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QUICK TIPS

INDEFINITE INTEGRATIONS

Review of Key Notes and Formulae

1. Definition: Reverse process of differentiation

$\int f(x)\,dx = g(x) + C \rightarrow$ Indefinite Integration Constant.

$f(x)$ → Integrand

$g(x)$ → Integral / Primitive / Anti-derivative

2. Standard Integration to Remember

Integrands	Integrals	Integrands	Integrals
(i) $\int x^n\,dx$	$\frac{x^{n+1}}{n+1}+C$	(xi) $\int \cot x\,dx$	$-\log_e\lvert\cos x\rvert+C$
(ii) $\int \frac{1}{x}\,dx$	$\log_e\lvert x\rvert+C$	(xii) $\int \tan x\,dx$	$\log_e\lvert\sec x\rvert+C$
(iii) $\int e^x\,dx$	e^x+C	(xiii) $\int \sec x\,dx$	$\log_e\lvert\sec x+\tan x\rvert+C$
(iv) $\int a^x\,dx$	$\frac{a^x}{\log_e a}+C$	(xiv) $\int \operatorname{cosec} x\,dx$	$\log_e\lvert\operatorname{cosec} x-\cot x\rvert+C$
(v) $\int \sin x\,dx$	$-\cos x+C$	(xv) $\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}+C$
(vi) $\int \cos x\,dx$	$\sin x+C$	(xvi) $\int \frac{-dx}{\sqrt{a^2-x^2}}$	$\cos^{-1}\frac{x}{a}+C$
(vii) $\int \sec^2 x\,dx$	$\tan x+C$	(xvii) $\int \frac{dx}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}+C$
(viii) $\int \operatorname{cosec}^2 x\,dx$	$-\cot x+C$	(xviii) $\int \frac{-dx}{a^2+x^2}$	$\frac{1}{a}\sec^{-1}\frac{x}{a}+C$

Integrands	Integrals	Integrands	Integrals
(ix) $\int \sec x \tan x\, dx$	$\sec x + C$	(xix) $\int \frac{dx}{\lvert x\rvert\sqrt{x^2 \quad a^2}}$	$\frac{1}{a}\sec^{-1}\frac{x}{a} + C$
(x) $\int \operatorname{cosec} x \cot x\, dx$	$-\operatorname{cosec} x + C$	(xx) $\int \frac{-dx}{\lvert x\rvert\sqrt{x^2 - a^2}}$	$\frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + C$

3. Methods of Integration

(A) Integration by substitution method:

For integral $\int f'\{g(x)\}\, g'(x)\, dx$, we create a new variable $t = g(x)$, so that $g'(x) = \frac{dt}{dx}$

★ Special Integrals in Substitution Method

(i) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log_e \left| \frac{a+x}{a-x} \right| + C$

(ii) $\int \frac{dx}{x^2 - a^2} dx = \frac{1}{2a} \log_e \left| \frac{x-a}{x+a} \right| + C$

(iii) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log_e \left| x + \sqrt{x^2 - a^2} \right| + C$

(iv) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log_e \left| x + \sqrt{x^2 + a^2} \right| + C$

Different forms:

FORM-I : $\int \frac{px+q}{ax^2+bx+c} dx$ OR $\int \frac{px+q}{\sqrt{ax^2+bx+c}}$

$\Rightarrow$ Put $px + q = K_1 \frac{d}{dx}(ax^2 + bx + c) + K_2$

Now, find K_1 & K_2 and integrate it.

(B) Integration by parts method:

We use this method when there is a product of two functions.

$$\int \underset{\text{I}}{f(x)} \cdot \underset{\text{II}}{g(x)}\, dx = f(x) \int g(x)\, dx - \int \left\{ \frac{d}{dx} f(x) \int g(x)\, dx \right\} dx$$

★ Order to follow: Inverse → Logarithm → Algebraic → Trigonometric → Exponential.

★ Some Special Integrals in By-parts Method

(i) $\int \sqrt{x^2+a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2 \log\left|x+\sqrt{x^2+a^2}\right|\right] + C$

(ii) $\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)\right] + C$

(iii) $\int \sqrt{x^2-a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2-a^2} - a^2 \log\left|x+\sqrt{x^2-a^2}\right|\right] + C$

Different forms:

FORM-I : $\int (px+q)\sqrt{ax^2+bx+c}\, dx$

$\Rightarrow$ Put $px+q = K_1\left[\frac{d}{dx}(ax^2+bx+c)\right] + K_2$

Now, find K_1 & K_2 and then integrate it.

(C) Integration by partial fraction method:

If degree of numerator < degree of denominator then partial fraction method will be applicable. Decompose $\frac{f(x)}{g(x)}$ into partial fraction.

Ex. (i) $\frac{px+q}{(x-a)(x-b)^2(x-c)^3} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2} + \frac{D}{(x-c)} + \frac{E}{(x-c)^2} + \frac{F}{(x-c)^3}$

(ii) $\frac{px2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Now, find all values of constants (assumed) and then integrate it.

TIPS AND TRICKS: (T-1)

Short trick to solve linear form integration.

$$\int\left(\frac{ax+b}{cx+d}\right)dx = \frac{ax}{c} - \frac{(ad-bc)}{c^2}\log_e|cx+d| + C$$

Illustration 1

Solve: $\int \frac{2x+7}{4x+5}\, dx$

Short-cut solution :

Using T-1 $\int \frac{2x+7}{4x+5}dx = \frac{2x}{4} - \frac{(10-28)}{16}\log_e|4x+5| + C$

$= \frac{x}{2} + \frac{9}{8}\log_e|4x+5| + C$

Illustration 2

Solve: $\int \frac{7x-15}{4x+11}$

Short-cut solution :

Using T-1 $\int \frac{7x-15}{4x+11}dx = \frac{7x}{4} - \frac{(77+60)}{16}\log_e|4x+11| + C$

$- \frac{7x}{4} - \frac{137}{16}\log_e|4x+11| + C$

TIPS AND TRICKS: (T-2)

Short trick to solve integration of the form:

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\left(\frac{d}{dx}\frac{(ax^2+bx+c)}{\sqrt{4ac-b^2}}\right) + C$$

where, $4ac - b^2 > 0$

Illustration 3

Solve: $\int \frac{dx}{5x^2+6x+5}$

Short-cut solution :

Using T-2 $\because\ 4ac - b^2 = 100 - 36 = 64 > 0$

$\Rightarrow \int \frac{dx}{5x^2+6x+5} = \frac{2}{\sqrt{64}}\tan^{-1}\left(\frac{10x+6}{\sqrt{64}}\right) + C = \frac{1}{4}\tan^{-1}\left(\frac{5x+3}{4}\right) + C$

Illustration 4

Solve: $\int \frac{dx}{x^2+4x+7}$

Short-cut solution :

Using T-2 $\because\ 4ac - b^2 = 28 - 16 = 12 > 0$

$$\Rightarrow \int \frac{dx}{x^2 + 4x + 7} = \frac{2}{\sqrt{12}} \tan^{-1}\left(\frac{2x+4}{\sqrt{12}}\right) + C = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + C$$

TIPS AND TRICKS: (T-3)

Short trick to solve integration of the form:

$\int x^n f(x)\,dx$; where $f(x)$ is **trigonometric or exponential function.**

Tabular Method : (Process as follows)

Ex: $I - \int \underset{II}{(\sin 2x)}\ \underset{I}{x^2}\ dx$

Derivative of I	Sign	Integral of II
x^2		$\sin 2x$
	+ ↘	$-\frac{1}{2}\cos 2x$
$2x$	− ↘	
		$-\frac{1}{4}\sin 2x$
2	+ ↘	
		$+\frac{1}{8}\cos 2x$
0		

{Multiply the terms as arrow indicates sign changes alternatively}

$$\Rightarrow I = -\frac{x^2}{2}\cos 2x + \frac{2x}{4}\sin 2x + \frac{2}{8}\cos 2x$$

Illustration 5

Solve: $\int \underset{I}{x^4}\ \underset{II}{e^x}\ dx$

Short-cut solution :

Using T-3

Differentiation	Sign	Integration
x^4	+	e^x
$4x^3$	−	e^x
$12x^2$	+	e^x
$24x$	−	e^x
24	+	e^x
0		e^x

$\Rightarrow x^4e^x - 4x^3e^x + 12x^2e^x - 24x\,e^x + 24e^x$

TIPS AND TRICKS: (T-4)

Short trick to solve integration of the form:

$$\int \frac{dx}{a + b\cos^2 x} = \frac{1}{a}\left\{\sqrt{\frac{a}{a+b}}\tan^{-1}\left(\sqrt{\frac{a}{a+b}}\tan x\right)\right\} + C$$

★ **Note:** In case of '$\sin^2 x$' use $\sin^2 x = 1 - \cos^2 x$

Illustration 6

$\int \frac{dx}{3 + 2\cos^2 x}$, solve the integration

Short-cut solution :

Using T-4 $\because a = 3, b = 2$

$$\Rightarrow \int \frac{1}{a + b\cos^2 x}dx = \frac{1}{3}\left\{\sqrt{\frac{3}{5}}\tan^{-1}\left(\sqrt{\frac{3}{5}}\tan x\right)\right\} + C$$

Illustration 7

$\int \frac{dx}{6 + 4\sin^2 x}$, solve the integration

Short-cut solution :

Using T-4 $\because \int \frac{dx}{10 - 4\cos^2 x}$

Here, $a = 10, b = -4$

$$\Rightarrow \int \frac{dx}{6+4\sin^2 x} = \int \frac{dx}{10-4\cos^2 x} = \frac{1}{10}\left\{\sqrt{\frac{10}{6}}\tan^{-1}\left(\sqrt{\frac{10}{6}}\tan x\right)\right\} + C$$

TIPS AND TRICKS: (T-5)

Short trick to solve integration of the form:

$$\int \frac{ae^{kx}+b}{ce^{kx}+d}\,dx = \frac{bx}{d} + \frac{1}{k}\frac{(ad-bc)}{cd}\log_e|ce^{kx}+d| + C$$

Illustration 8

Solve: $\int \frac{3e^x+5}{2e^x+7}\,dx$

Short-cut solution :

Using T-5 $\int \frac{3e^x+5}{2e^x+7}\,dx = \frac{5x}{7} + \frac{(21-10)}{14}\log_e|2e^x+7| + C$

Illustration 9

Solve: $\int \frac{7e^{5x}+10}{-8e^{5x}+3}\,dx$

Short-cut solution :

Using T-5 $\int \frac{7e^{5x}+10}{-8e^{5x}+3}\,dx = \frac{10x}{3} + \frac{1}{5}\left(\frac{101}{-24}\right)\log_e|3-8e^{5x}| + C$

TIPS AND TRICKS: (T-6)

Short trick of the form:

(i) $\int e^{mx}\sin(nx)\,dx = \frac{e^{mx}}{m^2+n^2}\left(m\sin(nx) - n\cos(nx)\right) + C$

(ii) $\int e^{mx}\cos(nx)\,dx = \frac{e^{mx}}{m^2+n^2}\left(m\cos(nx) + n\sin(nx)\right) + C$

Illustration 10

Solve: $I = \int e^{3x} \cdot \sin 2x \, dx$

Short-cut solution :

Using T-6 Here, $m = 3, n = 2$

$$\Rightarrow \int e^{mx} \sin(nx) dx = \frac{e^{3x}}{9+4}(3 \sin 2x - 2 \cos 2x) + C$$

Illustration 11

Solve: $I = \int e^{-5x} \cdot \cos 7x \, dx$

Short-cut solution :

Using T-6 Here, $m = -5, n = 7$

$$\Rightarrow \int e^{mx} \cos(nx) dx = \frac{e^{-5}}{49+25}(-5 \cos(7x) + 7 \sin(7x)) + C$$

TIPS AND TRICKS: (T-7)

Short trick to solve integration in the partial fraction form:

★ This trick is valid only when denominator can be factorized into the linear form.

Ex. $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

Step 1. Put $x - 1 = 0 \Rightarrow x = 1$ and substitute $x = 1$ in rest of the factors, multiplied by $\log_e$ (factor which has avoided).

Step 2. Similarly repeat the process for other factors which are in denominator i.e. $x = -2, x = 3$ and add all the parts.

$$\Rightarrow \frac{2(1)-1}{(1+2)(1-3)} \ln|x-1| + \frac{(2(-2)-1)}{(-2-1)(-2-3)} \ln|x+2| + \frac{2(3)-1}{(3-1)(3+2)} \ln|x-3| + C$$

Illustration 12

Solve: $\int \frac{x^2+4}{x^3-3x^2+2x}dx$

Short-cut solution :

Using T-7 $\quad \because \int \frac{x^2+4}{x(x-1)(x-2)}$

(Put $x=0, x=1, x=2$ systematically as discussed earlier)

$$\Rightarrow \int \frac{x^2+4}{x^3-3x^2+2x}dx = \int \frac{x^2+4}{x(x-1)(x-2)}dx = \frac{0+4}{(0-1)(0-2)}$$

$$\ln|x| + \frac{1+4}{1(1-2)}\ln|x-1| + \frac{4+4}{2(2-1)}\ln|x-2| + C$$

$$\Rightarrow\ 2\ln|x| - 5\ln|x-1| + 4\ln|x-2| + C$$

TIPS AND TRICKS: (T-8)

Short trick to solve integration of the form:

(i) $\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\log_e\left|\frac{x^n}{x^n+1}\right| + C; n \in N$

(ii) $\int \frac{dx}{x(x^n-1)} = \log_e\left|\frac{x^n-1}{x^n}\right| + C; n \in N$

Illustration 13

Solve: $\int \frac{dx}{x(x^6+1)}$

Short-cut solution :

Using T-8 (i) $\quad \because\ n=6$

$$\Rightarrow \int \frac{dx}{x(x^6+1)} = \frac{1}{6}\ln\left|\frac{x^6}{x^6+1}\right| + C$$

Illustration 14

Solve: $\int \frac{dx}{x(x^4-1)}$

Short-cut solution :

Using T-8 (ii) $\because\ n = 4$

$$\Rightarrow \int \frac{dx}{x\left(x^4-1\right)} = \ln\left|\frac{x^4-1}{x^4}\right| + C$$

TIPS AND TRICKS: (T-9)

Short trick to solve integration of the form:

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx = \left(\frac{ac+bd}{c^2+d^2}\right)x + \left(\frac{ad-bc}{c^2+d^2}\right)\ln(\text{Denominator}) + C$$

Illustration 15

Solve: $\int \frac{2\cos x + \sin x}{4\cos x + 3\sin x}\,dx$

Short-cut solution :

Using T-9 Here, $a = 2, b = 1, c = 4, d = 3$

$$\Rightarrow \int \frac{2\cos x + \sin x}{4\cos x + 3\sin x}\,dx = \left(\frac{8+3}{25}\right)x + \left(\frac{6-4}{25}\right)\ln|4\cos x + 3\sin x| + C$$

SHORTCUTS: (SC-1)

Reverse process of integration i.e. differentiation. If we have to find the unknowns then differentiate the given integration both sides and reach the answer.

Illustration 16

If $\int \frac{5\tan x}{\tan x - 2}\,dx = x + a\ln|\sin x - 2\cos x| + K$ **[AIEEE 2012]**

Then 'a' is equal to

(a) 1 (b) 2 (c) -1 (d) -2

Short-cut solution :

Using SC-1 Taking $\frac{d}{dx}$ of both sides

$$\Rightarrow \frac{5\tan x}{\tan x - 2} = 1 + \frac{a(\cos x + 2\sin x)}{\sin x - 2\cos x}$$

$$\Rightarrow \frac{5\tan x}{\tan x - 2} - 1 = \frac{a(\cos x + 2\sin x)}{\sin x - 2\cos x}$$

(Divide N^r and D^r by $\cos x$ in RHS)

$$\Rightarrow \frac{4\tan x + 2}{\tan x - 2} = \frac{a + 2a\tan x}{\tan x - 2}$$

On comparing both sides $\Rightarrow a = 2$ **Ans. (b)**

Illustration 17

If $I_n = \int \tan^n x\, dx\ (n > 1)$ and $I_4 + I_6 = a\tan^5 x + bx^5 + C$; then find a, b. **[JEE M 2017]**

Short-cut solution :

Using SC-1 $\because \int \tan^4 x\, dx + \int \tan^6 x\, dx = a\tan^5 x + bx^5 + C$

On differentiating both sides, we get

$\Rightarrow \tan^4 x + \tan^6 x = 5a\tan^4 x \cdot \sec^2 x + 5bx^4$

$\Rightarrow \tan^4 x + \tan^6 x = 5a\tan^4 x + 5a \cdot \tan^6 x + 5bx^4$

On comparing both sides $\Rightarrow a = \frac{1}{5}, b = 0$

SHORTCUTS: (SC-2)

Integrals of the form $\int \sin^n x \cos^m x\, dx$

Case-I: If n is odd, put $\cos x = t$

Case-II: If m is odd, put $\sin x = t$

Case-III: If m and n both odd, put $\sin x$ or $\cos x = t$

Case-IV: If $m + n$ is negative even integer, put $\tan x = t$.

Illustration 18

Solve: $\int \sin^{99} x \cos^3 x \, dx$

Short-cut solution :

Using SC-2 $\quad m = 3$ and $n = 99 \Rightarrow$ Case-III

Hence, put $\sin x = t \Rightarrow \dfrac{dt}{dx} = \cos x$

$$\Rightarrow \int t^{99}(1-t^2)\, dt = \frac{t^{100}}{100} - \frac{t^{102}}{102}$$

$$\Rightarrow \frac{(\sin x)^{100}}{100} - \frac{(\sin x)^{102}}{102} + C$$

Illustration 19

Solve: $\int \dfrac{dx}{\sin^{1/2} x \cos^{7/2} x}$

Short-cut solution :

Using SC-2 $\quad \because m + n = -4 \Rightarrow$ Case-IV

Hence, convert in terms of $\tan x$.

$$\Rightarrow \int \frac{\sec^4 x}{\sqrt{\dfrac{\sin x}{\cos x}}}\, dx = \int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}\, dx$$

Now, put $\tan x = t \Rightarrow \dfrac{dt}{dx} = \sec^2 x$

$$\Rightarrow \int \frac{1+t^2}{\sqrt{t}}\, dt = 2\sqrt{t} + \frac{2}{5}(t)^{5/2} = 2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2} + C$$

SHORTCUTS: (SC-3)

Integrals of the form:

(i) $\int e^x (f(x) + f'(x))\, dx = e^x f(x) + C$

(ii) $\int (x f'(x) + f(x))\, dx = x f(x) + C$

Illustration 20

Solve: $\int \frac{x+\sin x}{1+\cos x}\,dx$

Short-cut solution :

Using SC-3 (ii)

$$\int\left(\frac{x}{1+2\cos^2\frac{x}{2}-1}+\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1+2\cos^2\frac{x}{2}-1}\right)dx$$

$$\Rightarrow \int\Big(\underbrace{x\cdot\frac{1}{2}\sec^2\frac{x}{2}}_{f'(x)}+\underbrace{\tan\frac{x}{2}}_{f(x)}\Big)dx \;\Rightarrow\; x\tan\frac{x}{2}+C$$

Illustration 21

Solve: $\int \frac{e^x(x^2+5x+7)}{(x+3)^2}\,dx$

Short-cut solution :

Using SC-3 (i) $\int e^x\left\{\underbrace{\left(\frac{x+2}{x+3}\right)}_{f(x)}+\underbrace{\frac{1}{(x+3)^2}}_{f'(x)}\right\}dx \;\Rightarrow\; e^x\left(\frac{x+2}{x+3}\right)+C$

TECHNIQUE: (Tech.1)

Algebraic Twins : To find the integral of the form

$\int\frac{x^2+a^2}{x^4+\lambda x^2+a^4}dx$ and $\int\frac{x^2-a^2}{x^4+\lambda x^2+a^4}dx$

To evaluate the integral of these above forms, first we divide both numerator and denominator by x^2 and then express the denominator in the form $\left(x-\frac{a^2}{x}\right)^2\pm k^2$ and $\left(x+\frac{a^2}{x}\right)^2\pm k^2$ respectively. Then put $x-\frac{a^2}{x}=t$ and $x+\frac{a^2}{x}=t$ respectively.

Illustration 22

Evaluate $\int \frac{x^2+1}{x^4+x^2+1}\,dx$

Short-cut solution :

Using Tech.

$$I=\int \frac{x^2+1}{x^4+x^2+1}\,dx=\int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}\,dx=\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+3}$$

Put $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx=dt$

$$\therefore I=\int \frac{dt}{t^2+3}=\int \frac{dt}{t^2+(\sqrt{3})^2}=\frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}}+C$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right)+C$$

TECHNIQUE: (Tech. 2)

To find the integral of the form $\int \frac{\phi(x)}{P\sqrt{Q}}\,dx$, where P and Q both are pure quadratic expression in x. Such that $P=ax^2+b$, $Q=cx^2+d$.

Then, put $x=\frac{1}{t}$ and $c+dt^2=u^2$.

Illustration 23:

Evaluate $\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx$

Short-cut solution :

Using Tech.

Putting $x = \frac{1}{t}$ and $dx = -\frac{1}{t^2}dt$, we get

$$I = \int \frac{\left(-\frac{1}{t^2}\right)dt}{\left(1-\frac{1}{t^2}\right)\sqrt{1+\frac{1}{t^2}}} = -\int \frac{t\,dt}{(t^2-1)\sqrt{t^2+1}}$$

Let $t^2+1 = u^2$, we get $2t\,dt = 2u\,du$

$$I = -\int \frac{du}{u^2-(\sqrt{2})^2} = \frac{-1}{2\sqrt{2}}\log\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right| + c$$

$$= -\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{t^2+1}-\sqrt{2}}{\sqrt{t^2+1}+\sqrt{2}}\right| + c = -\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{\frac{1}{x^2}+1}-\sqrt{2}}{\sqrt{\frac{1}{x^2}+1}+\sqrt{2}}\right| + c$$

$$= -\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right| + c$$

Concept Booster Exercise

1. $\int \frac{x-1}{x+1} dx$ is equal to

(a) $x - \log_e |x+1| + C$ (b) $x - 2\log_e |x+1| + C$

(c) $-2\log_e |x+1| + C$ (d) $-\log_e |x+1| + C$

2. $\int \frac{dx}{(x+1)(x+2)(x+3)}$ is equal to

(a) $\frac{1}{2}\ln|(x+1)(x+3)| - \ln|x+2| + C$

(b) $\frac{1}{2}\ln|x+1| + C$

(c) $\frac{1}{2}\ln|x+2| - \ln|x+1| + C$

(d) $\frac{1}{2}\ln|x+3| - \ln|x+2| + C$

3. $\int \frac{dx}{2+\sin^2 x}$ is equal to

(a) $\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$ (b) $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$ (d) None of these

4. $\int 2x^3 e^{-x} dx$ is equal to

(a) $e^x(-x^3 - 3x^2 - 6x - 6)$ (b) $e^{-x}(x^3 + 3x^2 + 6x + 6)$

(c) $e^{-x}(-x^3 - 3x^2 - 6x - 6)$ (d) None of these

5. $\int \frac{10 \cdot e^{2x} + 5}{11 \cdot e^{2x} - 2} dx$ is equal to

(a) $\frac{-5x}{2} + \frac{75}{44}\log_e|11 \cdot e^{2x} - 2| + C$

(b) $\frac{75}{44}\log_e|11 \cdot e^{2x} - 2| + C$

(c) $\frac{-5x}{2}+\frac{2}{9}\log_e|10\cdot e^{2x}+5|+C$

(d) $\frac{2}{9}\log_e|10\cdot e^{2x}+5|+C$

6. $\int e^x\cdot\sin 3x\,dx$ is equal to

(a) $\frac{e^x}{9}(\sin 3x-3\cos 3x)+C$ (b) $\frac{e^x}{10}(\sin 3x-3\cos 3x)+C$

(c) $\frac{e^x}{8}(\sin 3x-3\cos 3x)+C$ (d) $\frac{e^x}{10}(\sin 3x+3\cos 3x)+C$

7. $\int\frac{dx}{x^2+3x+4}$ is equal to

(a) $2\tan^{-1}\left(\frac{2x+3}{\sqrt{7}}\right)+C$ (b) $\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{2x+3}{\sqrt{7}}\right)+C$

(c) $\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{2x-3}{\sqrt{7}}\right)+C$ (d) $\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{2x+3}{\sqrt{7}}\right)+C$

8. $\int\frac{1}{x(1+x^7)}dx$ is equal to

(a) $\log_e\left|\frac{x^7}{1+x^7}\right|+C$ (b) $\frac{1}{7}\log_e\left|\frac{x^7}{1+x^7}\right|+C$

(c) $-\frac{1}{7}\log_e\left|\frac{x^7}{1+x^7}\right|+C$ (d) $7\log_e\left|\frac{x^7}{1+x^7}\right|+C$

9. $\int\frac{\sin x}{\sin x-\cos x}dx$ is equal to

(a) $\frac{1}{2}\log|\sin x+\cos x|+C$ (b) $\log|\sin x-\cos x|+C$

(c) $\frac{1}{2}\log|\sin x-\cos x|+\frac{x}{2}+C$ (d) $\frac{1}{2}\log|\sin x+\cos x|+\frac{x}{2}+C$

10. If $\int\frac{\sin x}{\sin(x-\alpha)}dx=Ax+B\log\cdot\sin(x-\alpha)+C$, then value of (A, B) is **[AIEEE 2004]**

(a) $(\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$

11. If $f\left(\dfrac{x-4}{x+2}\right)=2x+1; x\in R-\{1,-2\}$, then $\int f(x)\,dx$ is equal to

[JEE M 2018]

(a) $12\ln|1-x|-3x+C$ (b) $-12\ln|1-x|-3x+C$
(c) $-12\ln|1-x|+3x+C$ (d) $12\ln|1-x|+3x+C$

12. If $\int \dfrac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax + B\log(9e^{2x}-4)+C$, then, '$A$' and '$B$' are

(a) $\dfrac{-3}{2},\dfrac{35}{36}$ (b) $\dfrac{-2}{3},\dfrac{35}{36}$
(c) $\dfrac{-3}{2},\dfrac{36}{35}$ (d) $\dfrac{-2}{3},\dfrac{36}{35}$

13. $\int \sin^5 x\cos^2 x\,dx$ is equal to

(a) $-\dfrac{\cos^7 x}{7}-\dfrac{2\cos^5 x}{5}-\dfrac{\cos^3 x}{3}$ (b) $-\dfrac{\cos^7 x}{7}+\dfrac{2\cos^5 x}{5}-\dfrac{\cos^3 x}{3}$
(c) $\dfrac{\cos^7 x}{7}+\dfrac{2\cos^5 x}{5}+\dfrac{\cos^3 x}{3}$ (d) None of these

14. $\int \dfrac{(x-1)e^x}{(x+1)^3}dx$ is equal to

(a) $\dfrac{e^x}{(x-1)^2}+C$ (b) $\dfrac{e^x}{(x+1)^2}+C$
(c) $\dfrac{e^x}{x-1}+C$ (d) $\dfrac{e^x}{x+1}+C$

Numerical Value Problems

15. Let $f(x)=\int e^x(x-1)(x-2)\,dx$, then '$f$' decreases in the interval (a, b), then $a+b$ is ________ . **[JEE M 2020]**

16. If $\int e^{3x}\cos 4x\,dx = e^{3x}(A\sin 4x + B\cos 4x)+C$, then $4A+3B$ is equal to ________ .

17. If $\int \dfrac{e^x+(1+x^2)e^x\tan^{-1}x}{1+x^2}dx = A\cdot e^x\tan^{-1}x + C$, then '$A$' is equal to ________ .

18. If $\int \dfrac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2}+B\sin^{-1}\left(\dfrac{x+3}{4}\right)+C$ their ordered pair is (A, B), then $|A+B|$ is ________ .

Solutions

1. **(b)** Using T-1 $\because\ a=1, b=-1, c=1, d=1$

$\Rightarrow\ x-\frac{2}{1}\log_e|x+1|+C$

2. **(a)** Using T-7

Putting $x=-1$

$\Rightarrow\ \frac{1}{2}\ln|x+1|$

Putting $x=-2$

$\Rightarrow\ -\ln|x+2|$

Putting $x=-3$

$\Rightarrow\ \frac{1}{2}\ln|x+3|$

Hence, $\int\frac{1}{(x+1)(x+2)(x+3)}dx$

$=\frac{1}{2}\ln(x+1)(x+3)-\ln(x+2)+C$

3. **(c)** Using T-4 $\int\frac{dx}{3-\cos^2 x}$ $(\because\ \sin^2 x=1-\cos^2 x)$

$\Rightarrow\ a=3, b=-1\ \Rightarrow\int\frac{dx}{3-\cos^2 x}=\frac{1}{3}\left\{\sqrt{\frac{3}{2}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\tan x\right)\right\}+C$

4. **(c)** Using T-3

x^3	$+$	e^{-x}
$3x^2$	$-$	$-e^{-x}$
$6x$	$+$	e^{-x}
6	$-$	$-e^{-x}$
0		e^{-x}

$\Rightarrow\ \int 2x^3 e^{-x}dx=-x^3e^{-x}-3x^2\cdot e^{-x}-6xe^{-x}-6e^{-x}$

$\Rightarrow\ e^{-x}(-x^3-3x^2-6x-6)$

5. **(a)** Using T-5 $\because\ a=10, b=5, c=11, d=-2, k=2$

$\Rightarrow\int\frac{10\cdot e^{2x}+5}{11\cdot e^{2x}-2}dx=\frac{5x}{-2}+\frac{1}{2}\left(\frac{-75}{-22}\right)\log_e|11\cdot e^{2x}-2|+C$

6. **(b)** Using T-6 (i) $m = 1, n = 3$

$$\Rightarrow \int e^x \sin(3x)\,dx = \frac{e^x}{1+9}(\sin(3x) - 3\cos(3x)) + C$$

7. **(d)** Using T-2 $\because\ 4ac - b^2 = 16 - 9 = 7 > 0$

$$\Rightarrow \int \frac{dx}{x^2 + 3x + 4} = \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x+3}{\sqrt{7}}\right) + C$$

8. **(b)** Using T-8 (i) $\because\ n = 7$

$$\Rightarrow \int \frac{1}{x(1+x^7)}\,dx = \frac{1}{7}\log_e \left|\frac{x^7}{1+x^7}\right| + C$$

9. **(c)** Using T-9 Here, $\int \frac{0 \cdot \cos x + 1 \cdot \sin x}{-\cos x + 1 \cdot \sin x}\,dx$

$\because\ a = 0, b = 1, c = -1, d = 1$

$$\Rightarrow \int \frac{\sin x}{\sin x - \cos x}\,dx = \left(\frac{1}{1+1}\right)x + \left(\frac{1}{2}\right)\ln|\sin x - \cos x| + C$$

10. **(b)** Using SC-1 $\int \frac{\sin x}{\sin(x-\alpha)}\,dx = Ax + B\log\sin(x-\alpha) + C$

Differentiating both sides

$$\Rightarrow \frac{\sin x}{\sin(x-\alpha)} = A + \frac{B \cdot \cos(x-\alpha)}{\sin(x-\alpha)}$$

$$\frac{\sin x}{\sin(x-\alpha)} = \frac{\sin x[A\cos\alpha + B\sin\alpha] + \cos x[B\cos\alpha - A\sin\alpha]}{\sin(x-\alpha)}$$

On comparing both sides $\Rightarrow A = \cos\alpha, B = \sin\alpha$

11. **(b)** Using T-1 Put $\frac{x-4}{x+2} = t \Rightarrow x = \frac{2t+4}{1-t}$

$$\Rightarrow f(t) = 2\left[\frac{2t+4}{1-t}\right] + 1 = \frac{3(t)+9}{1-t} \Rightarrow \int \frac{3x+9}{1-x}\,dx$$

$a = 3, b = 9, c = -1, d = 1$

$$\Rightarrow \frac{3x}{-1} - \left(\frac{3+9}{1}\right)\log_e|1-x| + C$$

12. **(a)** Using T-5 Here, $a = 4, b = 6, c = 9, d = -4$

Multiply numerator and denominator by $e^x \Rightarrow k = 2$

Now, $\int \frac{4e^{2x}+6}{9e^{2x}-4}\,dx = \frac{6x}{-4} + \frac{1}{2}\left(\frac{-16-54}{-36}\right)\log_e|9e^{2x}-4| + C$

$\Rightarrow A = \frac{-3}{2},\ B = \frac{35}{36}$

13. **(b)** Using SC-2 $n = 5, m = 2 \Rightarrow$ 'n' is odd

Hence, using case-I, put $\cos x = t \Rightarrow dx = -\frac{1}{\sin x}\frac{dt}{dx}$

$\Rightarrow \int \sin^5 x \cdot \frac{t^2}{-\sin x}\,dt = -\int t^2(1-t^2)^2\,dt$

$= -\int (t^6 - 2t^4 + t^2)\,dt$

$= -\frac{\cos^7 x}{7} + \frac{2\cos^5 x}{5} - \frac{\cos^3 x}{3}$

14. **(b)** Using SC-3

$\int e^x\left\{\underbrace{\frac{x+1}{(x+1)^3}}_{f(x)} + \underbrace{\frac{-2}{(x+1)^3}}_{f'(x)}\right\}dx = \frac{e^x}{(x+1)^2} + C$

15. **(3)** Since $f(x)$ is decreasing $\Rightarrow f'(x) < 0$

$\Rightarrow e^x(x-1)(x-2) < 0$

$\Rightarrow x \in (1, 2)$

Hence, $a + b = 3$

16. **(1)** Using T-6 (ii) Here, $m = 3, n = 4$

$\Rightarrow \int e^{3x}\cos(4x)\,dx = \frac{e^{3x}}{9+16}(3\cos 4x + 4\sin 4x) + C$

$\Rightarrow \frac{e^{3x}}{25}\{4\sin 4x + 3\cos 4x\} + C$

$\Rightarrow A = \frac{4}{25}$ and $B = \frac{3}{25} \Rightarrow 4A + 3B = 1$

17. (1) Using SC-3 $\because \int e^x \left\{ \tan^{-1} x + \frac{1}{1+x^2} \right\} dx$

As we know that $\int e^x \{(f(x)) + f'(x)\} \, dx = e^x f(x) + C$

$\Rightarrow e^x \tan^{-1} x + C$

$\Rightarrow A = 1$

18. (4) Using SC-1 $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

Differentiating both sides

$$\Rightarrow \frac{2x+5}{\sqrt{7-6x-x^2}} = \frac{A(-2x-6)}{2\sqrt{7-6x-x^2}} + \frac{B}{\sqrt{1-\left(\frac{x+3}{4}\right)^2}} \times \frac{1}{4}$$

$$\frac{2x+5}{\sqrt{7-6x-x^2}} = \frac{A(-x-3)}{\sqrt{7-6x-x^2}} + \frac{B/2}{\sqrt{7-6x-x^2}}$$

On comparing both sides $\Rightarrow A = -2, B = -2 \Rightarrow |A+B| = 4$