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The liquids and gases do not have their own shape and the shape of the container. Due to this reason, the liquid and gases have the ability to flow under the action of external force. Hence liquids and gases are called FLUIDS.

FLUIDS: **"The term fluid refer to a substance that flows under the action of applied force and does not have a shape of its own".**

HYDROSTATICS (Fluids statics): *"The study of fluids (liquids &gases) at rest is known as hydrostatics".*

➤The fluid in motion is termed as "***HYDRODYNAMICS".***

➤ Since fluids change their shape rapidly, therefore their mechanical behaviour cannot be described in the same manner (as in case of solids).

IDEAL FLUID: A FLUID WHICH IS IMCOMPRESSIBLE IS CALLED AND IDEAL FLUID.
This simply means that there will be no change in the volume.

In a liquid at rest, the forces exerted by the liquid are always perpendicular to the surfaces in contact with the liquid. The surface in contact with the liquid may be a solid surface (*e.g.*, bottom or walls of the containing vessel, a body immersed in the liquid) or a liquid surface.

(*i*) Liquid in contact with solid surface. Consider a liquid at rest in a vessel as shown in Fig. 19.1. Then force exerted by the liquid at each point on the bottom or walls is perpendicular to the surface at that point. To prove this fact, consider a point O at the bottom of the vessel. Suppose for a moment that force F exerted by the liquid at point O is not perpendicular to the

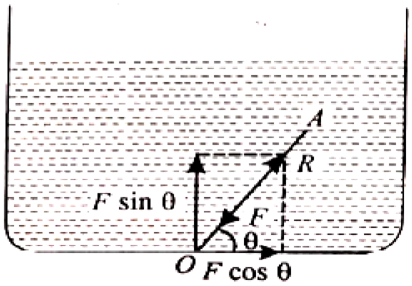


Fig. 19.1

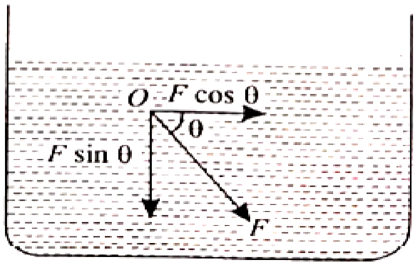


Fig. 19.2

bottom but makes an angle θ with the horizontal as shown in Fig. 19.1. According to Newton's third law, the surface exerts an equal and opposite force $R = F$. The reaction R (= F) has horizontal and vertical components $F\cos\theta$ and $F\sin\theta$ respectively. The horizontal component $F\cos\theta$ would cause the liquid to flow which contradicts our original assumption that the liquid is at rest. Therefore, $F\cos\theta$ must be zero *i.e.*, $\theta = 90°$. Similarly, the force exerted by the liquid is perpendicular to any point of the surface of the container in contact with it.

►►Therefore, **a liquid at rest always exerts a normal force on the solid surface in contact.**

(*ii*) Liquid in contact with liquid surface. Consider a liquid at rest in a vessel as shown in Fig. 19.2. Consider a point O inside the liquid. The force at point O must be perpendicular to the surface in contact with O. Suppose this were not true and force F at point O acts making an angle θ with the horizontal. The horizontal and vertical components of the force are $F\cos\theta$ and $F\sin\theta$ respectively. The horizontal component $F\cos\theta$ would cause the liquid to flow which contradicts our original assumption that the liquid is at rest. Therefore, $F\cos\theta$ must be zero *i.e.*, $\theta = 90°$.

►►Therefore, **the force acting on a liquid at rest are always perpendicular to the surfaceof the liquid.**

Fluids exerts thrust

Explanation: When a liquid (fluid) kept in a container, the molecules of the fluid is in random motion. Due to them thermal velocities, they are constantly colliding with the walls of the container and rebounding form them. they suffer a change in momentum due to which they transfer some momentum to the walls. Thus,

"Momentum transferred to the walls per unit time by the molecules of fluid accounts for the force (thrust) of the fluid on the walls of the container".

➤ **Thrust** - - - ***"The total force exerted by a liquid in any surface in contact with it is called thrust of the liquid".***

Thrust = pressure × surface area [as $P = F/A$]

PRESSURE OF THE LIQUID

While dealing with fluids, pressure exerted by the fluid is an important physical quantity. It is defined as under.

The normal force (thrust) exerted by liquid at rest on a unit area of the surface in contact with it is called **pressure of the liquid** *on the surface.*

If F is the normal force exerted by a liquid on a surface of area A, then pressure of the liquid on the surface is

$$\text{Pressure, } P = \frac{F}{A}$$

i.e., 1Pa = N/ m^2

➢ **Dimension formula: -** $[M L^{-1} T^{-2}]$

DISCUSSION

(*i*) Although force is a vector quantity, **pressure is a scalar quantity**. In a liquid at rest, pressure at a given point is exerted equally in all directions. Therefore, it makes no sense to associate direction with pressure.

(*ii*) The concept of pressure is particularly useful in dealing with fluids *i.e.*, liquids and gases. It is because fluids often lack definite shape and volume and it is convenient to use the quantities pressure and density rather than force and mass when studying hydrostatics and hydrodynamics.

APPLICATIONS OF PRESSURE:

(*i*) Nails and pins have pointed ends. Because of pointed ends, nails and pins have very small area. A small force on the head of a pin or nail will exert considerable pressure ($P = F/A$) to drive it into the surface easily.

(*ii*) Wide wooden or concrete sleepers are placed below the railway tracks. Because of this arrangement, the area over which the weight of the train acts is increased. As a result, the pressure on the ground is reduced, providing safety to railway tracks.

(*iii*) Buildings have wide foundations. Because of wide foundation, the area over which the weight of the building acts is increased. This reduces the pressure exerted by the building on the ground.

(*iv*) It is painful to walk barefooted on road covered with pebbles having sharp edges. The sharp-edged pebbles have very small area. Therefore, the weight of the body exerts considerable pressure on the pebbles through the feet. The pebbles exert equal reaction on the feet. For this reason, it is painful to walk barefooted on pebbles.

Expression for pressure exerted by a liquid column:

Consider a vessel filled with a liquid of density ‘ρ’. Imagine an area of cross section ‘A’ at a depth of ‘h’ below the free surface of the liquid.

The weight of the liquid column exerts downward thrust and hence a downward pressure.

F = mass of the liquid × g

= Volume × Density × g

= Area × h ×ρ ×g = A × (h ×ρ ×g)

∴

$$\text{Pressure} = \text{Force}/\text{Area} = \frac{A \times (h \times \rho \times g)}{A}$$

Or, **Pressure = h × ρ × g**

DISCUSSIONS

(*i*) The pressure exerted by a liquid at rest is independent of the size and shape of the container ($\because P = h\rho g$).

(*ii*) All the points at the same depth have the same pressure ($\because P \propto h$; ρ and g are constant). As depth increases, pressure increases and *vice-versa*.

Explanation of fluid pressure. The pressure of a liquid (or fluid) contained in a container can be explained on the basis of molecular theory of matter. A liquid is composed of molecules which are in a state of continuous random motion. They suffer collisions with each other and with the walls of the container. If we consider any surface placed within the liquid, quite a large number of molecules strike the surface and thereby impart momentum to the surface. The momentum transferred per second to the surface gives the average force or thrust on the surface. The liquid pressure is the thrust per unit area of the surface.

PASCAL'S LAW (Basic principle of Hydrostatic and was stated and formulate by Blaise Pascal).

A/ p's law, **"The pressure applied to an enclosed liquid is transmitted undiminished to every portion of the liquid and the wall of the containing vessel."**

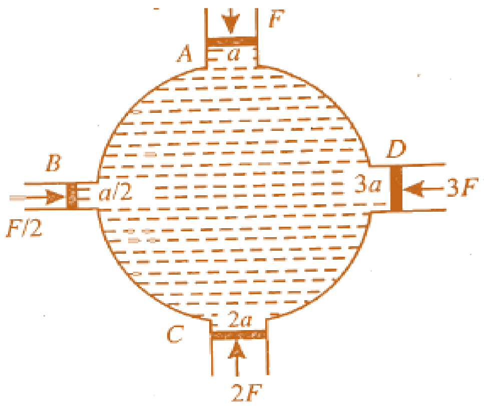
In other words,

"The pressure in a fluid in equilibrium to the same everywhere if the effect of gravity can be neglected.

Consider a vessel containing water and fitted with 4 frictionless piston A, B, C and D of different area of cross section a_1, a_2, a_3 and a_4 respectively.

Let a force F_1 be applied to push the piston 'A'.

Therefore, on the piston, $= \frac{F}{a_2}$



A / P's law, pressure is transmitted equally in all directions.

Therefore, it is found that the piston B, C and D can be held in their respective position if we apply force's F_2, F_3 and F_4 such that

$$\frac{F_2}{a_2} = \frac{F_3}{a_3} = \frac{F_4}{a_4} = \frac{F_1}{a_1}$$

where, $F_1 \neq F_2 \neq F_3 \neq F_4$

provided that $a_1 \neq a_2 \neq a_3 \neq a_4$.

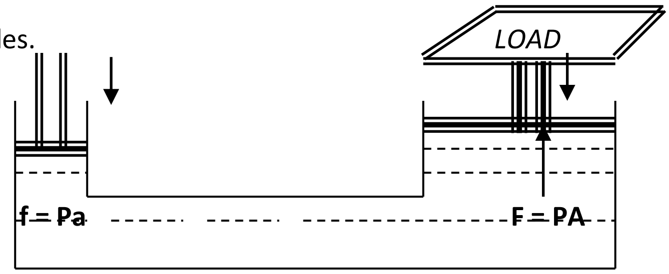
Application of Pascal's law

Hydraulic press (or lift)

The piston of small cross – sectional area 'a' exerts a force 'f' directly on a liquid.

The pressure P = f / a is transmitted undiminished through the connecting pipe to a large cylinder which has piston of area 'A'.

As per Pascal's law, pressure must be the same on both sides.



∴

$$\text{Pressure} = \frac{f}{a} = \frac{F}{A}$$

i.e., $\frac{F}{A} = \frac{f}{a}$

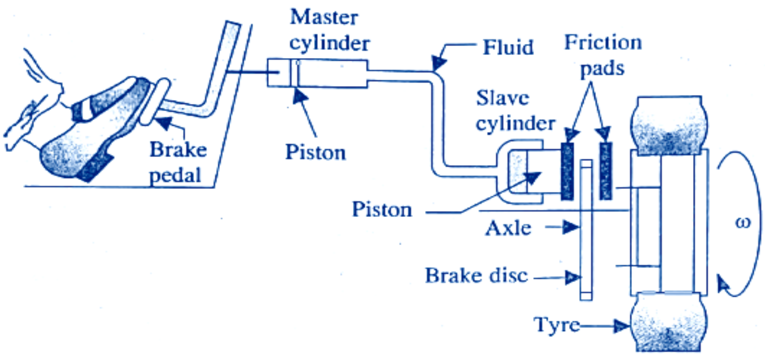
∴ $F = \frac{A}{a} \times f$ - - - - - - - - [1]

As $A > a$, ∴ from [1] **F > f**

- It means the force on the large piston will be much more than the applied force. This can be used to lift a heavy load placed on the platform of large piston or to press the things placed between the piston and the heavy platform.

Hydraulic Breaks

It consists of master cylinder fitted with piston. The piston 'P' is attached to brake pedal with help of lever system. The master cylinder is connected to the wheel through a tube. Wheel cylinder consists of two piston P_1 & P_2. This piston presses against the breake- -Shoes S_1 & S_2.



WORKING: --

When the brake pedal is pressed with a foot, the pushed into the master cylinder. The pressure so produced is transmitted equally to piston P_1& P_2 of the wheel cylinder. Due to this pressure, the piston P_1 and P_2 move outwardly and force the brake – shoes to expand outwardly. These brake shoes press against the inner rim wheel and retards the motion of the wheel.

- The function of master cylinder is to apply equal pressure to all the wheel.

Hydrostatic paradox (Contradictory statement)

Fig. 19.8 shows a vessel consisting of four sections (A, B, C and D) of different shapes but with its base horizontal. If a liquid is poured into the vessel, it will be seen that liquid rises to the same height in all the sections. It is because the pressure within a liquid is directly proportional ($P = h\rho g$) to its depth below the free surface. Since the base of the vessel is horizontal, the pressure of the liquid at all points on the base must be the same. Therefore, the liquid will rise to the same level regardless of the shape of the sections.

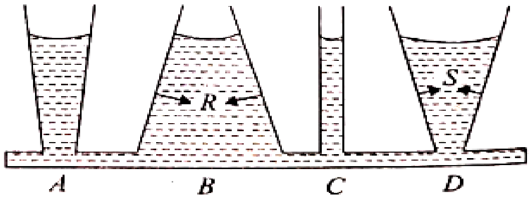


Fig. 19.8

At first glance, there appears to be a **paradox** here. The weight of the liquid is obviously different in each section and since liquid pressure depends upon the weight of the liquid above a reference point, it would seem that pressure should be different at the base of sections. No doubt the cross-sectional area of B at the base is greater than that at D but the force at the base of B is the sum of the weight of water above the base together with the downward component of reaction R of the sides of the section B. However, the force on the base of D is the weight of water above the base *minus* the upward component of the reactions S of the walls of section D. Therefore, the value of pressure at the base of section B is the same as that on the base of section D. This is also true for other sections.

Pascal's law, effect of gravity

Consider a vessel containing fluid, as the fluid is in equilibrium, so every volume element of the fluid is also in equilibrium.

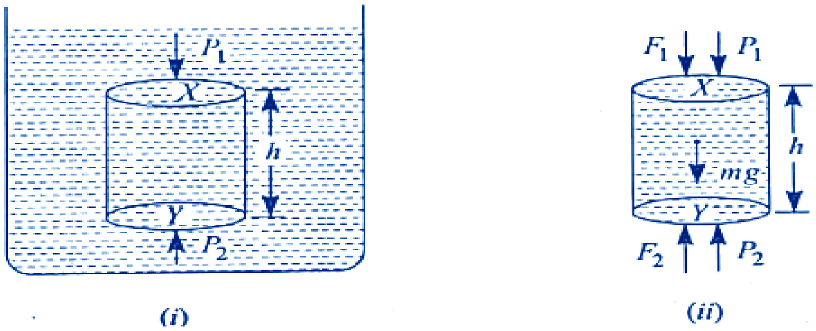
Consider one volume element in the form of a cylinder column of fluid height 'h' and of area of cross - section A. The various forces acting on the cylindrical

(*i*) The downward force $F_1 = P_1 A$ acting on the upper face.

(*ii*) The upward force $F_2 = P_2 A$ acting on the lower face.

(*iii*) The weight mg of the liquid in the considered cylinder of height h. This force acts downward. Now volume of this cylinder is $A\,h$.

$\therefore$ $mg = (\text{Volume} \times \text{density})\, g = (A\,h\rho)\, g = Ah\rho g$



Since the liquid is not moving and, therefore, is in equilibrium. Consequently, these three vertically directed forces must add to zero *i.e.*,

$$(F_1 + m\,g) - F_2 = 0$$

or $$P_1 A + m\,g = P_2 A$$

or $$P_1 A + A\,h\rho g = P_2 A$$

or $$P_1 + h\rho g = P_2$$

$\therefore$ $$P_2 - P_1 = h\rho g \quad ...(i)$$

$\therefore$ $P_2 - P_1 = h\rho g \longrightarrow$ (i) $\qquad P_2 = P_1 + h\rho g \longrightarrow$ (ii)

◙ If both the faces are act at the same level in the liquid, then h = 0

$\therefore$ $P_1 = P_2$

i.e., In the presence of gravity, the pressure at all point inside the liquid lying at the same horizontal plane.

➢ *If gravity effect neglected, then* **g = 0**

Therefore, from (i) $P_2 - P_1 = h\rho 0$

$P_2 - P_1 = 0$

$P_2 = P_1$

i.e., the **pressure** at every two points inside the liquid is **same** *in the absence of gravity*.

- 'P_2' is called **absolute pressure** at depth 'h' below the free surface of the liquid.
- Eq –(i) or (ii) shows that **Absolute pressure** (at depth h) is greater than P_1 by an amount equal to <u>h **ρ** g</u>.
- $P_2 - P_1 = h\rho g$, where $P_2 - P_1$ is the difference of **pressure** between two points separated by a depth 'h' and is known as **Gauge pressure**.
- since $P_2 = P_1 + h\rho g$

  If the pressure P_1 is increased in any way, the **pressure** P_2 at any depth must increase exactly the same amount.

Example 19.1. A large can crusher is shown in Fig. 19.10. The large piston has an area of 8 m² and exerts a force F_2 of magnitude 2×10^6 N on the cans. Calculate the magnitude of the force F_1 exerted by the small piston (area 10 cm²) on the fluid. Do not ignore the fact that the large piston is 1 m higher than the small piston. The fluid used is water.

Solution. The pressure P_2 in the fluid at the top of the crusher is

$$P_2 = \frac{F_2}{A_2} = \frac{2\times10^6}{8} = 2.5\times10^5 \text{ N/m}^2$$

The pressure P_1 on the fluid at the small piston is

$$P_1 = P_2 + h\rho g \text{ (Here } h = 1 \text{ m)}$$

$$= 2.5\times10^5 + 1\times10^3\times9.8 = 2.6\times10^5 \text{ N/m}^2$$

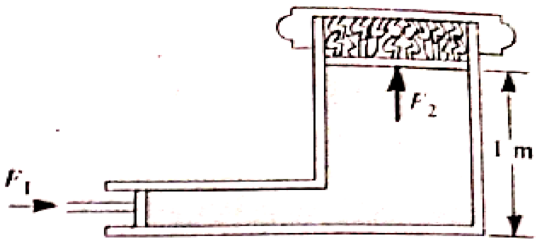


Fig. 19.10

Since the small piston has area $A_1 = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2$, the magnitude of force F_1 on this piston is

$$F_1 = P_1 A_1 = (2.6\times10^5)\times10^{-3} = 260 \text{ N}$$

Example 19.2. Two pistons of a hydraulic lift have diameters of 30 cm and 2.5 cm. What is the force exerted by the larger piston when 50 kg wt is placed on the smaller piston? If the stroke of the smaller piston is 4 cm, through what distance will the larger piston move after 10 strokes?

Solution. Area of smaller piston, $A_1 = \pi\left(\frac{2.5}{2}\right)^2$; Area of larger piston, $A_2 = \pi\left(\frac{30}{2}\right)^2$

Now $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\therefore \quad F_2 = F_1\times\frac{A_2}{A_1} = 50\times\frac{\pi(30/2)^2}{\pi(2.5/2)^2} = 7200 \text{ kg wt}$$

In one stroke, input work done = output work done

or $$F_1\times l_1 = F_2\times l_2$$

$$\therefore \quad l_2 = \frac{F_1 l_1}{F_2} = \frac{50\times4}{7200} = 0.028 \text{cm}$$

Distance through which larger piston moves in 10 strokes

$$= 10 l_2 = 10\times0.028 = 0.28 \text{ cm}$$

Example 19.3. A rectangular tank is 10 m long, 5 m broad and 3 m high. It is filled to bri with water of density 10^3 kgm^{-3}. Calculate the thrust at the bottom and walls of the tank du to hydrostatic pressure. Take $g = 9.8$ ms^{-2}.

Solution. Pressure at the bottom of the tank is

$$P_{bottom} = h\rho g = 3\times10^3\times9.8 = 29.4\times10^3 \text{ N/m}^2$$

$$\text{Area of bottom, } A_{bottom} = \text{Length}\times\text{Breadth} = 10\times5 = 50 \text{ m}^2$$

$$\therefore \quad \text{Thrust on the bottom} = P_{bottom}\times A_{bottom} = 29.4\times10^3\times50 = 1.47\times10^6 \text{ N}$$

Average pressure on the walls is

$$P_{walls} = \frac{0+h\rho g}{2} = \frac{0+29.4\times10^3}{2} = 14.7\times10^3 \text{ N/m}^2$$

$$\text{Area of broad walls} = 2\times\text{length}\times\text{height} = 2\times10\times3 = 60 \text{ m}^2$$

$$\text{Area of narrow walls} = 2\times\text{breadth}\times\text{height} = 2\times5\times3 = 30 \text{ m}^2$$

$$\text{Total area of walls, } A_{walls} = 60 + 30 = 90 \text{ m}^2$$

$$\text{Thrust on walls} = P_{walls}\times A_{walls} = 14.7\times10^3\times90 = 1.323\times10^6 \text{ N}$$

ABSOLUTE AND GAUGE PRESSURE

Consider a container of liquid open at the top as shown in Fig. 19.11. The free surface at the top experiences the pressure P_a of the atmosphere pressing down at the surface. Pascal's principle states that every point in the container experiences this added pressure. The total pressure at a point A at a depth h below the free surface is

$$P = P_a + h\rho g$$

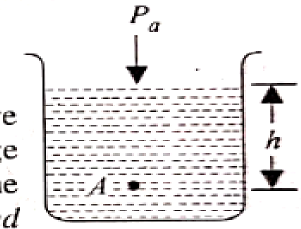


Fig. 19.11

Thus the absolute or actual pressure at point A is $P_a + h\rho g$.

Generally, when we use a gauge to measure pressure in a container, we are doing so with P_a surrounding us and the gauge. What the gauge measures is the *difference* between the pressure P in the container and the atmospheric pressure P_a. *The difference between the absolute pressure and the atmospheric pressure (i.e., $P - P_a$) is called the* **gauge pressure P_G**.

$$\therefore \quad P_G = P - P_a = h\rho g$$

$$\therefore \quad \text{Absolute pressure, } P = P_G + P_a \text{ where } P_G = h\rho g$$

Example 19.4. **A household hot water heating system has an expansion tank 12 m above the boiler. If the tank is open to the atmosphere, calculate the absolute and gauge pressure in the boiler. Given that $P_a = 1.01 \times 10^5$ Pa.**

Solution. Absolute pressure P in the boiler is

$$P = P_a + h\rho g$$

Here $P_a = 1.01 \times 10^5$ Pa ; $h = 12$ m ; $\rho = 1000$ kg m^{-3} ; $g = 9.8$ ms^{-2}

$$\therefore \quad P = 1.01 \times 10^5 + 12 \times 1000 \times 9.8 = 2.19 \times 10^5 \text{ Pa}$$

$$\text{Gauge pressure, } P_G = P - P_a = 2.19 \times 10^5 - 1.01 \times 10^5 = 1.18 \times 10^5 \text{ Pa}$$

If there is a pressure gauge on the furnace, it is always calibrated to read gauge rather than absolute pressure.

Example 19.5. **The interior of a submarine located at a depth of 50 m in sea water is maintained at sea level atmospheric pressure. Find the force acting on a window 20 cm square. The density of sea water is 1.03×10^3 kg m^{-3}.**

Solution. Pressure outside the submarine is

$$P = P_a + h\rho g$$

Pressure inside the submarine $= P_a$

$$\therefore \quad \text{Net pressure on window, } P' = P - P_a = h\rho g$$

$$\text{or} \quad P' = 50 \times 1.03 \times 10^3 \times 9.8 = 5.05 \times 10^5 \text{ Pa}$$

$$\text{Area of window, } A = 20 \times 20 = 400 \text{ cm}^2 = 0.04 \text{ m}^2$$

$$\therefore \quad \text{Force on window, } F = P'A = 5.05 \times 10^5 \times 0.04 = 2.02 \times 10^4 \text{ N}$$

Example 19.6. **A liquid stands at the same level in the U-tube when at rest. If A is the area of cross-section of the tube and g is the acceleration due to gravity, what will be the difference in height of the liquid in the two limbs when the system is given an acceleration 'a' towards the right as shown in Fig. 19.12.**

Solution. Fig. 19.12 shows the conditions of the problem. Let l be the length of the horizontal portion CD of the tube.

Mass of liquid in the portion CD

$$= \text{Volume} \times \text{density}$$

$$= (Al) \times \rho = A\, l\rho$$

Force on the above mass towards left

$$= A\, l\rho \times a$$

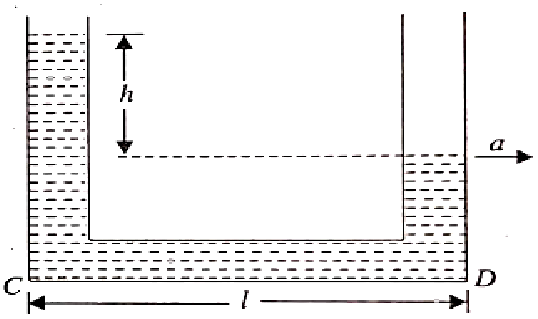


Fig. 19.12

Due to difference h in height of the liquid, the downward force exerted on the liquid in the horizontal portion $CD = h\rho g \times A$.

$$\therefore \quad h\rho g \times A = A\, l\rho \times a$$

$$\text{or} \quad h = \frac{al}{g}$$

Example 19.7. **The liquids shown in Fig. 19.13 in the two arms are mercury (specific gravity = 13.6) and water. If the difference of heights of the mercury columns is 2 cm, find the height h of the water column.**

Solution. Figure 19.13 shows the conditions of the problem. Let the atmospheric pressure be P_a in SI units.

Pressure at A, $P_A = P_a + h\rho g = P_a + h \times 1000 \times g$

Pressure at B, $P_B = P_a + h'\rho' g = P_a + 0.02 \times 13600 \times g$

Since A and B are at the same level, the pressures are equal at A and B.

$\therefore \quad P_a + h \times 1000 \times g = P_a + 0.02 \times 13600 \times g$

or $$h = \frac{0.02 \times 13600}{1000} = 0.27 \text{ m} = 27 \text{ cm}$$

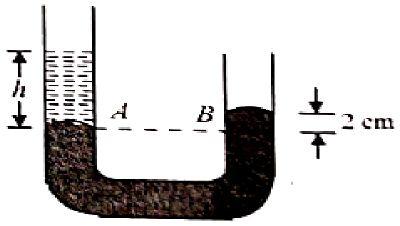


Fig. 19.13

Atmospheric pressure

The earth is surrounded by a gaseous envelope which is called the atmosphere which extends up to a height of many kilometers above the surface of the earth.

- Chief component of atmosphere at earth's Surface: -- ***N_2 = 78%; O_2 = 21%; Argon = 0.94%; CO_2 = 0.032%; H_2 = 0.01%***

  remainder includes ***Na, He, dust particles.***

  Now, as we go higher, the density of atmospheric pressure goes on decreasing.

- The pressure exerted by the ocean of the air on the earth's surface is called atmospheric pressure. Thus, ***"The atmospheric pressure at any point is numerically equal to weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere".***
- Standard (normal) atmospheric pressure is equal to the pressure due to a column of 76 cm (0.76 m) of mercury at 0^0C at sea level.

  At 0^0C, Density of mercury = 13.6×10^3 K g / m^3 and at sea level, g = 9.8 m/s^2

$\therefore$ Pressure = $h \times \rho \times g$

$\therefore$ 1 atmosphere (1atm) = $0.76 \times 13.6 \times 10^3 \times 9.8$

$= 1.013 \times 10^5$ N/m^2

1atm = 1.013 × 105 Pa (*In SI*)

MERCURY BAROMETER

A mercury barometer is an instrument used to measure the atmospheric pressure. A long tube (about 1 m) closed at one end is filled with mercury and then inverted into a trough of mercury (See Fig. 19.14). It is seen that the mercury level in the tube falls till the height h becomes nearly 76 cm above the mercury level in the trough. The upper end of the closed tube is nearly a vacuum and so its pressure can be taken as zero. We say that *atmospheric pressure is 76 cm of mercury.*

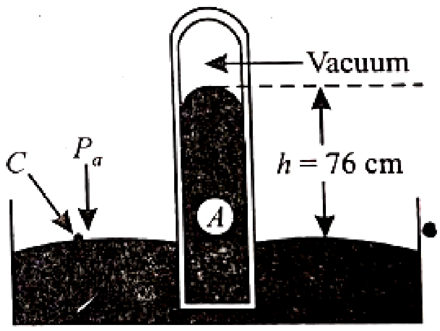


Fig. 19.14

Theory. To understand how simple barometer measures atmospheric pressure, refer to Fig. 19.14. At point C on the surface of mercury in the trough, the pressure is the atmospheric pressure P_a. According to Pascal's principle, this pressure is transmitted uniformly through the mercury in the trough. Therefore, at point A inside the tube, at the level of point C, the pressure is the same as at the point C (*i.e.,* atmospheric pressure). Now the pressure at point A inside the tube is due only to the height h of mercury in the tube and is $h\rho g$ where ρ is the density of mercury.

$\therefore$ Atmospheric pressure, $P_a = h\rho g$

Now at normal temperature and pressure at sea level, h = 76 cm = 0.76 m. Also density of mercury is $\rho = 13.6 \times 10^3$ kg/m^3.

$\therefore$ Standard atmospheric pressure is

$$P_a = (0.76) \times (13.6 \times 10^3) \times 9.8 = 1.01 \times 10^5 \text{ N/m}^2 \text{ or Pa}$$

(*i*) The atmospheric pressure is directly proportional (ρ and g being constant) to the height h of the mercury column. Therefore, atmospheric pressure can be conveniently expressed in terms of the height of mercury column. We say that standard atmospheric pressure is 76 cm of mercury. It means that the weight of 76 cm column of mercury is equal to the weight of a column of the atmosphere of the same cross-section, extending to the top of earth's atmosphere.

(*ii*) The vertical height of the mercury column remains constant even when the tube is tilted, unless the top of the tube is less than 76 cm above the level in the trough — in which case the mercury completely fills the tube.

HEIGHT OF ATMOSPHERE

When molecules of air or of any gas collide with a solid or liquid surface, they exert force against that surface. The force is of impulsive nature, much like the force of a tennis ball hitting the practice board. At sea level, collisions of air molecules with a solid surface cause an average force of 1.01×10^5 N to be exerted on each 1 m^2 of the surface. Thus atmospheric pressure at sea level is 1.01×10^5 N/m^2. Let us now calculate the height of air column that would cause a pressure of 1.01×10^5 N/m^2 on the surface of earth. Suppose the required height is h metres above earth's surface. Although the density of air goes on decreasing as we go up, we can assume it to be constant at 1.293 kg/m^3 throughout. Then our estimate of h is

$$h\rho g = 1.01 \times 10^5$$

or $$h \times 1.293 \times 9.8 = 1.01 \times 10^5$$

$\therefore$ $$h = \frac{1.01\times10^5}{1.293\times9.8} \simeq 8000 \text{ m} = 8 \text{ km}$$

Units. There are two common units of pressure worth mentioning at this point. One, called the **torr** is named after the inventor of the barometer, the Italian physicist Torricelli (1608 – 1647). The other, the **bar**, is commonly used in the science of meterology. These units have the values :

$$1 \text{ torr} = 1 \text{ mm of Hg} \; ; \; 1 \text{ bar} = 10^5 \text{ Pa (exactly)}$$

Buoyancy and ARCHIMEDE'S PRINCIPLE

When a body is partially or wholly immersed in a fluid, it displaces the fluid. The displaced fluid has a tendency to regains its original position. As a result of this an upwards force is exerted on the body by the displaced fluid. The upward force is called ***Buoyant force (or thrust)***.

- The thrust is clearly equal to the apparent loss in weight.

- The Buoyant force acts at the centre of buoyancy, this will coincide with the centre of Gravity (C.G) [IF the solid body is homogeneous and spherical]. So, the homogeneous spherical body immersed in a liquid remains in **Equilibrium.** However, if the body is not homogeneous & spherical, then the centre of gravity may not lie on the line of upward thrust and hence there may be **Torque** that causes rotation of the body.

The buoyant force arises from the fact that pressure in a fluid increases with the increase in depth. Therefore, upward pressure on the bottom surface of the immersed body is greater than the downward pressure on its top face. The resultant force is then directed up. Thus we find that when a body is immersed in a fluid, the force of gravity ($W = mg$) acts downward while the buoyant force F_B exerted by the fluid acts upward. The relative magnitudes of W and F_B will decide whether the body will sink or float in the fluid.

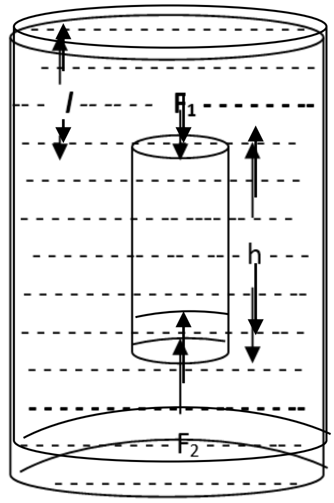
Archimedes's principle

It states that, ***"When a body is immersed wholly or partially in a liquid (at rest), it loses some of its weight. The loss in weight of the body in the liquid is equal to the weight of the liquid displaced by immersed part of the body".***

In other words, ***"If a body is wholly or partially immersed in a fluid, it experiences an upward thrust equal the weight of the fluid displaced and this upward force acts through the centre of gravity".***

PROOF:

Consider a body of mass 'M' completely immersed in a liquid of density 'ρ'.
Let 'A' be the area of cross-section of the both the lower and upper face each.
Let 'l' be the depth of upper face from the free surface of the liquid
h = height of the body.



Pressure *(acting on the upper face)*, $P_1 = l\rho g$ ------------------------ [1]

Pressure *(acting on the lower face)*, $P_2 = (l + h)\rho g$ ---------------- [2]

Downward force (or thrust) on the upper face, $F_1 = P_1A = l\rho gA$
Upward force (or thrust) on the lower face, $F_2 = P_2A = (l + h)\rho gA$
Net force on the body, $F = F_2 - F_1$
$= (l + h)\rho gA - l\rho gA$
$= l\rho gA + h\rho gA - l\rho gA$
$= (Ah)\rho g$
$= (\text{Area} \times \text{height})\rho g = \text{Volume} \times \rho g$

∴ **$F = V\rho g$**

Also, **F = (Volume × density) g = mass × g = m g** = Weight ***of the displaced liquid***

This upward thrust (F = mg) exerted by the fluid on the body is called a **Buoyant force**.

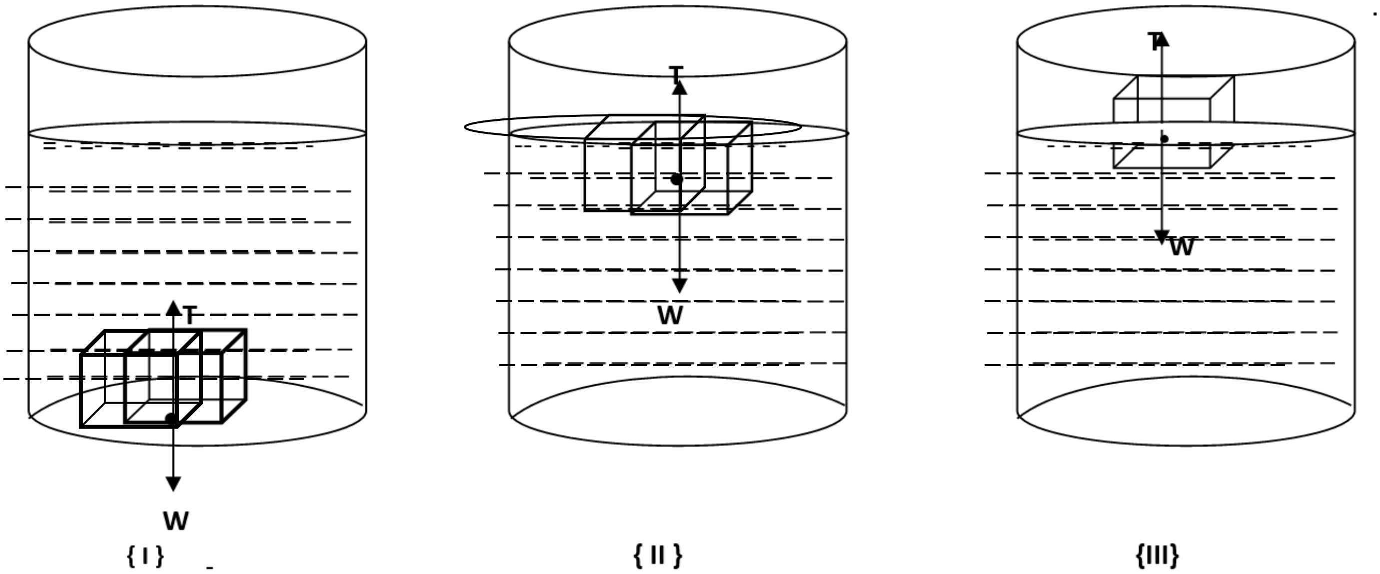
- Forces acting on the body are: ----- [1] Its weight Mg, (acts vertically downwards).
  [2] Net up thrusts (the buoyancy).
- **Apparent weight of the body = Mg – mg**
- **Actual weight of the body = Apparent weight + mg.**
- **Weight of the liquid displaced (mg) = Actual weight – Apparent weight.**
- **Conclusion: Apparent weight of the body is less than its actual weight by an amount equal to the weight of the liquid displaced by the body.**

Law of Floatation — [Extension of Archimedes' principle]

Consider a body of volume 'V' and density 'ρ', immersed completely in a liquid of density '**σ**'(Sigma)
Force acting on the body are --- [1] The weight of the body **W** (= **V ρ g),** through the centre of gravity [Downward].
--- [2] Upward thrust (**T**) (i.e., weight of the liquid displaced), through centre of gravity (upward) (i.e, centre of buoyancy)

∴ **Apparent weight = W - T**

- **Case I**: -- When ***W > T,*** Then **W - T = +ive value, so** a net force acts downwards. *The body sinks to the bottom of the* liquid. *In* this case **ρ > σ.**
- **Case II: -- When *W = T, Then* W - T = 0.** The body will not experience any resultant force upward or downwards. The *Body is in equilibrium inside the liquid i.****e.,*** *the body will just float****.*** In this case **ρ = σ.**
- **Case III: -- *When W > T,*** Then **W - T = -ive value, so** a net force acts upwards. The body continue to move up till the weight of the liquid displaced by its immersed part just equals the weight of the body. The body will then just float in the liquid. In this case **ρ < σ.**



DEFINATION: -- "*A body floats in a fluid if weight of the fluid displaced by the immersed portion of the body is equal to the weight of the body".*

RELATION BETWEEN DENSITY OF SOLID(ρ) AND LIQUID(σ): -

Weight of the floating solid = Weight of the liquid displaced

$V_1 \rho g = V_2 \sigma g$

$$\frac{\rho}{\sigma} = \frac{V_2}{V_1}$$

$$\frac{\text{Density of solid}}{\text{Density of liquid}} = \frac{\text{Volume of immersed portion of solid}}{\text{Total volume of the solid}}$$

= Fraction of volume of the body immersed in the liquid

FRACTION OF VOLUME SUBMERGED OF A FLOATING BODY

Consider a body floating in a liquid.

Let V = Total volume of the body

V' = Volume of body submerged in liquid

ρ = Density of the body

ρ_l = Density of liquid

Mass of liquid displaced by body = $V'\rho_l$

Weight of liquid displaced by body = $V'\rho_l g$

∴ Buoyant force, $F_B = V'\rho_l g$

Mass of body = Volume × density = $V\rho$

Weight of body = $V\rho g$

According to principle of floatation, F_B = Weight of body

or $V'\rho_l g = V\rho g$

$$\therefore \quad \frac{V'}{V} = \frac{\rho}{\rho_l} \quad ...(i)$$

$$\text{Volume of body outside the liquid} = V - V' = V - \frac{\rho}{\rho_l}V$$

$$= V\left(1 - \frac{\rho}{\rho_l}\right) \quad ...(ii)$$

Example 19.11. An object floats on water with 20% of its volume above the water line. What is the density of the object? Density of water = 1000 kg m^{-3}.

Solution. Suppose the volume of the entire object is V. Then the volume of the object under water is $0.8\ V$.

$$\text{Buoyant force, } F_B = 0.8\ V \times \rho_w \times g$$

If ρ is the density of the object, then,

$$\text{Weight of object, } W = \rho V g$$

According to principle of floatation, $F_B = W$

or $$0.8\ V \times \rho_w \times g = \rho\ V\ g$$

$\therefore$ $$\rho = 0.8 \times \rho_w = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Example 19.12. A glass tumbler is full to the brim with water. An ice cube floats on the top surface. (*i*) What fraction of the ice cube is immersed? (*ii*) What happens to water level after the ice is melted? Density of ice is 910 kg m^{-3} and that of water is 1000 kg m^{-3}.

Solution. (*i*) Suppose the volume of the entire cube is V_i and its volume under water is V_w. The density of ice is ρ_i and that of water is ρ_w.

$$\text{Weight of ice cube, } W_i = \rho_i V_i\ g$$

$$\text{Buoyant force, } F_B = \rho_w V_w\ g$$

According to principle of floatation. $F_B = W_i$

or $$\rho_w V_w g = \rho_i V_i g$$

or $$\rho_w V_w = \rho_i V_i$$

$\therefore$ $$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{910}{1000} = 0.91$$

(*ii*) As the cube melts, its volume shrinks from V_i to V_w as its density goes up from ρ_i to ρ_w. The final water level is just the same as before.

Example 19.13. A copper cube weighs 0.50 kg in air and 0.40 kg in water. Is the cube hollow or solid? Density of copper = 8.96×10^3 kg/m^3 ; density of water = 10^3 kg/m^3.

Solution. Suppose V is the volume of the copper cube.

$$\text{Loss of weight in water} = 0.50 \times g - 0.40 \times g = 0.1 \times g$$

$$\text{Weight of water displaced} = \text{Volume} \times \text{density} \times g = V \times 10^3 \times g$$

According to Archimedes' principle,

$$V \times 10^3 \times g = 0.1 \times g$$

$\therefore$ $$V = \frac{0.1}{10^3} = 10^{-4} \text{m}^3$$

$$\text{Density of copper cube} = \frac{\text{Mass}}{\text{Volume}} = \frac{0.5}{10^{-4}} = 0.5 \times 10^4 \text{kg/m}^3$$

Since it is less than the density of copper, the cube is hollow.

Example 19.14. An object that weighs 40 N in air, "weighs" 20 N when submerged in water and 30 N when submerged in a liquid of unknown density. What is the density of the liquid?

Solution. Suppose V is the volume of the object.

In water. $W = 40$ N ; $W_a = 20$ N ; $F_B = V\rho_w g$

Now $$F_B = W - W_a$$

or $$V\rho_w g = W - W_a \quad \therefore V = \frac{W - W_a}{\rho_w g} \quad ...(i)$$

In liquid. $W = 40$ N ; $W'_a = 30$ N ; $F'_B = V\rho_l g$

Now $$F'_B = W - W'_a$$

or $$V\rho_l g = W - W'_a$$

$\therefore$ $$\rho_l = \frac{W - W'_a}{V g} = \frac{W - W'_a}{W - W_a} \times \rho_w \quad \left(\because V = \frac{W - W_a}{\rho_w g}\right)$$

$$= \left(\frac{40 - 30}{40 - 20}\right) \times 1000 = 500 \text{ kg/m}^3$$