.

PC0113

27. Define a 'good word' as a sequence of letters that consists only of the letters A, B and C and in which A never immediately followed by B, B is never immediately followed by C, and C is never immediately followed by A. If the number of n-letter good words are 384, find the value of n.

PC0114

28. In how many different ways a grandfather along with two of his grandsons and four grand daughters can be seated in a line for a photograph so that he is always in the middle and the two grandsons are never adjacent to each other.

PC0115

29. If as many more words as possible be formed out of the letters of the word "DOGMATIC" then find the number of words in which the relative order of vowels and consonants remain unchanged.

PC0116

30. There are 10 different books in a shelf. Find the number of ways in which 3 books can be selected so that exactly two of them are consecutive.

PC0117

31. In how many ways can you divide a pack of 52 cards equally among 4 players. In how many ways the cards can be divided in 4 sets, 3 of them having 17 cards each & the 4^{th} with 1 card.

PC0118

32. Find the number of ways in which the letters of the word 'KUTKUT' can be arranged so that no two alike letters are together.

PC0119

33. How many 6 digits odd numbers greater than 60,0000 can be formed from the digits 5, 6, 7, 8, 9, 0 if

(i) repetitions are not allowed

(ii) repetitions are allowed.

PC0120

34. In how many other ways can the letters of the word **MULTIPLE** be arranged;

(i) without changing the order of the vowels

(ii) keeping the position of each vowel fixed &

(iii) without changing the relative order/position of vowels & consonants.

PC0121

Paragraph for Question 35 & 36

Consider the number N = 2910600.

On the basis of above information, answer the following questions :

35. Total number of divisors of N, which are divisible by 15 but not by 36 are-

(A) 92 (B) 94 (C) 96 (D) 98

PC0122

36. Total number of ways, in which the given number can be split into two factors such that their highest common factor is a prime number is equal to-

(A) 16 (B) 32 (C) 48 (D) 64

PC0122

37. (a) How many divisors are there of the number x = 21600. Find also the sum of these divisors.

PC0123

(b) In how many ways the number 7056 can be resolved as a product of 2 factors.

PC0124

(c) Find the number of ways in which the number 300300 can be split into 2 factors which are relatively prime.

PC0125

(d) Find the number of positive integers that are divisors of atleast one of the numbers 10^{10} ; 15^7 ; 18^{11}.

PC0126

Subjetive :

38. A committee of 10 members is to be formed with members chosen from the faculties of Arts, Economics, Education, Engineering, Medicine and Science. Number of possible ways in which the faculties representation be distributed on this committee, is ________.

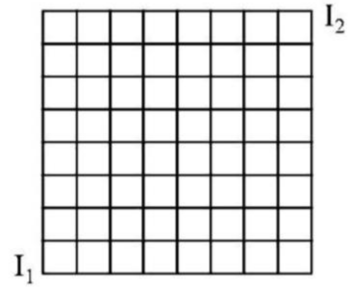
(Assume every department contains more than 10 members).

PC0127

39. If x_1, x_2, x_3 are the whole numbers and gives remainders 0,1,2 respectively, when divided by 3 then total number of different solutions of the equation $x_1 + x_2 + x_3 = 33$ are k, then $\frac{k}{11}$ is equal to

PC0128

40. On the normal chess board as shown, I_1 & I_2 are two insects which starts moving towards each other. Each insect moving with the same constant speed . Insect I_1 can move only to the right or upward along the lines while the insect I_2 can move only to the left or downward along the lines of the chess board. Find the total number of ways the two insects can meet at same point during their trip.



PC0129

41. Determine the number of paths from the origin to the point (9, 9) in the cartesian plane which never pass through (5, 5) in paths consisting only of steps going 1 unit North and 1 unit East.

PC0130

42. There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

PC0131

EXERCISE (S-2)

1. The straight lines l_1 , l_2 & l_3 are parallel & lie in the same plane. A total of m points are taken on the line l_1, n points on l_2 & k points on l_3. How many maximum number of triangles are there whose vertices are at these points ?

PC0132

2. (a) How many five digits numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if each digit is to be used atmost once.

PC0133

(b) Find the number of 4 digit positive integers if the product of their digits is divisible by 3.

PC0134

3. There are 3 cars of different make available to transport 3 girls and 5 boys on a field trip. Each car can hold up to 3 children. Find

(a) the number of ways in which they can be accomodated.

(b) the numbers of ways in which they can be accomodated if 2 or 3 girls are assigned to one of the cars.

In both the cases internal arrangement of children inside the car is considered to be immaterial.

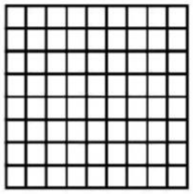
PC0135

4. Find the sum of all numbers greater than 10000 formed by using the digits 0 , 1 , 2 , 4 , 5 no digit being repeated in any number.

PC0136

Paragraph for Question 5 & 6

If 10 vertical equispaced (1 cm) lines and 9 horizontal equispaced lines (1 cm) are drawn in a plane as shown in the given figure.



On the basis of above information, answer the following questions :

5. Total number of rectangles with one side odd & one side even are given by-

(A) 600 (B) 700 (C) 800 (D) 900

PC0137

6. If squares of odd side length are selected from the above grid, then sum of their areas is equal to-

(A) $\sum_{r=1}^{4}(11-2r)(10-2r)(2r-1)^2\ cm^2$

(B) $\sum_{r=1}^{8}(9-2r)(7-2r)(2r+1)^2\ cm^2$

(C) $\sum_{r=1}^{8}(11-2r)(9-2r)(2r+1)^2\ cm^2$

(D) $\sum_{r=1}^{5}(11+2r)(9+2r)(2r-1)^2\ cm^2$

PC0137

7. How many 4 digit numbers are there which contains not more than 2 different digits?

PC0138

8. Find the number of words each consisting of 3 consonants & 3 vowels that can be formed from the letters of the word **"Circumference".** In how many of these c's will be together.

PC0139

9. Find the number of three elements sets of positive integers $\{a, b, c\}$ such that $a \times b \times c = 2310$.

PC0140

Instruction for question nos. 10 to 12 :

2 American men; 2 British men; 2 Chinese men and one each of Dutch, Egyptial, French and German persons are to be seated for a round table conference.

10. If the number of ways in which they can be seated if exactly two pairs of persons of same nationality are together is $p(6!)$, then find p.

PC0141

11. If the number of ways in which only American pair is adjacent is equal to $q(6!)$, then find q.

PC0141

12. If the number of ways in which no two people of the same nationality are together given by $r(6!)$, find r.

PC0141

13. For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence 1, 4, 7, 10...... Find the number of values of k for which S_k contain the term 361.

PC0142

14. A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice-cream cones if

(i) they are all of different flavours

(ii) they are non necessarily of different flavours

(iii) they contain only 3 different flavours

(iv) they contain only 2 or 3 different flavours?

PC0143

15. How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram can not have more than 5 candy bars and Shyam must have at least two. Assume all Candy bars to be alike.

PC0144

EXERCISE (JM)

1. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is - **[AIEEE-2010]**

(1) 3 (2) 36 (3) 66 (4) 108

PC0145

2. **Statement - 1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$. **[AIEEE-2011]**

Statement - 2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

PC0146

3. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points, then : **[AIEEE-2011]**

(1) $N > 190$ (2) $N \leq 100$ (3) $100 < N \leq 140$ (4) $140 < N \leq 190$

PC0147

4. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is - **[AIEEE-2012]**

(1) 879 (2) 880 (3) 629 (4) 630

PC0148

5. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[JEE (Main)-2013]**

(1) 256 (2) 220 (3) 219 (4) 211

PC0149

6. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[JEE (Main)-2013]**

(1) 7 (2) 5 (3) 10 (4) 8

PC0150

7. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is : **[JEE (Main)-2015]**

(1) 820 (2) 780 (3) 901 (4) 861

PC0151

8. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is : **[JEE (Main)-2015]**

(1) 275 (2) 510 (3) 219 (4) 256

PC0152

9. The number of integers greater than 6000 that can be formed, using the digits 3,5,6,7 and 8 without repetition, is : **[JEE (Main)-2015]**

(1) 120 (2) 72 (3) 216 (4) 192

PC0153

10. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

[JEE (Main)-2016]

(1) 58^{th} (2) 46^{th} (3) 59^{th} (4) 52^{nd}

PC0154

11. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[JEE (Main)-2017]**

(1) 484 (2) 485 (3) 468 (4) 469

PC0155

12. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is- **[JEE(Main)-2018]**

(1) less than 500
(2) at least 500 but less than 750
(3) at least 750 but less than 1000
(4) at least 1000

PC0156

13. Let S = {1,2,3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :- **[JEE(Main)-2019]**

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$ (3) $2^{50}-1$ (4) $2^{50}+1$

PC0157

14. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : **[JEE(Main)-2019]**

(1) 175 (2) 162 (3) 160 (4) 180

PC0158

15. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is : **[JEE(Main)-2019]**

(1) 288 (2) 306 (3) 360 (4) 310

PC0159

16. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is : **[JEE(Main)-2019]**

(1) 36 (2) 60 (3) 48 (4) 72

PC0160

17. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is : **[JEE(Main)-2019]**

(1) 210 (2) 190 (3) 170 (4) 180

PC0161

18. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is : **[JEE(Main)-2019]**

(1) 2^{20} (2) $2^{20} - 1$ (3) $2^{20} + 1$ (4) 2^{21}

PC0162

19. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is: **[JEE(Main)-2020]**

(1) $\frac{5}{2}(6!)$ (2) 5^6 (3) $\frac{1}{2}(6!)$ (4) 6!

PC0163

20. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _______. **[JEE(Main)-2020]**

PC0164

21. If a,b and c are the greatest value of ${}^{19}C_p$,${}^{20}C_q$ and ${}^{21}C_r$ respectively, then **[JEE(Main)-2020]**

(1) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$ (2) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$ (3) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$ (4) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$

PC0165

22. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is .... **[JEE(Main)-2020]**

PC0166

EXERCISE (JA)

1. Let S = {1,2,3,4}. The total number of unordered pairs of disjoint subsets of S is equal to -

[JEE 10, 5M, –2M]

(A) 25 (B) 34 (C) 42 (D) 41

PC0167

2. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is - **[JEE 2012, 3M, –1M]**

(A) 75 (B) 150 (C) 210 (D) 243

PC0168

Paragraph for Question 3 and 4 :

Let a_n denotes the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

3. The value of b_6 is **[JEE 2012, 3M, –1M]**

(A) 7 (B) 8 (C) 9 (D) 11

PC0169

4. Which of the following is correct ? **[JEE 2012, 3M, –1M]**

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

PC0169

5. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The number of such distinct arrangements (n_1,n_2,n_3,n_4,n_5) is **[JEE(Advanced)-2014, 3]**

PC0170

6. Let $n \geq 2$ b an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

[JEE(Advanced)-2014, 3]

PC0171

7. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 in always placed in envelope numbered 2. Then the number of ways it can be done is - **[JEE(Advanced)-2014, 3(–1)]**

(A) 264 (B) 265 (C) 53 (D) 67

PC0172

8. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is **[JEE (Advanced) 2015, 4M, –0M]**

PC0173

9. A debate club consists of 6 girls and 4 body. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 member) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

[JEE(Advanced)-2016, 3(–1)]

(A) 380 (B) 320 (C) 260 (D) 95

PC0174

10. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then $\frac{y}{9x}=$

[JEE(Advanced)-2017, 3]

PC0175

11. Let S = {1, 2, 3,.....,9}. For k = 1,2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

[JEE(Advanced)-2017, 3(–1)]

(A) 125 (B) 252 (C) 210 (D) 126

PC0176

12. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is ______ **[JEE(Advanced)-2018, 3(0)]**

PC0177

13. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boy and 2 girls.

(ii) Let α_2 be the total number of ways in which the committe can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committe can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the commitee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I	LIST-II
P. The value of α_1 is	**1.** 136
Q. The value of α_2 is	**2.** 189
R. The value of α_3 is	**3.** 192
S. The value of α_4 is	**4.** 200
	5. 381
	6. 461

The correct option is :-

(A) **P → 4; Q → 6, R → 2; S → 1**
(B) **P → 1; Q → 4; R → 2; S → 3**
(C) **P → 4; Q → 6, R → 5; S → 2**
(D) **P → 4; Q → 2; R → 3; S → 1**

[JEE(Advanced)-2018, 3(–1)]

PC0178

14. Five person A,B,C,D and E are seated in a ciruclar arrangement. If each of them is given a hat of one of the three colours red, blue and green ,then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is

[JEE(Advanced)-2019, 3(0)]

PC0179

ANSWER KEY

Do yourself-1

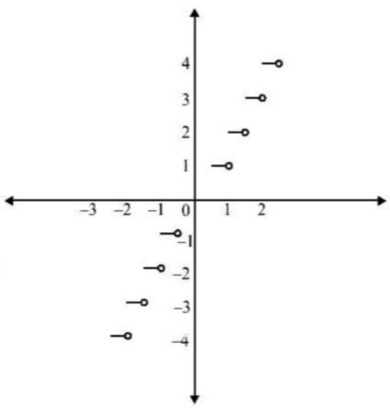
(i) 7 **(ii)** 3

Do yourself-2

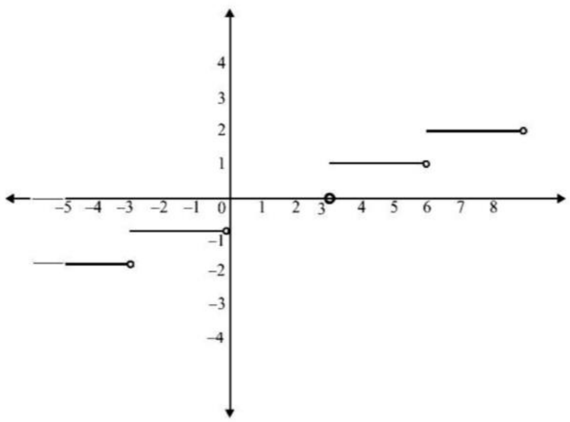
(i) 0 **(ii)** r = 4 **(iii)** ${}^{50}C_4$ **(iv)** 20 **(v)** 120, 48

(vi) **Sketch the graph of**

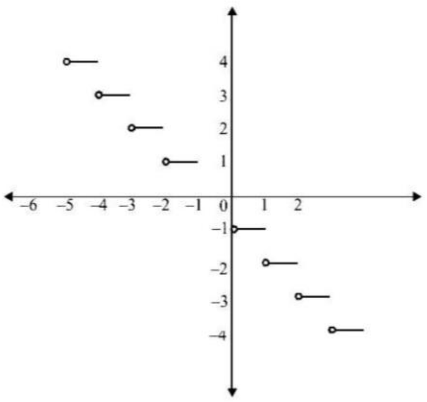
(a)



(b)



(c)



(vii) (a) $x \in [-6,-3)$ (b) $x \in \phi$ (c) $x \in (-33,-30] \cup [30,33)$

(d) $x \in (-15,-12] \cup (9,12]$ (e) $x \in (-3,-1]$

Do yourself-3

(i) 10 **(ii)** 450 **(iii)** 840, 40

Do yourself-4

(i) $\dfrac{16!}{(2!)^8 8!} \times 8!$ **(ii)** 360 **(iii)** ${}^nC_2.n!$

Do yourself-5

(i) $5^n - 4^n - 4^n + 3^n$

Do yourself-6

(i) 60, 6^{th} **(ii)** 60

Do yourself-7

(i) 36 **(ii)** $\frac{9!}{2} = 181440$ **(iii)** 5400 **(iv)** 2688

Do yourself-8

(i) $(p + 1)^n - 1$ **(ii)** $2^{10} - 1$

Do yourself-9

(i) 23 **(ii)** 36

Do yourself-10

(i) (a) ${}^{15}C_3$ (b) ${}^{7}C_3$ **(ii)** ${}^{12}C_2$ **(iii)** ${}^{23}C_2$

Do yourself-11

(i) 9

EXERCISE (O-1)

1. A	2. A	3. C	4. C	5. C	6. D	7. A	8. A
9. C	10. B	11. B	12. C	13. D	14. C	15. D	16. C
17. A	18. B	19. D	20. C	21. B	22. D	23. B	24. B
25. C	26. C	27. C	28. A	29. D	30. B		
31. (A) R; (B) S; (C) Q; (D) P			32. A	33. B	34. C	35. A	36. C
37. C	38. C	39. C	40. B	41. A	42. C	43. D	44. A
45. D	46. B	47. C	48. B	49. A	50. D		

EXERCISE (O-2)

1. A 2. A 3. D 4. A 5. B 6. C 7. C 8. A
9. A,B 10. A,B,D 11. B,C 12. C,D 13. B,D 14. A 15. B
16. 240, 240, 255, 480 17. B 18. A 19. D 20. A 21. C 22. D
23. (a) C ; (b) B ; (c) B 24. A,B,C,D 25. A,C,D 26. A,B,D
27. (A) T; (B) R; (C) P; (D) Q; (E) S 28. D

EXERCISE (S-1)

1. 120 2. (i) 24 ; (ii) 576; (iii) 360
3. (i) 840; (ii) 120; (iii) 400; (iv) 240; (v) 480; (vi) 40; (vii) 60; (viii) 240
4. (i) 120; (ii) 40; (iii) 40; (iv) 80; (v) 20
5. 20 6. 4^7 7. (a) 3^4 ; (b) 24 8. 36 9. 720 10. 576
11. 999 12. (ii) 792; (iv) r = 3 13. 1568 14. 172800
15. 3150 16. 960 17. 13, 156 18. 120, 216, 210 19. 2500
20. 967680 21. 420 22. 43200 23. ${}^{24}C_2 \cdot {}^{15}C_3$ 24. 1106
25. 5400 26. ${}^{8}C_4 \cdot 4!$ 27. n = 8 28. 528 29. 719
30. 56 31. $\frac{52!}{(13!)^4}$; $\frac{52!}{3!(17!)^3}$ 32. 30 33. 240, 15552
34. (i) 3359; (ii) 59; (iii) 359 35. C 36. C
37. (a) 72; 78120; (b) 23; (c) 32 ; (d) 435 38. 3003 39. 6
40. 12870 41. 30980

EXERCISE (S-2)

1. ${}^{m+n+k}C_3 - ({}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3)$ 2. (a) 744; (b) 7704 3. (a) 1680; (b) 1140
4. 3119976 5. C 6. A 7. 576 8. 22100 , 52
9. 40 10. 60 11. 64 12. 244 13. 24
14. (i) 15, (ii) 126, (iii) 60, (iv) 105 15. 440

EXERCISE (JM)

1. 4 **2.** 3 **3.** 2 **4.** 1 **5.** 3 **6.** 2 **7.** 2
8. 3 **9.** 4 **10.** 1 **11.** 2 **12.** 4 **13.** 1 **14.** 4
15. 4 **16.** 2 **17.** 3 **18.** 1 **19.** **1** **20.** **2454** **21.** 4 **22.** 490.00

EXERCISE (JA)

1. D **2.** B **3.** B **4.** A **5.** 7 **6.** 5 **7.** C
8. 5 **9.** A **10.** 5 **11.** D **12.** 625 **13.** C **14.** 30.00