AEP STUDY CIRCLE
SINCE 2001... IIT-NEET-CBSE
ACCENTS EDUCATIONAL PROMOTERS

23 YEAR' OF LEGACY

REVISION MODULE
CBSE-XII

Offline - Online
NEET IIT-JEE CBSE
The Success Destination
Learning Academy

For CBSE Class 12 students, the application of integrals is an essential topic with various practical applications. Some specific applications of integrals that align with the CBSE Class 12 curriculum:

Area Under a Curve:
CBSE often emphasizes the concept of finding the area under curves using integration. Students are expected to understand how to calculate the area between a curve and the x-axis or between two curves.

APPLICATION OF INTEGRALS
CBSE-XII
MATHEMATICS

We hope you find this sample paper a valuable tool in your preparation. Best of luck with your studies, and may you excel in your CBSE Class XII MATHEMATICS examination.

www.aepstudycircle.com
+91-9939586130
+91-7739650505
2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

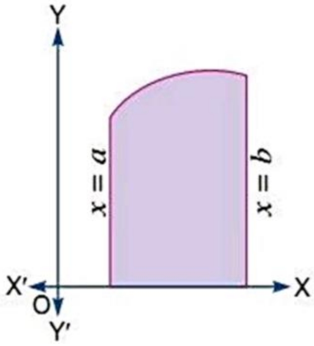
Application Of Integrals

Revision module

POINTS TO REMEMBER

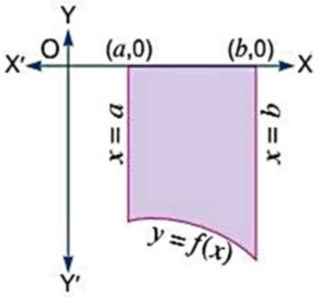
Area of Bounded Regions: Let $f(x)$ be a continuous function defined on $[a, b]$, then the area bounded by the curve $y = f(x)$, the x-axis and the straight lines $x = a$ and $x = b$ is given by

$$\int_a^b f(x)\,dx = \int_a^b y\,dx$$



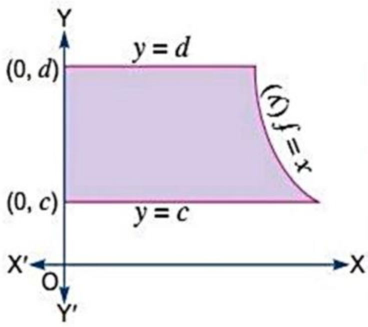
Theorem 1. If the curve $y = f(x)$ lies below x-axis, then the area bounded by $y = f(x)$, $y = 0$, $x = a$ and $x = b$ will be negative and in this situation we take the modulus of the area *i.e.*, the area is represented by

$$\left|\int_a^b f(x)dx\right|$$



Theorem 2. If the curve is given in the form $x = f(y)$, then the area of curve $x = f(x)$ bounded between $x = 0$, $y = c$ and $y = d$ is given by

$$\int_c^d f(y)dy$$



Multiple Choice Questions

Choose and write the correct option in the following questions.

1. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to [*NCERT Exemplar*]

(*a*) 4π sq units (*b*) $2\sqrt{2}\pi$ sq units (*c*) $4\pi^2$ sq units (*d*) 2π sq units

2. The area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to [*NCERT Exemplar*]

(*a*) $\pi^2 ab$ sq units (*b*) πab sq units (*c*) $\pi a^2 b$ sq units (*d*) πab^2 sq units

3. The area bounded by the curves $y=\sqrt{x}$, $2y+3=x$ and x-axis in the 1st quadrant is

(*a*) 9 sq units (*b*) $\frac{27}{4}$ sq units (*c*) 36 sq units (*d*) 18 sq units

4. The area of the region bounded by the curve $x^2=4y$ and the straight line $x=4y-2$ is [*NCERT Exemplar*]

(*a*) $\frac{3}{8}$ sq unit (*b*) $\frac{5}{8}$ sq unit (*c*) $\frac{7}{8}$ sq unit (*d*) $\frac{9}{8}$ sq units

5. The area of the region bounded by the curve $y=\sqrt{16-x^2}$ and x-axis is [*NCERT Exemplar*]

(*a*) 8π sq units (*b*) 20π sq units (*c*) 16π sq units (*d*) 256π sq units

6. Area of the region in the first quadrant enclosed by the x-axis, the line $y=x$ and the circle $x^2+y^2=32$ is [*NCERT Exemplar*]

(*a*) 16π sq units (*b*) 4π sq units (*c*) 32π sq units (*d*) 24 sq units

7. Area of the region bounded by the curve $y=\cos x$ between $x=0$ and $x=\pi$ is [*NCERT Exemplar*]

(*a*) 2 sq units (*b*) 4 sq units (*c*) 3 sq units (*d*) 1 sq unit

8. The area of the region bounded by the curve $x=2y+3$ and the lines $y=1$ and $y=-1$ is [*NCERT Exemplar*]

(*a*) 4 sq units (*b*) $\frac{3}{2}$ sq units (*c*) 6 sq units (*d*) 8 sq units

9. The area of the region bounded by the curve $y=x^2$ and the line $y=16$ is

(*a*) $\frac{37}{3}$ sq units (*b*) $\frac{256}{3}$ sq units (*c*) $\frac{64}{3}$ sq units (*d*) $\frac{128}{3}$ sq units

10. The area of the region bounded by the curve $y^2=9x$, $y=3x$ is

(*a*) 1 sq unit (*b*) $\frac{1}{2}$ sq unit (*c*) 4 sq units (*d*) 14 sq units

11. The area of the curve $y=\sin x$ between 0 and π is

(*a*) 2 sq units (*b*) 4 sq units (*c*) 12 sq units (*d*) 14 sq units

12. The area of the region bounded by the curve $ay^2=x^3$, the y-axis and the lines $y=a$ and $y=2a$ is

(*a*) 3 sq units (*b*) $\frac{3}{5}a^2|2\times 2^{2/3}-1|$ sq units

(*c*) $\frac{3}{5}a|2^{3/2}-1|$ sq units (*d*) 1 sq unit

13. The area of the region $\{(x,y): x^2+y^2\le 1\le x+y\}$ is

(*a*) $\frac{\pi}{5}$ (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{4}-\frac{1}{2}$ (*d*) $\frac{\pi^2}{2}$

14. The area of the region bounded by the curves $x=at^2$ and $y=2at$ between the ordinate corresponding to $t=1$ and $t=2$ is

(*a*) $\frac{56}{3}a^2$ sq units (*b*) $\frac{40}{3}a^2$ sq units (*c*) 5π sq units (*d*) None of these

15. The area of a minor segment of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{2}$ is

(*a*) $\frac{a^2}{12}(4\pi-3\sqrt{3})$ sq units (*b*) $\frac{a^2}{4}(4\pi-3)$ sq units

(*c*) $\frac{a^2}{12}(3\pi-4)$ sq units (*d*) None of these

16. The area of the region bounded by the curves $y = x^3$ and $y = x + 6$ and $x = 0$ is
(*a*) 7 sq units (*b*) 6 sq units (*c*) 10 sq units (*d*) 14 sq units

17. The area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$ is
(*a*) 4 sq units (*b*) 3 sq units (*c*) $\frac{4}{3}$ sq units (*d*) None of these

18. The area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$ is
(*a*) $\frac{\pi a^2}{4}$ sq units (*b*) $\frac{a^2}{4}$ sq units (*c*) πa^2 sq units (*d*) 4π sq units

19. The area of the region bounded by the triangle whose vertices are (–1, 1), (0, 5) and (3, 2) is
(*a*) $\frac{15}{2}$ sq units (*b*) 15 sq units (*c*) 4 sq units (*d*) 10 sq units

20. The area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -2$ and $x = 3$ is
(*a*) $\frac{4}{3}$ sq units (*b*) $\frac{7}{2}$ sq units (*c*) $\frac{17}{2}$ sq units (*d*) $\frac{16}{3}$ sq units

Answers

1. (*d*)	2. (*b*)	3. (*a*)	4. (*d*)	5. (*a*)	6. (*b*)	7. (*a*)
8. (*c*)	9. (*b*)	10. (*b*)	11. (*a*)	12. (*b*)	13. (*c*)	14. (*a*)
15. (*a*)	16. (*c*)	17. (*c*)	18. (*a*)	19. (*a*)	20. (*c*)	

Solutions of Selected Multiple Choice Questions

1. Area $= 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\,dx$

$$= 4\left(\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}} = 2\pi \text{ sq units.}$$

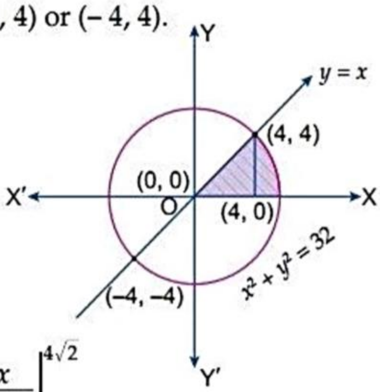
$\therefore$ Option (*d*) is correct.

6. We have, area enclosed by x-axis *i.e.*, $y = 0$, $y = x$ and the circle $x^2 + y^2 = 32$ in first quadrant.

Since, $x^2 + (x)^2 = 32$ $[\because y = x]$

$\Rightarrow$ $2x^2 = 32$ $\Rightarrow$ $x = \pm 4$

So, the intersection point of circle $x^2 + y^2 = 32$ and line $y = x$ are (4, 4) or (– 4, 4).



Now, $x^2 + y^2 = (4\sqrt{2})^2$

$\Rightarrow$ $x^2 + (0)^2 = 32$ $[\because y = 0]$

$\Rightarrow$ $x = \pm 4\sqrt{2}$

So, the circle intersects the x-axis at $(\pm 4\sqrt{2}, 0)$.

Area of shaded region $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$

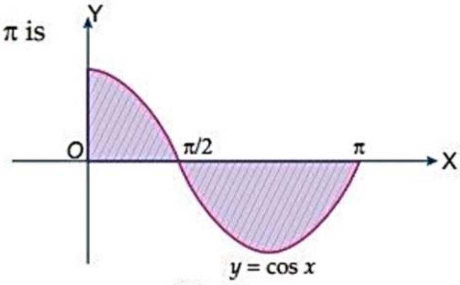
$$= \left|\frac{x^2}{2}\right|_0^4 + \left|\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right|_4^{4\sqrt{2}}$$

$$= \frac{16}{2} + \left[\frac{4\sqrt{2}}{2}\cdot 0 + 16\sin^{-1}\frac{(4\sqrt{2})}{(4\sqrt{2})} - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$$

$$= 8 + \left[16\cdot\frac{\pi}{2} - 2\cdot\sqrt{16} - 16\cdot\frac{\pi}{4}\right] = 8 + [8\pi - 8 - 4\pi] = 4\pi \text{ sq units.}$$

$\therefore$ Option (*b*) is correct.

7. Required area enclosed by the curve $y = \cos x$, $x = 0$ and $x = \pi$ is



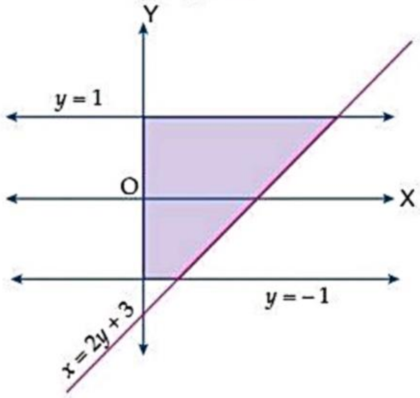
$$A = \int_0^{\pi/2} \cos x\, dx + \left| \int_{\pi/2}^{\pi} \cos x\, dx \right|$$

$$= \left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\sin\pi - \sin\frac{\pi}{2}\right|$$

$$= 1 + 1 = 2 \text{ sq units.}$$

$\therefore$ Option (*a*) is correct.

8. Required area, $A = \int_{-1}^{1} (2y + 3)\, dy$



$$= \left[\frac{2y^2}{2} + 3y\right]_{-1}^{1}$$

$$= \left[y^2 + 3y\right]_{-1}^{1}$$

$$= [1 + 3 - 1 + 3] = 6 \text{ sq units.}$$

$\therefore$ Option (*c*) is correct.

COMPETENCY-FOCUSED QUESTIONS AS PER NEP

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A) :** The area of the curve $y = \sin^2 x$ from 0 to π will be more than that of the curve $y = \sin x$ from 0 to π.

   Reason (R) : $x^2 > x$, if $x > 1$

2. **Assertion (A) :** The area of the ellipse $2x^2 + 3y^2 = 6$ will be more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.

   Reason (R) : The length of the semi-major axis of ellipse $2x^2 + 3y^2 = 6$ is more than the radius of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.

3. **Assertion (A) :** Area enclosed by the circle $x^2 + y^2 = 36$ is equal to 36π sq.units.

   Reason (R) : Area enclosed by the circle $x^2 + y^2 = r^2$ is πr^2.

4. **Assertion (A) :** Area enclosed by the curve $y = x^3$ and the line $y = x$ in first quadrant is $\frac{1}{4}$ sq. units.

   Reason (R) : Area is always the real number.

5. **Assertion (A) :** Area between the curve $x = y^2$ and the line $x = 3$ is $4\sqrt{3}$ sq. units.

   Reason (R) : Points of intersection of $y^2 = 4ax$ and the line $ax + by + c = 0$ is obtained by solving equations of curve and line.

Answers

1. (*d*) **2.** (*b*) **3.** (*a*) **4.** (*b*) **5.** (*b*)

Solutions of Assertion-Reason Questions

1. $\because\ \sin^2 x \le \sin x\ \ \forall\, x \in (0, \pi)$

$\therefore$ Area of $y = \sin^2 x$ will be lesser than the area of $y = \sin x$ in $x \in (0, \pi)$.

Clearly, Assertion (A) is false and Reason (R) is true.

$\therefore$ Option (*d*) is correct.

2. Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

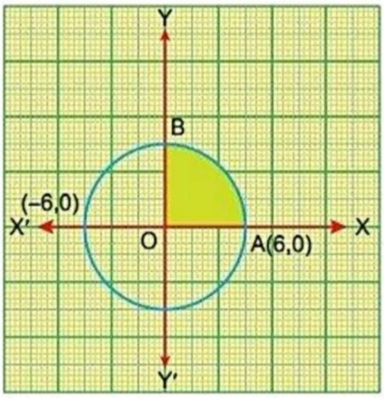
$\therefore$ Option (*b*) is correct.

3. $\because$ Required area $= 4 \times ar\,(OABO) = 4 \times \int_0^6 \sqrt{6^2 - x^2}\,dx$

$$= 4 \times \left[\frac{x}{2}\sqrt{6^2 - x^2} + \frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

$$= 4 \times \left[(0 + 18\sin^{-1}(1)) - (0 + 0)\right]$$

$$= 4 \times 18 \times \sin^{-1}(1) = 72 \times \frac{\pi}{2} = 36\,\pi \text{ sq. units}$$



Clearly A is correct statement.

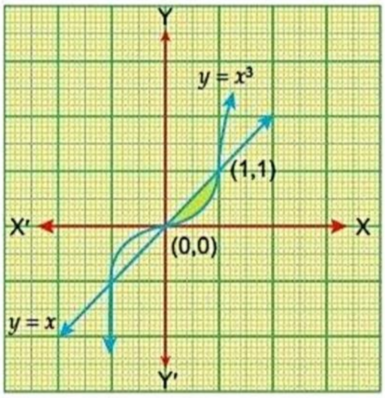
Also R is a correct statement and gives the correct explanation of statement A.

$\therefore$ Option (*a*) is correct.

4. We are given curve and line

$$y = x^3 \qquad \ldots(i)$$

$$y = x \qquad \ldots(ii)$$



From (*i*) and (*ii*), we have $\quad x^3 = x$

$\Rightarrow\ x^3 - x = 0 \quad \Rightarrow \quad x(x^2 - 1) = 0$

$\Rightarrow\ x(x-1)(x+1) = 0 \quad \Rightarrow \quad x = 0, -1, 1$

From (*ii*),

If $x = 0$, $y = 0$,

If $x = -1$, $y = -1$

If $x = 1$, $y = 1$

$\therefore$ Points of intersection are $(-1, -1)$, $(0, 0)$ and $(1, 1)$.

Required area $= \int_0^1 \left(y_{\text{line}} - y_{\text{parabola}}\right)dx$

$$= \int_0^1 x\,dx - \int_0^1 x^3\,dx = \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1$$

$$= \left(\frac{1}{2} - 0\right) - \left(\frac{1}{4} - 0\right)$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\text{sq. units}$$

Statement A is correct.

Also statement R is correct but does not correctly explain the statement A.

$\therefore$ Option (*b*) is correct.

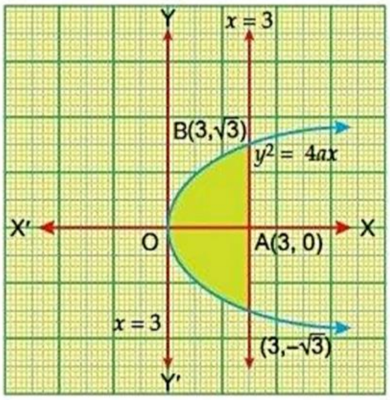
5. We are given equations of curve and line as

$x = y^2$ ...(i)

and $x = 3$ ...(ii)

From (i) & (ii), we have $y^2 = 3 \Rightarrow y = \pm\sqrt{3}$

$\therefore$ Points of intersection are $(3, \sqrt{3})$ and $(3, -\sqrt{3})$.



Required area $= 2 \times \text{ar}(OABO) = 2\int_0^3 \sqrt{x}\,dx$

$$= 2 \times \left[\frac{x^{3/2}}{3/2}\right]_0^3 = 2 \times \frac{2}{3}\left[x^{3/2}\right]_0^3$$

$$= \frac{4}{3}\left[3^{3/2} - 0^{3/2}\right] = \frac{4}{3} \times 3\sqrt{3} = 4\sqrt{3} \text{ sq. units.}$$

Statement A is correct.

Also statement R is correct but does not give correct explanation of statement A.

$\therefore$ Option (b) is correct.

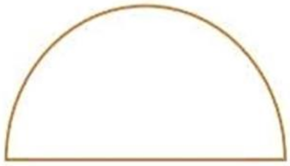
COMPETENCY-FOCUSED QUESTIONS AS PER NEP

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**

An architect designs a building whose lift (elevator) is from outside of the building attached to the walls. The floor (base) of the lift (elevator)) is in semicircular shape.



The floor of the elevator (lift) whose circular edge is given by the equation $x^2 + y^2 = 4$ and the straight edge (line) is given by the equation $y = 0$.

(i) Find the point of intersection of the circular edge and straight line edge.

(ii) Find the length of each vertical strip of the region bounded by the given curves.

(iii) (a) Find the area of a vertical strip between given circular edge and straight edge.

(b) Find the area of a horizontal strip between given circular strip and straight edge.

OR

(iii) Find the area of the region of the floor of the lift of the building (in square units).

Sol. (i) Given curve for circle and straight line are

$x^2 + y^2 = 4$ ...(1)

$y = 0$ ...(2)

$\therefore$ From (1) and (2), we have

$$x^2 = 4 \Rightarrow x = \pm 2$$

$\therefore$ Points of intersection are (2, 0) and (–2, 0).

(*ii*) Given curve, is circle whose equation is

$$x^2 + y^2 = 4$$

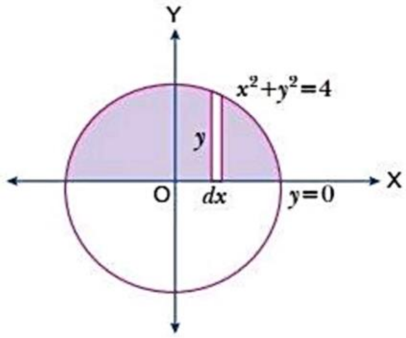
$\Rightarrow$ $y^2 = 4 - x^2 \Rightarrow y = \sqrt{4 - x^2}$

and $y = 0$

It represents x-axis.

$\therefore$ Length of the vertical strip is $y = \sqrt{4 - x^2}$, *i.e.*, $\sqrt{4 - x^2}$.

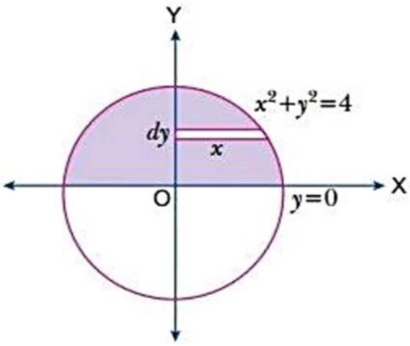
(*iii*) (*a*) We have,



Area of one vertical strip

$$= y.\, dx = \sqrt{4 - x^2}\,.\, dx$$

(*b*) We have,



Area of one horizontal strip

$$= x.\, dy$$

$$= \sqrt{4 - y^2}\,.\, dy$$

OR

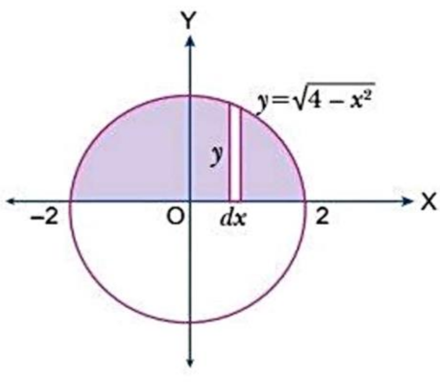
(*iii*) We have,

Area of the floor $= \int_{-2}^{2} y.dx = 2\int_{0}^{2} \sqrt{4 - x^2}.dx$

$$= 2\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

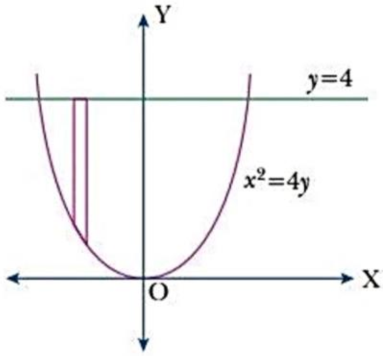
$$= 4\sin^{-1} 1 - 0$$

$$= 4 \times \frac{\pi}{2} = 2\pi \text{ sq. units}$$



2. Read the following passage and answer the following questions.

A student designs an open air Honeybee nest on the branch of a tree, whose plane figure is parabolic and the branch of tree is given by a straight line.



(i) Find point of intersection of the parabola and straight line.

(ii) Find the area of each vertical strip.

(iii) (a) Find the length of each horizontal strip of the bounded region.

(b) Find the length of each vertical strip.

OR

(iii) Find the area of region bounded by parabola $x^2 = 4y$ and line $y = 4$ (in square units).

Sol. (i) Given equation of parabola is $x^2 = 4y$ and equation of straight line $y = 4$. ...(i)

$\therefore$ From (i), we get

$$x^2 = 4 \times 4 = 16 \quad \Rightarrow x = \pm 4$$

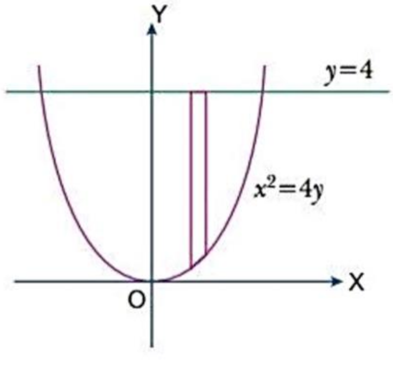
$\therefore$ Point of intersection are (4, 4) and (–4, 4).

(ii) Area of each (one) vertical strip

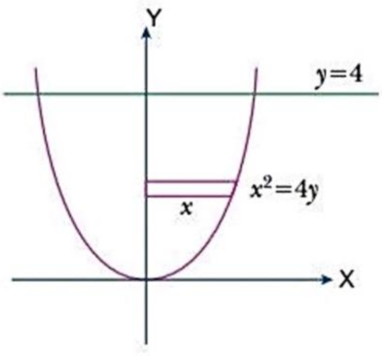
$$= y.\,dx$$

$$= 4\,dx - \frac{x^2}{4}\,dx$$

$$= \left(4 - \frac{x^2}{4}\right).\,dx$$



(iii) (a) We have,

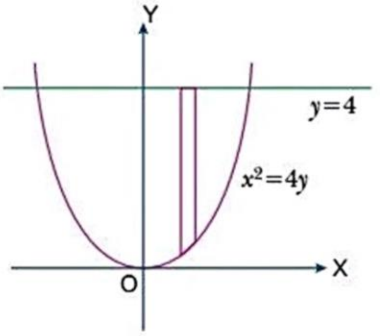


$$x^2 = 4y \qquad ...(i)$$

$$\Rightarrow \quad x = 2\sqrt{y}$$

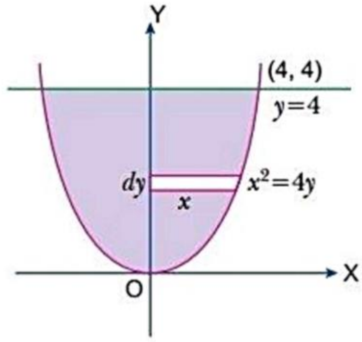
$\therefore$ Length of the horizontal strip be $2 \times 2\sqrt{y} = 4\sqrt{y}$.

(*b*) We have
Length of the vertical strip = $4 - \frac{x^2}{4} = \frac{1}{4}(16 - x^2)$



OR

(*iii*) We have
Area of required bounded region



$$= 2\int_0^4 x\, dy$$

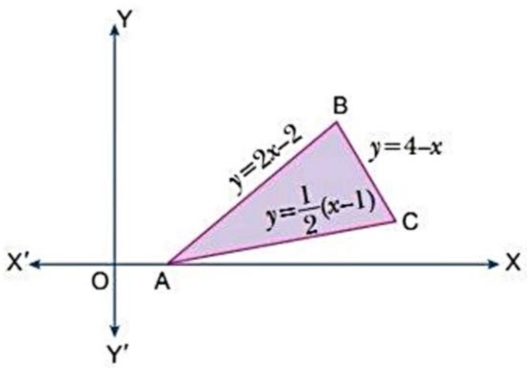
$$= 2\int_0^4 2\sqrt{y}\, dy$$

$$= 4\left[\frac{2}{3} y^{\frac{3}{2}}\right]_0^4 = \frac{8}{3}\left[(4)^{\frac{3}{2}} - 0\right] = \frac{64}{3} \text{ sq units}$$

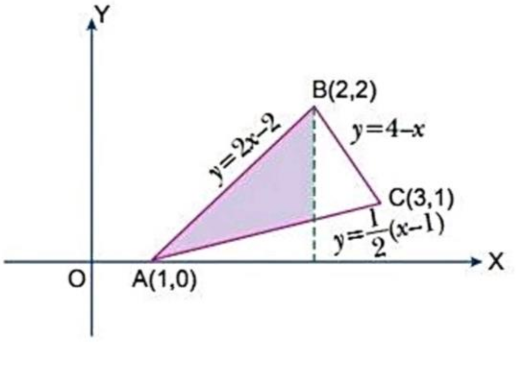
3. Read the following passage and answer the following questions.

A farmer has a triangular shaped field. His son, a science student observes the triangular field has three edges and can be drawn on a plain paper with three lines given by its equations.

(*i*) Find the area of the shaded region in the figure shown below.



(*ii*) Find the area of the triangle *ABC* given below.



Sol. We are given sides

$$y = 2x - 2 \quad ...(i)$$

$$y = \frac{1}{2}(x-1) \quad ...(ii)$$

and $y = 4 - x \quad ...(iii)$

From (*i*) and (*ii*), we get

$$2x - 2 = \frac{1}{2}(x-1)$$

$\Rightarrow \quad 4x - 4 = x - 1$

$\Rightarrow \quad 3x = 3 \quad \Rightarrow x = 1$

Using $x = 1$ in (*i*), we get

$$y = 2 \times 1 - 2 = 2 - 2 = 0$$

$\therefore \quad A \equiv (1, 0)$

From (*i*) and (*iii*), we get

$$2x - 2 = 4 - x$$

$\Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$

Using $x = 2$ in (*iii*), we get

$$y = 4 - 2 = 2$$

$\therefore \quad B \equiv (2, 2)$

From (*ii*) and (*iii*), we get

$$4 - x = \frac{1}{2}(x-1)$$

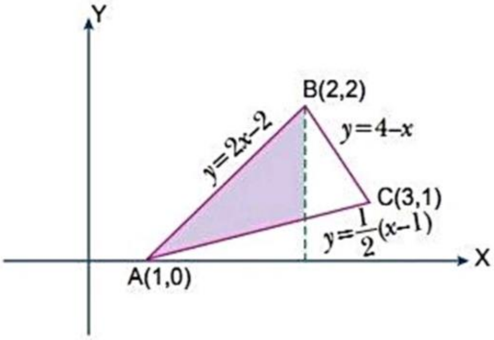
$\Rightarrow \quad 8 - 2x = x - 1$

$\Rightarrow \quad 9 = 3x \quad \Rightarrow \quad x = 3$

$\Rightarrow \quad y = 4 - x = 4 - 3 = 1$

$\therefore \quad C \equiv (3, 1)$

(*i*) We have area of shaded region



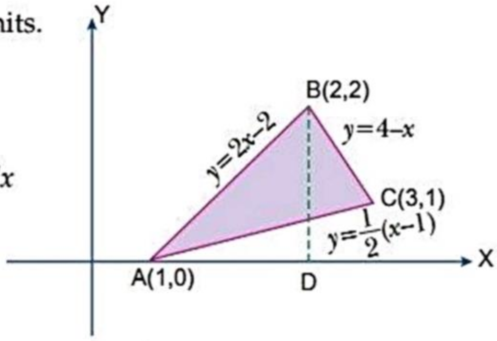
$$= \int_1^2 \left\{(2x-2) - \frac{1}{2}(x-1)\right\} dx$$

$$= \int_1^2 \left(\frac{3}{2}x - \frac{3}{2}\right) dx$$

$$= \frac{3}{2}\int_1^2 (x-1)\, dx = \frac{3}{2}\left[\frac{x^2}{2} - x\right]_1^2$$

$$= \frac{3}{2}\left[\frac{4}{2} - 2 - \frac{1}{2} + 1\right] = \frac{3}{2}\left(\frac{3}{2} - 1\right) = \frac{3}{4} \text{ sq. units.}$$

(*ii*) We have to find area of the triangle *ABC*.



$$= \int_1^2 (2x-2)\, dx + \int_2^3 (4-x)\, dx - \int_1^3 \frac{1}{2}(x-1)\, dx$$

$$= 2\left[\frac{x^2}{2} - x\right]_1^2 + \left[4x - \frac{x^2}{2}\right]_2^3 - \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^3$$

$$= 2\left[2 - 2 - \frac{1}{2} + 1\right] + \left[12 - \frac{9}{2} - 8 + 2\right] - \frac{1}{2}\left[\frac{9}{2} - 3 - \frac{1}{2} + 1\right] = \frac{3}{2} \text{ sq. units.}$$

Very Short Answer Questions

CONSTRUCTED RESPONSE QUESTIONS — AS PER NEP

1. Find the area bounded by $y = x^2$, x – axis and lines $x = -1$ and $x = 1$.

[*CBSE Sample Paper 2021*]

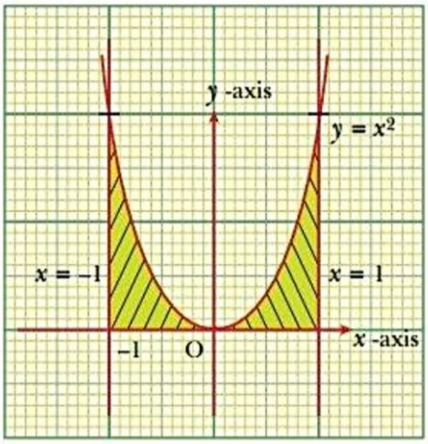
Sol. Area of required region

$$= \int_{-1}^{1} x^2\, dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{1}{3} + \frac{1}{3}$$

$$= \frac{2}{3} \text{ sq. unit}$$



2. Sketch the region $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X-axis. Find the area of the region using integration.

[*NCERT Exemplar*]

Sol. Given region is $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X-axis.

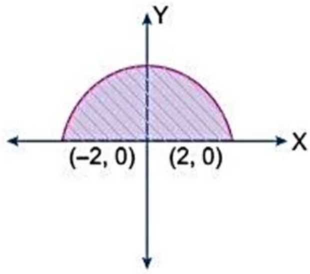
We have, $y = \sqrt{4 - x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$

$\therefore$ Area of shaded region, $A = \int_{-2}^{2} \sqrt{4 - x^2}\, dx = \int_{-2}^{2} \sqrt{2^2 - x^2}\, dx$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\cdot \sin^{-1}\frac{x}{2}\right]_{-2}^{2}$$

$$= \frac{2}{2}\cdot 0 + 2\cdot\frac{\pi}{2} + \frac{2}{2}\cdot 0 - 2\sin^{-1}(-1)$$

$$= 2\cdot\frac{\pi}{2} + 2\cdot\frac{\pi}{2} = 2\pi \text{ sq units.}$$



3. Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$.

[*NCERT Exemplar*]

Sol. Given equation of the curve is $y = \sqrt{a^2 - x^2}$.

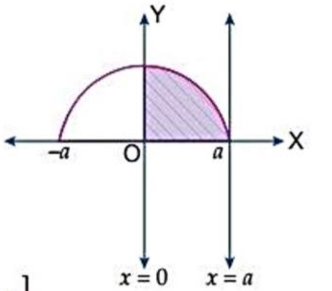
$\Rightarrow$ $y^2 = a^2 - x^2 \Rightarrow y^2 + x^2 = a^2$

$\therefore$ Required area of shaded region, $A = \int_{0}^{a} \sqrt{a^2 - x^2}\, dx$

$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{0}^{a}$$

$$= \left[0 + \frac{a^2}{2}\sin^{-1}(1) - 0 - \frac{a^2}{2}\sin^{-1}0\right]$$

$$= \frac{a^2}{2}\cdot\frac{\pi}{2} = \frac{\pi a^2}{4} \text{ sq units.}$$



4. **Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.** [*NCERT Exemplar*]

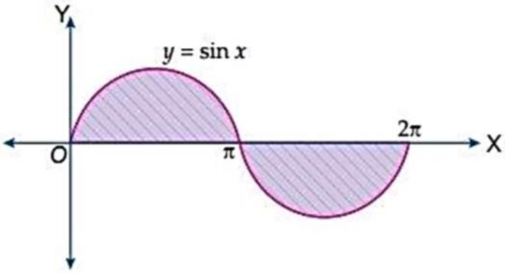
Sol. Required area $= \int_0^{2\pi} \sin x\,dx = \int_0^{\pi} \sin x\,dx + \left|\int_{\pi}^{2\pi} \sin x\,dx\right|$

$= -[\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$

$= -[\cos\pi - \cos 0] + |-[\cos 2\pi - \cos\pi]|$

$= -[-1-1] + |-(1+1)|$

$= 2 + 2 = 4$ sq units.



5. **Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.** [*CBSE 2023 (65/5/1)*]

Sol. We are given lines

$2x + y = 8$ ...(*i*)

$y = 2$ ...(*ii*)

and $y = 4$ ...(*iii*)

Form (*i*) and (*ii*), we have

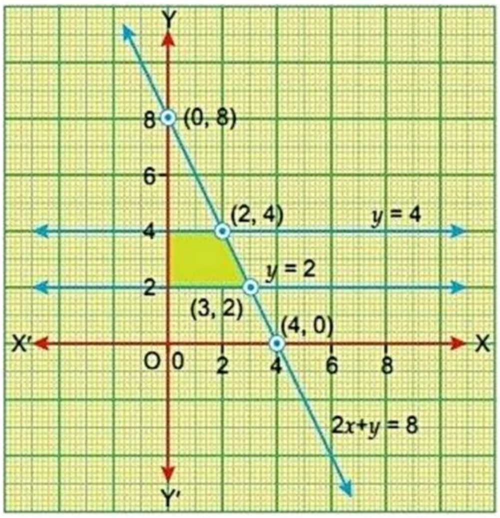
$2x + 2 = 8 \Rightarrow 2x = 6 \Rightarrow x = 3$

$\therefore$ Point of intersection of lines (*i*) and (*ii*) is (3, 2).

From (*i*) and (*iii*), we have

$2x + 4 = 8 \Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore$ Point of intersection of lines (*i*) and (*iii*) is (2, 4).



$\therefore$ Required area $= \int_2^4 x\,dy = \int_2^4 \frac{(8-y)}{2}dy$

$= \frac{1}{2}\left[\int_2^4 8\,dy - \int_2^4 y\,dy\right] = \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4$

$= \frac{1}{2}\left[8(4-2) - \frac{1}{2}(4^2 - 2^2)\right] = \frac{1}{2}\left[16 - \frac{1}{2}(12)\right]$

$= \frac{1}{2}[16 - 6] = \frac{1}{2} \times 10 = 5$ sq. units.

6. **Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.** [*CBSE 2021-22 (Term-2)*]

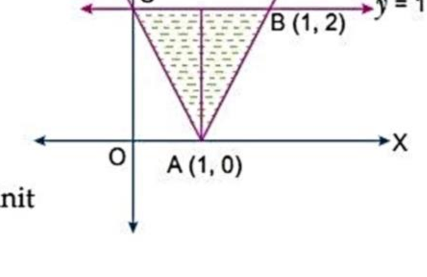
Sol. Given curves, $y = |x-1| = \begin{cases} 1-x, & x \le 1 \\ x-1, & x > 1 \end{cases}$

and, $y = 1$.

$\therefore$ Area of required (shaded) region is

$A = \int_0^1 (1-x)dx + \int_1^2 (x-1)dx$

$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2 = 1 - \frac{1}{2} + \frac{4}{2} - 2 - \frac{1}{2} + 1 = 1$ sq. unit



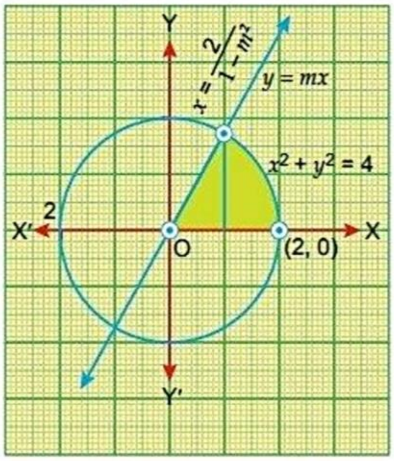
Long Answer Questions

1. **The area of the region bounded by the line $y = mx$ ($m > 0$), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.** **[CBSE 2023 (65/3/2)]**

Sol. Given line $y = mx, m > 0$ ...(i)

the curve, $x^2 + y^2 = 4$ ...(ii)

and x-axis *i.e.*, $y = 0$ ...(iii)



On plotting lines (i), (ii), and (iii), we have

On solving $y = mx$ and $x^2 + y^2 = 4$, we have

$$x^2 + m^2x^2 = 4$$

$$\Rightarrow \quad x^2 = \frac{4}{1+m^2}$$

$$x = \frac{2}{\sqrt{1+m^2}}$$

$$\therefore \text{ Area of shaded region} = \int_0^{2/\sqrt{1+m^2}} mx\,dx + \int_{2/\sqrt{1+m^2}}^{2} \sqrt{4-x^2}\,dx$$

$$\Rightarrow \quad \frac{\pi}{2} = m\left[\frac{x^2}{2}\right]_0^{2/\sqrt{1+m^2}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{2}{\sqrt{1+m^2}}}^{2}$$

$$\Rightarrow \quad \frac{\pi}{2} = m \times \frac{4}{2(1+m^2)} + 2\sin^{-1}(1) - \frac{1}{\sqrt{1+m^2}}\sqrt{4 - \frac{4}{1+m^2}} - 2\sin^{-1}\frac{1}{\sqrt{1+m^2}}$$

$$\Rightarrow \quad \frac{\pi}{2} = \frac{2m}{1+m^2} - \frac{2m}{1+m^2} + 2 \times \frac{\pi}{2} - 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right)$$

$$\Rightarrow \quad \frac{\pi}{2} = \pi - 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) \quad \Rightarrow \quad 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{4} \quad \Rightarrow \quad \frac{1}{\sqrt{1+m^2}} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 1 + m^2 = 2 \quad \Rightarrow \quad m^2 = 1 \quad \Rightarrow \quad m = 1 \qquad (\text{As } m > 0)$$

2. **Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.** **[CBSE 2023 (65/1/1)]**

Sol. Given curves,

$$x^2 = y \quad ...(i)$$

$$y = x + 2 \quad ...(ii)$$

and x-axis *i.e.* $y = 0$

On plotting these two curves, we have the region (shaded).

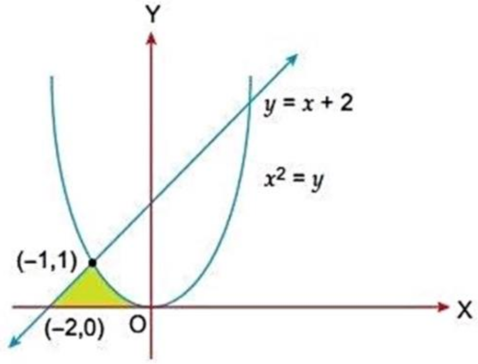
Points of intersection of the curves (i) and (ii), we get

$$x^2 = x + 2 \;\Rightarrow\; x^2 - x - 2 = 0 \;\Rightarrow x^2 - 2x + x - 2 = 0$$

$$\Rightarrow x(x-2) + 1\,(x-2) = 0$$

$$\Rightarrow (x+1)\,(x-2) = 0$$

$$\Rightarrow x = -1, 2$$



$\therefore$ Area of the shaded region $= \int_{-2}^{-1}(x+2)dx + \int_{-1}^{0}x^2dx = \left[\frac{x^2}{2}+2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$

$= \frac{(-1)^2}{2} + 2\times(-1) - \frac{(-2)^2}{2} - 2\times(-2) + \frac{0}{3} - \frac{(-1)^3}{3}$

$= \frac{1}{2} - 2 - \frac{4}{2} + 4 + \frac{1}{3} = \frac{1}{2} - 4 + 4 + \frac{1}{3} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ sq. unit

3. Using integration, find the area of region bounded by line $y = \sqrt{3}x$, the curve $y = \sqrt{4-x^2}$ and y-axis in first quadrant. **[*CBSE 2023 (65/2/1)*]**

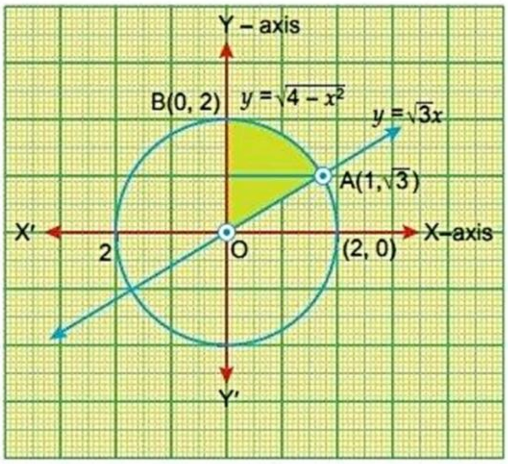
Sol. Given equation of curves

$y = \sqrt{3}x$ ...(*i*)

$y = \sqrt{4-x^2}$ ...(*ii*)

and, y-axis *i.e.* $x = 0$ ...(*iii*)

on plotting equation (*i*), (*ii*) and (*iii*), we have



Required region is shaded region *OABO*.

Point of intersection of $y = \sqrt{3}x$ and $y = \sqrt{4-x^2}$ is $A(1, \sqrt{3})$.

$\therefore$ Area of shaded region $= \int_{0}^{\sqrt{3}} \frac{y}{\sqrt{3}}dy + \int_{\sqrt{3}}^{2}\sqrt{4-y^2}\,dy$

$= \frac{1}{\sqrt{3}}\left[\frac{y^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{y}{2}\sqrt{4-y^2} + \frac{4}{2}\sin^{-1}\frac{y}{2}\right]_{\sqrt{3}}^{2}$

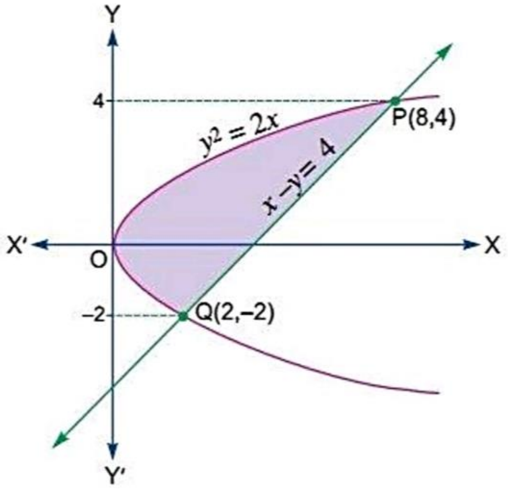
$= \frac{1}{\sqrt{3}}\left[\frac{(\sqrt{3})^2}{2} - 0\right] + \left[\frac{2}{2}\times 0 + 2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\frac{\sqrt{3}}{2}\right]$

$= \frac{1}{\sqrt{3}}\times\frac{3}{2} + 2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3}$ sq. units

4. **Find the area of the following region using integration:**
$\{(x, y): y^2 \le 2x \text{ and } y \ge x - 4\}$ [*CBSE 2023 (65/5/1)*]

Sol. We are given region

$$\{(x, y) \mid y^2 \le 2x \text{ and } y \ge x - 4\}$$
$$= \{(x, y) \mid y^2 \le 2x\} \cap \{(x, y) \mid y \ge x - 4\}$$



Given curves are $y^2 = 2x$ ...(*i*)

and $x - y = 4$ ...(*ii*)

Obviously, curve (*i*) is right handed parabola having vertex at (0, 0) and axis along +ve direction of x-axis while curve (*ii*) is a straight line.

For intersection point of curve (*i*) and (*ii*), we get

$$(x-4)^2 = 2x$$
$\Rightarrow$ $x^2 - 8x + 16 = 2x$
$\Rightarrow$ $x^2 - 8x - 2x + 16 = 0$ $\Rightarrow$ $x(x-8) - 2(x-8) = 0$
$\Rightarrow$ $(x-8)(x-2) = 0$ $\Rightarrow$ $x = 2, 8$
$\Rightarrow$ $y = -2, 4$

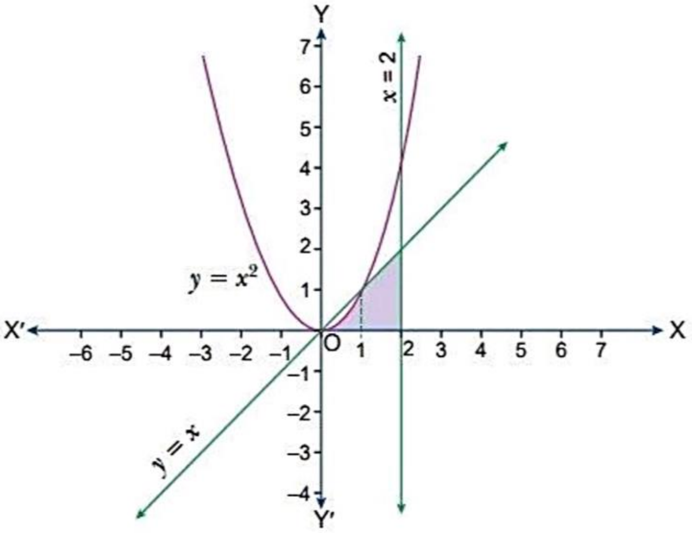
Intersection points are (2, −2), (8, 4).

Therefore, required area = area of shaded region

$$= \int_{-2}^{4} (y+4)\, dy - \int_{-2}^{4} \frac{y^2}{2}\, dy = \left[\frac{(y+4)^2}{2}\right]_{-2}^{4} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-2}^{4}$$
$$= \frac{1}{2}.[64-4] - \frac{1}{6}[64+8] = 30 - \frac{72}{6} = 18 \text{ sq units.}$$

5. **Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$ and find the area of the region using integration.**

Sol.



The points of intersection of the parabola $y = x^2$ and the line $y = x$ are (0, 0) and (1, 1).

Required area $= \int_0^1 y_{\text{parabola}}\, dx + \int_1^2 y_{\text{line}}\, dx$

$$= \int_0^1 x^2 dx + \int_1^2 x dx = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6} \text{ sq units}$$

6. Find the area of the region bounded by the curves $x^2+y^2=4$, $y=\sqrt{3}x$ and x – axis in the first quadrant. **[*CBSE Sample Paper 2021*]**

Sol. Solving $y=\sqrt{3}x$ and $x^2+y^2=4$, we get $x^2+3x^2=4$

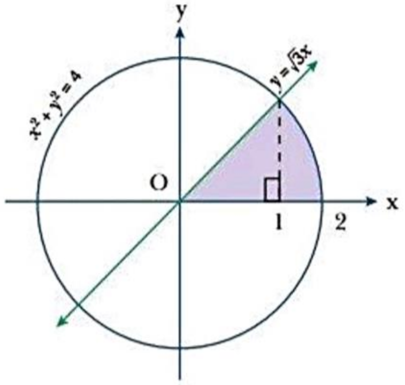
$\Rightarrow \quad x^2=1 \quad \Rightarrow \quad x=1$

Required area $=\sqrt{3}\int_0^1 x\,dx+\int_1^2\sqrt{2^2-x^2}\,dx$

$=\dfrac{\sqrt{3}}{2}[x^2]_0^1+\left[\dfrac{x}{2}\sqrt{2^2-x^2}+2\sin^{-1}\left(\dfrac{x}{2}\right)\right]_1^2$

$=\dfrac{\sqrt{3}}{2}+\left[2\times\dfrac{\pi}{2}-\dfrac{\sqrt{3}}{2}-2\times\dfrac{\pi}{6}\right]$

$=\dfrac{2\pi}{3}$ square units.



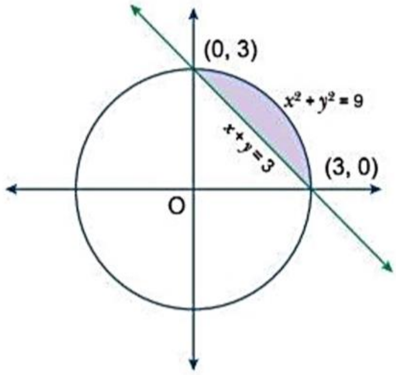
7. Using integration, find the area of the region $\{(x,y): x^2+y^2\le 9, x+y\ge 3\}$. **[*CBSE 2020 (65/1/2)*]**

Sol. Given region $\{(x,y): x^2+y^2\le 9, x+y\ge 3$

We have equation of the curve $x^2+y^2=9$ which is a circle with centre (0, 0) and radius 3 units and $x+y=3$, a straight line.

On plotting we have the required region as the shaded region.

$\therefore$ Area of the shaded (required) region



$=\int_0^3\left[\sqrt{9-x^2}-(3-x)\right]dx$

$=\int_0^3\sqrt{9-x^2}\,dx-\int_0^3(3-x)\,dx$

$=\int_0^3\sqrt{3^2-x^2}\,dx-\left[3x-\dfrac{x^2}{2}\right]_0^3=\left[\dfrac{x}{2}\sqrt{9-x^2}+\dfrac{9}{2}\sin^{-1}\left(\dfrac{x}{3}\right)\right]_0^3-\left[3\times3-\dfrac{9}{2}-0\right]$

$=\left(0+\dfrac{9}{2}\sin^{-1}(1)-0\right)-\left(9-\dfrac{9}{2}\right)=\dfrac{9\pi}{4}-\dfrac{9}{2}=\dfrac{9}{4}(\pi-2)$ sq. units.

8. Using integration, find the area of the region bounded by the triangle whose vertices are (2, –2), (4, 5) and (6, 2). **[*CBSE 2020 (65/1/1)*]**

Sol. Let given vertices of the triangle be $A(2,-2)$, $B(4,5)$ and $C(6,2)$.

On plotting these points, we get the ΔABC.

Now, equation of line AB be

$\dfrac{y-5}{x-4}=\dfrac{7}{2}$

$\Rightarrow \quad y-5=\dfrac{7}{2}(x-4)=\dfrac{7}{2}x-14$

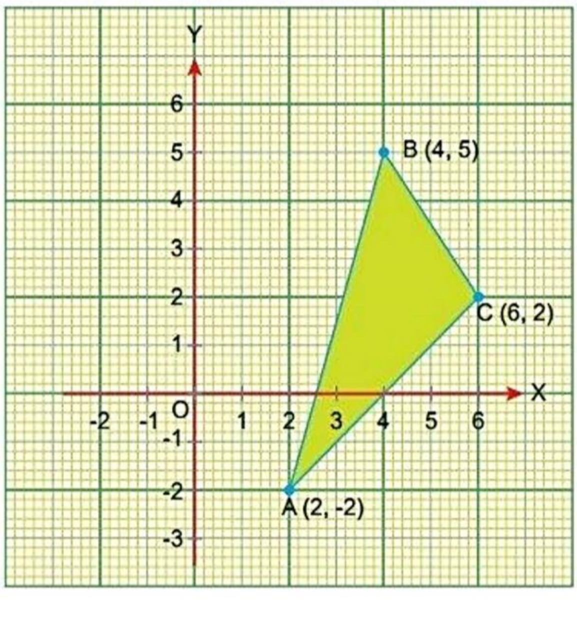
$\Rightarrow \quad y=\dfrac{7}{2}x-9$

$\therefore \quad y_{AB}=\dfrac{7}{2}x-9$

Equation of BC be

$\dfrac{y-5}{x-4}=\dfrac{3}{-2}$

$\Rightarrow \quad y-5=-\dfrac{3}{2}(x-4)=-\dfrac{3}{2}x+6$



$\Rightarrow \quad y_{BC} = -\frac{3}{2}x + 11$

Also, equation of AC be

$$\frac{y+2}{x-2} = \frac{4}{4} = 1 \Rightarrow \quad y + 2 = x - 2 \quad \Rightarrow \quad y = x - 4$$

$\therefore \quad y_{AC} = x - 4$

$\therefore$ Area of required triangle (shaded region ΔABC)

$$= \int_2^4 y_{AB}dx + \int_4^6 y_{BC}dx - \int_2^6 y_{AC}dx$$

$$= \int_2^4 \left(\frac{7}{2}x - 9\right)dx + \int_4^6 \left(-\frac{3}{2}x + 11\right)dx - \int_2^6 (x-4)\,dx$$

$$= \left[\frac{7}{4}x^2 - 9x\right]_2^4 + \left[-\frac{3}{4}x^2 + 11x\right]_4^6 - \left[\frac{x^2}{2} - 4x\right]_2^6$$

$$= \frac{7}{4}\times(4^2 - 2^2) - 9(4-2) - \frac{3}{4}\times(6^2 - 4^2) + 11\times(6-4) - \frac{(6^2 - 2^2)}{2} + 4(6-2)$$

$$= \frac{7}{4}\times 12 - 18 - \frac{3}{4}\times 20 + 22 - 16 + 16$$

$= 21 - 18 - 15 + 22 = 43 - 33 = 10$ sq. units.

9. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$. [*CBSE Delhi* 2012, 2019 (65/4/1)]

Sol. Given lines are

$3x - 2y + 1 = 0$ ...(*i*)

$2x + 3y - 21 = 0$ ...(*ii*)

$x - 5y + 9 = 0$ ...(*iii*)

For intersection of (*i*) and (*ii*) applying (*i*) × 3 + (*ii*) × 2, we get

$9x - 6y + 3 + 4x + 6y - 42 = 0$

$\Rightarrow \quad 13x - 39 = 0 \quad \Rightarrow \quad x = 3$

Putting it in (*i*), we get

$9 - 2y + 1 = 0$

$\Rightarrow \quad 2y = 10 \quad \Rightarrow \quad y = 5$

Intersection point of (*i*) and (*ii*) is (3, 5).

For intersection of (*ii*) and (*iii*) applying (*ii*) – (*iii*) × 2, we get

$2x + 3y - 21 - 2x + 10y - 18 = 0$

$\Rightarrow \quad 13y - 39 = 0 \quad \Rightarrow \quad y = 3$

Putting $y = 3$ in (*ii*), we get

$2x + 9 - 21 = 0$

$\Rightarrow \quad 2x - 12 = 0 \quad \Rightarrow \quad x = 6$

Intersection point of (*ii*) and (*iii*) is (6, 3).

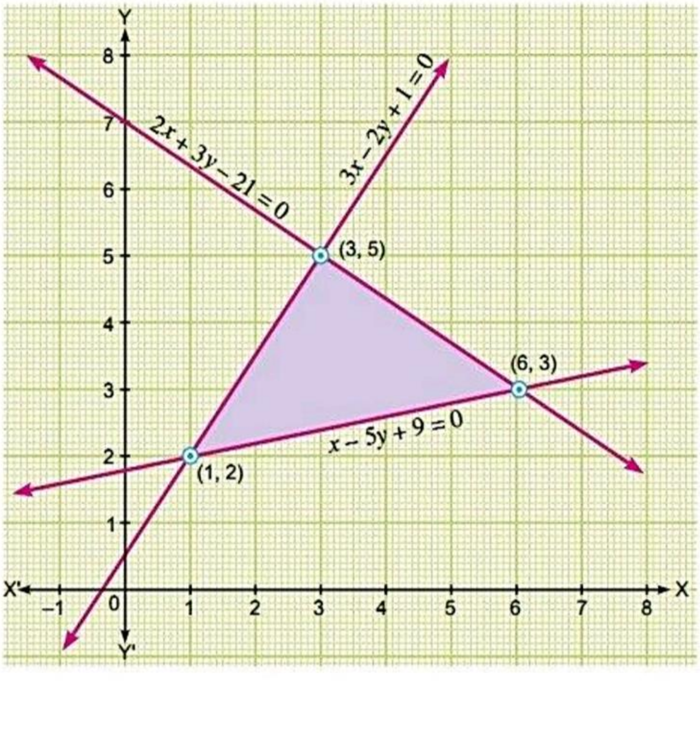
For intersection of (*i*) and (*iii*) applying (*i*) – (*iii*) × 3, we get

$3x - 2y + 1 - 3x + 15y - 27 = 0$

$\Rightarrow \quad 13y - 26 = 0 \quad \Rightarrow \quad y = 2$

Putting $y = 2$ in (*i*), we get

$3x - 4 + 1 = 0 \quad \Rightarrow \quad x = 1$



Intersection point of (*i*) and (*iii*) is (1, 2).

With the help of point of intersection we draw the graph of lines (*i*), (*ii*) and (*iii*).

Shaded region is required region.

$$\therefore \text{ Area of required region } = \int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{-2x+21}{3}dx - \int_1^6 \frac{x+9}{5}dx$$

$$= \frac{3}{2}\int_1^3 x\,dx + \frac{1}{2}\int_1^3 dx - \frac{2}{3}\int_3^6 x\,dx + 7\int_3^6 dx - \frac{1}{5}\int_1^6 x\,dx - \frac{9}{5}\int_1^6 dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}\right]_1^3 + \frac{1}{2}[x]_1^3 - \frac{2}{3}\left[\frac{x^2}{2}\right]_3^6 + 7[x]_3^6 - \frac{1}{5}\left[\frac{x^2}{2}\right]_1^6 - \frac{9}{5}[x]_1^6$$

$$= \frac{3}{4}(9-1) + \frac{1}{2}(3-1) - \frac{2}{6}(36-9) + 7(6-3) - \frac{1}{10}(36-1) - \frac{9}{5}(6-1)$$

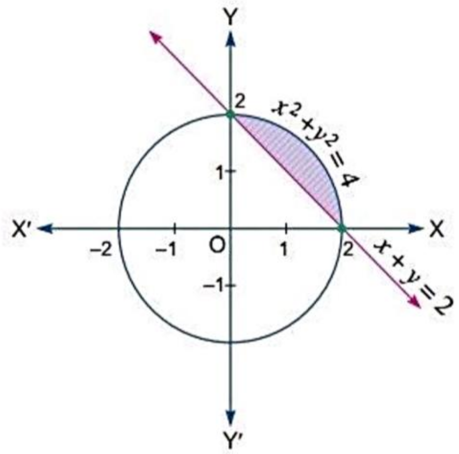
$$= 6+1-9+21-\frac{7}{2}-9 = 10-\frac{7}{2} = \frac{20-7}{2} = \frac{13}{2} \text{ sq units.}$$

10. Find the area of the region $\{(x, y) : x^2 + y^2 \le 4, x + y \ge 2\}$. **[*CBSE (AI)* 2012]**

Sol. Let $R = \{(x, y) : x^2 + y^2 \le 4, x + y \ge 2\}$

$\Rightarrow \quad R = \{(x, y) : x^2 + y^2 \le 4\} \cap \{(x, y) : x + y \ge 2\}$

i.e., $R = R_1 \cap R_2$, where $R_1 = \{(x, y) : x^2 + y^2 \le 4\}$ and $R_2 = \{(x, y) : x + y \ge 2\}$



For region R_1

Obviously $x^2 + y^2 = 4$ is a circle having centre at (0,0) and radius 2.

Since (0,0) satisfy $x^2 + y^2 \le 4$. Therefore region R_1 is the region lying interior of circle $x^2 + y^2 = 4$.

For region R_2

x	0	2
y	2	0

$x + y = 2$ is a straight line passing through (0, 2) and (2, 0). Since (0, 0) does not satisfy $x + y \ge 2$ therefore R_2 is that region which does not contain origin (0, 0) *i.e.*, above the line $x + y = 2$.

Hence, shaded region is required area.

Now, area of required region

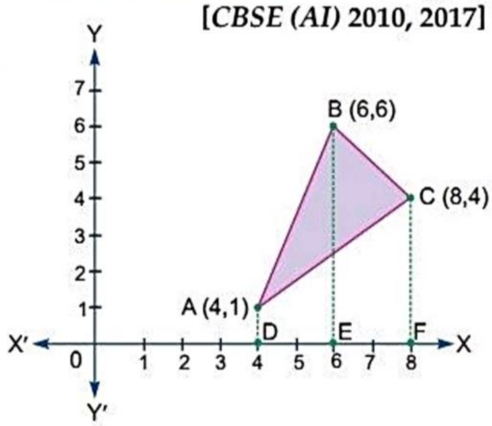
$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx = \left[\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}4\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 - 2[x]_0^2 + \left[\frac{x^2}{2}\right]_0^2$$

$$= [2\sin^{-1}1 - 0] - 2[2-0] + \left[\frac{4}{2} - 0\right]$$

$$= 2 \times \frac{\pi}{2} - 4 + 2 = (\pi - 2) \text{ sq units.}$$

11. Using integration, find the area of the triangle *ABC*, co-ordinates of whose vertices are *A*(4, 1), *B*(6, 6) and *C*(8, 4). **[*CBSE (AI)* 2010, 2017]**

Sol. Given triangle *ABC*, coordinates of whose vertices are *A*(4, 1), *B*(6, 6) and *C*(8, 4).



Equation of *AB* is given by

$$y - 6 = \frac{6-1}{6-4}(x-6) \text{ or } y = \frac{5}{2}x - 9$$

Equation of *BC* is given by

$$y - 4 = \frac{4-6}{8-6}(x-8) \text{ or } y = -x + 12$$

Equation of AC is given by

$$y-4=\frac{4-1}{8-4}(x-8) \text{ or } y=\frac{3}{4}x-2$$

$\therefore$ Area of ΔABC = area of trap. $DABE$ + area of trap. $EBCF$ – area of trap. $DACF$

$$= \int_4^6\left(\frac{5}{2}x-9\right)dx+\int_6^8(-x+12)\,dx-\int_4^8\left(\frac{3}{4}x-2\right)dx$$

$$= \frac{5}{2}\left[\frac{x^2}{2}\right]_4^6-9[x]_4^6-\left[\frac{x^2}{2}\right]_6^8+12[x]_6^8-\frac{3}{4}\left[\frac{x^2}{2}\right]_4^8+2[x]_4^8$$

$$= \frac{5}{4}(36-16)-9(6-4)-\frac{1}{2}(64-36)+12(8-6)-\frac{3}{8}(64-16)+8$$

$$= \frac{5}{4}\times 20-18-\frac{28}{2}+24-\frac{3}{8}\times 48+8$$

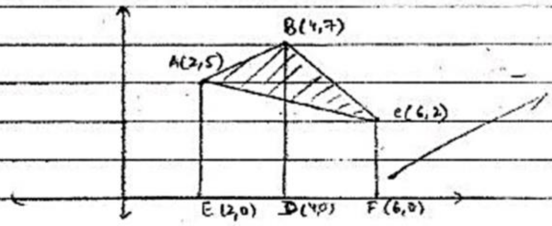
$$= 25-18-14+24-18+8 = 7 \text{ sq units.}$$

12. Using integration, find the area of triangle ABC, whose vertices are $A(2, 5)$, $B(4, 7)$ and $C(6, 2)$.

[*CBSE 2019 (65/1/1)*]

Sol.

Q28



Eq^n of general line:

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

Eq^n of line AB :

$(y-5)=\frac{2}{2}(x-2)$

$y_1 = x+3$

Eq^n of line BC:

$(y-2)=\frac{5}{-2}(x-6)$

$\Rightarrow y-2=-\frac{5}{2}x+15$

$\Rightarrow y_2=\frac{-5x+17}{2}$

Eq^n of line AC

$(y-5)=\frac{3}{-4}(x-2)$

$y-5=-\frac{3}{4}x+\frac{3}{2}$

$y_3=\frac{-3x}{4}+\frac{13}{2}$

Area of shaded region is required area A = ar(ABDE) + ar(BCFD) – ar(ACFE)

$A=\int_2^4 y_1\,dx+\int_4^6 y_2\,dx-\int_2^6 y_3\,dx$

$=\int_2^4(x+3)\,dx+\int_4^6\left(\frac{-5x+17}{2}\right)dx-\int_2^6\left(\frac{-3x}{4}+\frac{13}{2}\right)dx$

$A=\left[\frac{x^2}{2}+3x\right]_2^4+\left[17x-\frac{5x^2}{4}\right]_4^6-\left[\frac{13x}{2}-\frac{3x^2}{8}\right]_2^6$

$=\left[8+12-2-6\right]+\left[102-45-68+20\right]-\left[39-\frac{27}{2}-13+\frac{3}{2}\right]$

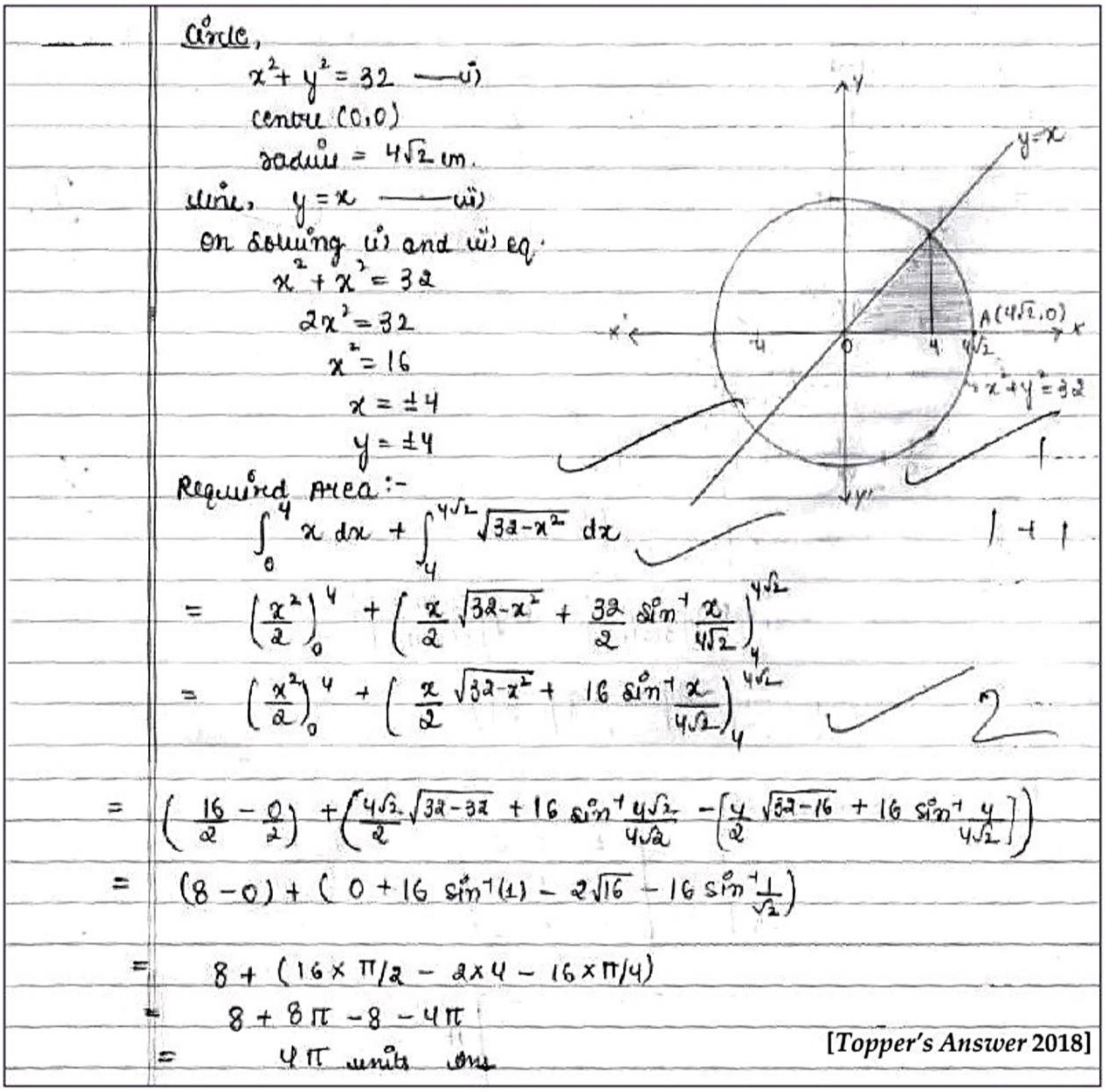
$= 12+9-14$

$A = 7$ sq.units

[*Topper's Answer* 2019]

13. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. **[*CBSE Delhi* 2014, *2018*, *CBSE* 2020 (65/4/1)]**

Sol.



Circle,
$x^2 + y^2 = 32$ —— (i)
centre (0, 0)
radius = $4\sqrt{2}$ cm.
Line, $y = x$ —— (ii)
On solving (i) and (ii) eq.
$x^2 + x^2 = 32$
$2x^2 = 32$
$x^2 = 16$
$x = \pm 4$
$y = \pm 4$
Required Area :-
$\int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$
$= \left(\frac{x^2}{2}\right)_0^4 + \left(\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right)_4^{4\sqrt{2}}$
$= \left(\frac{x^2}{2}\right)_0^4 + \left(\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right)_4^{4\sqrt{2}}$
$= \left(\frac{16}{2} - \frac{0}{2}\right) + \left(\frac{4\sqrt{2}}{2}\sqrt{32 - 32} + 16\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}} - \left[\frac{4}{2}\sqrt{32 - 16} + 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]\right)$
$= (8 - 0) + \left(0 + 16\sin^{-1}(1) - 2\sqrt{16} - 16\sin^{-1}\frac{1}{\sqrt{2}}\right)$
$= 8 + (16 \times \pi/2 - 2 \times 4 - 16 \times \pi/4)$
$= 8 + 8\pi - 8 - 4\pi$
$= 4\pi$ units Ans

[*Topper's Answer* 2018]

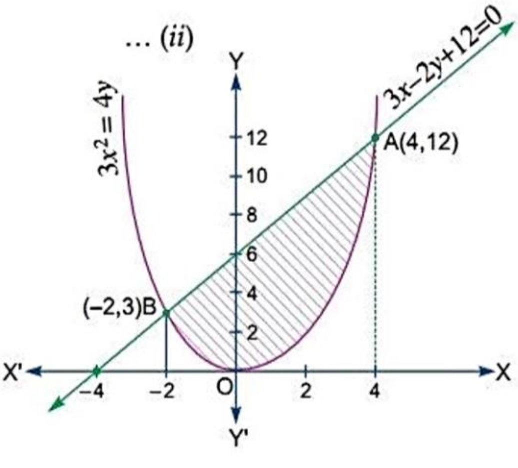
14. Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$. **[*CBSE (AI)* 2009, *(F)* 2013]**

Sol. Given equation of parabola $4y = 3x^2 \Rightarrow y = \frac{3x^2}{4}$ … (*i*)

and the line $3x - 2y + 12 = 0 \Rightarrow \frac{3x + 12}{2} = y$ … (*ii*)

The line intersect the parabola at (–2, 3) and (4, 12).

Hence, the required area will be the shaded region.



Required area $= \int_{-2}^{4} \frac{3x + 12}{2}dx - \int_{-2}^{4} \frac{3x^2}{4}dx$

$= \left[\frac{3}{4}x^2 + 6x - \frac{x^3}{4}\right]_{-2}^{4}$

$= (12 + 24 - 16) - (3 - 12 + 2)$

$= 20 + 7 = 27$ square units.

15. Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines whose equations are $x = 2y$ and $x = 3y - 3$. **[*CBSE Sample Paper* 2016]**

Sol. Obviously $4y = x^2$ is upward parabola having vertex at origin.

Now $4y = x^2$

$$\Rightarrow \quad 4\frac{dy}{dx} = 2x \;\Rightarrow\; \frac{dy}{dx} = \frac{1}{2}x \;\Rightarrow\; \left.\frac{dy}{dx}\right]_{(2,1)} = \frac{1}{2}\times 2 = 1$$

$\Rightarrow$ Slope of tangent at (2, 1) to given curve $4y = x^2$ is 1.

Equation of tangent $= \dfrac{y-1}{x-2} = 1$

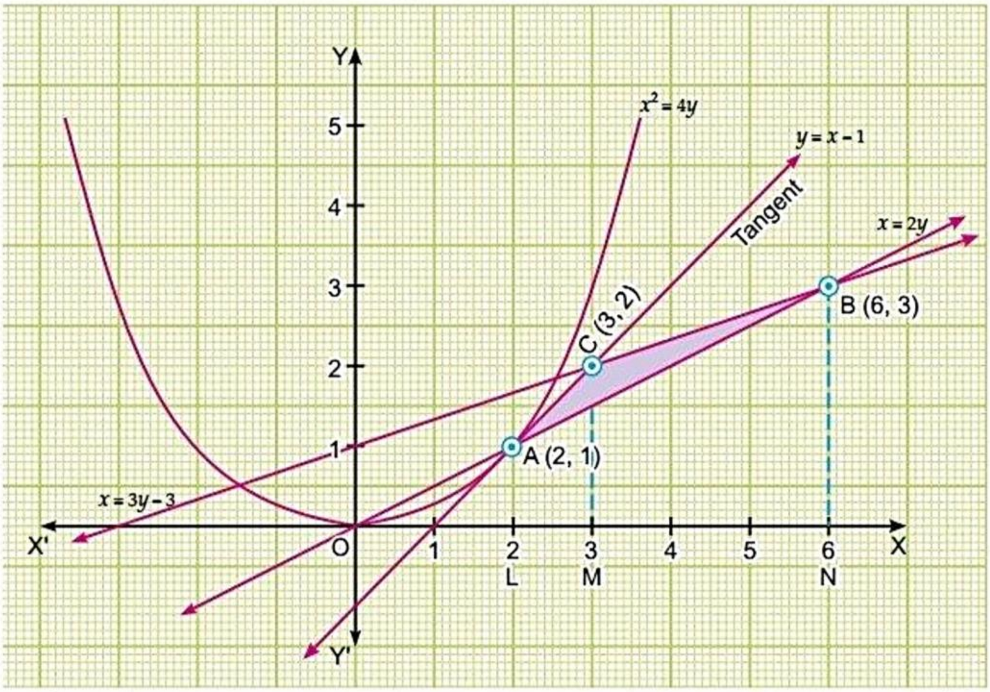
$\Rightarrow \quad y - 1 = x - 2 \quad \Rightarrow \quad y = x - 1$

Now, for graph of $x = 2y$

x	0	2
y	0	1

Also for graph of $x = 3y - 3$

x	0	3
y	1	2



After plotting the graph, we get shaded region *ABC* as required region, area of which is to be calculated.

After solving the respective equation, we get

Coordinate of $A \equiv (2, 1)$; $B \equiv (6, 3)$; $C \equiv (3, 2)$

Now, the required area = area of shaded region *ABC*

$$= ar(\text{region } ALMC) + ar(\text{region } CMNB) - ar(\text{region } ALNB)$$

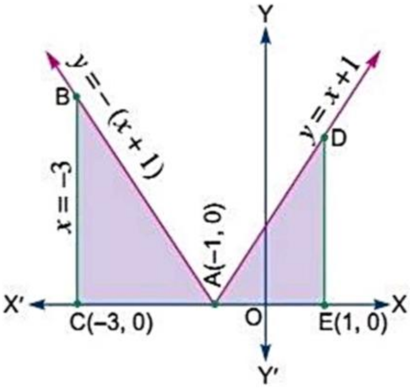
$$= \int_2^3 (x-1)\,dx + \int_3^6 \frac{x+3}{3}dx - \int_2^6 \frac{x}{2}dx = \left[\frac{x^2}{2} - x\right]_2^3 + \frac{1}{3}\left[\frac{x^2}{2} + 3x\right]_3^6 - \frac{1}{2}\left[\frac{x^2}{2}\right]_2^6$$

$$= \left[\left(\frac{9}{2} - 3\right) - \left(\frac{4}{2} - 2\right)\right] + \frac{1}{3}\left[\left(\frac{36}{2} + 18\right) - \left(\frac{9}{2} + 9\right)\right] - \frac{1}{4}(36 - 4)$$

$$= \frac{3}{2} + \frac{1}{3}\left(36 - \frac{27}{2}\right) - 8 = \frac{3}{2} + \frac{45}{6} - 8 = 1 \text{ square unit.}$$

16. Sketch the graph $y = |x+1|$. Evaluate $\int_{-3}^{1}|x+1|\,dx$. What does this value represent on the graph?

Sol. We have, $y = |x+1| = \begin{cases} x+1, \text{ if } x+1 \geq 0 \text{ i.e., } x \geq -1 \\ -(x+1), \text{ if } x+1 < 0 \text{ i.e., } x < -1 \end{cases}$



So, we have $y = x + 1$ for $x \geq -1$ and $y = -x - 1$ for $x < -1$. Clearly, $y = x + 1$ is a straight line cutting x and y-axes at $(-1, 0)$ and $(0, 1)$ respectively. So, $y = x + 1$, $x \geq -1$ represents that portion of the line which lies on the right side of $x = -1$. Similarly, $y = -x - 1$, $x < -1$ represents that part of the line $y = -x - 1$ which is on the left side of $x = -1$. A rough sketch of $y = |x+1|$ is shown in fig.

Now, $\int_{-3}^{1}|x+1|\,dx = \int_{-3}^{-1} -(x+1)\,dx + \int_{-1}^{1}(x+1)\,dx$

$$= -\left[\frac{(x+1)^2}{2}\right]_{-3}^{-1} + \left[\frac{(x+1)^2}{2}\right]_{-1}^{1} = -\left[0 - \frac{4}{2}\right] + \left[\frac{4}{2} - 0\right] = 4 \text{ sq units}$$

This value represents the area of the shaded portion shown in figure.

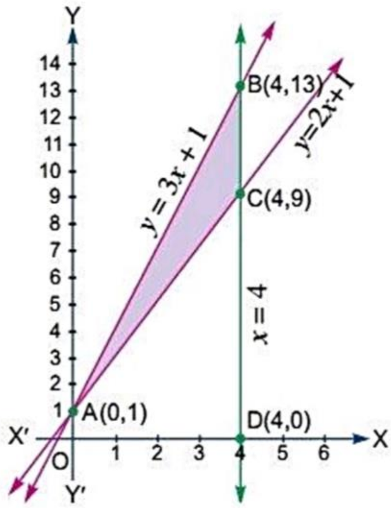
17. Using integration find the area of the triangular region whose sides have equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$. **[*CBSE Delhi 2008; 2011*]**

Sol. The given lines are

$$y = 2x + 1 \qquad \ldots(i)$$

$$y = 3x + 1 \qquad \ldots(ii)$$

$$x = 4 \qquad \ldots(iii)$$



For intersection point of (*i*) and (*iii*)

$y = 2 \times 4 + 1 = 9$

Coordinates of intersecting point of (*i*) and (*iii*) is (4, 9).

For intersection point of (*ii*) and (*iii*)

$y = 3 \times 4 + 1 = 13$

i.e., coordinates of intersection point of (*ii*) and (*iii*) is (4, 13).

For intersection point of (*i*) and (*ii*)

$2x + 1 = 3x + 1 \quad \Rightarrow \quad x = 0$

$\therefore \quad y = 1$

i.e., coordinates of intersection point of (*i*) and (*ii*) is (0, 1).

Shaded region is required triangular region.

$\therefore$ Required area = area of trapezium *OABDO* – area of trapezium *OACDO*

$$= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx = \left[3\frac{x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$$

$$= [(24 + 4) - 0] - [(16 + 4) - 0] = 28 - 20 = 8 \text{ sq units.}$$

Questions for Practice

Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) The area bounded by the curve $y = x\,|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

(*a*) 0 sq. units (*b*) $\frac{1}{3}$ sq. unit (*c*) $\frac{2}{3}$ sq. unit (*d*) $\frac{4}{3}$ sq. units

(*ii*) The area bounded by the curve $y = |\sin x|$, x-axis and ordinates $x = \pi$ and $x = 10\pi$ is equal to

(*a*) 8 sq. units (*b*) 10 sq. units (*c*) 18 sq. units (*d*) 20 sq. units

(*iii*) The area of the region bounded by the parabola $y^2 = x$ and the straight line $2y = x$ is

(*a*) $\frac{4}{3}$ sq. units (*b*) 1 sq. unit (*c*) $\frac{2}{3}$ sq. unit (*d*) $\frac{1}{3}$ sq. unit

(*iv*) The area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$ is

(*a*) $\frac{9}{2}$ sq. units (*b*) 4 sq. units (*c*) 2 sq. units (*d*) None of these

(*v*) Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(*a*) π sq. units (*b*) $\frac{\pi}{2}$ sq. units (*c*) $\frac{\pi}{3}$ sq. units (*d*) $\frac{\pi}{4}$ sq. units

Very Short Answer Questions

2. Find the area bounded by the curve $y = x^2$, $x = 2$, $x = 3$ and x-axis.
3. Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$.
4. Find the area under the curve $y = \sqrt{x-1}$ between the lines $x = 1$ and $x = 5$.

Long Answer Questions

5. Find the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$. [*NCERT Exemplar*]
6. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$. [*CBSE Delhi 2010, 2013*]
7. Using integration, find the area of the region $\{(x, y)\} : 9x^2 + y^2 \le 36$ and $3x + y \ge 6$. [*CBSE (F)* 2009]
8. Find the area of the region $\{(x, y) : x^2 \le y \le x\}$.
9. Find the area of the region bounded by the curve $y = \frac{3}{4}x^2$ and the line $3x - 2y + 12 = 0$.
10. Using integration, find the area of the triangle *ABC*, where *A* is (2, 3), *B* is (4, 7) and *C* is (6, 2).
11. Make a rough sketch of the region given below and find its area, using integration:
$\{(x, y) : 0 \le y \le x^2 + 3; 0 \le y \le 2x + 3, 0 < x \le 3\}$
12. Using integration, find the area of the triangle *ABC*, whose vertices have coordinates
A (2, 0), *B* (4, 5) and *C* (6, 3). [*CBSE (F)* 2012]

13. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$. **[*CBSE Delhi* 2010; *(F)* 2014]**

14. Using integration, find the area of the triangle formed by negative x-axis and tangent and normal to the circle $x^2+y^2=9$ at $(-1, 2\sqrt{2})$. **[*CBSE East* 2016]**

15. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq units, then using integration, find the value of m. **[*CBSE Ajmer* 2015]**

16. Using the method of integration, find the area of the triangular region whose vertices are (2, –2), (4, 3) and (1, 2). **[*CBSE (North)* 2016]**

17. Find the area of the triangle whose vertices are (–1, 1), (0, 5) and (3, 2), using integration. **[*CBSE* 2019 (65/3/1)]**

18. Using integration, find the area of the triangle whose vertices are (2, 3), (3, 5) and (4, 4). **[*CBSE* 2019 (65/1/3)]**

19. Using integration, find the area of the triangle ABC with vertices as $A(-1, 0)$, $B(1, 3)$ and $C(3, 2)$. **[*CBSE (F) 2009*]**

20. Using integration, find the area of the triangle ABC, where A is (2, 3), B is (4, 7) and C is (6, 2).

Answers

1. *(i)* *(c)* *(ii)* *(c)* *(iii)* *(a)* *(iv)* *(a)* *(v)* *(a)*
2. $\frac{19}{3}$ sq. units
3. $\frac{4}{3}$ sq. units
4. $\frac{16}{3}$ sq. units
5. $\frac{15}{2}$ sq. units
6. $\frac{9}{8}$ sq. units
7. $3(\pi-2)$ sq. units
8. $\frac{1}{6}$ sq. unit
9. 27 sq. units
10. 9 sq. units
11. $\frac{50}{3}$ sq. units
12. 7 sq. units
13. $\left(\frac{3\pi}{2}-3\right)$ sq. units
14. $9\sqrt{2}$
15. $m = 2$
16. $\frac{13}{2}$ sq. units
17. $\frac{15}{2}$ sq. units
18. $\frac{3}{2}$ sq. units
19. 4 sq. units
20. 9 sq. units