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CBSE-XI PHYSICS

OPTICS

UNIT : 06

CHAPTER : 01

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REFRACTION

T.I.REFLECTION

REFRACTION
(Refracting – Surface)

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RAY-OPTICS

Optics: The branch of physics which deals with the study of nature, production and propagation of light. The subject of optics can be divided into two main branches: **rays' optics and wave optics**.

1. **Ray or geometrical optics**: It concerns itself with the particle nature of light and is based on

(i) the rectilinear propagation of light and

(ii) the laws of reflection and refraction of light.

It explains the formation of images in mirrors and lenses, the aberrations of optical images and the working and designing of optical instruments.

2. **Wave or physical optics**: It concerns itself with the wave nature of light and is based on the phenomena like

(i) interference

(ii) diffraction and

(iii) polarisation of light

In the previous chapter, we have learnt that light is an electromagnetic wave in which the electric and magnetic fields vary harmonically in space and time. The visible light consists of waves with wavelengths ranging from 4000 Å to 7500 Å.

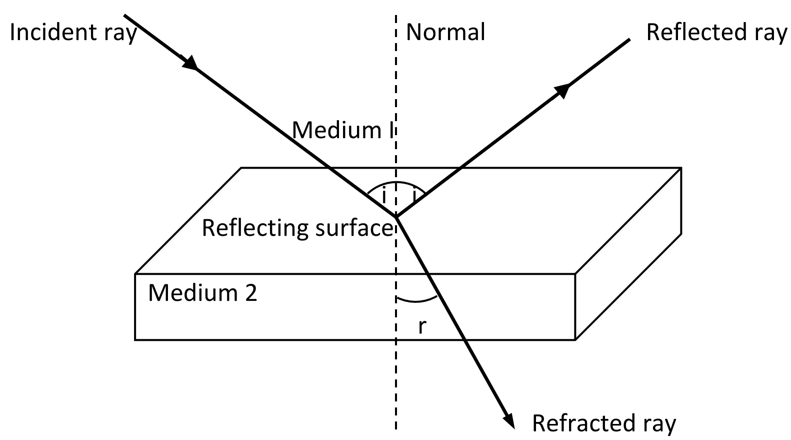
BEHAVIOUR OF LIGHT AT THE INTERFACE OF TWO MEDIA

When light travelling in one medium falls on the surface of a second medium, the following three effects may occur.

(i) A part of the incident light is turned back into the first medium. This is called reflection of light.

(ii) A part of the incident light is transmitted into the second medium along a changed direction. This is called refraction of light.

(iii) The remaining third part of light energy is absorbed by the second medium. This is called absorption of light.

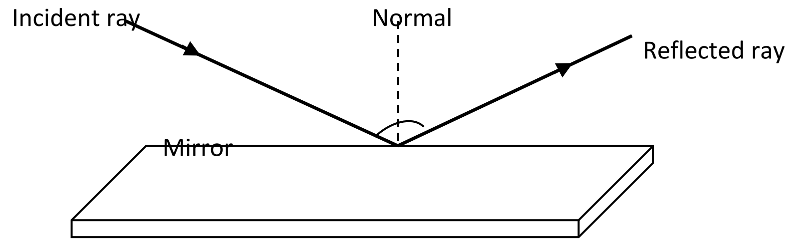


[Reflection and refraction of light]

Laws of reflection of light: Reflection of light takes place according to the following two laws:

(i) The angle of incidence is equal to the angle of reflection, i.e., $\angle i = \angle r$.

(ii) The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

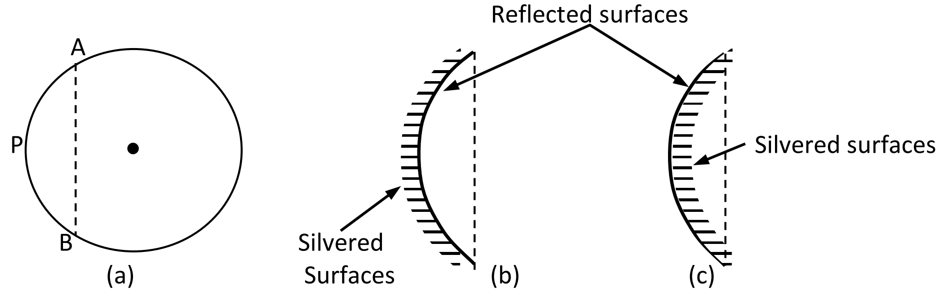


[The incident ray, reflected ray and the normal to the reflecting surface lie on the same plane]

The above laws of reflection are valid both in case of plane and curved reflecting surfaces.

SPHERICAL MIRRORS

consider a hollow glass sphere being cut by a plane. The section APB, cut by the plane, forms a part of a sphere and is known as a spherical surface. If either side of this spherical surface is silvered, we get a spherical mirror.



[(a) A hollow sphere cut by a plane (b) concave mirror (c) convex mirror]

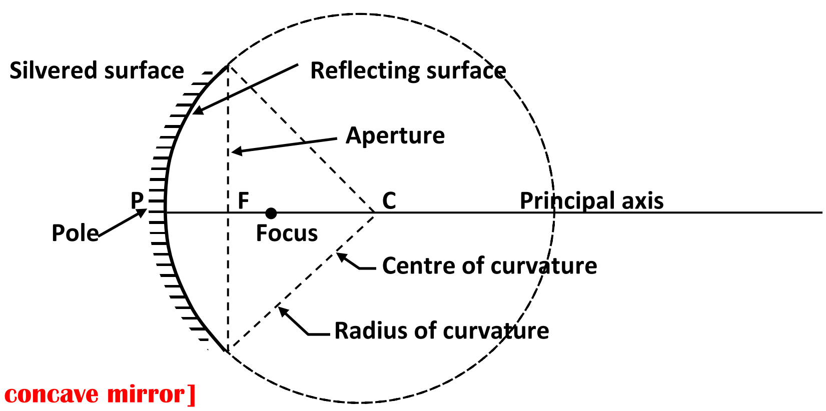
A spherical mirror is a reflecting surface which forms part of a hollow sphere.

Types of spherical mirror:

(i) Concave mirror: A spherical mirror in which the outer bulged surface is silvered polished and the reflection of light takes place from the inner hollow surface is called a concave mirror.

(ii) Convex mirror: A spherical mirror in which the inner hollow surface is silvered polished and the reflection of light takes place from the outer bulged surface is called a convex mirror.

TERM LOGY, CONNECTION WITH SPHERICAL MIRRORS: Let APB be a principal section of a spherical mirror, i.e., the section cut by a plane passing through pole and centre of curvature of the mirror.



[Characteristics of a concave mirror]

1. Pole: It is the middle point P of the spherical mirror.

2. Centre of curvature: It is the centre C of the sphere of which the mirror forms a part.

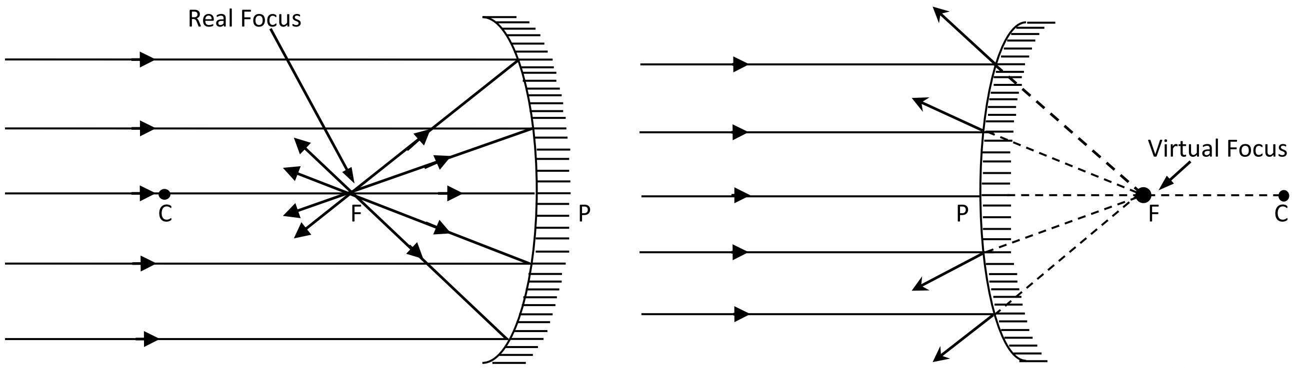
3. Radius of curvature: It is the radius (R = AC or BC) of the sphere of which the mirror forms a part.

4. Principal axis: The line PC passing through the pole and the centre of curvature of the mirror is called its principle axis.

5. Linear aperture: It is the diameter AB of the circular boundary of the spherical mirror.

6. Angular aperture: it is the angle ACB subtended by the boundary of the spherical mirror at its centre of curvature C.

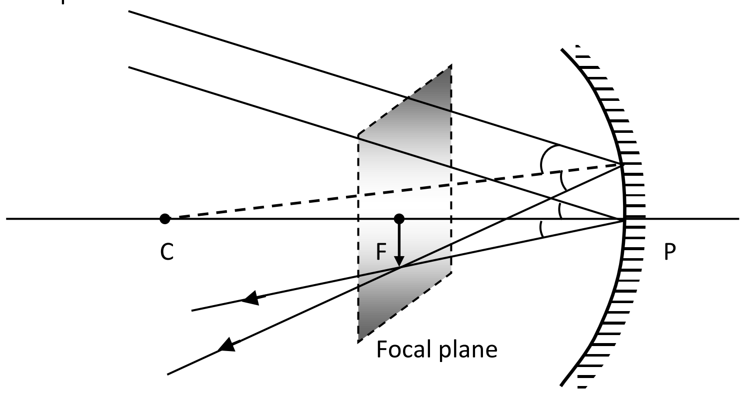
7. Principal focus: A narrow beam of light parallel to the principal axis either actually converges to or appears to diverge from a point F on the principal axis after reflection from the spherical mirror. This point is called the principle focus of the mirror. A concave mirror has a real focus while a convex mirror has a virtual focus, as shown in Fig.



8. Focal length: It is the distance (f = PF) between the focus and the pole of the mirror.

9. Focal plane: The vertical plane passing through the principle focus and perpendicular to the principal axis is called focal plane. When a parallel beam of light is incident on a concave mirror at a small angle to the principle axis, it is converged to a point in the focal plane of the mirror, as shown in Fig.

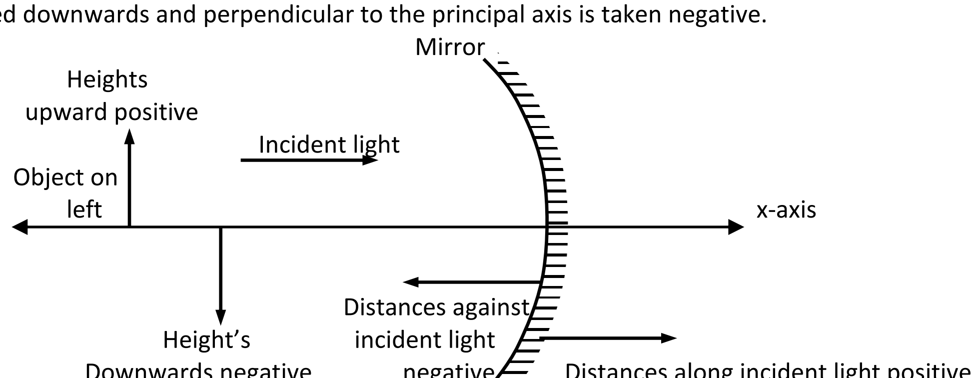
● A line joining any point of the spherical mirror to its centre of curvature will be normal to the mirror at that point.



[Focal plane of concave mirror]

New Cartesian Sign Convention for spherical Mirrors: According to this sign convention:

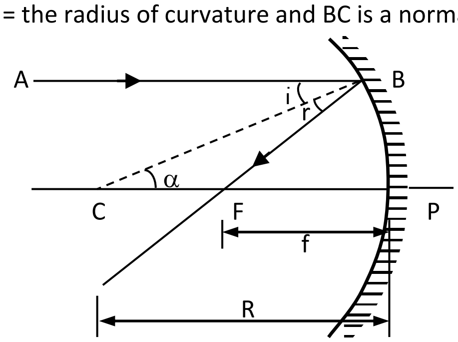
●1. All ray diagrams are drawn with the incident light travelling from left to right.
●2. All distances are measured from the pole of the mirror.
●3. All distances measured in the direction of incident light are taken to be positive.
●4. All distances measured in the opposite direction of incident light are taken to be negative.
●5. Heights measured upwards and perpendicular to the principal axis is taken positive.
●6. Heights measured downwards and perpendicular to the principal axis is taken negative.

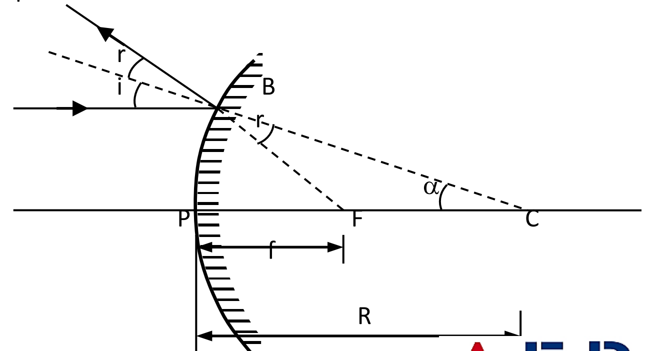


[New Cartesian sign convention]

According to the sign convention, the focal length and radius of curvature are negative for a concave mirror and positive for a convex mirror.

Relation between f and R: Consider a ray AB parallel to the parallel axis, incident at point B of a spherical mirror (concave or convex) of small aperture. After reflection from the mirror, this ray converges to point F (in case of a concave mirror) or appears to diverge from point F (in case of a convex mirror), obeying the laws of reflection. Thus, F is the focus of the mirror, C is the centre of curvature, CP = the radius of curvature and BC is a normal to mirror at point B.





According to the law of reflection $\angle i = \angle r$

As AB is parallel to PC, $\angle \alpha = \angle i$

$\therefore$ In Δ BFC, $\angle r = \angle \alpha$

Hence CF = FB

For the mirror of small aperture,

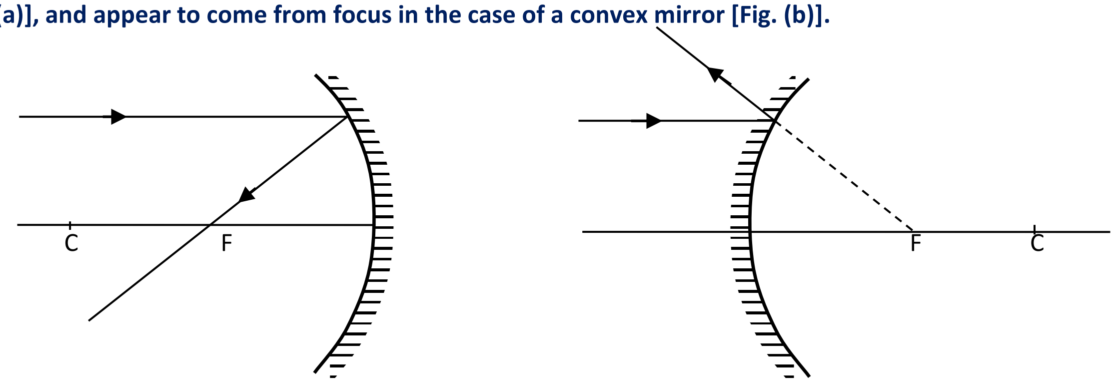
FB $\simeq$ FP $\therefore$ CF $\simeq$ FP

Hence CP = CF + FP = FP + FP = 2FP or R = 2f or f = R/2

or Focal length = ½ × Radius of curvature.

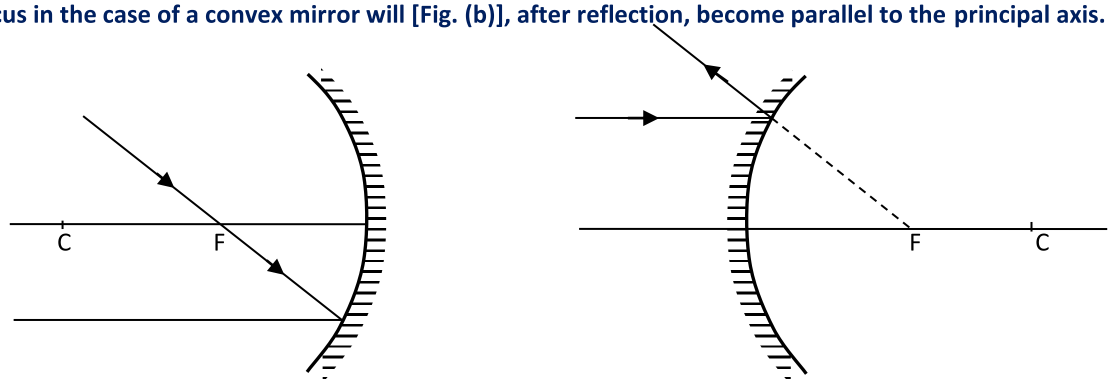
Rules for drawing images formed by spherical mirrors: The position of the image formed by spherical mirrors can be found by considering any two of the following rays of light coming from a point on the object.

(i) A ray proceeding parallel to the principal axis will, after reflection, pass through the principle focus in the case of a concave mirrors (Fig. (a)], and appear to come from focus in the case of a convex mirror [Fig. (b)].



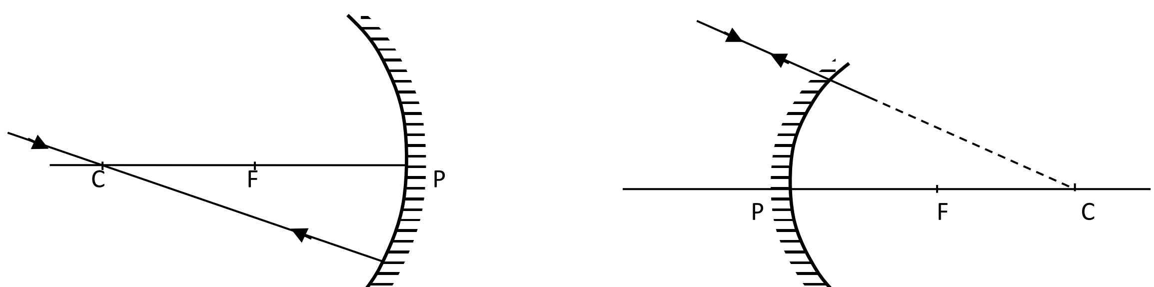
[(a) A ray parallel to the principal axis passes through F after reflection from a concave mirror. (b) A ray parallel to the principal axis appears to come from F after reflection from a convex mirror.]

(ii) A ray passing through the principle focus in the case of a concave mirror [Fig. (a)], and directed towards the principle focus in the case of a convex mirror will [Fig. (b)], after reflection, become parallel to the principal axis.



[(a) A ray through F becomes parallel to the principal axis after reflection from a concave mirror. (b) A ray directed towards F becomes parallel to the principal axis after reflection from a convex mirror.]

(iii) A ray passing through the centre of curvature in the case of concave mirror [Fig. (a)], and directed towards the centre of curvature in the case of a convex mirror [Fig. (b)] falls normally ($\angle i = \angle r = 0°$) and is reflected back along the same path.



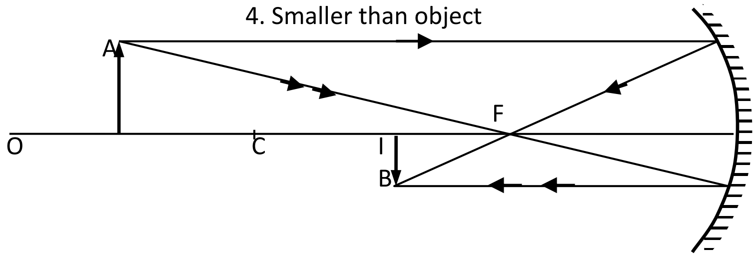
[(a) A ray passing through C is reflected back along of same path after reflection from a concave mirror. (b) A ray directed towards C is reflected back along of same path after reflection from a convex mirror.]

(iv) For the ray incident at any angle at the pole, the reflected ray follows the laws of reflection.

- **Formation of images by concave mirrors:**

(a) Object beyond C: The image is

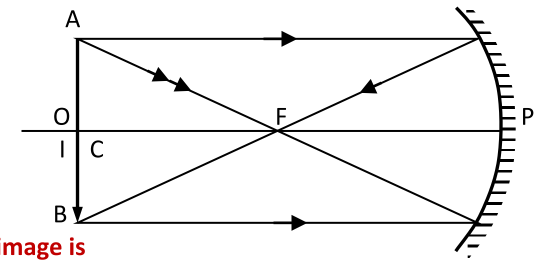
1. Between C and F   2. Real
3. Inverted   4. Smaller than object



(a)

(b) Object at C: The image is

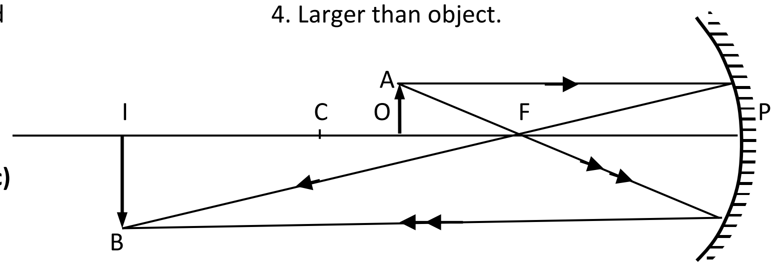
1. At C   2. Real
3. Inverted   4. Same size as object.



(b)

(c) Object between F and C: The image is

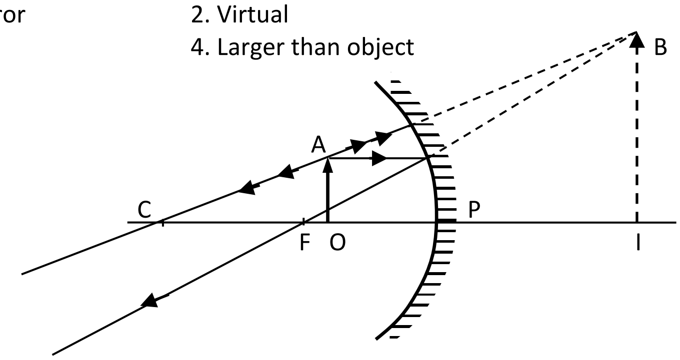
1. Beyond C   2. Real
3. Inverted   4. Larger than object.



(c)

(d) Object between F and P: The image is

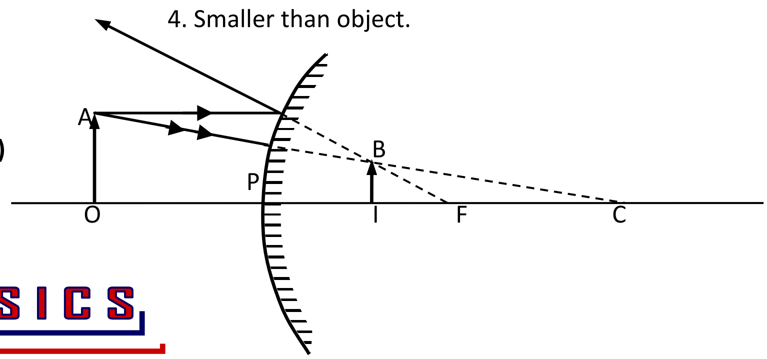
1. Behind the mirror   2. Virtual
3. Erect   4. Larger than object



(d)

- **Formation of image by convex mirror:** For any position of the object between ∞ and pole P, the image is

1. Behind the mirror   2. Virtual
3. Erect   4. Smaller than object.



(e)

THE MIRROR FORMULA

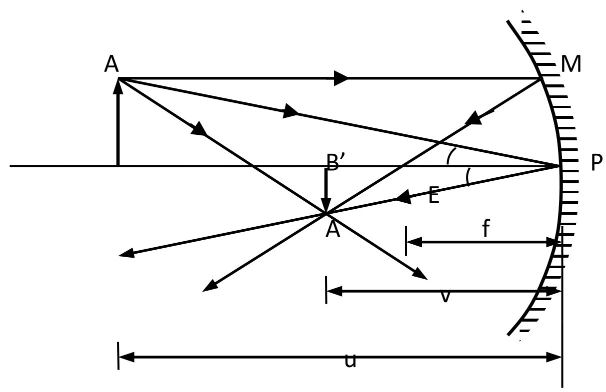
The mirror formula is a mathematical relationship between object distance u, image distance v and the focal length f of a spherical mirror. This relation is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

In order words, we can say that

$$\frac{1}{\text{Object distance}} + \frac{1}{\text{Image distance}} = \frac{1}{\text{Focal length}}$$

This formula is applicable to all concave and convex mirrors, whether the image formed is real or virtual.

Derivation of mirror formula for a concave mirror when it forms a real image: Consider an object AB placed on the principal axis beyond and centre of curvature C of a concave mirror of small aperture, as shown in Fig. A ray AM from the object travels parallel to the principal axis and after reflection from the mirror it passes through focus F. Another ray AP is incident on the pole P of the mirror and is reflected along PA' in accordance with the laws of reflection so that ∠APB = ∠B' PA'. The two reflected rays meet at point A'. Thus, A is the real image of A. The image of any point on AB will lie on a corresponding point of A'B'. Hence A'B' is the real image of AB formed by reflection from the concave mirror.



Using Cartesian sign convention, we find

Object distance, BP = – u

Image distance, B'P = – v

Focal length, FP = f

Radius of curvature CP = – R = – 2f

Now Δ A'B' C ~ Δ ABC

∴ $\frac{A'B'}{AB} = \frac{CB'}{BC} = \frac{CP - B'P}{BP - CP} = \frac{-R + v}{-u + R}$ ... (1)

As ∠A' PB' = ∠APB, therefore,

ΔA'B'P ~ ΔABP

Consequently,

$\frac{A'B'}{AB} = \frac{B'P}{BP} = \frac{-v}{-u} = \frac{v}{u}$ ... (2)

From equations (1) and (2), we get

$\frac{-R + v}{-u + R} = \frac{v}{u}$

or – uR + uv = – uv + vR

or vR + uR = 2 nv

Dividing both sides by uvR, we get

$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$

But R = 2 f ∴ $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ This proves the mirror formula for a concave mirror, when it forms a real image.

Derivation of Mirror formula for a concave mirror when the image formed is virtual: Consider an object AB placed on the principal axis of a concave mirror (of small aperture) between its pole P and focus F. As shown in Fig. a virtual and erect image A'B' is formed behind the mirror, after reflection from the concave mirror.

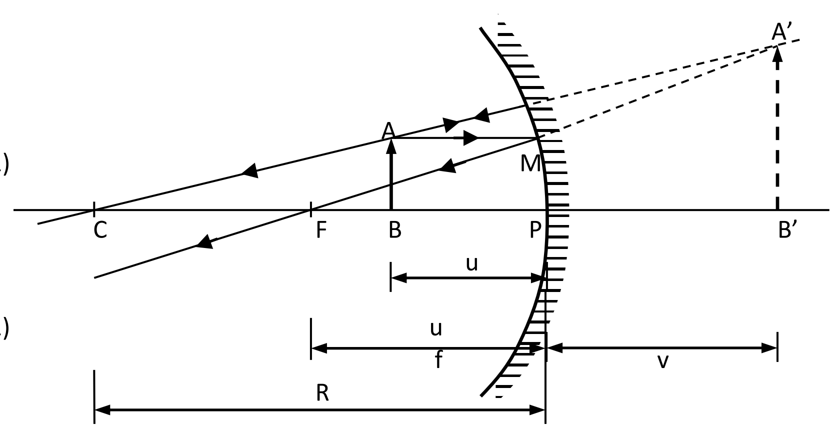
Using the Cartesian sign convention, we find that

Object distance, BP = – u

Image distance, PB' = v

Focal length, FP = – f

Radius of curvature CP = – R = – 2f



Now ΔABC ~ ΔA'B'C, therefore

$\frac{AB}{A'B'} = \frac{CB}{CB'} = \frac{CP - BP}{CP + PB'} = \frac{-2f + u}{-2f + v}$ ... (1)

Also ΔMPF ~ ΔA'B'F, therefore,

$\frac{MP}{A'B'} = \frac{FP}{FB'} = \frac{FP}{FP + PB'}$

or $\frac{AB}{A'B'} = \frac{-f}{-f + v}$ ... (2)

From equation (1) and (2), we get

$\frac{-2f + u}{-2f + v} = \frac{-f}{-f + v}$

or $2f^2 - fu - 2fv + uv = 2f^2 - fv$

or $-fv - fu + nv = 0$ or $nv = fv + fu$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This proves the mirror formula for a concave mirror when it forms a virtual image.

Derivation of mirror formula for a convex mirror: Consider an object AB placed on the principal axis of a convex mirror of small aperture, as shown in the Fig. A ray AM from the object travels parallel to the principal axis and after reflection from the mirror, it appears to come from the focus F. Another ray AP is incident on the pole P of the mirror and is reflected along PQ in accordance with the laws of reflection, so that ∠APB = ∠PBQ. The two reflected rays appear to diverge from a common point A'. Thus A' is the virtual image of A. The image of any point on AB will lie on a corresponding point of A'B'. Hence A'B' is the virtual image of AB formed by reflection from the convex mirror.

Using Cartesian sign convention, we find

Object distance, BP = – u

Image distance, PB' = + v

Focal length, FP = + f

Radius of curvature, PC = +R = + 2f

Now ΔA'B'C ~ Δ ABC

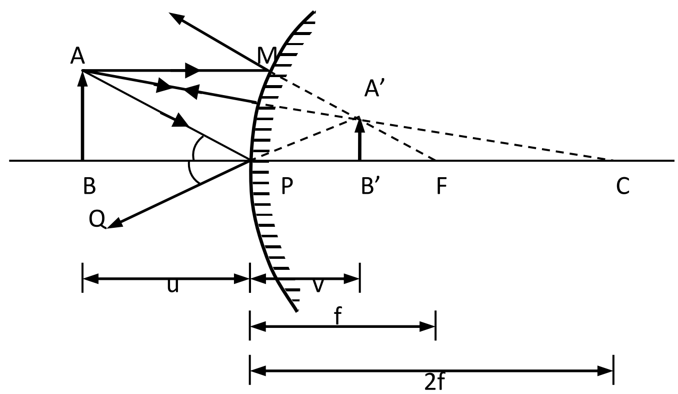
$$\therefore \frac{A'B'}{AB} = \frac{B'C}{BC} = \frac{PC - PB'}{BP + PC} = \frac{R - v}{-u + R} \quad \text{... (1)}$$

As ∠A'B'P = ∠A'B'P = ∠ APB,

Therefore, ΔA'B'P ~ ΔAPB,

Consequently,

$$\frac{A'B'}{AB} = \frac{PB'}{BP} = \frac{v}{-u} \quad \text{... (2)}$$



[To derive mirror formula for a convex mirror]

From equation (1) and (2), we get

$$\frac{R - v}{-u + R} = \frac{v}{-u}$$

or $-uR + uv = -uv + vR$

or $vR + uR = 2nv$

Dividing both sides by nvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But R = 2 f

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a convex mirror.

● **Linear magnification**: The ratio of the height of the image of that of the object is called linear or transverse magnification or just magnification and is denoted by m.

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{h_2}{h_1}$$

Concave mirror: Fig. shows the ray diagram for the formation of image A'B' of a finite object AB by a concave mirror.

Now, ΔAPB ~ ΔA' PB'

$$\therefore \frac{A'B'}{AB} = \frac{B'P}{BP}$$

Applying the new Cartesian sign convention, we get

A'B' = $-h_2$ [Downward image height]

AB = $+h_1$ [Upward object height]

B' = – v [Image distance on left]

BP = – u [Object distance on left]

$$\therefore \frac{-h_2}{h_1} = \frac{-v}{-u}$$

Magnification, $m = \frac{h_2}{h_1} = -\frac{v}{u}$

Convex mirror: Fig. shows the formation of image A'B' of a finite object AB by a convex mirror.

Now, ΔA'B'P ~ ΔABP

$$\therefore \quad \frac{A'B'}{AB} = \frac{PB'}{BP}$$

Applying the new Cartesian sign convention, we get

$A'B' = +h_2$, $\quad AB = +h_1$

$PB' = +v$, $\quad BP = -u \quad \therefore \quad \frac{h_2}{h_1} = \frac{v}{-u}$

Magnification, $m = \frac{h_2}{h_1} = -\frac{v}{u}$

Linear magnification in terms of u and f: The mirror formula is

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Multiplying both sides by v, we get

$$\frac{v}{u} + 1 = \frac{v}{f}$$

or $\quad -\frac{v}{u} = 1 - \frac{v}{f} = \frac{f - v}{f}$

$\therefore \quad m = -\frac{v}{u} = \frac{f - v}{f}$

Linear magnification in terms of v and f: As

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Multiplying both sides by u, we get

$$1 + \frac{u}{v} = \frac{u}{f}$$

or $\quad -\frac{u}{v} = 1 - \frac{u}{f} = \frac{f - u}{f}$

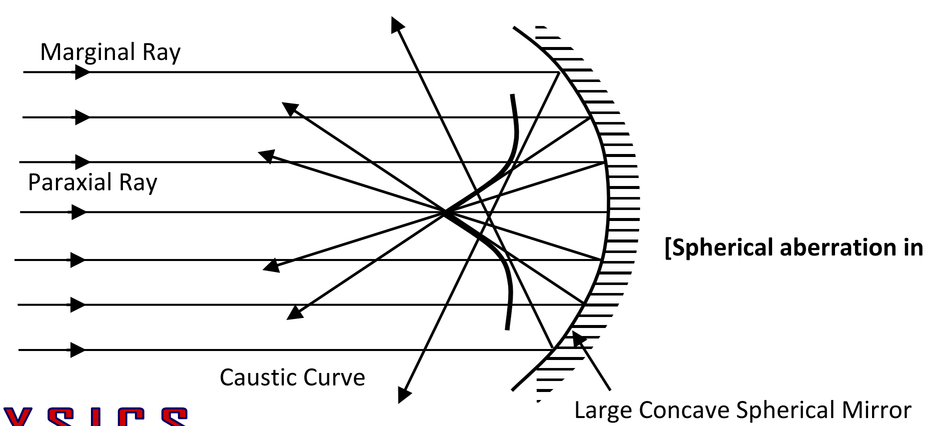
$\therefore \quad m = -\frac{v}{u} = \frac{f}{f - u}$

CONCEPTUALS:

- The same mirror formula is valid for both concave and convex mirrors whether the image formed is real or virtual.
- If $|m| > 1$, the image is magnified.
- If $|m| < 1$, the image is diminished.
- If $|m| = 1$, the image is of the same size as the object.
- If m is positive (or v is positive). The image is virtual and erect.
- If m is negative (or v is negative), the image is real and inverted.

SPHERICAL ABERRATION

Spherical aberration: ***The inability of a spherical mirror of large aperture to bring all the rays of wide beam of light falling on it to focus at a sing point is called spherical aberration***. As shown in Fig., only the paraxial rays are focussed at the principal focus F. the marginal rays meet the principal axis at a point closer to the pole than the principal focus. The different rays are reflected on to surface knows as the caustic curve. This results in blurred image of the object.

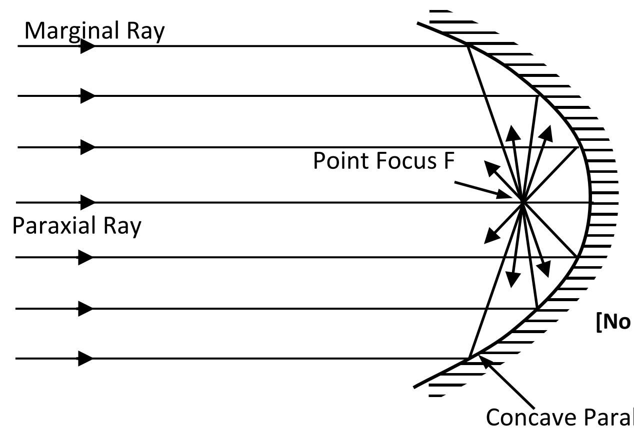


[Spherical aberration in a spherical mirror]

Spherical aberration can be reduced by following methods:

1. By using spherical mirrors of small apertures.
2. By using stoppers so as to cut off the marginal rays.
3. By using parabolic mirrors.

As shown in Fig. a parabolic mirror focuses all the rays in a wide parallel beam to a single point on the principle axis and thus spherical aberration is reduced.



[No spherical aberration in a parabolic mirror]

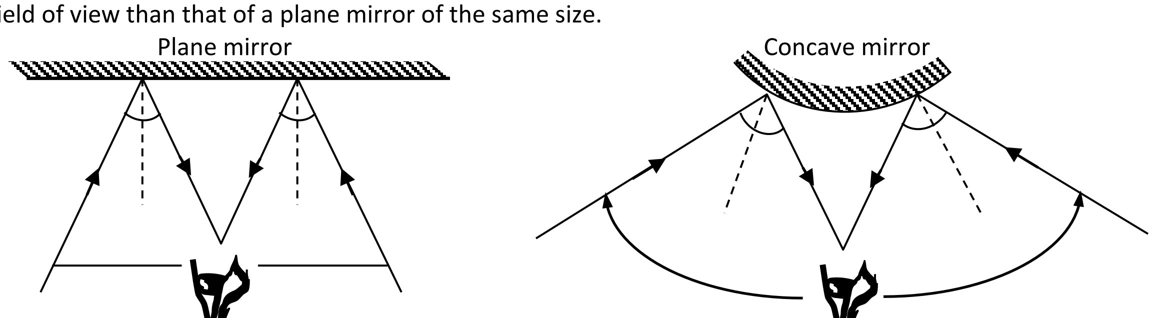
USES OF CURVED MIRRORS

Uses of concave mirrors:

1. A concave mirror is used as shaving or make-up mirror because it forms a magnified and erect image of the face when it is held closer to the face.
2. Doctors use concave mirrors as head mirror: The mirror is strapped to the doctor's forehead and light from a lamp after reflection from the mirror is focussed into the throat or ear of the patient.
3. A small concave mirror with a small hole at its centre is used in the doctor's ophthalmoscope. The doctor looks through the hole from behind the mirror while a beam of light from a lamp reflected from it is directed into the pupil of patient's eye which makes the retina visible.
4. Concave mirrors are used as reflectors in head-lights of cars, railway engines, torch lights, etc. The source is placed at the focus of a concave mirror. The light rays after reflection travel over a large distance as a parallel intense beam.

Uses of convex mirrors:

A convex mirror is used as a rear-view mirror in automobiles. The reason is that it always forms a small and erect image and it has a larger field of view than that of a plane mirror of the same size.



(a) Small field of view (b) Large field of view

Uses of parabolic mirrors:

1. A concave parabolic mirror can-focus a wide parallel beam to a single point. This property is used by dish antennas to collect and bring to focus microwave signals from satellites.
2. When a source of light is placed at the focus of a paraboloidal mirror, the reflected beam is accurately parallel and is thrown over a very large distance. Due to this property, paraboloidal mirrors are used as reflectors in search lights, car head lights, etc.
3. They are used in astronomical telescopes of large aperture for overcoming spherical aberration.

Examples based on Formation of Images by Spherical Mirrors

Formulae used

1. For any spherical mirror, $f = R/2$
2. Mirror formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}$
3. Magnification, $m = \frac{h_2}{h_1} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$

4. Magnification m is –ve for real images and +ve for virtual images.
5. f and R are –ve for a concave mirror and +ve for a convex mirror.
6. For a real object u is –ve, v is –ve for real image and +ve for virtual image.
7. Do not give any sign to unknown quantity. The sign will automatically appear in the final result.

Units used

The quantities f, u, v, h_1 and h_2 are all in m or cm while magnification m has no units.

Q. 1. ***An object is placed (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature and magnification of image in each case.***

Sol. As R is negative for a concave mirror, so

$$f = R = -15 = -7.5 \text{ cm}$$

(i) Here object distance, u = – 10 cm

By mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-0.75} - \frac{1}{-10} = \frac{-2.5}{7.5 \times 10} = -\frac{1}{30}$$

or $v = -30$ cm

As v is – ve, a real image is formed 30 cm from the mirror on the same side as the object.

Magnification, $m = -\frac{v}{u} = -\frac{-30}{-10} = -3$

The image is magnified, real and inverted.

(ii) Here object distance, u = – 5 cm

By mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-7.5} - \frac{1}{-5} = \frac{-5 + 7.5}{7.5 \times 5} = \frac{1}{15}$$

or $v = +15$ cm

As v is +ve, a virtual image is formed 15 cm behind the mirror.

Magnification, $m = -\frac{v}{u} = -\frac{15}{-5} = 3$

The image is magnified, virtual and erect.

Q. 2. ***If you sit in a parked car, you glance in the rear-view mirror R = 2 m and notice a jogger approaching. If the jogger is running at a speed of 5 ms^{-1}, how fast is the image of the jogger moving when the jogger is (a) 39 m (b) 29 m (c) 19 m (d) 9 m away?***

Sol. As the rear-view mirror is convex, so

$R = +2$m, $\qquad f = R/2 = +1$ m

From mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \qquad \therefore \qquad v = \frac{fu}{u - f}$$

When, $u = -39$ m,

$$v = \frac{1 \times (-39)}{-39 - 1} = \frac{39}{40} \text{ m}$$

As the jogger moves at a constant speed of 5 ms^{-1}, the position of the jogger after 1 s,

$$u = -39 + 5 = -34 \text{ m}$$

Difference in the position of the image in 1 s is

$$v = v' = \frac{39}{40} - \frac{34}{35} = \frac{1365 - 1360}{1400}$$

$$= \frac{5}{1400} = \frac{1}{280} \text{ m}$$

$\therefore$ Average speed of the image $= \frac{1}{280}$ ms^{-1}

For u = – 29 m, – 19 m and – 9 m, the speed of image will be

$$\frac{1}{150} \text{ ms}^{-1}, \frac{1}{60} \text{ ms}^{-1} \text{ and } \frac{1}{10} \text{ ms}^{-1} \text{ respectively}$$

The speed becomes very high as the jogger approaches the car. The change in speed can be experienced by anybody while travelling in a bus or a car.

Q. 3. ***An object 0.05 m high is placed at a distance of 0.5 m from a concave mirror of radius of curvature 0.2 m. Find the position, nature and the size of the image formed.*** 11

Sol. Here, $h_1 = 0.05$ m = 5 cm, u = – 0.5 m = – 50 cm

$$f = -\frac{R}{2} = -\frac{0.2}{2} = -0.1 \text{ m} = -10 \text{ cm}$$

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

∴ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} - \frac{1}{-50}$

$= \frac{-5+1}{50} = \frac{-4}{50}$

or $v = -\frac{50}{4} = -12.5$ cm

Now $m = \frac{h_2}{h_1} = -\frac{v}{u} = \frac{-12.5}{50} = -\frac{1}{4}$

∴ $h_2 = -\frac{1}{4} \times h_1 = -\frac{1}{4} \times 5 = -1.25$ cm

As v is –ve, so a real inverted image of height 1.25 cm is formed at a distance of 12.5 cm from the concave mirror on the same side as the object.

Q. 4. ***A square wire of side 3.0 cm is placed 25 cm away from a concave mirror of focal length 10 cm. What is the area enclosed by the image of the wire? (The centre of the wire is on the axis of the mirror, with its two sides normal to the axis.).***

Sol. Here, u = – 25 cm, f = – 10 cm

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

∴ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{25}$

$= \frac{-5+2}{50} = -\frac{3}{50}$

or $v = \frac{-50}{3}$ cm

Now $m = -\frac{v}{u} = -\frac{50}{3 \times 25} = -\frac{2}{3}$

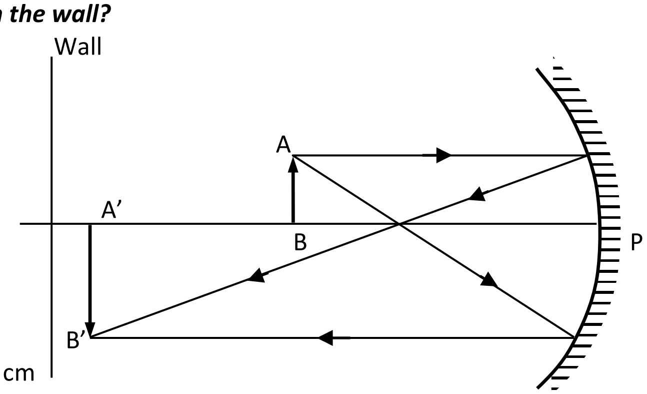
$= \frac{\text{Side of image of wire } (h_2)}{\text{Side of square wire } (h_1)}$

∴ Side of image of wire, h_2

$= -\frac{2}{3} \times h_1 = -\frac{2}{3} \times 3 = -2$ cm

Area enclosed by the image of wire = $(2)^2 = 4$ cm^2

Q. 5. ***A concave mirror of focal length 10 cm is placed at a distance of 35 cm from a wall. How far from the wall should an object be placed to get its image on the wall?***



Sol. Here, f = – 10 cm, v = – 35 cm

From mirror formula,

$\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = -\frac{1}{10} + \frac{1}{35} = -\frac{1}{14}$

∴ Distance of the object from wall

= 35 – 14 = 21 cm

Q. 6. ***An object is placed at a distance of 40 cm on the principal axis of a concave mirror of radius of curvature 30 cm. By how much does the image move if the object is shifted towards the mirror through 15 cm?***

Sol. In first case:

$u = -40$ cm, $R = -30$ cm or $f = -15$ cm

From mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{15} + \frac{1}{40} = -1/24 \text{ or } v = -24 \text{ cm}$$

In second case: The object is shifted towards the mirror by 15 cm, so

$u' = -(40 - 15) = -25$ cm

From mirror formula,

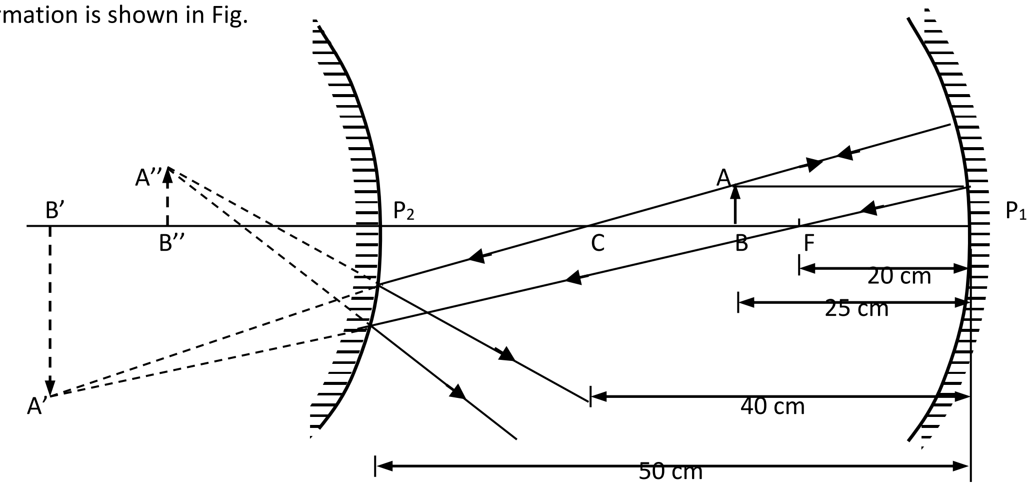
$$\frac{1}{v'} = \frac{1}{f} - \frac{1}{u'} = -\frac{1}{15} + \frac{1}{25} = -\frac{2}{75} \quad \text{or} \quad v' = -37.5 \text{ cm}$$

Distance through which the image shifts

$= -v' - v = -37.5 + 24 = -13.5$ cm i.e., the image shifts 13.5 cm farther from the mirror.

Q. 7. ***An object is placed exactly midway between a concave mirror of radius of curvature 40 cm and a convex mirror of radius of curvature 30 cm. The mirrors face each other and are 50 cm apart. Determine the nature and position of the image formed by successive reflections first at the concave mirror and then at the convex mirror.***

Sol. The image formation is shown in Fig.



(i) For concave mirror, $u_1 = -25$ cm, $f_1 = -20$ cm

For mirror formula,

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100}$$

$\therefore$ $v_1 = -100$ cm

As v_1 is negative, the image of A'B' is real and is formed in front of concave mirror each that $P_1 B' = 100$ cm

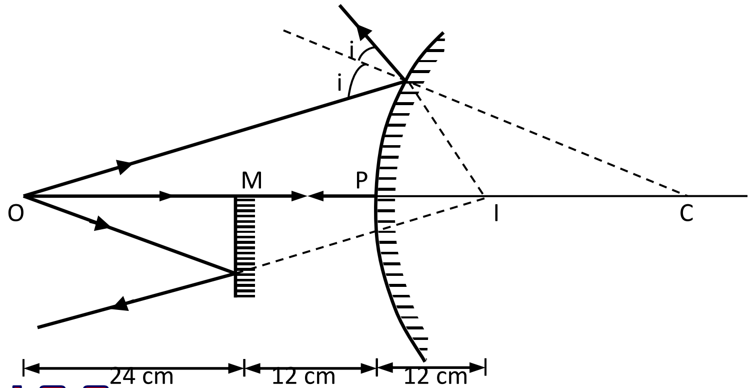
(ii) For convex mirror: The image A'B' acts as virtual object.

$\therefore$ $u_2 = +(100 - 50) = 50$ cm, $f_2 = +50$ cm

Hence $$\frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2} = \frac{1}{15} - \frac{1}{50} = \frac{7}{150} \quad \text{or} \quad v_2 = +21.43 \text{ cm}$$

As v_2 is positive, the final image A"B" is virtual and is formed behind the convex mirror such that $P_2B'' = 21.43$ cm

Q. 8. ***An object is placed at a distance of 36 cm from a convex mirror. A plane mirror is placed in between so that the two virtual images so formed coincide. If the plane mirror is at a distance of 24 cm from the object, find the radius of curvature of the convex mirror.***



Sol. The image I of the object O formed by plane mirror should be at 24 cm behind the mirror or 12 cm behind the convex mirror. For no parallax between the images formed by the two mirrors, the image formed by the convex mirror should also lie at I. Therefore, for convex mirror

$u = OP = -36$ cm; $v = PI = +12$ cm

$\therefore \quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{36} + \frac{1}{12} = \frac{-1+3}{36} = \frac{1}{18}$

or $\quad f = 18$ cm

Radius of curvature of convex mirror = 36 cm

Q. 9. ***An object is kept in front of a concave mirror of focal length 15 cm. The image formed is three times the size of the object. Calculate the two possible distances of the object from the mirror.***

Sol. As the mirror is concave, so $f = -15$ cm

When the image formed is real

$m = \frac{h_2}{h_1} = -\frac{v}{u} = -3 \quad$ or $\quad v = +3u$

As $\quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\therefore \quad \frac{1}{u} + \frac{1}{3u} = -\frac{1}{15}$

or $\quad \frac{4}{3u} = -\frac{1}{15}$

or $\quad u = -\frac{15 \times 4}{3} = -20$ cm

When the image formed is virtual.

$m = \frac{h_2}{h_1} = -\frac{v}{u} = +3 \quad$ or $\quad v = -3u$

As $\quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\therefore \quad \frac{1}{u} - \frac{1}{3u} = -\frac{1}{15} \quad$ or $\quad \frac{2}{3u} = -\frac{1}{15}$

$\therefore \quad u = -\frac{15 \times 2}{3} = -10$ cm

Q. 10. ***When the distance of an object from a concave mirror is decreased from 15 cm to 9 cm, the image gets magnified 3 times than that in first case. Calculate the focal length of the mirror.***

Sol. Magnification, $\quad m = \frac{f}{f-u}$

In first case,

$u = -15$ cm $\quad \therefore \quad m = \frac{f}{f+15}$

In second case,

$u = -9$ cm $\quad \therefore \quad m' = \frac{f}{f+9}$

But $m' = 3m$

or $\quad \frac{f}{f+9} = \frac{3 \times f}{f+15} \quad$ or $\quad f + 15 = 3f + 27 \quad$ or $\quad f = -6$ cm

Q. 11. ***Two objects A and B when placed on after another in front of a concave mirror of focal length 10 cm, form images of same size. Size of object A is 4 times that of B. If object A is placed at a distance of 50 cm from the mirror, what should be the distance of B from the mirror?***

Sol. For object A, $m = \frac{h_2}{h_1} = \frac{f}{f-u_1}$

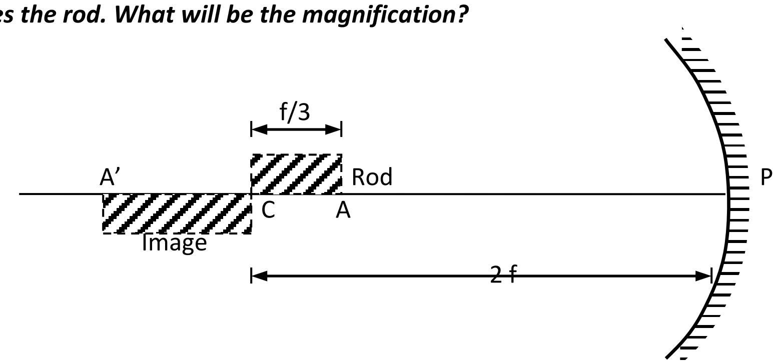
For object B, $m' = \frac{h_2'}{h_1'} = \frac{f}{f-u_2}$

$\therefore \quad \frac{m}{m'} = \frac{h_2}{h_1} \times \frac{h_1'}{h_2'} = \frac{f-u_2}{f-u_1}$

As $h_1 = 4h_1'$, $h_2 = h_2'$, $f = -10$ cm $\quad$ and $\quad u_1 = -50$ cm, therefore,

$\frac{1}{4} = \frac{-10-u_2}{-10+50} \quad$ or $\quad u_2 = -20$ cm

Q. 12. ***A thin rod of length f/3 is placed along the optic axis of a concave mirror of focal length f such that its magic which is real and elongated, just touches the rod. What will be the magnification?***



Sol. The image of the rod placed along the optical axis will touch the rod only when one end of the rod AC is at the centre of curvature of the concave mirror (PC = 2f, AC = f/3). Then the image of the end C of the rod will be formed at the same point C.

For the end A of the rod, we have

$$u = PA = PC - AC = 2f - \frac{f}{3} = \frac{5f}{3}$$

From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f} - \frac{3}{5f} = \frac{2}{5f}$

Thus, the image of A is formed at A' at a distance 5f/2 from the pole P (PA' = 5f/2).

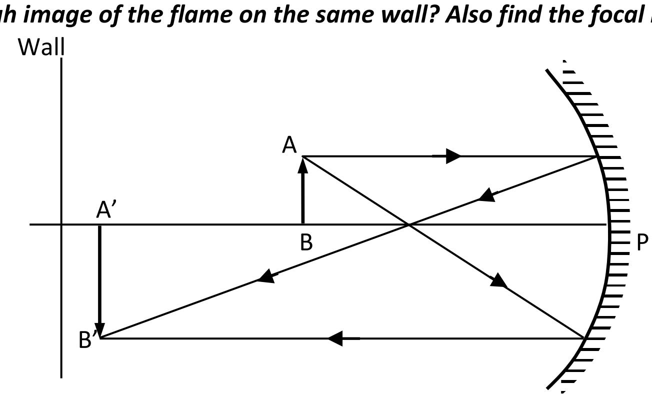
Length of the image $= A'C = PA' - PC = \frac{5f}{2} - 2f = \frac{f}{2}$

$\therefore$ Magnification $= \frac{CA'}{CA} = \frac{f/2}{f/3} = 1.5$

Problems For Practice

Q. 3. ***A candle flame 3 cm high is placed at a distance of 3 m from a wall. How far from the wall must a concave mirror be placed so that it may form 9 cm high image of the flame on the same wall? Also find the focal length of the mirror.***

Sol.



Let BP = x

Then A'P = 3 + x metre

So $u = -x$ m and $v = -(x + 3)$ m

As $m = \frac{h_2}{h_1} = -\frac{v}{u}$ $\therefore$ $\frac{-9\text{ cm}}{3\text{ cm}} = -\frac{x+3}{x}$

or $x = 1.5$ m $\therefore$ $u = -1.5$ m and $v = -4.5$ m

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{1.5} - \frac{1}{4.5} = -\frac{8}{9}$ or $f = -1.225$ m

Q. 5. ***Calculate the distance of an object of height h from a concave mirror of focal length 10 cm, so as to obtain a real image of magnification 2.***

Sol. Here f = – 10 cm and = – 2 for real image

But $m = \frac{f}{f-u}$ $\therefore$ $-2 = \frac{-10}{-10-u}$ or $20 + 2u = -10$ or $u = -15$ cm

Q. 7. ***When an object is placed at a distance of 60 cm from a convex spherical mirror, the magnification produced is 1/2. Where the object should be placed to get a magnification of 1/3?***

Sol. Here $u = -60$ cm

In first case, $m = -\frac{v}{u}$

$\therefore \quad \frac{1}{2} = -\frac{v}{-60}$ or $v = +30$ cm

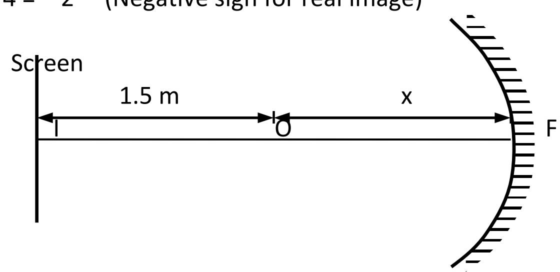
Now $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{60} + \frac{1}{30} = \frac{1}{60}$ or $u = -120$ cm

Q. 8. An object of 1 cm² face area is placed at a distance of 1.5 m from a screen. How far from the object should a concave mirror be placed so that it forms 4 cm² image of object on the screen? Also, calculate the focal length of the mirror.

Sol. Let $u = OP = -x$ cm and $v = IP = -(x + 1.5)$ m

Areal magnification $= \frac{4\ cm^2}{1\ cm^2} = 4$

$\therefore$ Linear magnification $= -\sqrt{4} = -2$ (Negative sign for real image)



As $m = -\frac{v}{u}$ $\therefore$ $-2 = -\frac{-(x + 1.5)}{-x}$ or $x = 1.5$ m

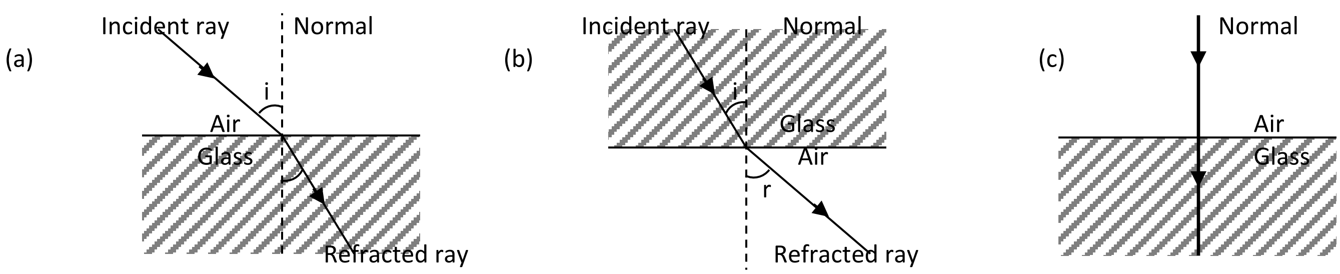
$\therefore \quad u = -1.5$ m, $v = -3$ m

Now $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{1.5} - \frac{1}{3} = -1$

$\therefore \quad f = -1$ m

REFRACTION OF LIGHT

When light travels in the same homogenous medium, it travels along a straight path. However, when it passes obliquely from one transparent medium to another, the direction of its path changes at the interface of the two media. This is called refraction of light.



[Refraction of light (a) from rarer to denser medium (b) from denser to rarer medium (c) no refraction for normal incidence]

The phenomenon of the change in the path of light as it passes obliquely from one transparent medium to another is called refraction of light.

The path along which the light travels in the first medium is called incident ray and that in the second medium is called refracted ray. The angles which the incident ray and the refracted ray make with the normal at the surface of separation are called angle of incident (i) and angle of refraction (r) respectively.

------------ It is observed that:

1. When a ray of light passes from an optically rarer medium to a denser medium, it bends towards the normal ($\angle r < \angle i$), as shown in Fig. (a).

2. When a ray of light passes from an optically denser to a rarer medium, it bends away from the normal ($\angle r > \angle i$), as shown in Fig. (b).

3. A ray of light travelling along the normal passes undeflected, as shown in Fig. (c). Here $\angle i = \angle r = 0°$.

LAWS OF REFRACTION OF LIGHT

The phenomenon of refraction of light obeys the following two laws:

First law: The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in the same plane.

Second law: The ratio of the sine of the angle of incidence and the sine of the angle of refraction is constant for a given pair of media.

Mathematically, sin i = $^1\mu_2$, a constant.

The ratio $^1\mu_2$ is called refractive index of second medium with respect to first medium. The second law was first deduced by a Dutch scientist Willibord Snell in 1621, so it is also known as Snell's law of refraction.

REFRACTIVE INDEX

Refractive index in terms of speed of light:

The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

$$\text{Refractive index} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

or $$\mu = \frac{c}{v}$$

● Refractive index of a medium with respect to vacuum is also called **absolute refractive index.**

Refractive index in terms of wavelength: Since the frequency (ν) remains unchanged when light passes from one medium to another, therefore, $\mu = \frac{c}{v} = \frac{\lambda_{vac} \times \nu}{\lambda_{med} \times \nu} = \frac{\lambda_{vac}}{\lambda_{med}}$

The refractive index of a medium may be defined as the ratio of wavelength of light in vacuum to its wavelength in that medium.

Relative refractive index: The relative refractive index of medium 2 with respect to medium 1 is defined as the ratio of speed of light (v_1) in medium 1 to the speed of light (v_2) in medium 2 and is denoted by $^1\mu_2$.

Thus $$^1\mu_2 = \frac{v_1}{v_2}$$

As refractive index is the ratio of two similar physical quantities, so it has no units and dimensions.

Factors on which the refractive index of a medium depends: These are as follows:

1. Nature of the medium.
2. Wavelength of the light used.
3. Temperature
4. Nature of the surrounding medium.

Refractive index is a characteristic of the pair of the media and also depends on the wavelength of light, but is independent of the angle of incidence.

Conceptual ›:

- **Optical density is a quantity quite different from mass density. Optical density is the ratio of the speed of light in two media while mass density is the mass per unit volume. Interestingly, an optically denser medium may have mass density less than an optically rarer medium. For example, the mass density of turpentine is less than that of water but turpentine is optically denser than water.**

CAUSE OF REFRACTION

Light travels with different speeds in different media. The bending of light or refraction occurs due to the change in the speed of light as it passes from one medium of another. Larger the change in the speed of light as it passes from one medium to another, the more is the bending due to refraction. The Snell's law of refraction may be written as

$$^1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

From the above equation, we can note the following results:

(i) **If $v_1 > v_2$, then $^1\mu_2 > 1$ and sin i > sin r or i > r** i.e., the refracted ray bends towards the normal. The medium 2 is said to be optically denser than medium 1. Hence a ray of light bends towards the normal as it refracts from a rarer medium into a denser medium. (ii) **If $v_1 < v_2$, then $1\mu_2 < 1$ and sin i < sin r or i < r** i.e., the refracted ray bends away from the normal. The medium 2 is said to optically rarer than medium 1. Hence a ray of light bends away from the normal as it refracts from a denser medium into a rarer medium.

Physical significance of refractive index: The refractive index of a medium gives the following two information:

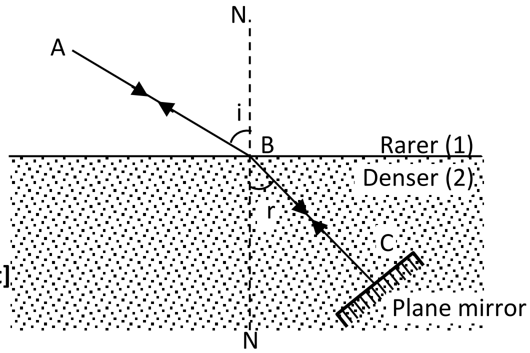
(i) The value of refractive index gives information about the direction of bending of refracted ray. It tells whether the ray will bend towards or away from the normal.

(ii) The refractive index of a medium is related to the speed of light. It is the ratio of the speed of light in vacuum to that in the given medium. For example, refractive index of glass is 3/2. This indicates that the ratio of the speed of light in glass to that in vacuum is 2: 3 or the speed of light in glass is two-third of its speed in vacuum.

PRINCIPLE OF REVERSIBILITY OF LIGHT

This principle states that if the final path of a ray of light after it has suffered several reflections and refractions is reversed, it retraces its path exactly.

[Principle of reversibility of light]



As shown in Fig., consider a ray of light AB incident on a plane surface XY, separating rarer medium 1 (air) from denser medium 2 (water). It is refracted along BC.

Let angle of incidence, ∠ABN = i

and angle of refraction, ∠ CBN' = r

From Snell's law of refraction

$$\frac{\sin i}{\sin r} = {}^1\mu_2 \quad \dots (1)$$

Suppose a plane mirror is placed perpendicular to the path of ray BC. This reverses the beam along its own path. Therefore, for the reversed ray, we have

Angle of incidence, ∠CBN' = r

Angle of refraction, ∠ABN = i

Again, from Snell's law

$$\frac{\sin r}{\sin i} = {}^2\mu_1 \quad \dots (2)$$

Multiplying equations (1) and (2), we get

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = {}^1\mu_2 \times {}^2\mu_1$$

or $\mathbf{1 = {}^1\mu_2 \times {}^2\mu_1}$ **or** $\mathbf{{}^1\mu_2 = \dfrac{1}{{}^2\mu_1}}$

Thus, the refractive index of medium 2 with respect to medium 1 is reciprocal of the refractive index of medium 1 with respect to medium 2.

REFRACTION THROUGH A RECTANGULAR GLASS SLAB AND LATERAL SHIFT

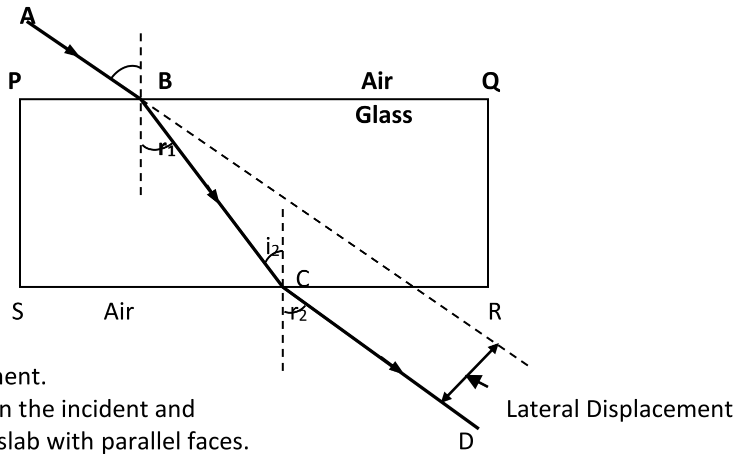
Consider a rectangular glass slab PQRS, as shown in Fig. A ray AB is incident on the face PQ at an angle of incidence i_1. On entering the glass slab, it bends towards normal and travels along BC at an angle of refraction r_1. The refracted ray BC is incident on face SR at an angle of incidence i_2. The emergent ray CD bends away from the normal at an angle of refraction r_2.

Using Snell's law for refraction at face PQ,

$$\frac{\sin i_1}{\sin r_1} = {}^a\mu_g \quad \dots (1)$$

For refraction at face SR,

$$\frac{\sin i_2}{\sin r_2} = {}^g\mu_a = \frac{1}{{}^a\mu_g} \quad \dots (2)$$



Multiplying (1) and (2), we get

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1$$

As PQ ∥ SR, therefore, $i_2 = r_1$; hence

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = 1$$

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = 1$$

or $\quad \sin i_1 = \sin r_2 \quad$ or $\quad i_1 = r_2$

Thus, the emergent ray CD is parallel to the incident ray AB, but it has been laterally displaced with respect to the incident ray. This shift in the path of light on emerging from a refracting medium with parallel faces is called lateral displacement.

Hence lateral shift is the perpendicular distance between the incident and an emergent ray, when light is incident obliquely on a refracting slab with parallel faces.

●**Expression for lateral displacement:** Fig. shows the path of the ray undergoing refraction through the slab PQRS. Let t be the thickness of the slab and x, the lateral displacement of the emergent ray. Then from right Δ BEC, we have

$$\frac{x}{BC} = \sin(i - r)$$

or $\quad x = BC \sin(i - r)$

From right ΔBFC, we have

$$\frac{BF}{BC} = \cos r \quad \text{or} \quad BC = \frac{BF}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad x = \frac{t}{\cos r} \sin(i - r) \qquad \text{... (1)}$$

$$= \frac{t\,[\sin i \cos r - \cos i \sin r]}{\cos r}$$

$$= t\left(\sin i = \frac{\cos i \sin r}{\cos r}\right)$$

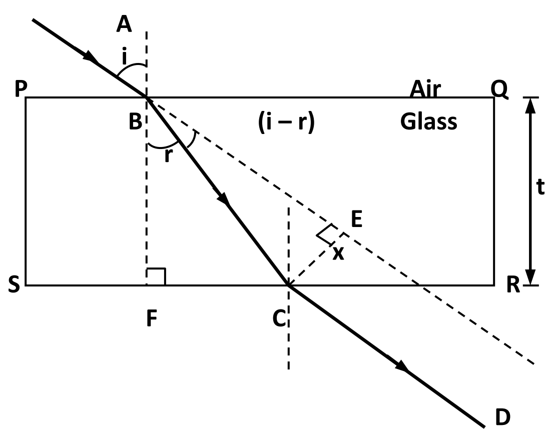
From Snell's law, $\mu = \frac{\sin i}{\sin r}$

or $\quad \sin r = \frac{\sin i}{\mu}$

and $\quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$

Hence $\quad x = t\left[\sin i - \frac{\cos i \,.\, \sin i}{\mu\left(1 - \frac{\sin^2 i}{\mu^2}\right)^{1/2}}\right]$

or $\quad x = t \sin i\left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}}\right] \qquad \text{... (2)}$



[Calculation of lateral displacement]

Clearly, x tends to a maximum value when $i \to 90°$, so that $\sin i \to 1$ and $\cos i \to 0$. Thus

$$x_{mas} = t \sin 90° = t$$

i.e., the displacement of the emergent ray cannot exceed the thickness of the glass slab.

From equation (1), it may be noted that the lateral shift produced by a glass slab increases with

(i) The increase in the thickness of the glass slab,

(ii) The increase in the value of the angle of incidence, and

(iii) The increase in the value of the refractive index of the slab.

REFRACTION THROUGH A COMBINATION OF MEDIA

$${}^1\mu_2 \times {}^2\mu_3 \times 3\mu_1 = 1$$

Refraction through a combination of media: Fig. shows the refraction of a ray of light from air (1) to water (2), glass (3) and finally to air. As all boundaries are parallel planes, emergent ray is parallel to the incident ray. Thus, the angle of emergence is equal to the angle of incidence.

For a ray going from medium 1 to medium 2,

$$^{1}\mu_2 = \frac{\sin i}{\sin r_1}$$

For a ray going from medium 2 to medium 3,

$$^{2}\mu_3 = \frac{\sin r_1}{\sin r_2}$$

For a ray going from medium 3 to medium 1,

$$^{3}\mu_1 = \frac{\sin r_2}{\sin i}$$

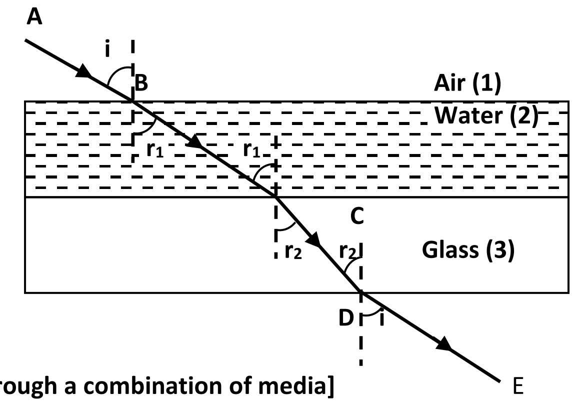
Multiplying the above three equations, we get

$$^{1}\mu_2 \times {}^{2}\mu_3 \times {}^{3}\mu_1 = 1$$

Moreover, $$^{1}\mu_3 = \frac{1}{^{1}\mu_2 \times {}^{3}\mu_1}$$

or $$^{2}\mu_3 = \frac{^{1}\mu_3}{^{1}\mu_2} \qquad \left(\because\ {}^{3}\mu_1 = \frac{1}{^{1}\mu_3}\right)$$

or $$^{w}\mu_g = \frac{^{a}\mu_g}{_{a}\mu_w}$$



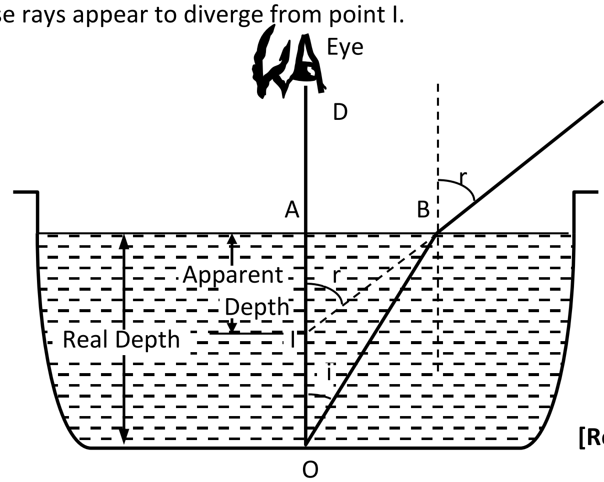
[Refraction through a combination of media]

Thus, by knowing the refractive indices of any two media like glass and water with respect to air, the refractive index of glass with respect to water or vice versa can be calculated.

PRACTICAL APPLICATIONS OF REFRACTION

Real and apparent depths: It is on account of refraction of light that the apparent depth of an object placed in denser medium is less than the real depth.

Fig. shows a point object O placed at the bottom of a beaker filled with water. The rays OA and OB starting from O are refracted along AD and BC, respectively. These rays appear to diverge from point I.



[Real and apparent depths]

So, I is the virtual image of O. Clearly, the apparent depth AI is smaller than the real depth AO. That is why a water tank appears shallower or an object placed at the bottom appears to be raised.

From Snell's law, we have

$$^{\omega}\mu_a = \frac{\sin i}{\sin r} = \frac{\sin \angle AOB}{\sin \angle AIB} = \frac{AB/BO}{AB/BI} = \frac{BI}{BO}$$

As the size of the pupil is small, the ray BC will enter the eye only if B is close to A. Then

$$BI \simeq AI \text{ and } BO \simeq AO$$

$$\therefore \quad ^{a}\mu_\omega = \frac{1}{^{\omega}\mu_a} = \frac{AO}{AI}$$

or $$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

or $$\text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index}}$$

As the refractive index of any medium (other than vacuum) is greater than unity, so the apparent depth is less than the real depth.

Normal shift: The height through which an object appears to be raised in a denser medium is called normal shift. Clearly

$$\text{Normal shift} = \text{Real depth} - \text{Apparent depth}$$

or $$d = AO - AI = AO - \frac{AO}{\mu}$$

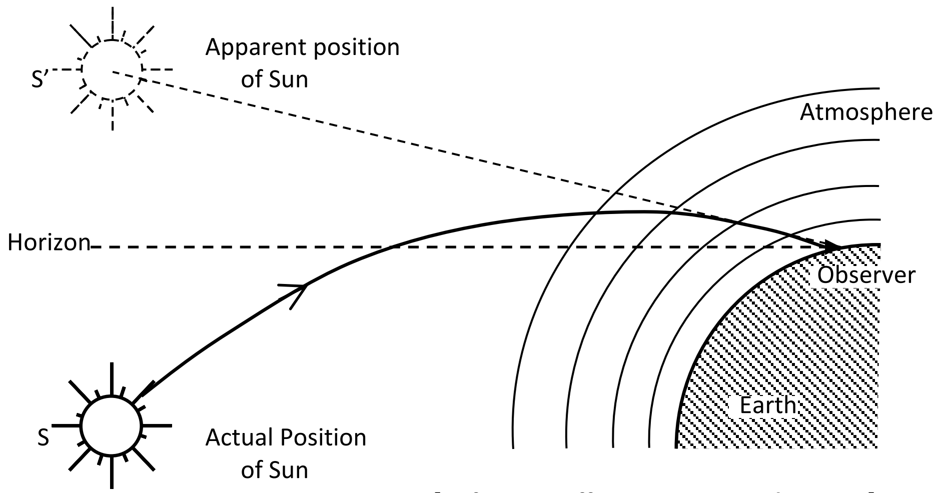
$= AO\left(1 - \frac{1}{\mu}\right)$

or $d = t\left(1 - \frac{1}{\mu}\right)$

Clearly, the normal shift in the position of an object when seen through a denser medium depends on two factors:
1. The real depth of the object or the thickness (t) of the refracting medium.
2. The refractive index of the denser medium. The higher the value of μ, greater is the apparent shift 'd'

.

Apparent shift in the position of the sun at sunrise and sunset: Due to the atmosphere refraction, the sun is visible before actual sunrise and after actual sunset.



[Refraction effect at sunset and sunrise]

With altitude, the density and hence refractive index of air-layers decreases. The light rays starting from the sun S travel from rarer to denser layers. They bend more and more towards the normal.

However, an observer sees an object in the direction of the rays reaching his eyes. So to an observer standing on the earth, the sun which is actually in a position S below the horizon, appears in the position S', above the horizon. The apparent shift in the direction of the sun is by about 0.5°. Thus the sun appears to rise early by about 2 minutes and for the same reason, it appears to set late by about 2 minutes and for the same reason, it appears to set late by about 2 minutes. This increases the length of the day by about 4 minutes.

Apparent flattening of the sun at sunrise and sunset: The sun the horizon appears flattened. This is due to atmospheric refraction. The density and refractive index of the atmosphere decrease with altitude, so the rays from the top and bottom portions of the sun on the horizon are refracted by different degrees. This causes the apparent flattening of the sun. But the rays from the sides of the sun on a horizontal plane are generally refracted by the same amount, so the sun still appears circular along its sides.

Examples based on (i) Refraction of Light (ii) Lateral shift and (iii) Real and Apparent Depths

Formulae used

1. Refractive index = $\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

   or $\mu = \frac{c}{v}$

2. $\mu = \frac{\text{Wavelength in vacuum}}{\text{Wavelength in medium}} = \frac{\lambda}{\lambda'}$

3. Snell's law, ${}^1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$

   or $\mu_1 \sin i = \mu_2 \sin r$

4. ${}^1\mu_2 = \frac{1}{{}^2\mu_1}$

5. ${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$ or ${}^2\mu_3 = \frac{{}^1\mu_3}{{}^1\mu_2}$

6. Lateral shift of a ray through a rectangular slab,

   $x = \frac{t}{\cos r}\sin(i - r)$

$= t \sin i \left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}}\right]$

7. $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{t}{\text{Apparent depth}}$

Apparent depth $= \frac{t}{\mu}$

8. Apparent shift $= t\left(1 - \frac{1}{\mu}\right)$

9. Total apparent shift for compound media

$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + ...$

Units used All distances are in metre, angles in degrees and refractive index μ has no units.

Q. 1. ***A ray of light of frequency 5×10^{14} Hz is passed through a liquid. The wavelength of light measured inside the liquid is found to be 450×10^{-9} m. Calculate the refractive index of the liquid.***

Sol. Here $v = 5 \times 10^{14}$ Hz, $\lambda = 450 \times 10^{-9}$ m,

$c = 3 \times 10^8$ ms^{-1}

Refractive index of the liquid,

$\mu = \frac{c}{v} = \frac{c}{v\lambda}$

$= \frac{3 \times 10^8}{5 \times 10^{14} \times 450 \times 10^{-9}} = 1.33$

Q. 2. ***A light of wavelength 6000 Å in air, enters a medium with refractive index 1.5. What will be the wavelength of light in that medium?***

Sol. In air, λ = 6000 Å = 6×10^{-7} m, $c = 3 \times 10^8$ ms^{-1}

Refractive index of the medium, $\mu = 1.5$

When light travels from air to the refracting medium, its frequency remains unchanged.

$\therefore$ $v' = v = \frac{c}{\lambda} = \frac{3 \times 10^8}{6 \times 10^{-7}} = 5 \times 10^{14}$ Hz

Wavelength of light in the medium,

$\lambda = \frac{\lambda}{\mu} = \frac{6000\ \text{Å}}{1.5} = 4000$ Å

Q. 3. ***The refractive index of glass is 1.5 and that of water is 1.3. If the speed of light in water is 2.25×10^8 ms^{-1}, what is the speed of light in glass?***

Sol. Here $^a\mu_g = \frac{c}{v_g} = 1.5$

and $^a\mu_w = \frac{c}{v_\omega} = 1.3$

$\therefore$ $\frac{c}{v_\omega} \times \frac{v_g}{c} = \frac{1.3}{1.5}$

or $v_g = \frac{1.3}{1.5} \times v_\omega = \frac{1.3}{1.5} \times 2.25 \times 10^8$ $= 1.95 \times 10^8$ ms^{-1}

Q. 4. ***A ray of light passes through a plane boundary separating two media whose refractive indices are $\mu_1 = 3/2$ and $\mu_2 = 4/3$. (i) If the ray travels from medium 1 to medium 2 at an angle of incidence of 30°, what is the angle of refraction? (ii) If the ray travels from medium 2 to medium 1 at the same angle of incidence, what is the angle of refraction?***

Sol. Here $\mu_1 = \frac{3}{2}$, $\mu_2 = \frac{4}{3}$, $i = 30°$

(i) When the ray travels from medium 1 to medium 2,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

or $\frac{\sin 30°}{\sin r} = \frac{4/3}{3/2} = \frac{8}{9}$

or $\sin r = \frac{9}{8} \times \sin 30° = \frac{9}{8} \times \frac{1}{2} = 0.5625$

$\therefore$ $r = 34° 14'$

(ii) When the ray travels from medium 2 to medium 1,

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

or $\frac{\sin 30^\circ}{\sin r} = \frac{3/2}{4/3} = \frac{9}{8}$

or $\sin r = \frac{8}{9} \times \sin 30^\circ = \frac{8}{9} \times \frac{1}{2} = 0.4445$

∴ $r = 26^\circ\ 24'$

Q. 5. ***A rectangular glass slab rests in the bottom of a trough of water. A ray of light incident on water surface at an angle of 50° passes through water into glass. Calculate the angle of refraction in glass. Given that μ for water is 4/3 and that for glass is 3/2.***

Sol. Here $^a\mu_\omega = \frac{4}{3}$, $^a\mu_g = \frac{3}{2}$

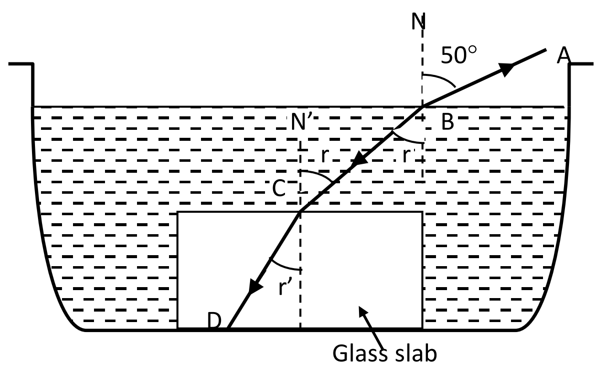
∴ $^\omega\mu_g = \frac{^a\mu_g}{^a\mu_\omega} = \frac{3/2}{4/3} = \frac{9}{8}$

Angle of incidence on water surface, i = 50°

∴ $\frac{\sin 50^\circ}{\sin r} = \frac{4}{3}$

∴ $\sin r = \frac{3}{4} \sin 50^\circ$

$= \frac{3}{4} \times 0.766 = 0.5745$



∴ Angle of refraction, r = 35.06°

For refraction at water-glass interface, we have

$\frac{\sin 35.06^\circ}{\sin r'} = \frac{9}{8}$

or $\sin r' = \frac{8}{9} \times 0.5745 = 0.5107$

∴ $r' = 30.7^\circ$

Q. 6. ***A ray of light is incident at an angle of 60° on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5. Calculate the lateral shift produced.***

Sol. Here i = 60°, μ = 1.5, t = 0.1 m

By Snell's law, $\mu = \frac{\sin i}{\sin r}$

∴ $\sin r = \frac{\sin i}{\mu} = \frac{\sin 60^\circ}{1.5}$

$= \frac{0.866}{1.5} = 0.5773$

or $r = 35^\circ\ 16'$

Lateral shift produced,

$x = \frac{t}{\cos r} \sin (i - r)$

$= \frac{0.1}{\cos 35^\circ\ 16'} \sin (60^\circ - 35^\circ 16')$

$= \frac{0.1}{\text{Cos } 35^\circ 16'} \times \sin 24^\circ\ 44'$ $= \frac{0.1 \times 0.4184}{0.8164}$ $= 0.0513$ m.

Q. 7. ***The apparent depth of an object at the bottom of tank filled with a liquid of refractive index 1.3 is 7.7 cm. What is the actual depth of the liquid in the tank?***

Sol. Refractive index,

$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

∴ $1.3 = \frac{\text{Real depth}}{7.7}$

Hence, Real depth = 1.3 × 7.7 cm = 10.01 cm

Q. 8. ***The velocity of light in glass is 2×10^8 ms^{-1} and that in air is 3×10^8 ms^{-1}. By how much would an ink dot appear to be raised, when covered by a glass plate 6.0 cm thick?***

Sol. Here

$v = 2 \times 10^8 \text{ ms}^{-1}, \quad c = 3 \times 10^8 \text{ ms}^{-1}$

Refractive index of glass,

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

Real depth = 6.0 cm

∴ $$\text{Apparent depth} = \frac{\text{Real depth}}{\mu} = \frac{6.0}{1.5} = 4.0 \text{ cm}$$

Distance through which the ink dot appears to be raised = 6.0 – 4.0 = 2.0 cm

Q. 9. ***A mark is made on the bottom of a beaker and a microscope is focussed on it. The microscope is raised through 1.5 cm. To what height water must be poured into the beaker to bring the mark again into focus? Given that μ for water is 4/3.***

Sol. Here apparent shift, d = 1.5 cm 22

Let t be the height through which water must be poured into the beaker. Then

$$d = t\left(1 - \frac{1}{\mu}\right)$$

∴ $$1.5 = t\left(1 - \frac{1}{4/3}\right)$$

or $t = 1.5 \times 4 = 6.0$ cm

Q. 10. ***The bottom of a container is a 4.0 cm thick glass (μ = 1.5) slab. The container contains two immiscible. Liquids A and B of depths 6.0 cm and 8.0 cm respectively. What is the apparent position of a scratch on the outer surface of the bottom of the glass slab when viewed through the container? Refractive indices of A and B are 1.4 and 1.3 respectively.***

Sol. The total apparent shift in the position of the image due to all the three media is given by

$$d = t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + t_3\left(1 - \frac{1}{\mu_3}\right)$$

Given t_1 = 4.0 cm, t_2 = 6.0 cm, t_3 = 8.0 cm

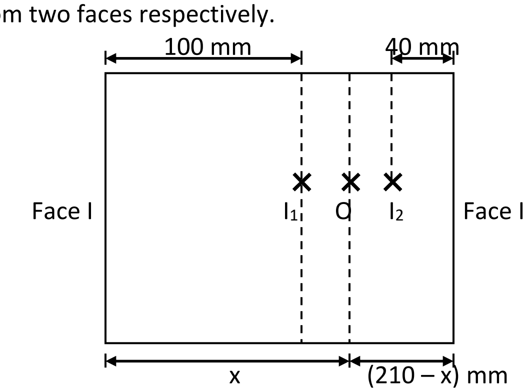
$\mu_1 = 1.5$, $\mu_2 = 1.4$, $\mu_3 = 1.3$

∴ $$d = 4.0\left(1 - \frac{1}{1.5}\right) + 6.0\left(1 - \frac{1}{1.4}\right) + 8.0\left(1 - \frac{1}{1.3}\right)$$

$= 1.33 + 1.71 + 1.85 = 4.89$ cm

Q. 11. ***A transparent cube of side 210 mm contains a small dir bubble. It apparent distance, when viewed through one face of the cube is 100 mm and when viewed through the opposite face is 40 mm. What is the actual distance of the bubble from the second face and what is the refractive index of the material of the cube?***

Sol. The situation is shown in Fig. Let O be the air bubble inside the transparent cube at distance x from face I, then its distance from face II will be (210 – x) mm. Let I_1 and I_2 be its images as seen from two faces respectively.



For face I:

Real depth = x mm, apparent depth = 100 mm

∴ $\mu = x$ ... (i)

For face II:

Real depth = (210 – x) mm,

Apparent depth = 40 mm

∴ $$\mu = \frac{210 - x}{40} \quad \text{... (ii)}$$

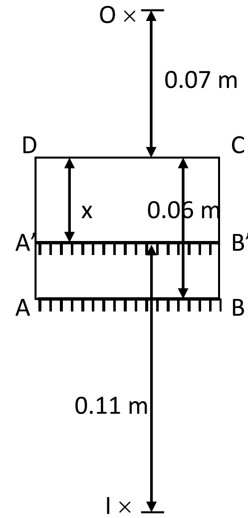
From equations (i) and (ii), we have

$$\frac{x}{100} = \frac{210 - x}{40} \quad \text{or} \quad x = 150 \text{ mm}$$

∴ Actual distance of the bubble from face II = 210 – 15 = 60 mm

Also, $$\mu = \frac{x}{100} = \frac{150}{100} = 1.50$$

Q. 12. ***One face of a glass cube of side 0.06 m is silvered. An object is placed at a distance of 0.07 m from the face opposite to the silvered face. Looking from the object side, the image of the object appears to be 0.11 m behind the silvered face. Calculate the refractive index of the material of the glass.***



Sol. The situation is shown in Fig. AB is silvered face of the glass cube of side 0.06 m. Object O is placed at distance of 0.07 m from face CD, the face opposite to silvered face AB. When seen from the object side, the silvered face AB appears at the raided position A'B'. Suppose the apparent silvered face A'B'. Suppose the apparent silvered face A'B' lies at depth x below the face CD. As the image I of the object O appears at 0.11 m behind the silvered face A'B',

therefore, $0.07 + x = 0.11$

or $x = 0.11 - 0.07 = 0.04$ m

∴ Real depth of silvered face = DA = 0.06 m

Apparent depth of silvered face = DA' = 0.04 m

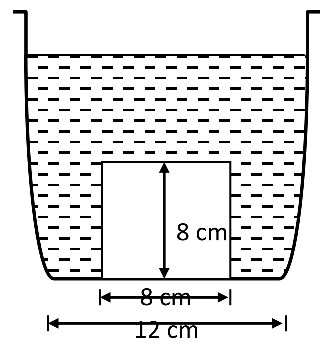
$$\mu = \frac{DA}{DA'} = \frac{0.06}{0.04} = 1.5$$

Q. 13. ***A cylindrical vessel of diameter 12 cm contains 800 π cm³ of water. A cylindrical glass piece of diameter 8.0 cm and height 8.0 cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays (Fig.), locate its image. The index of refraction of glass is 1.50 and that of water is 1.33.***

Sol. Volume of water = 800π cm³

Volume of cylindrical glass piece = $\pi\left(8/2\right)^2 \times 8 = 128\pi$ cm³

23



Total volume of water and glass piece = $800\pi + 128\pi = 928\pi$ cm³

Height of water level from the bottom = $\frac{\text{Volume}}{\pi r^2} = \frac{928\pi}{\pi \times (6)^2} = 25.78$ cm

∴ Depth of water above the glass piece = 25.78 – 8.0 = 17.78 cm

Total apparent shift of the bottom

$$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$$

$$= 17.78\left(1 - \frac{1}{1.33}\right) + 8.0\left(1 - \frac{1}{1.50}\right)$$

$$= 4.44 + 2.66 = 7.1 \text{ cm}$$

Thus, the image is seen at 7.1 cm above the bottom.

Problems For Practice

Q. 1. ***A film of oil of refractive index 1.20, lies on water of refractive index 1.33. A light ray is incident at 30° in the oil on the oil-water boundary. Calculate the angle of refraction in water.***

Sol. Refractive index of water relative to oil is

$$^{o}\mu_w = \frac{^{a}\mu_w}{^{a}\mu_o} = \frac{1.33}{1.20} = 1.11$$

From Snell's law, $^{o}\mu_w = \frac{\sin i}{\sin r}$

∴ $1.11 = \frac{\sin 30°}{\sin r}$ or $\sin r = \frac{1}{2 \times 1.11} = 0.45$

or $r \approx 27°$

Q. 2. ***A printed page is kept pressed by a glass cube (μ = 1.5) of edge 6.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top?***

Sol. Normal shift,

$$d = t\left(1 - \frac{1}{\mu}\right) = 6.0\left(1 - \frac{1}{1.5}\right) = 2.0 \text{ cm}$$

Q. 3. ***A travelling microscope is focussed on a mark made on a paper. When a slab of 1.47 cm thickness is placed on the mark, the microscope has to be raised through 0.49 cm to focus the mark again. Calculate the refractive index is glass.***

Sol. As $d = t\left(1 - \frac{1}{\mu}\right)$ $\therefore$ $0.49 = 1.47\left(1 - \frac{1}{\mu}\right)$

or $\frac{1}{\mu} = 1 - \frac{1}{3} = \frac{2}{3}$ or $\mu = 1.5$

Q. 4. ***The velocity of light in a transparent medium is 1.8×10^8 ms^{-1}, while that in vacuum is 3×10^8 ms^{-1}. Find by how much the bottom of the vessel containing the liquid appears to be raised if the depth of the liquid is 0.25 m.***

Sol. Here $v = 1.8 \times 10^8$ ms^{-1}, $c = 3 \times 10^8$ ms^{-1}

$\therefore$ Refractive index, $\mu = \frac{c}{v} = \frac{3 \times 10^8}{1.8 \times 10^8} = \frac{5}{3}$

Real depth = 0.25 m

$\therefore$ Apparent depth = $\frac{\text{Real depth}}{\mu} = \frac{0.25}{5/3} = 0.15$ m

Distance through which bottom appears to be raised = 0.25 – 0.15 = .1 m

Q. 5. ***Calculate the index of refraction of a liquid from the following into glass:***
(a) Reading for the bottom of an empty beaker: 11.324 cm (b) Reading for the bottom of the beaker, when partially filled with the liquid: 11.802 cm (c) Reading for the upper level of the liquid in the beaker: 12.895 cm

Sol. Real depth = Reading from the upper level of the liquid – Reading from the bottom of the empty beaker
= 12.895 – 11.324 = 1.571 cm

Apparent depth = Reading from the upper level of the liquid – Reading from the bottom of the beaker when partially filled with liquid
= 12.895 – 11.802 = 1.093 cm

$\mu = \frac{1.571}{1.093} = 1.437$

Q. 6. ***While determining the refractive index of a liquid experimentally, the microscope was focussed at the bottom of a beaker, when its reading was 3.965 cm. On pouring liquid upto a height 2.537 cm inside the beaker, the reading of the refocused microscope was 3.348 cm. Find the refractive index of the liquid.***

Sol. Real depth = 2.537 cm

Apparent shift in the position of bottom of the beaker = 3.965 = 3.348 = 0.617 cm

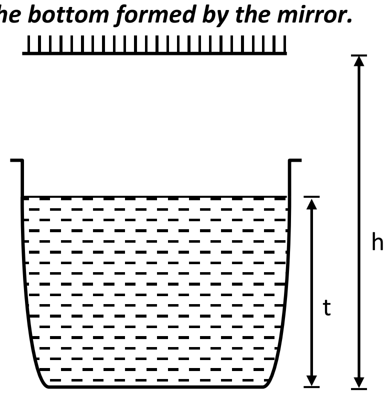
Apparent depth = 2.537 – 0.617 = 1.920 cm

$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{2.537}{1.920} = 1.321$

Q. 7. ***A vessel contains water upto a height of 20 cm and above it an oil upto another 20 cm. The refractive indices of water and oil are 1.33 and 1.30 respectively. Find the apparent depth of vessel when viewed from above.***

Sol. Total apparent shift = $t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$

Q. 8. ***In Fig. a plane mirror lies at a height h above the bottom of a beaker containing water (refractive index μ) upto a height t. Find the position of the image of the bottom formed by the mirror.***



Sol. Apparent shift in the bottom of the beaker is

$d = t\left(1 - \frac{1}{\mu}\right)$

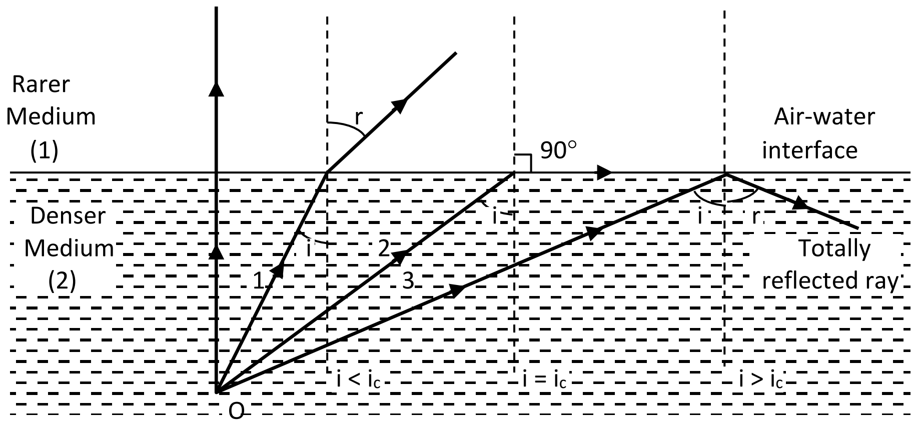
Apparent distance of the bottom from the mirror

$= h - d = h - t\left(1 - \frac{1}{\mu}\right) = h - t + \frac{t}{\mu}$

Hence the image is formed behind the mirror at distance of $h - t + \frac{t}{\mu}$

TOTAL INTERNAL REFLECTION

If light passes from an optically denser medium to a rarer medium, then at the interface, the light is partly reflected back into the denser medium and partly refracted to the rarer medium. This reflection is called internal reflection. Under certain conditions, the whole of the incident light can be made to be reflected back into the denser medium. This gives rise to an interesting phenomenon called total internal reflection.



[Total internal reflection]

when a ray of light (ray 1) travels at a small angle of incident from a denser medium to a rarer medium, say from water to air, the refracted ray bends away from the normal so that the angle of refraction is greater than the angle of corresponding angle of incidence increases, the corresponding angle of refraction also increases. Then for a certain angle of incidence (ray 2), the angle of refraction becomes 90°, i.e., the refracted ray goes along the surface of separation.

The angle of incidence in a denser medium for which the angle of refraction in the rarer medium is 90° is called critical angle of the denser medium and is denoted by i_c.

If the angle of incidence is increased beyond i_c (ray 3), no light is refracted into the rarer medium (since the angle of refraction cannot be greater than 90°), but whole of it is reflected back into the denser medium in accordance with the laws of reflection. This phenomenon is known as total internal reflection.

The phenomenon in which a ray of light travelling at an angle of incidence greater than the critical angle from denser to a rarer medium is totally reflected back into the denser medium is called total internal reflection.

Necessary conditions for total internal reflection:

1. Light must travel from an optically denser to an optically rarer medium.
2. The angle of incidence in the denser medium must be greater than the critical angle for the two media.

Relation between critical angle and refractive index. From Snell's law,

$$\frac{\sin i}{\sin r} = {}^2\mu_1 = \frac{1}{{}^1\mu_2}$$

When $i = i_c$, $r = 90°$. Therefore,

$$\frac{\sin i_c}{\sin 90°} = \frac{1}{{}^4\mu_2} \quad \text{or} \quad {}^1\mu_2 = \frac{1}{\sin i_c}$$

If the rarer medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$ (say) and we get

$\mu = 1/\sin i_c$ Thus the refractive index of any medium is equal to the reciprocal of the sine of its critical angle.

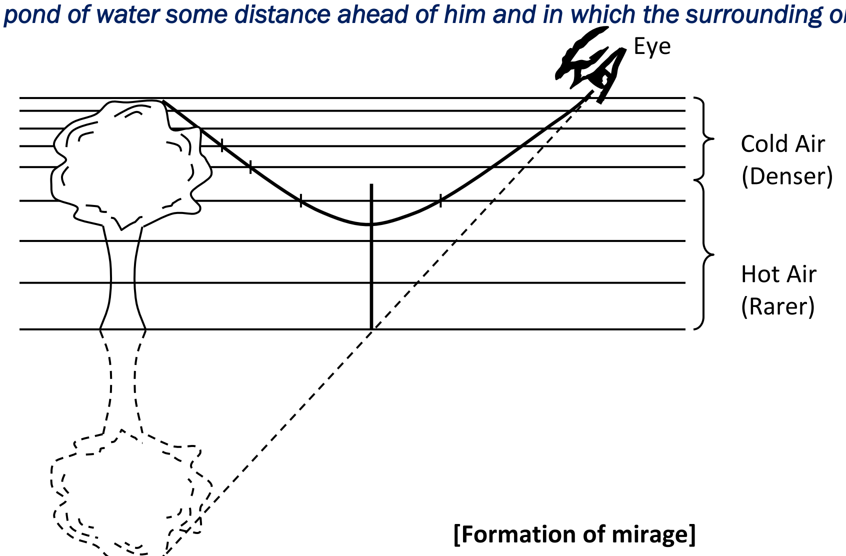
CRITICAL ANGLES OF SOME TRANSPARENT MEDIA

Substance	*Refractive index*	*Critical angle*
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.65	37.31°
Diamond	2.42	24.41°

APPLICATIONS OF TOTAL INTERNAL REFLECTION

1. **Sparkling of diamond**: The brilliancy of diamond is due to total internal reflection. As the refractive index of diamond is very large, its critical angle is very small, about 24.4°. The faces of diamond are so cut that the light entering the crystal suffers total internal reflections repeatedly and hence gets collected inside but it comes out through only a few faces. Hence the diamond sparkles when seen in the direction of emerging light.

2. Mirage: *It is an optical illusion observed in deserts or over hot extended surfaces like a coal-tarred road, due to which a traveller sees a shimmering pond of water some distance ahead of him and in which the surrounding objects like trees, etc, appear inverted.*



[Formation of mirage]

On a hot summer day, the surface of the earth becomes very hot. The layers of air near the earth are more heated than the higher ones. Hence the density and refractive index of air layers increase as we move high up. As the rays of light from a distance object like a tree travel towards the earth through layers of decreasing refractive index, they bend more and more away from the normal. A state is reached when the angle of incidence becomes greater than the critical angle, the rays are totally reflected. These rays then move up through layers of increasing refractive index and therefore undergo refraction in a direction opposite to that in the first case. These rays reach the observer's eyes and he sees an inverted image of the object, as if formed in a pond of water.

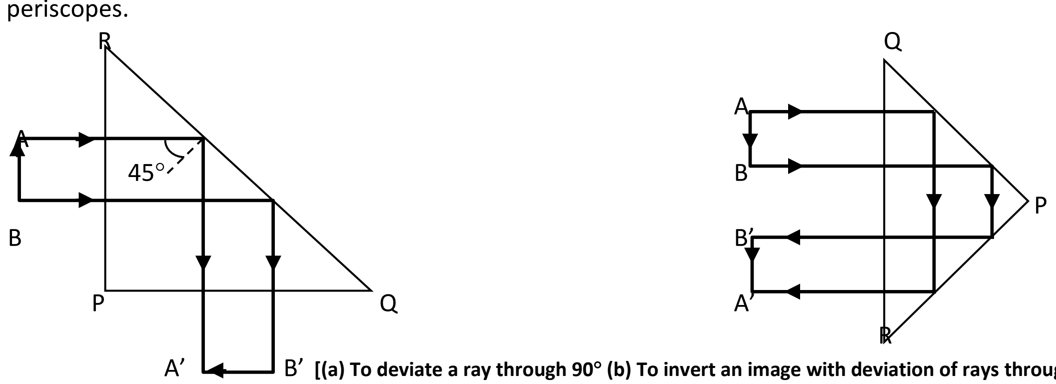
3. Totally reflecting prisms: 4. Optical fibres:

TOTALLY REFLECTING PRISMS

A right-angled isosceles prism, i.e., a 45° - 90° - 45° prism is called a totally reflecting prism. Whenever a ray falls normally on any face of such a prism, it is incident on the inside face at 45° that is at an angle greater than the critical angle of glass (about 42°); hence this ray is always totally internally reflected.

These prisms may be used in three ways:

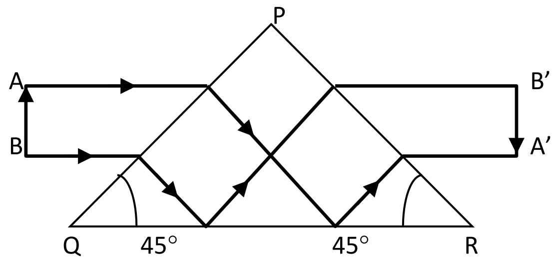
(i) To deviate a ray through 90°: As shown in Fig. (a), as the light is incident normally on one of the faces containing right angle, it enters the prism without deviation. It is incident on the hypotenuse face at an angle of 45°, greater than the critical angle. The light is totally internally reflected. Having been deviated through 90°, the light passes through third face without any further deviation. Such prisms are used in periscopes.



[(a) To deviate a ray through 90° (b) To invert an image with deviation of rays through 180°]

(ii) To invert an image with deviation of rays through 180°: As shown in Fig (b), the light is incident normally on the hypotenuse face, it first suffers total internal reflection from one shorter face and then from the other shorter face. The final beam emerges through the hypotenuse face, parallel to the incident beam. The deviation is 180°, Such a prism is called a proprium

(iii) To invert an image without deviation of rays (Erecting prism): As shown in Fig., the light enters at one shorter face at an angle. After refraction, it is totally reflected from the hypotenuse face and then refracted out of the other shorter face to become parallel to the incident beam. The rays do not suffer any deviation, only their order is reversed. The incident ray, which is on the top, emerges from the bottom of the prism. Such prisms are called erecting prisms and are used in binoculars and in projection lanterns.



[To invert an image without deviation of rays]

Advantages of totally reflecting prisms over plane mirrors: The totally reflecting prisms have many advantages over plane mirrors are reflectors:

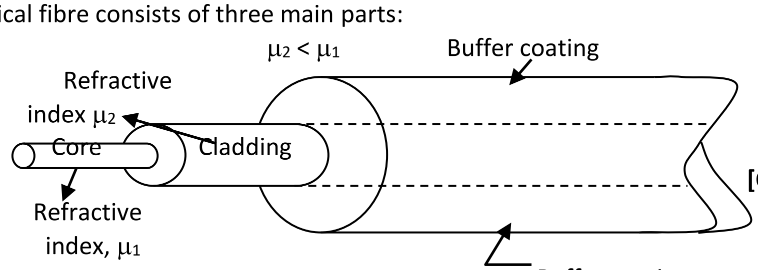
1. In prisms, the light is totally reflected, while there is always some loss of intensity in case of plane mirrors.
2. The reflecting properties of prisms are permanent, while these are affected by tarnishing in case of plane mirrors.
3. No multiple images are formed in prisms, while a plane mirror forms a number of faint images in addition to a prominent image.

OPTICAL FIBRES

These days we find in the market some decorative lamps provided with fine plastic fibres. At their one ends; the fibres are fixed over an electric lamp while their free ends form a fountain like structure. When the lamp is switched on, the light travels from the bottom of each fibre and appears at the tip of free and as a bright dot of light. The plastic fibres in these lamps are optical fibres. The working of optical fibres is based on the phenomenon of total internal reflection.

An optical fibre is a hair-thin long strand of quality glass or quartz surrounded by a glass coating of slightly lower reflective index. It is used as a guided medium for transmitting an optical signal from ne place to another.

Construction: An optical fibre consists of three main parts:



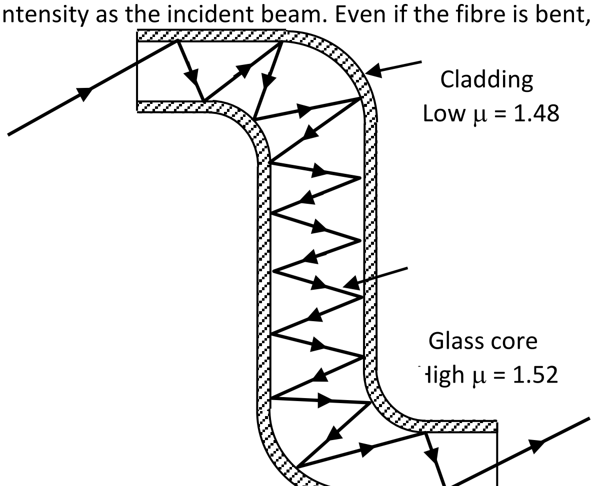
[Optical fibre structure]

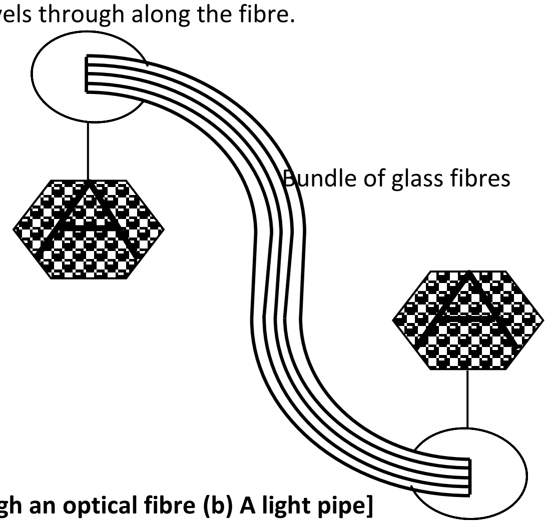
(i) Core: The central cylindrical core is made of high quality glass/silica/plastic of refractive index μ_1 and has a diameter about γ10 to 100 μm.

(ii) Cladding: the core is surrounded by a glass/plastic jacket of refractive index $\mu_2 < \mu_1$. In a typical optical fibre, the refractive indices of core and cladding may be 1.52 and 1.48 respectively.

(iii) Buffer coasting: For providing safety and strength, the core cladding of optical fibres is enclosed in a plastic jacket.

Propagation of light through an optical fibre: As shown in Fig. (a), when light is incident on one end of the fibre at a small angle, it goes inside and suffers repeated total internal reflections because the angle of incidence is greater than the critical angle of the fibre material with respect to its outer coating. As there is no loss of intensity in total internal reflection, the outcoming beam is of as much intensity as the incident beam. Even if the fibre is bent, light easily travels through along the fibre.





[(a) Propagation of light through an optical fibre (b) A light pipe]

A bundle of optical fibres is called a light pipe. A single fibre cannot be used to see the complete image of an object.

But, if the image is broken into a large number of fine dots and each portion of the image is seen through a separate fibre, the complete image can be seen. A light pipe can be used to transmit such an image accurately.

Applications of optical fibres:

1. As a light pipe, optical fibres are used in medical and optical examination. A light pipe is inserted into the stomach through the mouth. Light transmitted through the outer layers of the light pipe is scattered by the various parts of stomach into the central portion of the light pipe is scattered by the various parts of stomach into the central portion of the light pipe to produce a final image with excellent details. The technique is called endoscopy.
2. They are used in transmitting and receiving electrical signals in telecommunications. The electrical signals are first converted to light by suitable transducers. Each fibre can transmit about 2000 telephone conversations without much loss of intensity.
3. They are used for transmitting optical signals and two-dimensional pictures.
4. In the form of photometric sensors, they are used for measuring the blood flow in the heart.
5. In the form of refractometers, they are used to measure refractive indices of liquids.

Examples based on Total Internal Reflection

Formulae used /; 1. Critical angle, i_c = Angle of incident in denser medium for which angle of refraction is 90° in rarer medium.

2. Refractive index of denser medium, $\mu = \dfrac{1}{\sin i_c}$ 3. Total internal reflection occurs when $i > i_c$

Units used:Angle i_c is in degrees and μ has no units.

Q. 1. ***Find the value of critical angle for a material of refractive index √3.***

Sol. Here $\mu = \sqrt{3}$

$$\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.5773$$

∴ Critical angle, $i_c \simeq 35.3°$

Q. 2. ***Calculate the speed of light in a medium, whose critical angle is 45°.***

Sol. Here $i_c = 45°$

As $\mu = \dfrac{c}{v} = \dfrac{1}{\sin i_c}$

∴ $v = c \sin i_c = 3 \times 10^8 \times \sin 45° = 3 \times 10^8 \times 0.707 = 2.121 \times 10^8\ ms^{-1}$

Q. 3. ***The velocity o light in a liquid is 1.5 × 10⁸ ms⁻¹ and in air, it is 3 × 10⁸ ms⁻¹. If a ray of light passes from this liquid into air, calculate the value of critical angle.***

Sol. Here $v = 1.5 \times 10^8\ ms^{-1}$, $c = 3 \times 10^8\ ms^{-1}$

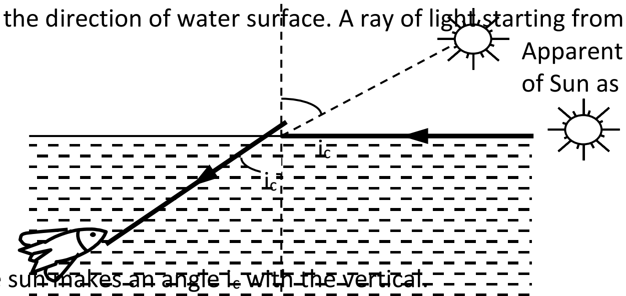
Refractive index of the liquid,

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2 \quad \text{or} \quad \sin i_c = \frac{1}{\mu} = \frac{1}{2}$$

∴ Critical angle, $i_c = 30°$

Q. 4. ***Determine the direction in which a fish under water sees the setting sun. Refractive index of water is 1.33.***

Sol. Fig. shows the setting sun in the direction of water surface. A ray of light starting from the sun enters the eye of the fish.



The apparent position of the sun makes an angle i_c with the vertical.

From Snell's law

$$\frac{\sin 90°}{\sin i_c} = 1.33 \quad \text{or} \quad \sin i_c = \frac{1}{1.33} = 0.7518$$

∴ $i_c = \sin^{-1}(0.7518) = 48.7°$Angle between the apparent position of the sun and the horizontal = 90 – 48.7 = 41.3°

Q. 5. ***Determine the critical angle for a glass-air surface, if a ray of light which is incident in air on the surface is deviated through 15°, when its angle of incidence is 40°.***

Sol. The path of the refracted ray is shown in Fig.

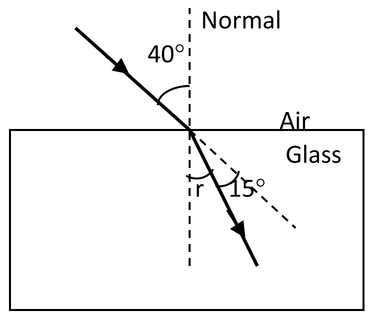
Clearly, $r + 15 = 40°$ or $r = 25°$

Refractive index of glass,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 40°}{\sin 25°}$$

$$= \frac{0.6428}{0.4226} = 1.52$$

∴ $\sin i_c = \frac{1}{\mu} = \frac{1}{1.52} = 0.6579$ Critical angle, $i_c = 41.14°$



Q. 6. ***A Glass slab is immersed in water. Find the critical angle at glass-water interface. Given $^a\mu_g = 1.5$ and $^a\mu_w = 1.33$.***

or

A ray of light passes from glass ($\mu_g = 3/2$) to water ($\mu_w = 4/3$). What is the critical angle of incidence?

Sol. From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_w}{\mu_g}$$

∴ $$\frac{\sin i_c}{\sin 90°} = \frac{1.33}{1.5}$$

or $\sin i_c = 0.8867$ ∴ $i_c = \sin^{-1}(0.8867) = 62° 28'$

Q. 7. ***The critical angle of incidence in a glass slab placed in air is 45°. What will be the critical angle when it is immersed in water of refractive index 1.33?***

Sol. $^a\mu_g = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2} = 1.414$

Refractive index of glass w.r.t. water will be

$$^\omega\mu_g = \frac{^a\mu_g}{^a\mu_w} = \frac{1.414}{1.33}$$

When glass slab is immersed in water, the critical angle i_c, is given by

$$\sin i_c' = \frac{1}{^\omega\mu_g}$$

$$= \frac{1}{\frac{1.414}{1.33}} = \frac{1.33}{1.414} = 0.9432$$

∴ $i_c' = 70° 36'$

Q. 8. ***A ray of light incident on the horizontal surface of a glass slab of 70° just grazes the adjacent vertical surface after refraction. Calculate the critical angle and refractive index of the glass.***

Sol. As shown in Fig., the refracted ray will graze the vertical surface BC only when the ray QR is incident at critical angle i_c. Clearly,

$r + i_c = 90°$ or $r = 90° - i_c$

Using Snell's law for attraction at face AB, we get

$$\frac{\sin 70°}{\sin r} = \mu$$

or $\sin r = \frac{\sin 70°}{\mu}$

or $\sin (90° - i_c) = \frac{\sin 70°}{\mu}$

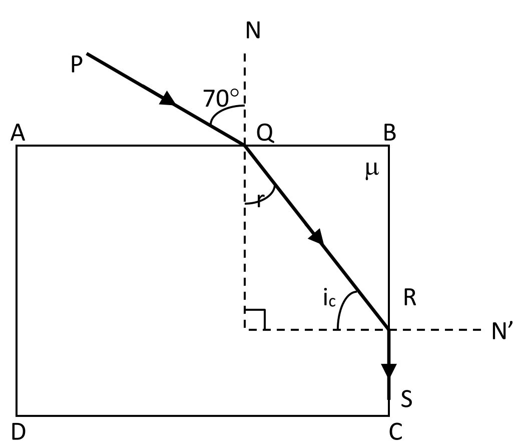
or $\cos i_c = \frac{\sin 70°}{\mu}$

For refraction at face BC, we have

$$\sin i_c = \frac{1}{\mu}$$

∴ $\tan i_c = \frac{\sin i_c}{\cos i_c} = \frac{1/\mu}{\sin 70°/\mu}$

$$= \frac{1}{\sin 70°} = \frac{1}{0.9397} = 1.0642$$



or $\quad i_c = 46°47'$

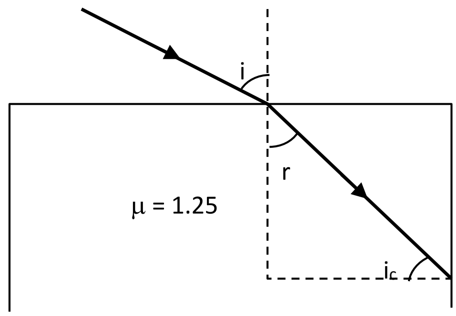
Hence $\quad \mu = \dfrac{1}{\sin i_c} = \dfrac{1}{\sin 46°\, 47'}$

$= \dfrac{1}{0.7288} = 1.372$

Q. 9. ***For a situation shown in Fig., find the maximum angle i for which the light suffers total internal reflection at the vertical surface.***

Sol. The critical angle i_c is given by

$\sin i_c = \dfrac{1}{\mu} = \dfrac{1}{1.25} = \dfrac{4}{5}$



As $\quad i_c + r = 90°$, therefore

$\sin r = \sin(90° - i_c) = \cos i_c$

$= \sqrt{1 - \left(\dfrac{4}{3}\right)^2} = \dfrac{3}{5}$

From Snell's law, $\dfrac{\sin i}{\sin r} = 1.25$

or $\quad \sin i = 1.25 \times \sin r = 1.25 \times \dfrac{3}{5} = 0.75 \quad$ or $\quad i = 48.6°$

If the angle of incidence at vertical surface is greater than i_c, then i will be less than 48.6°. Hence the maximum value of i, for which total internal reflection occurs at the vertical surface is 48.6°.

Q. 10. ***A ray of light travelling in glass (refractive index, $^a\mu_g = 3/2$) is incident on a horizontal glass-air surface at the critical angle i_c. If a thin layer of water (refractive index $^a\mu_w = 4/3$) is now poured on the glass-air surface, at what angle will the ray of light emerge into air at the water-air surface?***

Sol. Here $\quad \sin i_c = \dfrac{1}{^a\mu_g} = \dfrac{1}{3/2} = \dfrac{2}{3}$

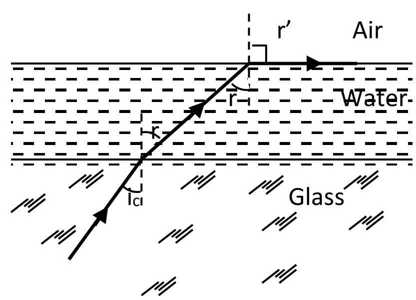
Refractive index of water relative to glass is

$^s\mu_w = \dfrac{^a\mu_w}{^a\mu_g} = \dfrac{4/3}{3/2} = \dfrac{8}{9}$

From Snell's law,

$\dfrac{\sin i_c}{\sin r} = {^g\mu_w} \quad$ or $\quad \dfrac{2/3}{\sin r} = \dfrac{8}{9}$

∴ $\quad \sin r = \dfrac{9}{8} \times \dfrac{2}{3} = \dfrac{3}{4}$



Also, $\quad {^w\mu_a} = \dfrac{1}{^a\mu_w} = \dfrac{1}{4/3} = \dfrac{3}{4}$

For refraction at water-air interface,

$\dfrac{\sin r}{\sin r'} = {^w\mu_a} = \dfrac{3}{4}$

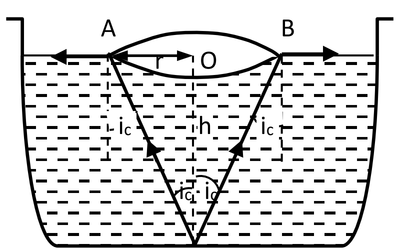
or $\quad \sin r' = \dfrac{4}{3} \times \sin r = \dfrac{4}{3} \times \dfrac{3}{4} = 1 \quad ∴ \quad r' = 90°$

Thus, the ray of light emerges into air grazing the water surface.

Q. 11. ***A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index 5/3. A person is viewing the source from above the surface. There is an opaque disc is radius 1 cm floating on the surface. The centre of the disc lies vertically above the source O. The liquid from the vessel is gradually drained out through a tap. What is the maximum height of the liquid for which the source cannot be seen at all?***

Sol. As shown in Fig., suppose the height OS = h is such that ∠OSA = i_c. Then any other ray from S will be totally internally reflected because then the angle of incidence would be greater than i_c.

In ΔOSA, $\quad \sin i_c = \dfrac{OA}{OS} = \dfrac{r}{\sqrt{r^2 + h^2}}$



Also, $\sin i_c = \frac{1}{\mu}$

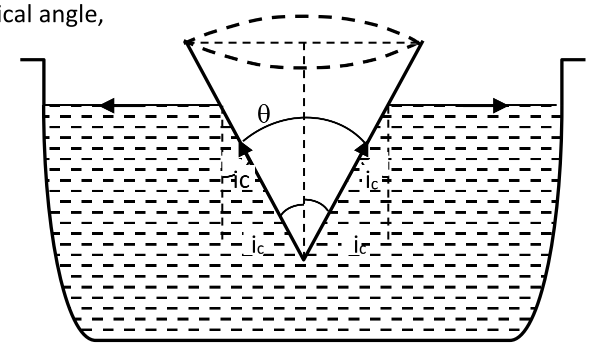
$\therefore$ $\frac{1}{\mu} = \frac{r}{\sqrt{r^2 + h^2}}$ or $\frac{1}{\mu^2} = \frac{r^2}{r^2 + h^2}$

$r^2 + h^2 = \mu^2 r^2$ or $h^2 = r^2 (\mu^2 - 1)$

$\therefore$ $h = r\sqrt{\mu^2 - 1} = 1\sqrt{\left(\frac{5}{3}\right)^2 - 1} = \frac{4}{3} = 1.33$ cm

Q. 12. ***The refractive index of water is 4/3. Obtain the value of the semi vertical angle of the cone within which the entire outside view would be confined for a fish under water. Draw an appropriate any diagram.***

Sol. Clearly, the fish can see the outside view of the cone with semi vertical angle,



$$\frac{\theta}{2} = i_c$$

But $\mu = \frac{1}{\sin i_c}$

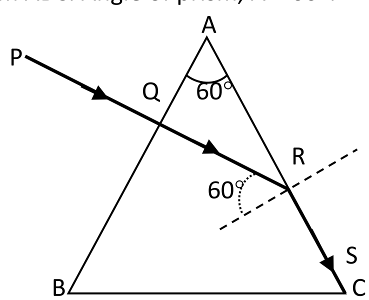
or $\frac{4}{3} = \frac{1}{\sin i_c}$

or $\sin i_c = \frac{3}{4} = 0.75$

$\therefore$ $\theta/2 = i_c = \sin^{-1}(0.75) = 48.6°$

Q. 13. ***A narrow beam of light is incident normally on one face of an equiangular glass prism of refractive index 1.45. When the prism is immersed in a suitable liquid, the ray makes a grazing emergence along the other face. Find the refractive index of this liquid and draw a diagram showing the path of rays.***

Sol. Fig., shows an equiangular glass prism ABC. Angle of prism, A = 60°.



The ray PQ, incident normally on face AB, goes undeviated along QR. On face AC, its angle of incidence = 60°. When the prism is immersed in the liquid, the emergent ray RS grazes the face AC.

$\therefore$ Critical angle, $i_c = 60°$

Refractive index of prism glass w.r.t. the liquid,

$$^{l}\mu_g = \frac{1}{\sin i_c} = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}} = 1.155$$

But $^{l}\mu_g = \frac{^{a}\mu_g}{^{a}\mu_l}$ $\therefore$ $^{a}\mu_l = \frac{^{a}\mu_g}{^{l}\mu_g} = \frac{1.45}{1.555} = 1.255$

Problems For Practice

Q. 1. ***The critical angle for water is 48.2°. Find the refractive index.***

Sol. Refractive index,

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 48.2°} = \frac{1}{0.7455} = 1.34$$

Q. 2. ***Find the critical angle for a ray of light going from paraffin oil to air.***

Sol. $\sin i_c = \frac{1}{\mu} = \frac{1}{1.44} = 0.6944$ $\therefore$ $i_c = 43.98°$

Q. 3. ***Refractive index of glass is 1.5. Calculate the velocity of light in glass if velocity of light in vacuum is 3×10^6 ms^{-1}. Also calculate the critical angle for glass-air interface.***

Sol. $v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8$ ms^{-1}

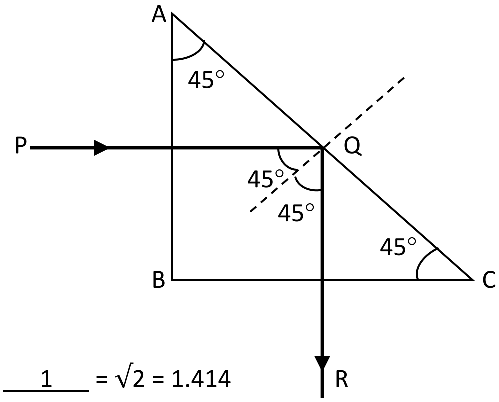
Also, $\sin i_c = \frac{1}{\mu} = \frac{1}{1.5} = \frac{2}{3} = 0.6667$ ∴ $i_c = 41° 49'$

Q. 4. An optical fibre (μ = 1.72) is surrounded by a glass coating (μ = 1.50). Find the critical angle for total internal reflection at the fibre-glass interface.

Sol. $\sin i_c = \frac{1.50}{1.72} = 0.8721$ ∴ $i_c = 60.7°$

Q. 5. What is the small index of refraction of the material or a right-angled prism with equal sides for which a ray of light entering one of the sides normally will be totally reflected?

Sol. As shown in Fig., the ray PQ enters through the side AB normally and is incident on AC at an angle of 45°. It will be totally reflected along QR if the critical angle for the material of the prism is 45°.



∴ $\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2} = 1.414$

Q. 6. Find the maximum angle of refraction when a ray of light is refracted from glass (μ = 1.5) to air.

Sol. When the ray of light is incident at the critical angle i_c, the angle of refraction is maximum and is equal to 90°.

Q. 7. Calculate the critical angle for glass-air surface, if a ray of light which is incident in air on the glass surface is deviated through 15°, when angle of incidence is 45°.

Sol. Here $i = 45°, r = 45° - 15° = 30°$

$\mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}} \times \frac{2}{1} = \sqrt{2}$

∴ $\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$ or $i_c = 45°$

Q. 8. A luminous object O is located at the bottom of a big pool of liquid of refractive index μ and depth h. The object O emits rays upwards in all directions, so that a circle of light is formed at the surface of the liquid by the rays which are refracted into the air. What happens to the rays beyond the circle? Determine the radius and the area of the circle.

Sol. Refer to fig., The light rays emerge through a circle of radius r.

Radius, $r = h \tan i_c = h \cdot \frac{\sin i_c}{\cos i_c}$

$= h. \frac{1/\mu}{\sqrt{1 - 1/\mu^2}} = \frac{h}{\sqrt{\mu^2 - 1}}$

Area of patch $= \pi r^2 = \frac{\pi h^2}{\mu^2 - 1}$

Q. 9. Glycerine (refractive index 1.4) is poured into a large jar of radius 0.2 m to a depth of 0.1 m. There is a small light source at the centre of the bottom of the jar. Find the area of the surface of glycerine through which the light passes.

Sol. Let S be the light source. If light falls on the surface at critical angle i_c, it grazes along the surface, as shown in Fig.

$\sin i_c = \frac{1}{\mu} = \frac{1}{1.4} = 0.7143$

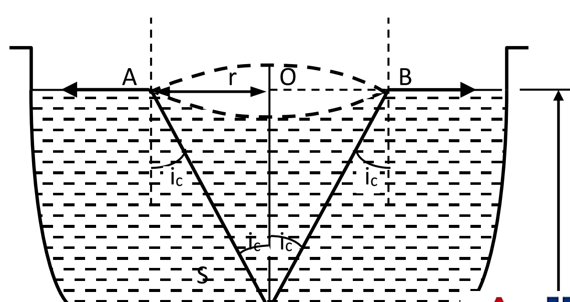
∴ $i_c = \sin^{-1}(0.7143) = 45.6°$

and $\tan i_c = \tan 45.6° = 1.0.21$

But $\tan i_c = \frac{OA}{OS}$

or $1.021 = \frac{r}{0.1}$

or $r = 0.1 \times 1.021 = 0.1021$ m



Area of the surface through which light passes = $\pi r^2 = 3.14 \times (0.1021)^2 = 0.0327\ m^2$

Q. 10. ***A liquid of refractive index 1.5 is poured into a cylindrical jar of radius 20 cm upto a height of 20 cm. A small bulb is lighted at the centre of the bottom of jar. Find the area of liquid surface through which the light of the bulb passes into air.***

Sol. Proceeding as in the above problem, we get

$$\sin i_c = \frac{1}{\mu} = \frac{1}{1.5} = 0.6667 \quad \text{or} \quad i_c = 41.8^\circ$$

$$r = h \tan i_c = 20 \times \tan 41.8^\circ = 20 \times 0.8941$$
$$= 17.882\ \text{cm}$$

Area of the surface through which light passes,

$$= \pi r^2 = 3.14 \times (17.882)^2 = 1004.6\ \text{cm}^2$$

SPHERICAL LENSES

A lens is a piece of a refracting medium bounded by two surfaces, at least one of which is a curved surface.

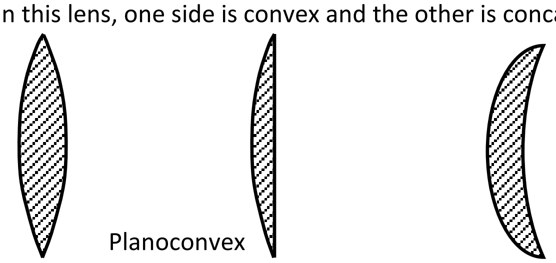
The commonly used lenses are the spherical lenses. These lenses have either both surfaces spherical or one spherical and the other a plane one. Lenses can be divided into two categories:

(i) Convex or converging lenses, and (ii) Concave or diverging lenses

(i) Convex or converging lens: It is thicker at the centre than at the edges. It converges a parallel beam of light on refraction through it. It has a real focus.

◙*Types of convex lenses:*

(a) Double convex or biconvex lens. In this lens, both surfaces are convex.
(b) Planoconcave lens: In this lens, one side is convex and the other is plane.
(c) Concavo-convex: In this lens, one side is convex and the other is concave.



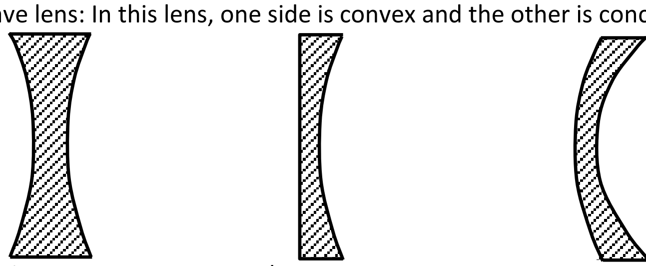
Biconvex Planoconvex Concavo-convex

[Different types of convex lenses]

(ii) Concave of diverging lens: It is thinner at the centre than at the edges. It diverges a parallel beam of light on refraction through it. It has a virtual focus.

◙*Types of concave lenses:*

(a) Double concave or biconcave lens. In this lens, both sides are concave.
(b) Plano concave lens: In this lens, one side is plane and the other is concave.
(c) Convexo-concave lens: In this lens, one side is convex and the other is concave.



Biconcave Planoconcave Convexo-concave

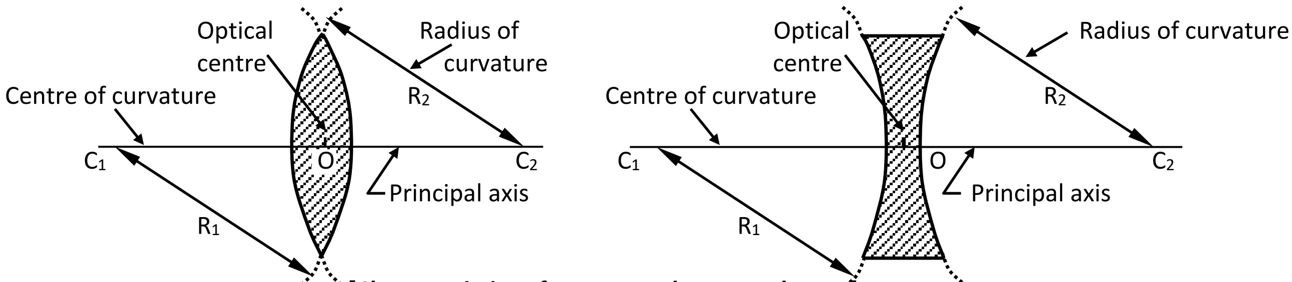
[Different types of concave lenses]

DEFINITIONS IN CONNECTION WITH SPHERICAL LENSES

(i) Centre of curvature (C): The centre of curvature of the surface of a lens is the centre of the sphere of which it forms a part. Because a lens has two surfaces, so it has two centres of curvature.

(ii) Radius of curvature (R): The radius of curvature of the surface of a lens is the radius of the sphere of which the surface forms a part.

(iii) Principal axis (C_1 C_2): It is the line passing through the two centres of curvature of the lens.



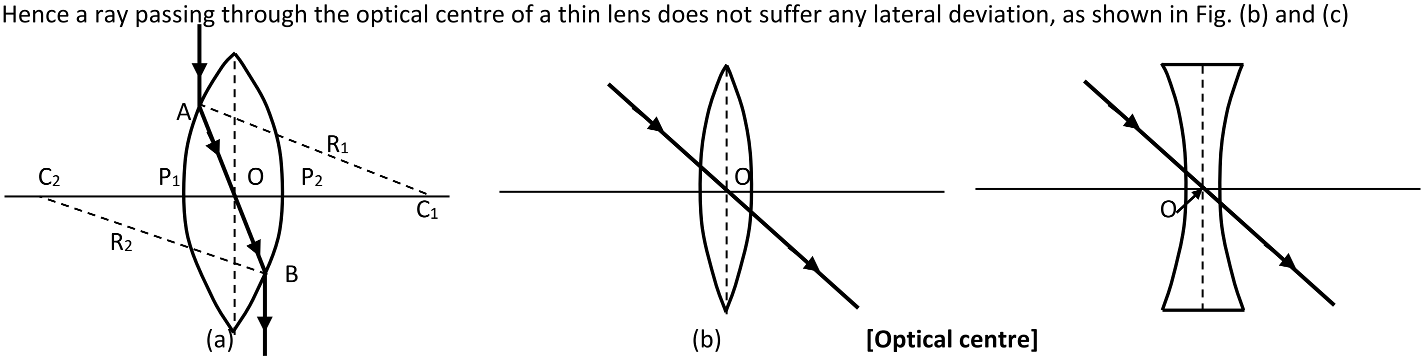
[Characteristics of convex and concave lenses]

(iv) Optical centre: If a ray of light is incident on a lens such that after refraction through the lens the emergent ray is parallel to the incident ray, then the point at which the refracted ray intersects the principal axis is called the optical centre of the lens. In Fig. (a), O is the optical centre of the lens. It divides the thickness of the lens in the ratio of the radii of curvature of its two surfaces. Thus:

$$\frac{OP_1}{OP_2} = \frac{P_1C_1}{P_2C_2} = \frac{R_1}{R_2}$$

If the radii of curvature of the two surfaces are equal, then the optical centre coincides with the geometric centre of the lens.

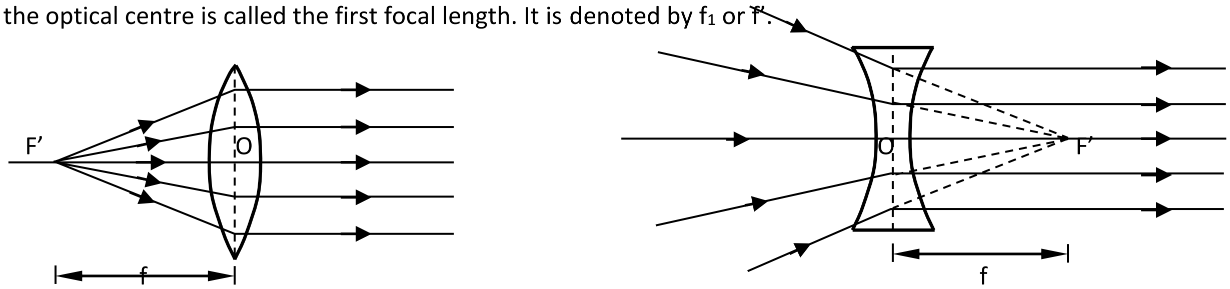
For the ray passing through the optical centre, the incident and emergent rays are parallel. However, the emergent ray suffers same lateral displacement relative to the incident ray. This lateral displacement decreases with the decreases in thickness of the lens. Hence a ray passing through the optical centre of a thin lens does not suffer any lateral deviation, as shown in Fig. (b) and (c)



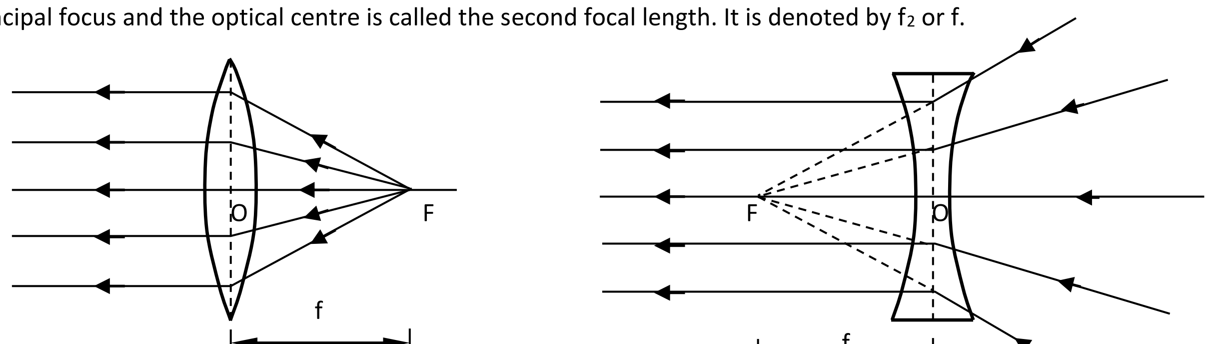
(a) (b) **[Optical centre]**

(v) Principal foci and focal length:

First principal focus: It is a fixed point on the principal axis such that rays starting from this point (in convex lens) or appearing to go towards this point, after refraction through the lens, become parallel to the principal axis. It is represented by F_1 or F'. The plane passing through this point and perpendicular to the principal axis is called the first focal plane. The distance between first principal focus and the optical centre is called the first focal length. It is denoted by f_1 or f'.



Second principal focus: It is fixed point on the principal axis such that the light rays incident parallel to the principal axis, after refraction through the lens, either converge to this point (in convex lens) or appear to diverge from this point (in concave lens). The plane passing through this point and perpendicular to principal axis is called the second focal plane. The distance between the second principal focus and the optical centre is called the second focal length. It is denoted by f_2 or f.



[Second principal focus and second focal length]

Generally, the focal length of a lens refers to its second focal length. It is obvious from the above figures that the foci of a convex lens is real and those of a concave lens are virtual. Thus the focal length of a convex lens is taken positive and the focal length of a concave lens is taken negative.

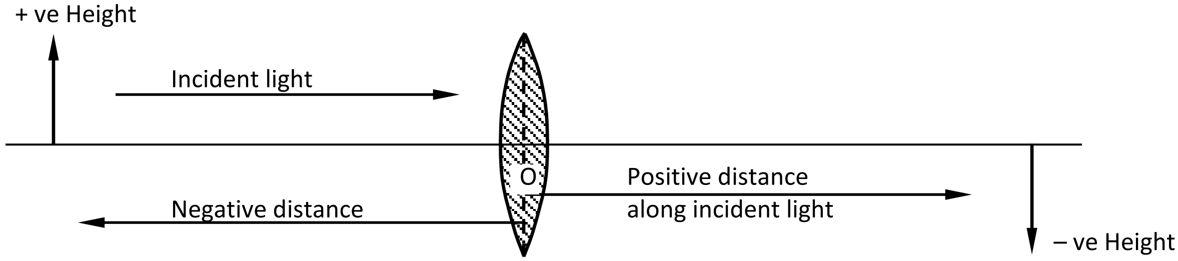
If the medium on both sides of a lens is same, then the numerical values of the first and second focal lengths are equal. Thus

$f = f'$

(vi) Aperture: It is the diameter of the circular boundary of the lens.

NEW CARTESIAN SIGN CONVENTION FOR SPHERICAL LENSES

1. All distances are measured from the optical centre of the lens.
2. The distances measured in the same direction as the incident light are taken positive.
3. The distances measured in the direction opposite to the direction of the incident light are taken negative.
4. Heights measured upwards and perpendiculars to the principal axis are taken positive.
5. Heights measured downwards and perpendiculars to the principal axis are taken negative.



[New Cartesian sign convention for a spherical lens]

◈*Consequences of the sign convention:*

1. The focal length of a converging lens is positive and that of a diverging lens is negative.
2. Object distance is always negative.
3. The distance of real image is positive and that of virtual image is negative.
4. The object height h_1 is always positive. Height h_2 of virtual erect image is positive and that of real inverted image is negative.
5. The linear magnification $m = h_2/h_1$ is positive for a virtual image and negative for a real image.

Before deriving formulae for spherical lenses, we first consider refraction by single spherical surface.

REFRACTION AT A CONVEX SPHERICAL SURFACE

◈*New Cartesian sign convention for refraction at a spherical surface*:

1. All distances are measured from the pole of the spherical surface.
2. The distances measured in the direction of incident light are positive.
3. The distances measured in the opposite direction of incident light are negative.

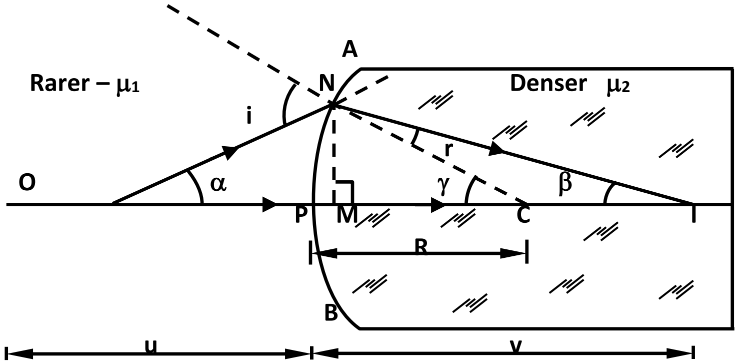
◈*Assumption used in the study of refraction at a spherical surface:*

1. The object taken is a point object placed on the principal axis.
2. The aperture of the spherical refractive surface is small.
3. The incident and refracted rays make small angles with the principal axis so that the sines or tangents of these angles may be taken equal to the angles themselves.

◈**Refraction at a convex spherical surface**:

(i) The object lies in rarer medium and the image formed is real: In Fig. AMB is a convex refracting surface which separates a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_1 from a denser medium of refractive index μ_2. Let P be the pole, C be the centre of curvature and R = PC be the radius of curvature of this surface. Suppose a point object O is placed on the principal axis in the rarer medium. Starting from the point object O, a ray ON is incident at an angle i. After refraction, it bends towards the normal CN at an angle of refraction r. Another ray OP is incident normally on the convex surface and passes undeviated. The two refracted rays meet at point I. So, I is the real image of point object O

Draw NM perpendicular to the principal axis. Let α, β and γ be the angles, as shown in Fig.



[Refraction from rarer to denser medium, when the image is real]

In ΔNOC, i is an exterior angle, therefore,

$$i = \alpha + \gamma$$

Similarly, from Δ NIC, we have

$$\gamma = r + \beta$$

or $$r = \gamma - \beta$$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$\therefore$ $\alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP}$ [$\because$ P is close to M]

$$\beta \simeq \tan\beta = \frac{NM}{MI} \simeq \frac{NM}{PI}$$

And $\gamma \simeq \tan\gamma = \frac{MP}{MC} \simeq \frac{NM}{PC}$

From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and r are small, therefore,

$$\frac{i}{r} = \frac{\mu_2}{\mu_1}$$

or $\mu_1 i = \mu_2 r$

or $\mu_1 [\alpha + \gamma] = \mu_2 [\gamma - \beta]$

or $\mu_1 \left(\frac{NM}{OP} + \frac{NM}{PC}\right) = \mu_2 \left(\frac{NM}{PC} - \frac{NM}{PI}\right)$

or $\mu_1 \left(\frac{1}{OP} + \frac{1}{PC}\right) = \mu_2 \left(\frac{1}{PC} - \frac{1}{PI}\right)$

or $\frac{\mu_1}{OP} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$

Using new Cartesian sign convention, we find

Object distance, OM = – u

Image distance, PI = + v

Radius of curvature, PC = + R

$\therefore$ $\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

or $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

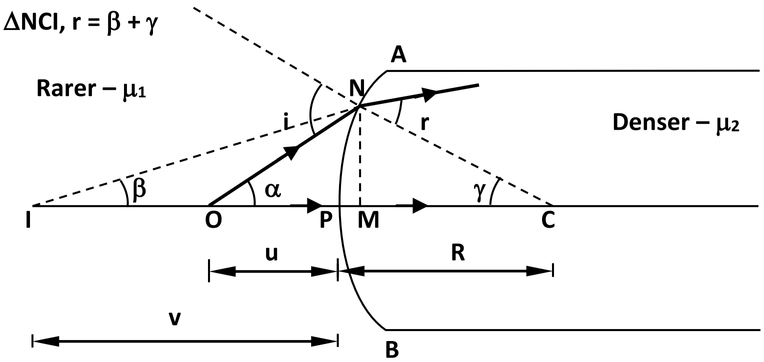
If first medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$, we have

$$\boxed{\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}}$$

(ii) The object lies in the rarer medium and the image formed is virtual: When the object O in the rarer medium lies close to the pole P of the convex refract- ing surface, the two refracted rays appear to diverge from a point I on the principal axis, as shown in Fig. So, I is the virtual image of the point object O.

From ΔNOC, $i = \alpha + \gamma$

From ΔNCI, $r = \beta + \gamma$



[Refraction from rarer to denser medium, when the image is virtual]

Suppose all the rays are paraxial. Then the angles i, r, α, β and γ will be small

$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} = \frac{NM}{OP}$ [∵ M is close to P]

$\beta \simeq \tan\beta = \frac{NM}{IM} = \frac{NM}{IP}$

and $\quad \gamma \simeq \tan\gamma = \frac{NM}{MC} = \frac{NM}{PC}$

For Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and r are small, so

$$\frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \mu_1 i = \mu_2 r$$

or $\quad \mu_1(\alpha + \gamma) = \mu_2(\beta + \gamma)$

or $\quad \mu_1\left(\frac{NM}{OP} + \frac{NM}{PC}\right) = \mu_2\left(\frac{NM}{IP} + \frac{NM}{PC}\right)$

or $\quad \mu_1\left(\frac{1}{OP} + \frac{1}{PC}\right) = \mu_2\left(\frac{1}{IP} + \frac{1}{PC}\right)$

or $\quad \frac{\mu_1}{OP} - \frac{\mu_2}{IP} = \frac{\mu_2 - \mu_1}{PC}$

Using new Cartesian sign convention, we find that

Object distance, OP = – u
Image distance, IP = – v
Radius of curvature, PC = + R

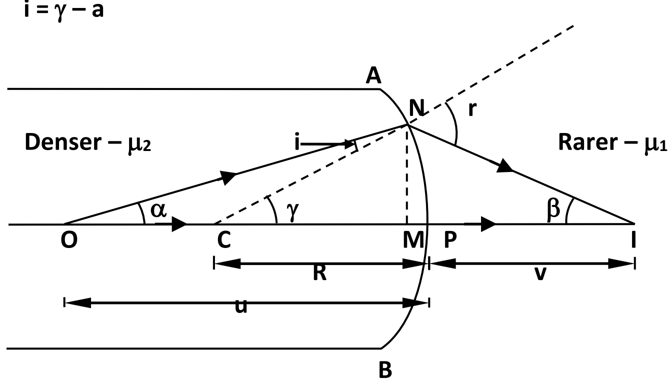
$\therefore \quad \frac{\mu_1}{-u} - \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{R}$

or $\quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

(iii) The object lies in the denser medium and the image formed is real: Fig. shows a convex refracting surface which is convex towards the rarer medium. The point object O lies in the denser medium. The two refracted rays meet at point I. So I is the real image of the point object O.

From ΔNOC, $\gamma = i + \alpha$ or $i = \gamma - a$

From ΔNIC, $r = \beta + \gamma$



[Refraction from denser to rarer medium when the image is real]

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

∴ $\alpha \simeq \tan\alpha = \frac{NM}{OM} = \frac{NP}{OP}$ [∵ M is close to P]

$\beta \simeq \tan\beta = \frac{NM}{MI} = \frac{NM}{PI}$

and $\gamma \simeq \tan\gamma = \frac{NM}{CM} = \frac{NM}{CP}$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

As i and r are small angles, so

$$\frac{i}{r} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \mu_2 i = \mu_1 r$$

or $\mu_2(\gamma - \alpha) = \mu_1(\beta + \gamma)$

or $\mu_2\left(\frac{NM}{CP} - \frac{NM}{OP}\right) = \mu_1\left(\frac{NM}{PI} + \frac{NM}{CP}\right)$

or $\mu_2\left(\frac{1}{CP} - \frac{1}{OP}\right) = \mu_1\left(\frac{1}{PI} + \frac{1}{CP}\right)$

or $\frac{-\mu_1}{PI} - \frac{\mu_2}{OM} = \frac{\mu_1 - \mu_2}{CP}$

Using the new Cartesian sign convention, we have

Object distance, OP = – u
Image distance, PI = + v
Radius of curvature, CP = – R

∴ $\frac{-\mu_1}{v} - \frac{\mu_2}{-u} = \frac{\mu_1 - \mu_2}{-R}$

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

(iv) The object lies in the denser medium and the image formed is virtual: If the point object O placed on the principal axis lies close to the pole of the refracting surface, then the two refracted rays appear to come from the point I, as shown in Fig. So, I is the virtual image of the point object O.

From ΔNOC, $i + \gamma = \alpha$ or $i = \alpha - \gamma$

From ΔNIC, $r + \gamma = \beta$ or $r = \beta - \gamma$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

∴ $\alpha \simeq \tan\alpha = \frac{NM}{OM} = \frac{NP}{OP}$ [∵ M is close to P]

$\beta \simeq \tan\beta = \frac{NM}{IM} = \frac{NM}{IP}$

$\gamma \simeq \tan\gamma = \frac{NM}{CM} = \frac{NM}{CP}$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

As i and r are small angles, so

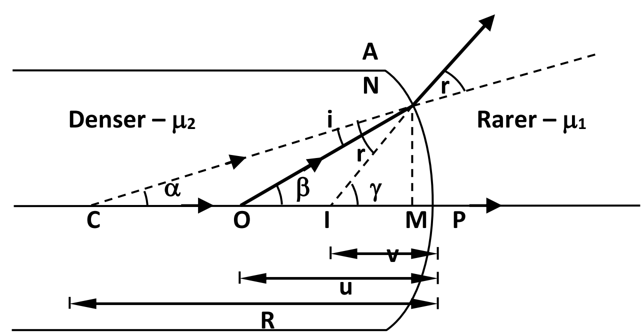
$$\frac{i}{r} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \mu_2 i = \mu_1 r$$

or $\mu_2(\alpha - \gamma) = \mu_1(\beta - \gamma)$

or $\mu_2\left(\frac{NM}{OP} - \frac{NM}{CP}\right) = \mu_1\left(\frac{NM}{IP} - \frac{NM}{CP}\right)$

or $\mu_2\left(\frac{1}{OP} - \frac{1}{CP}\right) = \mu_1\left(\frac{1}{IP} + \frac{1}{CP}\right)$

or $\frac{-\mu_1}{IP} + \frac{\mu_2}{OP} = -\frac{\mu_1 - \mu_2}{CP}$



[Refraction from denser to rarer medium when the image is virtual]

Using the new Cartesian sign convention, we have Object distance, OP = – u   Image distance, IP = – v

Radius of curvature, CP = – R

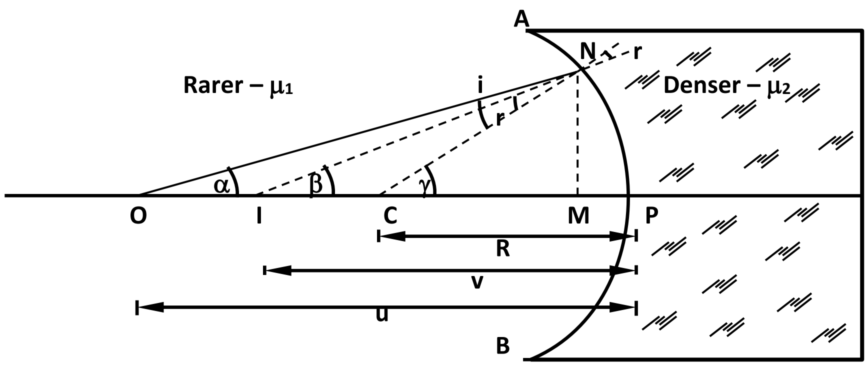
∴ $\frac{-\mu_1}{-v} + \frac{\mu_2}{-u} = -\frac{\mu_1 - \mu_2}{-R}$

or $\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$

REFRACTION AT A CONCAVE SPHERICAL SURFACE

(i) **The object lies in the rarer medium**: In Fig. APB is a concave refracting surface separating two media of refractive indices μ_1 and μ_2.

Let P = Pole of the concave surface APB
C = Centre of curvature of the concave surface
O = Point object placed on the principal axis. **I = Virtual image of point object O**



[Refraction at a concave surface when the object lies in the rarer medium]

In Δ NOC, γ is an exterior angle, therefore

$$Y = \alpha + i$$

or $i = \gamma - \alpha$

Similarly, from Δ NIC, we have

$$\gamma = \beta + r$$

or $r = \gamma - \beta$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$\therefore$ $\alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP}$ **[∵ M is close to P]**

$$\beta \simeq \tan\beta = \frac{NM}{IM} \simeq \frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma = \frac{NM}{CM} \simeq \frac{NM}{CP}$$

From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and i are small angles, therefore

$$\frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \mu_1 i = \mu_2 r$$

or $\mu_1 [\gamma - \alpha] = \mu_2 [\gamma - \beta]$

or $\mu_1 \left(\frac{NM}{CP} - \frac{NM}{OP}\right) = \mu_2 \left(\frac{NM}{CP} + \frac{NM}{IP}\right)$

or $\mu_1 \left(\frac{1}{CP} - \frac{1}{OP}\right) = \mu_2 \left(\frac{1}{CP} + \frac{1}{IP}\right)$

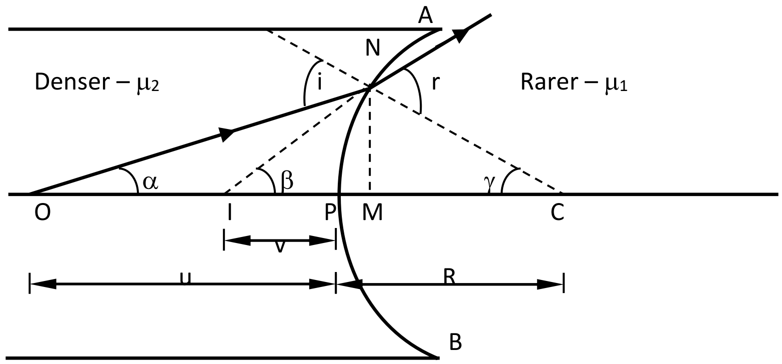
or $\frac{-\mu_1}{OP} + \frac{\mu_2}{IP} = \frac{\mu_2 - \mu_1}{CP}$

Using the new Cartesian sign convention, we have
Object distance, OP = – u
Image distance, PI = – v
Radius of curvature, CP = – R

$\therefore$ $\frac{-\mu_1}{-u} + \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{-R}$

or $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

(iv) **The object lies in the denser medium:** As shown in Fig. when the point object O is placed in the denser medium, the refracted rays appear to diverge from a point I in the denser medium. So, I is the virtual image of the point object O.



[Refraction at a concave surface when the object lies in the denser medium]

From ΔNOC, $i = \alpha + \gamma$

From ΔNIC, $r = \beta + \gamma$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$\therefore$ $\alpha \simeq \tan\alpha = \dfrac{NM}{OM} = \dfrac{NM}{OP}$ [$\therefore$ M is close to P]

$\beta \simeq \tan\beta = \dfrac{NM}{IM} = \dfrac{NM}{IP}$

$\gamma \simeq \tan\gamma = \dfrac{NM}{MC} = \dfrac{NM}{PC}$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

As i and r are small angles, so

$$\frac{i}{r} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \mu_2\, i = \mu_1\, r$$

or $\mu_2 [\alpha + \gamma] = \mu_1 [\beta + \gamma]$

or $\mu_2 \left(\dfrac{NM}{OP} + \dfrac{NM}{PC}\right) = \mu_1 \left(\dfrac{NM}{IP} + \dfrac{NM}{PC}\right)$

or $\mu_2 \left(\dfrac{1}{OP} + \dfrac{1}{PC}\right) = \mu_1 \left(\dfrac{1}{IP} + \dfrac{1}{PC}\right)$

or $\dfrac{-\mu_1}{IP} + \dfrac{\mu_2}{OP} = \dfrac{\mu_1 - \mu_2}{PC}$

Using the new Cartesian sign convention, we find

Object distance, OP = – u

Image distance, PI = – v

Radius of curvature, CP = + R

$\therefore$ $\dfrac{-\mu_1}{-v} + \dfrac{\mu_2}{-u} = \dfrac{\mu_1 - \mu_2}{R}$

or $\dfrac{\mu_1}{v} - \dfrac{\mu_2}{u} = \dfrac{\mu_1 - \mu_2}{R}$

conceptual..............................

- For both convex and concave spherical surfaces, the refraction formulae are same, only proper signs of u, v and R are to be used.
- For refraction from rarer to denser medium, the refraction formula is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{... (1)}$$

- For refraction from denser to rarer medium, we interchange μ_1 and μ_2 and obtain the refraction formula,

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{... (2)}$$

- If the rarer medium is air ($\mu_1 = 1$) and the denser medium has refractive index μ (i.e., $\mu_2 = \mu$), then for refraction from rarer to denser medium, from (1) we get the relation:

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R} \quad \text{... (3)}$$

For refraction from denser to rarer medium, we put $\mu_1/\mu_2 = 1/\mu$ in (2) and get the relation:

$$\frac{1/\mu}{v} - \frac{1}{u} = \frac{(1/\mu) - 1}{R} \qquad \text{... (4)}$$

◈ For an object placed in air, the refraction formula (3) is applicable, i.e., $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$

As R is positive for a convex surface, v will be negative if the value of u is less than $R/(\mu - 1)$. In that case, the image will be formed in air and will be virtual.

As R is negative for a concave surface, the value of v will also be negative for all negative values of u. Thus, image will always be formed in air and will be virtual.

◈ The factor $\frac{\mu_2 - \mu_1}{R}$ is called power factor of the spherical refracting surface. It gives a measure of the degree to which the refracting surface can converge of diverge the rays of light passing through it.

Examples based on Refraction through Spherical Surfaces

1. For refraction from rarer to denser medium,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

2. For refraction from denser to rarer medium,

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

3. Power of a surface, $P = \frac{\mu_2 - \mu_1}{R} = \frac{\mu - 1}{R}$ (For air)

4. First principal focal length, $f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$

5. Second principal focal length, $f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$

∴ $\frac{f_2}{f_1} = -\frac{\mu_2}{\mu_1}$

Units used Distances u, v, f and R are in metre, power P is in dioptre (D), refractive indices μ_1, μ_2 and μ have no units.

Q. 1. Light from a point source in air falls on a convex spherical glass surface (μ = 1.5, radius of curvature = 20 cm). The distance of light source from the glass surface is 100 cm. At what position is the image formed?

Sol. Here $\mu_1 = 1$, $\mu_2 = 1.5$, $\mu = -100$ cm

$R = +20$ cm [R is + ve for a convex refracting surface]

As $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

∴ $\frac{1.5}{v} + \frac{1}{100} = \frac{1.5 - 1}{20} = \frac{1}{40}$

or $\frac{3}{2v} = \frac{1}{40} - \frac{1}{100} = \frac{5 - 2}{200} = \frac{3}{200}$

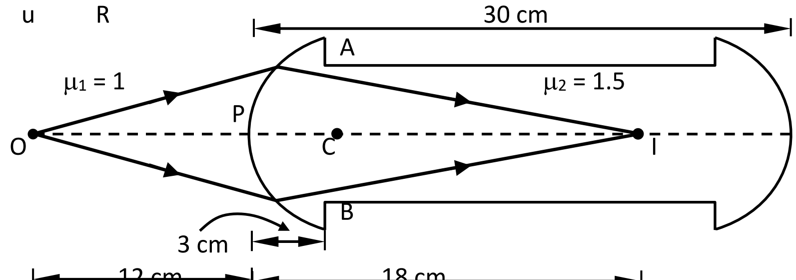
∴ $v = +100$ cm; Thus, the image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

Q. 2. A glass dumbbell of length 30 cm and refractive index 1.5 has ends of 3 cm radius of curvature. Find the position of the image formed due to refraction at one end only, when the object is situated in air at a distance of 12 cm from the end of the dumbbell along the axis.

Sol. Refraction occurs from air to glass at convex spherical surface APB. Therefore,

$u = -12$ cm, $R = +3$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

As $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



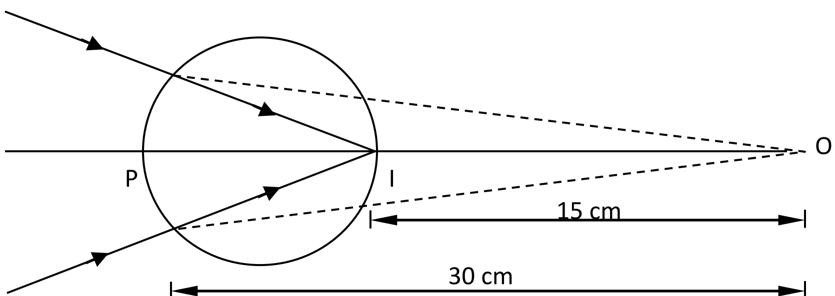
∴ $\frac{1.5}{v} + \frac{1}{12} = \frac{1.5-1}{3} = \frac{0.5}{3}$

or $\frac{1.5}{v} = \frac{0.5}{3} - \frac{1}{12} = \frac{2-1}{12} = \frac{1}{12}$

or $v = 1.5 \times 12 = 18$ cm; As v is positive, so a real image is formed at 18 cm from the end P of the dumbbell.

Q. 3. ***The diameter of a glass sphere is 15 cm. A beam of light strikes the sphere, which converges at point 30 cm behind the pole of the spherical surface. Find the position of the image if $\mu = 1.5$***

Sol. In the absence of glass sphere, the light rays will converge at point O. So O acts as virtual object for the image I for the second surface.



∴ $u = OI = OP - IP = 30 - 15 = 15$ cm,

$\mu_1 = 1, \quad \mu_2 = 1.5$

$R = +\frac{15}{2} = +7.5$ cm

As the light passes from rarer to denser medium, so

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ or $\frac{1.5}{v} - \frac{1}{30} = \frac{1.5-1}{7.5} = \frac{0.5}{7.5} = \frac{1}{15}$

or $\frac{1.5}{v} = \frac{1}{15} + \frac{1}{30} = \frac{1}{10}$ or $v = +10 \times 1.5 = +15$ cm Thus the image is formed at the other end (I) of the diameter.

Q. 4. ***What curvature must be given to the bounding surface of $\mu = 1.5$ for virtual image of an object in the medium of $\mu = 1$ at 10 cm to be formed at a distance of 40 cm. Also calculate power of the surface and two principal focal lengths of the surface.***

Sol. Here $u = -10$ cm, $v = -40$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

As the object is placed in the rarer medium, so

$\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$ or $\frac{1.5-1}{R} = \frac{1.5}{-40} + \frac{1}{10} = \frac{2.5}{40} = \frac{1}{16}$

or $R = 16 \times 0.5 = 8$ cm

As R is positive, the refracting surface is convex.

Power of surface,

$P = \frac{\mu_2 - \mu_1}{R} = \frac{1.5-1}{8 \text{ cm}} = \frac{0.5}{0.08 \text{ m}} = 6.25$ D

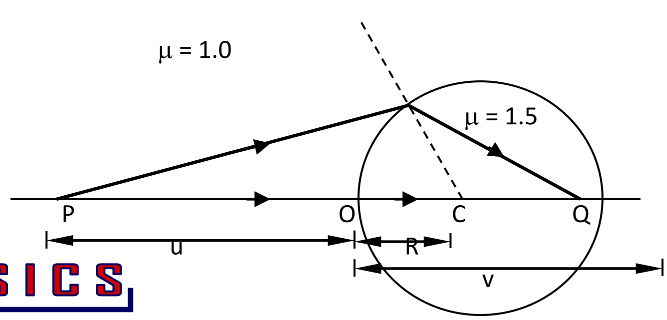
First principal focal length,

$f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1} = \frac{1 \times 8}{0.5} = -16$ cm

Second principal focal length,

$f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1} = \frac{1.5 \times 8}{0.5} = 24$ cm

Q. 5. ***A spherical surface of radius of curvature R separates air ($\mu = 1.0$) from glass ($\mu = 1.5$). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = OQ [Fig]. Find the distance of object from the spherical surface.***



Sol. Here $\mu_1 = 1.0$, $\mu_2 = 1.5$, PO = OQ = x (say)

Clearly, u = – x and v = + x

As the light passes from rarer to denser medium, so

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{or} \quad \frac{1.5}{+x} - \frac{1}{-x} = \frac{1.5 - 1}{R}$$

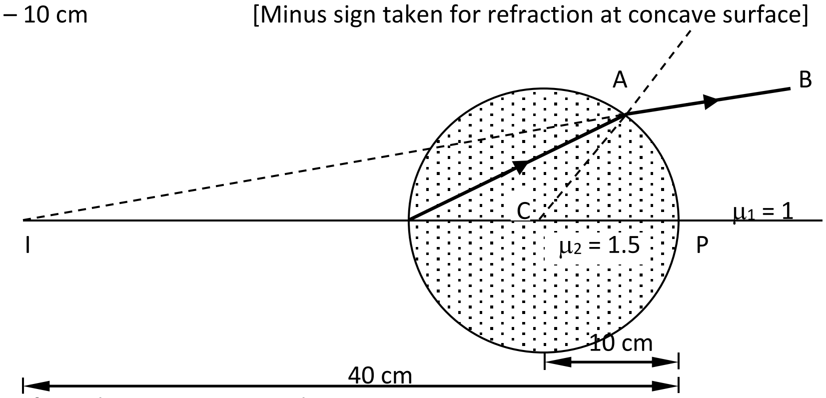
or $\frac{2.5}{x} = \frac{0.5}{R}$ or x = 5 R

Q. 6. ***A mark placed on the surface of a glass sphere is viewed through glass from an oppositely directed position. If the diameter of the sphere is 20 cm; find the position of the image. Refractive index of glass is 1.5.***

Sol. Fig. shows a glass sphere of radius 20 cm. The mark O on its surface acts as object. The incident ray OA is in glass and refracted ray AB is in air. I is the image of O. Thus

$\mu_1 = 1$, $\mu_2 = 1.5$, u = OP = – 20 cm

R = – 10 cm [Minus sign taken for refraction at concave surface]



As light passes from denser to rarer medium, so

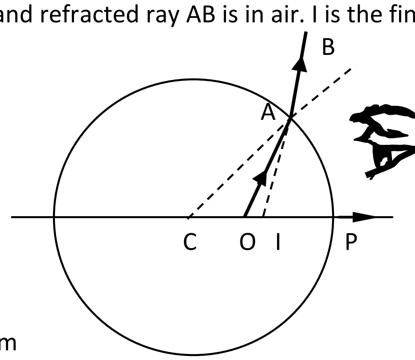
$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{or} \quad \frac{1}{v} + \frac{1.5}{20} = \frac{1 - 1.5}{-10}$$

or $\frac{1}{v} = \frac{1}{20} - \frac{3}{40} = \frac{2-3}{40} = -\frac{1}{40}$ or v = – 40 cm

Negative sign shows that the image is virtual. It is formed on the same side of the refracting surface as the object as a distance of the 40 cm from the pole P.

Q. 7. ***A small air bubble in a glass sphere of radius 2 cm appears to be 1 cm from the surface when looked at, along a diameter. If the refractive index of glass is 1.5, find the true position of the air bubble.***

Sol. Here incident ray OA is in glass and refracted ray AB is in air. I is the final image of the air bubble at O.



Here $\mu_1 = 1$, $\mu_2 = 1.5$

$R = -5 \times 10^{-2}$ m = – 5 cm

$u = OP = CP - CO = -(5 \times 10^{-2} - 2 \times 10^{-2})$

$= -3 \times 10^{-2}$ m = – 3 cm

As $\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$

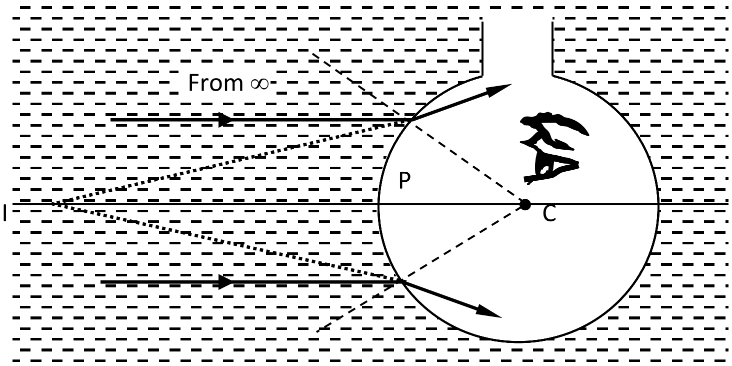
∴ $\frac{1}{v} - \frac{1.5}{-3} = \frac{1 - 1.5}{-5} = \frac{1}{10}$

or $\frac{1}{v} = \frac{1}{10} - \frac{1}{2} = -\frac{4}{10}$ or v = – 2.5 cm

Thus, the soap bubble appears at 2.5 cm from P.

Q. 8. ***An empty spherical flask of diameter 15 cm is placed in water of refractive index 4/3. A parallel beam of light strikes the flask. Where does it get focussed, when observed from within the flask?***

Sol. Fig. shows a spherical flask placed inside water. The centre of the flask is the centre of curvature of the spherical refracting surface. A parallel beam of light falling on the flask diverges and appears to come from point I.



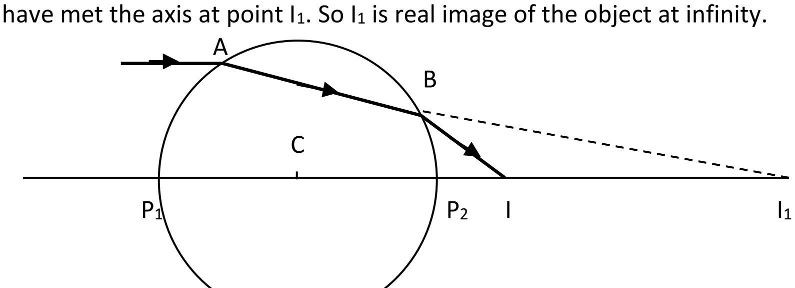
Here $\mu_1 = 1$, $\mu_2 = 4/3$, $u = -\infty$, $R = +\frac{15}{2}$ cm

As the light travels from denser to rarer medium, so

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{or} \quad \frac{1}{v} - \frac{4/3}{-\infty} = \frac{1 - 4/3}{15/2} \quad \text{or} \quad v = -45/2 = -22.5 \text{ cm}$$

Q. 9. ***A sunshine recorder globe of 30 cm diameter is made of glass of refractive index $\mu = 1.5$. A ray enters the globe parallel to the axis. Find the position from the centre of the sphere where the ray crosses the axis.***

Sol. For refraction at surface AP_1. The ray SA parallel to the axis is incident on glass globe at point A. If the glass medium were continuous, it would have met the axis at point I_1. So I_1 is real image of the object at infinity.



∴ $\mu_1 = 1$, $\mu_2 = 1.5$, $\mu = -\infty$, $R = +\frac{30}{2} = 15$ cm

Let $P_1 I_1 = v'$. As refraction takes place from rarer to denser medium, we use the relation

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v'} - \frac{1}{-\infty} = \frac{1.5 - 1}{15} \quad \text{or} \quad v' = \frac{1.5 \times 15}{0.5} = 45 \text{ cm}$$

For refraction at surface BP_2: The ray AB (before meeting point I_1) suffers another refraction at surface BP_2. The real image I_1 acts as virtual object for refraction at surface BP_2 and I is the real image.

∴ $u = P_2 I_1 - P_1 I_1 - P_1 P_2 = 45 - 30 = 15$ cm

$R = -15$ cm

Let $P_2 I = v$. As refraction occurs from denser to rarer medium, so we use the relation

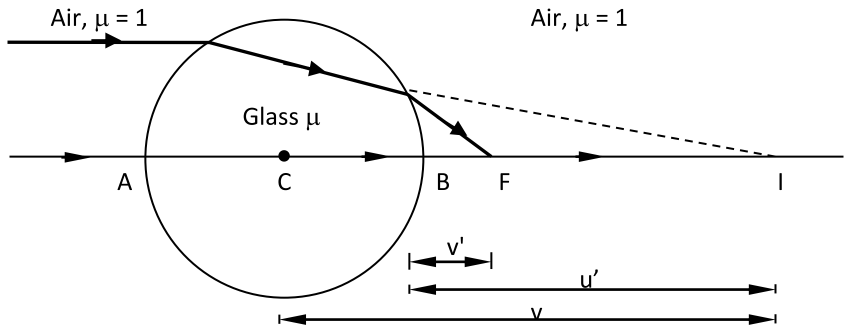
$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

$$\frac{1}{v} - \frac{1.5}{15} = \frac{1 - 1.5}{-15} \quad \text{or} \quad \frac{1}{v} = \frac{2}{15} \quad \text{or} \quad v = 7.5 \text{ cm}$$

Distance of image I from the centre of the sphere is $CI = CP_2 + P_2 I = 15 + 7.5 = 22.5$ cm.

Q. 10. ***A parallel beam falls on a solid glass sphere at normal incidence. Prove that the distance of the image from the outer edge in terms of refractive index μ and radius R of sphere is $\frac{R(2-\mu)}{2(\mu-1)}$.***

Sol. Fig. shows the refraction of a parallel beam of light through a glass sphere of refractive index μ at normal incidence.



For refraction at first surface from air to glass,

$\mu_1 = 1, \quad \mu_2 = \mu, \quad u = \infty, \quad R = +R$

As $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

∴ $\dfrac{\mu}{v} - \dfrac{1}{\infty} = \dfrac{\mu - 1}{R}$ or $v = \dfrac{\mu R}{\mu - 1}$

This image formed at I acts as virtual object for second surface. For refraction at second surface from glass to air,

$u' = BI = AI - AB = v - 2R$

$= \dfrac{\mu R}{\mu - 1} - 2R = \dfrac{\mu R - 2R(\mu - 1)}{\mu - 1} = \left(\dfrac{2 - \mu}{\mu - 1}\right) R$

Now $\mu_1 = \mu, \quad \mu_2 = 1, R = -R$

∴ $\dfrac{1}{v'} - \dfrac{\mu}{u'} = \dfrac{1 - \mu}{R}$ or $\dfrac{1}{v'} - \dfrac{\mu(\mu - 1)}{R(2 - \mu)} = \dfrac{\mu - 1}{R}$

or $\dfrac{1}{v'} = \dfrac{\mu - 1}{R} + \dfrac{\mu(\mu - 1)}{R(2 - \mu)}$

$= \dfrac{(\mu - 1)(2 - \mu + \mu)}{R(2 - \mu)} = \dfrac{2(\mu - 1)}{R(2 - \mu)}$

∴ $v' = \dfrac{R(2 - \mu)}{2(\mu - 1)}$

LENS MAKER'S FORMULA

This formula relates the focal length of a lens to the refractive index of the lens material and the radii of curvature of its two surfaces.

● This formula is so called because it is used by manufacturers to design lenses of required focal length from a glass of given refractive index.

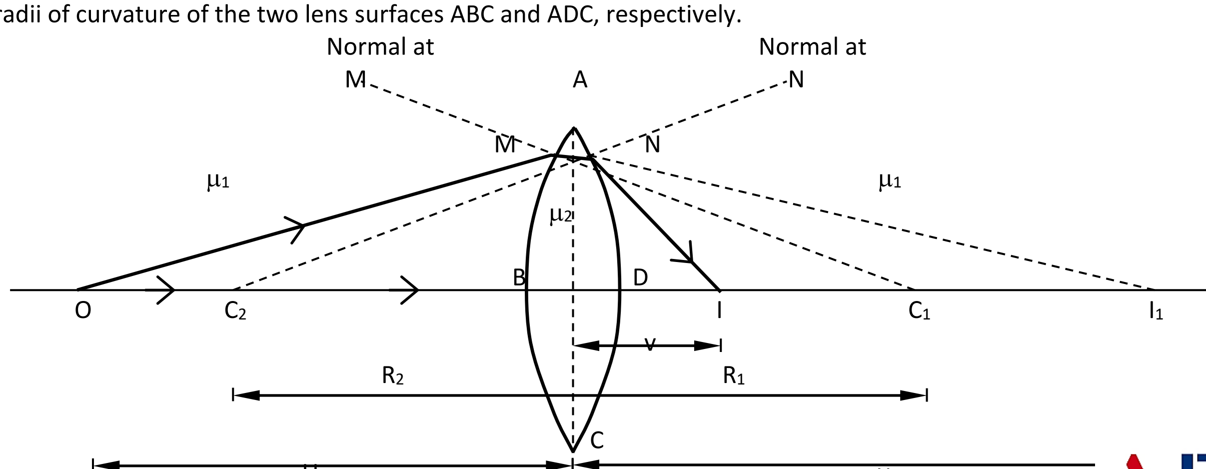
◈New Cartesian sign convention for spherical lenses:

(i) All distances are measured from the optical centre of the lens.
(ii) The distances measured in the direction of incident light are positive.
(iii) The distances measured in the opposite direction of incident light are negative.

◈Assumptions made in the derivation of Lens maker's formula:

(i) The lens used in thin so that the distances measured from its surfaces may be taken equal to those measured fro its optical centre.
(ii) The object is a point object placed on the principle axis. (iii) The aperture of the lens is small.
(iv) All the rays are paraxial, i.e., they make very small angles with the normals to the lens faces and with the principal axis.
Lens maker's formula for a double convex lens:

•Lens's maker's formula for a double convex lens: consider a thin double convex lens of refractive index μ_2 placed in a medium of refractive index μ_1. Here $\mu_1 < \mu_2$. Let B and D be the poles, C_1 and C_2 be the centres of curvature, and R_1 and R_2 be the radii of curvature of the two lens surfaces ABC and ADC, respectively.



[Refraction through a double convex lens]

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. The ray OM is incident on the first surface ABC. It is refracted along MN, bending towards the normal at this surface. If the second surface ADC were absent, the ray MN would have met the principal axis at I_1. So, we can treat I_1 as the real image formed by first surface ABC in the medium of refractive index μ_2.

For refraction at surface ABC, we can write the relation between the object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad \text{... (1)}$$

But actually, the ray MN suffers another refraction at surface ADC, bending away from the normal at point N. The emergent ray meets the principal axis at point I which is the final image of formed by the lens. For refraction at second surface, I_1 acts as a virtual object placed in the medium of refractive index μ_2 and I is the real image formed in the medium of refractive index μ_1. Therefore, the relation between the object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \qquad \text{... (2)}$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{... (3)}$$

If the object is placed at infinity ($u = \infty$), the image will be formed at the focus, i.e., v = f. Therefore,

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{... (4)}$$

This is lens maker's formula.

When the lens is placed in air, $\mu_1 = 1$, and $\mu_2 = \mu$. The lens maker's formula takes the form:

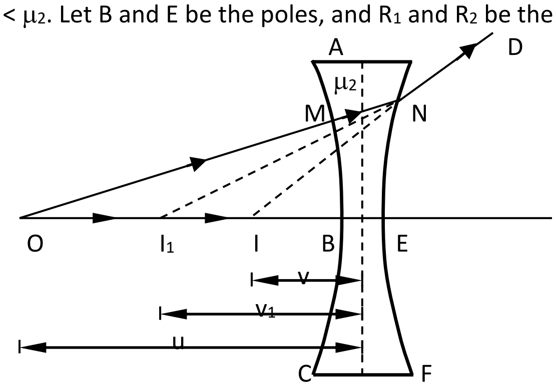
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

From equations (3) and (4), we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is the thin lens formula which gives relationship between u, v and f of lens.

●**Lens maker's formula for a double concave lens**: consider a thin double concave lens of refractive index μ_2 placed in a medium of refractive μ_1. Here $\mu_1 < \mu_2$. Let B and E be the poles, and R_1 and R_2 be the radii of curvature of the two-lens surface ABC and DEF, respectively.



[Refraction through a double concave lens]

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. First the spherical surface ABC forms its virtual image I_1. As refraction occurs from rarer to denser medium, so we can write the relation between object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad \text{... (1)}$$

But the lens material is not continuous. The ray MN suffers another refraction at N and emerges along IN. So, I is the final virtual image of the point object O. The image I_1 acts as an object for refraction at surface DEF from denser to rarer medium. So, the relation between object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_2}{v} - \frac{\mu_1}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \qquad \text{... (2)}$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If an object is placed at infinity, then the image is formed at the focus i.e., v = f, so

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ **This is lens maker's formula.**

When the lens is placed in air, $\mu_1 = 1$, and $\mu_2 = \mu$. The lens maker's formula takes the form:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Examples based on Lens Maker's Formula

Formulae used 1. For the lens of material of refractive index μ_2 placed in a medium of refractive index μ_1,

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

2. When the lens is placed in air, $\mu_1 = 1$ and $\mu_2 = \mu$

∴ $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

3. f and R are positive for convex surface and negative for concave surfaces.

Units used Focal length f and radii of curvature R_1 and R_2 are in metre, refractive indices μ_1, μ_2 and μ have no units.

Q. 1. The radius of curvature of each face of biconcave lens, made of glass of refractive index 1.5 is 30 cm. Calculate the focal length of the lens in air.

Sol. Here $\mu = 1.5$ $R_1 = -30$ cm, $R_2 = +30$ cm

Using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{-30} - \frac{1}{30}\right)$$

$$= -0.5 \times \frac{2}{30} = -\frac{1}{30}$$ ∴ f = – 30 cm

Q. 2. The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. If focal length is 12 cm, what is the refractive index is glass?

Sol. Here f = + 12 cm, $R_1 = +10$ cm

$R_2 = -15$ cm, μ = ?

As $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∴ $$\frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = (\mu - 1) \times \frac{5}{30}$$

or $$\mu - 1 = \frac{6}{12} = 0.5$$ ∴ $\mu = 1.5$

Q. 3. A biconvex lens has a focal length half the radius of curvature of either surface. What is the refractive index of lens material?

Sol. Here f = R/2, $R_1 = R$, $R_2 = -R$

As $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∴ $$\frac{2}{R} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$ or $\mu = 2$

Q. 4. The radii of curvature of a double convex lens of glass (μ = 1.5) are in the ratio 1:2. This lens renders the rays parallel coming from an illuminated filament at a distance of 6 cm. Calculate the radii of curvature of its surfaces.

Sol. Here f = + 6 cm, $\mu = 1.5$, $R_1 = +R$

$R_2 = -2R$

As $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∴ $$\frac{1}{6} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{2R}\right)$$ or $$\frac{1}{6} = 0.5 \times \frac{3}{2R}$$

or $$R = \frac{0.5 \times 3 \times 6}{2} = 4.5 \text{ cm}$$

∴ $R_1 = +R = +4.5$ cm and $R_2 = -2R = -9.0$ cm

Q. 5. ***The radii of curvature of a double convex lens of glass (μ = 1.5) are in the ratio 1:2. This lens renders the rays parallel coming from an illuminated filament at a distance of 6 cm. Calculate the radii of curvature of its surfaces.***

Sol. Here f = + 6 cm, μ = 1.5, R_1 = + R, R_2 = – 2R

As $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore$ $\frac{1}{6} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{2R}\right)$ or $\frac{1}{6} = 0.5 \times \frac{3}{2R}$

or $R = \frac{0.5 \times 3 \times 6}{2} = 4.5$ cm

$\therefore$ R_1 = + R = + 4.5 cm and R_2 = – 2R = – 9.0 cm

Q. 6. ***Find the radius of curvature of the convex surface of a plano-convex lens, whose focal length is 0.3 m and the refractive index of the material of the lens is 1.5***

Sol. Here μ = 1.5, f = + 0.3 m, $R_1 = \infty$, R_2 = – R

Using length maker's formula,

$\frac{1}{f} = (\mu - 1)\ \frac{1}{R_1} - \frac{1}{R_2}$

or $\frac{1}{+0.3} = (1.5 - 1)\left(\frac{1}{\infty} + \frac{1}{R}\right)$

or $\frac{1}{0.3}\left(= 0.5 \times \frac{1}{R}\right)$ or R = 0.15 m

Q. 7. ***A convex lens of focal length 0.2 m and made of glass (μ = 1.50) is immersed in water (μ = 1.33). Find the change in the focal length of the lens.***

Sol. For glass lens in air,

${}^a\mu_g = 1.5$, $f_a = 0.2$ m

As $\frac{1}{f_a} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore$ $\frac{1}{0.2} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ or $\frac{1}{R_1} - \frac{1}{R_2} = 10$

For the same lens in water, ${}^a\mu_w = 1.33$

$\frac{1}{f_w} = ({}^w\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{1.5}{1.33} - 1\right) \times 10 = 0.17 \times 10$

or $f_w = \frac{133}{170} = 0.78$

$\therefore$ Change in focal length = $f_\omega - f_a$ = 0.78 – 0.20 = 0.58 m

Q. 8. ***If the refractive index from air to glass is 3/2 and that from air to water is 4/3, find the ratio of focal lengths of a glass lens is water and in air.***

Sol. Here ${}^a\mu_g = \frac{3}{2}$, ${}^a\mu_w = \frac{4}{3}$

${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

Let f_w and f_a be the focal lengths of glass lens in water and air respectively. Then

$\frac{1}{f_w} = ({}^w\mu_g - 1)\ \frac{1}{R_1} - \frac{1}{R_2}$ ... (i)

$\frac{1}{f_a} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ... (ii)

Dividing (ii) and (i), we get

$\frac{f_w}{f_a} = \frac{{}^a\mu_g - 1}{{}^w\mu_g - 1} = \frac{3/2 - 1}{9/8 - 1} = 4 : 1$

Q. 9. ***A double convex lens has a focal length of 25 cm in air. When it is dipped into a liquid of refractive index 4/3, its focal length is increased to 100 cm. Find the refractive index of the lens material.***

Sol. For the lens in air: f_a = 25 cm

Let μ = refractive index of lens material relative to air

Then $\frac{1}{f_a} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\dfrac{1}{25} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ ... (i)

For the lens in liquid: $f_1 = 100$ cm, $\mu_1 = \dfrac{4}{3}$, $\mu_2 = \mu$

$\therefore$ $\dfrac{1}{f_e} = \left(\dfrac{\mu_2}{\mu_1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

or $\dfrac{1}{100} = \left(\dfrac{\mu}{4/3} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ ... (ii)

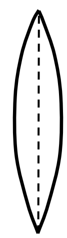
Dividing (i) by (ii), we get

$\dfrac{100}{25} = \dfrac{\mu - 1}{3\mu/4 - 1}$ or $3\mu - 4 = \mu - 1$ or $\mu = \dfrac{3}{2} = 1.5$

Q. 10. ***An equiconvex lens is focal length 15 cm is cut into two equal halves as shown in Fig. What is the focal length of each half?***

Sol. For the equiconvex lens, let

$R_1 = +R$, $R_2 = -R$



Then from lens maker's formula,

$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R} + \dfrac{1}{R}\right) = \dfrac{2(\mu - 1)}{R}$ ... (i)

For each half lens, $R_1 = R$, $R_2 = -\infty$

$\therefore$ $\dfrac{1}{f'} = (\mu - 1)\left(\dfrac{1}{R} - \dfrac{1}{-\infty}\right) = \dfrac{\mu - 1}{R}$ ... (ii)

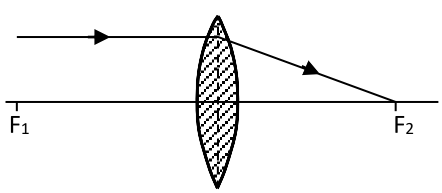
Dividing (i) by (ii), we get $\dfrac{f'}{f} = 2$

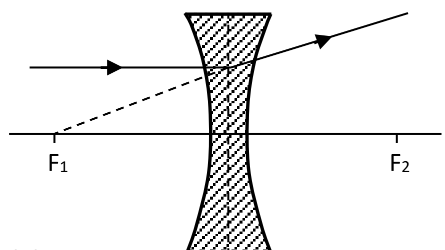
or $f' = 2f = 2 \times 15 = 30$ cm

RULES FOR DRAWING IMAGES FORMED BY SPHERICAL LENSES

The position of the image formed by any spherical lens can be found by considering any two of the following rays of light coming from a point on the object.

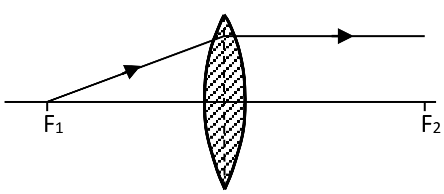
(i) A ray from the object parallel to the principal axis after refraction passes through the second principal focus F_2 [in a convex lens, as shown in Fig. (a)] or appears to diverge [in a concave lens, as shown in Fig. (b)] from the first principal focus F_1.

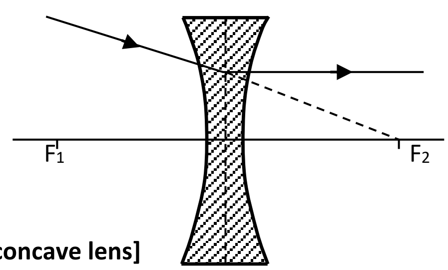




[Path of ray incident parallel to the principal axis of (a) convex lens (b) concave lens]

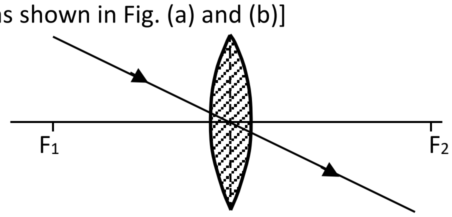
(ii) A ray of light passing through the first principal focus [in a convex lens, as shown in Fig. (a) or appearing to meet at it (in a concave lens, as shown in Fig. (b)] emerges parallel to the principal axis after refraction.

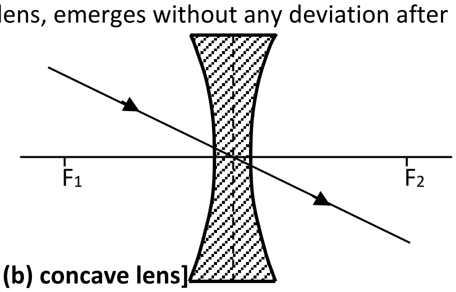




[Path of a ray passing through focus of (a) convex lens (b) concave lens]

(iii) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction, as shown in Fig. (a) and (b)]





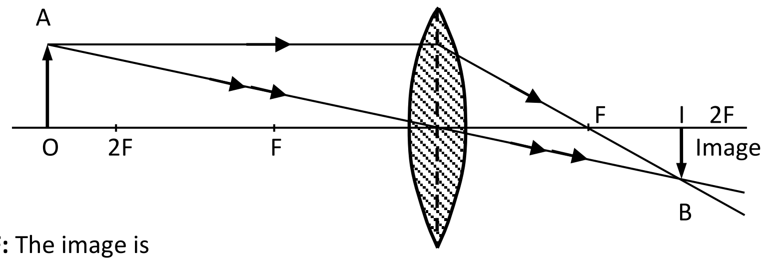
[Path of a ray passing through the optical centre (a) convex lens (b) concave lens]

Formation of images by spherical lenses

(a) Object beyond 2F: The image is

(i) between F and 2F (ii) real
(iii) inverted (iv) smaller

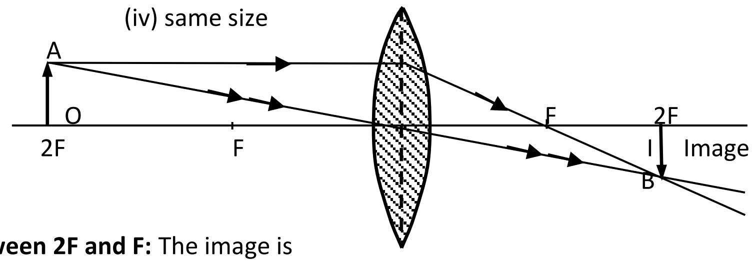
(a)



(b) Object at 2F: The image is

(i) at 2F (ii) real
(iii) inverted (iv) same size

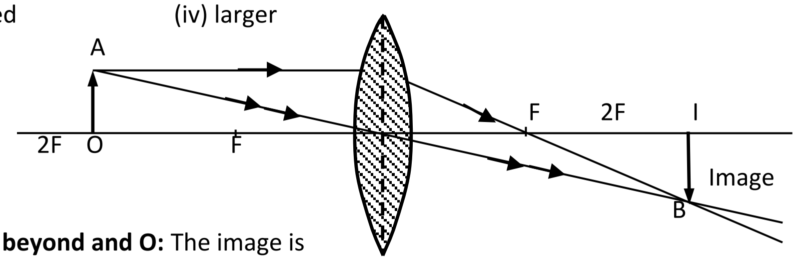
(b)



(c) Object between 2F and F: The image is

(i) beyond 2F (ii) real
(iii) inverted (iv) larger

(c)



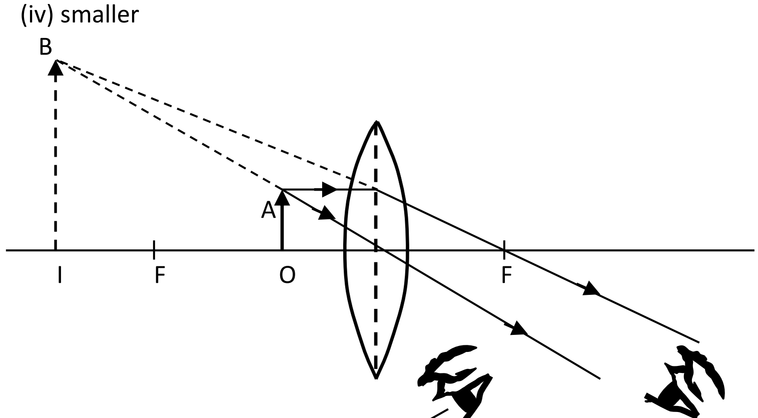
(d) Object beyond and O: The image is

(i) behind object (ii) virtual
(iii) erect (iv) larger

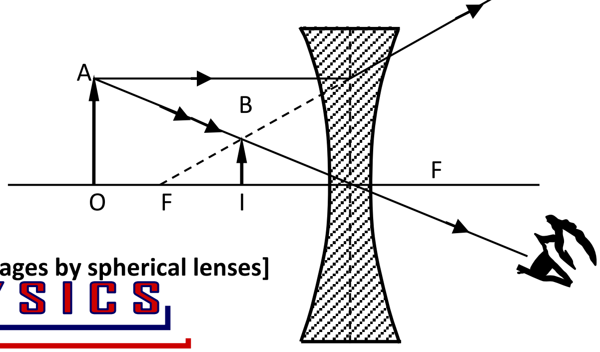
(e) Object in any position: The image is

(i) in front of object (ii) virtual
(iii) erect (iv) smaller

(d)



(e)



THIN LENS FORMULA

Thin lens formula is a mathematical relation between the object distance u, image distance v and focal length f of a spherical lens. This relation is: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

In words, we can say that

$$\frac{1}{\text{Image distance}} - \frac{1}{\text{Object distance}} = \frac{1}{\text{Focal length}}$$

This formula is valid for both convex and concave lenses for both real and virtual images..

●Assumptions used in the derivation of lens formula:

(i) The lens used is thin.

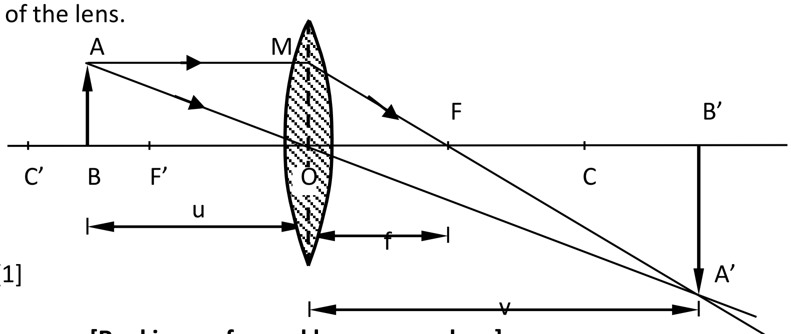
(ii) The aperture of the lens is small.

(iii) The incident and refracted rays make small angles with the principal axis.

(iv) The object is a small object placed on the principal axis.

●Derivation of thin lens formula for a convex lens when it forms a real image: As shown in Fig. consider an object AB placed perpendicular to the principal axis of a thin convex lens between its F′ and C′. A real, inverted and magnified image A′B′ is formed beyond C on the other side of the lens.

[1]



[Real image formed by a convex lens]

ΔA′B′O and ΔABO are similar,

∴ $\frac{A'B'}{AB} = \frac{OB'}{BO}$ ... (1)

Also ΔA′B′F and ΔMOF are similar,

∴ $\frac{A'B'}{MO} = \frac{FB'}{OF}$

But MO = AB,

∴ $\frac{A'B'}{AB} = \frac{FB'}{OF}$ ... (2)

From (1) and (2), we get

$\frac{OB'}{BO} = \frac{FB'}{OF} = \frac{OB' - OF}{OF}$

using new Cartesian sign convention, we get

Object distance, BO = – u

Image distance, OB′ = + v

Focal length, OF = + f

∴ $\frac{v}{-u} = \frac{v - f}{f}$

or vf = – uv + uf or uv = uf – vf

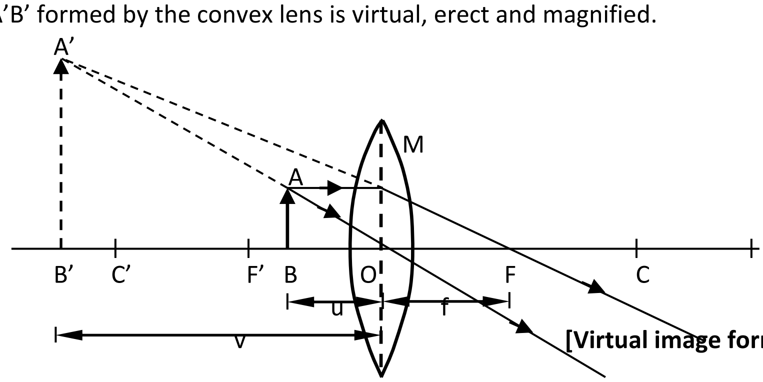
Dividing both sides by uvf, we get

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

This proves the lens formula for a convex lens when it forms a real image.

●Derivation of this lens formula for a convex lens when it forms a virtual image: As shown in optical centre O and the focus F of a convex lens, the image A′B′ formed by the convex lens is virtual, erect and magnified.

[2]



[Virtual image formed by a convex lens]

Triangles A′B′O and ABI are similar,

∴ $\frac{A'B'}{AB} = \frac{B'O}{BO}$ ... (1)

Also, triangles A′B′F and MOF are similar

∴ $\frac{A'B'}{AB} = \frac{B'F}{OF}$

But MO = AB, therefore

$$\frac{A'B'}{AB} = \frac{B'F}{OF} \qquad \text{... (2)}$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{B'F}{OF} = \frac{B'O + OF}{OF}$$

Using new cartesian sign convention,

$$BO = -u,\ B'O = -v,\ OF = +f$$

$$\therefore \quad \frac{-v}{-u} = \frac{-v + f}{f}$$

or $\quad -vf = uv - uf$

or $\quad uv = uf - vf$

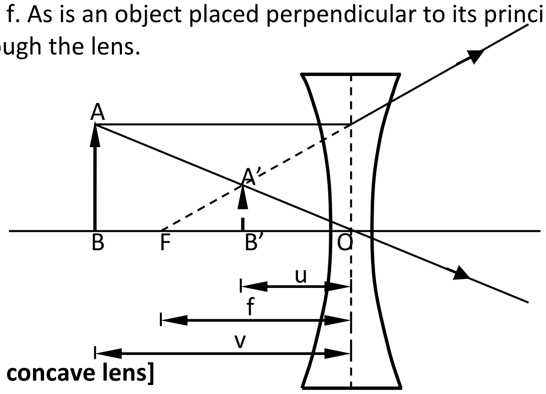
Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a convex lens when it forms a virtual image.

●Derivation of thin lens formula for a concave lens: As shown in Fig., suppose O be the optical centre and F be the principal focus of concave lens of focal length f. As is an object placed perpendicular to its principal axis. A virtual, erect and diminished image A'B' is formed due to refraction through the lens.

[3]



[Virtual image formed by a concave lens]

As $\quad \Delta A'B'O \sim \Delta ABO$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'O}{BO} \qquad \text{... (1)}$$

Also, $\quad \Delta A'B'F \sim \Delta MOF$

$$\therefore \quad \frac{A'B'}{MO} = \frac{FB'}{FO}$$

But MO = AB, therefore,

$$\frac{A'B'}{AB} = \frac{FB'}{FO} \qquad \text{... (2)}$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{FB'}{FO} = \frac{FO - B'O}{FO}$$

Using new Cartesian sign convention, we get

$$BO = -v,\ B'O = -v, \quad FO = -f$$

$$\therefore \quad \frac{-v}{-u} = \frac{-f + v}{-f}$$

or $\quad vf = uf - uv \quad$ or $\quad uv = uf - vf$

dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a concave lens.

LINEAR MAGNIFICATION

The linear magnification produced by a lens is defined as the ratio of the size of image formed by the lens to the size of the object. It is denoted by m. Thus

$$m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{h_2}{h_1}$$

Convex lens: Fig. [1] shows a ray diagram for the formation of image A'B' of a finite object AB by a convex lens.

Now $\quad \Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$$

Applying the new cartesian sign convention, we get

$A'B' = -h_2 \quad$ [Downward image height]

$AB = +h_1 \quad$ [Upward object height]

$OB = -u \quad$ [Image distance on left]

$OB' = +v \quad$ [Image distance on right]

$$\therefore \quad \frac{-h_2}{+h_1} = \frac{+v}{-u} \quad \text{or} \quad \frac{h_2}{h_1} = \frac{v}{u}$$

Magnification, $m = \frac{h_2}{h_1} = \frac{v}{u}$

Concave lens: Fig. [3] shows the formation of a virtual image A'B' of a finite object AB by a concave lens.

Now $\Delta AOB \sim \Delta A'OB'$

$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$

Applying the new cartesian sign convention, we get $A'B' = +h_2$, $AB = +h_1$

$OB' = -v$, $OB = -u$ $\quad \therefore \quad \frac{+h_2}{+h_1} = \frac{-v}{-u}$

$\therefore$ **Magnification,** $\mathbf{m = \frac{h_2}{h_1} = \frac{v}{u}}$

Linear magnification in terms of u and f: The thin lens formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Multiplying both sides by u, we get

$$\frac{u}{v} - 1 = \frac{u}{f} \quad \text{or} \quad \frac{u}{v} = 1 + \frac{u}{f} = \frac{f+u}{f}$$

$$\therefore \quad m = \frac{v}{u} = \frac{f}{f+u}$$

Linear magnification in terms of v and f. The thin lens formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Multiplying both sides by v, we get

$$1 - \frac{v}{u} = \frac{v}{f}$$

$$\therefore \quad m = \frac{v}{u} = 1 - \frac{v}{f} = \frac{f-v}{f}$$

Hence $\mathbf{m = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}}$

Conceptual.................................

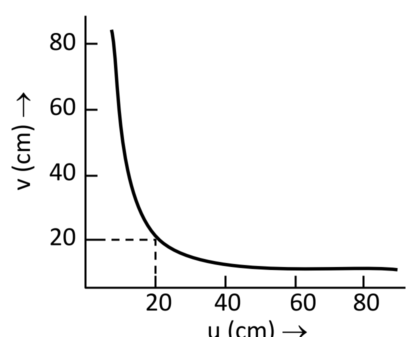
- **The same thin lens formula is valid for both convex and concave lenses and for both real and virtual images.**
- **When |m| > 1, the image is magnified.**
- **When |m| < 1, the image is diminished.**
- **When |m| = 1, the image is of the same size as the object.**
- **When m is positive (or v is negative), the image is virtual and erect.**
- **When m is negative (or v is positive), the image is real and inverted.**

Examples based on Thin Lens Formula & Linear Magnification

1. Focal length of any lens is given by the thin lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$
2. Magnification, $m = \frac{h_2}{h_1} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$
3. In cartesian sign convention, u is taken negative
4. In case of convex lens, v is positive for real image and negative for virtual image and f is positive.
5. In case of concave lens, u, v and f are all negative.
6. Magnification m is positive for virtual image and negative for real image.

Units used Distances u, v and f are in cm or m.

Q. 1. ***A lens forms a real image of an object. The distance of an object to the lens is 4 cm and the distance of the image from the lens is v cm. The given graph shows the variation of v with u. (i) What is the nature of the lens?***



Sol. (i) As the lens forms a real image, if must be a convex lens.

(ii) From the graph, when u = 20 cm, we have v = 20 cm.

For the convex lens forming a real image, u is negative and v and f are positive.

∴ u = – 20 cm, v = + 20 cm

Using thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{20} - \frac{1}{-20} = \frac{1}{10} \quad \text{or} \quad f = +10 \text{ cm}$$

Q. 2. ***A needle placed 45 cm from a lens forms an image on a screen placed 90 cm on the other side of the lens. Identify the type of the lens and determine its focal length. What is the size of image if the size of image if the size of the needle is 5.0 cm?***

Sol. Here u = – 45 cm, v = + 90 cm

Using the thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{90} + \frac{1}{45} = \frac{1+2}{90} \quad \therefore \quad f = +30 \text{ cm}$$

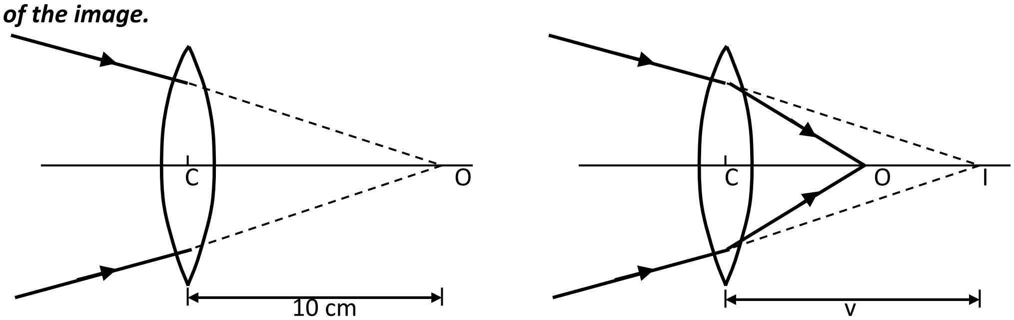
Positive value of indicates that the lens is converging

Magnification, $m = \frac{h_2}{h_1} = \frac{v}{u}$ or $\frac{h_2}{5} = \frac{90}{-45}$ [∵ h_1 = 5 cm]

∴ Size of image, h_2 = – 10 cm   Negative sign indicates that the image is real and inverted.

Q. 3. ***Converging light rays are falling on a convex lens as shown in Fig. (a). If the focal length of the lens is 30 cm, then find the position of the image.***

Sol.



The incident light rays appear to converge at point O. So O acts as virtual object for the lens and I is its real image as shown in Fig. (b). Here u = + 10 cm, f = + 30 cm

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} \quad \text{or} \quad v = +7.5 \text{ cm}$$

Thus, the image is formed at 7.5 cm from the lens on the same side as the virtual object O.

Q. 4. ***A convergent beam of light passes through a diverging lens of focal length 0.2 m and comes to focus at distance 0.3 m behind the lens. Find the position of the point at which the beam would converge in the absence of the lens.***

Sol. As the focal length of a diverging lens is negative and the distance measured in the direction of incident and the distance measured in the direction of incident ray is positive,

f = – 0.2 m, v = + 0.3 m

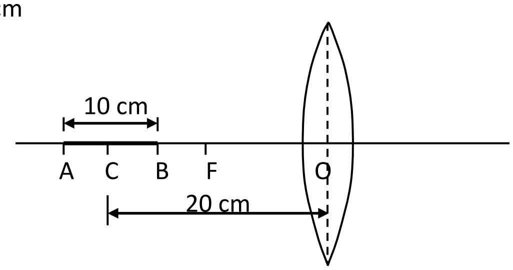
As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ∴ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{0.3} + \frac{1}{0.2} = \frac{50}{6}$

or u = + 0.12 m

So, in the absence of the lens the beam would converge at a point 0.12 m from the position of the lens.

Q. 5. ***A needle 10 cm long is placed along the axis of a convex lens of focal length 10 cm such that the middle point of the needle is at a distance of 20 cm from the lens. Find the length of the image of the needle.***

Sol. Fig. shows a needle AB of length 10 cm, placed on the axis of a convex lens.



Here CO = 20 cm, AO = 20 + 5 = 25 cm and BO = 20 – 5 = 15 cm

For the image of end, A of the needle u_1 = AO = – 25 cm, f = + 10 cm

Using thin lens formula,

$$\frac{1}{v_1} = \frac{1}{f} + \frac{1}{u_1} = \frac{1}{+10} + \frac{1}{-25} = \frac{3}{50}$$

or v_1 = 50/3 = 16.67 cm

For the image of end B of the needle

u_2 = BO = – 15 cm, f = + 10 cm

∴ $\frac{1}{v_2} = \frac{1}{f} + \frac{1}{\mu_2} = \frac{1}{+10} + \frac{1}{-15} = \frac{1}{30}$ or v_2 = 30 cm

Hence the length of the image of needle AB = $v_2 - v_1$ = 30 – 16.67 = 13.37 cm

Q. 6. ***A double convex lens made of glass of refractive index 1.5 has its both surfaces of equal radii of curvature of 20 cm each. An object of 5 cm height is placed at a distance of 10 cm from the lens. Find the position, nature and size of the image.***

Sol. Here $\mu = 1.5$, $R_1 = +20$ cm, $R_2 = -20$ cm

Using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{20} + \frac{1}{20}\right) = 0.5 \times \frac{2}{20} = \frac{1}{20}$$

or $f = +20$ cm   Now, $u = -10$ cm, $f = +20$ cm

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20} \quad \text{or} \quad v = -20 \text{ cm}$$

Magnification, $m = \frac{h_2}{h_1} = \frac{v}{u}$ or $\frac{h_2}{5 \text{ cm}} = \frac{-20}{-10}$

or $h_2 = 2 \times 5 = 10$ cm

Hence a virtual and erect image of height 10 cm is formed at a distance of 20 cm from the lens on the same side as the object.

Q. 7. ***A double convex lens has 10 cm and 15 cm as its two radii of curvature. The image of an object placed 30 cm from the lens, is formed at 20 cm from the lens on the other side. Find the refractive index of the material of the lens. What will be the focal length? What will be the focal length of the lens, if it is immersed in water of refractive index 1.33 cm?***

Sol. Here $u = -30$ cm, $v = 20$ cm, $R_1 = +10$ cm, $R_2 = -15$ cm

Using thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{20} - \frac{1}{-30} = \frac{5}{60} = \frac{1}{12} \quad \text{or} \quad f = 12 \text{ cm}$$

Using lens maker's formula,

$$\frac{1}{f} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $\frac{1}{12} = ({}^a\mu_g - 1)\ \frac{1}{10} + \frac{1}{15} = ({}^a\mu_g - 1) \times \frac{1}{6}$

or ${}^a\mu_g = 1 + 6/12 = 1.5$

When the lens is immersed in water,

$$\frac{1}{f_w} = \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $\frac{1}{12} = ({}^a\mu_g - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = ({}^a\mu_g - 1) \times \frac{1}{6}$

or ${}^a\mu_g = 1 + \frac{6}{12} = 1.5$

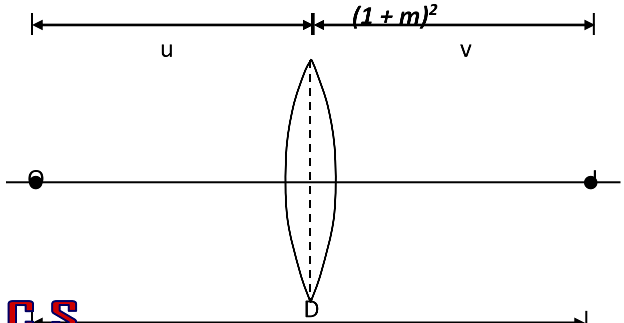
When the lens is immersed in water,

$$\frac{1}{f_w} = \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{10} + \frac{1}{15}\right) = \frac{0.17}{1.33} \times \frac{1}{6}$$

or $f_w = \frac{1.33 \times 6}{0.17} = 46.94$ cm

Q. 8. ***The distance between an object and a screen is D. A convex lens of focal length f if placed between the object and the screen. (i) Prove that in order to obtain the image of the object on the screen the minimum value of D should be 4 f. (ii) If m be the magnification of the image, prove that*** $f = \frac{mD}{(1+m)^2}$.



Sol. (i) From fig. it is clear, that if D is the distance between object and the screen, then

$u = -(D - v) = -D + v$

From the lens formula,

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$ or $v = \frac{uf}{u+f} = \frac{(-D+v)f}{-D+v+f}$

or $-vD + v^2 + vf = -Df + vf$

or $v^2 - Dv + Df = 0$ or $v = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$

As v must be positive for real image, so

$D^2 - 4Df \geq 0$ or $D \geq 4f$ i.e., to obtain image of the object on the screen, the minimum value of D should be 4 f.

(ii) Taking u negative in the formula $f = \frac{uv}{u-v}$, we get

$f = \frac{uv}{u+v} = \frac{uv\,(u+v)/u^2}{(u+v)^2/u^2} = \frac{v/u\,(u+v)}{\left(1+\frac{v}{u}\right)^2} = \frac{mD}{(1+m)^2}$

Q. 9. ***An illuminated object and a screen are placed 90 cm apart. What is the focal length and nature of the lens required to produce a clear image on the screen, twice the size of the object?***

Sol. As the image is real, the lens must be a convex lens and it should be placed between the object and the screen.

Let distance between object and convex lens = x,

then $u = -x$, $v = 90 - x$

Now $m = \frac{v}{u} = -2$ [Minus sign as image is real]

or $\frac{90-x}{-x} = -2$ or $90 - x = 2x$ or $x = \frac{90}{3} = 30$

∴ $u = -30$ cm, $v = +60$ cm

Now $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{60} - \frac{1}{-30} = \frac{3}{60} = \frac{1}{20}$ or $f = 20$ cm

Q. 10. ***The image obtained with a convex lens is erect and its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object. If the focal length of the lens is 20 cm, calculate the object and image distances.***

Sol. Here f = 20 cm, m = + 4 for a virtual image.

To calculate u, we have $m = \frac{f}{u+f}$ or $4 = \frac{20}{u+20}$ or $u = -15$ cm

To calculate v, we have

$m = \frac{f-v}{f}$ or $4 = \frac{20-v}{20}$ or $v = -60$ cm

Q. 11. ***A luminous object and a screen are placed on an optical bench and a converging lens is placed between them to throw a sharp image of the object on the screen, the linear magnification of the image of found to be 2.5. The lens is now moved 30 cm nearer the screen and a sharp image is again formed. Calculate the focal length of the lens.***

Sol. In Fig. let O and I be the positions of object and screen, respectively. Let L_1 and L_2 be the two conjugate positions of the lens, then

$OL_1 = L_2I = x$ (say)

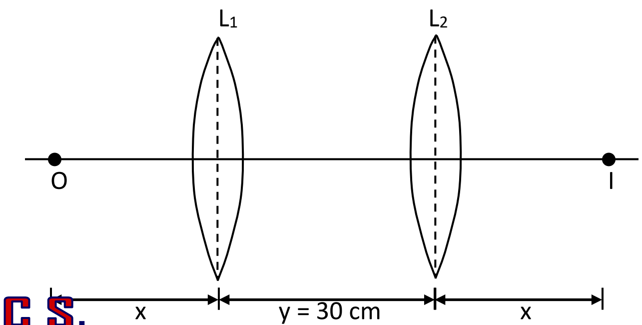
Because the u and v values are just interchanged

For the lens in position L_1, $u = OL_1 = -x$, $v = L_1I = 30 + x$

But magnification, $m = \frac{v}{U} = -2.5$ [minus sign taken as image is real]

or $30 + x / -x = -2.5$, $x = 20$ cm

∴ $u = -20$ cm and $v = 30 + 20 = 50$ cm

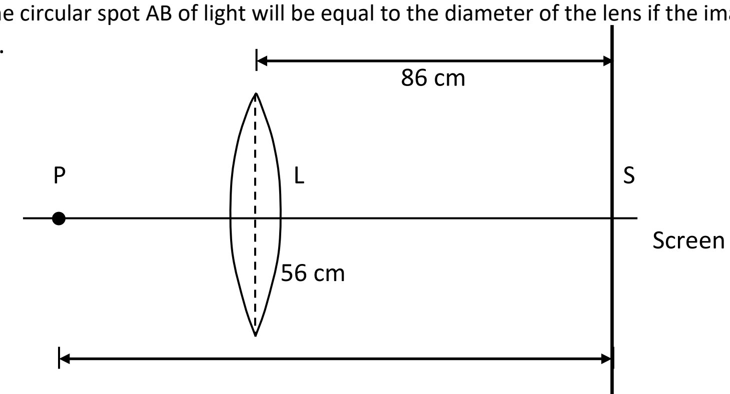


As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ∴ $\frac{1}{50} + \frac{1}{20} = \frac{1}{f}$

or $f = \frac{50 \times 20}{50 + 20} = 14.3$ cm

Q. 12. ***In Fig. a convex lens L is placed at a distance of 36 cm from a screen. If a point-source P is placed at 56 cm from the screen, then a circular spot of light of diameter equal to the diameter of the lens is formed. Show the image formation by a ray diagram. Calculate upto what distance the source be displaced so that its clear image can be formed on the screen.***

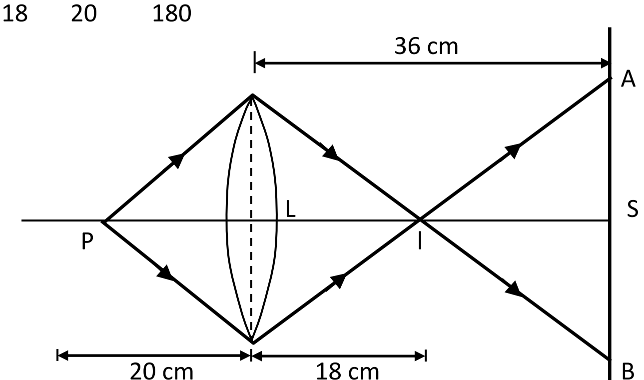
Sol. As is clear from Fig. the circular spot AB of light will be equal to the diameter of the lens if the image I is formed exactly in the middle of the lens and the screen.



∴ $u = -20$ cm, $v = +18$ cm

Using thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{18} + \frac{1}{20} = +\frac{19}{180}$$

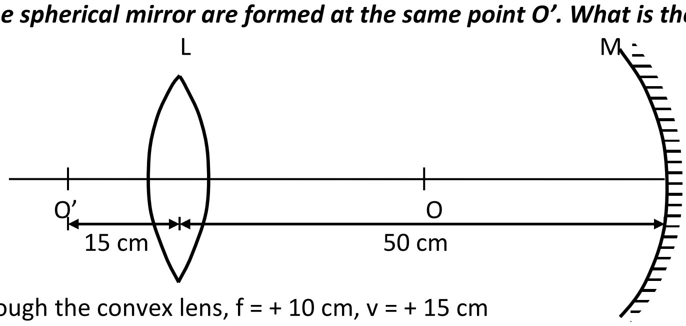


To obtain a clear image on the screen, the distance u of the source from the lens has to be changed. In that case, v = + 36 cm

∴ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{36} - \frac{19}{180} = -\frac{14}{180}$ or $u = -12.8$ cm

i.e., the source P should be at a distance of 12.86 cm from the lens. It must be displaced by 20 – 12.86 = 7.14 cm towards the lens.

Q. 13. ***In the accompanying diagram, the direct image formed by the lens (f = 10 cm) of an object placed at O and that formed after reflection from the spherical mirror are formed at the same point O'. What is the radius of curvature of the mirror?***



Sol. For refraction through the convex lens, f = + 10 cm, v = + 15 cm

∴ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{15} - \frac{1}{10} = \frac{-1}{30}$

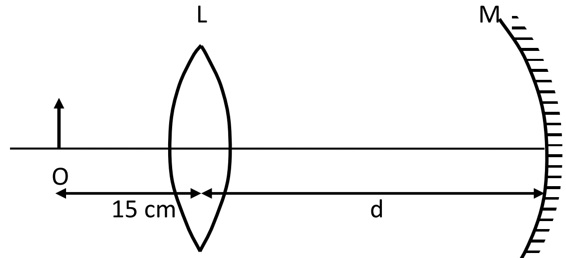
or $u = -30$ cm i.e., LO = 30 cm

The image formed first by reflection from the mirror and then by refraction through the lens will be located at O' only if the image formed by reflection from the mirror is formed at O i.e., if distance OM = R

Hence LO + OM = 30 cm + R = 50 cm

or R = 20 cm

Q. 14. ***Calculate the distance d, so that a real image of an object at O, 15 cm in front of a convex lens of focal length 10 cm be formed at the same point O. The radius of curvature of the mirror is 20 cm. Will the image be inverted or erect?***



Sol. The final image will be formed at the same point O if the concave mirror reverses the path of light incident on it. For this the image formed by the lens must be located at the centre of curvature of mirror M. Then the light will fall normally on and will retrace its path after reflection.

For refraction through the convex lens,

$u = -15$ cm $f = +10$ cm

$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-15} = \frac{1}{30}$ or $v = 30$ cm

For concave mirror, $R = 20$ cm

Hence $d = v + R = 30 + 20 = 50$ cm ; The final image formed at O will be an inverted image.

Q. 15. ***In the following ray diagram are given the positions of an object O, image I and two lenses L_1 and L_2. The focal length of L_1 is also given. Find the focal length of L_2.***

Sol. For the convex lens: $f = +15$ cm, $u = -40$ cm, $v = ?$

From thin lens formula,

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{15} - \frac{1}{40} = \frac{5}{120} = \frac{1}{24}$ or $v = +24$ cm

The image I' formed by the convex lens serves an object for the concave lens.

$\therefore$ For the concave lens:

$u = +(24 - 14) = +10$ cm, $v = +30$ cm, $f = ?$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{30} - \frac{1}{10} = -\frac{2}{30} = -\frac{1}{15}$ or $f = -15$ cm

Q. 16. ***From the ray diagram shown below, calculate the focal length of the concave lens.***

Sol. **For the convex lens:**

$f = +20$ cm, $u = -60$ cm, $v = ?$

From thin lens formula,

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{60} = +\frac{2}{60} = +\frac{1}{30}$

or $v = +30$ cm

The image I' formed by the convex lens serves as an object for the concave lens. But the rays converging on the concave lens become parallel after refraction through it and form image at infinity.

$\therefore$ **For the concave lens:**

$u = +(30 - 10) = +20$ cm, $v = \infty$, $f = ?$

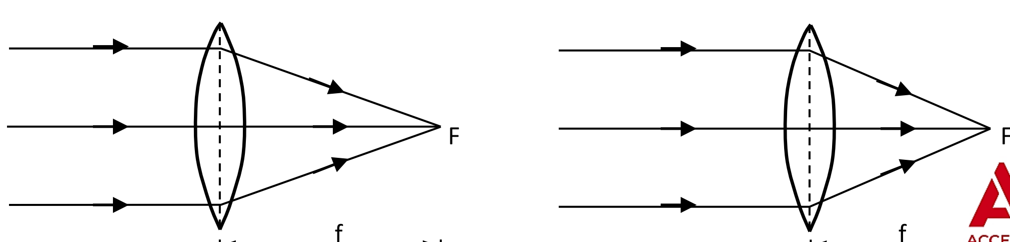
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{\infty} - \frac{1}{20} = -\frac{1}{20}$ or $f = -20$ cm

POWER OF A LENS

The ***power of a lens is a measure of the degree of convergence of divergence of the light rays falling on it.***

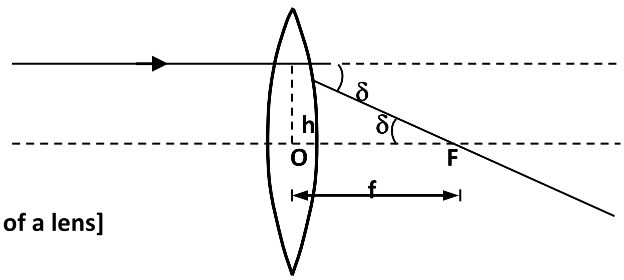
●A convex lens of shorter focal length bends light rays towards the principal axis through a larger angle, by focussing them closer to the optical centre. Hence smaller lens focal length of lens, more is ability to bend light rays and greater is its power.

The power of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical centre.



[(a) Large f, small bending power, (b) small f, large bending power]

In fig., a beam of light is incident at distance h from the optical centre O of a convex lens of focal length f. It converges the beam by angle δ.



[Power of a lens]

Clearly, $\tan\delta = \frac{h}{f}$

If h = 1, then $\tan\delta = \frac{1}{f}$ or $P = \frac{1}{f}$

Thus, the power of a lens may also be defined as the reciprocal of its focal length.

SI unit of power: The SI unit of power is dioptre, denoted by D. If f = 1 m, then

$$P = \frac{1}{1\text{ m}} = 1\text{m}^{-1} = 1 \text{ dioptre (D)}$$

On dioptre is the power of a lens whose principal focal length is 1 metre.

The focal length of a converging lens is positive and that of a diverging lens is negative, Thus, the power of a converging lens is positive and that of a diverging lens is negative. We can measure the power of a lens directly by a device called dioptremetre. Thus, when an optician prescribes a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length, f = 1/(+2.5 D) = + 0.40 m = + 40 cm. Similarly, a power of – 4.0 D means a concave lens of focal length – 25 cm.

By using lens maker's formula, the power of a lens can be expressed in terms of its refractive index μ and radii of curvature R_1 and R_2 as follows:

$$P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As the power of a lens is reciprocal of its focal length, so it characteristics the focal properties of the lens, such as nature, size and position of image, etc.

COMBINATION OF THIN LENSES

The purpose of using a lens combination is (i) To magnify an image

(ii) To increase the sharpness of the final image by minimising certain defects or aberrations in it.

(iii) To erect the final image

(iv) To increase the field of view.

Different lens combinations are used in the objectives of cameras, microscopes, telescopes and other optical instruments.

Total magnification: When lenses are used in combination, each lens magnifies the image formed by the preceding lens. Hence the total magnification m is equal to the product of the magnifications m_1, m_2 and m_3 ...., produced by the individual lenses.

$$M = m_1 \times m_2 \times m_3 \times \ldots\ldots\ldots$$

●***Equivalent lens:*** A single lens which forms the image of an object at the same position as is formed by a combination of lenses is called an equivalent lens.

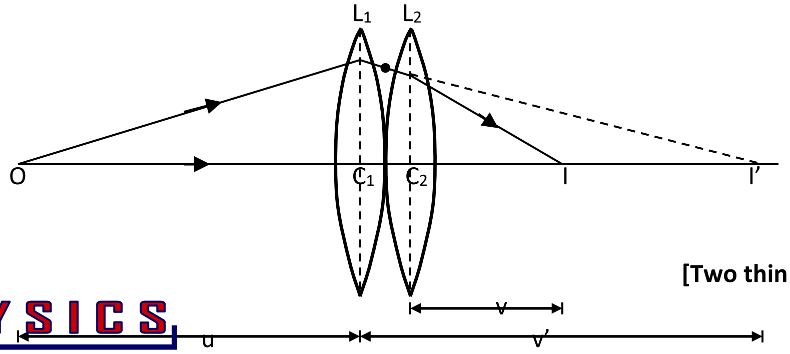
Equivalent focal length and power of two thin lenses in contact: As shown in Fig. let L_1 and L_2 be two thin lenses of focal length f_1 and f_2 respectively, placed coaxially in contact with one another. Let O be a point object on the principal axis of the lens system.

Let $OC_1 = u$. In the absence of second lens L_2, the first lens L_1 will form a real image I' of O at distance $C_1I' = v'$. Using thin lens formula.

$$\frac{1}{f_1} = \frac{1}{v'} - \frac{1}{u} \qquad \ldots (1)$$

The image I' acts as a virtual object (u = v') for the second lens L_2 which finally forms its real image I at distance v. Thus

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v'} \qquad \ldots (2)$$



[Two thin lenses]

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \qquad \text{... (3)}$$

For the combination of thin lenses in contact, if f is the equivalent focal length, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{... (4)}$$

From equation (3) and (4), we find that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

∴ Equivalent power, $P = P_1 + P_2$

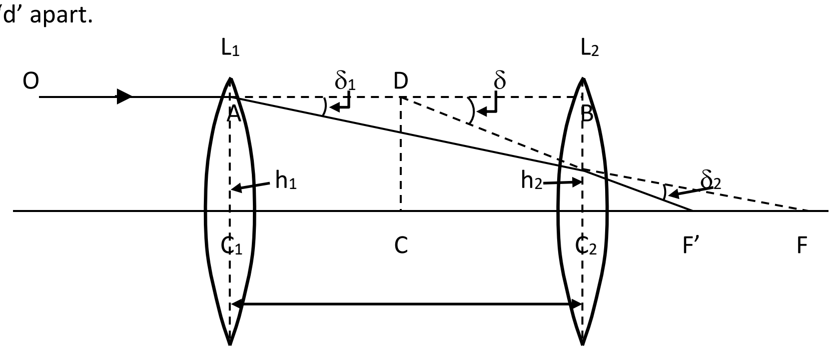
For n thin lenses in contact, we have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots \frac{1}{f_n}$$

∴ Equivalent power,

$$P = P_1 + P_2 + P_3 + \ldots\ldots + P_n$$

Thin lenses separated by a small distance: consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2, respectively. The two lenses are placed coaxially, distance 'd' apart.



[Two thin lenses separated by a small distance]

Suppose a ray OA traversing parallel to the principal axis is incident on lens L_1. It is refracted along AF, F being the second principal focus of L_1. The deviation produced by L_1 is

$$\delta_1 \simeq \tan \delta_1 = \frac{h_1}{f_1}$$

The emergent ray is further refracted by second lens L_2 along BF'. Since the incident ray OA is parallel to the principal axis, F' should be second principal focus of the combination. The deviation produced by the second lens L_2 is

$$\delta_2 \simeq \tan \delta_2 = \frac{h_2}{f_2}$$

The final emergent ray BF', when produced backwards, meets the incident ray at point D. Obviously, δ is the final deviation produced. A single thin lens placed at C will produce the same deviation as by the combination of two lenses. Thus distance CF' is the second focal length of the combination. If f is the focal length of the combination, then

$$\delta = \frac{h_1}{F}$$

It is obvious from fig. that

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

As $\Delta AC_1F \sim \Delta BC_2F$, therefore, we have

$$\frac{AC_1}{C_1F} = \frac{BC_2}{C_2F} \quad \text{or} \quad \frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or $$h_2 = \frac{f_1 - d}{f_1} . h_1$$

Hence $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{f_1 - d}{f_1 f_2} . h_1 \quad \text{or} \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

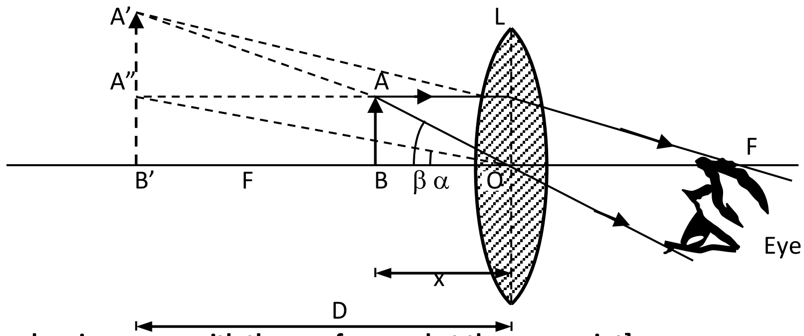
In terms of powers of the lenses,

$$\mathbf{P = P_1 + P_2 - d.\ P_1 .\ P_2}$$

SIMPLE MICROSCOPE

A **simple microscope or a magnifying glass is just a convex lens of short focal length, held close to the eye.**

Working principle: When the final image is formed at the least distance of distinct vision: When an object AB is placed between the focus F and optical centre O of a convex lens; a virtual erect and magnified image A'B' is formed on the same side of the lens as the object. Since a normal eye can see an object clearly at the least distance of distinct vision D (= 25 cm), the position of the lens is so adjusted that the final image is formed at the distance D from the lens, as shown in Fig.



[A simple microscope with the eye focussed at the near point]

Magnifying power: The magnifying power of a simple microscope is defined as the ratio of the angles subtended by the image and the object at the eye, when both are at the least distance of distinct vision from the eye. Thus

$$\text{Magnifying power} = \frac{\text{Angle subtended by the image at the least distance of distinct vision}}{\text{Angle subtended by the object at the least distance of distinct vision}}$$

As the eye is held close to the lens, the angles subtended at the lens may be taken to be the angles subtended at the eye. The image A'B' is formed at the least distance of distinct vision 'D'. Let ∠A' OB' = β. Imagine the object AB to be displaced to position A"B' at distance D from the lens. Let ∠A" OB' = α. Then magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{AB/OB}{A''B'/OB} = \frac{AB/OB}{AB/OB'} \qquad [\because A''B' = AB]$$

$$= \frac{OB'}{OB} = \frac{-D}{-x}$$

or $$m = \frac{D}{x}$$

Let f be the focal length of the lens. As the image is formed at the least distance of distinct vision from the lens, so

$$v = -D$$

Using thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We get , $$\frac{1}{-D} - \frac{1}{-x} = \frac{1}{f}$$

or $$\frac{1}{x} = \frac{1}{D} + \frac{1}{f}$$

or $$\frac{D}{x} = 1 + \frac{D}{f}$$

∴ $$m = 1 + \frac{D}{f}$$

Thus, shorter the focal length of the convex lens, the greater is its magnifying power.

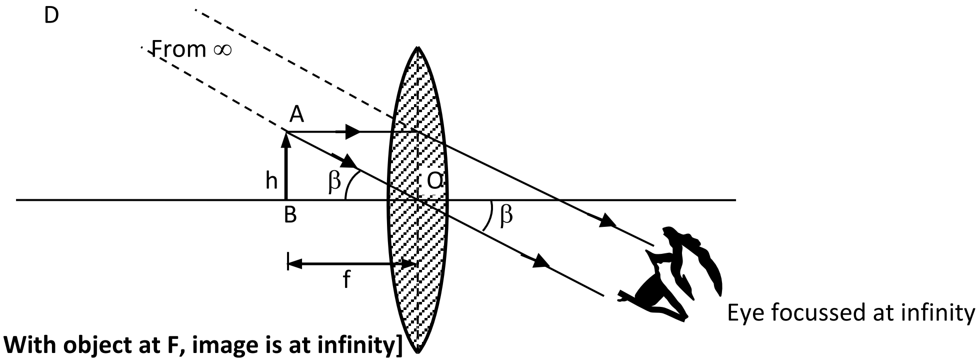
Working principle: When the final image is formed at infinity: When we see an image at the near point, it causes some strain in the eye. Often the object is placed at the focus of the convex lens, so that parallel rays enter the eye, as shown in Fig. (a). The image is formed at infinity, which is more suitable and comfortable for viewing by the relaxed eye.

Magnifying power: It is defined as the ratio of the angle formed by the image (when situated at infinity) at the eye to the angle formed by the object at the eye, when situated at the least distance of distinct vision.

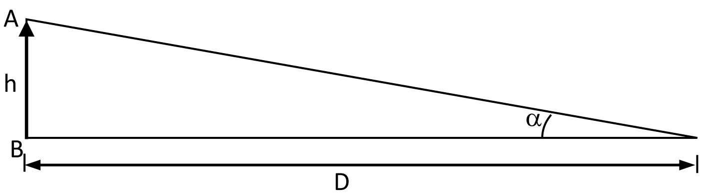
$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad [\alpha, \beta \text{ are small}]$$

$$\tan\beta = \frac{h}{f}$$

$$\tan \alpha = \frac{h}{D}$$



[(a) With object at F, image is at infinity]



[(b) Object at the near point]

∴ $m = \frac{h/f}{h/D}$ or $m = \frac{D}{f}$

This magnification is one less than the magnification when the image is formed at the near point. But viewing is more comfortable when the eye is focussed at infinity.

Uses of simple microscope:

1. Watch makers and jewellers use a magnifying glass for having a magnified view of the small parts of watches and the fine jewellery work.
2. In magnifying the printed letters in a book, textures of fibres or threads of a cloth, engravings, details of stamp, etc.
3. Magnifying glass is used in science laboratories for reading vernier scales, etc.

Conceptual................................

- **Least distance of distinct vision (D): The minimum distance from the eye, at which the eye can see the objects clearly and distinctly without and strain is called the least distance of distinct vision. For a normal eye, its value is 25 cm.**
- **Near point: The nearest point from the eye, at which an object can be clearly seen by the eye is called its near point. The near point of a normal eye is at a distance of 25 cm.**
- **Far point: The farthest point from the eye, at which an object can be seen clearly by the eye, is called the far point of the eye. For a normal eye, the far point is at infinity.**
- **Accommodation: It is the ability of the eye lens due to which it can change its focal length so that images of objects at various distances can be formed on the same retina.**
- **Power of accommodation: The power of accommodation of the eye is the maximum variation of its power for focussing on near and far object. For a normal eye, the power of accommodation is about 4 dioptres.**
- **The magnifying power is expressed with a unit X. So if a magnifying glass produces an angular magnification of 10, it is called a 10 X magnifier.**
- **A simple microscope has a limited maximum magnification of about 10, for realistic focal lengths. For much larger magnification, we use two convex lenses, one enhancing (compounding) the effect of the other. This is known as the compound microscope.**

Examples based on Simple Microscope

Formulae used

1. When the final image is formed at the least distance of distinct vision, the magnifying power is

$$m = 1 + \frac{D}{f}$$

2. When the final image is formed at infinity, the magnifying power is $m = \frac{D}{F}$

Units used

Magnification m has no units. D = 25, for a normal eye.

Q. 1. A thin convex lens of focal length 5 cm is used as a simple microscope by a person with normal near point (25 cm). What is the magnifying power of the microscope?

Sol. Here f = 5 cm, D = 25 cm

Magnifying power, $m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$

Q. 2. A simple microscope is a combination of two lenses, in contact, of powers + 15 D and + 5 D. Calculate the magnifying power of the microscope, if the final image is formed at 25 cm from the eye.

Sol. power of combination, $P = P_1 + P_2 = 15 + 5 = +20$ D

∴ Focal length of combination

$= \frac{1}{P} = \frac{1}{20}$ m = 5 cm

∴ Magnifying power, $m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$

Q. 3. An object is to be seen through a simple microscope of power 10 D. Where the object should be placed so as to produce maximum angular magnification? The least distance for distinct vision is 25 cm.

Sol. Angular magnification is maximum when the final image is formed at the near point.

∴ $v = -25$ cm, $f = \frac{1}{P} = \frac{1}{10}$ m = 10 cm

Now $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{10} = \frac{-7}{50}$ or $u = -50/7 = -7.1$ cm

Q. 4. A simple microscope is rated 5 X for a normal relaxed eye. What will be its magnifying power for a relaxed farsighted eye whose near point is 40 cm?

Sol. For normal eye:

D = 25 cm, m = 5

As $m = \frac{D}{f}$ ∴ $5 = \frac{25}{f}$ or f = 5

For relaxed farsighted eye:

D′ = 40 cm, f = 5 cm

∴ $m = \frac{D'}{f} = \frac{40}{5} = 8$ Thus the magnifying power of the simple microscope is 8 X in the second case.

Q. 5. A converging lens of focal length 6.25 cm is used as a magnifying glass. If the near point of the observer is 25 cm from the eye and the lens is held close to eye, calculate (i) the distance of the object from the lens and (ii) the angular magnification. Find the angular magnification, when the final image is formed at infinity.

Sol. Here f = 6.25 cm, v = – D = – 25 cm

(i) Using thin lens formula,

$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{6.25}$

$= -\frac{1}{25} - \frac{4}{25} = -\frac{5}{25} = -\frac{1}{5}$ or $u = -5$ cm

(ii) Angular magnification,

$m = 1 + \frac{D}{f} = 1 + \frac{25}{6.25} = 1 + 4 = 5$

When the final image is formed at infinity, the angular magnification becomes $M = \frac{D}{f} = \frac{25}{6.25} = 4$

Q. 6. A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm. (i) What is the closest and the farthest distance at which he can read the book when viewing through the magnifying glass? (ii) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Sol. (i) For the closest distance: v = – 25 cm, f = 5 cm, u = ?

As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

∴ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{5} = \frac{-1-5}{25} = \frac{-6}{25}$

or $u = -\frac{25}{6}$ cm $= -4.2$ cm

This is the closest distance at which the man can read the book.

For the farthest image: v = ∞, f = 5 cm, u =?

∴ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{\infty} - \frac{1}{5} = 0 - \frac{1}{5} = -\frac{1}{5}$

or $u = -5$ cm

This is the farthest distance at which the man can read the book.

(ii) Maximum angular magnification

$$= \frac{D}{u_{min}} = \frac{25}{25/6} = 6$$

Minimum angular magnification

$$= \frac{D}{u_{max}} = \frac{25}{5} = 5$$

Q. 7. A figure divided into squares each of size 1 mm² is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.

(i) What is the magnification (image size/object size) produced by the lens? How much is the area of each square in the virtual image?

(ii) What is the angular magnification (magnifying power) of the lens?

(iii) Is the magnification in (i) equal to the magnifying power in (ii)? Explain.

Sol. (i) Here, area of each square (or object) = 1 mm^2

$= 1\ mm^2$

$u = -9$ cm, $f = +10$ cm

As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore$ $\frac{1}{u} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{9} = \frac{9-10}{90} = -\frac{1}{90}$

or $v = -90$ cm

Magnitude of magnification, $m = \left|\frac{v}{u}\right| = \frac{90}{9} = 10$

Area of each square in the virtual image

$= (10)^2 \times 1 = 100\ mm^2 = 1\ cm^2$

(ii) Magnifying power,

$$m = \frac{D}{|u|} = \frac{25}{9} = 2.8$$

(iii) No. Magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus magnification magnitude is $\left|\frac{v}{u}\right|$ and magnifying power is $\frac{25}{|u|}$.

Only when the image is located at the near point $|v| = 25$ cm, the two quantities are equal.

Q. 8. (i) At what distance should the lens be held from the figure in example 118 in order to view the squares distinctly with the maximum possible magnifying power?

(ii) What is the magnification (image size/object size) in this case?

(iii) Is the magnification equal to magnifying power in this case? Explain.

Sol. (i) Maximum magnifying power is obtained when the image is formed at the near point (25 cm).

$\therefore$ $v = -25$ cm, $f = +10$ cm, $u = ?$

As $\frac{1}{u} - \frac{1}{u} = \frac{1}{f}$

$\therefore$ $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{10} = \frac{-2-5}{50} = \frac{-7}{50}$

or $u = -\frac{50}{7} = -7.14$ cm So lens should be held 7.14 cm away from the figure.

(ii) Magnitude of magnification,

$$m = \frac{v}{|u|} = \frac{25}{50/7} = 3.5$$

(iii) Magnifying power $= \frac{D}{|u|} = \frac{25}{50/7} = 3.5$ Yes, the magnifying power is equal to the magnifying power is equal to the magnifying of magnification because image is formed at the least distance of distinct vision.

COMPOUND MICROSCOPE

A compound microscope is an optical device used to see magnified images of tiny objects. A good quality compound microscope can produce magnification of the order of 1000.

Construction: It consists of two convex lenses of short focal length, arranged co-axially at the ends of two sliding metal tubes

1. Objective: It is convex lens of very short focal length f_0 and small aperture. It is positioned near the object to be magnified.

2. Eyepiece or ocular: It is convex lens of comparatively larger focal length f_e and larger aperture than the objective ($f_e > f_0$). It is positioned near the eye for viewing the final image.

The distance between the two lenses can be varied by using rack and pinion arrangement.

Working: **(a) When the final image is formed at the least distance of distinct vision.** The object AB to be viewed is placed at distance u_0, slightly larger than the focal length f_0 of the objective O. The objective forms a real, inverted and magnified image A'B', of the object AB on the other side of the lens O, as shown in Fig. (a). The separation between the objective O and the eyepiece E, is so adjusted that the image A'B' lies within the focal length f_e of the eyepiece. The image A'B' acts as an object for the eyepiece which essentially acts like a simple microscope. The eyepiece E forms a virtual and magnified final image A" B" of the object AB. Clearly, the final image A" B" is inverted with respect to the object AB.

Magnifying power: The magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the final virtual image to the angle subtended at the eye by the object, when both are at the least distance of distinct vision from the eye.

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h'/u_e}{h/D} = \frac{h'}{h}\cdot\frac{D}{u_e} = m_0 m_e$$

Here $m_0 = \frac{h'}{h} = \frac{v_0}{u_0}$

As the eye piece acts as a simple microscope, so

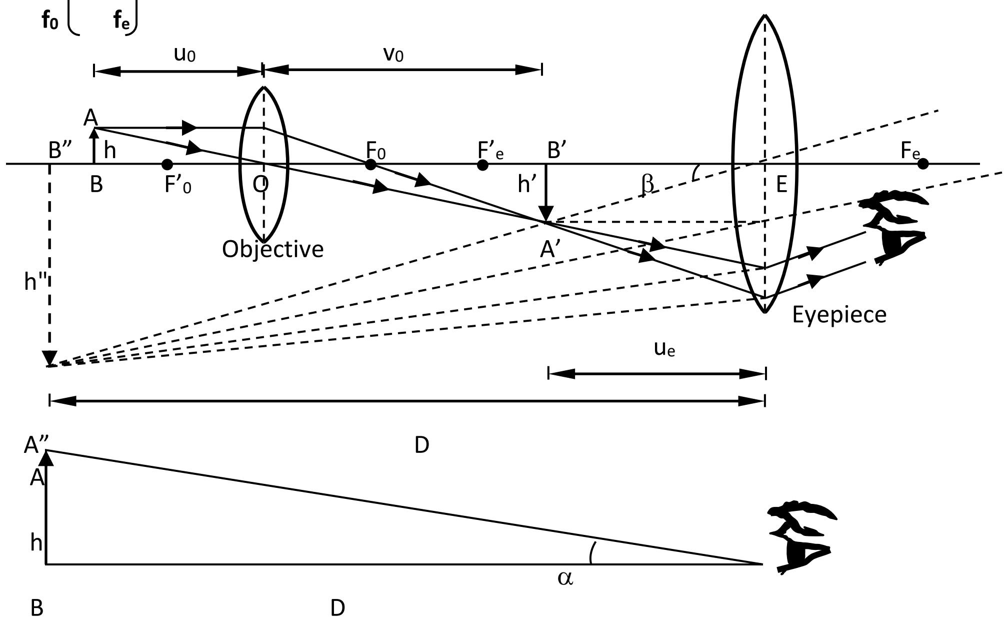
$$m_e = \frac{D}{u_e} = 1 + \frac{D}{f_e} \qquad \therefore \qquad m = \frac{v_0}{u_0}\left(1 + \frac{d}{f_e}\right)$$

As the object AB is placed close to the focus F_0 of the objective, therefore, $u_0 \simeq -f_0$

Also, image A'B' is formed close to the eye lens whose focal length is short, therefore $v_0 \simeq L$ = the length of the microscope tube or the distance between the two lenses

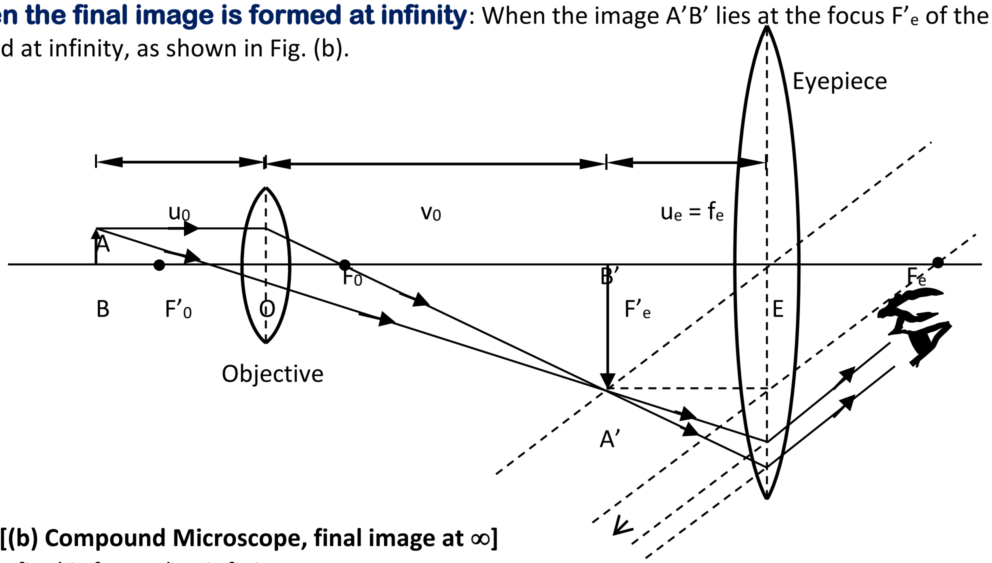
$$\therefore \qquad m_0 = \frac{v_0}{u_0} = \frac{L}{-f_0}$$

$$\therefore \qquad m = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \qquad \text{[for final image at D]}$$



[(a) Compound Microscope, final image at D]

(b) When the final image is formed at infinity: When the image A′B′ lies at the focus F'_e of the eye piece i.e., $u_e = f_e$, the image A′′ is formed at infinity, as shown in Fig. (b).



[(b) Compound Microscope, final image at ∞]

When the final is formed at infinity, $m_e = \frac{D}{f_e}$

∴ $m = -\frac{L}{f_0} \times \frac{D}{f_e}$ [For final image at ∞]

Obviously, **magnifying power of the compound microscope is large when both f_0 and f_e are small**.

Conceptual.................................

- **In a compound microscope, the objective is a convex lens of short focal length and small aperture, while the eyepiece is a convex lens of short focal length and large aperture.**
- **In actual practice, each of the objective and the eyepiece consists of combination of lenses. To eliminate chromatic aberration, an objective consists of two lenses in contact. To minimise chromatic and spherical aberrations, an eyepiece consists of two lenses separated by a certain distance.**
- **In a compound microscope, the objective and the eyepiece are placed a fixed distance apart. For focussing on an object, the distance of the objective from that object is changed with the help of a rack and pinion arrangement.**
- **For large magnifying power, both f_0 and f_e have to be small. Also, f_e is taken larger than f_0 so as to increase the field of view of the microscope.**
- **The visibility and quality of the image can be improved by illuminating the object and by using oil immersion objective.**
- **When the final image is formed at the least distance D of distinct vision, the length of the compound microscope,**
  $$L = v_0 + u_e$$
- **When the final image is formed at infinity, the length of the compound microscope, $L = v_0 + f_e$.**

Examples based on Compound Microscope

Formulae used

1. Magnifying power, $m = m_o \times m_e$
2. When the final image is formed at the least distance of distinct vision,
$$m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$
3. When the final image is formed at infinity,
$$m = \frac{v_0}{u_0} \cdot \frac{D}{f_e} = -\frac{L}{f_0} \cdot \frac{D}{f_e}$$

Units used: The distances u_0, u_e, v_0, v_e, D and L are all in metre or cm and magnification m has no units. D = 25 cm, for the normal eye.

Q. 1. ***A compound microscope has a magnification of 30. The focal length of its eye piece is 5 cm. Assuming the final image to be formed at least distance of distinct vision (25 cm), calculate the magnification produced by the objective.***

Sol. Here m = 30, f_e = 5 cm, D = 25 cm

Magnifying power of a compound microscope is

$$m = m_0 \times m_e = m_0\left(1 + \frac{D}{f_e}\right)$$

or $30 = m_0\left(1 + \frac{25}{5}\right)$ or $m_0 = 5$

Q. 2. ***A compound microscope with an objective of 1.0 cm focal length and an eyepiece of 2.0 cm focal length has a tube length of 20 cm. Calculate the magnifying power of the microscope, if the final image is formed at the near point of the eye.***

Sol. Here $f_0 = 1.0$ cm, $f_e = 2.0$ cm, $L = 20$ cm, $D = 25$ cm

When the final image is formed at the near point of the eye, the magnifying power is

$$m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

$$= \frac{20}{1.0}\left(1 + \frac{25}{2}\right) = 20 \times 13.5 = 270$$

Q. 3. ***The focal length of the eyepiece and the objective of a compound microscope are 5 cm and 1 cm respectively and the length of the tube is 20 cm. Calculate the magnifying power of the microscope, when the final image is formed at infinity. The value of least distance of distinct vision is 25 cm.***

Sol. As the final image is being formed at infinity, the image formed by the objective must lie at the focus of the eyepiece i.e., at a distance of 5 cm from the eyepiece.

∴ Image distance for the objective

= Tube length (L) – f_e = 20 – 15 = 15 cm

Using cartesian sign convention for the objective,

$v_0 = +15$ cm, $f_0 = +1$ cm, u_0 ?

$$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0}$$

$$= \frac{1}{15} - \frac{1}{1} = -\frac{14}{15}$$

or $u_0 = -\frac{15}{14}$ cm

When the final image is formed at infinity, the magnifying power of the microscope will be,

$$m = \frac{v_0}{u_0} \times \frac{D}{f_e}$$

$$= \frac{15}{15/14} \times \frac{25}{2} \qquad \text{[Numerically]}$$

$= 70$

Q. 4. ***The total magnification produced by a compound microscope is 20, while that produced by the eyepiece along is 5. When the microscope is focussed on a certain object, the distance between objective and eyepiece is 14 cm. Find the focal length of objective and eyepiece, if distance of distinct vision is 20 cm.***

Sol. Here $m = 20$, $m_e = 5$, $D = 20$ cm

∴ $m_0 = \frac{m}{m_e} = \frac{20}{5} = 4$

As the eyepiece acts as a simple microscope, so $m_e = 1 + \frac{D}{f_e}$ or $5 = 1 + \frac{20}{f_e}$

∴ $f_e = 5$ cm

Also, $m_e = \frac{v_e}{u_e}$

or $5 = \frac{-25}{u_e}$ or $u_e = -5$ cm

Distance between objective and eyepiece = 14 cm

or $|u_e| + |v_0| = 14$

or $5 + v_0 = 14$ or $v_0 = 9$ cm

Now $m_0 = \frac{v_0}{u_0}$

or $-4 = \frac{9}{u_0}$ [Negative sign form real image] or $u_0 = -\frac{9}{4}$ cm

Focal length f_0 of the objective is given by

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$$= \frac{1}{9} + \frac{1}{9/4} = \frac{1}{9} + \frac{4}{9} = \frac{5}{9}$$

or $f_0 = 1.8$ cm

Q. 6. ***A compound microscope is used to enlarge an object kept at a distance of 0.30 m from its objective, which consists of several convex lenses and has focal length 0.02 m. If a lens of focal length 0.1 m is removed from the objective, find out the distance by which the eye-piece of the microscope must be moves to refocus the image.***

Sol. For the objective:

$u_0 = -0.03$ m $= -3$ cm, $f_0 = 0.02$ m $= 2$ cm

$\therefore \quad \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \qquad$ or $\qquad v_0 = +6$ cm

The image is formed at 6 cm behind the objective. Let f_0' be the new focal length of the objective when a lens of focal length 0.1 m or 10 cm is removed from it.

Then, $\frac{1}{f_0'} = \frac{1}{f_0} - \frac{1}{10} = \frac{1}{2} - \frac{1}{10} = \frac{2}{5}$

or $\quad f_0' = +\frac{5}{2}$ cm

If v_0' is the new distance of the image formed by the objective, then

$\frac{1}{v_0'} = \frac{1}{u_0} + \frac{1}{f_0'} = -\frac{1}{3} + \frac{2}{5} = \frac{1}{15}$

Distance through which the eyepiece should be moves to refocus image

$= v_0' - v_0 = 15 - 6 = 9$ cm

TELESCOPE

A telescope is on optical device which enables us to see distant objects clearly. It provides angular magnification of the distant objects.

●**Different types of telescopes:** Broadly, the telescope can be divided into two categories:

1. Refracting telescopes: These make use to lenses to view distant objects. These are of two types:

(a) Astronomical telescope: It is used to see heavenly objects like the sun, start, planets, etc. The final image formed is inverted one which is immaterial in the case of heavenly bodies because of their round shape.

(b) Terrestrial telescope: It is used to see distant objects on the surface of the earth. The final image formed is erect one. This is an essential condition of viewing the objects on earth's surface correctly.

2. Reflecting telescopes: These make use of converging mirrors to view the distant objects. For example, Newtonian and Cassegrain telescope.

●ASTRONOMICAL TELESCOPE

Astronomical telescope is a refractive type telescope used to see heavenly bodies like stars, planets, satellites, etc.

Construction: It consists of two converging lenses mounted co-axially at the outer ends of two sliding tubes.

1. Objective: It is a convex lens of large focal length and a much larger aperture. It faces the distant object. In order to form bright image of the distant objects, the aperture of the objective is taken large so that it can gather sufficient light from the distant objects.

2. Eyepiece: It is a convex lens of small focal length and small aperture. It faces the eye. The aperture of the eyepiece is taken small so that whole light of the telescope may enter the eye for distinct vision.

Working: (a) When the final image is formed at the least distance of distinct vision: As shown in the least distance of distinct vision. As shown in Fig. the parallel beam of light coming from the distant object falls on the objective at some angle α. The objective focuses the beam in its focal plane and forms a real, inverted and diminished image A'B'. This image A'B' acts as an object for the eyepiece. The distance of the eyepiece is so adjusted that the image A'B' lies within its focal length. The eyepiece magnifies this image so that final image A" B" is magnified and inverted with respect to the object. The final image is seen distinctly by the eye at the least distance of distinct vision.

Magnifying power: the magnifying power of a telescope is defined as the ratio of the angle subtended at the eye of the final image formed at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective. Thus $\quad \angle A'OB' = \alpha \quad$ Also, let $\angle A''EB'' = \beta$

$\therefore \quad$ Magnifying power, $\quad m = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta$ are small$]$

$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$

According to the new Cartesian sign convention, $\quad OB' = +f_0$ = focal length of the objective

$B'E = -u_e$ = distance of A'B' from the eyepiece, acting as an object for it

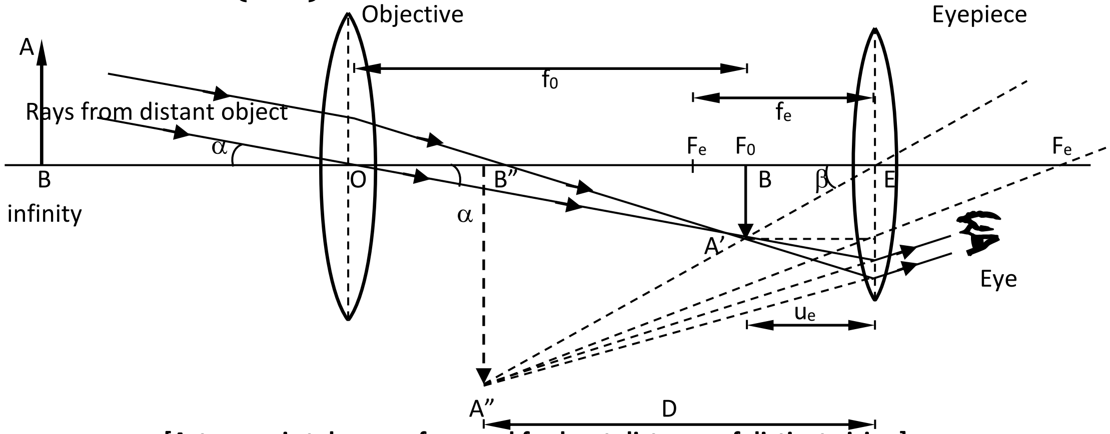
$\therefore \quad m = -\frac{f_0}{u_e}$

Again, for the eyepiece: $u = -u_e$ and $v = -D$

As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore$ $\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$

or $\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$



[Astronomic telescope focussed for least distance of distinct vision]

Hence, $m = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for the magnifying power indicates that the final image formed is real and inverted.

(b) When the final image is formed at infinity: Normal adjustment. As shown in Fig., when a parallel beam of light is incident on the objective, it forms a real, inverted and diminished image A'B' in its focal plane. The eyepiece is so adjusted that the image A'B' exactly lies at its focus. Therefore, the final image is formed at infinity, and is highly magnified and inverted with respect to the object.

Magnifying power in normal adjustment: It is defined as the ratio of the angle subtended at the eye by the object seen directly, when both the image and the object lie at infinity.

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective. Thus $\angle A'OB' = \alpha$ And let $\angle A'EB' = \beta$

$\therefore$ Magnifying power,

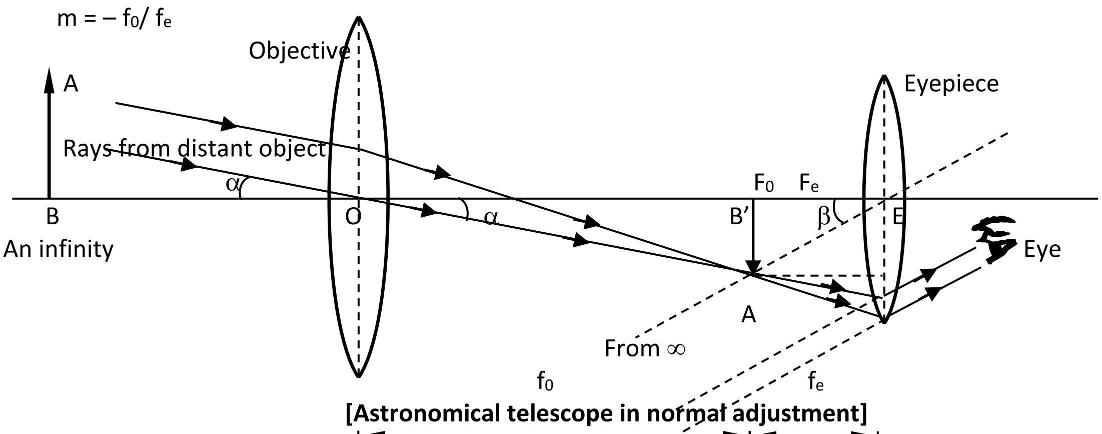
$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$ [$\because \alpha, \beta$ are small angles]

$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$

Applying new Cartesian sign convention, $OB' = +f_0$ = Distance of A'B' from the objective along the incident light

$B'E = -f_e$ = Distance of A'B from the eyepiece against the incident light

$\therefore$ $m = -f_0/f_e$



[Astronomical telescope in normal adjustment]

Conceptual................................

In a telescope, the objective has large focal length and large aperture while the eyepiece has small focal length and small aperture.

- **A telescope is focussed on the distant object by varying distance between the objective and the eyepiece with the help of rack and pinion arrangement.**
- **The objective of the telescope should have large aperture because then a much wider beam of light is incident on it and is converged into a small cone which, on entering the eye, produces sufficient illumination on the retina. So even two distant faint stars which cannot be seen by naked eyes, become visible through such a telescope.**
- **In a telescope, the image is not actually magnified. A telescope simply increases the visual angle. The visual angle β for the image is much larger than the visual angle α for the object. Consequently, the angular magnification β/α is quite large.**
- **In normal adjustment, the distance between the objective and the eyepiece = $f_0 + f_e$.**
  When the final image is formed at the least distance of distinct vision, the magnifying power of the telescope is larger than that in the case of normal adjustment because the factor $\left(1 + \frac{f_e}{D}\right) > 1$.
- **An astronomical telescope forms an inverted image. As the celestial objects are oval in shape, so it does not matter whether the final image is inverted or erect.**

TERRESTRIAL TELESCOPE

It is a refracting type telescope used to see erect images of distant earthly objects. It uses an additional convex lens between objective and eyepiece for obtaining an erect image.

As shown in Fig. the objective forms a real, inverted and diminished image, A'B' of the distant object in its focal plane. Now the erecting lens is held at twice its focal length from the focal plane of the objective. This lens forms a real, inverted and equal size image A"B" of A'B'. This image is now erect with respect to the distant object. The eyepiece is so adjusted that the image A"B" lies at the principal focus. Hence the final image is formed at infinity and is highly magnified and erect with respect to the distant object.

As the erecting lens does not cause any magnification, the angular magnification of the terrestrial telescope is same as that of the astronomical telescope.

$\therefore \quad m = \frac{f_0}{f_e}$'

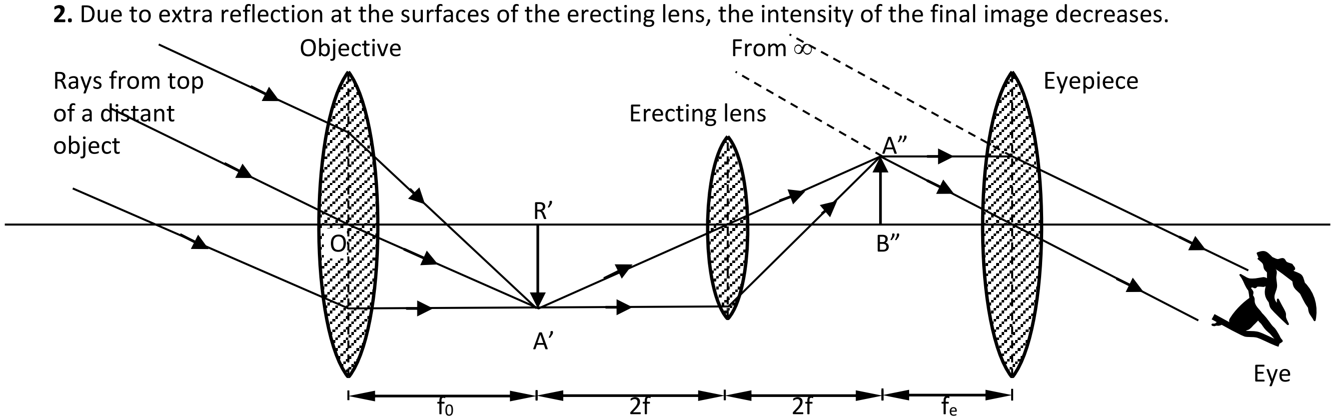
When the image is formed at infinity,

$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$, When the image is formed at the least distance of distinct vision.

Drawbacks:

1. The length of the terrestrial telescope is much larger than the astronomical telescope. In normal adjustment, the length of a terrestrial telescope = $f_0 + 4f + f_e$, where f is the focal length of the erecting lens.

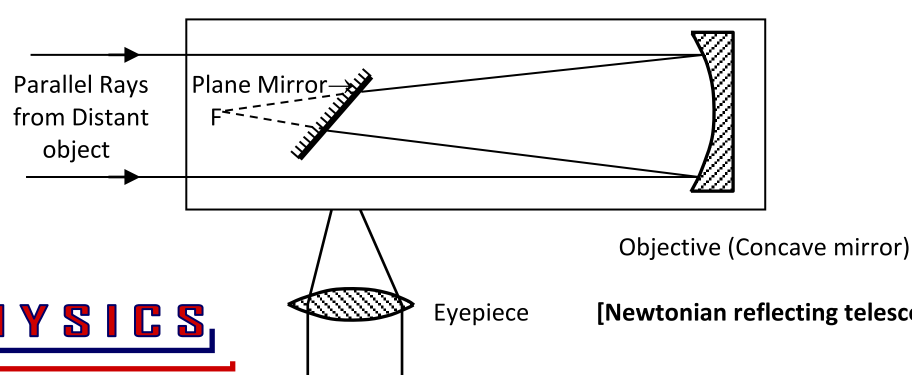
2. Due to extra reflection at the surfaces of the erecting lens, the intensity of the final image decreases.



[Terrestrial telescope]

REFLECTING TELESCOPES

Newtonian reflecting telescope: The first reflecting telescope was set up by Newton is 1668. As shown in Fig., it consists of a large concave mirror of large focal length as the objective, made of an alloy of copper and tin.



[Newtonian reflecting telescop

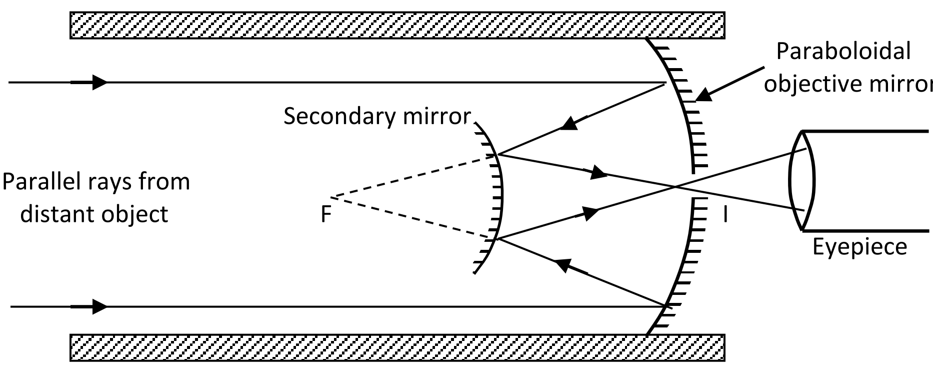
A beam of light from the distant start is incident on the objective. Before the rays are focussed at F, a plane mirror inclined at 45° intercepts them and turns them towards an eyepiece adjusted perpendicular to the axis of the instrument. The eye-piece forms a highly magnified, virtual and erect image of the distant object.

Cassegrain reflecting telescope: Fig. shows Cassegrainian type reflecting telescope. It consists of a large concave paraboloidal (primary) mirror having a hole at its centre. There is a small convex (secondary) mirror near the focus of the primary mirror. The eyepiece is placed on the axis of the telescope near the hole of the primary mirror.

The parallel rays from the distant object are reflected by the large concave mirror. Before these rays come to focus at F, they are reflected by the small convex mirror and are converged to a point I just outside the hole. The final image formed at I is viewed through the eyepiece. As the first image at F in inverted with respect to the distant object and the second image I is erect with the respect to the first image at F is inverted with respect to the distant object and the second image I is erect with the respect to the first image F, hence the final image is inverted with respect to the object.

Let f_0 be the focal length of the objective and f_e that of the eyepiece. For the final image formed at the least distance of distinct vision, $m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

For the final image formed at infinity, $m = \frac{f_0}{f_e} = \frac{R/2}{f_e}$



[Cassegrain reflecting telescope]

Advantages of a reflecting type telescope: A reflecting type telescope has the following advantages over a refracting type telescope:

1. A concave mirror of large aperture has high gathering power and absorbs very less amount of light than the lenses of large apertures. The final image formed in reflecting telescope is very bright. So even very distant or faint stars can be easily viewed.
2. Due to large aperture of the mirror used, the reflecting telescopes have high resolving power.
3. As the objective is a mirror and not a lens, it is free from chromatic aberration (formation of coloured image of a white object).
4. The use of paraboloidal mirror reduces the spherical aberration (formation of non-point, blurred image of a point object).
5. A mirror requires grinding and polishing of one surface only. So it costs much less to construct a reflecting telescope than a refracting telescope of equivalent optical quality.
6. A lens of large aperture tends to be very heavy and, therefore, difficult to make and support by its edges. On the other hand, a mirror of equivalent optical quality weighs less and can be supported over it entire back surface.

FOR YOUR KNOWLEDGE...............................

- The largest refracting telescope is at the Yerkes Observatory in Wisconsin, USA. It uses an objective lens of diameter 102 cm.
- The largest reflecting telescope in the world are the pair of Keck telescopes in Hawaii, USA. They use reflecting mirrors of diameter 10 m each.
- The largest telescope in India is in Kavalur, Tamilnadu. It is a Cassegrain reflecting telescope having objective of diameter 2.34 m. It was ground, polished, set up and is being used by the Indian Institute of Astrophysics, Bangalore.
- Prism binocular: It is a double telescope that uses two sets of totally reflecting prisms. This makes the final image erect which is very desirable for observations on earth. Binoculars are much more compact and easier to use than a refracting telescope, and allow use of both eyes.

Examples based on Telescopes

1. Astronomical telescope: (i) In normal adjustment, $m = f_0/f_e$
Distance between objective and eyepiece = $f_0 + f_e$

(ii) When the final image is formed at the least distance of distinct vision, $m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

Distance between objective and eyepiece $= f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$

2. Terrestrial telescope: (i) In normal adjustment, $m = \frac{f_0}{f_e}$

Distance between objective and eyepiece $= f_0 + 4f + f_e'$ Where f = focal length of the erecting lens.

3. Galileo's telescope: In normal adjustment, $m = \frac{f_0}{f_e}$ Distance between objective and eyepiece $= f_0 - f_e$

4. Reflecting telescope: $m = \frac{f_0}{f_e} = \frac{R/2}{f_e}$

where f_0 = focal length of concave mirror, f_e = focal length of eyepiece

Units used Lengths f_0, f_e, f and D are in all in cm or metre.

Q. 1. ***The magnifying power of an astronomical telescope in the normal adjustment position is 100. The distance between the objective and the eyepiece is 101 cm. Calculate the focal lengths of the objective and the eyepiece.***

Sol. Here $m = f_0 = 100$ or $f_0 = 100 f_e$

But $f_0 + f_e = 101$ cm

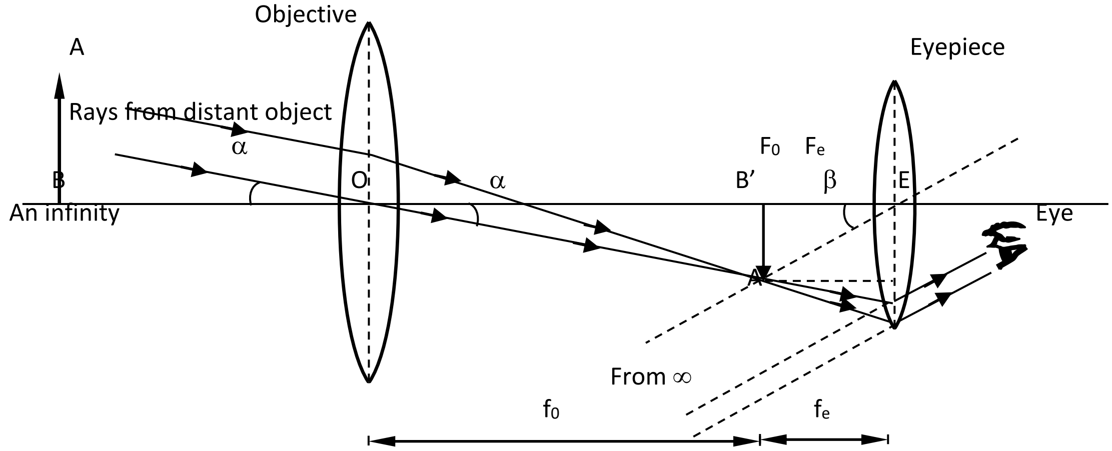
or $100 f_e + f_e = 101$

$\therefore$ $f_e = 1$ cm

and $f_0 = 100$ cm

Q. 2. ***Draw a labelled ray diagram of an astronomical telescope, forming the image at infinity. An astronomical telescope uses two lenses of powers 10 diopter, 1 diopter.***
(i) State with reason, which lens is preferred as objective and eye-piece.
(ii) Calculate the magnifying power of the telescope, if the final image is formed at the near point.
(iii) How do the light gathering power and resolving power of a telescope change, if the aperture of the objective lens is doubled?

Sol.



(i) The lens of power 1 dioptre should be used as objective because of its larger focal length and the lens of 10 dioptre should be used as eyepiece because of its smaller focal length.

(ii) Here, $f_0 = \frac{1}{1\text{ D}} = 1$ m $= 100$ cm

$f_e = \frac{1}{10\text{ D}} = 0.1$ m $= 10$ cm

$\therefore$ $m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) = \frac{100}{10}\left(1 + \frac{10}{25}\right) = 14$

(iii) Light gathering capacity of a telescope $\propto$ Area of the objective

i.e., $Q \propto \frac{\pi D^2}{4}$ or $Q \propto D^2$

When aperture (D) is doubled, light gathering capacity increases 4 times.

R.P. of a telescope $\propto$ D When aperture (D) is doubled, resolving power also gets doubled.

Q. 3. ***An amateur astronomer wishes to estimate roughly the size of the sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eye-piece of focal length 10 cm. By adjustment the distance of the eye-piece from the objective, he obtains an image of the sun on a screen 40 cm behind the eyepiece. The diameter of the sun's image is measured to be 6.0 cm. What is the estimate of the sun's size, given that the average earth-sun distance is 1.5 $\times$ 10^{11} m.***

Sol. Here $f_0 = 200$ cm, $f_e = 10$ cm,

$v_e = +40$ cm (for real image)

As $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

∴ $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{40} - \frac{1}{10}$

$= \frac{1-4}{40} = \frac{-3}{40}$

or $u_e = -\frac{40}{3}$ cm

Magnification produced by eyepiece is $m_e = \frac{v_e}{|u_e|} = \frac{40}{40/3} = 3$

∴ Diameter of the image formed by the objective is d = 6/3 cm = 2 cm

If D is the diameter of the sun (in m), then the angle subtended by it on the objective will be

$\alpha = \frac{D}{1.5 \times 10^{11}}$ rad

Angle subtended by the image at the objective will be equal to this angle and is given by

$\alpha = \frac{\text{Size of image}}{f_0}$

$= \frac{2}{200} = \frac{1}{100}$ rad

∴ $\frac{D}{1.5 \times 10^{11}} = \frac{1}{100}$

Or $D = \frac{1.5 \times 10^{11}}{100} = 1.5 \times 10^9$ m

Q. 4. ***A telescope objective of focal length 1 m forms a real image of the moon 0.92 cm in diameter. Calculate the diameter of the moon taking its mean distance from the earth to be 38 × 10⁴ km. If the telescope uses an eyepiece of 5 cm focal length, what would be the distance between the two lenses for (i) the final image to be formed at infinity and (ii) the final image (virtual) at 25 cm from the eye.***

Sol. Let d be the diameter of the moon. The angle subtended by the moon at the objective of the telescope is

$\alpha = \frac{\text{Diameter of moon}}{\text{Distance of moon from earth}}$

$= \frac{d}{38 \times 10^4 \text{ km}} = \frac{d}{3.8 \times 10^8 \text{ m}}$

The angle subtended by the image formed by the objective in its focal plane will also be equal to α and is given by

$\alpha = \frac{\text{Diameter of moon's image}}{\text{Focal length of the objective}}$

= 0.92 cm = 0.0092 rad

∴ $\frac{d}{3.8 \times 10^8 \text{ m}} = 0.0092$

or $d = 3.8 \times 10^8 \times 0.0092 = 3.5 \times 10^6$ m

(i) When the image is formed at infinity, the distance between the two lenses is

$L = f_0 + f_e = 100 \text{ cm} + 5.0 \text{ cm} = 105$ cm

(ii) When the image is formed by the eyepiece at the least distance of distinct vision, we have

$v_e = -D = -25$ cm, $f_e = +5$ cm

Using thin lens formula,

$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = \frac{-6}{25}$

or $u_e = -\frac{25}{6} = -4.17$ cm

Therefore, the distance between the two lenses is

$L' = f_0 + |u_e| = 100 + 4.17 = 104.17$ cm

Q. 5. ***A telescope has an objective of focal length 50 cm and eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away from the object. Calculate (a) the separation between the objective and eyepiece and (b) the magnification produced.***

Sol. For the image formed by the objective, we have

$u_0 = -200$ cm, $f_0 = +50$ cm

As $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$

∴ $\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{50} - \frac{1}{200} = \frac{4-1}{200} = \frac{3}{200}$

or $v_0 = \frac{200}{3}$ cm

Magnification produced by the objective is

$m_0 = \frac{v_0}{u_0} = \frac{200}{3 \times (-200)} = -\frac{1}{3}$

The image formed by objective acts as an object for the eyepiece. So

$v_e = -25$ cm, $f_e + 5$ cm

∴ $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5} = \frac{-1-5}{25} = \frac{-6}{25}$

or $u_e = -\frac{25}{6}$ cm

Magnification produced by the eyepiece, $m_e = \frac{v_e}{u_e} = \frac{-25}{-25/6} = 6$

(a) The separation between the objective and eyepiece

$= v_0 + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.83$ cm

(b) Magnification produced,

$m = m_0 \times m_e = -½ \times 6 = -2.$

Q. 6. ***An astronomical telescope consisting of an objective of focal length 60 cm and an eyepiece of focal length 3 cm is focussed on the moon, so that the final image is formed at the least distance of distinct vision (25 cm) from the eyepiece. Assuming that the diameter of the moon subtends an angle of 1 °/2 at the objective, calculate (a) the angular magnification and (b) the actual size of the image seen.***

Sol. (a) When the final image is formed at the least distance of distinct vision, the magnifying power of the telescope is

$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) = \frac{60}{3}\left(1 + \frac{3}{25}\right)$

$= 20 \times \frac{28}{25} = 22.4$

(b) The angular size 'β' of the image is given by

$m = \beta/\alpha$

or $\beta = m\,\alpha = 22.4 \times 1°/2 = 11.2°$

$= 11.2 \times \frac{\pi}{180} = \frac{11.2 \times 22}{180 \times 7}$ rad

Hence the actual (linear) size of the image

$= \beta D = \frac{1.6 \times 22}{180} \times 25$ cm $= 4.9$ cm

Q. 7. ***A terrestrial telescope has an objective of focal length 180 cm and an eyepiece of focal length 5.0 cm. The erecting lens has a focal length of 3.5 cm. What is the separation between the objective and the eyepiece? What is the magnifying power of the telescope? Can we use the telescope for viewing astronomical objects?***

Sol. In a terrestrial telescope, the inverted image formed by the objective is made erect by positioning it at the 2f point of an erecting lens of focal length f.

In normal adjustment, the separation between the objective and the eyepiece is

$L = f_0 + 4f + f_e = 180 + 4 \times 3.5 + 5.0 = 199$ cm

Magnifying power,

$m = \frac{f_0}{f_e} = \frac{180}{5} = 36$

Yes, the telescope can be used to view astronomical objects through there is no need to make the 'inverted' image of a star 'upright'. But the final image is less bright than in an equivalent astronomical telescope because of the extra loss of some light due to reflection and absorption by the erecting lens.