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XI CBSE

PHYSICS
SURFACE TENSION

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SURFACE TENSION

UNIT:VII CHAP:02

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SURFACE TENSION
UNIT-07 CH:02

TOOLS OF SI

According to the moplecular theory , matter is made up of very minute parcles called molecules Which attracts each other. The intermolecular forces may be classified as :
(a) Force of Cohesion (b) Force of Adhesion

Force of Cohesion

" The force of attraction between the molecules of a same substance or same kind is called is called Cohesive force ."

- In case of solids , Cohesive is very large and hence solids have definite shape and size .
- The force of cohesion in case of liquid is weaker than solid and hence liquids do not have definite shape but have definite volume.
- The force of cohesion is negligible in case of gases because of this fact , gases have neither fixed shape nor volume .

Ex- (i) Two drops of liquid mixed into one when brought in mutual contact because of the cohesive force.
(ii) It is very difficult to break a drop of mercury in to small droplets because of large cohesive force between mercury molecules.

Force of Adhesion

" Adhesive force is the force of attraction between molecules of different substance ."

- Force of Adhesion is different for different substance.

Ex- (i) Gum has a greater Adhesive force for solid surface than water .
(ii) Water wets glass. This is because the force of adhesion between the molecules of water and the glass is stronger than the force of cohesion between water molecules.
(iii) Adhesive force help us to write the paper with ink.

Practical Examples and Illustrations :

(*a*) The ink sticks on paper. It is because the adhesive force between ink and paper is greater than the cohesive force of ink molecules.

(*b*) Water wets the glass. It is because the adhesive force between water molecules and glass molecules is greater than the force of cohesion between water molecules.

(*c*) Mercury does not wet the glass because the adhesive force between mercury molecules and glass molecules is less than the cohesive force between mercury molecules.

(*d*) We are able to write on the blackboard with a piece of chalk because the adhesive force between chalk molecules and wood molecules is much greater than the cohesive force between chalk molecules.

Some facts about molecular forces :

(*i*) On the average, the molecules in solids and liquids are separated by a distance of the order of 10^{-9} m.

(*ii*) The molecules of a liquid or solid attract each other if distance between them is 10^{-9} m or less.

(*iii*) If the distance between the molecules becomes greater than 10^{-9} m, the attraction between the molecules becomes negligible.

(*iv*) The maximum distance (10^{-9} m) upto which two molecules attract each other is called *molecular range*.

(*v*) The force of attraction between molecules is due to electrical interaction between the charges.

(*vi*) Molecular forces do not obey inverse square law.

Molecular Range

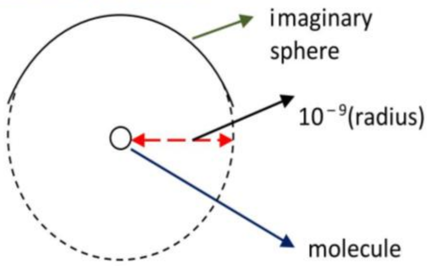
" It is the maximum distance up to which a molecule can exert some appreciable force of attraction for another molecule ."

- It is different for different substance.
- Molecular range for solid & liquid is 10^{-9} m.

Sphere of influence (or sphere of molecular activity)

"It is a sphere drawn around a particular molecule as centre and molecular range as radius ."

- All the molecules in this sphere attract the molecule at the centre.



Surface film

"Surface film is a topmost layer of liquid at rest with thickness equal to the molecular range."

Surface Tension: *"Surface tension is a property of liquid at rest by virtue of which a liquid surface tends to occupy a minimum surface area and behaves like a stretched membrane ."*

In other words ,

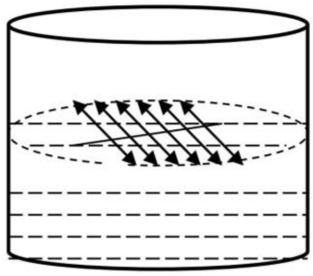
"S T is a property by virtue of which the free surface of a liquid at rest behaves like a stretched elastic membrane tending to contract to possess minimum surface area ."

Measurement of ST :-

Surface Tension of a liquid is measured as force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line .

i.e.,

$$S = \frac{F}{l}$$ Where, F = Force of S T



i.e. Force of ST = $\frac{\text{Total force in either side of imaginary line}}{\text{Length of the line}}$.

DIRECTION :- The direction of this force is perpendicular to the line and tangential to the liquid.

UNIT Of ST -- (I) Cgs → $S = \frac{F}{l} = \frac{dyn}{cm} = dyn / cm$.

(II) S I → $S = \frac{F}{l} = \frac{N}{m} = N / m$.

Dimensional Formula : $S = \frac{F}{l} = \left[\frac{MLT^{-2}}{L}\right] = [ML^0T^{-2}]$.

- ST is a scalar quantity because it has no specific direction for a given liquid .

Inter molecular binding energy :

Two molecules of a substance are held together by an attractive force. In order to separate the two molecules of a substance, some energy must be supplied to it from out side . Thus,

"The minimum energy required to separated two molecules from each other's influence is called intermolecular binding energy."

Explanation of some phenomenon on the basis of ST :-

1.] Lead Balls are spherical in shape;

EXPLANATION - When molten lead is poured through holes into water from a certain height , then falling lead drops solidify and take the form of small sphere in order to have minimum surface area .

2.] Rain drops and a globule of mercury placed on glass plate are spherical ;.

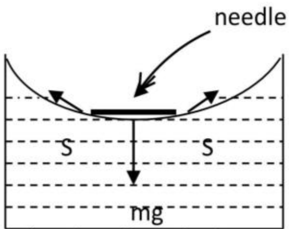
EXPLANATION - The liquid tries to acquire minimum surface area due to ST and hence take form of drops. However the drops get flattened because force of gravity is more than force of ST.

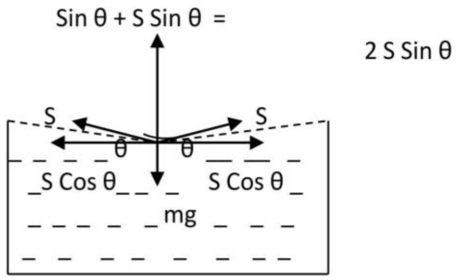
3.] Hair of a shaving brush / painting brush, when dipped in water spread out , but as soon as it is taken out , its hair stick together .

EXPLANATION – When the brush is dipped in water , there is water all around its hair and they are spread out . But it is taken out , the water film is formed between the hair and they contract due to ST.

4.] A greased needle placed gently or the free surface of water in a Beaker does not sink.

EXPLANATION -





When greased needle is placed gently on the free surface of water , the stretched membrane of water is depressed under the weight (mg) of the needle . The force of ST acts in inclined manner making angle θ with the horizontal . The horizontal component S Cos θ of ST cancel each other .
The vertical component Sin θ of ST get added up and balance the weight (mg) of the needle .

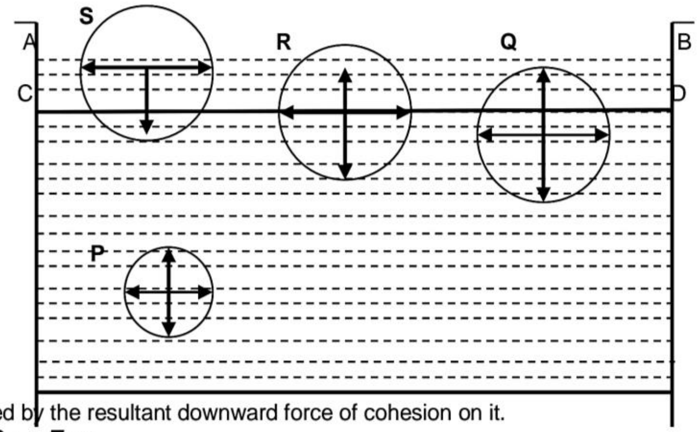
| | y , insects can walk on the free surface of water.

5.] Bits of Camphor move irregularly when placed in water surface .

EXPLANATION- When camphor dissolves in water , the ST of water decreases . Due to irregular shape , the camphor may dissolve unevenly on different sides . Thus, there exist unbalanced force due to the ST on the piece of the Camphor and hence the piece of Camphor moves in a random direction.

Molecular Theory of surface tension:

The Liquid enclosed between free surface AB of the liquid and an imaginary plane CD at a distance 'r' (equal to molecular range) from the free surface of the liquid Form a **liquid film**.



Consider 4 liquid molecules P, Q ,R, and S .
Draw the sphere of influence around these molecules.

The molecules in the upper half of the sphere of sphere of influence exert a resultant upward force of cohesion and those in lower half of sphere of influence exert a resultant downward force of cohesion, on the molecule at the centre.

MOLECULE 'P' & 'Q' :(P is well within the liquid , Q is just below the free surface of the liquid)
Since , the sphere of influence of molecules (P & Q) lies inside the liquid, the no. of molecules in the upper half of sphere of influence is the same as that in its lower half. So

P & Q are attract at equally in all direction .Therefore the resultant upward force of cohesion on molecule P and Q is balanced by the resultant downward force of cohesion on it.

∴ ***Net force of cohesion on the molecules P & Q = Zero***

MOLECULE 'R' : (Little below the upper surface of surface film and a part of its sphere of influence is outside the free liquid surface).
The no. of molecules attracting molecule R downward is more than the no. of molecules attracting it upward .

∴ ***Net force of cohesion on the molecule R = Downward force***

MOLECULE 'S' : (Lies half outside the liquid.)
The no. of molecules in the lower half of the sphere is twice the no. of molecules in the upper half of the sphere of influence

∴ Molecule 'S' experience -- ***maximum Downward force***

In fact only gas or vapour molecules in the upper half of sphere of influence of S attracting it upward.

Conclusion : [1] No resultant force acts on a molecule, so long it lies below the surface film .But for all those molecules lying in the surface film , a resultant downward force of cohesion act (with increases as we move from the bottom CD of the surface film to its top AB) . Therefore, free surface of the liquid behaves like a stretched membrane.

[2] To take a molecule into the surface film anywhere below it , some work is to be done against the down-ward force of cohesion . This work done appears as potential energy of the molecule . Thus, all the molecules in the surface film possess potential energy , which shows that the surface film has P . E .

** **PE of the surface film α No. of molecules in the surface film .**

** **PE of molecules lying near the free surface is greater than the molecule in the interior of the liquid .**

[3] Every system tends to be in the state of stable equilibrium for this PE of the system must be minimum Hence , in order to have minimum PE , the liquid surface must have minimum no. of molecules. For this, the surface film should have minimum volume , which would be so, if the surface film has least area . Thus free surface of the liquid tends to occupy minimum surface area.

[4] Since for a given volume , surface area of a sphere is minimum, the free surface of a liquid rest tries to assume a spherical sphere.

➔ **ST is a joint property of the interface separating two substance at least one of the which is a fluid .It is not the property of a single fluid alone .**

(*i*) The molecules on the surface of a liquid have more potential energy than those within the liquid. It is because to take the molecule from the interior of the liquid to its surface, work must be done against the downward molecular forces. This work done is stored as potential

(*ii*) Every system tends to arrange itself in such a way that it has minimum potential energy. Therefore, the free surface of a liquid also tends to have minimum potential energy. This is possible only if the area of the liquid surface is minimum. *In order to acquire minimum area, the surface of a liquid tends to contract and hence behaves as a stretched membrane.*

(*iii*) For a given volume, a sphere has the minimum surface area. Therefore, every surface of a liquid tries to become spherical. But due to the presence of gravitational force it fails to do so unless the gravitational force is negligible.

Example 19.19. **Water is kept in a beaker of radius 5.0 cm. Consider a diameter of the beaker on the surface of the water. Find the force by which the surface on one side of the diameter pulls the surface on the other side. Surface tension of water is 0.075 N/m.**

Solution. Length of diameter, $l = 2 \times 5 = 10$ cm $= 0.1$ m

Now surface tension, $T = \frac{F}{l}$ or $F = T \times l = 0.1 \times 0.075 = 7.5 \times 10^{-3}$ N

Example 19.20. **A wire ring of 3 cm radius resting flat on the surface of a liquid is raised. The pull required is 3.03 gf more before the film breaks than it is afterwards. Find the surface tension of the liquid.**

Solution. When the wire ring is resting flat on the surface of the liquid, a thin film of liquid is in contact with the ring. The surface tension force acts along the circumference of the ring (downward). Therefore, in order to raise the ring, the force needed is equal to the sum of the weight of the ring and the force due to surface tension. However, when the film breaks, the force required to raise the ring is equal to the weight of the ring because no force is to be applied against surface tension. Clearly, the difference between the forces applied before and after breaking of the film is equal to the force due to surface tension.

$\therefore$ Surface tension force $= 3.03$ gf $= 3.03 \times 980$ dynes

The film has two surfaces, one inside and one outside the ring. Therefore, the total length over which the surface tension acts is $2(2\pi r) = 4\pi r$. If T is the surface tension of the liquid,

Surface tension force $= 4\pi r \times T$

$\therefore$ $4\pi r \times T = 3.03 \times 980$

or $T = \frac{3.03 \times 980}{4\pi \times 3} = 78.80$ dynes/cm

Example 19.21. **The maximum force, in addition to the weight required to pull a wire 5 cm long from the surface of water at 20°C is 728 dynes. Calculate the surface tension of water.**

Solution. There is a film on either side of the wire. Therefore, the length over which the surface tension acts is $l = 2 \times 5 = 10$ cm.

$\therefore$ Surface tension of water, $T = \frac{F}{l} = \frac{728}{10} = 72.8$ dynes/cm

Surface Energy:-

The free surface of a liquid at rest is always in a state of Tension . The force of ST tends to decrease the surface area to the minimum . If the surface area of the liquid is to be increased , work shall have to done against the force of ST . This work done is started in the liquid surface film as its potential energy . Thus,

" Potential energy per unit area of the surface film is called the surface energy ."

or,

" Surface energy is the amount of workdone in increasing the area of a surface film through unit under isothermal condition (the temperature remains unchanged)."

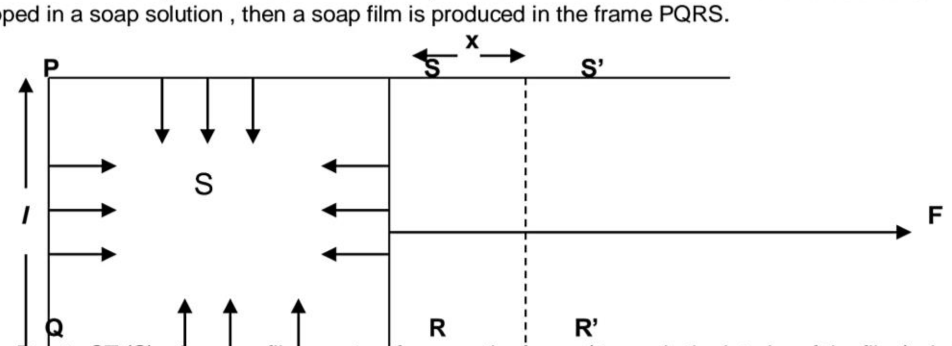
$\therefore$ Surface energy = $\frac{\text{work done in increasing the surface area}}{\text{Increase in surface area .}}$

UNIT -- (i) Cgs = erg / cm
(ii) S I = Joule / m

Dimensional Formula = $[\,M\,L^0\,T^{-2}\,]$

Relation between surface energy (E) and surface tension (S)

Consider a Rectangular frame of wire PQRS , whose arm RS can slide over the arm PS and QR . If this frame is dipped in a soap solution , then a soap film is produced in the frame PQRS.



Due to ST (S) , the soap film exerts a force on the frame (towards the interior of the film) . Let '*l*' be length of the arm RS.

$\therefore$ Force acting on the arm RS towards the film is

$F = S \times x$ [since, $S = \frac{F}{2l}$] [since soap has two surface that is why the length is taken twice .]

Let the arm RS be displaced to a new position R` S` through a distance 'x' that the temperature of the film remains the same .

∴ Work done , W = F x = 2 l S x

To increase the area of the soap film , we have to pull the sliding wire RS outward with a force F.

∴ Increase in surface area of the film , A = 2 l x .

This work done is equal to the PE of the liquid film ,

∴ Potential Energy , $E = \frac{\text{Work done}}{\text{increase in surface area}}$

$$E = \frac{2lSx}{2lx}$$

$$E = S$$

- **Surface energy (E) is numerically equal to surface tension .**

> *Thus surface tension T is numerically equal to the surface energy σ.* This provides second definition of *T*.
> *Hence* **surface tension** *is the work done (against force of surface tension) at constant temperature to increase the surface area of the liquid by unit area.*

Units. The SI unit of work done is Nm (or joule) and that of area is m^2. Therefore, SI unit of surface energy is N/m (or J/m^2). Note that surface tension and surface energy have the same units and dimensions.

Discussion.

(*i*) The surface tension (*T*) is numerically equal to the surface energy (σ).
(*ii*) The surface tension may be defined as surface force per unit length or surface energy per unit area.

Example 19.23. Calculate the work done against surface tension forces in blowing a soap bubble of diameter 1 cm. The surface tension of soap solution is 2.5×10^{-2} N/m.

Solution. Here $T = 2.5 \times 10^{-2}$ N/m ; $r = \frac{1}{2}$ cm = 0.5 cm

Original surface area of bubble = 0

Final surface area of bubble, $A = 2\times(4\pi r^2) = 8\pi r^2$

The factor 2 is necessary because the soap bubble has two free surfaces.

∴ Increase in surface area = $A - 0 = A = 8\pi r^2 = 8\pi\times(0.5)^2\times10^{-4}\,m^2$

∴ Work done = $T \times$ Increase in surface area

$= (2.5 \times 10^{-2}) \times [8\pi \times (0.5)^2 \times 10^{-4}] =$ **1.57×10^{-5} J**

Example 19.24. Find the work required to break up a drop of water of radius 0.5 cm into drops of water each of radii 1 mm. The surface tension of water is 7×10^{-2} N/m.

Solution. The original radius of drop is *R* (= 0.5 cm) and the final radius of each drop is *r* (= 1 mm = 0.1 cm).

Since the volume of a drop is $(4/3)\pi$ (radius)^3

∴ Number of drops formed $= \dfrac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \dfrac{R^3}{r^3} = \dfrac{(0.5)^3}{(0.1)^3} = 125$

Final surface area of drops $= 125\times(4\pi r^2)$

$= 125 \times [4\pi \times (0.1)^2 \times 10^{-4}]\,m^2 = 5\pi\times10^{-4}\,m^2$

Original surface area of drop $= 4\pi R^2 = 4\pi\times\left[(0.5)^2\times10^{-4}\right]m^2 = \pi \times 10^{-4}\,m^2$

Work done = $T \times$ Increase in surface area

$= (7 \times 10^{-2}) \times [5\pi \times10^{-4} - \pi\times10^{-4}] =$ **8.8×10^{-5} J**

Example 19.25. **What amount of energy will be liberated if 1000 droplets of water each 10^{-8} m diameter coalesce to form a large spherical drop? Surface tension of water is 7.2×10^{-2} N/m.**

Solution. The radius of each smaller drop is r (= 0.5×10^{-8} m) and that of the large drop is R. Since mass is conserved,

$$\therefore \quad \left(\frac{4}{3}\pi R^3\right)\rho = 1000\left(\frac{4}{3}\pi r^3\right)\rho$$

$$\text{or} \quad R^3 = 1000\ r^3$$

$$\text{or} \quad R = 10\ r = 10 \times (0.5 \times 10^{-8}) = 5 \times 10^{-8}\ \text{m}$$

$$\text{Decrease in surface area} = 1000 \times 4\pi r^2 - 4\pi R^2$$

$$= 4\pi\ (1000 \times 25 \times 10^{-18} - 25 \times 10^{-16})\ \text{m}^2$$

$$= 4\pi \times 225 \times 10^{-16}\ \text{m}^2$$

$$\therefore \quad \text{Energy liberated} = T \times \text{Decrease in surface area}$$

$$= (7.2 \times 10^{-2}) \times (4\pi \times 225 \times 10^{-16})$$

$$= 2.035 \times 10^{-14}\ \text{J}$$

Example 19.26. **A film of water is formed between two straight parallel wires each 10 cm long at a separation of 0.5 cm. Calculate the work required to increase 1 mm distance between the wires. Surface tension of water = 72×10^{-3} N/m.**

Solution. Initial surface area of film = Length × Separation = $10 \times 0.5 = 5\ \text{cm}^2 = 5 \times 10^{-4}\ \text{m}^2$

$$\text{Final surface area of film} = 10 \times (0.5 + 0.1) = 6\ \text{cm}^2 = 6 \times 10^{-4}\ \text{m}^2$$

$$\text{Increase in surface area} = (6 - 5) \times 10^{-4} = 1.0 \times 10^{-4}\ \text{m}^2$$

The film has two surfaces. Therefore, net increase in the surface area of the film is

$$\Delta A = 2 \times (1.0 \times 10^{-4}) = 2 \times 10^{-4}\ \text{m}^2$$

$$\therefore \quad \text{Work done} = \text{Surface tension} \times \text{Increase in area}$$

$$= (72 \times 10^{-3}) \times (2 \times 10^{-4}) = 144 \times 10^{-7}\ \text{J}$$

Example 19.27. **Suppose that 64 raindrops combine into a single drop. Calculate the ratio of the total energy of the 64 drops to that of a single drop.**

Solution. Let R be the radius of single drop and r the radius of each smaller drop.

$$\text{Volume of single drop} = \text{Volume of 64 drops}$$

$$\text{or} \quad \frac{4}{3}\pi R^3 = 64 \times \left(\frac{4}{3}\pi r^3\right)$$

$$\therefore \quad R = 4r$$

$$\text{Surface energy of 64 drops, } S_1 = 64 \times 4\pi r^2 \times T$$

$$\text{Surface energy of bigger drop, } S_2 = 4\pi R^2 \times T$$

$$\therefore \quad \frac{S_1}{S_2} = \frac{64 \times 4\pi r^2 \times T}{4\pi R^2 \times T} = \left(\frac{r}{R}\right)^2 \times 64 = \frac{64}{16} = 4$$

Angle of Contact

There are two types of liquids – ***one which wet the container*** and the ***other which do not wet the container***.

- The liquid (Ex- water) which wet the container has ***concave meniscus***.
- The liquid (Ex- Mg) which does not wet the container has ***convex meniscus***.

When a liquid surfaces touches a solid surface , the shape of the liquid surface Near the contact . This effect is due to the presence of cohesive force And adhesive forces .

The **angle of contact** *between a liquid and a solid is defined as the angle (θ) which the tangent to the liquid surface at the point of contact (i.e., point A in Fig. 19.29) makes with the solid surface inside the liquid.*

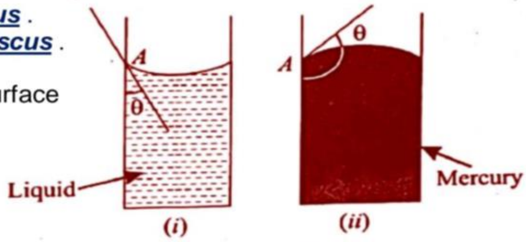


Fig. 19.29

1. When angle of contact is acute. Fig. 19.29 (*i*) shows that angle of contact between the liquid (*not water*) and the solid surface (glass wall) is acute (*i.e.*, $\theta < 90°$). In this case, the resultant adhesive force is greater than the resultant cohesive force. Therefore, the glass wall pulls up liquid and liquid tends to stick (*i.e.*, wet) to the glass. As a result, the meniscus of the liquid becomes concave. To sum up, when angle of contact is acute, the :

(*a*) liquid will wet the solid.
(*b*) meniscus of the liquid will be concave.
(*c*) liquid will rise in the capillary tube made of such a solid.

2. When the angle of contact is obtuse. Fig. 19.29 (*ii*) shows that angle of contact between mercury and the solid is obtuse (*i.e.*, $\theta > 90°$). In this case, the resultant cohesive force is greater than the resultant adhesive force. Therefore, the mercury gets depressed near the glass wall and mercury surface pulls from the glass wall (*i.e.*, mercury does not wet the glass). As a result, the meniscus of mercury becomes convex. To sum up, when angle of contact is obtuse, the :

(*a*) liquid will not wet the solid.
(*b*) meniscus of the liquid will be convex.
(*c*) liquid will get depressed in the capillary tube made of such a solid.

3. When angle of contact is zero. The angle of contact for pure water and clean glass is zero. In this case, the resultant adhesive force is equal to the resultant cohesive force. Therefore, in the glass tube, the meniscus of water will be exactly hemispherical.

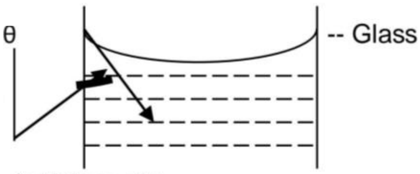
Factors on which the value of angle of contact depends :

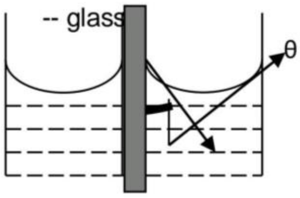
---- 1] depends upon the nature of the liquid & solid in contact.
---- 2] depends upon the medium which exist above the free surface.

Factors on which the value of angle of contact depends not depends :

---- 1] independent of the inclination of the solid to liquid surface.

- **'θ' is fixed for a given pair of solid & liquid & surrounding medium.**
- The angle of contact remains the same whether the liquid is contained in glass vessel or a glass plate is inserted in the liquid or a drop of given liquid rests on the glass plate. (Simply , angle of contact does not depends on the manner of contact.)





- **Values of θ** - - - - -

1] **Value of θ** Of Mercury and water = about140^0
2] **Value of θ** Of Glass and water = about 89^0
3] **Value of θ** Of pure water and perfectly clean glass = 0^0
4] Value of **θ of pure** water and silver = about 90^0
5] Value of **θ** Chromium and water = 160^0

Shape of the liquid Meniscus : (LAPLACE'S THEORY:)

When a liquid and solid are brought in contact with each other , the liquid surface generally becomes curve .
"The curved surface of the liquid is called the meniscus of liquid."

- **The curvature is due to two forces** ---- 1.] Force of cohesion (**between liquid and liquid molecule**)
---- 2.] Force of adhesion (**between solid and liquid molecule**)

- If bore of the capillary tube is sufficiently small , then curvature of the liquid surface can be marked. When a capillary Tube(hair like tube of very fine bore) is dipped in a liquid, the liquid surface curved near the point of contact .

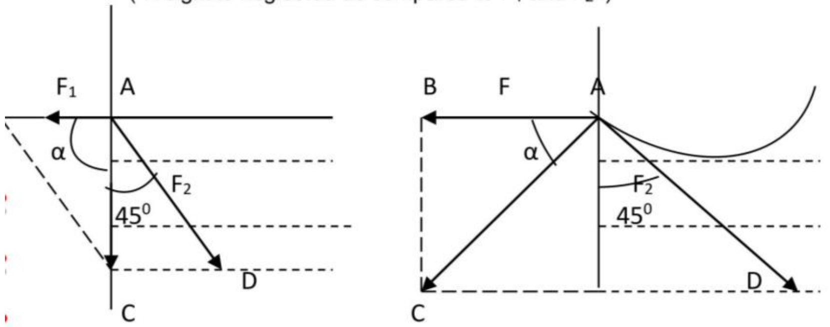
Consider a molecule A which is in Contact with solid (i.e., wall of the capillary tube). **The various forces acting on molecule A** .----------

----I.] Force F_1 (due to Force of adhesion) , acts outwards at right angle to the wall of the tube ,(represented by AB)

--- II.] Force F_2 (due to force of cohesion) , acts at an angle of 45^0 to the vertical , This force is represented by AD.

---III] mg (weight) of the molecule A (vertically downward along the wall of the tube).

(Weight is neglected as compared to F_1 and F_2 .)



Thus, ***there are only two forces F_1 and F_2 acting on the liquid molecules*** .
These forces are inclined at an angle of 135^0 (90 + 45) .

The resultant force represented by AC will depends upon the values of F_1 and F_2 . Let the resultant force make an angle α with F_1 .

A / law of law of vector

$$\tan\alpha = \frac{F_2 \sin 135}{F_1 + F_2 \cos 135} = \frac{F_2 \sin(180-45)}{F_1 + F_2 \cos(180-45)}$$

$$= \frac{F_2 \sin 45}{F_1 + F_2(-\cos 45)} = \frac{F_2\sqrt{2}}{F_1 - F_2/\sqrt{2}}$$

$$\boxed{\tan\alpha = \frac{F_2}{\sqrt{2}F_1 - F_2}}$$

Special Case ;

1.] If $F_2 = \sqrt{2}F_1$, then, $\tan\alpha = \frac{F_2}{\sqrt{2}F_1 - F_2} = \frac{F_2}{F_2 - F_2} = \frac{F_2}{0} = \infty = \tan 90^0$

∴ $\alpha = 90^0$.

Thus, the resultant force acts vertically downward and hence the **meniscus is plane and horizontal** .
EX- Pure water contained in silver coated capillary tube.

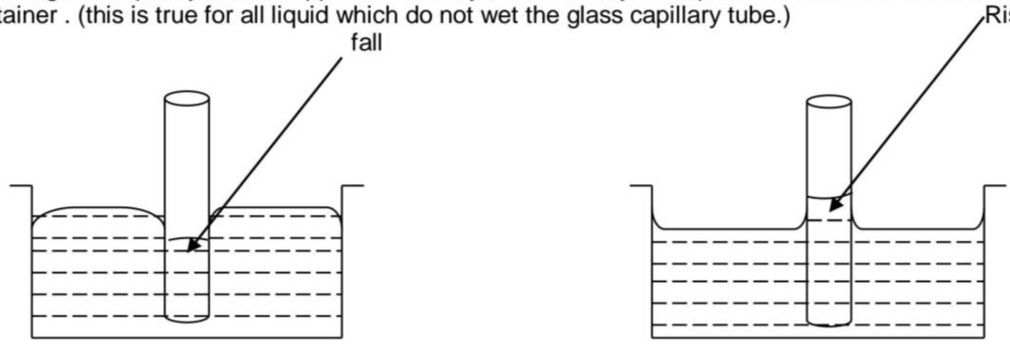
2.] If $F_2 < \sqrt{2}F_1$ then tan α = **+ive value** ; ∴ **α = acute angle** .
Thus , the resultant force is directed outside the liquid hence the **meniscus is concave upward.**
EX: Water in glass capillary tube.

3] If $F_2 > \sqrt{2}F_1$, then tan α = **--ive value** ; ∴ **α = obtuse angle**
Thus, the resultant force is directed inside the liquid and hence the **meniscus is convex upward**.
Ex:- Mercury in glass capillary

CAPALLARITY:

A glass tube of very fine uniform bore through out the length of tube is called Capillary tube .

- When a glass capillary tube is dipped in mercury , the mercury is depressed below the free surface of the liquid in the container . (this is true for all liquid which do not wet the glass capillary tube.)



- When a glass capillary tube is dipped in water , the water rises up in the tube (true for all liquids which wet capillary tube).

The narrower the bore of the tube , greater is the rise or fall of the liquid in the tube .

Thus,

"The phenomenon of rise and fall of liquid in a capillary tube is called capillarity".

Ex :-[i] The oil in a lamp rise in the wick to the top by capillary action.
[ii] The action of towel in soaking up the moisture and fine droplets from the body is due to capillary action ofcotton.
[iii] Ink absorbed by the blotting paper due to capillary action.
[iv] Sandy soil is more dry than clay .It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries
[v] Swelling of wood in rainy season is due to rise of moisture from air , in the pores of wood.

[vi] Leaves , trunk and branches of a tree posses fine capillaries . Water rises even up to the topmost leaves by the capillary action.
[vii] Water stored in the earthen pot oozes out of the surface and gets evaporated and help to keep the water cool.

Examples based on Surface Tension and Surface Energy

- **Formulae Used**
  1. Surface tension = $\frac{\text{Force}}{\text{Length}}$ or $\sigma = \frac{F}{l}$
  2. Increase in surface energy or work done,
     W = Surface tension × increase in area of the liquid surface.
- **Units Used**
  The unit of surface tension is Nm^{-1} and that of increase in surface energy or work done is joule.

Q. 1.A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N (Which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

Sol. Here $F = 1.5 \times 10^{-2}$ N, $l = 30$ cm $= 0.3$ m
As the soap film has two free surfaces, so the force F acts over twice the length of the slider. Hence

$$\sigma = \frac{F}{2l} = \frac{1.5 \times 10^{-2}}{2 \times 0.30} = 2.5 \times 10^{-2}\ Nm^{-1}$$

Q. 2.A wire ring of 3 cm radius is rested on the surface of liquid and the raised. The pull required is 3.03 g more before the film breaks than it is afterwards. Find the surface tension of the liquid.

Sol. The additional pull F of 3.03 g wt. is equal to the force of surface tension.
$F = 3.03$ g wt $= 3.03 \times 981$ dyne
As the liquid touches the ring both along the inner and outer circumference, so force on the ring due to surface tension, $F = 2 \times 2\pi r \times \sigma = 4\pi r \sigma$

$\therefore \quad 4\pi r\sigma = 3.03 \times 981$

$$\sigma = \frac{3.03 \times 981}{4\pi r} = \frac{3.03 \times 981}{4 \times 3.14 \times 3} = 78.84 \text{ dyne cm}^{-1}$$

Q. 3.Calculate the work done in blowing a soap bubble from a radius of 2 cm to 3 cm. The surface tension of the soap solution is 30 dyne cm^{-1}.

Sol. Here $r_1 = 2$ cm, $r_2 = 3$ cm, $\sigma = 30$ dyne cm^{-1}
Increase in surface area $= 2 \times 4\pi (r_2^2 - r_1^2) = 8\pi (3^2 - 2^2) = 40\pi$ cm^2
Work done = σ × Increase in surface area $= 30 \times 40 \times 3.142 = 3770.4$ erg.

Q. 4.The surface tension of a soap solution is 0.03 Nm^{-1}. How much work is done to produced a soap bubble of radius 0.05 m?

Sol. Work done = Total surface area × surface tension
$= 2 \times 4\pi r^2 \times \sigma = 2 \times 4 \times 3.14 \times (0.05)^2 \times 0.03 = 1.884 \times 10^{-3}$ J

Q. 5.A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ.

Sol. Radius of larger drop = D/2
Let radius of each small drop = r
Now volume of 27 small drops = Volume of the larger drop

$$27 \times 4 \times \pi r^3 = 4\pi \left(\frac{D}{2}\right)^3 \quad \text{or} \quad r = \frac{D}{6}$$

Initial surface area of larger drop

$$= 4\pi R^2 = 4\pi \left(\frac{D}{2}\right)^2 = \pi D^2$$

Final surface area of 27 small drops $= 27 \times 4\pi r^2$

$$= 27 \times 4\pi \left(\frac{D}{6}\right)^2 = 3\pi D^2$$

$\therefore$ Increase in surface area $= 3\pi D^2 - \pi D^2 = 2\pi D^2$
Change in energy = Increase in surface area × Surface tension $= 2\pi D^2 \sigma$

Q. 6. A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded. Surface tension of mercury = 32×10^{-2} Nm^{-1}.

Sol. Volume of 10^6 droplets = Volume of larger drop

$$10^6 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$R = 10^{-2} R = 10^{-2} \times 1.0 = 10^{-2}$ cm $= 10^{-4}$ m
Surface area of larger drop $= 4\pi R^2 = 4\pi \times (10^{-2})^2 = 4\pi \times 10^{-4}$ m^2
Surface area of 10^6 droplets$= 4\pi r^2 \times 10^6 = 4\pi \times (10^{-4})^2 \times 10^6 = 4\pi \times 10^{-2}$ m^2
$\therefore$ Increase in surface area $= 4\pi \times 10^{-4} (100 - 1) = 4\pi \times 99 \times 10^{-4}$ m^2
$\therefore$ Work done in spraying a spherical drop of mercury = Surface tension × increase in surface area
$= 32 \times 10^{-2} \times 4\pi \times 99 \times 10^{-4} = 3 \times 9.8 \times 10^{-2}$ J

Q. 7. A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}.

Sol. Volume of 1000 droplets = Volume of larger drop

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$r = \frac{R}{10} = \frac{2 \times 10^{-3}\,m}{10} = 2 \times 10^{-4}$ m

Surface area of larger drop = $4\pi R^2 = 4\pi \times (2 \times 10^{-3})^2 = 16\,\pi \times 10^{-6}\,m^2$

Surface area of 1000 droplets = $4\pi r^2 \times 1000 = 4\pi \times (2 \times 10^{-4})^2 \times 1000 = 16\,\pi \times 10^{-5}\,m^2$

$\therefore$ Increase in surface area = $16\,\pi \times 10^{-6}\,(10 - 1) = 144\,\pi \times 10^{-6}\,m^2$

The resultant increase in surface energy = Surface tension $\times$ increase in surface area

$= 0.07 \times 144 \times 22 \times 10^{-6} = 3168 \times 10^{-8}$ J

Q. 8. ***Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?***

Sol. Surface energy of first bubble = Surface tension $\times$ surface area

$= 2 \times 4\pi r_1^2\,\sigma = 8\pi\,r_1^2\,\sigma$

Similarly, surface energy of second bubble

$= 8\pi r_2^2\,\sigma$

Let r be the radius of the coalesced bubble. Then, surface energy of coalesced bubble = $8\pi r^2\,\sigma$

By the conservation of energy,

$8\pi r^2\,\sigma = 8\pi r_1^2\,\sigma + 8\pi r_2^2\,\sigma = 8\pi\,(r_1^2 + r_2^2)\,\sigma$

or $r^2 = r_1^2 + r_2^2 = 3^2 + 4^2 = 25$ or $r = 5$ cm

Q. 9. ***If 500 erg of work is done in blowing a soap bubble to a radius r, what additional work is required to be done to blow it to a radius equal to 3r?***

Sol. Work done in blowing the soap bubble from radius 0 to r is

$W = \sigma \times 2 \times 4\pi\,r^2$ … (i)

Additional work required to increase the radius from r to 3r will be

$W' = \sigma \times$ Increase in surface area

$= \sigma \times 2 \times 4\pi\,[(3r)^2 - r^2]$

$W' = \sigma \times 2 \times 4\pi \times 8r^2$ … (ii)

Dividing equation (ii) by (i), we get

$\frac{W'}{W} = 8$

or $W' = 8W = 8 \times 500$ erg = 4000 erg

Q. 10. ***Soapy water drips from a capillary. When the drop breaks away, the diameter of its neck is 1 mm. The mass of drop is 0.0123 g. Find the surface tension of soapy water.***

Sol. When the drop breaks away from the capillary, weight of drop

= Force of surface tension acting on the capillary

or $mg = \pi D \times \sigma$

Where D = diameter of the drop

or $\sigma = \frac{mg}{\pi D} = \frac{1.29 \times 10^{-5} \times 9.8}{3.14 \times 1 \times 10^{-3}} = 4.03 \times 10^{-2}\,Nm^{-1}$

Q. 11. ***A glass plate of length 10 cm, breadth 4 cm and thickness 0.4 cm, weighs 20 g in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight?***

Sol. Here l = 10 cm, b = 4 cm, t = 0.4 cm, m = 20 g, $\sigma = 70$ dyne cm^{-1}

Various force acting on the plate are

(i) Weight of the plate acting vertically downwards,

= mg = 20×980 dyne = 20 g f

(ii) Force due to surface tension acting vertically downwards,

$F = \sigma \times$ Length of plate in contact with water

$= \sigma \times 2$ (length + thickness) $= 70 \times 2\,(10 + 0.4) = 70 \times 20.8$ dyne

$= \frac{70 \times 20.8}{980}$ g f = 1.4857 g f

(iii) Upwards thrust due to liquid = Weight of the liquid displaced

= Volume of liquid displaced $\times$ density $\times$ g= $(l \times b/2 \times t) \times \rho \times g$

$= (10 \times 4/2 \times 0.4)\,1 \times 980$ dyne $= \frac{8 \times 980}{980}$ g f = 8 g. f

$\therefore$ Apparent weight = 20 + 1.4857 – 8 = 13.4857 g f.

Q. 12. ***If a number of little droplets of water of surface tension σ, all of the same radius r combine to form a single drop of radius R and the energy released is converted into kinetic energy, find the velocity acquired by the bigger drop.***

Sol. Volume of bigger drop = Volume of n smaller drops

$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ or $n = \frac{R^3}{r^3}$

Mass of bigger drop, m = Volume $\times$ density

$= \frac{4}{3}\pi R^3 \times 1 = \frac{4}{3}\pi R^3$

Energy released, W = S.T. $\times$ Decrease in surface area

$= \sigma \times 4\pi\,(nr^2 - R^2) = 4\pi\sigma\left(\frac{R^3}{r^3}r^2 - R^2\right)$

$= 4\pi\,R^3\,\sigma\left(\frac{1}{r} - \frac{1}{R}\right) = 3 \times 4\,\pi R^3\sigma\left(\frac{R - r}{rR}\right)$

$= 3m\sigma\left(\frac{R - r}{rR}\right)$

But K.E. produced = W

$\therefore \quad \frac{1}{2} mv^2 = 3m\sigma \left(\frac{R-r}{rR}\right)$ or $v = \sqrt{\frac{6\sigma(R-r)}{rR}}$

Q. 13. ***If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by*** $\Delta\theta = \frac{3\sigma}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

where σ is the surface tension of water and J is the mechanical equivalent of heat.

Sol. Let n be the number of little droplets which coalesce to form single drop. Then

Volume of n little droplets = Volume of single drop

or $\quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $nr^3 = R^3$

Decrease in surface area = $n \times 4\pi r^2 - 4\pi R^2$

$= 4\pi [nr^2 - R^2] = 4\pi \left(\frac{nr^3}{r} - R^2\right)$

$= 4\pi \left(\frac{R^3}{R} - R^2\right) = 4\pi R^3 \left(\frac{1}{r} - \frac{1}{R}\right) \quad [\because nr^3 = R^3]$

Energy evolved,

W = Surface tension × decrease in surface area

$= 4\pi\sigma R^3 \left(\frac{1}{r} - \frac{1}{R}\right)$

Heat produced, $Q = \frac{W}{J} = \frac{4\pi\sigma R^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

;

But $Q = ms\Delta\theta$ = Volume of single drop × density of water × specific heat of water × $\Delta\theta$

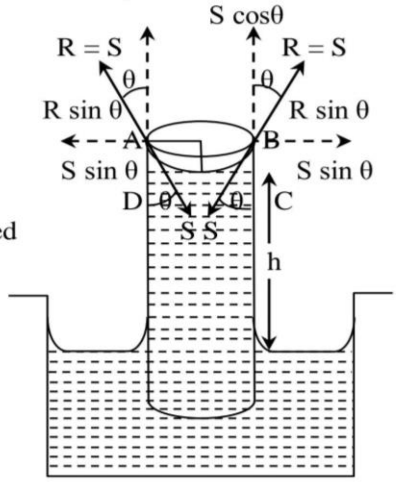
$= \frac{4}{3}\pi R^3 \times 1 \times 1 \times \Delta\theta$

Hence $\frac{4}{3}\pi R^3 \Delta\theta = \frac{4\pi\sigma R^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

or $\quad \Delta\theta = \frac{3\sigma}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

ASCENT FORMULA :--- [Rise of liquid in a capillary tube]

Consider capillary tube of radius 'r' (opened at both the ends). Let it be dipped in the liquid (water) of surface tension 'S' and liquid rises in the tube to height 'h'.



Force of ST tends of move the area ($2\pi r^2 \to$ Hemi – sphere) of the free surface minimum (i.e., πr^2 – area of circle.)

Force of ST acts downwards along tangent to the liquid meniscus at the point of contact, such as A and B.

An equal reaction to this force (ST) acts on the liquid meniscus in upward direction (R).

The process of rise in water level continue until the weight of the lifted water balanced the net upward force due to ST.

Circumference of the circle of contact is **$2\pi r$**.

Resolve R (= S) into two component.

1. **R cosθ (=S cosθ)** which acts at every point of the meniscus in the upward direction and is **responsible for the rise of liquid** in the capillary-tube.
2. **R sinθ (=S sinθ)** acts at **right angle** to the length of the capillary tube (It has no effect on the rise of liquid in the tube).

** **S sinθ** cancel each other as it acts at the end of diameter of the circle of contact.

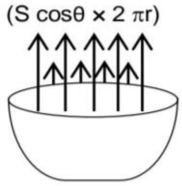
➤**Since all the vertical components acts in the same direction.**

∴ Total vertical force acting on the circular meniscus

F = Circumference of the circle of contact **α R cosθ**.

$F = 2\pi r \times S\cos\theta \quad \ldots (i) \quad [\because R = S]$

(S cosθ × 2 πr)



➤**Due to F = 2πr × S cosθ,**

liquid rises in the tube and continues to pull the liquid upward until equilibrium is achieved (i.e., $F = 2\pi r S\cos\theta$ is balanced by the weight ***w*** of the liquid.

Now, **Volume of the liquid in the capillary tube =**

V = Vol. of cylinder of length (***h***) and radius (***r***) +vol. of cylinder ABCD of radius (***r***) and length (***r***) -vol. of Hemi-sphere of radius (R).

$= \quad \pi r^2 h + [\pi r^2 \times r - \frac{1}{2} \times 4/3\, \pi r^3]$

$= \pi r^2 h + [\pi r^3 - 2/3\, \pi r^3]$

$= \pi r^2 h + \frac{\pi r^3}{3}$ $\therefore$ $\mathbf{V = \pi r^2 [h + r/3]}$

Also, **mass** of the liquid raised in capillary tube.

$m = V \times P$ [∵ Density of the liquid = P]

$m = \pi r^2 (h + r/3) \times P$

$\therefore$ **Weight** of the **liquid raised in the capillary** tube.

$W = mg = \pi r^2 (h + r/3)\, P \times g$

In equilibrium, $W = F$

$\therefore \pi r^2 (h + r/3)\, Pg = 2 \pi r S \cos\theta$

$$(h + r/3) = \frac{2 S \cos\theta}{r \rho g} \quad \text{or,} \quad h = \frac{2 S \cos\theta}{r \rho g} - \frac{r}{3}$$

Since r << h

Then r/3 can be neglected as compound to 'h'

$$h = \frac{2 S \cos\theta}{r \rho g}$$

◈◈ Conclusion:

Since $h = \frac{2 S \cos\theta}{r \rho g}$

1. Rise of liquid in capillary tube is inversely proportional to r, i.e., $h \propto 1/\rho$ and $h \propto 1/g$.
2. Directly proportional to --- ☞ Force of ST (i.e., S) and ☞ Cos θ.

◈ For those liquid which wet the glass θ (angle of contact) is acute.

$\therefore$ Cosθ = + ive value i.e., h = +ive

This means that liquid which wet glass raises in glass capillary tube.

◈ For those liquid which do not wet glass, θ is obtuse.

$\therefore$ Cosθ = -ve value i.e., h = -ve

Which the level of liquid in the tube is below the liquid level outside the tube.

◈ For pure water and clean glass: θ is nearly equals to zero.

$\therefore$ $\cos\theta = \cos 0^\circ$ $\therefore$ height, $h = \frac{2s}{r \rho g}$

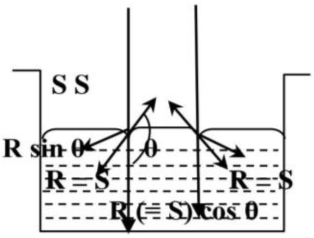
DESCENT FORMULA:-

If a capillary tube is dipped in a liquid which does not wet the capillary tube (like mercury), the angle of contact (θ) is obtuse and hence Cos θ = -ve. In that case the liquid (Hg) is depressed i.e., its level in the capillary tube will be lower than the level of mercury is the trough.

$\therefore$ **Total downward force** = $2 \pi r \times S \cos\theta$

Total downward pressure = $\frac{2 \pi r \times S \cos\theta}{\pi r^2}$

$$h = \frac{2 S \cos\theta}{r \rho g}$$



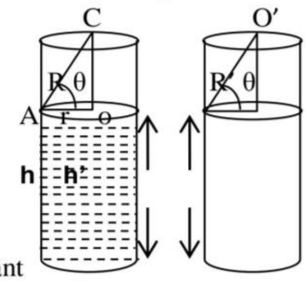
◈◈ Capillary Rise in a tube of insufficient length:

(In sufficient length means that the liquid can rise to a greater height than length of the tube.)

When the height to which a liquid can rise in a capillary tube is more than the length of the tube, the liquid does not flow.

Explanation: Height through which a liquid rise in the capillary tube of radius 'r'.

$$h = \frac{2 S \cos\theta}{r \rho g} \quad \dots (i)$$



➤**This equation has been derived on the assumption that the capillary tube is so narrow that the radius of liquid meniscus is equal to the radius of the capillary tube.**

But $r/R = \cos\theta$ [In rt. ∠ˢ Δ AOC $\cos\theta = r/R$]

From (i) $\therefore$ $h = 2S/\cancel{r}\rho g \times \cancel{r}/R$, $h = 2S/\rho gR$, $hR = 2S/\rho g$ = constant

When capillary tube is cut and its length is less than h (i.e., h') liquid rises up to the top of the tube and spreads in such a way that Radius (R') of the meniscus increases and it becomes more flat so that

Where C = Centre of curvature of then the meniscus.

R = Radius of meniscus

$hR = h'R'$ = constant

i.e., If h > h'

then R < R' [liquid does not over flow]

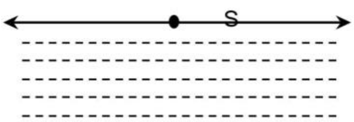
◈◈Excess of pressure inside a curved surface

There is always an excess of pressure on concave side of curved surface over that on the convex side.

Explanation: A liquid molecule on the surface of liquid is attracted by force of cohesion in all direction & force of ST acts tangentially to the liquid surface at rest. The direction of resultant force on a molecule on the surface of liquid depends upon the shape of the liquid surface.

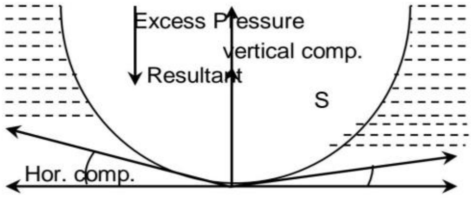
Case I: Plane surface – If the surface of the liquid is plane, the molecule on the liquid surface is attracted equally the surface of the liquid. The resultant force due to ST 'S' is zero. The pressure therefore is communicated to the inner side or outside of the liquid surface.

Pressure on the liquid side = Pressure on the vapour side of the plane surface.

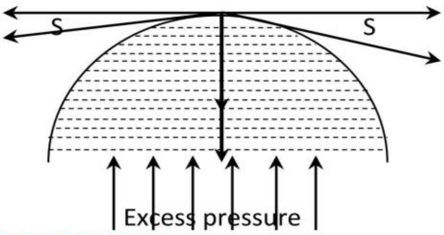


Case II: If the liquid is curved (i.e., *Convex or Concave*)

[A] ***Concave surfaces***: If the surface is concave upward, there will be resultant force due to ST acting on the molecule (Horizontal component cancel out) Since the molecule on the surface is in equilibrium, there must be an excess pressure on the concave side on downward direction to balance the resultant force of ST.



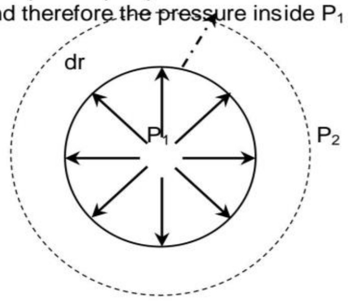
Convex surface: If the surface is convex, the resultant force due to ST acts in downward direction. Since the molecule on the surface are in equilibrium there must be an excess pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of ST.



◈◈ Excess of pressure inside a liquid drop:-

Consider a liquid drop of radius 'r' having ST 'S'. Due to ST, the molecule lying on the surface of liquid drop experience a resultant force acting inward perpendicular to the surface. Since size of the liquid cannot be reduced to zero and therefore the pressure inside P_1 must be greater than the pressure outside it P_2).

This pressure ($P_1 - P_2$) inside the drop will provide a force acting outward perpendicular to the surface to balance the resultant force due to ST.



Now,

Due to excess pressure P (= $P_1 - P_2$), the drop expands, let radius increases by 'dr'.

Work done by the excess pressure, W = force × displacement

Or, W = excess pressure × area × increase in radius

$= P\,(4\pi r^2) \times dr$ ... (*i*)

Increase in surface area of the drop = Final surface are – initial surface area

$= 4\pi\,[r + dr]^2 - 4\,\pi r^2$

$= 4\pi\,[r^2 + dr^2 + 2r\,dr] - 4\pi r^2$

$= 4\pi\,[r^2 + dr^2 + 2r\,dr - r^2] = 4\pi\,[2r\,dr]$

$= 8\pi\,r\,dr$ ... [2] [Neglecting dr^2 being very-very small]

Increase in potential energy = S × area

$= 8\pi\,r\,dr \times S$

Now, Work done = Increase in potential energy [Work-energy theorem]

$[4\pi r^2 \times dr]\,P = 8\pi\,r\,dr \times S$

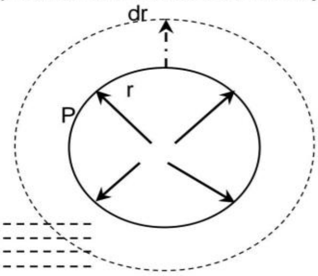
$$P = \frac{2S}{r}$$ (Where $P = P_1 - P_2$)

◈◈ Excess of pressure inside the soap bubble:

In case of liquid bubble, there are two surfaces (inner & outer).Consider a soap bubble of radius 'r', the molecule lying on the surface will experience a resultantforce acting inward perpendicular to the surface due to ST.

[Since size of bubble cannot be reduced to zero, and therefore, the pressure inside the P_1 must be greater than the pressure outward it (P_2).]

The excess pressure ($P_1 - P_2$) inside the bubble will provide a force acting outward perpendicular to the surface of the bubble to counter balance the resultant force due to ST. Due to excess pressure ($P_1 - P_2$) = P, the drop expands, let its radius increase by 'dr'



Work done by the excess pressure, W = force × displacement

$= [4\pi r^2 \times P] \times dr$ ... (*i*)

Increase in surface area of the bubble = $2\,[4\pi\,(r + dr)^2 - 4\pi r^2]$

$= 16\,\pi\,r\,dr$ [Neglecting 'dr' being very-very small]

Increase in potential energy = S × area $= 16\pi\,r\,dr \times S$

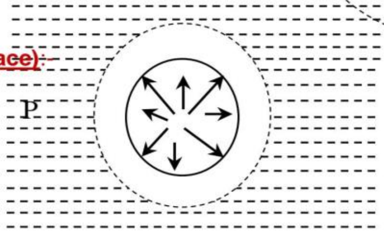
Now, Work done = Increase in PE [work-energy theorem]

$[4\pi r^2 \times dr]\,P = 16\,\pi\,r\,dr \times S$

$$P = \frac{4S}{R}$$

◈◈ Excess pressure inside an air bubble in a liquid (one surface):-

------ Similar to that of liquid drop in air.



$$P = \frac{2S}{r}$$