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PHYSICS VISCOSITY FLUID FRICTION

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FLUID FRICTION

UNIT:VII CHAP:03

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VISCOSITY
Fluid friction

VISCOSITY: [Fluid friction] --- **"Viscosity is the property of a fluid (liquid or gas) by virtue of which it opposes the relative motion between its different layers".**

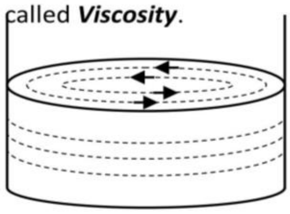
Or

"Viscosity is the property of a fluid by virtue of which an internal frictional fore comes into play when the fluid is in motion and opposes the relative motion of its different layers".

Illustration:

[1] Pour equal amounts of water and honey in two similar funnels. It will be observed that **water flows out of the funnel very quickly**. On the other hand, **honey is extremely slow in flowing down**. This difference in the behaviour of two liquids **indicates that the internal frictional forces** in the two liquids have different values.

[2] Consider a liquid contained in a cylindrical vessel. Let the liquid be set into rotation with the help of a glass rod. The rotation of the liquid can be observed as a series of co-axial cylindrical rotating layers. It will be seen that the innermost cylindrical layer has maximum speed. The speed of the successive layers goes on decreasing as we approach the walls of the cylindrical vessel. Thus, there is a relative motion between different layers of the liquid. This observation can be made more clearly by sprinkling some saw dust on the surface of rotating liquid. When the string of the liquid is stopped, the liquid comes to rest after some time. This indicates that there must be some '***Internal friction force***' which opposes the relative motion between different layers. This force is called ***Viscous force***. The phenomenon is called ***Viscosity***.

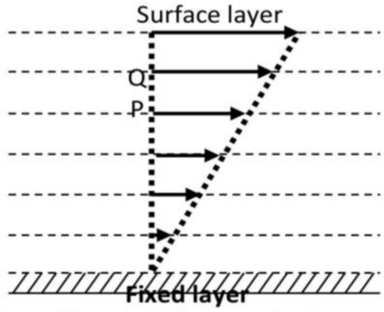


- **The external force required to maintain the relative motion between the different layers of the fluids is a measure of the fluid.**
- **Both liquids and gases exhibit viscosity, although liquids are much more viscous than gases.**
- **The greater the viscosity, the less easy it is for the fluid to flow and stickier it feels.**

Cause of Viscosity: Viscosity is **due to the intermolecular forces** which are effective when the different layers of the liquid are moving with different velocities. **These forces are of Vander Waal's force and vary inversely as the seventh power of intermolecular distance**. ***Due to these forces, every fast moving liquid layer tends to accelerate the adjoining slow moving layer and every slow moving layer tends to retard the adjoining fast moving layer of the liquid. As a result a backward tangential dragging force called viscous drag comes into play which accounts for viscosity of liquid.***

Coefficient of Viscosity: Consider a liquid flowing steadily over a solid horizontal surface.

The layer of the liquid in contact with the solid horizontal surface is at rest. So, this layer is a fixer layer. The velocities of other layer increases uniformly with the increase in distance from the fixed layer. Consider two layer P & Q at distance x and $x + dx$ respectively from the fixed layer. The layer P tries to retard Q and layer Q moving at faster rate tries to accelerate P. ***Thus two layer tends to destroy their relative motion as if there is a backward dragging force acting tangentially*** **therefore to maintain the flow of liquid, an external force equal & opposite to backward dragging force must be applied.**

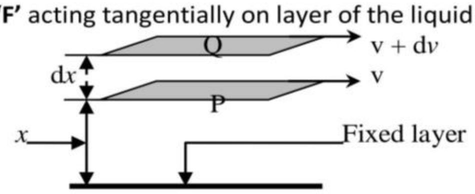


Let 'v' and 'v + dv' be their respective velocities of 'P' & 'Q'. The **velocity Gradient *("Rate of change of velocity with distance in the direction of increasing distance, is called velocity gradient {dv/dx})* dv/dx** is in a direction perpendicular to the direction of flow of the liquid.

According to ***Newton's law of viscous flow***, the viscous force **'F'** acting tangentially on layer of the liquid is proportional to -------

[1] Area 'A' of the layer i.e., $F \propto A$

[2] The velocity gradient i.e., $F \propto dv/dx$



∴ **F α A dv/dx**

Or **F = - η A dv/dx** ... (i) where, η = constant, called coefficient of viscosity of the liquid. It depends upon the nature of the liquid. [Negative sign shows that the direction of viscous force 'F' is opposite to the direction of motion or velocity]

If A = 1 and dv/dx = 1, then from (i) η = -F (numerically)

Thus, **"Coefficient of viscosity of a liquid is defined as the viscous force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of the liquid."**

Or

"Coefficient of viscosity of a liquid is defined as the tangential force required maintaining a unit velocity gradient between two layers each of unit area."

UNIT of coefficient of viscosity: From (i) $\eta = \frac{F}{A\,[dv/dx]}$ [Magnitude]

In cgs Unit: The unit of η is '***poise***' and 1 poise = $\frac{1 \text{ dyne}}{1\ cm^2 \times (1\ cm\ s^{-1}/cm)}$ = ***dyne cm^{-2} sec***.

"Coefficient of viscosity is said to be 1 poise, if 1 dyne tangential force is required to maintain a velocity gradient of 1 cm s^{-1}/ cm between two layers of each of area 1 cm^2."

In SI Unit: The unit of η is Pascal – second (***pa s***) or '***deca poise***' 1 deca poise = $\frac{1 \text{ newton}}{1\ m^2 \times (1\ m\ s^{-1}/m)}$ = ***Nm s^{-2}***

"Coefficient of viscosity is said to be 1 decapoise, if 1 N tangential force is required to maintain a velocity gradient 1 ms^{-1}/m between two layers of each of area 1 m^2."

➢ In addition to poise and decapoise, centipoises and micro poise are commonly used.
1 centipoises = 10^{-2} poise; 1 micro poise = 10^{-6} poise

Relation between DECAPOISE and POISE:
1 deca poise = N ms^{-2} = $10^5 \times (10^2)^{-2}$ dyne sec cm^{-2} = 10 poise

Dimensional formula of coefficient of viscosity:

We know that, $F = -\eta A \frac{dv}{dx}$

Or, $\eta = \frac{F}{A\,[dv/dx]}$ [Magnitude]

$[\eta] = \frac{[M L T^{-2}]\,[L]}{[L]^2\,[LT^{-1}]}$ = $[M L^{-1} T^{-1}]$

Comparisons between Viscosity and Friction:

Points of Similarity:
(1) Both arise due to intermolecular forces
(2) Both opposes motion.
(3) Both come into play due to relative motion.

Points of Dissimilarity:
(1) Forces of viscosity depends upon the area of layers. The force of friction does not depends upon the area of the surfaces in contact.
(2) Forces of viscosity depend upon the relative velocity of two layers. The force of friction does not depends upon the relative velocity of two bodies in contact.

Example 20.1. A square plate of side 20 cm moves parallel to another plate with a velocity of 20 cm s^{-1}; both plates immersed in water. If the viscous force is 4×10^{-3} N and viscosity of water is 0.001 decapoise, what is their distance apart?

Solution. Area of plate, $A = 20 \times 20 = 400\ cm^2 = 400 \times 10^{-4}\ m^2$

Relative velocity, $dv = 20\ cm\ s^{-1} = 0.2\ m\ s^{-1}$

Coefficient of viscosity, $\eta = 0.001$ decapoise

Now $F = \eta A \frac{dv}{dx}$

or $4 \times 10^{-3} = 0.001 \times \frac{400 \times 10^{-4} \times 0.2}{dx}$

or $4 \times 10^{-3} = \frac{0.08 \times 10^{-4}}{dx}$

∴ $dx = \frac{0.08 \times 10^{-4}}{4 \times 10^{-3}} = 0.002\ m = 0.2\ cm$

Example 20.2. The relative velocity between two layers of water is 8 cm/s. If the perpendicular distance between the layers is 0.1 cm, find the velocity gradient.

Solution. Velocity gradient $= \frac{dv}{dx}$

Here $dv = 8$ cm/s ; $dx = 0.1$ cm

$\therefore$ Velocity gradient $= \frac{8}{0.1} = 80\ s^{-1}$

Example 20.3. Calculate the horizontal force required to move a metal plate of area $2 \times 10^{-2}\ m^2$ with a velocity of $4.5 \times 10^{-2}\ ms^{-1}$ when it rests on a layer of oil 1.5×10^{-3} m thick. Coefficient of viscosity of oil = 2 Nsm^{-2}.

Solution. $F = \eta A \frac{dv}{dx}$

Here $\eta = 2\ Nsm^{-2}$; $A = 2 \times 10^{-2}\ m^2$; $\frac{dv}{dx} = \frac{4.5\times10^{-2}}{1.5\times10^{-3}} = 30\ s^{-1}$

$\therefore$ $F = 2 \times 2 \times 10^{-2} \times 30 = 1.2$ N

Example 20.4. A metal plate of area 0.1 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley as shown in Fig. 20.3. A liquid with a film thickness of 0.3 mm is placed between the plate and the table. When released, the plate moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid. The pulley may be considered massless and frictionless.

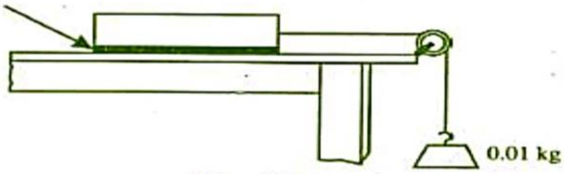


Fig. 20.3

Solution. The plate will move towards the right due to tension $T (= mg)$ in the string.

Now $T = mg = 0.01 \times 9.8 = 9.8 \times 10^{-2}$ N

Since the plate moves with a constant speed ($= 0.085\ ms^{-1}$), the viscous drag (F) is equal to tension in the string *i.e.*, $F = T = 9.8 \times 10^{-2}$ N.

Now $F = \eta A \frac{dv}{dx}$

Here $F = 9.8 \times 10^{-2}$ N ; $A = 0.1\ m^2$; $dv = 0.085\ ms^{-1}$; $dx = 3 \times 10^{-4}$ m

$\therefore$ $9.8 \times 10^{-2} = \eta \times 0.1 \times \frac{0.085}{3\times10^{-4}}$ or $\eta = 3.46 \times 10^{-3}$ Pas

◈ **STROKES LAW**: When a body (say spherical) falls through a viscous medium (liquid or gas) at rest, It drags the fluid Immediately in contact with it . But the layer of the fluid at a large distance from the falling body remains undisturbed. Thus, the falling body produces relative motion between different layers of the medium. As a result of this, the falling body experiences a viscous force, which opposes the motion of the falling body. This backward dragging force increases with the increases in velocity of the moving body.

Stroke performed many experiment on the motion of small spherical bodies in different fluids. He concluded that the viscous dragging force 'F' acting on a small spherical body of radius 'r' depends upon.........

.... **[1]** Coefficient of viscosity 'η' of the fluid. **i.e. $F \propto \eta$**
.... **[2]** Velocity of the spherical body 'v'. **i.e. $F \propto v$**
.... **[3]** Radius of spherical body 'r' **i.e. $F \propto r$**

$\therefore F = 6 \pi \eta rv$ ------------ Strokes law of Viscosity

Proof: Let (i) $F \propto \eta^a$; (ii) $F \propto r^b$; (iii) $F \propto v^c$

Combining these, $F \propto \eta^a r^b v^c$

$F = K \eta^a r^b v^c$ ... (i) [Where, K = dimension constant]

Write down the dimensions on either side, we get

$[M L T^{-2}] = [M L^{-1} T^{-1}]^a [L]^b [LT^{-1}]^c$

$[M^1 L^1 T^{-2}] = [M^a L^{-a+b+c} T^{-a-c}]$

Equating power of M, L, T, we get (principal of homogeneity)

$\underline{a = 1}$,

-a + b + c = 1 => -1 + b + c = 2 => b + c = 2 => b + 1 = 2 => $\underline{b = 1}$

And, -a – c = 2 => -1 – c = -2 => $\underline{c = 2 - 1 = 1}$

Substituting the values of a, b & c in equation [1], we get

$$F = K\,\eta^1\,r^1\,v^1$$

$$F = K\,\eta\,r\,v$$

The value of 'K' was found (experimentally) to be 6π, therefore,

$$F = 6\,\pi\,\eta\,rv$$

Stroke's law is valid under the following assumption..

1. **The viscous medium is homogenous and of infinity extent.**
2. **The spherical body is perfectly rigid and smooth.**
3. **The medium is continuous i.e., the size of the moving body is much larger than the distance between the molecules of the medium.**
4. **The body does not slip in the medium.**
5. **The velocity of the spherical body is less than critical velocity, So, the motion of the body through the fluid does not give rise to turbulent flow.**

Importance of stroke's law:

1. This law accounts the formation of cloud.
2. This law accounts, why the speed of rain drop is less than of a body falling freely with a constant velocity from the height of clouds.
3. This helps a man coming down with the help of parachute.

TERMINAL VELOCITY:

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

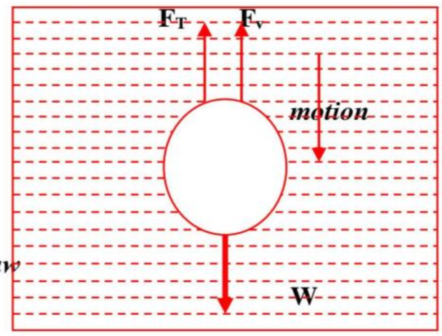
Explanation: When a small spherical body falls freely through a viscous medium, three forces acts on it..........

[i] Weight of the body acting vertically downward.
[ii] Upward thrust due to buoyancy equal to the weight of displaced liquid.
[iii] Viscous drag acting in the direction opposite to the motion of body.

According to Stroke's law, **F α v i.e., the opposing viscous drag goes on increasing with the increase in velocity of the body.**

As the body falls through a medium, its velocity goes on increasing due to gravity. Therefore, the opposing viscous drag which acts upwards also goes on increasing. A stage reaches when the true weight of the body is just equal to the sum of the upward thrust due to buoyancy and the upward viscous drag. At this stage, there is no net force to accelerate the body. Hence ***it starts falling with a constant velocity, which is called terminal velocity***.

Expression for terminal velocity:



Consider a spherical body of radius 'r' falling through a viscous fluid having density 'σ' and coefficient of viscosity 'η'.
Let 'ρ' be density of the material of the body.

True weight of the body, W = Volume × density × g

$= 4/3\,\pi\,r^3 \times \rho \times g$

Upward thrust due to buoyancy, F_T = weight of the medium displaced

= Vol. Of the medium displaced × density × g

$= 4/3\,\pi\,r^3 \times \sigma \times g$

If 'v' is the terminal velocity of the body, then according to Stroke's law

Upward viscous drag, $F_v = 6\,\pi\,\eta\,rv$

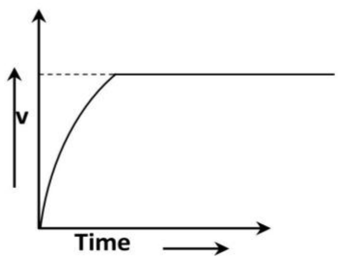
When body attains terminal velocity, then $F_T + F_v = W$

$$4/3\,\pi\,r^3 \times \sigma \times g + 6\,\pi\,\eta\,rv = 4/3\,\pi\,r^3 \times \rho \times g$$

$$6\,\pi\,\eta\,rv = 4/3\,\pi\,r^3 \times \rho \times g - 4/3\,\pi\,r^3 \times \sigma \times g$$

$$6\,\pi\,\eta\,rv = 4/3\,\pi\,r^3\,(\rho - \sigma)\,g$$

$$V = \frac{2\,r^2\,(\rho - \sigma)\,g}{9\,\eta}$$



Discussion of the result:-

(i) The **terminal velocity varies directly as the square of the radius of the body** and **inversely as coefficient of viscosity of the medium**. Thus, ***a given sphere shall attain less velocity in a more viscous medium.*** It also depends upon density of the body and the medium.

(ii) **If $\rho < \sigma$** the value of **v** is **negative** i.e. the body will move up with a constant velocity. It is due to this reason that the gas bubbles rise up through soda water bottle.

(iii) By knowing r, ρ, σ and terminal velocity v (by nothing the time taken by the body to cover a known distance in the liquid), the coefficient of viscosity of the liquid can be calculated.

☞ *When the rain drops are small in size, their terminal velocities are small and so they remains suspended in air in the form of clouds. When a no. of drop combines, they grow in size. Since the terminal velocity is proportional to the square of the radius of the drop therefore the terminal velocity increases considerably and they start falling down in the form of rain.*

Effect of temperature on the viscosity:

The viscosity of liquid decreases with increase in temperature and increases with the decreases in temperature that is, $\eta = 1/\sqrt{T}$. The variation of viscosity of liquid is given by ---- $\eta_T = \eta_0 / (1 + \alpha t + \beta t)$, where η_T & η_0 at the coefficient of viscosities at t° C & 0° C resp. and α & β are constant.

On the other hand, the value of viscosity of gas increases with the increase in temperature and vice-versa. i.e., $\eta = \sqrt{T}$ (from Kinetic theory of gases).

Effect of pressure on the viscosity:

With the increase in pressure, the viscosity of liquids increases but the viscosity of water decreases, whereas the viscosity of gases remains unchanged.

Some application of viscosity:

1. **Liquids of high viscosity are used in shock absorber and buffers of trains.**
2. **Since viscosity of liquid varies with temperature, so the proper choice of lubricant is made depending upon the season.**
3. **The knowledge of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.**
4. **The phenomenon of viscosity on air and liquid is used to dampen the motion of some instruments.**
5. **It plays an important role in the circulation of blood through arteries and veins.**

Example 20.5. **A drop of water of radius 0.01 mm is falling through a medium whose density is 1.21 kg/m^3 and coefficient of viscosity is 1.8×10^{-5} Nsm^{-2}. Find the terminal velocity of the drop and viscous force on the drop.**

Solution. Radius of drop, $r = 0.01$ mm $= 0.01 \times 10^{-3}$ m

Density of drop, $\rho = 1000$ kg/m^3

Density of medium, $\sigma = 1.21$ kg/m^3

Viscosity of medium, $\eta = 1.8 \times 10^{-5}$ Ns m^{-2}

$$v_T = \frac{2}{9}\times\frac{r^2(\rho-\sigma)g}{\eta}$$

$$= \frac{2}{9}\times\frac{(0.01\times10^{-3})^2\times(1000-1.21)\times9.8}{1.8\times10^{-5}} = 1.2\times10^{-2}\ \text{ms}^{-1}$$

Viscous force on drop, $F = 6\pi\ \eta\ r\ v_T$

$$= 6\pi\times(1.8\times10^{-5})\times(0.01\times10^{-3})\times1.2\times10^{-2} = 4.07\times10^{-11}\ \text{N}$$

Example 20.6. **A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water after a free fall through a height h in earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given : coefficient of viscosity of water = 1.0×10^{-3} Nsm^{-2}; $g = 10$ ms^{-2} and density of water = 1.0×10^3 kg/m^3.**

Solution. The velocity acquired by the sphere in falling freely through a height h is

$$v = \sqrt{2gh}$$

As per the conditions of the problem, this is the terminal velocity of the sphere in water *i.e.*, Terminal velocity of sphere in water is

$$v_T = \sqrt{2gh} \qquad ...(i)$$

By Stokes' law, the terminal velocity v_T of sphere in water is given by ;

$$v_T = \frac{2}{9}\times\frac{r^2(\rho-\sigma)g}{\eta}$$

$$= \frac{2}{9}\times\frac{(1.0\times10^{-3})^2\times(1.0\times10^4-1.0\times10^3)\times10}{1.0\times10^{-3}} = 20\ \text{ms}^{-1}$$

From eq. (i), $h = \frac{v_T^2}{2g} = \frac{(20)^2}{2\times10} = 20$ m

Example 20.7. Two identical drops of water are falling through air with a steady velocity of 10 cm s^{-1}. If the drops combine to form a single drop, what would be the terminal velocity of the single drop?

Solution. Terminal velocity $\propto$ (radius)^2.

Suppose r is the radius of each of the small drop and R is the radius of bigger drop. Let v_T and v_T' be the terminal velocities of the drops before and after combination respectively. Then,

$$\frac{v_T}{v_T'} = \frac{r^2}{R^2}$$

Now $$2\times\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \quad \text{or} \quad \left(\frac{r}{R}\right)^2 = \frac{1}{(2)^{2/3}}$$

$\therefore$ $$v_T' = v_T\left(\frac{R}{r}\right)^2 = 10\times(2)^{2/3} = 15.9 \text{ cm s}^{-1}$$

Example 20.8. With what terminal velocity will an air bubble 0.8 mm in diameter rise in a liquid of viscosity 0.15 Ns m^{-2} and specific gravity 0.9? Density of air is 1.293 kg m^{-3}.

Solution. The terminal velocity of the bubble is given by ;

$$v_T = \frac{2}{9}\times\frac{r^2(\rho-\sigma)g}{\eta}$$

Here $r = 0.4$ mm $= 4\times10^{-4}$ m ; $\sigma = 0.9\times10^3$ kg m^{-3} ; $\rho = 1.293$ kg m^{-3} ; $\eta = 0.15$ Ns m^{-2}

$\therefore$ $$v_T = \frac{2}{9}\times\frac{(4\times10^{-4})^2\times(1.293-0.9\times10^3)\times9.8}{0.15}$$
$$= -0.0021 \text{ ms}^{-1} = -0.21 \text{ cm s}^{-1}$$

The negative sign shows that the bubble will rise up.

Example 20.9. In Millikan's experiment for determining electronic charge, an oil drop of density 0.95 g/cm^3 falls with a terminal velocity 1.142×10^{-2} cm/s through air of density 0.0013 g/cm^3 and viscosity 181×10^{-6} g cm^{-1} s^{-1}. Find the radius of the drop.

Solution. $$v_T = \frac{2}{9}\times\frac{r^2(\rho-\sigma)g}{\eta}$$

or $$r^2 = \frac{9}{2}\times\frac{\eta v_T}{(\rho-\sigma)g}$$

Here $\rho = 0.95$ g/cm^3 ; $\sigma = 0.0013$ g/cm^3 ; $v_T = 1.142\times10^{-2}$ cm/s ; $\eta = 181\times10^{-6}$ g cm^{-1} s^{-1}

$\therefore$ $$r^2 = \frac{9}{2}\times\frac{(181\times10^{-6})\times(1.142\times10^{-2})}{(0.95-0.0013)\times981} = 10^{-8}$$

or $$r = \sqrt{10^{-8}} = 10^{-4} \text{ cm}$$

FLUID FLOW

The study of fluids in motion is called fluid dynamics. The general flow of a fluid can be very complicated. For example, consider the flow of a river in flood or the rise of smoke from a burning cigarette. While each drop of water or each smoke particle is governed by Newton's laws of motion, the resulting equations can be very complex. Fortunately, many features of fluid motion can be understood with certain simplifying assumptions. While discussing fluid flow, we *generally* make the following assumptions :

(*i*) The fluid is *nonviscous i.e.*, there is no internal friction between the adjacent layers of the fluid.

(*ii*) The fluid is *incompressible*. This means that density of the fluid is constant.

(*iii*) The fluid motion is *steady*. The steady flow of a fluid means that the velocity, density and pressure at each point in the fluid do not change with time.

Stream-line, Laminar and Turbulent Flow:

[I] Stream-line flow: "When the flow of fluid is such that the velocity (v) of every particle at any point of the fluid is constant, then the flow is said to be steady or streamline".

Or

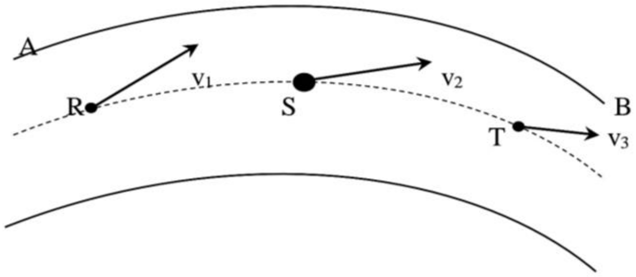
"Streamline flow of a liquid is that flow in which every particle of the liquid follows exactly the path its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point".

Explanation: Consider a liquid flowing through a tube AB. Let v_1, v_2, v_3 be the velocity of the particle of the liquid at points R, S, and T respectively. If the velocity of all the particle of all the particle of the liquid pass through R is v_1, through S is velocity v_2 and through T is velocity v_3, the flow of the liquid is said to be orderly or streamline.

- **In streamline flow, every particle of the liquid follow the path of its preceding particle and the velocity of all the particles crossing a particular point is the same**.
- **In streamline flow, the velocity of different particle at different points in their path may not be same.**

- The path followed by the particles of a fluid in a streamline flow is called stream-line. Thus, **"A streamline is the actual path followed by the procession of particle in a steady flow, which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point"**.

➯ If the lateral pressure on the streamline is the same throughout, then the streamline is straight. If the lateral pressure is different, then the streamline is curved. In case of curved streamline, the pressure is greater on convex than on concave side.



- **Tube of flow**: ***"A tube of flow is the bundle of stream lines having the same of the liquid particles over any Cross-section perpendicular to the direction of flow"***.
- In the stream line flow of a liquid, the energy needed to drive the liquid is used up only in overcoming the viscous drag between the layers. But the stream line flow is possible only so long as the liquid velocity does not exceed a certain limiting value for it. This limiting value of the velocity is called ***critical velocity***.

Properties of streamlines:

- **1. The tangent at any point of the streamline gives the direction of velocity of the liquid at that point.**
- **2. Two streamline cannot intersect. If two streamlines intersect, then it would mean two different directions of velocity at a given point. This is physically impossible. Hence two streamlines cannot intersect each other.**
- **3. At a particular point of the streamline, the velocity of liquid is constant. However, at different points of the streamline, the velocity may be different.**
- **4. Crowding of stream lines represent a faster flow of the liquid.**

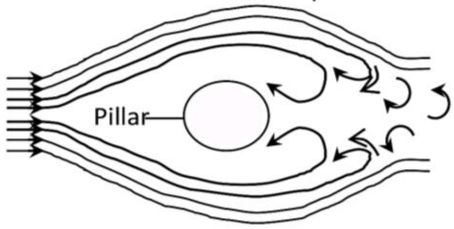
[II] Laminar flow: "The flow in which the liquid moves in the form of horizontal layers of different velocity is called laminar flow".

- The particles of one layer do not enter into another layer in laminar flow.
- When the velocity of flow of a liquid is less than the critical velocity of that liquid, than the liquid flows steadily. Each layer of the liquid slides over the other layer. It behaves as if different laminate are flowing over one another.
- In general, laminar flow is a streamline flow.

[I] Turbulent flow: "The flow of fluid in which velocity of all particles crossing a given point is not same and the motion of the fluid becomes disorderly or irregular is called turbulent flow".

- When velocity of a liquid exceeds the critical velocity, the paths and velocities of the liquid particles begins to change continuously and haphazardly. The flow looses all its orderliness and is called turbulent flow.
- In turbulent flow, most of the energy needed to drive the liquid is dissipated in setting up eddies and whirlpools in it and only small amount of energy is available for forward flow.

Examples: [1] Eddies are seen by the sides of the pillar of a river bridge.
[2] After rising a short distance, smooth column of smoke from incense sticks break up into an irregular and random pattern.
[3] Transverse flapping of flag.
[4] Moving car and airplane produce turbulent flow.



CRITICAL VELOCITY:

"The critical velocity of a liquid is that velocity of liquid, up to which its flow is streamline and above which its flow is turbulent".

- It is the maximum possible velocity possessed by a liquid in streamline flow.
- Osborne Reynolds [1883] Proved experimentally that the critical velocity for the viscous liquid is given by:

$$V_C = \frac{N_R\,\eta}{\rho D}$$

Here, N_R = Reynolds Number
η = Coefficient of viscosity
D = Diameter of the tube through which liquid is flowing.
ρ = Density of the liquid.

- Greater the viscous force exerted by the surrounding fluid, it is more likely that the flow will be steady. 6
- The turbulent flow is less likely for a viscous fluid flowing at low rates.

The critical velocity V_C of a liquid flowing through a tube depends upon
----(i) coefficient of velocity of liquid (η)
----(ii) density of liquid (ρ) and
----(iii) the radius of the tube (r).

Using the method of dimensions, we can obtain an expression for critical velocity. Let

$V_C = K\,\eta^a \rho^b r^c$ ... (i) Where K is a dimensionless of V_C in term of η, ρ and r.

In dimensional form, the equation (i) can be written as

$[M^0 LT^{-1}] = [ML^{-1}T^{-1}]^a [ML^{-3}]^b [L]^c = [M^{a+b} L^{-a-3b+c} T^{-a}]$

Applying the principle of homogeneity of dimensions, we get

Power of M, a + b = 0
Or b = -a ...... (ii)
Power of L, -a – 3b + c = 1
Or -a + 3b + c = 1
Or 2a + c = 1 ...... (iii)
Power of T, -1 = -a or a = 1
From (ii), b = -1
From (iii), c = 1 – 2a = 1 – 2 × 1 = -1
Putting these values in (i), we get

$V_C = K\,\eta^1 \rho^{-1} r^{-1} = K\,\eta/\rho r$ ...... (iv)

For the flow to be streamline value of V_C should be as large as possible. For this, η should be large ρ r should be small. If the value of Reynolds number lies between 0 to 2000, the flow of liquid is stream line or laminar. For values of N_R above 3000, the flow of liquid is turbulent and for the values of N between 2000 to 3000, the flow of liquid is unstable changing from stream line to turbulent flow.

Physical signification of Reynold's number:

Reynold's number describes the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

Consider a tube of small area of cross – section A, through which a fluid of density ρ is flowing with velocity v.

The mass of fluid flowing through the tube per second,

Δm = volume flowing per second × density
$= Av \times \rho$

Therefore, Inertial force per unit area = $\frac{\text{rate of change of momentum}}{\text{Area}}$

$$= \frac{(\Delta m)v}{A} = \frac{(Av\rho)\,v}{A} = v^2\rho$$

Viscous force, $F = \eta A \frac{v}{r}$ [in magnitude]

Where 'r' is the radius of the tube and v/r is the velocity gradient between the layers of the liquid flow

Therefore, Viscous force per unit area = $F/A = \eta v / r$

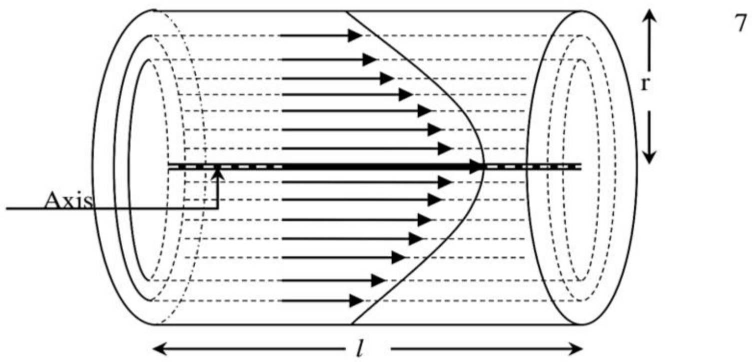
Thus, **Reynold's number** = $\frac{\textbf{Inertial force per unit area}}{\textbf{Viscous force per unit area}}$

$$N_R = \frac{v^2\rho}{\eta v/r} = \frac{v\rho r}{\eta}$$

- **Reynolds number is dimensionless $[N_R]$** = $\frac{[v_c]\,[\rho]\,[D]}{[\eta]} = \frac{[LT^{-1}]\,[ML^{-3}]\,[L]}{[ML^{-1}T^{-1}]} = [M^0 L^0 T^0]$

FLOW OF A LIQUID THROUGH A PIPE:

Consider a liquid flowing steadily through a horizontal pipe. It will be observed that the liquid flows in cylindrical layers co-axial with the pipe. The cylindrical layer which is in contact with the wall of the pipe is at rest. The velocity of the different layers goes on increasing gradually as we move from the wall towards the axis of the tube. It is maximum along the axis of the tube.



POISEUILLE'S EQUATION

Poiseuille studied the stream line flow in the capillary tubes and concluded that the volume {V} of the liquid flowing per second through a capillary tube depends upon-----
...(i) Coefficient of viscosity, η;... (ii) Radius of the tube, r; ....(iii) The pressure gradient, $\frac{P}{l}$

Proof :

Let (i) $V \propto \eta^a$; (ii) $V \propto r^b$; (iii) $V \propto (P/l)^c$

Combining all these factors, we get $V \propto \eta^a r^b (P/l)^c$

$V = K \eta^a r^b (P/l)^c$ ... (i)

[Where K is a dimensionless constant of proportion

Writing down the dimension of either side, we get

$[M L^3 T^{-1}] = [M L^{-1} T^1]^a [L]^b [ML^{-2} T^{-2}]^c$

$[M L^3 T^{-1}] = [M^{a+c} L^{-a+b-2c} T^{-a-2c}]$

Equating powers of M, L & T, we get [Principle of homogeneity]

Powers of M; $a + c = 0$ => $a = -c$ => $a = -1$

Powers of L; $-a + b - 2c = 3$ => $-(-1) + b - 2(1) = 3$ => $b = 4$

Powers of T; $-a - 2c = -1$ => $c - 2c = -1$ => $c = 1$

From equation (i), $V = k\eta^{-1} r^4 (p/l)^1$ or, $v = k \frac{pr^4}{\eta l}$

Mathematical calculation shows that $k = \pi/8$

$\therefore$ $$V = \frac{\pi pr^4}{8 \eta l}$$ which is Poiseuille's equation.

➢ **P's equation holds good under the following condition:--**

1. The capillary tube is horizontal. So, the gravity has no effect on the flow of liquid through the tube.
2. The cylindrical layer in contact with the wall of the capillary tube is at rest. The velocity of the liquid layer goes on increasing as we move from the wall towards the axis of the tube. So, the layer along the axis moves with max velocity.
3. The liquid flow is streamlined and parallel to the axis of the tube. So, the acceleration of the liquid at any point is zero.
4. The pressure over any cross-section of the tube is constant. So, there is no radial flow of the liquid.
5. The liquid is viscous.
6. The liquid can withstand small shearing stress.

** These condition can be nearly realized in actual practice if the capillary tube is of fine bore and the liquid flows with small velocity.

➢ According to poiseuille's equation $V = \frac{\pi pr^4}{8 \eta l} = P/R$; where, $R = \frac{8 \eta l}{\pi r^4}$ = called **Liquid resistance**.

➢ When two capillary tubes of length l_1 & l_2, radius r_1 & r_2 are joined end (in series), through which a liquid is flowing the equivalent liquid resistance (R_S) is

$$R_S = R_1 + R_2 = \frac{8 \eta l}{\pi r^4} + \frac{8 \eta l}{\pi r_2^4}$$

➢ When two capillary tubes are joined in parallel through which the liquid is flowing, then the effective liquid resistance is

$$1/R_P = 1/R_1 + 1/R_2 = \pi \frac{r_1^4}{8 \eta l} + \pi \frac{r_2^4}{8 \eta l}$$

ENERGIES POSSESSED BY A LIQUID:

A liquid may possess the following three types of energies:-

(a) Pressure energy (b) Kinetic energy (c) Potential energy

Pressure energy: ***"Pressure energy is the energy possessed by a liquid by virtue of its pressure"***.

- It is the potential energy associated with pressure i.e., it is measure of work done in pushing the liquid against pressure without imported any velocity to it.

Let us consider a liquid of density 'ρ' contained in a wide tank 'T' having side tube near the bottom of the tank. A frictionless piston of cross-sectional area 'a' is fitted in the side tube. Let 'h' is the height of free surface of the liquid in the tank above the axis of the tube. Therefore,

Hydrostatic pressure on the piston $P = h \rho g$

Let the liquid be pushed in the tank by moving the piston very slowly to the left through a small distance 'x' (Since the process is very slow, hence the kinetic energy acquired by the liquid in this process is negligible.

Volume of the liquid pushed into the tank = $a\,x$

Mass of the liquid pushed into the tank = $a\,x\,\rho$

Force on the piston = P a.

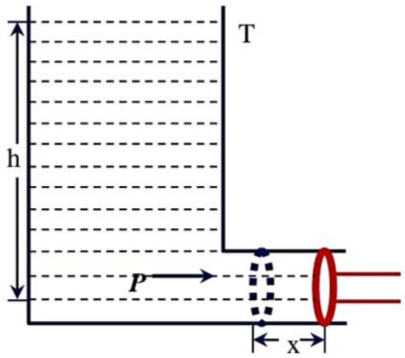
Work done in pushing the liquid into the tank = $Pa \times x$

This work done in pushing the liquid against he pressure P into the tank without imparting it any velocity becomes the pressure energy of mass 'a x p' and volume 'ax' of the liquid.

Therefore, Pressure energy per unit mass = $P\,a\,x / a\,x\,\rho = P/\rho$

Also, Pressure energy per unit volume of the liquid = $P\,a\,x / a\,x$ = P [hydrostatic Pressure]

Hence, ***"Pressure energy per unit volume of the liquid is equal to the hydrostatic pressure due to the liquid"***.



Kinetic energy: ***"Kinetic energy is the energy possessed by a liquid by virtue of its motion or velocity"***.

Consider a mass 'm' of liquid moving with velocity 'v'; density 'ρ'; and volume 'V'.

K.E. of the liquid = $\frac{1}{2} mv^2$

K.E. per unit mass of the liquid = $\frac{1}{2} mv^2/m = \frac{1}{2} v^2$

K.E. per unit volume of the liquid = $\frac{1}{2} mv^2/V = \frac{1}{2} \rho v^2$

Potential energy: ***"Potential energy is the energy possessed by a liquid by virtue of its height or position above the surface of earth or any reference level taken as zero"***.

Consider a mass 'm' of a liquid at a height 'h' above the zero level and V be its volume.

P.E. of the liquid = mgh

P.E. per unit mass of the liquid = $mgh/m = gh$

P.E. per unit volume of the liquid = $mgh/V = \rho gh$

**** Total energy per unit mass of a flowing = $P/\rho + \frac{1}{2} v^2 + gh$**

**** Total energy per unit volume of a flowing liquid = $P + \frac{1}{2} \rho v^2 + \rho gh$**

** The three forms of energy are inter convertible.

BERNOULLI'S THEOREM:

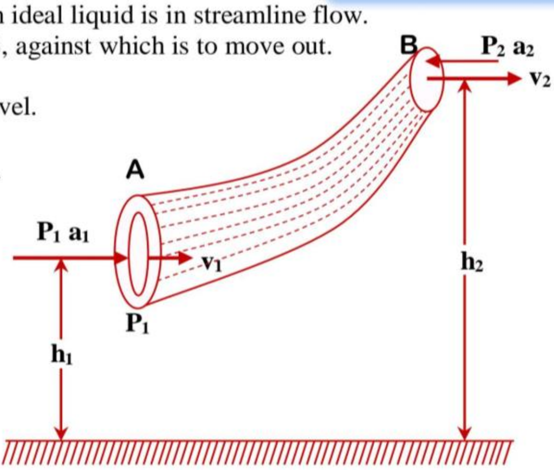
[1738, DANIEL BERNOULLI, established a theorem for the streamline flow of an ideal fluid by making use of the principal of conservation of energy.]

"For the stream line flow for an ideal fluid (non-viscous and incompressible), the sum of the pressure energy, Kinetic energy and potential energy per unit mass is always constant at every cross-section throughout the flow".

Mathematically, $P + \frac{1}{2} \rho v^2 + \rho gh$ = constant

Proof: Consider a tube AB of varying cross-section through which an ideal liquid is in streamline flow.

Proof: Consider a tube AB of varying cross-section through which an ideal liquid is in streamline flow.



Let P_1 = pressure applied on the liquid at A; P_2 = Pressure at the end B, against which is to move out.

a_1 & a_2 = Area of cross-section of the tube at A & B resp.

h_1 & h_2 = Mean height of section A & B from the reference level.

v_1& v_2 = Normal velocity of the liquid flow at A & B.

ρ = Density of the ideal liquid flowing through the tube.

Since the liquid flows from A to B, therefore,

$$\boxed{P_1 > P_2}$$

The mass 'm' of the liquid crossing per second through any section of the tube in accordance with the equation of continuity.

$a_1 v_1 \rho = a_2 v_2 \rho = m$

or, $a_1 v_1 = a_2 v_2 = m/\rho = V$

As, $a_1 > a_2$; therefore, $v_2 > v_1$

Now, force on the liquid at section A = $P_1 a_1$

Force on the liquid at section B = $P_2 a_2$

Work done / second by the liquid at section A = $P_1 a_1 \times v_1 = P_1 \times V$

Work done / second by the liquid at section B = $P_2 a_2 \times v_2 = P_2 \times V$

Net work done / second on the liquid by the pressure energy in moving the liquid from section A to B = $P_1 V - P_2 V$.

When mass 'm' of the liquid flows in one second from A to B, its height increases from 'h_1' to 'h_2' and its velocity increases from 'v_1' to 'v_2'.

Therefore, Increases in Potential energy / second of the liquid from A to B = $mgh_2 - mgh_1$

Increases in Kinetic energy / second of the liquid from A to B = $\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$

According to work energy theorem,

Work done / second by the pressure energy = Increase in P.E. / sec. + Increases in K.E. / sec.

Or, $P_1 V - P_2 V = [mgh_2 - mgh_1] + [\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2]$

=> $P_1 V + mgh_1 + \frac{1}{2} mv_1^2 = P_2 V + mgh_2 + \frac{1}{2} mv_2^2$

=> $\cancel{m}[P_1 V/m + gh_1 + \frac{1}{2} v_1^2] = \cancel{m}[P_2 V/m + gh_2 + \frac{1}{2} v_2^2]$

=> $P_1/\rho + gh_1 + \frac{1}{2} v_1^2 = P_2/\rho + gh_2 + \frac{1}{2} v_2^2$

$$\boxed{P/\rho + gh + \frac{1}{2} v^2 = \text{Constant}} \quad \text{———— [A]}$$

P/ρ = Pressure energy per unit mass; gh = Potential energy per unit mass; $\frac{1}{2} v^2$ = Kinetic energy per unit mass.

From [A] Pressure energy per unit mass + P.E. per unit mass + K.E. per unit mass = constant

i.e., ***the sum of pressure energy; potential energy and kinetic energy per unit mass are constant at all cross-sections in the stream line flow of an ideal liquid. [Proves Bernoulli's theorem].***

ANOTHER form of B's theorem:

[1] Multiplying both sides of [A] by 'ρ', we get

$$\boxed{P + \rho gh + \frac{1}{2} \rho v^2 = \text{constant}} \quad \text{———— [B]}$$

i.e., ***For a stream line flow of an ideal fluid, the sum of pressure energy, kinetic energy and potential energy per unit volume is always constant at all cross-sectional of the liquid***.

[2] Dividing [A] by 'g' we get,

$$\boxed{P/\rho g + h + v^2/2g = \text{constant}} \quad \text{———— [C]}$$

Here, $P/\rho g$ = pressure head; h = Gravitational head; $v^2/2g$ = velocity head

(Every term of the equation [C] has a dimension of length. So, each term is called '*a head*'.)

i.e., ***For a stream line flow of an ideal liquid, the sum of pressure head, velocity head and gravitational head at every cross-section of the liquid is constant***.

[3] If the liquid is flowing through a horizontal tube, the ends of the tube are at same level. Therefore, there is no gravitational head i.e., $h = 0$

From [B] $$\boxed{P + \frac{1}{2} \rho v^2 = \text{Constant}} \quad \text{———— [D]}$$

Here P = Static pressure; $\frac{1}{2} \rho v^2$ = Dynamic pressure.

If P increases, v decreases and vice versa.

i.e., ***In a stream line flow of an ideal liquid through a horizontal tube, the velocity increases where pressure decreases and vice versa.***

Limitations of BERNOULLI'S THEOREM

In the derivation of Bernoulli's equation, certain assumptions are made which are not justified in actual practice. As a result, Bernoulli's equation has the following limitations :

(*i*) In the derivation of Bernoulli's equation, it is assumed that the liquid is nonviscous *i.e.*, the liquid has zero viscosity (no friction). However, a real liquid does have some viscosity so that a part of mechanical energy is lost to overcome liquid friction. This fact is not taken into account in this equation.

(*ii*) In the derivation of Bernoulli's equation, it is assumed that rate of flow of liquid is constant (*i.e.*, $a_1v_1 = a_2v_2$). But this is not correct in actual practice. Thus in the case of a liquid flowing through a pipe, the velocity of flow is maximum at the centre and goes on decreasing towards the walls of the pipe. Therefore, we should take average velocity of the liquid.

(*iii*) In the derivation of Bernoulli's equation, it is assumed that there is no loss of energy when liquid is in motion. In practice, this is not true. For example, a part of K.E. of flowing liquid is converted into heat and is lost forever.

(*iv*) If the liquid is flowing along a curved path, the energy due to centrifugal force must be considered.

Applications of BERNOULLI'S THEOREM

(*i*) **Aerofoil lift.** An aerofoil (*e.g.*, an aircraft wing) is shaped so that air flows faster along the top surface than the lower one as shown in Fig. 20.12. This is shown by the closeness of the streamlines above the aerofoil compared with those below. From Bernoulli's theorem, the pressure of the air below is greater than that above. It is this difference in pressure that provides a net upward force. This is called **dynamic lift.** Actually Bernoulli's theorem is only one aspect of the lift on a wing. Wings are usually tilted slightly upward so that air striking the bottom surface is directed downward. The change in momentum of the rebounding air molecules results in an additional upward force on the wing.

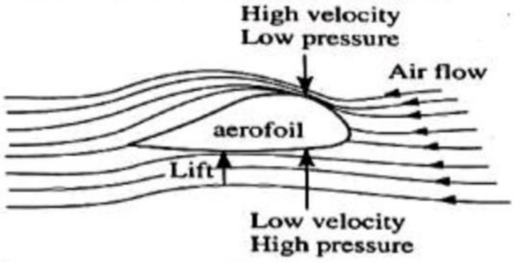


Fig. 20.12

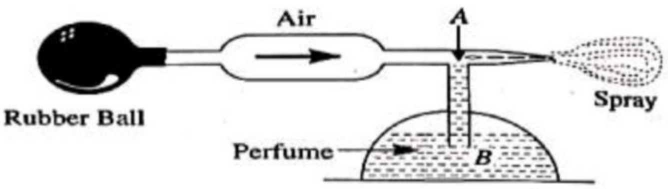


Fig. 20.13

(*ii*) **Atomiser or sprayer.** Fig. 20.13 shows a perfume atomiser. Similar devices are used to spray paint or insecticide. When the bulb in the atomiser is squeezed, air rushes through the narrow neck of the atomiser. This reduces the pressure in the narrow channel (point *A*) so that atmospheric pressure pushes the perfume up the tube leading to the narrow channel. The perfume is then dispersed into a fine spray of droplets.

(*iii*) **Blowing off roof in wind storm.** When wind flows with a high velocity above the roof of a house (See Fig. 20.14), it causes a lowering of pressure above the roof. However, pressure below the roof is still atmospheric. If the wind blows fast enough, the internal air pressure (atmospheric) can blow the roof off. Let us calculate the upward force on the roof during wind storm. Air above the roof pushes down with a force $F_1 = P_1A$ where P_1 is the air pressure above the roof and A is the area of the roof. The air inside the house pushes up with a force $F = P_a A$ where P_a is the atmospheric pressure of the air inside the house. The net upward force is

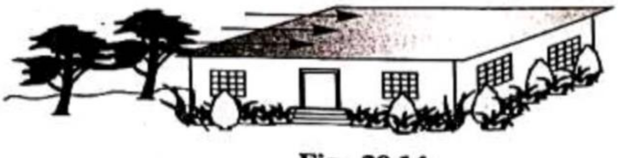


Fig. 20.14

$$F_{net} = F - F_1 = P_a\ A - P_1\ A = A\ (P_a - P_1)$$

A small pressure difference over a large area can produce a large upward force.

(*iv*) **A suction effect** is experienced by a person standing close to the platform at a station when a fast train passes. The fast-moving air between the person and the train produces a decrease in pressure and the excess air pressure on the other side pushes the person towards the train.

TORRICELLI'S THEOREM

This theorem applies to a liquid flowing from a drum with a horizontal opening near the base and may be stated as under :

This theorem states that if the difference in levels between the hole and the upper liquid surface in a drum is h, then the velocity with which the liquid emerges from the hole is $\sqrt{2gh}$.

Note that this is the same velocity which a freely falling object will acquire in falling from rest through a vertical distance h.

Proof. Suppose an ideal liquid flows through a hole H at the bottom of a wide drum as shown in Fig. 20.16. Let ρ be the density of the liquid and h the depth of the hole below the free surface of the liquid. According to Bernoulli's theorem, at any point of the liquid, we have,

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{Constant} \quad ...(i)$$

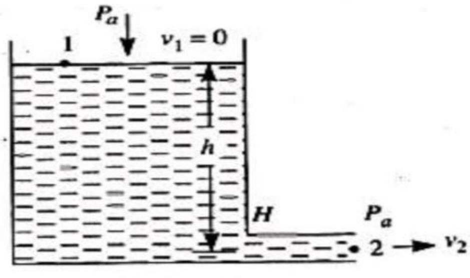


Fig. 20.16

We select points 1 and 2 for applying Bernoulli's theorem. The point 1 is at the surface of the liquid in the drum and point 2 is the place where liquid leaves the hole. Suppose we measure the height from the hole level.

At point 1. $P = P_a$; $\rho gh = \rho gh$; $\frac{1}{2}\rho v^2 = \frac{1}{2}\rho (0)^2 = 0$

It is bec he drum is wide and, therefore, $v_1 = 0$. Therefore, eq. (i) evaluated at point 1 becomes :

$$P_a + \rho g h = \text{Constant} \quad ...(ii)$$

At point 2. $P = P_a$; $\rho gh = 0$; $\frac{1}{2}\rho v^2 = \frac{1}{2}\rho v_2^2$

Therefore, eq. (i) evaluated at point 2 becomes :

$$P_a + \frac{1}{2}\rho v_2^2 = \text{Constant} \quad ...(iii)$$

$$\therefore \quad P_a + \rho g h = P_a + \frac{1}{2}\rho v_2^2$$

or

$$v_2 = \sqrt{2gh}$$

Although it is seen to be a special case of Bernoulli's theorem, it was discovered by Torricelli about 100 years before Bernoulli gave his theorem. *Note that velocity* $(v_2 = \sqrt{2gh})$ *of the liquid emerging from the hole depends only upon the depth (h) of the hole below the free surface of the liquid.*

Horizontal range. The liquid flows out of the hole H in the form of a parabolic jet (See Fig. 20.17) and strikes the ground at a distance R from the base of the drum. The distance R is the horizontal range of the liquid coming out of the hole. At the hole H, the velocity v_2 of the emerging liquid is along the horizontal direction.

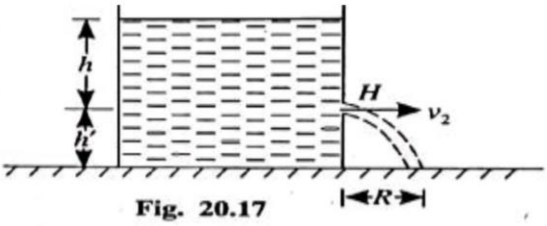


Fig. 20.17

$$\therefore \quad R = v_2 t = \sqrt{2gh} \times t$$

Here t is the time taken by the parabolic jet to strike the ground after emerging from the hole H.

Let h^1 = Height of hole above the bottom of the drum

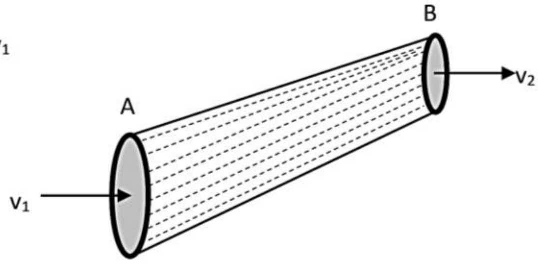
Therefore, the vertical distance covered by the jet in time t is h'. Now at point H, vertical velocity is zero.

$$\therefore \quad h' = 0 + \frac{1}{2}g t^2 \text{ or } t = \sqrt{\frac{2h'}{g}}$$

$$\therefore \text{ Horizontal range,} \quad R = \sqrt{2gh} \times \sqrt{\frac{2h'}{g}} = 2\sqrt{hh'}$$

EQUATION OF CONTINUITY:

Let us consider the steady and irrotational flow of an incompressible and non-viscous liquid through a pipe AB of varying cross-section. Let a_1 & a_2 = Cross-sectional areas of the pipe at A & B resp.
Let the liquid enters the end A normally with velocity v_1 and leave the end B normally with velocity v_2.



Volume of liquid flowing per second through end A = $a_1 v_1$
[It is assumed that there is no change in the velocity and cross-sectional area in one second]

Mass of liquid flowing per second through A = $a_1 v_1 \rho_1$ where, ρ_1 = density of the liquid at A.
Mass of liquid flowing per second through B = $a_2 v_2 \rho_2$ where, ρ_2 = density of the liquid at B.

Since there is no **source or sink** in the pipe where liquid can be created or destroyed. **So, mass of liquid crossing any crossing any section of the pipe per second must be same. Therefore,**

Mass of liquid flowing per second through A = Mass of liquid flowing per second through B

or, $a_1 v_1 \rho_1 = a_2 v_2 \rho_2$

Since the liquid is incompressible, therefore, $= \rho_2$

∴ $a_1 v_1 = a_2 v_2$

a v = constant This is called equation of continuity for the stream line flow of an incompressible and non-viscous liquid.

DISCUSSION:

1. **v α 1/a** the velocity of flow is inversely proportional to the cross – sectional area. The velocity of flow increases as the cross-sectional area decreases and vice-versa.
2. In case of pipe the cross-sectional area is proportional to the square of the radius R of the pipe.
   ∴ **$v \propto 1/R^2$**
3. The cross-sectional area available to flowing water goes on increasing as we approach the bottom of a canal or river. So, the velocity of the flow decreases. This explains as to **why the deep water runs slowly**.
4. The number of streamline starting from A is the same as that ending on B. At the end A, the streamline are wide apart and velocity of flow of the liquid is less. At the end B, the streamline are crowded and the velocity of flow is more. So, we concluded that ***crowded streamlines indicate region of high velocity while the widely spaced streamline indicates the low velocity regions***.
5. Since the velocity of the liquid increases from A to B therefore there must be some accelerating force. So, the pressure is greater than the pressure at the end B. T may be concluded from here that the pressure is greater at a point where the velocity is small and vice versa.
6. The falling of water becomes narrow, as the velocity of falling stream of water increases (due to acceleration due to gravity) so its area cross-section decreases, according to equation of continuity.
7. In case a non-viscous liquid is flowing with stream line flow, the velocity of flow is independent of the nature of the liquid.

Example 20.10. **Water flows through a horizontal pipe of varying cross-section at the rate of 10 m³/minute. Determine the velocity of water at a point where the radius of the pipe is 10 cm.**

Solution. Rate of discharge, $Q = 10\ m^3/minute = \frac{1}{6}\ m^3/s$

Cross-sectional area, $a = \pi r^2 = \pi(0.1)^2 = 0.0314\ m^2$

Now, $Q = a\,v$

∴ Velocity of water, $v = \frac{Q}{a} = \frac{1/6}{0.0314} = 5.3\ ms^{-1}$

Example 20.11. **How fast can a raindrop having a diameter 3.0 mm fall before the flow of air around it becomes turbulent? Critical Reynolds number for laminar flow around a sphere is 10. The viscosity of air = 0.019 × 10⁻³ Pas and density of air = 1.29 kg/m³.**

Solution. We can consider the raindrop as a sphere. As it falls at speed v, the air flows past it with the same speed. The Reynolds number is

$$N_R = \frac{\rho v D}{\eta}$$

The critical velocity above which turbulence will occur ($N_R = 10$) is

$$v_{max} = \frac{\eta N_R}{\rho D} = \frac{10\times0.019\times10^{-3}}{1.29\times3.0\times10^{-3}} = 4.9\times10^{-2}\ ms^{-1} = 4.9\ cms^{-1}$$

Example 20.12. The flow rate of water from a tap of diameter 1.25 cm is 3 litres per minute. The coefficient of viscosity of water is 10^{-3} Pa-s. Characterise the flow.

Solution. Reynolds number, $N_R = \frac{\rho v D}{\eta}$

Here $\rho = 10^3$ kg/m^3 ; $V = 3$ litres/min $= \frac{3\times10^3}{60}$ c.c./s $= 50\times10^{-6}$ m^3/s ; $D = 1.25\times10^{-2}$ m

$$\text{Velocity of water, } v = \frac{V}{\text{Area}} = \frac{50\times10^{-6}}{\pi/4\times(1.25\times10^{-2})^2} = 0.4074 \text{ ms}^{-1}$$

$$\therefore \quad N_R = \frac{10^3\times0.4074\times(1.25\times10^{-2})}{10^{-3}} = 5092.5$$

Since the value of N_R is greater than 3000, the flow of water from the tap is turbulent.

Example 20.13. Water flows through a pipe of internal diameter 20 cm at the speed of 1 ms^{-1}. What should the diameter of the nozzle be if the water is to emerge at the speed of 4 ms^{-1}?

Solution. Here $d_1 = 20$ cm $= 0.2$ m ; $v_1 = 1$ ms^{-1} ; $v_2 = 4$ ms^{-1} ; $d_2 = ?$

Now $$a_1v_1 = a_2v_2$$

or $$\left(\frac{\pi}{4}d_1^2\right)v_1 = \left(\frac{\pi}{4}d_2^2\right)v_2$$

or $$\frac{d_2^2}{d_1^2} = \frac{v_1}{v_2} = \frac{1}{4}$$

or $$\frac{d_2}{d_1} = \frac{1}{2}$$

$$\therefore \quad d_2 = \frac{d_1}{2} = \frac{0.2}{2} = 0.1 \text{ m} = \textbf{10 cm}$$

Example 20.14. At what speed will the velocity head of a stream of water be equal to 40 cm?

Solution. Velocity head, $\frac{v^2}{2g} = h \quad \therefore \quad v = \sqrt{2gh}$

Here $g = 9.8$ m s^{-2} ; $h = 40$ cm $= 0.40$ m

$$\therefore \quad v = \sqrt{2\times9.8\times0.40} = \textbf{2.8 ms}^{-1}$$

Example 20.15. A pipe is running full of water. At a certain point A, it tapers from 60 cm diameter to 20 cm diameter at B ; the pressure difference between A and B is 100 cm of water column. Find the rate of flow through the pipe.

Solution. $$\frac{a_A}{a_B} = \left(\frac{d_A}{d_B}\right)^2 = \left(\frac{0.6}{0.2}\right)^2 = 9$$

$$P_A - P_B = h\rho g = (1)\times(1000)\times9.8 = 9800 \text{ N/m}^2$$

Now $$v_A a_A = v_B a_B \quad \therefore \quad v_B = v_A\left(\frac{a_A}{a_B}\right) = 9\,v_A$$

Using Bernoulli's theorem for a horizontal pipe,

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 = \frac{P_B}{\rho} + \frac{1}{2}v_B^2$$

or $$\frac{1}{\rho}(P_A - P_B) = \frac{1}{2}\left(v_B^2 - v_A^2\right)$$

or $$P_A - P_B = \frac{\rho}{2}\left(v_B^2 - v_A^2\right)$$

or $$9800 = \frac{1000}{2}\left(81v_A^2 - v_A^2\right) \qquad (\because v_B = 9\,v_A)$$

or $$9800 = 40000\,v_A^2$$

$$\therefore \quad v_A = \sqrt{\frac{9800}{40000}} = 0.495 \text{ ms}^{-1}$$

$$\therefore \quad \text{Rate of discharge, } Q = v_A a_A = 0.495\times\frac{\pi}{4}(0.6)^2 = \textbf{0.14 m}^3\textbf{/s}$$

Example 20.16. The reading of a pressure meter attached to a closed pipe is 2.5 × 10^5 Nm^{-2}. On opening the valve of the pipe, the reading of the pressure meter reduces to 2.0 × 10^5 Nm^{-2}. Calculate the speed of water flowing through the pipe.

Solution. $P_1 = 2.5\times10^5$ Nm^{-2} ; $v_1 = 0$ ($\because$ Initially the pipe was closed)

$P_2 = 2.0\times10^5$ Nm^{-2} ; $v_2 = ?$; $\rho = 1000$ kg m^{-3} (for water)

According to Bernoulli's theorem for a horizontal pipe,

$$P_1 + \frac{1}{2}\rho\,v_1^2 = P_2 + \frac{1}{2}\rho\,v_2^2$$

or $$P_1 = P_2 + \frac{1}{2}\rho\,v_2^2 \qquad (\because v_1 = 0)$$

or $$v_2^2 = \frac{2(P_1 - P_2)}{\rho} = \frac{2\left(2.5\times10^5 - 2.0\times10^5\right)}{1000} = 100$$

$$\therefore \quad v_2 = \sqrt{100} = \textbf{10 ms}^{-1}$$

Example 20.24. In giving a patient blood transfusion, the bottle is set up so that the level of blood is 1.3 m above the needle, which has an internal diameter of 0.36 mm and is 3 cm in length. If 4.5 cm^3 of blood passes through the needle in one minute, calculate the viscosity of blood. The density of blood is 1020 kgm^{-3}.

Solution. Volume of blood flowing per second is

$$Q = \frac{\text{Total volume}}{\text{Time}} = \frac{4.5}{60} = 0.075\ cm^3s^{-1}$$

Length of needle, l = 3 cm

Radius of needle, $R = \frac{0.36}{2}$ mm = 0.18 mm = 0.018 cm

Density of blood, ρ = 1020 kg m^{-3} = 1.02 g cm^{-3}

Pressure difference, p = 1.3 m of column of blood

$= h\rho g = (1.3 \times 100) \times (1.02) \times (980)$ dynes cm^{-2}

Now $$Q = \frac{\pi p R^4}{8 \eta l}$$

$\therefore$ Viscosity of blood, $$\eta = \frac{\pi p R^4}{8 Q l}$$

$$= \frac{\pi \times (1.3 \times 100) \times (1.02) \times 980 \times (0.018)^4}{8 \times 0.075 \times 3} = \mathbf{0.0238\ poise}$$

Example 20.25. Water is conveyed through a horizontal tube 0.08 m in diameter and 4 km length at the rate of 20 litres per second. Assuming only viscous resistance, calculate the pressure difference required to maintain the flow ($\eta = 10^{-3}$ Nsm^{-2}).

Solution. According to Poiseuille's formula,

Rate of flow, $$Q = \frac{\pi p R^4}{8 \eta l}$$

$\therefore$ Required pressure difference, $$p = \frac{8 Q \eta l}{\pi R^4}$$

Here Q = 20 litres/s = 2×10^{-2} m^3/s ; $\eta = 10^{-3}$ Nsm^{-2} ; l = 4 km = 4000 m ; R = 0.04 m

$\therefore$ $$p = \frac{8 \times (2 \times 10^{-2}) \times 10^{-3} \times 4000}{\pi \times (0.04)^4} = \mathbf{7.96 \times 10^4\ N/m^2}$$

Example 20.26. Two capillaries of same length and radii in the ratio of 1 : 2 are connected in series and a liquid flows through this system under streamline conditions. If the pressure across the two extreme ends of the combination is 1 m of water, what is the pressure difference across the first capillary?

Solution. Let l and R be the length and radius of the first capillary respectively. Then the corresponding values for the second capillary are l and $2R$. Let p_1 and p_2 be the pressure differences across the first and second capillary respectively. Since the capillaries are connected in series, the rate of flow of liquid through them is the same *i.e.*,

$$\frac{\pi p_1 R^4}{8 \eta l} = \frac{\pi p_2 (2R)^4}{8 \eta l}$$

or $$p_1 = 16\, p_2$$

It is given that $p_1 + p_2$ = 1 m.

$\therefore$ $$p_1 + \frac{p_1}{16} = 1\ m$$

$\therefore$ $$p_1 = \frac{16}{17}\ m = \mathbf{0.94\ m}$$

CONCEPTUAL QUESTIONS

Q.1. The diameter of ball A is half that of ball B. What will be their ratio of terminal velocities in water?

Ans. The terminal velocity is directly proportional to the square of radius of the ball. Therefore, the ratio of terminal velocities will be 1 : 4.

Q.2. Why do the clouds seem floating in the sky?

Ans. The terminal velocity of a raindrop is directly proportional to the square of radius of the drop. When falling, large drops have high terminal velocities while small drops have small terminal velocities. The small drops fall so slowly that cloud seems to be floating.

Q.3. Why a parachute descends slowly whereas a stone dropped from the same height falls rapidly?

Ans. The open parachute has a large surface area. Therefore, the viscous force of air on the parachute is larger than that on a falling stone. For this reason, parachute descends slowly *i.e.*, it has a small terminal velocity.

Q.4. What is viscosity?

Ans. Viscosity is fluid friction caused (in liquids) by cohesive forces between molecules and (in a gas) by collisions between molecules. The viscosity of liquids decreases as temperature rises, but the opposite is true for gases.

Q.5. What is the principle on which continuity equation is based?

Ans. The continuity equation is based on the principle of conservation of mass. We know that fluid is conserved ; as a fluid moves and deforms, new fluid is neither created nor destroyed. The continuity equation is a mathematical statement of this fact that fluid (*i.e.*, matter itself) is conserved.

Q.6. What is the principle on which Bernoulli's theorem is based?

Ans. Bernoulli's theorem is based on the principle of law of conservation of energy. The work done on a fluid as it flows from one place to another is equal to the change in its mechanical energy.

Q.7. What are the implications of Bernoulli's theorem when the fluid is at rest?

Ans. Bernoulli's theorem states that :

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \quad ...(i)$$

The equations of hydrostatics are special cases of Bernoulli's theorem when the velocity is zero everywhere. Thus when v_1 and v_2 are zero, eq. (*i*) becomes :

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

or

$$P_1 - P_2 = \rho g (h_2 - h_1)$$

Q.8. Name three physical principles which apply to both liquids and gases.

Ans. The three principles which apply to both liquids and gases are : Bernoulli's theorem, Pascal's principle and Archimedes' principle.

Q.9. What causes turbulent flow in a fluid?

Ans. The turbulent flow in a fluid may be caused due to too great a speed, abrupt change in areas or direction of flow.

Q.10. When a fluid flows through a narrow constriction, its speed increases. How does it get the energy for this extra speed?

Ans. When a fluid flows through a narrow constriction, its speed increases. This is easily noticed by the increased speed of a river when it flows through the narrow parts. The fluid must speed up in the constricted region if the flow is to be continuous. Bernoulli wondered how the fluid got energy for this extra speed. He rightly reasoned that it is acquired at the expense of a lowered internal pressure.

Q.11. Why does a flag flutter when strong winds are blowing on a certain day?

Ans. When strong winds blow on the surface of a flag, the velocity of the wind is different at the different points on the flag. According to Bernoulli's theorem, the pressure will also be different at these points. This pressure difference causes the flag to flutter.

Q.12. According to Bernoulli's theorem, the pressure of water should remain uniform in a pipe of uniform cross-section. But actually it goes on decreasing. Why?

Ans. It is due to the viscosity of water. When water flows, work is done against the viscous force. This work done is taken from the pressure energy. Hence pressure of water falls as it flows through a pipe.

Q.13. If two ships are moving parallel and close to each other, they experience an attractive force. Why?

Ans. When the two ships come close to each other, the air velocity between the narrow-gap increases and so pressure decreases. The pressure on the outer surfaces of the ships is then greater than the pressure in the gap. Therefore, the ships are pulled towards each other and sometimes they may collide.

Q.14. The accumulation of snow on an aeroplane wing may reduce the lift. Explain.

Ans. Due to accumulation of snow on the wings of the aeroplane, the structure of the wings no longer remains as that of aerofoil. As a result, the net upward force (*i.e.*, lift) is decreased.

Q.15. Show that the quantity $\frac{\rho v D}{\eta}$ (Reynolds number) is dimensionless.

Ans. Dimensions of $\rho = [ML^{-3}]$; Dimensions of $v = [LT^{-1}]$

Dimensions of $D = [L]$; Dimensions of $\eta = [ML^{-1}T^{-1}]$

$$\therefore \text{ Dimensions of } \frac{\rho v D}{\eta} = \frac{[ML^{-3}][LT^{-1}][L]}{[ML^{-1}T^{-1}]} = \frac{[ML^{-1}T^{-1}]}{[ML^{-1}T^{-1}]} = [M^0L^0T^0]$$

Therefore, Reynolds number is dimensionless.

Q.16. A liquid flowing out of a small hole in a vessel results in a backward thrust on the vessel. Why?

Ans. Since the area of cross-section of the hole is small, the speed of the liquid flowing out of the vessel will be large. Therefore, the fluid possesses large momentum at the hole. As no external force acts on the system, in order to conserve linear momentum, the vessel acquires a velocity in the backward direction *i.e.* a backward thrust is exerted on the vessel.

Q.17. Water is coming out of a hole made in the wall of tank filled with fresh water. If the size of the hole is increased, will the velocity of efflux of water change?

Ans. Velocity of efflux, $v = \sqrt{2gh}$

Since the velocity of efflux is independent of the area of hole, it will remain the same.

Q.18. Water flows faster than honey. Why?

Ans. According to Poiseuille's formula, the liquid flowing per second (Q) through a pipe is given by :

$$Q = \frac{\pi p R^4}{8 \eta l}$$

It is clear that for given P, R and l, $Q \propto 1/\eta$ where η is the coefficient of viscosity. Since η for water is less compared to honey, Q for water is greater than for honey. For this reason, water flows faster than honey.

VERY SHORT ANSWER QUESTIONS

Q.1. What is viscosity?

Ans. The internal friction between the layers of a fluid (liquid or gas) which opposes the motion of the fluid is called viscosity.

Q.2. What is the (*i*) SI unit (*ii*) dimensional formula of viscosity?

Ans. (*i*) Decapoise (*ii*) $[M L^{-1} T^{-1}]$

Q.3. Define one decapoise.

Ans. The coefficient of viscosity of a liquid is *one decapoise* if a tangential viscous drag of 1 N/m^2 acts between two parallel liquid layers moving with a velocity gradient of 1 ms^{-1} per metre.

Q.4. If liquid in a flask is shaken violently and kept on a table, the liquid comes to rest after some time. Why?

Ans. It is due to the internal friction between the layers of the liquid (*i.e.*, viscosity).

Q.5. Hotter liquids flow faster than cold ones. Why?

Ans. The coefficient of viscosity of a liquid decreases with the increase in temperature. Therefore, a hot liquid will flow faster than the cold one.

Q.6. If water in one flask and glycerine in the other are violently shaken and kept on the table, then which one will come to rest earlier?

Ans. The glycerine comes to rest earlier because the viscosity of glycerine is greater than that of the water.

Q.7. How will you compare the viscosity of two liquids?

Ans. Take two identical measuring cylinders. Fill each cylinder with a different liquid to the same height. Allow identical ball-bearings to fall through each cylinder. The ball-bearing will fall more slowly through a liquid of higher viscosity.

Q.8. What is the effect of temperature on the viscosity of (*i*) liquids (*ii*) gases?

Ans. (*i*) The viscosity of liquids decreases with the increase in temperature. (*ii*) The viscosity of gases increases with the increase in temperature.

Q.9. What is the (*i*) viscosity (*ii*) compressibility of an ideal liquid?

Ans. (*i*) zero (*ii*) zero.

Q.10. Why are machine parts jammed in winter?

Ans. Due to low temperature in winter, the viscosity of the lubricants used in machine parts increases. As a result, the machine parts get jammed.

Q.11. Why does a bigger rain drop fall faster than a smaller rain drop?

Ans. The terminal velocity of a rain drop (sphere) is directly proportional to the square of radius of the drop (See Art. 20.7). For this reason, a bigger rain drop falls faster than a smaller rain drop.

Q.12. What is the weight of a body falling with a terminal velocity through a viscous medium?

Ans. Zero. It is because when the body is falling through a viscous medium with the terminal velocity, the upward force (upthrust + viscous drag) is equal to the weight of the body.

Q.13. A body is falling through a viscous medium with terminal velocity. What is the acceleration of the body?

Ans. Zero. It is because when a body attains terminal velocity, it continues to move with the same velocity throughout the viscous medium.

Q.14. Why is parachute used while jumping from an aeroplane?

Ans. Viscous drag, $F = 6\pi\eta rv$. The open parachute has a large surface area (*i.e.*, large r) and experiences a large viscous force while descending. As a result, the terminal velocity becomes very small and the person does not get hurt.

Q.15. What is the terminal velocity of a body in a freely falling system?

Ans. Zero. It is because terminal velocity $v_T \propto g$ and for a freely falling system, $g = 0$.

Q.16. What is steady or streamline flow?

Ans. In a streamline or uniform flow, all the particles of the fluid that pass any given point follow the same path at the same speed (*i.e.*, they have the same velocity).

Q.17. What is turbulent flow?

Ans. In a turbulent flow (also known as disorderly flow), the speed and direction of the fluid particles passing any point vary with time.

Q.18. What is meant by critical velocity of a liquid?

Ans. The velocity of a liquid flow upto which its flow is streamline (steady) and above which its flow becomes turbulent is called critical velocity of the liquid.

Q.19. What is pressure head?

Ans. The pressure head at a location = $P/\rho g$ and has the dimensions of length.

Q.20. What is velocity head?

Ans. The velocity head at a location in a liquid flow = $v^2/2g$ and has the dimensions of length.

Q.21. What is the principle on which equation of continuity for steady flow of fluid is based?

Ans. It is based on the law of conservation of mass.

Q.22. What is Bernoulli's theorem?

Ans. It states that for the steady flow of an incompressible and non-viscous fluid, the total energy (pressure energy, potential energy and kinetic energy) of the fluid remains constant throughout the fluid flow.

Q.23. Upon which principle is Bernoulli's theorem based?

Ans. It is based on the principle of conservation of energy.

Q.24. What is the importance of Reynolds number?

Ans. It determines the nature of liquid flow (laminar or turbulent) through a pipe. It is a pure number (*i.e.*, dimensionless).

Q.25. What is an incompressible fluid?

Ans. An incompressible fluid is that in which changes in pressure produce no change in the density of the fluid.

SHORT ANSWER QUESTIONS

Q.1. What are the differences between solid friction and fluid friction?

Ans. (*i*) Solid friction is independent of the area of contact between the two surfaces. However, fluid friction (viscosity) is directly proportional to the area of liquid layers in contact.

(*ii*) Solid friction between two surfaces is independent of relative velocity between them. However, fluid friction is directly proportional to the relative velocity between the two liquid layers.

(*iii*) Solid friction is directly proportional to the normal reaction between the surfaces in contact. However, fluid friction is independent of the normal reaction between two layers of the liquid.

Q.2. What are the similarities between solid friction and fluid friction?

Ans. (*i*) Both come into play whenever there is a relative motion (*ii*) Both oppose the relative motion (*iii*) Both arise from intermolecular forces.

Q.3. What is the effect of (*i*) density (*ii*) pressure on the viscosity of liquids and gases?

Ans. (*i*) The viscosity of liquids increases with the increase in density while for gases, it decreases with the increase in density.

(*ii*) The viscosity of liquids (except water) increases with the increase in pressure while it is almost independent of pressure for gases. Note that viscosity of water decreases with the increase in pressure.

Q.4. The relative velocity between two parallel layers of water is 8 cms^{-1} and the perpendicular distance between them is 0.1 cm. What is the velocity gradient?

Ans. Velocity gradient $= \frac{dv}{dx} = \frac{8 \text{ cms}^{-1}}{0.1 \text{ cm}} = 80 \text{ s}^{-1}$

Q.5. Two hail stones with radii in the ratio of 1 : 2 fall from a great height through the atmosphere. What is the ratio of their terminal velocities?

Ans. Terminal velocity, $v \propto r^2$.

$$\therefore \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad \therefore \quad v_1 : v_2 = 1 : 4$$

Q.6. Why are oils of different viscosities used in automobiles in different seasons?

Ans. The viscosity of lubricating oils (liquids) decreases with the increase in temperature. Therefore, lubricating oil used in summer may not be suitable in winter. For this reason, oils of different viscosities are used as lubricants in different seasons.

Q.7. Why rain drops falling under gravity do not acquire very high velocity?

Ans. The rain drops falling under gravity acquire terminal velocity v_T (constant velocity) when viscous drag plus upthrust becomes equal to the weight of rain drop. Now v_T is directly proportional to the square of the radius of the drop (spherical body). Since the radius of rain drop is small, it does not acquire very high velocity.

Q.8. An object is thrown horizontally through air. What will be its terminal velocity?

Ans. Zero. It is because in this case, only retarding force due to air friction acts on the object. Therefore, the velocity of the object along the horizontal will ultimately become zero.

Q.9. What is terminal velocity? Upon which factors it depends?

Ans. When a spherical body falls through a fluid (liquid or gas), its velocity increases till the viscous drag plus upthrust becomes equal to the weight of the body. From now onwards, the body moves with constant maximum velocity called terminal velocity v_T. It is given by ;

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta} g$$

It is clear that terminal velocity depends upon radius (r) and density (ρ) of the body, density (σ) of the fluid and coefficient of viscosity (η) of the fluid.

Q.10. Why do air bubbles in water rise up?

Ans. The terminal velocity of a spherical body is directly proportional to the difference in the densities of the body and the fluid (*i.e.*, $\rho - \sigma$). Now here density of water (σ) is greater than the density of air (ρ), the terminal velocity is *negative*. For this reason, air bubbles rise up in water.

Q.11. Two identical drops of water are falling through air with a steady velocity v. If the drops combine to form a single drop, what would be the terminal velocity of the single drop?

Ans. Volume of bigger drop = Volume of 2 small drops

or $$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \text{ or } R=(2)^{1/3}\times r$$

We know that terminal velocity $\propto$ (radius)^2.

$$\therefore \quad \frac{v_T'}{v_T} = \frac{R^2}{r^2}=\frac{(2)^{2/3}r^2}{r^2}=(2)^{2/3}$$

$$\therefore \quad v_T' = (2)^{2/3}v_T = (2)^{2/3}v$$

Q.12. The velocity of a small ball of mass m and density ρ when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is σ, what is viscous force acting on the ball?

Ans. Since the ball has attained terminal velocity,

$\therefore$ Viscous force = Weight of ball – upthrust

$$= mg-\frac{m}{\rho}\times\sigma\times g = mg\left(1-\frac{\sigma}{\rho}\right)$$

Q.13. Distinguish between streamline and turbulent flow of a liquid.

Ans. (*i*) In streamline flow, all the particles of the liquid that pass any given point have the same velocity. In turbulent flow, the velocity of the liquid particles passing any point varies with time.

(*ii*) In streamline flow, the velocity of the liquid is less than the critical velocity of the liquid. In turbulent flow, liquid moves with a velocity greater than its critical velocity.

Q.14. Why two streamlines cannot cross each other?

Ans. The tangent at any point on a streamline gives the direction of flow of liquid at that point. If two streamlines cross each other at a point, then two tangents can be drawn at that point. It means that the liquid has two velocities along two different directions at this point. This is against the definition of streamline flow. Therefore, two streamlines cannot cross each other.

Q.15. Why does velocity increase when water flowing in broader pipe enters a narrow pipe?

Ans. For a steady flow, the equation of continuity tells us that av = constant. Here a is the area of cross-section of the pipe and v is the velocity of liquid flow. When water enters a narrow pipe, the area of cross-section (a) decreases and hence velocity (v) of water flow increases.

Q.16. Water flowing through a cylindrical pipe has radius $2R$ at point A and radius R at point B farther along the direction of flow. If the velocity of water at point A is v, then what is its velocity at point B?

Ans. According to equation of continuity, av = Constant.

$$\therefore \quad \pi(2R)^2\times v_A = \pi(R)^2\times v_B \quad \text{or} \quad v_B = 4\,v_A = 4\,v$$

Q.17. Deep water runs slow. Why?

Ans. According to equation of continuity, av = Constant *i.e.*, $v \propto 1/a$. For deep water, area of X-section (a) becomes large so that v is small. For this reason, deep water runs slow.

Q.18. Under what conditions Bernoulli's theorem can be used?

Ans. (*i*) The fluid is incompressible *i.e.*, density remains constant (*ii*) The fluid is non-viscous (*iii*) The flow is steady and the velocity of the fluid is less than the critical velocity.

Q.19. Why does the speed of a liquid increase and its pressure decrease when a liquid passes through constriction in a pipe?

Ans. When a liquid passes through a horizontal pipe, according to Bernoulli's theorem,

$$P_1+\frac{1}{2}\rho v_1^2 = P_2+\frac{1}{2}\rho v_2^2$$

Therefore, where pressure is high, velocity of liquid flow is low and *vice-versa*. When liquid passes through a constriction, the velocity increases ($\because$ av = constant) and pressure decreases.

Q.20. If the velocity head of a stream of water is equal to 10 cm, what is its speed of flow?

Ans. Velocity head, $h = \frac{v^2}{2g}$. Here $h = 10\times 10^{-2}$ m ; $g = 9.8$ ms^{-2}

$$\therefore \quad \text{Speed of flow, } v = \sqrt{2gh}=\sqrt{2\times9.8\times10\times10^{-2}} = 1.4 \text{ ms}^{-1}$$

Q.21. At what speed will the velocity head of a stream of water be equal to 40 cm of Hg?

Ans. Velocity head, $h = v^2/2g$ or $v = \sqrt{2gh}$

Here $g = 980$ cms^{-2}; h = 40 cm of Hg = 40 × 13.6 cm of water

$$\therefore \quad v = \sqrt{2\times980\times40\times13.6} = 1032.6 \text{ cms}^{-1} = 10.326 \text{ ms}^{-1}$$

Q.22. Air streams horizontally pass an air-plane. The speed over the top surface is 60 ms^{-1} and that under the bottom surface is 45 ms^{-1}. What is the difference in pressure? (Density of air = 1.293 kg m^{-3}.)

Ans. According to Bernoulli's theorem for horizontal flow,

$$\frac{P_1}{\rho}+\frac{1}{2}v_1^2 = \frac{P_2}{\rho}+\frac{1}{2}v_2^2$$

$$\therefore \quad P_2-P_1 = \frac{1}{2}\rho(v_1^2-v_2^2)=\frac{1}{2}\times1.293[(60)^2-(45)^2] = 1018 \text{ Nm}^{-2}$$