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WAVE OPTICS
DIFFRACTION

UNIT:VI
CHAPTER:II

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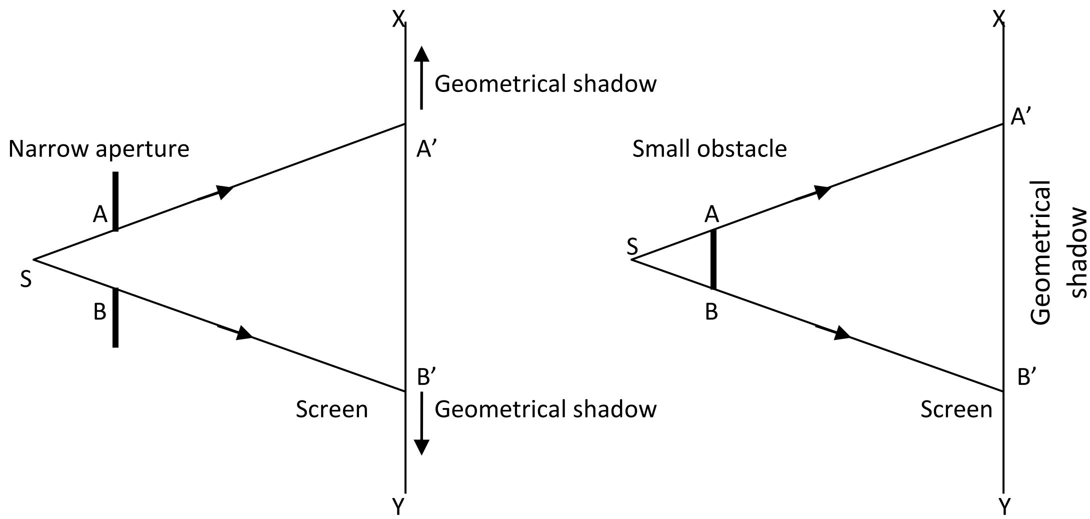
DIFFRACTION OF LIGHT

Light travels in a straight line. However, when light passes through a small hole, there is a certain amount of spreading of light. Similarly, when light passes by an obstacle, it appears to bend round the edges of the obstacle, it appears to bend round the edges of the obstacle and enters its geometrical shadow.

The phenomenon of bending of light around the corners of small obstacle or apertures and its consequent spreading into the regions of geometrical shadow is called diffraction of light.

To understand diffraction more clearly, consider a narrow aperture AB illuminated with light from a source S, as shown in Fig. (a) XY is a screen placed at large distance from AB. **According to rectilinear propagation of light, only the portion A' B' of the screen should be illuminated. However, it is seen that light enters the region of the geometrical shadow beyond A' and B'. The shadow is not sharp.**



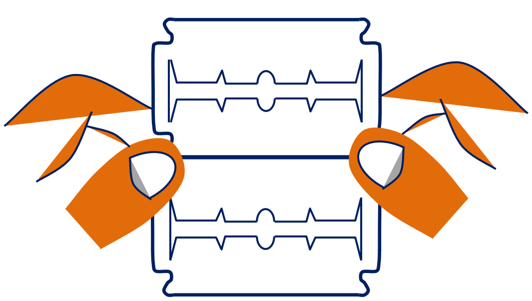
[Diffraction of light round corners of (a) a small aperture (b) a small object]

Similarly, when an obstacle AB (e.g., a very small disc) is placed in the path of light, we expect a dark shadow A'B' on the screen, as shown in Fig. (b). However, we observe a circular bright band at the centre, surrounded by dark and bright rings alternately. This shows that light bends around the edges, i.e., light shows diffraction.

Experiment 1: As shown in Fig., hold two blades so that their edges are parallel and form a narrow slit in between. Look through the slit on the straight filament of a clear glass bulb. With slight adjustment of the slit, a diffraction pattern of alternate bright and dark bands is seen.

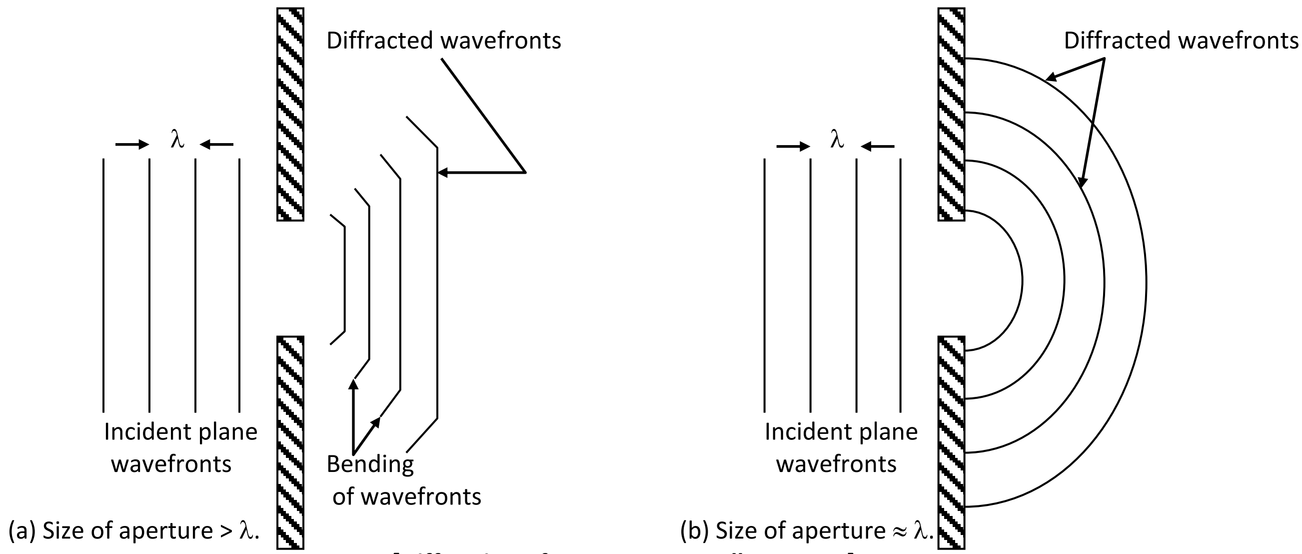
2. Look at a street lamp through a piece of fine cloth. The lamp appears as an enlarged disc. The threads in mutually perpendicular directions enclose a number of slits which form a pattern of several weaker images of the slits.

3. A pinhole placed at a distance of 2 m from a sodium lamp form a pattern of several weaker images of the slits.



[Experiment 1: A single slit formed by two blades]

Size of aperture or obstacle for observing diffraction: Suppose plane waves are made to fall on a screen having a small aperture. The waves emerging out of the aperture are observed to be slightly curved at the edges. This is diffraction. If the size of the aperture is large compared to the wavelength of the waves, the amount of bending is small [Fig. (a)]. If the size of the aperture is small, comparable to the wavelength λ of the waves, then the diffracted waves are almost spherical [Fig. (b)]. Hence the diffraction effect is more pronounced if the size of the aperture or the obstacle is of the order of the wavelength of the waves.



[Diffraction of a wave at a small aperture]

As the wavelength of light ($\approx 10^{-6}$ m) is much smaller than the size of the objects around us, so diffraction of light is not easily seen. But sound waves have large wavelength. They get easily diffracted by the objects around us.

FRESNEL AND FRAUNHOFER DIFFRACTION

Two types of diffraction: The diffraction phenomenon can be divided into two categories:

1. **Fresnel's diffraction:** In Fresnel's diffraction, the source and screen are placed close to the aperture or the obstacle and light after diffraction appears converging towards the screen and hence no lens is required to observe it. The incident wave fronts are either spherical or cylindrical.

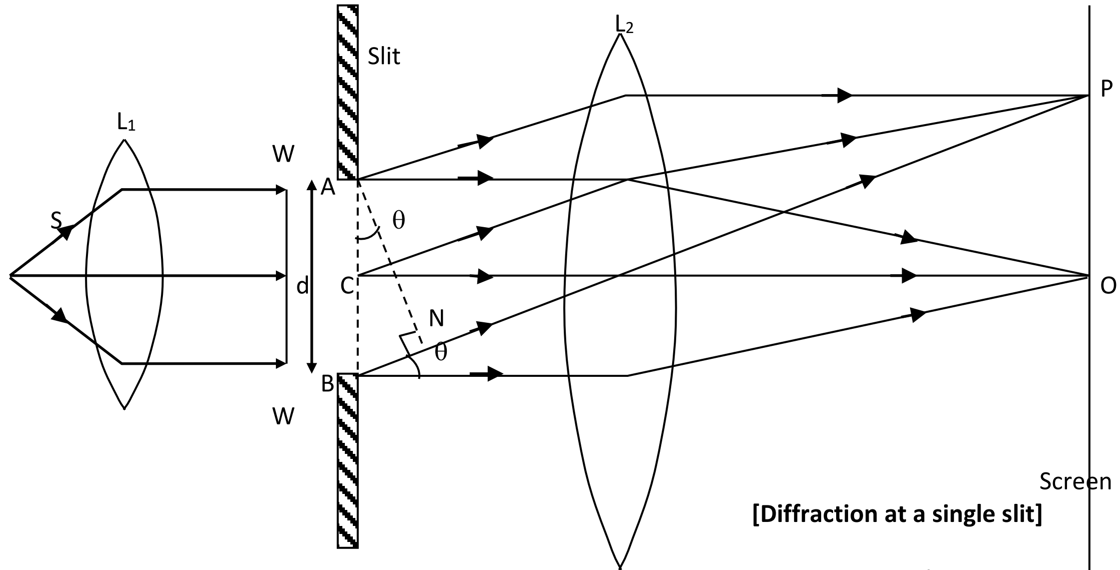
2: In **Fraunhofer's diffraction**, the source and screen are placed at large distances (effectively at infinity) from the aperture or the obstacle and converging lens is used to observe the diffraction pattern. The incident wavefront is planar one.

DIFFRACTION AT A SINGLE SLIT

A source S of monochromatic light is placed at the focus of a convex lens L_1. A parallel beam of light and hence a plane wavefront WW gets incident on a narrow rectangular slit AB of width d.

The incident wavefront disturbs all parts of the slit AB simultaneously. According to Huygens's theory, all parts of the slit AB will become source of secondary wavelets, which all start in the same phase. These wavelets spread out in all directions, thus causing diffraction of light after it emerges through slit AB. Suppose the diffraction pattern is focussed by a convex lens L_2 on a screen placed in its focal plane.

Central maximum: All the secondary wavelets going straight across the slit AB are focussed at the central point O of the screen. The wavelets from any two corresponding points of the two halves of the slit reach the point O in the same phase, they add constructively to produce a central bright fringe. For detailed explanation of diffraction fringes, see for your knowledge box on page.



[Diffraction at a single slit]

Calculation of path difference: Suppose the secondary wavelets diffracted at an angle θ are focussed at point P. The secondary wavelets start from different parts of the slit in same phase but they reach the point P in different phases. Draw perpendicular AN from A on to the ray from B. Then the path difference between the wavelets from A and B will be

$$p = BP - AP = BN = AB \sin\theta = d \sin\theta.$$

Positions of minima: Let the point P be so located on the screen that the path difference, p = λ and angle θ = θ_1. Then from the above equation, we get

$$d \sin\theta_1 = \lambda$$

We can divide the slit AB into two halves AC and CB. Then the path difference between the wavelets from A and C will be λ/2. Similarly, corresponding to every point in the upper half AC, there is a point in the lower half CB for which the path difference is λ/2. Hence the wavelets from the two halves reach the point p always in opposite phases. They interfere destructively so as to produce a minimum.

Thus, the condition for first dark fringe is $d \sin\theta_1 = \lambda$

Similarly, the condition for second dark fringe will be $d \sin\theta_2 = 2\lambda$

Hence the condition for nth dark fringe can be written as $d \sin\theta_n = n\lambda$, n = 1, 2, 3, ...

The directions of various minima are given by $\theta_n \simeq \sin\theta_n = n\dfrac{\lambda}{d}$ [As λ < < d, so $\sin\theta_n \approx \theta_n$]

Positions of secondary maxima: Suppose the point P is so located that $p = \dfrac{3\lambda}{2}$

When $\theta = \theta'_1$, then $d \sin\theta'_1 = \dfrac{3}{2}\lambda$

We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be λ/2. The wavelets from these points will interfere destructively. However, the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum.

Thus, the condition for the first secondary maximum is

$$d \sin\theta'_1 = 3/2\,\lambda$$

Similarly, the condition for the second secondary maximum is $d \sin\theta'_2 = \dfrac{5}{2}\lambda$

Hence the condition for nth secondary maximum can be written as $d \sin\theta'_n = (2n + 1)\dfrac{\lambda}{2}$, n = 1, 2, 3, ...

The directions of secondary maxima are given by $\theta'n \simeq \sin\theta'n = (2n + 1)\dfrac{\lambda}{2d}$

The intensity of secondary maxima decreases as n increases.

Intensity distribution curve: If we plot a graph between the intensities of maxima and minima against the diffraction angle θ, we get a graph of the type shown in Fig. It has a broad central maximum in the direction (θ = 0°) of incident light. On either side of the central maximum, it has secondary maxima of decreasing intensity at positions,

$$\theta = \pm(2n + 1)\frac{\lambda}{2d}$$

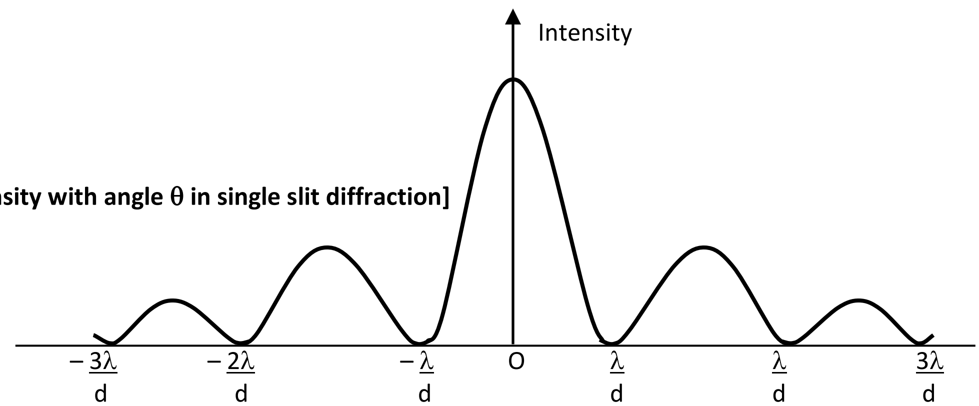
and minima at positions, $\theta = \pm n \frac{\lambda}{d}$ [$n \neq 0$]

The intensities of secondary maxima relating to the intensity of central maximum are in ratio,

$1 : \frac{1}{21} : \frac{1}{61} : \frac{1}{121}$ ....

Thus, the intensity of the first secondary maximum is just 4% of that of the central maximum.

[Variation of intensity with angle θ in single slit diffraction]

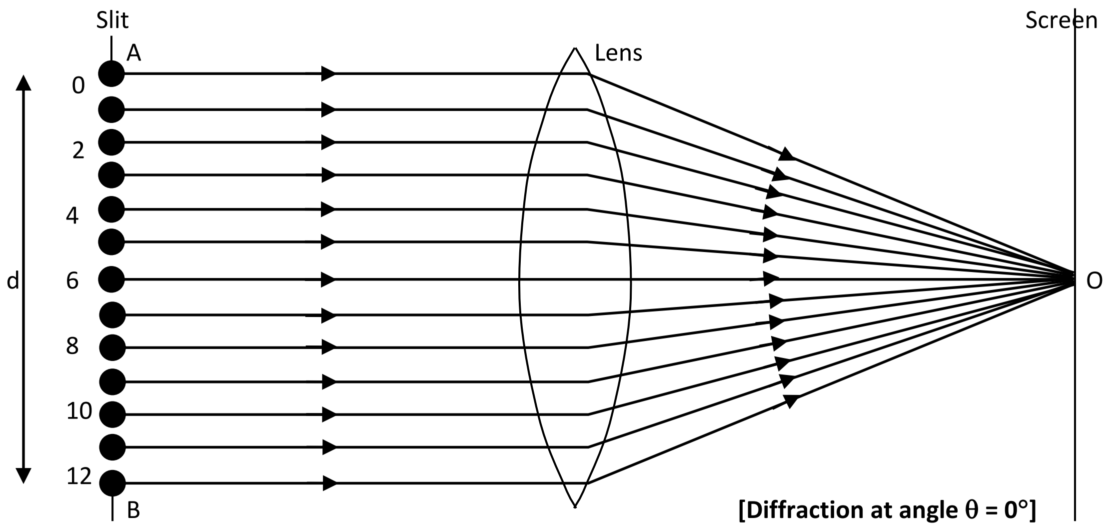


Intensity of secondary maxima decreases with the order of the maximum: The reason is that the intensity of the central maximum is due to the constructive interference of wavelets form all parts of the slit, the first secondary maximum is due to the contribution of wavelets from one third part of the sit (wavelets from remaining two parts interfere destructively), the second secondary maximum is due to the contribution of wavelets form the one fifth part only (the remaining four parts interfere destructively), the second secondary maximum is due to the contribution of wavelets from the one fifth part only (the remaining four parts interfere destructively) and so on. Hence the intensity of secondary maximum decreases with the increase in the order n of the maximum.

FOR YOUR KNOWLEDGE..

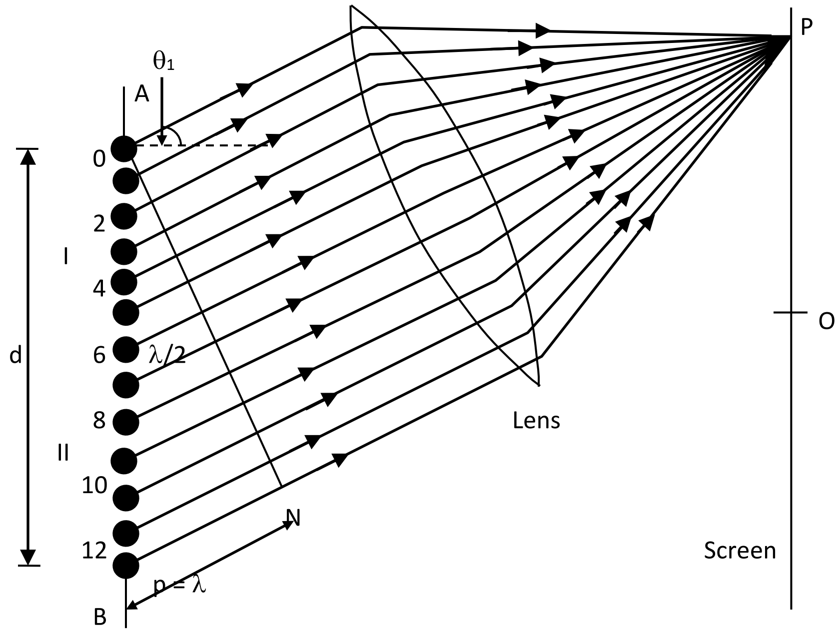
- **Explanation of diffraction fringes**

  Central maximum: All the wavelengths going straight ($\theta = 0°$) across the slit are focussed at the central point O of the screen, as shown in Fig. The wavelets from any two corresponding points such as (0, 12), (2, 10), (4, 8) etc. from the two halves of the slit have zero path difference. They undergo constructive interference to produce central bright fringe.



[Diffraction at angle θ = 0°]

First dark fringe: If angle θ is such that the path difference, $p = d \sin\theta = \lambda$, then the path difference between the rays' form A



[Diffraction at an angle θ given by $d \sin\theta = \lambda$]

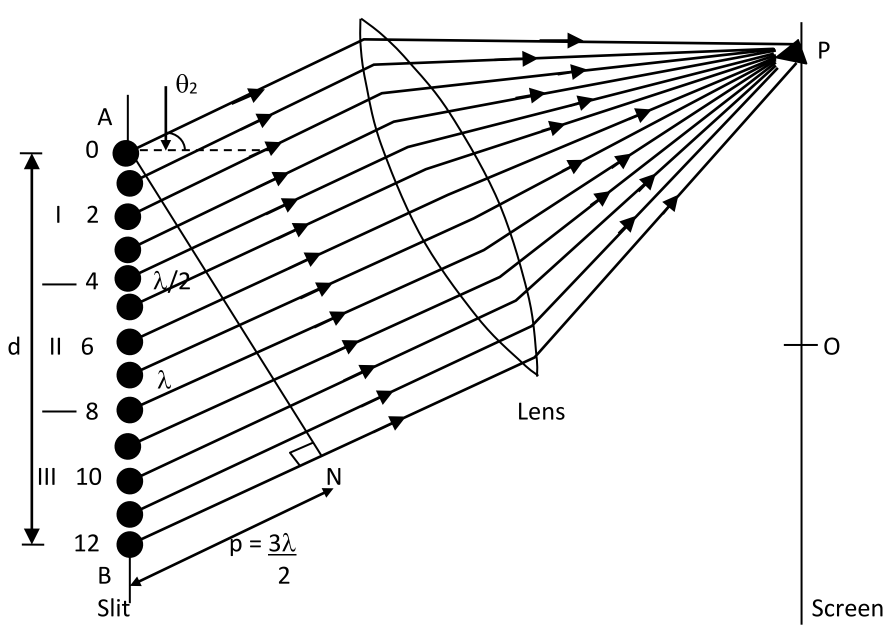
and B when they reach P is λ, as shown in Fig. If we divide the slit into two halves I and II, of 6 parts each, then obviously the wavelets form 0 and 6 will have a path difference of $\lambda/2$ or a phase difference of π. They interfere destructively. Similarly, the wavelet pairs (1, 7), (2, 8), (3, 9), (4, 10), (5, 11) and (6, 12) of the two halves will interfere destructively. Hence the condition for first dark fringe is

$$d \sin\theta = \lambda$$

First secondary maximum: Suppose the angle θ is such that the path difference $p = d \sin\theta = 3\lambda/2$.

We can divide the slit into three equal regions I, II and III, as shown in Fig. The path difference between any two corresponding points of regions I and II will be $\frac{\lambda}{2}$ or phase difference will be π. The wavelets from these points will interfere destructively. The wavelets from III region of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than the central maximum. The condition for the first secondary maximum can be written as

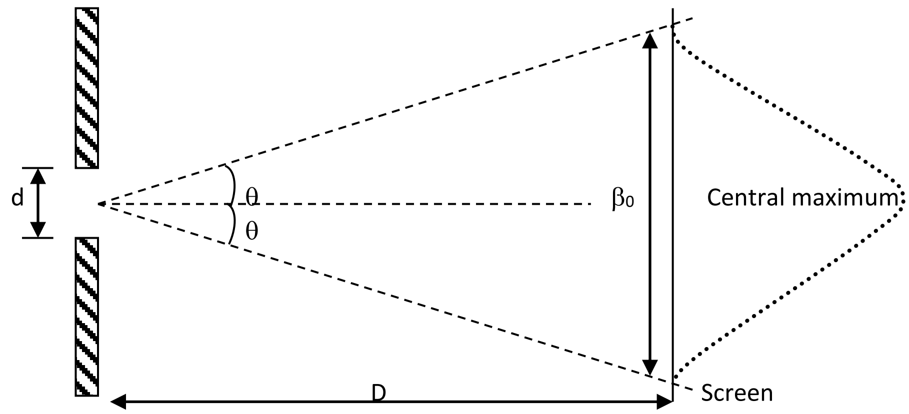
$$d \sin\theta = \frac{3}{2}\lambda$$



[Diffraction at an angle θ given by $d \sin\theta = \frac{3}{2}\lambda$]

WIDTH OF CENTRAL AND SECONDARY MAXIMA

Angular width of central maximum: The angular width of the central maximum is the angular separation between the directions of the first minima on the two sides of the central maximum, as shown in Fig.



[Angular and linear widths of central maximum)]

The directions of first minima on either side of central maximum are given by

$$\theta = \frac{\lambda}{d}$$

This angle is called half angular width of central maximum.

∴ Angular width of central maximum = $2\theta = \frac{2\lambda}{d}$

Linear width of central maximum: If D is the distance of the screen from the single slit, then the linear width of central maximum will be $\beta_0 = D \times 2\theta = \frac{2D\lambda}{d}$ $\left(2\theta\ (\text{rad}) = \frac{\text{Arc}}{\text{Radius}} = \frac{\beta_0}{D}\right)$

Linear width of a secondary maximum: the angular width of nth secondary maximum is the angular separation between the directions of nth and (n + 1)th minima.

Direction of nth minimum, $\theta_n = n\frac{\lambda}{d}$

Direction of (n + 1)th minimum, $\theta_{n+1} = (n+1)\frac{\lambda}{d}$

∴ Angular width of nth secondary maximum = $\theta_{n+1} - \theta_n = (n+1)\frac{\lambda}{d} - n\frac{\lambda}{d} = \frac{\lambda}{d}$

Hence the linear width of nth secondary maximum = Angular width × D

or $\beta = \frac{D\lambda}{d}$

Clearly, $\beta_0 = 2\beta$

Thus, the central maximum of a diffraction pattern is twice as wide as any secondary maximum.

Clearly, **width of a secondary maximum** $\propto \frac{1}{\text{slit width}}$

As the slit width is increased, the secondary maxima get narrower. If the slit is sufficiently wide, the secondary maxima disappear and only the central maximum is obtained which is the sharp image of the slit. Thus a distinct diffraction pattern is possible only if the slit is very narrow.

VALIDITY OF RAY OPTICS: FRESNEL'S DISTANCE

Ray optics as a limiting case of wave optics: Fresnel's distance and Fresnel's zone: A parallel beam of light of wavelength λ on passing through an aperture of size *d* gets diffracted into a beam of angular width,

$$\theta = \frac{\lambda}{d}$$

If a screen is placed at distance D, this beam spreads over a linear width, $x = \frac{D\lambda}{d}$

If the diffraction spread x is small, the concept of ray optics will be valid.

If we have an aperture of size d = 10 mm and use light of wavelength $\lambda = 6 \times 10^{-7}$ m, then the beam after travelling a distance of 3 m will get diffracted through a width

$$x = \frac{D\lambda}{d} = \frac{3 \times 6 \times 10^{-7}}{10 \times 10^{-3}}$$
$$= 18 \times 10^{-5} \text{ m} = 0.18 \text{ mm}$$

This diffraction spreading is not quite large. Thus, ray optics is valid in many common situations. It is useful here to define what is called Fresnel's distance, D_F.

The distance at which the diffraction spread of a beam is equal to the size of the aperture is called Fresnel's distance.

i.e., when $x = d$, $D = D_F$

$\therefore$ $d = \frac{D_F \lambda}{d}$ or $D_F = \frac{d^2}{\lambda}$

If $D < D_F$, then there will not be too much broadening by diffraction i.e., the light will travel along straight lines and the concepts of ray optics will be valid.

As $D < D_F$ or $D < \frac{d^2}{\lambda}$ or $d > \sqrt{\lambda D}$

For a given value of D, the quantity $\sqrt{\lambda D}$ is called the size of Fresnel zone and is denoted by d_F.

i.e., $d_F = \sqrt{\lambda D}$

Hence the concepts of ray optics can be conveniently used without introducing any appreciable error if the size of the aperture is greater than the size of the Fresnel zone,

i.e., $d > d_F$ .....

Examples based on Diffraction of Light and Fresnel's distance

◆ Formula Used

1. For diffraction at a single slit of width d,

(i) Condition for nth minimum is

$d \sin\theta = n\lambda$ where $n = 1, 2, 3, \ldots$

(ii) Condition of nth secondary maximum is

$d \sin\theta = (2n + 1)\frac{\lambda}{2}$, where $n = 1, 2, 3, \ldots$

(iii) Angular position or direction of nth minimum, $\theta_n = \frac{n\lambda}{d}$

(iv) Distance of nth minimum from the centre of the screen,

$x_n = \frac{nD\lambda}{d}$

(v) Angular position of nth secondary maximum,

$\theta_n' = (2n + 1)\frac{\lambda}{2d}$

(vi) Distance of nth secondary maximum from the centre of the screen, $x_n' = (2n + 1)\frac{D\lambda}{2d}$

(vii) Width of central maximum, $\beta_0 = 2\beta = \frac{2D\lambda}{d}$

(viii) Angular spread of central maximum on either side, $\theta = \pm\frac{\lambda}{d}$

(ix) Total angular spread of central maximum, $2\theta = \frac{2\lambda}{d}$

2. For diffraction at a circular aperture of diameter d,

(i) Angular spread of central maximum,

$\theta = \frac{1.22\lambda}{d}$

(ii) Linear spread, $x = D\theta$

(iii) Areal spread, $x^2 = (D\theta)^2$

Where D is the distance at which the effect is considered.

3. Fresnel distance, $D_F = \frac{d^2}{\lambda}$

4. Size of Fresnel zone, $d_F = \sqrt{\lambda D}$

◆ Units Used

Angles θ, θ_n and θ_n' are in radian, wavelength λ in metre, distances d, d_F, D and D_F in metre and n is a pure number.

Q. 1. ***Fraunhofer diffraction from a single slit of width 1.0 μm is observed with light of wavelength 500 nm. Calculate the half angular width of the central maximum.***

Sol. The Fraunhofer diffraction is the diffraction of plane wavefronts from a single slit.

Here $d = 1.0\ \mu m = 1.0 \times 10^{-6}$ m, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m

Half angular width θ of the central maximum is given by

$$\sin\theta = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{1.0 \times 10^{-6}} = 0.5 \qquad \therefore \qquad \theta = 30°$$

Q. 2. ***Light of wave length 600 nm falls normally on a slit of width 1.2 μm producing Fraunhoffer diffraction pattern on a screen. Calculate the angular position of the first minimum and the angular width of the central maximum.***

Sol. Here $\lambda = 600$ nm $= 600 \times 10^{-9}$ m, $d = 1.2\ \mu m = 1.2 \times 10^{-6}$ m

The angular position θ of the first dark fringe is given by

$$\sin\theta = \frac{\lambda}{d} = \frac{600 \times 10^{-9}}{1.2 \times 10^{-6}} = \frac{1}{2} \qquad \therefore \qquad \theta = 30°.$$

Angular width of central maximum = $2\theta = 60°$

Q. 3. ***Microwaves of frequency 24,000 MHz are incident normally on a rectangular slit of width 5 cm. Calculate the angular spread of the central maximum of the diffraction pattern of the slit.***

Sol. Here $f = 24{,}000$ MHz $= 24 \times 10^9$ Hz, d = 5 cm, 5×10^{-2} m

Angular spread of central maximum is

$$2\theta = \frac{2\lambda}{d} = \frac{2c}{d\,v} = \frac{2 \times 3 \times 10^8}{5 \times 10^{-2} \times 24 \times 10^9} = \frac{1}{2}\text{ rad}$$

Q. 4. ***A slit of width 'd' is illuminated by red light of wavelength 6500 Å. For what value of 'd' will (i) the first minimum fall at an angle of diffraction of 30° and (ii) the first maximum fall at an angle of diffraction of 30°.***

Sol. (i) For first minimum of the diffraction pattern,

$$d \sin\theta = \lambda$$

$$\therefore \quad d = \frac{\lambda}{\sin\theta} = \frac{6{,}500 \times 10^{-10}\text{ m}}{\sin 30°}$$

$$= \frac{6{,}500 \times 10^{-10}\text{ m}}{0.5} = 1.3 \times 10^{-6}\text{ m}$$

(ii) For first secondary maximum of the diffraction pattern,

$$d \sin\theta = \frac{3\lambda}{2}$$

$$\therefore \quad d = \frac{3\lambda}{2\sin\theta} = \frac{3 \times 6{,}500 \times 10^{-10}\text{ m}}{2 \times \sin 30°}$$

$$= 1.95 \times 10^{-6}\text{ m.}$$

Q. 5. ***Light of wavelength 550 nm is incident as parallel beam on a slit of width 0.1 mm. Find the angular width and the linear width of the principal maxima in the resulting diffraction pattern on a screen kept at a distance of 1.1 m from the slit. Which of these widths would not change if the screen were moved to a distance of 2.2 m from the slit?***

Sol. Here $\lambda = 550$ nm $= 550 \times 10^{-9}$ m, d = 0.1 mm $= 0.1 \times 10^{-3}$ m, D = 1.1 m

Angular width of principal maximum, $\quad 2\theta = \frac{2\lambda}{d} = \frac{2 \times 550 \times 10^{-9}}{0.1 \times 10^{-3}} = 1.1 \times 10^{-2}$ rad

Linear width of principal maximum, $\quad \beta_0 = \frac{2D\lambda}{d} = 1.1 \times 0.011 = 0.0121$ m = 12.1 mm.

When the screen is moved to a distance of 2.2 m, the angular width would not change because it is independent of this distance D.

Q. 6. ***A screen is placed 2 m away from a single narrow slit. Calculate the slit width if the first minimum lies 5 mm on either side of central maximum. Incident plane waves have a wavelength of 5000 Å.***

Sol. Here D = 2 m, $x_1 = 5$ mm $= 5 \times 10^{-3}$ m, $\lambda = 5000$ Å $= 5 \times 10^{-7}$ m

Linear width of central maximum, $\beta_0 = 2x_1 = 2 \times 5 \times 10^{-3}$ m $= 10^{-2}$ m

Slit width, $d = \frac{2D\lambda}{\beta_0} = \frac{2 \times 2 \times 5 \times 10^{-7}}{10^{-2}}$

$= 2 \times 10^{-4}$ m = 0.2 mm.

Q. 7. ***A parallel beam of light of wavelength 600 nm is incident normally on a slit of width 'd'. If the distance between the slits and the screen is 0.8 m and the distance of 2nd order maximum from the centre of the screen is 15 mm, calculate the width of the slit.***

Sol. Distance of 2nd order maximum from the centre of the screen,

$$x_2' = \frac{5}{2}\frac{D\lambda}{d} \quad \text{or} \quad d = \frac{5}{2}\frac{D\lambda}{x_2'}$$

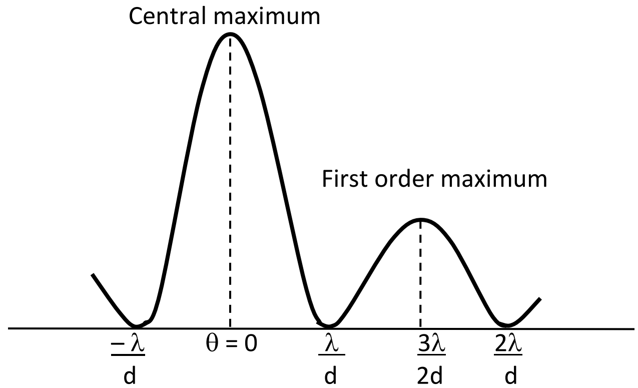
Given $\lambda = 600\text{ nm} = 6 \times 10^{-7}$ m, D = 0.8 m,

$x_2' = 15\text{ mm} = 15 \times 10^{-3}$ m

∴ $$d = \frac{5 \times 0.8 \times 6 \times 10^{-7}}{2 \times 15 \times 10^{-3}} = 8 \times 10^{-5}\text{ m} = 8\ \mu\text{m}$$

Q. 8. ***Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0.25 mm when light of wavelength 5890 Å is incident on it normally.***

Sol. From Fig., it is clear that the angular separation between central maximum and first order minimum is



$$\theta = \frac{3\lambda}{2d} - 0 = \frac{3\lambda}{2d}$$

or $$\theta = \frac{3 \times 5890 \times 10^{-10}}{2 \times 0.25 \times 10^{-3}} = 3.534 \times 10^{-3}\text{ rad}$$

Q. 9. ***Parallel light of wavelength 5000 Å falls normally on a single slit. The central maximum spreads out to 30° on either side of the incident light. Find the width of the slit. For what width of the slit the central maximum would spread out to 90° from the direction of the incident light?***

Sol. Angular spread of central maximum on either side of incident light is given by

$$\sin\theta = \frac{\lambda}{d}$$

∴ Slit width, $$d = \frac{\lambda}{\sin\theta} = \frac{5000 \times 10^{-10}}{\sin 30°} = 10^{-6}\text{ m}$$

For θ = 90°, we have $$d = \frac{\lambda}{\sin 90°} = \frac{5000 \times 10^{-10}}{1} = 5 \times 10^{-7}\text{ m.}$$

Q. 10. ***A slit of width 0.025 mm is placed in front of a lens of focal length 50 cm. The slit is illuminated with light of wavelength 5900 Å. Calculate the distance between the centre and first dark band of diffraction pattern obtained on a screen placed at the focal plane of the lens.***

Sol. Here $\lambda = 5900\text{ Å} = 59 \times 10^{-8}$ m, f = 50 cm = 0.50 m, d = 0.025 mm = 2.5×10^{-5} m

For first dark band, $\sin\theta = \frac{\lambda}{d}$

As the diffraction pattern is obtained in the focal plane of lens, therefore

$$\tan\theta = \frac{x}{f}$$

where x is the distance between the centre and the first dark band.

For small θ, $\tan\theta \simeq \sin\theta$ or $\frac{x}{f} = \frac{\lambda}{d}$

∴ $$x = \frac{\lambda}{d} \times f = \frac{59 \times 10^{-8} \times 0.50}{2.5 \times 10^{-5}}$$

$= 11.8 \times 10^{-3}\text{ m} = 11.8\text{ mm.}$

Q. 11. ***Two wavelength of sodium light 590 nm, 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture 2×10^{-4} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of first maximum of the diffraction pattern obtained in the two cases.***

Sol. Here $\lambda_1 = 590 \text{ nm} = 590 \times 10^{-9}$ m

$\lambda_2 = 596 \text{ nm} = 596 \times 10^{-9}$ m, $d = 2 \times 10^{-4}$ m, $D = 1.5$ m

Distance of first secondary maximum from the centre of the screen is

$$x = \frac{3}{2}\frac{D\lambda}{d}$$

For the two wavelengths, we have

$$x_1 = \frac{3}{2}\frac{D\lambda_1}{d} \quad \text{and} \quad x_2 = \frac{3}{2}\frac{D\lambda_2}{d}$$

Spacing between the first two maximum of sodium lines

$$= x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 \times 10^{-9} - 590 \times 10^{-9})$$

$$= \frac{3 \times 1.5 \times 6 \times 10^{-3}}{4}$$

$$= 6.75 \times 10^{-3} \text{ m} = 6.75 \text{ mm}$$

Q. 12. ***In Young's double slit experiment, the distance d between the slits S_1 and S_2 is 1 mm. What should the width of each slit be so as to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern?***

Sol. The linear separation between n bright fringes in an interference pattern on the screen is given by

$$x_n = \frac{n\lambda D}{d}$$

As $x_n << D$, the angular separation between n bright fringes should be

$$\theta_n = \frac{x_n}{D} = \frac{n\lambda}{d}$$

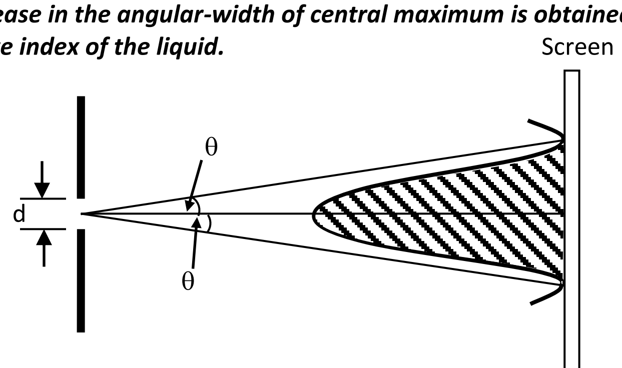
For 10 bright fringes, we get, $\theta_{10} = \frac{10\,\lambda}{d}$

The angular width of the central maximum in the diffraction pattern due to slit of width a is

$$2\,\theta_1 = \frac{2\lambda}{a}$$

We want $10\,\frac{\lambda}{d} < 2\,\frac{\lambda}{a}$ or $a \le \frac{d}{5} = \frac{1}{5}$ mm = 0.2 mm.

Q. 13. ***Angular width of a central maximum in the Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30 %. Calculate the wavelength of this light. The same decrease in the angular-width of central maximum is obtained when the original apparatus is immersed in a liquid. Find refractive index of the liquid.***



Sol. In single slit diffraction, first minimum occurs at

$$d \sin\theta = \lambda \quad \text{or} \quad \sin\theta = \frac{\lambda}{d}$$

As $\lambda << d$, so $\theta \simeq \sin\theta = \frac{\lambda}{d}$

Angular width of central maximum is $\phi = 2\theta = \frac{2\,\lambda}{d}$

$$\therefore \quad \frac{\phi_2}{\phi_1} = \frac{\lambda_2}{\lambda_1}$$

or $\lambda_2 = \frac{\phi_2}{\phi_1} . \lambda_1 = \frac{70}{100} \times 6000 = 4200$ Å $\quad [\because \phi_2 = 70\,\%$ of $\phi_1]$

When the apparatus is immersed in the liquid, the decrease in angular width is same. This indicates that the wavelength of light in the liquid is also 4200 Å.

$$\mu = \frac{\lambda}{\lambda_l} = \frac{6000}{4200} = 1.43$$

Q. 14. ***A laser operates at a frequency of 3×10^{14} Hz and has an aperture of 10^{-2} m. What will be the angular spread?***

Sol. Here $\nu = 3 \times 10^{14}$ Hz, $d = 10^{-2}$ m, $c = 3 \times 10^8$ m s^{-1}

Wavelength, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3 \times 10^{14}} = 10^{-6}$ m

$\therefore$ Angular spread, $\theta = \frac{1.22\,\lambda}{d} = \frac{1.22 \times 10^{-6}}{10^{-2}}$

$= 1.22 \times 10^{-4}$ rad.

Q. 15. ***A laser beam has a wavelength of 7×10^{-7} m and aperture 10^{-2} m. The beam is sent to moon, the distance of which from earth is 4×10^5 km. Find (i) the angular spread and (ii) areal spread when the beam reaches the moon.***

Sol. Here $\lambda = 7 \times 10^{-7}$ m, $d = 10^{-2}$ m, $D = 4 \times 10^5$ km $= 4 \times 10^8$ m

For the circular aperture, we have

(i) Angular spread,

$\theta = \frac{1.22\,\lambda}{d}$

$= \frac{1.22 \times 7 \times 10^{-7}}{10^{-2}}$

$= 8.54 \times 10^{-5}$ rad.

(ii) Areal spread

$= (D\theta)^2 = (4 \times 10^8 \times 8.54 \times 10^{-5})^2$

$= 1.197 \times 10^9$ m^2

Q. 16. ***A laser light beam of power 20 mW is focused on a target by a lens of focal length 0.05 m. If the aperture of the laser be 1 mm and the wavelength of its light 7000 Å, calculate the angular spread of the laser, the area of the target hit by it, and the intensity of the impact on the target.***

Sol. Here P = 20 mW = 20×10^{-3} W, f = 0.05 m, d = 1 mm = 10^{-3} m, λ = 7000 Å = 7000×10^{-10} m

(a) Angular spread of the laser beam,

$\theta = \frac{1.22\,\lambda}{d} = \frac{1.22 \times 7000 \times 10^{-10}}{1 \times 10^{-3}}$

$= 8.54 \times 10^{-4}$ radian.

(b) Linear spread of the laser = f . $\theta = 5 \times 10^{-2} \times 8.54 \times 10^{-4}$ m

$\therefore$ Linear spread of the laser, i.e., area of the target hit by it

$= (5 \times 8.54 \times 10^{-6})^2$

$= 1.832 \times 10^{-15}$ m^2

(c) Intensity of impact of the laser on the target

$= \frac{\text{Power of laser}}{\text{Area hit}} = \frac{20 \times 10^{-3}}{1.832 \times 10^{-15}}$

$= 10.97 \times 10^{12}$ Wm^{-2}

Q. 17. ***Calculate the distance that a beam of light of wavelength 500 nm can travel without significant broadening, if the diffraction aperture is 3 mm wide.***

Or

For what distance is ray optics a good approximation when the aperture is 3 mm wide and the wavelength is 500 nm?

Sol. Here d = 3 mm = 3×10^{-3} m, λ = 500 nm = 500×10^{-9} m

The distance up to which a beam of light can travel without significant broadening is called Fresnel distance and its value is given by $D_F = \frac{d^2}{\lambda} = \frac{(3 \times 10^{-3})^2}{500 \times 10^{-9}} = 18$ m

Q. 18. ***Light of wavelength 5×10^{-7} m is diffracted by an aperture of width 2×10^{-3} m. For what distance travelled by the diffracted beam does the spreading due to diffraction become greater than the width of the aperture?***

Sol. Fresnel distance,

$D_F = \frac{d^2}{\lambda} = \frac{(2 \times 10^{-3})^2}{5 \times 10^{-7}} = 8$ m

So, at a distance greater than 8 m, the spreading due to diffraction becomes greater than the width of the aperture.

Q. 19. ***Light of wavelength 600 nm is incident on an aperture of size 2 mm. Calculate the distance up to which light can travel such that its spread is less than the size of the aperture.***

Sol. Fresnel distance,

$D_F = \frac{d^2}{\lambda} = \frac{(2 \times 10^{-3})^2}{600 \times 10^{-9}} = 6.67$ m.

So, at a distance less than 6.67 m, the spreading of light is less than the size of the aperture.

DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION

Interference	Diffraction
1. Interference is the result of superposition of secondary waves starting from two difference wavefronts originating from two coherent sources.	Diffraction is the result of superposition of secondary waves starting from different parts of the same wavefront.
2. All bright and dark fringes are of equal width.	The width of central bright fringe is twice the width of any secondary maximum.
3. All bright fringes are of same intensity.	Intensity of bright fringes decreases as we move away from central bright fringe on either side.
4. Regions of dark fringes are perfectly dark. So, there is a good contrast between bright and dark fringes.	Regions of dark fringes are not perfectly dark. So, there is a poor contrast between bright and dark fringes.
5. At an angle of λ/d, we get a bright fringe in the interference pattern of two narrow slits separated by a distance d.	At an angle of λ/d, we get the first dark fringe in the diffraction pattern of a single slit of width d.

DIFFRACTION AS A LIMIT ON RESOLVING POWER

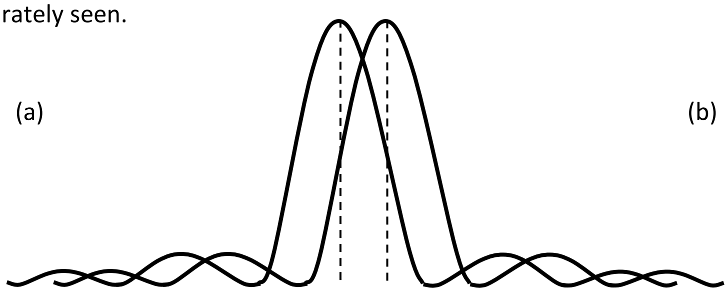
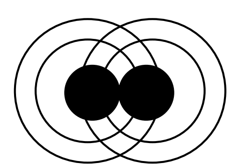
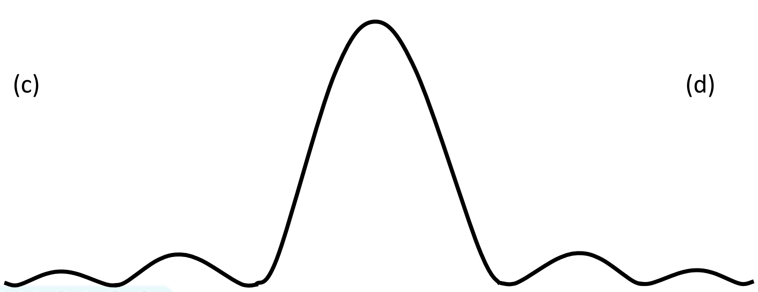
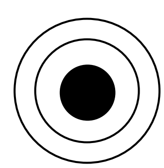
Diffraction as a limit on resolving power: All optical instruments like lens, telescope, microscope etc., act as apertures. Light on passing through them undergoes diffraction. This puts the limit on their resolving power. Suppose a lens is used to form the image of an object. We can think of the lens as a circular aperture. The image of each point is set of alternate bright and dark circular fringes with a bright disc at the centre. The size of this disc depends on the aperture of the lens and the wavelength of light used. If we have two nearby point objects, their images may give rise to diffraction patterns which overlap on each other, making the identification or resolution of the two objects difficult.

Limit of resolution: **The smallest linear or angular separation between two points objects at which they can be just separately seen or resolved by an optical instrument is called the limit of resolution of the instrument**.

Resolving power: The resolving power of an optical instrument is its ability to resolve or separate the images of two nearby point objects so that they can be distinctly seen. **It is equal to the reciprocal of the limit of resolution of the optical instrument.**

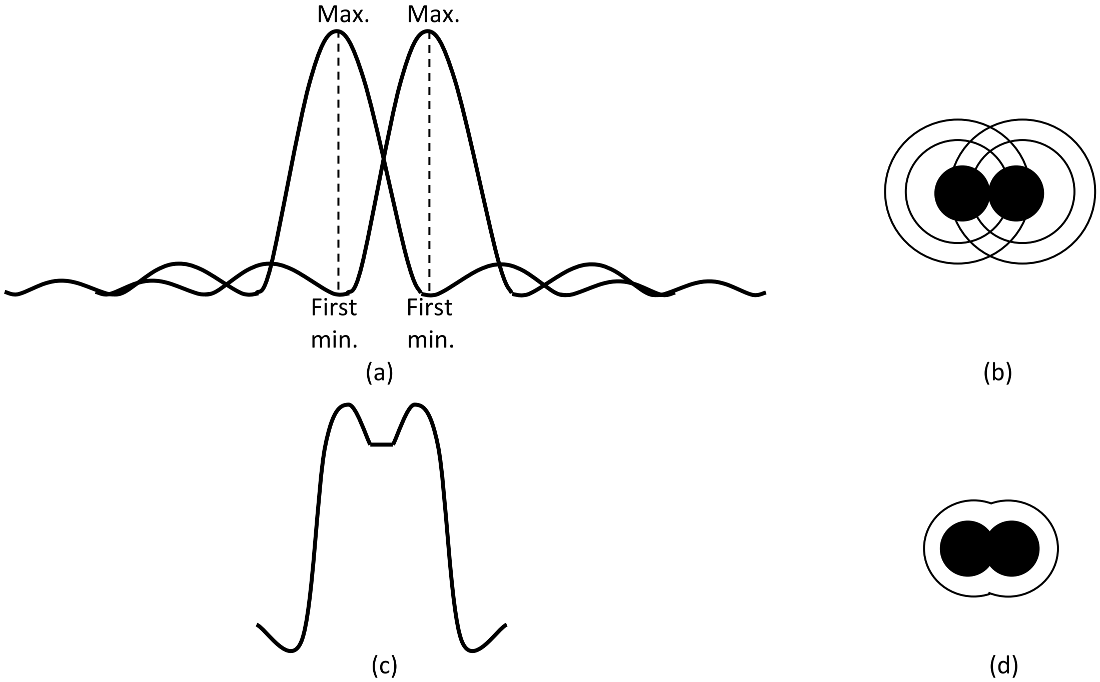
The smaller the limit of resolution of an optical instrument

Rayleigh's criterion for resolution: If we look through a telescope at two stars lying closed together, their different patterns overlap and the resultant pattern is little broader but otherwise similar to that of a single star, as shown in Fig. So the two stars cannot be resolved or separately seen.

(a)

(b)

(c)

(d)


[(a), (b) Overlapping of diffraction patterns of two close print objects. (c), (d) their resultant diffraction patterns]

According to Rayleigh's criterion, the images of two-point objects are just resolved when the central maximum of the diffraction pattern of one fall over the first minimum of the diffraction pattern of the other, as shown in Fig. When seen through the telescope, the resultant diffraction has a well-marked depression at the top, showing that these are really two stars and not one. Thus, the images of two stars have been just resolved.



[Rayleigh's criterion for resolution]

RESOLVING POWER OF MICROSCOPE AND TELESCOPE

Resolving power of a microscope: **The resolving power of a microscope is defined as reciprocal of the smallest distance between two point objects at which they can be just resolved when seen through the microscope**.

The smallest distance between two points objects at which they can be just resolved by the microscope, or the limit of resolution, is given by

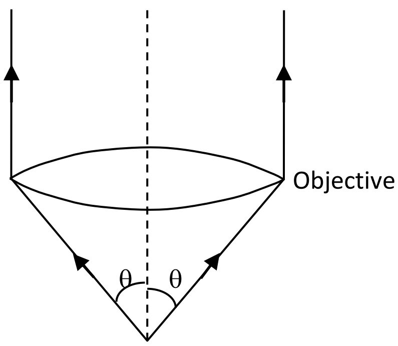
$$d = \frac{\lambda}{2\mu \sin\theta}$$

∴ Resolving power of a microscope $= \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$

Here, λ = the wavelength of light used,

θ = half the angle of cone of light from each point object or the angle subtended by each point object on the radius of the objective [Fig]

μ = the refractive index of the medium between the point object and the objective of the microscope.



The factor $\mu \sin\theta$ is called the numerical aperture (for eye, $\mu \sin\theta = 0.004$).

The smaller the limit of resolution 'd', the greater will be the resolving power. The resolving power of a microscope increases when an oil of high refractive index (μ) is used between the object and the objective (called the oil immersion objective) of the microscope.

Resolving power of a telescope: The resolving power of a telescope of defined as the reciprocal of the smallest angular separation between two distant objects whose images can be just resolved by it.

The smallest linear angular separation between two distant objects whose images can be just resolved by the telescope, or the limit of resolution, is given by

$$d\theta = \frac{1.22\,\lambda}{D}$$

∴ Resolving power of a telescope = $\frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$

Here λ = the wavelength of light,
D = the diameter of the telescope objective, and

$d\theta$ = the angle subtended by the two distant objects at the objective.

Thus, larger the aperture of the objective and smaller the wavelength of light used, the greater will be the resolving power of the telescope.

Resolving power of human eye: The diameter of the pupil of human eye is about 2 mm. If we take λ = 5000 Å, then the smallest angular separation between two distant point objects that the human eye can resolve will be

$$d\theta = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 5000 \times 10^{-10}}{2 \times 10^{-3}}$$

$$= 0.305 \times 10^{-3} \text{ rad} \approx 1'$$

Thus, the human eye can see two points objects distinctly if they subtended, at the eye, and angle equal to one minute of arc. This angle is called the limit of resolution of the eye. The reciprocal of this angle of limit of resolution gives the resolving power of the eye.

Further, if d is the separation between two point objects at a distance of 1 km which can be just resolving by human eyes, then

$$0.305 \times 10^{-3} = \frac{d}{10^3}$$

or $d = 0.305$ m = 30.5 cm

Thus, the human eyes can see two objects separated by 30 cm just resolved from a distance of 1 km.

Examples based on Resolving Power of (i) Telescope (ii) Microscope

◆ **FORMULA USED**

1. Limit of resolution of a telescope, $d\theta = \frac{1.22\,\lambda}{D}$
2. Resolving power of a telescope = $\frac{1}{d\,\theta} = \frac{D}{1.22\,\lambda}$

   Where D = diameter of the objective lens.
3. Limit of resolution of a microscope, $d = \frac{\lambda}{2\,\mu \sin\theta}$
4. Resolving power of a microscope = $\frac{1}{d} = \frac{2\,\mu \sin\theta}{\lambda}$

   Where θ = half angle of cone of light from the point object. The factor $\mu \sin\theta$ is called numerical aperture (N.A.).

◆ **UNITS USED**

Lengths λ, D and d are in metre while angle θ and $d\theta$ are in radian.

Q. 1. ***Assume that light of wavelength 6000 Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 inch?***

Sol. The limit of resolution of a telescope,

$$d\theta = \frac{1.22\,\lambda}{D}$$

Here D = 100 inch = 254 cm, λ = 6000 Å = 6×10^{-5} cm

∴ $d\theta = \frac{1.22 \times 6 \times 10^{-5}}{254} = 2.9 \times 10^{-7}$ rad.

Q. 2. ***A telescoping is used to resolve two stars separated by 4.6×10^{-6} rad. If the wavelength of light used is 5460 Å, what would be the aperture of the objective of the telescope?***

Sol. Here $d\theta = 4.6 \times 10^{-6}$ rad, λ = 5460 Å = 5460×10^{-10} m

As $d\theta = \frac{1.22\,\lambda}{D}$

∴ Aperture of the telescope, $D = \frac{1.22\,\lambda}{d\theta} = \frac{1.22 \times 5460 \times 10^{-10}}{4.6 \times 10^{-6}} = 0.1488$ m

Q. 3. ***The objective of an astronomical telescope has a diameter of 150 mm and a focal length of 4.0 m. The eyepiece has a focal length of 25.0 mm. Calculate the magnifying and resolving powers of the telescope. What is the distance between the objective and the eyepiece? Take $\lambda = 6000$ Å***

Sol. Assume that the final image is formed at infinity. Then

Magnifying power, $m = \frac{f_0}{f_e} = \frac{4}{25 \times 10^{-4}} = 160$

Resolving power $= \frac{D}{1.22\,\lambda} = \frac{150 \times 10^{-3}}{1.2 \times 6000 \times 10^{-10}}$

$= 2.049 \times 10^5$

Distance between objective and eyepiece

$= f_0 + f_e = 4 + 25 \times 10^{-3} = 4.025$ m.

Q. 4. ***Calculate the separation of two points on the moon that can be resolved using 600 cm telescope. given the distance of the moon from the earth is 3.8×10^{10} cm. The wavelength most sensitive to the eye is 5.5×10^{-5} cm.***

Sol. Here D = 600 cm, $\lambda = 5.5 \times 10^{-5}$ cm

Limit of resolution, $d\theta = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-5}}{600} = 1.1 \times 10^{-7}$ rad

Let x be the distance between two points on the moon and d be the distance between the moon and the objective of the telescope. Then $d\,\theta = \frac{x}{D}$

or $x = d \times d\theta = 3.8 \times 10^{10} \times 1.1 \times 10^{-7} = 4180$ cm.

Q. 5. ***A telescope has an objective of diameter 60 cm. The focal lengths of the objective and eye-piece are 2.0 m and 1.0 cm respectively. The telescope is directed to view two distant almost point sources of light (e.g. two stars of a binary). The sources are at roughly the same distance (= 10^4 light years) along the line of sight but are separated transverse to the line of sight by a distance of 10^{10} m. Will the telescope resolve the two objects i.e. will it see two distinct stars?***

Sol. Separation between the two stars is $y = 10^{10}$ m

Distance of the stars from the earth is

$x = 10^4$ light years $= 10^4 \times 9.47 \times 10^{15}$ m

∴ Angle subtended by the line joining the two stars on the objective lens (or on eye) is

$d\,\theta = y = \frac{10^{10}}{10^4 \times 98.47 \times 10^{15}} = 0.106 \times 10^{-9}$ rad

Now diameter of objective, D = 60 cm = 0.60 m

For mean yellow colour, $\lambda = 600$ nm $= 6 \times 10^{-7}$ m

According to Rayleigh's criterion, the limit of resolution of the telescope is

$d\theta' = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 6 \times 10^{-7}}{0.60}$

$= 1.22 \times 10^{-6}$ rad.

As the angle $d\theta$ subtended by the transverse separation of the two stars is much too small compared to the limit of resolution $d\theta'$, hence the two stars of the binary cannot be resolved by the given telescope.

Q. 6. ***Calculate the resolving power of a microscope if its numerical aperture is 0.12 and the wavelength of light used is 6000 Å.***

Sol. Here

N.A. = 0.12, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m

R.P. of microscope $= \frac{2 \times \text{N.A.}}{\lambda} = \frac{2 \times 0.12}{6 \times 10^{-7}} = 4 \times 10^5\ \text{m}^{-1}$.

Q. 7. ***Calculate the numerical aperture of a microscope required to just resolve two points separated by a distance of 10^{-4} cm, using light of wavelength 5.8×10^{-5} cm.***

Sol. Here $\lambda = 5.8 \times 10^{-5}$ cm, $d = 10^{-4}$ cm

As $d = \frac{\lambda}{2 \times \text{N. A.}}$

$\text{N.A.} = \frac{\lambda}{2\,d} = \frac{5.8 \times 1.5^{-5}}{2 \times 10^{-4}} = 0.29$

Q. 8. ***The smallest object detail that can be resolved with a certain microscope with light of wavelength 6000 Å is 3.5 × 10⁻⁵ cm. Find the numerical aperture of the objective (i) when used dry and (ii) when immersed in an oil of refractive index 1.5.***

Sol. Here $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m, $d = 3.5 \times 10^{-5}$ cm $= 3.5 \times 10^{-7}$ m

(i) When the objective is used dry,

$$\text{N.A.} = \frac{\lambda}{2d} = \frac{6 \times 10^{-7}}{3.5 \times 10^{-7}} = 0.86$$

(ii) When the objective is immersed in an oil of refractive index 1.5,

$$\text{N.A.} = \mu \times \text{dry aperture} = 1.5 \times 0.86 = 1.44$$

Q. 9. ***Assuming the diameter of the eye pupil to be 2.0 mm, calculate the smallest angular separation at which two point objects can be distinctly seen when viewed in light of wavelength 6000 Å.***

Sol. Here $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m, $d = 2.0$ mm $= 2.0 \times 10^{-3}$ m

The limit of resolution of the eye,

$$d\theta = \frac{1.22\,\lambda}{d} = \frac{1.22 \times 6 \times 10^{-7}}{2.0 \times 10^{-3}} = 3.66 \times 10^{-4} \text{ rad}$$