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RAY OPTICS
OPTICAL INSTRUMENTS

IN THIS CHAPTER ....

Light is a form of energy which makes objects visible to our eyes. The branch of physics, which deals with nature of light, its sources, properties, measurement, effects and vision is called **optics**. This branch can be divided into two sub-branches namely (i) **geometrical optics** or **ray optics**, and (ii) **wave optics**.

Ray optics deals with the propagation of light in terms of rays, whereas wave optics deals with the wave phenomenon.

Reflection of Light

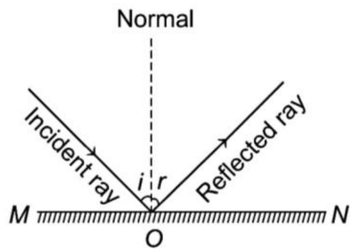
It is the phenomenon of bouncing back of light rays in the same medium on striking a smooth surface.

After reflection, the frequency, speed and wavelength do not change, but a phase change may occur depending on the nature of reflecting surface.

Laws of Reflection

(i) The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.

(ii) The angle of incidence, $\angle i$ = angle of reflection, $\angle r$.



Note In geometrical optics, angles are measured with normal.

Image

If rays emanating from a point actually meet at another point after reflection and/or refraction, that point is called the **image of the first point**.

(i) The image is **real**, if the rays actually converge to the point.

(ii) The image is **virtual**, if the rays do not actually meet but appear to diverge from the point when produced backwards.

Reflection from Plane Mirror (Surface)

In case of reflection from plane surface such as plane mirror

- The image is always erect, virtual, of same size and at the same distance from the mirror as the distance of the object from mirror.
- The image is laterally inverted.
- If keeping the incident ray fixed, the plane mirror is rotated through an angle θ, the reflected ray turns through an angle 2θ in that direction only.
- If the object is fixed and the mirror moves relative to the object with a speed v, the image moves with a speed $2v$ relative to the object.
- If the mirror is fixed and the object moves relative to the mirror with a speed v, the image also moves with the same speed v relative to the mirror.
- Deviation produced by plane mirror is given by $\delta = 180° - 2i$.
- The minimum size of a mirror required to see the full image of a person, is half the height of the person.
- If a plane mirror is rotated about an axis perpendicular to plane of mirror, then reflected ray image do not rotate.
- The minimum height of a plane mirror to see object's full height in it is $H/2$, where H is the height of object.
- Minimum height of the plane mirror fixed on the wall of a room in which an observer at the centre of the room can see the full image of the wall behind him, is one-third the height of wall.
- If plane mirror moves a distance x towards or away from the object, the image will move a distance $2x$ towards or away from the mirror/object.
- If both plane mirror and object are moved by a distance x, each in opposite directions, the image will be displaced by a distance $3x$ in the direction of the displacement of the mirror.
- If a luminous object is placed in front of a thick glass mirror, multiple images are formed due to multiple reflections. The second image formed by the first reflection by the polished surface is much brighter than the others. The intensity of the other images rapidly fade away.
- If an object is placed between two mirrors facing each other at an angle θ, the number of images is given by $\frac{360°}{\theta} = N$ and actual number of images are n, where

  (a) $n = N - 1$, if N is even integer,

  (b) $n = N$, if N is odd integer and object is not on the bisector of mirrors,

  (c) $n = N - 1$, if N is odd integer and object is on the bisector of mirrors and

  (d) If $\frac{360°}{\theta} = N$ is a fraction, the number of images will be equal to its lower boundary of integral part.

Example 1. *Find velocity of image when object and mirror both are moving toward each other with velocity $2\,ms^{-1}$ and $3\,ms^{-1}$, respectively.*

(a) $8\ ms^{-1}$ (b) $-8\ ms^{-1}$

(c) $-5\ ms^{-1}$ (d) $5\ ms^{-1}$

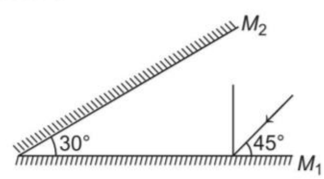
Sol. (a) Here, $v_{OM} = v_{IM}$

$$v_O - v_M = (v_I - v_M)$$

$$\Rightarrow \quad (+2ms^{-1}) - (-3ms^{-1}) = v_I + (-3\ ms^{-1})$$

$$\Rightarrow \quad v_I = 8\,ms^{-1}$$

Example 2. *Two plane mirrors are inclined at 30° as shown in figure. A light ray is incident at angle 45°. Find total deviation produced by combination of mirror after two successive reflections.*



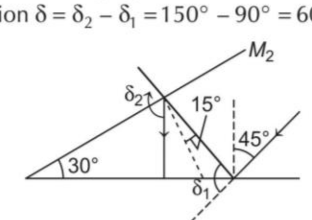
(a) 60° ↑ (b) 58° ↑

(c) 50° ↑ (d) 68° ↑

Sol. (a) Deviation at mirror M_1, $\delta_1 = 180° - 2 \times 45° = 90°$ ↑

Deviation at mirror M_2, $\delta_2 = 180° - 2 \times 15° = 150°$ ↑

Total deviation $\delta = \delta_2 - \delta_1 = 150° - 90° = 60°$ ↑

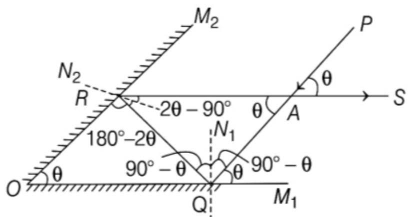


Example 3. *The plane mirrors M_1 and M_2 are inclined to each other such that a ray of light incident on mirror M_1 and parallel to the mirror M_2 is reflected from mirror M_2 parallel to the mirror M_1. The angle between the two mirror is* **[JEE Main 2019]**

(a) 45° (b) 75°

(c) 90° (d) 60°

Sol. (*d*) The given condition is shown in the figure given below, where two plane mirrors inclined to each other such that a ray of light incident on the first mirror M_1 and parallel to the second mirror M_2 is finally reflected from second mirror M_2 parallel to the first mirror.



where, PQ = incident ray parallel to the mirror M_2, QR = reflected ray from the mirror M_1, RS = reflected ray from the mirror M_2 which is parallel to the M_1 and θ = angle between M_1 and M_2.

According to geometry,

$$\angle PAS = \angle PQM_1 = \theta \text{ (angle on same line)}$$

$$\angle AQN_1 = \text{angle of incident} = 90° - \theta$$

$$\angle N_1QR = \text{angle of reflection} = (90° - \theta).$$

Therefore, for triangle ΔORQ, (according to geometry)

$$\angle\theta + \angle\theta + \angle ORQ = 180°$$

$$\angle ORQ = 180° - 2\theta$$

For normal N_2, angle of incidence (i) = angle of reflection (r)

$$= 2\theta - 90°$$

$$\therefore\ \angle ORA = i + r = 2\theta - 90° + 2\theta - 90° = 4\theta - 180°$$

Therefore, for the triangle ΔRAQ,

$$4\theta - 180° + 180° - 2\theta + \theta = 180° \Rightarrow 3\theta = 180° \Rightarrow \theta = 60°$$

Spherical Mirrors

Mirrors whose reflecting surface is a part of a hollow sphere is called spherical mirror. It is of two types namely; concave mirror and convex mirror.

Some terms related to spherical mirrors are as follows

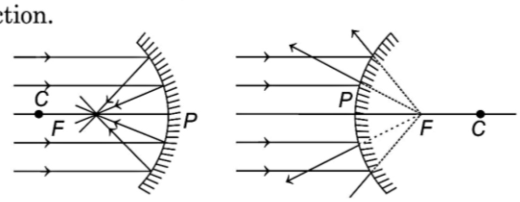
Pole (P) of the spherical mirror is geometric centre of its reflecting surface.

Centre of curvature (C) of a mirror is the centre of the sphere of which the mirror is a part.

Radius of curvature (R) is the distance between the pole and centre of curvature.

Principal axis of a spherical mirror is the line joining the pole and centre of curvature.

Principal focus (F) is a point on the principal axis of the mirror at which the light rays coming parallel to principal axis actually meet or appear to meet after reflection.



(a) Concave mirror (b) Convex mirror

Note For concave mirror focus, is infront of the mirror, while for convex mirror, focus is behind the mirror. Focus of concave mirror is real, while focus of convex mirror is virtual.

Focal length (f) is the distance between pole and focus of a spherical mirror.

i.e. $$f = \frac{R}{2}$$

Rules for Image Formation in Spherical Mirrors

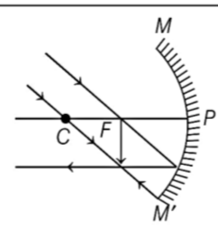
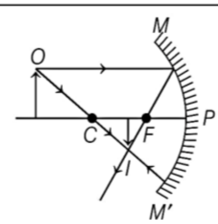
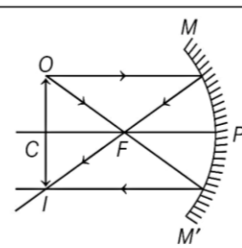
- A ray going through centre of curvature which strikes the mirror normally is reflected back along the same path.
- A ray parallel to principal axis after reflection either actually passes through the principal focus F or appear to diverge from it.
- A ray passing through the principal focus F or a ray which appears to converge at F is reflected parallel to the principal axis.
- A ray striking at pole P is reflected symmetrically back in the opposite side.

Sign Convention for Mirrors

According to cartesian sign convention,

- All distances are measured from the pole (P).
- Distances measured in the direction of incident ray are taken as positive, while in the direction opposite of incident ray are taken negative.
- Height measured upwards with respect to X-axis and normal to the principal axis (X-axis) of the mirror are taken as positive. Heights measured downwards are taken as negative.

Image Formation by Concave Mirror

Position of Object	Ray Diagram	Details of Image
At infinity		Real, inverted, very small, at F
Between infinity and C		Real, inverted, diminished, between F and C
At C		Real, inverted, equal in size, at C

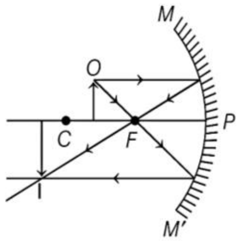
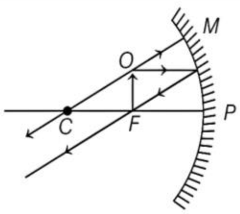
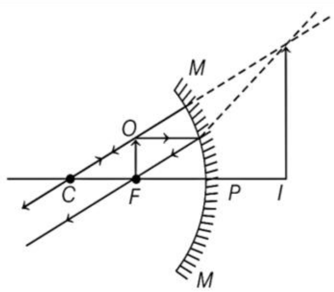
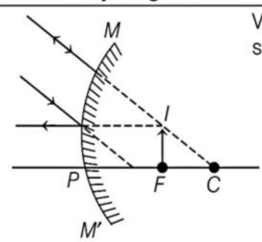
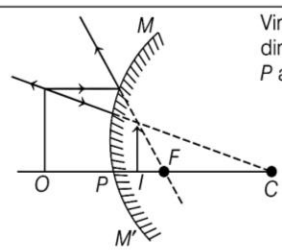
Position of Object	Ray Diagram	Details of Image
Between F and C		Real, inverted and very large, between $2F$ and infinity
At F		Real, inverted, very large, at infiinity
Between F and P		Virtual, erect, large in size, behind the mirror

Image Formation by Convex Mirror

Position of Object	Ray Diagram	Details of Image
At infinity		Virtual, erect, very small, at F
In front of mirror		Virtual, erect, diminished, between P and F

Mirror Formula

If an object is placed at a distance u from the pole of a mirror and its image is formed at a distance v from the pole, then according to the mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

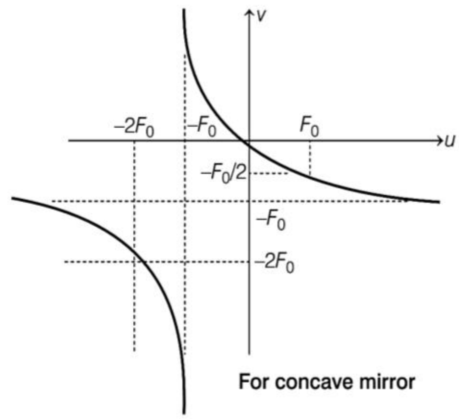
where, f is the focal length of the mirror.

Following are few important points related to mirror formula

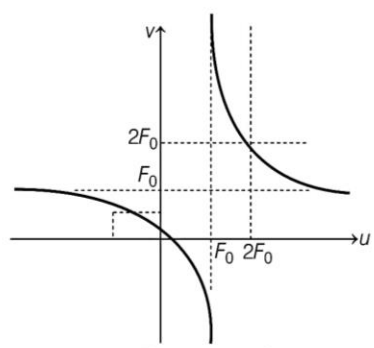
- In case of spherical mirrors, if object distance x_1 and image distance x_2 are measured from focus instead of pole, $u = (f + x_1)$ and $v = (f + x_2)$. The mirror formula reduces to,

  $$x_1 x_2 = f^2$$

  This formula is called **Newton's formula**.
- ***v* versus *u* graph for a plane mirror** For a spherical mirror v versus u graph is a rectangular hyperbola as shown in figure

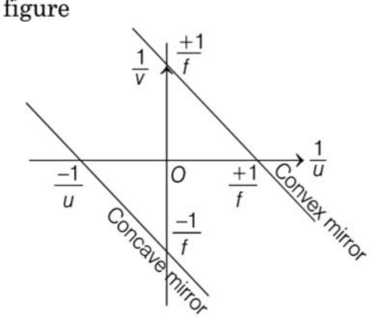


For concave mirror



For convex mirror

- **$\frac{1}{v}$ *versus* $\frac{1}{u}$ graph for a spherical mirror** For a spherical mirror $\frac{1}{v}$ *versus* $\frac{1}{u}$ graph is a straight line as shown in figure



$\frac{1}{v}$ *versus* $\frac{1}{u}$ graph for a spherical mirror

Magnification

It is the ratio of the height of the image (I) to height of the object (O).

Lateral (linear or transverse) **magnification,**

$$m = \frac{I}{O} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

Axial (longitudinal) **magnification,**

$$m_{ax} = -\frac{dv}{du} = \frac{v^2}{u^2} = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Also, length of image,

$$= \left(\frac{v}{u}\right)^2 \times \text{length of object } (L_O)$$

$$\Rightarrow \qquad L_I = \left(\frac{f}{u-f}\right)^2 \cdot L_O$$

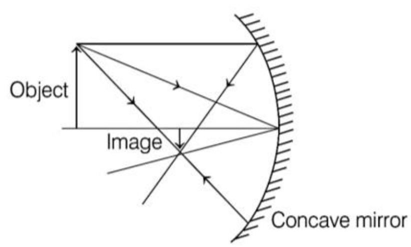
Areal magnification,

$$m_{ar} = \frac{A_I}{A_O} = \frac{v^2}{u^2}$$

$$= \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

where, A_I = area of image and A_O = area of object.

Example 4. *Lower half of concave mirror's reflecting surface is covered with an opaque (non-reflective) material. The intensity of an image of an object placed in front of the mirror is*



(a) *twice the original image*
(b) *half the original image*
(c) *same as the original image*
(d) *one-fourth the original image*

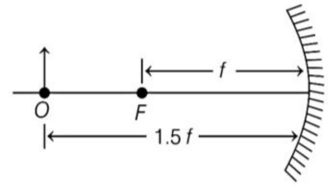
Sol. (b) One may think that the image will now show only half of the object, but according to the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low (in this case half).

Example 5. *An object of length 2.5 cm is placed at 1.5f from a concave mirror, where f is the focal length of the mirror. The length of the object is perpendicular to the principal axis. Find the length of the image. Is the image erect or inverted?*

(a) – 5 cm (b) 5 cm
(c) 6 cm (d) – 6 cm

Sol. (a) The focal length, $F = -f$

and $u = -1.5f$



From mirror formula, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or $$\frac{1}{-1.5f} + \frac{1}{v} = -\frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{1.5f} - \frac{1}{f} = -\frac{1}{3f}$$

Now, $$m = \frac{v}{u} = \frac{3f}{1.5f} = -2$$

or $$\frac{h_2}{h_1} = -2$$

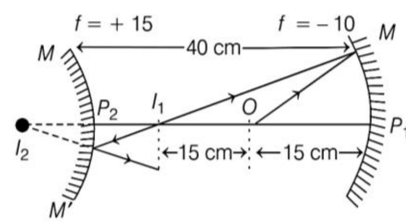
or $$h_2 = -2h_1 = -5 \text{ cm}$$

The image is 5 cm long. The minus sign shows that it is inverted.

Example 6. *A concave mirror of focal length 10 cm and a convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirrors, on their common axis and 15 cm from the concave mirror. Find the position of the image produced by the successive reflections, first at concave mirror and then at convex mirror.*

(a) -30 cm, 6 cm (b) -30 cm, $+10$ cm
(c) $+30$ cm, $+15$ cm (d) $+6$ cm, $+30$ cm

Sol. (a) According to given problem, for concave mirror.



Given, $u = -15$ cm

and $f = -10$ cm

So, $$\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}, \text{ i.e. } v = -30 \text{ cm}$$

i.e. Concave mirror will form real, inverted and enlarged image I_1 of object O at a distance 30 cm from it, *i.e.* at a distance $40 - 30 = 10$ cm from convex mirror.

For convex mirror, the image I_1 will act as an object and so for it $u = -10$ cm and $f = +15$ cm.

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{15}, \text{ i.e. } v = +6 \text{ cm}$$

So, final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual as shown in above figure.

Refraction of Light

It is the phenomena of bending of ray of light when they pass from one transparent medium to another depending on their optical densities.

Laws of Refraction

(i) The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

(ii) The ratio of the sine of angle of incidence i to the sine of angle of refraction r is constant, *i.e.*

$$\frac{\sin i}{\sin r} = n_{21} \text{ or } {}^1n_2$$

where, n_{21} is a constant, called the **refractive index** of the second medium with respect to the first medium.

This is known as **Snell's law of refraction.**

Note n_{21} is a characteristic of the pair of media (and also depend on the wavelength of light) but is independent of the angle of incidence.

Following are few important points related to refraction of light.

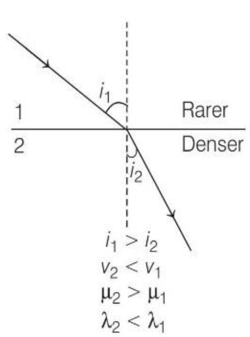
- When light travels from vacuum (or air) to any transparent medium, then refractive index of medium with respect to vacuum (or air) is called it's absolute refractive index, *i.e.* $\mu = \dfrac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \dfrac{c}{v}$
- If $n_{21} > 1, r < i$, then the refracted ray bend towards the normal. In such a case, medium 2 is said to be **optically denser than medium 1.**
- If $n_{21} < 1, r > i$, the refracted ray bend away from the normal. This is the case when incident ray in a denser medium refracts into a rarer medium.
- We can also write Snell's law as $\mu \sin i$ = constant

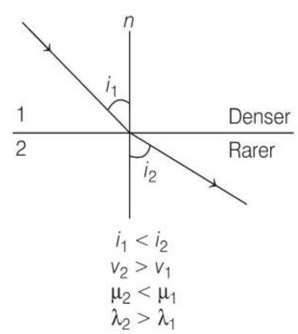
  For two media, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

  Also, $${}_1\mu_2 = \frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_1}{\mu_2}$$

  where, v_1 is the speed of light in medium 1 and v_2 in medium 2.

  Similarly, λ_1 and λ_2 are the corresponding wavelengths.





If $\mu_2 > \mu_1$, then $v_1 > v_2$ and $\lambda_1 > \lambda_2$, *i.e.*, in a rarer medium, speed and hence, wavelength of light is more.

- For n mediums, ${}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_4 \times \ldots \times (n-1)\mu_n = {}_1\mu_n$

  It is called **reversibility of light**. According to this principal, when a light ray, after suffering n number of reflections and refractions, will has its final path reversed, *i.e.*, it travels back along its entire initial path.
- Experiments show that, if the boundaries of the media are **parallel** the emergent ray CD, although laterally displaced. If reaches in the same medium, is parallel to the incident ray, however it is laterally displaced.

Example 7. *Refractive index of glass with respect to water is 9/8. Refractive index of glass with respect to air is 3/2. Find the refractive index of water with respect to air.*

(a) 3/4 (b) 4/3 (c) 9/8 (d) 8/9

Sol. (b) Given, ${}_w\mu_g = 9/8$ and ${}_a\mu_g = 3/2$

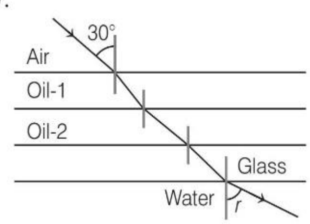
As, $${}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1$$

$$\therefore \quad \frac{1}{{}_w\mu_a} = {}_a\mu_w = {}_a\mu_g \times {}_g\mu_w = \frac{{}_a\mu_g}{{}_w\mu_g}$$

$\therefore$ Refractive index of water with respect to air,

$${}_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3}$$

Example 8. *Light is incident from air on oil at an angle of 30°. After moving through oil-1, oil-2 and glass it enters water. If the refractive indices of glass and water are 1.5 and 1.3, respectively, find the angle which the ray makes with normal in water.*



(a) $\sin^{-1}\left(\frac{1}{2.6}\right)$ (b) $\sin^{-1}\left(\frac{3}{2.6}\right)$

(c) $\sin^{-1}\left(\frac{1}{3.6}\right)$ (d) $\sin^{-1}(2.6)$

Sol. (a) As we know, $\mu \sin i$ = constant

$$\Rightarrow \quad \mu_{\text{air}} \sin i_{\text{air}} = \mu_{\text{glass}} \sin r_{\text{glass}}$$

$$\sin r_{\text{glass}} = \frac{\mu_{\text{air}}}{\mu_{\text{glass}}} \sin i_{\text{air}} \quad \ldots\text{(i)}$$

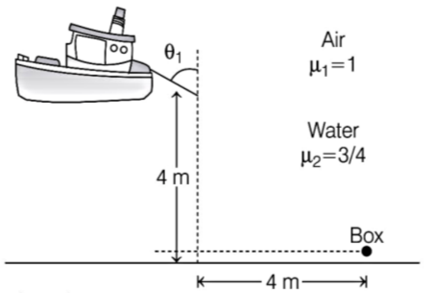
Again, $$\mu_{\text{glass}} \sin i_{\text{glass}} = \mu_{\text{water}} \sin i_{\text{water}} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\sin 30° = 1.3 \sin r$$

$$\sin r = \frac{1}{2 \times 1.3} = \frac{1}{2.6}, \; r = \sin^{-1}\left(\frac{1}{2.6}\right)$$

Example 9. *A searchlight on a yacht is being used at night to illuminate a sunken box, as shown in figure. At what angle of incidence, θ_1 should the light be aimed?*



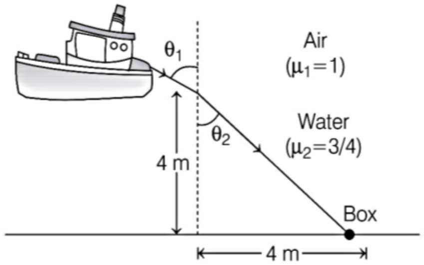
(a) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (b) $\sin^{-1}(\sqrt{2})$

(c) $\cos^{-1}\left(\frac{3}{2\sqrt{2}}\right)$ (d) $\cos^{-1}(\sqrt{2})$

Sol. (a) Here, $\tan\theta_2 = \frac{4}{4}, \Rightarrow \theta_2 = 45°$

Given, $\mu_1 = 1$ (for air) and $\mu_2 = 3/4$ (for water)

From Snell's law, $\mu_1 \cdot \sin\theta_1 = \mu_2 \cdot \sin\theta_2$



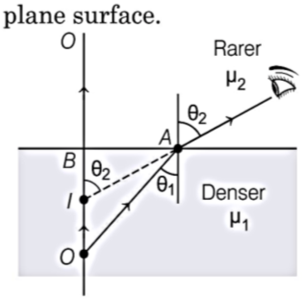
$$\Rightarrow \quad 1\cdot\sin\theta_1 = \frac{4}{3}\cdot\sin 45° \Rightarrow \sin\theta_1 = \frac{4}{3}\cdot\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\theta_1 = \frac{2\sqrt{2}}{3} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Apparent Shift of an Object due to Refraction

Due to bending of light at the interface of two different media, the image formation due to refraction creates an illusion of shifting of the object position.

***Case* I** The object O is in denser medium (glass, water, etc.) and is seen from rarer medium (air) normally through plane surface.

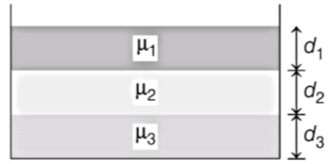


$$AI = d_{\text{apparent}} = \frac{d}{\mu}$$

$$\therefore \quad \text{Shift} = OI = AO - AI = d\left(1 - \frac{1}{\mu}\right)$$

where, d is actual depth of the object in the denser medium of refractive index μ.

Lateral magnification If immiscible liquids of refractive indices μ_1, μ_2 and μ_3 ...... (with $\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1, d_2 and d_3...... and so on respectively. Then, the apparent depth (for normal incidence) when seen from top of the first liquid will be

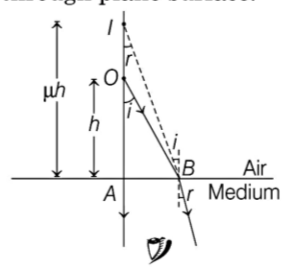


$$d_{\text{app}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + \ldots\ldots\ldots$$

Apparent shift

$$= d_1\left(1 - \frac{1}{\mu_1}\right) + d_2\left(1 - \frac{1}{\mu_2}\right) + \ldots + d_n\left(1 - \frac{1}{\mu_n}\right)$$

***Case* II** The object O is in rarer medium (air) and is seen from denser medium (glass, water, etc.) *i.e.* normally through plane surface.



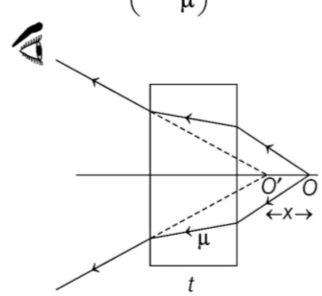
$\therefore$ Apparent height $(AI) = \mu$ (Actual height)

and Shift $= (\mu - 1)h$

Refraction through a Glass Slab

Normal shift If a glass slab is placed in the path of a converging or diverging beam of light, then point of convergence or point of divergence appears to be shifted as shown in figure.

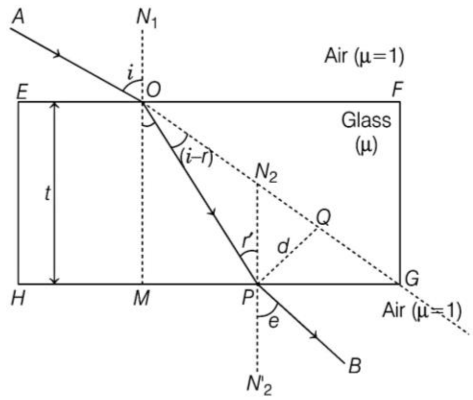
Normal shift, $\quad OO' = x = \left(1 - \frac{1}{\mu}\right)t$



Normal shift by a glass slab

Note Due to slab, if the $\mu_{\text{slab}} > \mu_{\text{medium}}$, then the shift is along the direction of incident ray and if, $\mu_{\text{slab}} > \mu_{\text{medium}}$, then the shift will be opposite direction.

Lateral Shift Refraction of a ray AO incident on the slab at an angle of incidence i trought the glass slab $EFGH$ is as shown below

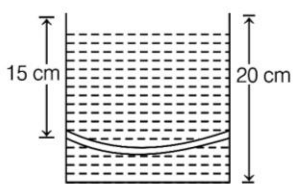


- If e is angle of emergence, then from Snell's law at faces EF and HG, $e = i$. *i.e.* the emergent ray is parallel to the incident ray.
- If PQ be the perpendicular dropped from P on incident ray produced, then the lateral displacement caused by the plate $= \dfrac{t\sin(i-r)}{\cos r}$.

Example 10. *A vessel has a concave mirror of focal length 30 cm placed at the bottom and it is filled with water of refractive index 4/3 upto 20 cm. The position of the image of the sun from the surface of water is*

(a) 7.5 cm above surface of water
(b) 7.5 cm below surface of water
(c) 15 cm above surface of water
(d) 15 cm below surface of water

Sol. (a) Apparent depth $= \dfrac{\text{Real depth}}{\mu} = \dfrac{20}{4/3} = \dfrac{20\times 3}{4} = 15$ cm



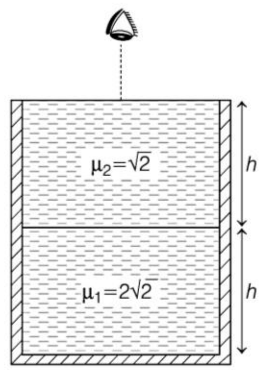
Shift in position $= 20 - 15 = 5$ cm
Apparent, $R = 60 - 5 = 55$ cm
Apparent, $f = 27.5$ cm

$\therefore$ Image of the sun is formed $(27.5 - 20 = 7.5$ cm) above the surface of water.

Example 11. *A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be* **[JEE Main 2020]**

(a) $\dfrac{h}{\sqrt{2}}$ (b) $\dfrac{3}{4}h\sqrt{2}$ (c) $\dfrac{h}{3\sqrt{2}}$ (d) $\dfrac{h}{2(\sqrt{2}+1)}$

Sol. (b)



When a vessel contains immiscible liquids, then apparent depth will be

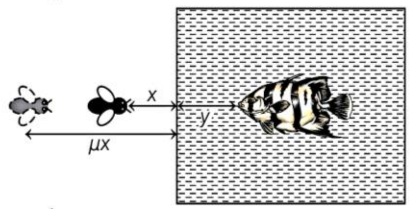
$$d_{\text{apparent}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \dots$$

In given case,

$$d_{\text{apparent}} = \frac{h}{\mu_1} + \frac{h}{\mu_2} = \frac{h}{2\sqrt{2}} + \frac{h}{\sqrt{2}} = \frac{3h}{2\sqrt{2}}$$

or $$d_{\text{apparent}} = \frac{3}{4}h\sqrt{2}$$

Example 12. *A fish in an aquarium, approaches the left wall at a rate of 3 ms^{-1} and observes a fly approaching it at 8 ms^{-1}. If the refractive index of water is 4/3, find the actual velocity of the fly.*



(a) 3.75 ms^{-1}
(b) 2.75 ms^{-1}
(c) 0.75 ms^{-1}
(d) 4.75 ms^{-1}

Sol. (a) For the fish, apparent distance of the fly from the wall of the aquarium is μx. If x is actual distance, then apparent velocity will be $= \dfrac{d(\mu x)}{dt}$.

$$(v_{app})_{fly} = \mu\, v_{fly}$$

Now, the fish observes the velocity of the fly to be 8 ms^{-1}. Therefore, apparent relative velocity $= 8\, ms^{-1}$

$$v_{fish} + (v_{app})_{fly} = 8\text{ ms}^{-1}$$

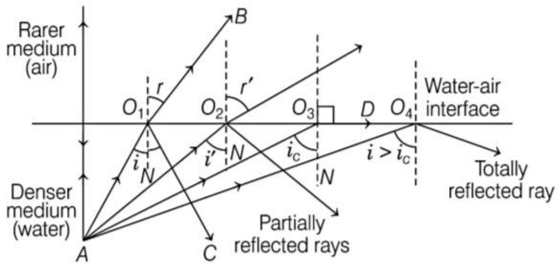
$$\Rightarrow \quad 3 + \mu v_{fly} = 8$$

$$v_{fly} = 5 \times \frac{3}{4} = 3.75\text{ ms}^{-1}$$

Total Internal Reflection

The angle of incidence ($\angle i$) in denser medium for which the angle of refraction ($\angle r$) in rarer medium is 90° is called the critical angle ($\angle C$) for the given pair of media. *i.e.*

$$C = \sin^{-1}\left(\frac{\mu_r}{\mu_d}\right)$$



Refraction and internal reflection of rays from a point A in the denser medium (water) incident at different angles at the interface with a rarer medium (air)

Now, if the angle of incidence ($\angle i$) in the rarer medium is greater than the critical angle ($\angle C$), then the ray instead of suffering refraction is reflected back in the same (denser) medium. This phenomenon is called **total internal reflection**.

For total internal reflection to take place following set of conditions must be obeyed

(i) The ray must travel from denser medium to rarer medium.

(ii) The angle of incidence ($\angle i$) must be greater than critical angle ($\angle C$).

Applications of Total Internal Reflection

- **Mirage** It is the phenomena, in which an inverted image of distant tall objects cause an optical illusion of water. This type of mirage is especially common in hot deserts.
- **Diamond** The critical angle for diamond-air interface (≅ 24.4°) is very small, therefore once light enters a diamond, it is very likely to undergo total internal reflection inside it. Due to this, diamond shines brilliantly.
- **Prism** Few prisms are designed to bend light by 90° or by 180°, making use of total internal reflection. Such prisms are also used to invert images, without changing their size.
- **Optical Fibres** These fibres are fabricated with high quality composite glass/quartz. Each fibre consists of a core and cladding such that refractive index of the material of the core is higher than that of the material of cladding.

  When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out of the other end.

  Thus, these are extensively used for transmitting audio and video signals through long distances.

Example 13. *An isotropic point source (bulb) is placed at a depth h below the water surface. A floating opaque disc is placed on the surface of water, so that the bulb is not visible from the surface. What is the minimum radius of the disc? (Take, refractive index of water = μ)*

(a) $\frac{2h}{\mu}$ (b) $\frac{h}{\sqrt{\mu-1}}$ (c) $\frac{h}{2\mu-1}$ (d) $\frac{\mu}{2h}$

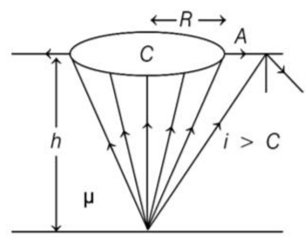
Sol. (b) As shown in figure, light from bulb will not emerge out of the water, if at the edge of disc.

$$i > C$$

$$\sin i > \sin C \quad \text{...(i)}$$

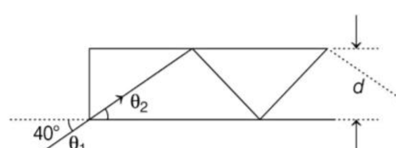
Now, if R is the radius of disc and h is the depth of bulb from it

$$\sin i = \frac{R}{\sqrt{R^2+h^2}} \text{ and } \sin C = \frac{1}{\mu}$$



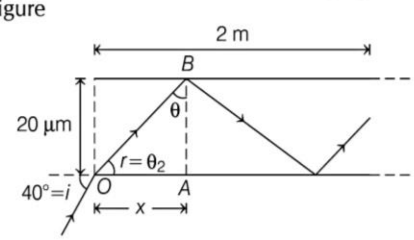
So, Eq. (i) becomes, $\frac{R}{\sqrt{R^2+h^2}} > \frac{1}{\mu}$ or $R > \frac{h}{\sqrt{\mu-1}}$

Example 14. *In figure, the optical fibre is $l = 2$ m long and has a diameter of $d = 20$ μm. If a ray of light is incident on one end of the fibre at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (Take, refractive index of fibre is 1.31 and $\sin 40° = 0.64$)* **[JEE Main 2019]**



(a) 55000 (b) 66000 (c) 45000 (d) 57000

Sol. (d) Total internal reflection occurs through given glass rod as shown in figure



From Snell's law, $n_1 \sin i = n_2 \sin r$
where, $n_1 = 1, n_2 = 1.31$ and $i = 40°$.
So, we get

$$1 \sin 40° = 1.31 \sin r$$

$$\Rightarrow \quad \sin r = \frac{0.64}{1.31} = 0.49 \approx 0.5$$

So, $r = 30°$
From ΔOAB, $\theta = 90° - r = 60°$

Now, $\tan\theta = \dfrac{x}{20\ \mu m}$

$$\Rightarrow \quad x = 20\sqrt{3}\ \mu m \qquad [\because \tan 60° = \sqrt{3}]$$

One reflection occurs in $20\sqrt{3}\ \mu m$.

$\therefore$ Total number of reflections occurring in 2m,

$$n = \frac{2}{20\sqrt{3} \times 10^{-6}}$$

$$= 57735 \text{ reflections}$$

$$\approx 57000 \text{ reflections}$$

Refraction from a Spherical Surface

If an object is placed in a medium of refractive index μ_1, at a distance u from the pole of a spherical surface of radius of curvature R and after refraction, its image is formed in a medium of refractive index μ_2 at a distance v, then

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Note This equation holds for any curved spherical surface.

Example 15. *A convex refracting surface of radius of curvature 40 cm separates two media of refractive indices 4/3 and 2. An object is placed in the first medium $\left(\mu = \frac{4}{3}\right)$ at a distance of 2 m from the refracting surface. The position of image is*

(a) 100 cm
(b) – 150 cm
(c) 200 cm
(d) – 80 cm

Sol. (c) Here, $\mu_1 = \frac{4}{3}, \mu_2 = 2, u = -2\text{ m} = -200$ cm

and $R = +40$ cm

As, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{2}{v} - \frac{4/3}{-200} = \frac{2 - 4/3}{40}$$

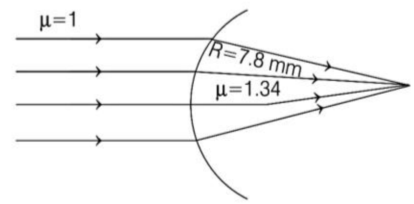
$$\Rightarrow \quad \frac{2}{v} + \frac{1}{150} = \frac{1}{60}$$

$$\Rightarrow \quad v = 200 \text{ cm}$$

Example 16. *The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm) and this surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.* ***[JEE Main 2019]***

(a) 4.0 cm (b) 2 cm
(c) 3.1 cm (d) 1 cm

Sol. (c) The given condition is shown in the figure below



where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.
Radius of curvature of this surface = 7.8 mm
From the image formation formula for spherical surface, *i.e.* relation between object, image and radius of curvature

$$\frac{\mu_R}{v} - \frac{\mu_I}{u} = \frac{\mu_R - \mu_I}{R} \quad \text{... (i)}$$

Given, $\mu_R = 1.34, \mu_I = 1, u = \infty$ (– ve) and $R = 7.8$
Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

or
$$\frac{1.34}{v} = \frac{0.34}{7.8}$$

$$\Rightarrow \quad v = \frac{1.34 \times 7.8}{0.34} \text{ mm}$$

$$\Rightarrow \quad v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

$$\Rightarrow \quad v = 31.2 \text{ mm}$$
$$= 3.12 \text{ cm} \simeq 3.1 \text{ cm}$$

Lens

It is a transparent medium bounded by two curved surfaces at least one of which should be spherical.

Lenses are of two types namely; Convex or convergent lens and Concave or divergent lens.

Some of the terms related to lenses are as follows

Optical centre is a point within or outside the lens, at which incident rays refract without deviation in its path.

Principal axis is the straight line joining the centres of curvature of the two surfaces of the lens.

Principal focus Lens has two principal foci

(i) **First principal focus** It is a point on the principal axis of lens, the rays starting from which (convex lens) or appear to converge at which (concave lens) become parallel to principal axis after refraction.

(ii) **Second principal focus** It is the point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or appear to diverge (concave lens) after refraction from the lens.

Both the foci of convex lens are real, while that of concave lens are virtual.

Focal length is the distance between focus and optical centre of lens.

Rules for Image Formation in Lens

- The rays coming parallel to principal axis of lens passes through the focus or appear to diverge after refraction.
- The rays coming through the first focus of lens go parallel to the principal axis of lens after refraction.
- The rays of light passing through optical centre go straight after refraction without changing their path.

Image Formation by a Convex Lens

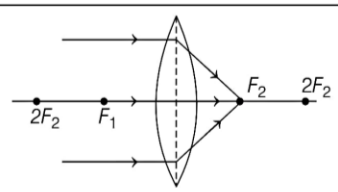
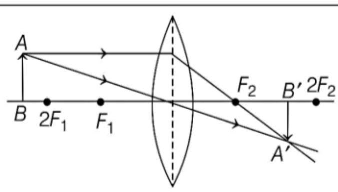
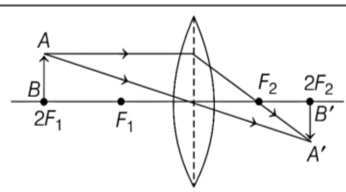
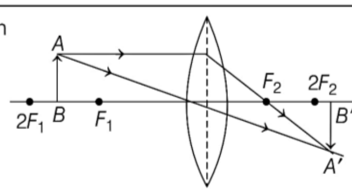
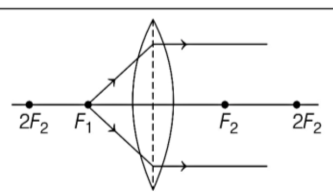
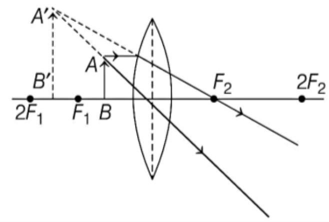
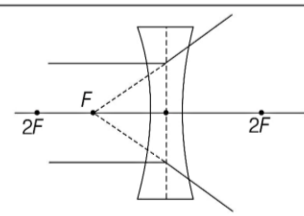
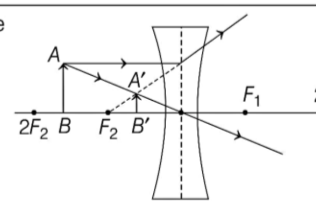
Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
At infinity		At the principal focus F_2 or in the focal plane	Real, inverted and extremely diminished
Beyond $2F_1$		Between F_2 and $2F_2$	Real, inverted and diminished
At $2F_1$		At $2F_2$	Real, inverted and of same size as the object
Between F_1 and $2F_1$		Beyond $2F_2$	Real, inverted and magnified
At F_1		At infinity	Real, inverted and highly magnified
Between F_1 and optical centre		On the same side as the object	Virtual, erect and magnified

Image Formation by Concave Lens

Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
At infinity		At the focus	Virtual erect and point size
Anywhere on the principal axis		Between optical centre and F_2	Virtual, erect, diminished

Lens Maker's Formula

If R_1 and R_2 are the radii of curvature of first and second refracting surfaces of a thin lens of focal length f and refractive index μ (with respect to surrounding medium), the relation between f, μ, R_1 and R_2 is known as lens maker's formula and is given by,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the lens is thin, then lens maker's formula is given as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is usually known as **thin lens formula.**

Example 17. *The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm and its focal length is 12 cm. The refractive index of glass lens is*

(a) 1.33 (b) 1.05
(c) 1.5 (d) 1.0

Sol. (c) Given, $f = +12\text{cm}, R_1 = +10\text{cm}, R_2 = -15\text{cm}$.

Refractive index of air is taken as unity.

From the Lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Putting the values in above expression, we get

$$\frac{1}{12}=(\mu-1)\left(\frac{1}{10}-\frac{1}{-15}\right)$$

$$\Rightarrow \quad \frac{1}{12}=(\mu-1)\left(\frac{1}{10}+\frac{1}{15}\right)$$

$$=(\mu-1)\left(\frac{3+2}{30}\right)=\frac{(\mu-1)}{6}$$

$$\Rightarrow \quad \frac{6}{12}=\mu-1 \Rightarrow \mu=\frac{1}{2}+1=\frac{3}{2}=1.5$$

Example 18. *One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side-by-side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is* **[JEE Main 2019]**

(a) $\frac{2R}{\mu_1-\mu_2}$ (b) $\frac{R}{2-(\mu_1-\mu_2)}$

(c) $\frac{R}{2(\mu_1-\mu_2)}$ (d) $\frac{R}{\mu_1-\mu_2}$

Sol. (d) Focal length of two lenses in contact is given as

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

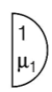
where, f_1 and f_2 are the focal lengths of the respective lenses. Focal length of a lens is given as

$$\frac{1}{f}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

∴ Focal length of plano-convex lens, *i.e.* lens 1,

$$R_1=\infty \text{ and } R=-R$$

$$\Rightarrow \quad \frac{1}{f_1}=\frac{\mu_1-1}{R}$$

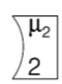


$$\text{or} \quad f_1=\frac{R}{(\mu_1-1)} \quad \text{...(i)}$$

Similarly, focal length of plano-concave lens, *i.e.* lens 2,

$$R_1=-R \text{ and } R_2=\infty$$

$$\Rightarrow \quad \frac{1}{f_2}=-\frac{(\mu_2-1)}{R}$$

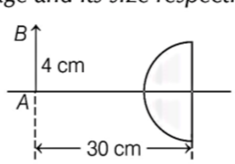


$$\text{or} \quad f_2=\frac{-R}{(\mu_2-1)} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{f}=\frac{\mu_1-1}{R}-\frac{\mu_2-1}{R}=\frac{\mu_1-\mu_2}{R} \Rightarrow f=\frac{R}{\mu_1-\mu_2}$$

Example 19. *A linear object of length 4 cm is placed at 30 cm from the plane surface of hemispherical glass of radius 10 cm. The hemispherical glass is surrounded by water. The final position of image and its size respectively, are*



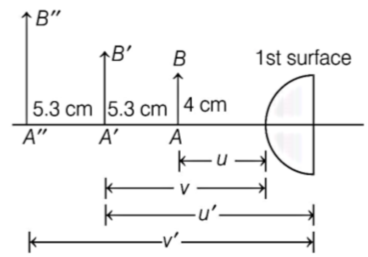
(a) – 30 cm, 5.3 cm (b) – 20 cm, 4.3 cm

(c) – 15 cm, 5.3 cm (d) – 10 cm, 3.6 cm

Sol. (a) For 1st surface,

$$\mu_1=\frac{4}{3}, \mu_2=\frac{3}{2}, u=-20 \text{ cm}$$

$$\text{and} \quad R=+10 \text{ cm}$$



$$\text{Using,} \quad \frac{\mu_2}{v}-\frac{\mu_1}{\mu}=\frac{(\mu_2-\mu_1)}{R}$$

$$\Rightarrow \quad \frac{(3/2)}{v}-\frac{(4/3)}{(-20)}=\frac{(3/2-4/3)}{10}$$

$$\Rightarrow \quad v=-30 \text{ cm}$$

$$\text{Using,} \quad \frac{A'B'}{AB}=\frac{\mu_1 v}{\mu_2 u}$$

$$\Rightarrow \quad \frac{A'B'}{(4\text{cm})}=\frac{(4/3)(-30)}{(3/2)(-20)}$$

$$\Rightarrow \quad A'B'=5.3 \text{ cm}$$

$A'B'$ behaves as the object for plane surface

$$\mu_1=\frac{3}{2}, \mu_2=\frac{4}{3}$$

$$\text{and} \quad R=\infty, u'=-40 \text{ cm}$$

$$\Rightarrow \quad \frac{\mu_1}{v'}=\frac{\mu_2}{u'}$$

$$\Rightarrow \quad \frac{(4/3)}{v'}=\frac{(3/2)}{(-40)}$$

Solving it, we get $v'=-35.4$ cm

$$\text{Now, using,} \quad \frac{A''B''}{A'B'}=\frac{(\mu_1 v')}{(\mu_2 u')}$$

$$\frac{A''B''}{(5.3)}=\frac{(3/2)(-35.4)}{(4/3)(-40)} \Rightarrow A''B''=5.3 \text{ cm}$$

The final images in all the above cases are shown in figure.

Lens Immersed in a Liquid

If a lens (made of glass) of refractive index μ_g is immersed in a liquid of refractive μ_l then its focal length f_l is given by

$$\frac{1}{f_l}=({}_l\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

If f_a is the focal length of lens in air, then

$$\frac{1}{f_a}=({}_a\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{f_l}{f_a}=\frac{({}_a\mu_g-1)}{({}_l\mu_g-1)}=\frac{({}_a\mu_g-1)}{\left(\frac{{}_a\mu_g}{{}_a\mu_l}-1\right)}$$

There are three cases based on this

(i) If $\mu_g > \mu_l$, *i.e.* lens is immersed in a liquid whose refractive index is less than that of material of lens, then f_l and f_a are of same sign and $f_l > f_a$, *i.e.* the nature of lens remains unchanged, but its focal length increases.

(ii) If $\mu_g = \mu_l$, *i.e.* lens is immersed in a liquid whose refractive index is same as that of material of lens, then $f_l = \infty$. It means lens behaves as a plane glass plate and becomes invisible in the medium.

(iii) If $\mu_g < \mu_l$, *i.e.* lens is immersed in a liquid whose refractive is greater than that of material of lens, then f_l and f_a have opposite signs and the nature of lens changes. *i.e.* A convex lens diverges the light rays and concave lens converges the light rays *i.e.* convex lens behaves like a concave lens and concave lens behaves like a convex lens.

Note **Newton's formula**

(i) In case of thin convex lens, if an object is placed at a distance x_1, from first focus and its image is formed at distance x_2, from the second focus from $x_1 x_2 = f^2$.

(ii) If some portion of a lens is covered with black paper, full image will be formed by brightness will be reduced.

Example 20. *A thin lens made of glass (refractive index $= 1.5$) of focal length $f = 16$ cm in immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_l, then the ratio f_l / f is closest to the integer* ***[JEE Main 2020]***

(a) 1 (b) 5
(c) 9 (d) 17

Sol. (c) Focal length of lens of refractive index n_2 placed in a medium of refractive index n_1 is

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a glass-lens in air, focal length,

$$\frac{1}{f_{air}} = \frac{1}{f} = \left(\frac{n_g}{n_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{... (i)}$$

When lens is dipped in a liquid, its focal length,

$$\frac{1}{f_{liquid}} = \frac{1}{f_l} = \left(\frac{n_g}{n_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{f_l}{f} = \frac{\left(\frac{n_g}{n_a} - 1\right)}{\left(\frac{n_g}{n_l} - 1\right)}$$

As, $f = 16$ cm, $n_g = 1.5$ and $n_l = 1.42$, we have

$$\frac{f_l}{f} = \frac{\left(\frac{1.5}{1} - 1\right)}{\left(\frac{1.5}{1.42} - 1\right)}$$

$$\Rightarrow \quad \frac{f_l}{f} = \left(\frac{\frac{3}{2} - 1}{\frac{150}{142} - 1}\right) = \left(\frac{\frac{1}{2}}{\frac{4}{71}}\right) = \frac{71}{8} \approx 8.9$$

So, $\frac{f_l}{f}$ is closest to integer 9.

Magnification Produced by a Lens

It is defined as, the ratio of the size of the image to that of the object. Mathematically,

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{I}{O} = \frac{v}{u}$$

or

$$m = \frac{f}{f+u} = \frac{f-u}{f}$$

For erect (and virtual image), m is positive and for an inverted (and real) image, m is negative.

Power of a Lens

It is the measure of the convergence or divergence which a lens introduces in the light falling on it. Mathematically,

$$P = \frac{1}{\text{Focal length (in m)}} \quad \text{[Dioptre]}$$

or

$$P = \frac{100}{f\text{ (in cm)}}$$

It is also defined as, the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical centre.

i.e. $\tan \delta = \frac{h}{f}$

Note Power of lens is positive for a converging lens and negative for a diverging lens.

Example 21. *A biconvex lens has a radius of curvature of magnitude 20 cm, which one of the following options describe best the image formed of an object height 2 cm placed 30 cm from the lens? (Take, refractive index of lens $\mu = 3/2$)*

(a) *Real, inverted, height = 4 cm*
(b) *Virtual, upright, height = 1 cm*
(c) *Real, inverted, height = 1 cm*
(d) *Virtual, inverted, height = 0.3 cm*

Sol. (a) Given, $R_1 = +20$ cm, $R_2 = -20$ cm and $\mu = \frac{3}{2}$

Using lens Maker formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{20} - \frac{1}{20}\right) = \frac{1}{20}$$

$$\Rightarrow \quad f = 20 \text{ cm}$$

Here, $u = -30$ cm, $f = 20$ cm, $v = ?$

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ or $\frac{1}{20} = \frac{1}{v} - \frac{1}{-30}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{60} \quad \text{or} \quad v = 60 \text{ cm}$$

Magnification, $m = \dfrac{v}{u} = \dfrac{h_i}{h_o}$

or $\dfrac{60}{-30} = \dfrac{h_i}{2}$

$\Rightarrow \quad h_i = -4$ cm

It means the image is real, inverted and of height $(h) = 4$ cm.

Example 22. *A convex lens is made of glass of refractive index 1.5. If radius of curvature of each of its two surfaces is 20 cm, find the ratio of the power of the lens, when placed in air to its power, when immersed inside a liquid of refractive index 1.25.*

(a) $\dfrac{2}{5}$ (b) $\dfrac{5}{2}$ (c) $\dfrac{4}{3}$ (d) $\dfrac{4}{9}$

Sol. (*b*) Given, $R_1 = 20$ cm $= 0.20$ m; $R_2 = -20$ cm $= -0.2$ m

When lens is placed in air If P_1 is power of the lens when placed in air, then

$$P_1 = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{0.2} - \frac{1}{-0.2}\right)$$
$$= 0.5(5 + 5) = 5 \text{ D}$$

When lens is placed inside the liquid Let μ' be the refractive index of the material of the lens w.r.t. the liquid, then power of the lens, when placed inside the liquid is given by

$$P_2 = (\mu' - 1)\left(\frac{2}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu' = \dfrac{\mu_{\text{glass}}}{\mu_{\text{liquid}}} = \dfrac{1.5}{1.25} = 1.2$

$\therefore \quad P_2 = (1.2 - 1)\left(\dfrac{1}{0.2} - \dfrac{1}{-0.2}\right) = 0.2\,(5 + 5) = 2$ D

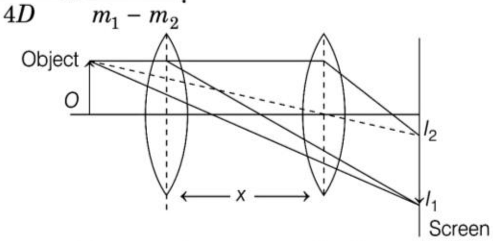
Hence, $\dfrac{P_1}{P_2} = \dfrac{5}{2}$

Displacement Method to Determine the Focal Length of a Convex Lens

The focal length of convex lens can be determined by displacement method. If an object and a screen placed at a distance $D(> 4f)$ apart, for two different position of lens two images (I_1 and I_2) of an object are formed at the screen.

Let distance between the two positions of the lens is x, then the focal length of the lens is given by

$$f = \frac{D^2 - x^2}{4D} = \frac{x}{m_1 - m_2}.$$



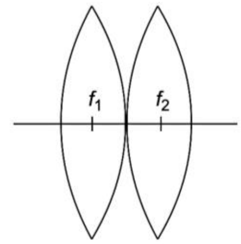
Two convex lenses separated by distance x

where, $m_1 = \dfrac{I_1}{O}$, $m_2 = \dfrac{I_2}{O}$ and $m_1 m_2 = 1$.

Also, size of object, $O = \sqrt{I_1 I_2}$.

Combination of Thin lenses

If two or more lenses are placed in contact, then equivalent focal length of the combination,



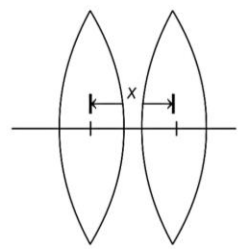
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots = \sum_{i=1}^{n} \frac{1}{f_i}$$

Power of combination, $P = P_1 + P_2 + \ldots = \sum_{i=1}^{n} P_i$

Magnification of combination,

$$M = m_1 \times m_2 \times \ldots = \sum_{i=1}^{n} m$$

Similarly, if two lenses of focal lengths f_1 and f_2 are separated by a distance x, then its equivalent focal length,



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

Power of combination, $P = P_1 + P_2 - x\,P_1 P_2$

Total magnification remains unchanged, *i.e.*

$$m = m_1 \times m_2$$

Note If a lens is made of a number of layers of different refractive indices, then number of images of an object formed by the lens is equal to number of different media.

Example 23. *An object is placed at a distance of 10 cm to the left on the axis of a convex lens L_1 of focal length 20 cm. A second convex lens L_2 of focal length 10 cm is placed co-axially to the right of the lens L_1 at a distance of 5 cm from it. Find the position of the final image and its magnification.*

(a) $16\dfrac{2}{3}$ *cm on the right of the second lens,* 3.33

(b) $16\dfrac{2}{3}$ *cm on the right of the second lens,* 1.33

(c) $16\dfrac{2}{3}$ *cm on the right of the first lens,* 1.33

(d) *None of the above*

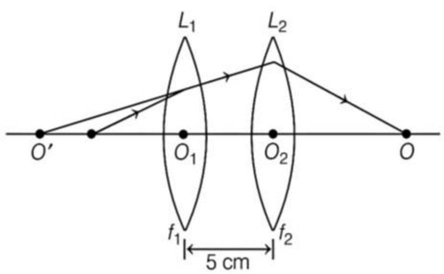
Sol. (*b*) Here, **for 1st lens** $u_1 = -10\text{cm}$

and $f_1 = 20$ cm

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{20} - \frac{1}{10}$$

$$\Rightarrow \quad v_1 = -20\text{cm}$$



i.e. the image is virtual and hence lies on the same side of the object. This will behave as an object for the second lens.

For 2nd lens, $\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$

Here, $u_2 = -(20+5)$, $f_2 = 10$ cm

$$\frac{1}{v_2} + \frac{1}{25} = \frac{1}{10}$$

$$\Rightarrow \quad v_2 = \frac{50}{3} = 16\frac{2}{3}\text{ cm}$$

i.e. Final image is at a distance of $16\frac{2}{3}$ cm on the right of the second lens.

The magnification of the image is given by

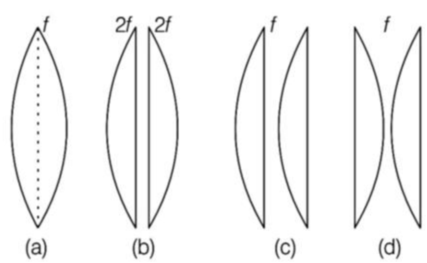
$$m = \frac{v_1}{u_1}\frac{v_2}{u_2}$$

$$= \frac{20}{10}\frac{50}{3\times 25}$$

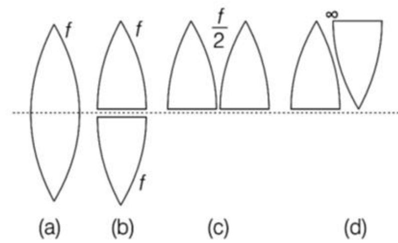
$$= \frac{4}{5} = 1.33$$

Cutting of a Lens

- If a symmetrical convex lens of focal length *f* is cut into two parts along its optical axis, then focal length of each part (a plano-convex lens) is 2*f*. However, if the two parts are joined as shown in figure, the focal length of combination is again *f*.



- If a symmetrical convex lens of focal length *f* is cut into two parts along the principal axis, then focal length of each part remains changed at *f*. If these two parts are joined with curved ends on one side, focal length of the combination is $\frac{f}{2}$. But on joining two parts in opposite sense, the net focal length becomes ∞ (or net power = 0).



Silvering of a Lens

- Let a plano-convex lens is having a curved surface of radius of curvature *R* and has refractive index μ. If its plane surface is silvered, it behaves as a concave mirror of focal length,

$$f = \frac{R}{2(\mu - 1)}$$

- If the curved surface of plano-convex lens is silvered, then it behaves as a concave mirror of focal length,

$$f = \frac{R}{2\mu}$$

- If one surface of a symmetrical double convex lens is silvered, then the lens behaves as a concave mirror of focal length,

$$f = -\frac{R}{2(2\mu - 1)}$$

Combination of Lenses and Mirrors

- When several lenses or mirrors are combined coaxially, image by first serves as an object for second and so on. The net magnification is given by

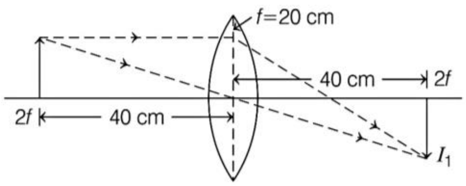
$$m = m_1 \times m_2 \times m_3 \times \ldots$$

- In some cases, object and image are formed at same place after interacting the (lens + mirror) combination. For this, after refraction from lens, rays must retrace their path, *i.e.* must be incident normally on the mirror after refraction from the lens. In other word, we can say that image by lens should be formed at centre of curvature *C* of mirror.

Example 24. *An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be* **[JEE Main 2019]**

(a) 20 cm from the convergent mirror, same size as the object
(b) 40 cm from the convergent mirror, same size as the object
(c) 40 cm from the convergent lens, twice the size of the object
(d) 20 cm from the convergent mirror,twice size of the object

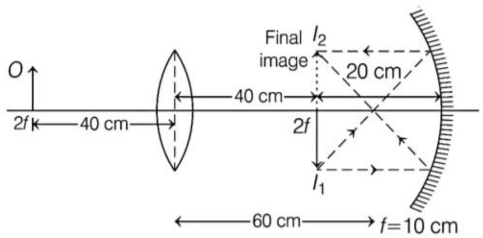
Sol. (*) In given system of lens and mirror, position of object O in front of lens is at a distance $2f$.
i.e. $u = 2f = 40$ cm



So, image I_1 formed is real, inverted and at a distance, $v = 2f = 2 \times 20 = 40$ cm, (behind lens) magnification,

$$m_1 = \frac{v}{u} = \frac{40}{-40} = -1$$

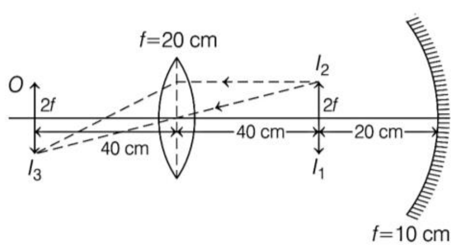
Thus, size of image is same as that of object.
This image I_1 acts like a real object for mirror.



As, object distance for mirror is $u = C = 2f = -20$ cm
where, C = centre of curvature.
So, image I_2 formed by mirror is at $2f$.
$\therefore$ For mirror $v = 2f = 2(-10) = -20$ cm

$$\text{Magnification, } m_2 = -\frac{v}{u} = -\frac{(-20)}{(-20)} = -1$$

Thus, image size is same as that of object.
The image I_2 formed by the mirror will act like an object for lens.



As the object is at $2f$ distance from lens, so image I_3 will be formed at a distance $2f$ or 40 cm. Thus, magnification,

$$m_3 = \frac{v}{u} = \frac{40}{-40} = -1$$

So, final magnification,

$$m = m_1 \times m_2 \times m_3 = -1$$

Hence, final image I_3 is real, inverted of same size as that of object and coinciding with object.

Refraction Through a Prism

A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle A with each other.

These surfaces are called as **refracting surfaces** and the angle between them is called **angle of prism A**.

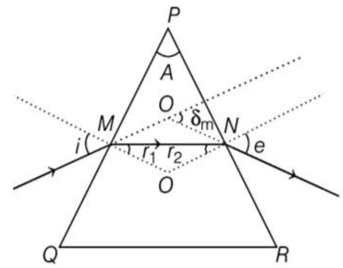


Figure shows the refraction of monochromatic light through a prism. Here, i and e represent the angle of incidence and angle of emergence respectively, r_1 and r_2 are two angles of refraction.

If μ is the refractive index of the material of the prism, then

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin e}{\sin r_2}$$

The angle between the incident ray and the emergent ray is known as the **angle of deviation** δ.

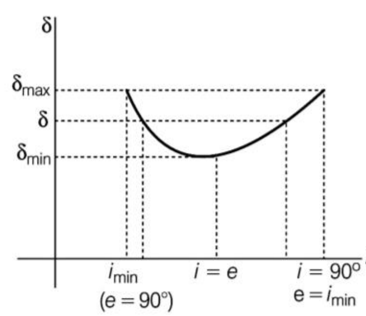
For refraction through a prism, it is found that

$$i + e = A + \delta$$

and $$r_1 + r_2 = A$$

Minimum Deviation

The variation of angle of deviation δ with the angle of incidence i of the ray incident on the first refracting face of the prism is shown in figure



At minimum deviation δ_m, the refracted ray inside the prism becomes parallel to its base. At this value,

$$i = e$$

Further at δ_m, refractive index of prism is given as,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\left(\sin\frac{A}{2}\right)}$$

This expression is also called **prism formula**.

Note For thin prism, $\delta_m = (\mu - 1)A$.

Example 25. *A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), then new angle of minimum deviation of a parallel beam of light is*

(a) 20° (b) 10°20′ (c) 30° (d) 60°23′

Sol. (b) Using the relation, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$$= \frac{\sin\left(\dfrac{60°+40°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)} = \frac{\sin 50°}{\sin 30°}$$

$$= \frac{0.7660}{0.50} = 1.532$$

$$\approx 1.53$$

Let δ'_m be the new angle of minimum deviation of a parallel beam of light when the prism is placed in water ${}_a\mu_w = 1.33$, ${}_a\mu_g = 1.532$

$$\therefore \quad {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.532}{1.33} = 1.152$$

$${}_w\mu_g = \frac{\sin\left(\dfrac{A+\delta'_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

or $$1.152 = \frac{\sin\left(\dfrac{60+\delta'_m}{2}\right)}{\sin\dfrac{60}{2}}$$

or $$\sin\left(30° + \frac{\delta'_m}{2}\right) = 1.152 \times \frac{1}{2} = 0.5759 = \sin 35.17°$$

or $$30° + \frac{\delta'_m}{2} = 35.17°$$

or $$\frac{\delta'_m}{2} = 35.17° - 30 = 5.17°$$

Angle of minimum deviation, $\delta'_m = 2 \times 5.17° = 10.34° = 10°20'$

Dispersion of Light

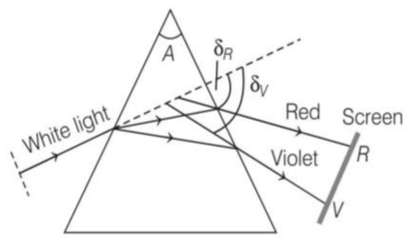
The phenomenon of splitting of light into different colours is known as dispersion. The pattern of colour components of light thus obtained is called the **spectrum of light.**

Dispersion takes place because the refractive index of medium for different frequencies (colours) is different. *e.g.* Bending of red light is least while it is most for the violet. Equivalently, red light travels faster than violet light in a glass prism.

Rainbow is an example of dispersion of sunlight by water drops in atmosphere. It is a phenomenon based on the combined effect of dispersion, refraction and reflection of sunlight by spherical water droplets of rain.

Dispersion of Light by a Prism

Dispersion of light is the phenomenon of splitting of white light into its constituent wavelengths on passing through a dispersive medium, *e.g.* prism. The reason for dispersion is the variation of refractive index of prism with wavelength. As, $\lambda_V > \lambda_R$, hence $\mu_V > \mu_R$ and consequently, $\delta_V > \delta_R$.



Angular Dispersion It is the angular separation between the two extreme rays.

Angular dispersion, $\theta = \delta_V - \delta_R$

$$= (\mu_V - \mu_R)A$$

Dispersive Power The dispersive power of a prism material is measured by the ratio of angular dispersion to the mean deviation suffered by light beam.

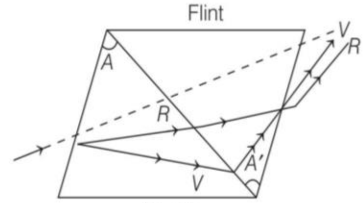
$\therefore$ Dispersive power, $\quad \omega = \dfrac{\delta_V - \delta_R}{\delta} = \dfrac{\mu_V - \mu_R}{\mu - 1}$

where, μ is the mean value of refractive index of prism.

The dispersive power of a prism depends only on its material and is independent of angle of prism and angle of incidence or size of the prism.

Dispersion without Deviation (Direct Vision Prism)

To produce dispersion without mean deviation, the combination of two prisms of different materials such that



$$A' = \left(\frac{\mu - 1}{\mu' - 1}\right)A$$

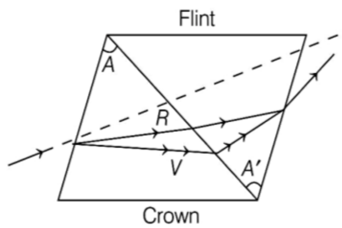
Net dispersion caused $= (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

$$= \delta(\omega - \omega')$$

Deviation without Dispersion (Achromatic Prism)

To produce deviation without dispersion, the combination of two prisms of different materials is as shown below

such that $A' = \dfrac{[\mu_V - \mu_R]}{[\mu'_V - \mu'_R]}A$



Resultant deviation produced $= \delta\left[1 - \dfrac{\omega}{\omega'}\right]$.

Example 26. *Calculate the dispersive power for crown glass from the given data*

$$\mu_V = 1.523$$

and $$\mu_R = 1.5145$$

(a) 0.01639 (b) 1.05639
(c) 0.05639 (d) 2.05639

Sol. (a) Here, $\mu_V = 1.523$ and $\mu_R = 1.145$

Mean refractive index, $\mu = \dfrac{1.523 + 1.5145}{2} = 1.51875$

Dispersive power is given by

$$\omega = \frac{(\mu_V - \mu_R)}{(\mu - 1)}$$

$$= \frac{1.523 - 1.5145}{(1.51875 - 1)} = 0.01639$$

Example 27. *A prism of crown glass with refracting angle of 5° and mean refractive index = 1.51 is combined with a flint glass prism of refractive index = 1.65 to produce no deviation. Find the angle of flint glass.*

(a) 3.92° (b) 4.68°
(c) 5.32° (d) 7.28°

Sol. (a) Let A' be the angle of flint glass prism.

Here, $A = 5°$ and $\mu = 1.51$ for crown glass prism.
Deviation produced by flint glass,

$$\delta' = (\mu - 1)A = (1.51 - 1) \times 5 = 2.55°$$

For no deviation, $\delta' = \delta$

or $$0.65\, A' = 2.55$$

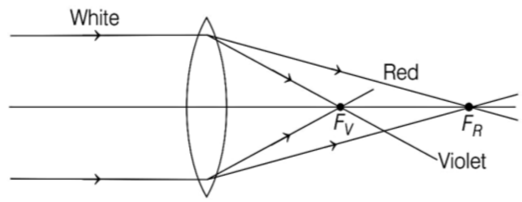
$$A' = \frac{2.55}{0.65} = 3.92°$$

Defects of Images

Actual image formed by an optical system is usually imperfect, the defects of images are called **aberrations.**

There are of two types namely; chromatic aberration and monochromatic aberration.

(i) **Chromatic aberration** The image of an object formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration. This defect arises due to the fact that focal length of a lens is different for different colours.



Chromatic aberration

The difference between f_R and f_V is a measure of longitudinal chromatic aberration (LCA), thus

$$\text{LCA} = f_R - f_V = \omega f_Y$$

where, $$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}$$

Here, $$\mu_Y = \frac{\mu_V + \mu_R}{2}$$

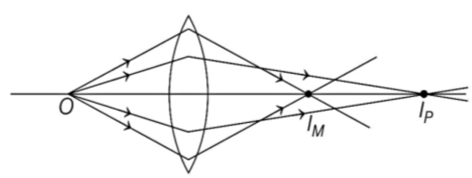
To get achromatism, we use a pair of two lenses in contact. The combination will be free from chromatic aberration.

$$\therefore \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

This is the **condition of achromatism.**

(ii) **Monochromatic aberration** This is the defect in image due to optical system. Monochromatic aberration is of many types such as; spherical, coma, distortion, curvature and antigmatism.

Spherical aberration arises due to spherical nature of lens (or mirror). It is the inability of the lens to form a point image of an axial point object.



Spherical aberration

Scattering of Light

As sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles. Light of shorter wavelength is scattered much more than light of larger wavelengths.

Rayleigh Scattering

According to this, the amount of scattering is inversely proportional to the fourth power of the wavelength.

Some phenomena based on scattering are bluish colour that predominates in a clear sky, sun appears reddish to us, etc.

Optical Instruments

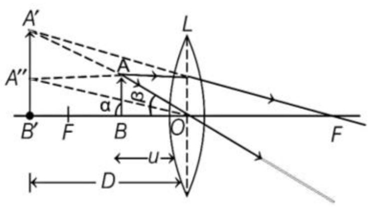
These are those instruments which are used to assist the eye in viewing an object, *e.g.* microscope, telescope, etc.

Microscope

It is an optical instrument which forms a magnified image of a small nearby object and thus increases the visual angle subtended by the image at the eye, so that the object is seen to be bigger and distinct.

Simple Microscope

It is a convex lens of short focal length which is fixed in a frame provided with handle.



Magnification of simple microscope

(a) When final image is formed at least distance of distinct vision,

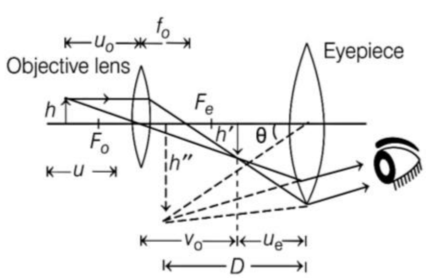
$$M = 1 + \frac{D}{f}$$

(b) For relaxed eye, $M = \frac{D}{f}$

where, D = least distance of distinct vision.

Compound Microscope

Figure shows a simplified version of a compound microscope. It consists of two converging lenses arranged coaxially. The one facing the object is called **objective** and the one close to eye is called **eyepiece**. The objective has a smaller aperture and smaller focal length than those of the eyepiece.



Magnification of compound microscope

(a) For relaxed eye, $M_\infty = -\frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$

In this position, length of microscope,

$$L_\infty = v_o + f_e$$

(b) When final image is formed at least distance of distinct vision.

$$M_D = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

Length of microscope,

$$L_D = v_o + u_e$$

where, v_o = distance of first image from objective lens,

u_o = distance of object from objective lens

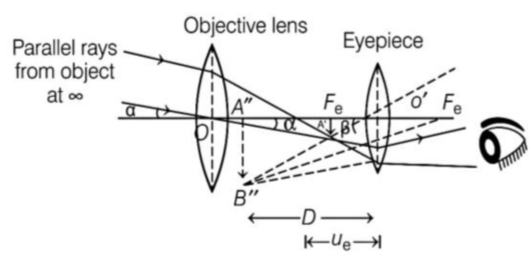
and f_e = focal length of eyepiece.

Telescope

It is an optical instrument which increases the visual angle at the eye by forming the image of a distant object at the least distance of distinct vision, so that the object is seen distinct and bigger.

Astronomical Telescope

It consists of two converging lenses placed coaxially. The one facing the distant object is called the **objective** and has a large aperture and large focal length. The other is called the **eyepiece**, which is placed closed to eye. The eyepiece tube can slide within the objective tube, so that the separation between the objective and the eyepiece may be varied.



Magnification of astronomical telescope

(a) For relaxed eye, $M_\infty = -\frac{f_o}{f_e}$

In this position, length of telescope,

$$L_\infty = f_o + f_e$$

(b) When final image is formed at least distance of distinct vision

$$M_D = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Length of telescope,

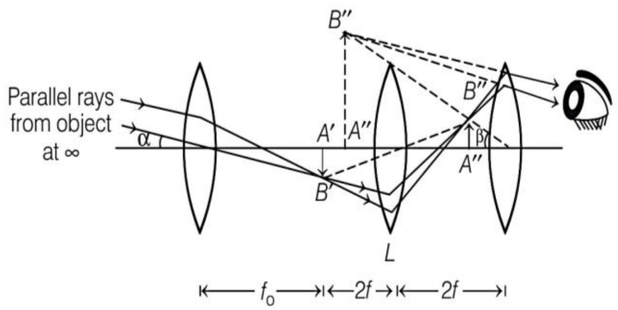
$$L_D = f_o + u_e$$

where, f_o = focal length of objective lens

and f_e = focal length of eyepiece.

Terrestrial Telescope

In an astronomical telescope, the final image is inverted with respect to the object. To remove this difficulty, a convex lens of focal length f is included between the objective and the eyepiece in such a way that the focal plane of the objective is a distance $2f$ away from this lens.



Magnification of terrestrial telescope,

(a) For relaxed eye, $M_\infty = f_o / f_e$

In this position, length of telescope,

$$L_\infty = f_o + 4f + f_e$$

(b) When final image is formed at least distance of distinct vision,

$$M_o = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Length of telescope,

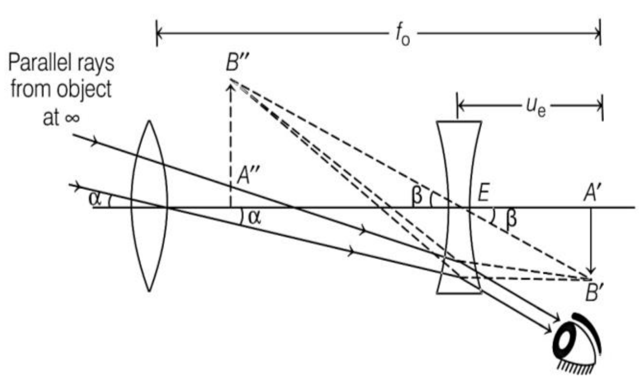
$$L_\infty = f_o + 4f + u_e$$

f_o = focal length of objective lens

and f_e = focal length of eyepiece.

Galilean Telescope

A simple model of Galilean telescope is shown in figure. A convergent lens is used as the objective and a divergent lens as the eyepiece.



Magnification of Galilean telescope

(a) For relaxed eye, $M_\infty = \dfrac{f_o}{f_e}$

In this position, length of telescope,

$$L_\infty = f_o - f_e$$

(b) When final image is formed at least distance of distinct vision, $M_D = \dfrac{f_o}{f_e}\left(1 - \dfrac{f_e}{D}\right)$

Length of telescope, $L_D = f_o - u_e$

Example 28. *An object is seen through a simple microscope of focal length 20 cm. Find the angular magnification produced, if the image is formed at 30 cm from the lens.*

(a) 2.08 (b) 2.05
(c) 3.08 (d) 1.5

Sol. (a) Given, $f = +30$ cm and $v = -30$ cm

Using the formula, we have, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, we have

$$\frac{1}{-30} - \frac{1}{u_o} = \frac{1}{20}$$

$$u_o = 12\text{cm}$$

The angular magnification, $M = \dfrac{D}{u_o} = \dfrac{25}{12} = 2.08$

Example 29. *A Galilean telescope is 27 cm long when focussed to form an image at infinity. If the objective has a focal length of 30 cm, what is the focal length of the eyepiece?*

(a) 3 cm (b) –3 cm
(c) 2 cm (d) –2 cm

Sol. (a) Given, $f_o = +30$ cm

Length of telescope when final image forms at infinity,

$$L_\infty = 27 \text{ cm}$$

$$\therefore \quad L_\infty = f_o - f_e$$

$$27 = 30 - f_e$$

$$\Rightarrow \quad f_e = 3 \text{ cm}$$

Practice Exercise

ROUND I Topically Divided Problems

Reflection from Plane and Spherical Mirror

1. An object is placed a symmetrically between two plane mirrors inclined at an angle of 72°. The number of image formed is

(a) 5 (b) 4
(c) 2 (d) infinite

2. Two plane mirrors are inclined to each other at an angle θ. A ray of light is reflected first at one mirror and then at the other. The total deviation of the ray is

(a) 2θ (b) $240° - 2\theta$
(c) $360° - 2\theta$ (d) $180° - \theta$

3. A candle is placed before a thick plane mirror. When looked obliquely in the mirror, a number of images are seen from the surfaces of the plane mirror. Then

(a) first image is brightest
(b) second image is brightest
(c) third image is brightest
(d) all images beyond second are brighter

4. An object is approaching a plane mirror at $10\ cms^{-1}$. A stationary observer sees the image. At what speed will the image approach the stationary observer?

(a) $10\ cms^{-1}$
(b) $5\ cms^{-1}$
(c) $20\ cms^{-1}$
(d) $15\ cms^{-1}$

5. A soldier directs a laser beam on an enemy by reflecting the beam from a mirror. If the mirror is rotated by an angle θ, by what angle will reflected beam rotate?

(a) $\theta/2$ (b) θ
(c) 2θ (d) None of these

6. It is necessary to illuminate the bottom of a well by reflected solar beam when the light is incident at an angle of $\alpha = 40°$ to the vertical. At what angle β to the horizontal should a plane mirror be placed?

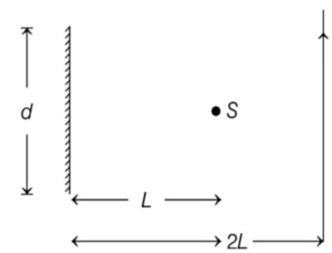
(a) 70° (b) 20°
(c) 50° (d) 40°

7. A car is moving with at a constant speed of $60\ kmh^{-1}$ on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at distance of 100 m and is approaching with a speed of $5\ km\ h^{-1}$. In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement (s) is/are correct? **[NCERT Exemplar]**

(a) The speed of the car in the rear is $65\ km\ h^{-1}$.
(b) In the side mirror, the car in the rear would appear to approach with a speed of $5\ km\ h^{-1}$ to the driver of the leading car.
(c) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases.
(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases.

8. A point source of light S is placed at a distance L in front of the centre of plane mirror of width d, which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below.

The distance over which the man can see the image of the light source in the mirror is **[JEE Main 2019]**



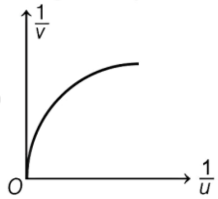
(a) $\frac{d}{2}$

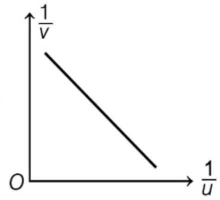
(b) d

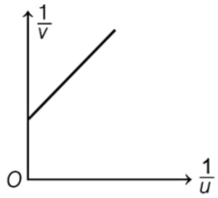
(c) $3d$

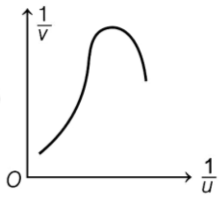
(d) $2d$

9. From a spherical mirror, the graph of $1/v$ *versus* $1/u$ is given by

(a) 

(b) 

(c) 

(d) 

10. A convex mirror forms an image one-fourth the size of the object. If object is at a distance of 0.5 m from the mirror, the focal length of mirror is
(a) 0.17 m (b) – 1.5 m (c) 0.4 m (d) – 0.4 m

11. With a concave mirror, an object is placed at a distance x_1 from the principal focus, on the principal axis. The image is formed at a distance x_2 from the principal focus. The focal length of the mirror is
(a) $x_1 x_2$ (b) $\frac{x_1 + x_2}{2}$
(c) $\sqrt{\frac{x_1}{x_2}}$ (d) $\sqrt{x_1 x_2}$

12. A person of 6 feet in length can see his full size erect image in a mirror 2 feet in height. This mirror has to be
(a) plane or convex (b) plane or concave
(c) necessarily convex (d) necessarily concave

13. A point object is placed at a distance of 30 cm from a convex mirror of a focal length 30 cm. The image will form at
(a) infinity
(b) pole
(c) 15 cm behind the mirror
(d) no image will be formed

14. A dentist has a small mirror of focal length 16 mm. He views the cavity in the tooth of a patient by holding the mirror at a distance of 8 mm from the cavity. The magnification is
(a) 1 (b) 1.5 (c) 2 (d) 3

15. A car is fitted with a convex mirror of focal length 20 cm. A second car 2 m broad and 1.6 m height is 6 m away from the first car. The position of the second car as seen in the mirror of the first car is
(a) 19.35 cm (b) 17.45 cm
(c) 21.48 cm (d) 15.49 cm

16. A plane mirror is reflecting a ray of incident light is rotated through an angle of about an axis through the point of incidence in the plane of the mirror perpendicular to the plane of incident, then
(a) the reflected ray rotates through an angle 2θ
(b) the reflected ray rotates through an angle of θ
(c) the reflected ray does not rotate
(d) None of the above

17. A man has a concave shaving mirror or focal length 0.2 m. How far should the mirror be held from his face in order to give an image of two fold magnification?
(a) – 0.1 m (b) 0.2 m
(c) 0.3 m (d) 0.4 m

18. A spherical mirror forms diminished virtual image of magnification 1/3. Focal length is 18 cm. The distance of the object is
(a) 18 cm (b) – 36 cm
(c) 48 cm (d) infinite

19. Sun subtends an angle of 0.5° at the centre of curvature of a concave mirror of radius of curvature 15 m. The diameter of the image of the sun formed by the mirror is
(a) 8.55 cm (b) 7.55 cm
(c) 6.55 cm (d) 6.55 cm

20. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is equal to
(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)^{1/2}$
(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

21. A convex mirror of radius of curvature 1.6 m has an object placed at a distance of 1 m from it. The image is formed at a distance of
(a) (8/13) m in front of the mirror
(b) (8/13) m behind the mirror
(c) (4/9) m in front of the mirror
(d) (4/9) m behind the mirror

22. A small object is placed 10 cm in front of a plane mirror. If you stand behind the object, 30 cm from the mirror and look at its image, for what distance must you focus your eyes?
(a) 20 cm (b) 60 cm
(c) 80 cm (d) 40 cm

23. A spherical mirror forms an image of magnification $m = \pm 3$. The object distance, if focal length of mirror is 24 cm, may be
(a) 32 cm, 24 cm (b) 32 cm, 16 cm
(c) 32 cm only (d) 16 cm only

24. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Then, choose the correct option.

(a) Location of the image is 6.7 cm.
(b) Magnification is 5/9.
(c) If the needle is moved farther from the mirror, the image goes on decreasing.
(d) All of the above

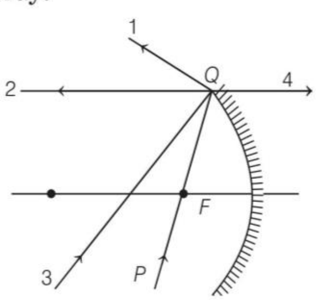
25. When an object is at distances of u_1 and u_2 from the poles of a concave mirror, images of the same size are formed. The focal length of the mirror is

(a) $|u_1 + u_2|$ (b) $|u_1 - u_2|$
(c) $\left|\frac{u_1 + u_2}{2}\right|$ (d) $\left|\frac{u_1 - u_2}{2}\right|$

26. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is **[JEE Main 2019]**

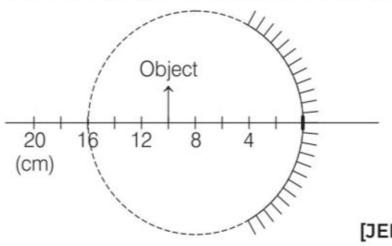
(a) 0.16 m (b) 1.60 m (c) 0.32 m (d) 0.24 m

27. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray?



(a) 1 (b) 2 (c) 3 (d) 4

28. A spherical mirror is obtained as shown in the above figure from a hollow glass sphere. If an object is in front of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale)

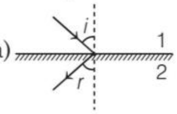


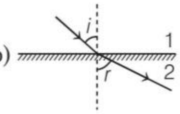
[JEE Main 2020]

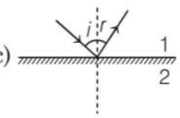
(a) Erect, virtual and unmagnified
(b) Inverted, real and magnified
(c) Erect, virtual and magnified
(d) Inverted, real and unmagnified

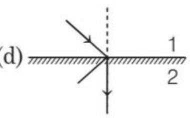
Refraction of Light

29. There are certain materials developed in laboratories which have a negative refractive index (figure). A ray incident from air (medium 1) into such a medium (medium 2) shall follow a path given by **[NCERT Exemplar]**

(a) 

(b) 

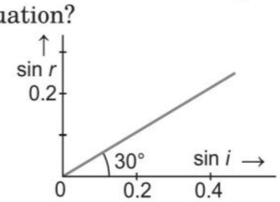
(c) 

(d) 

30. Monochromatic light of wavelength λ_1 travelling in medium of refractive index n_1 enters a denser medium of refractive index n_2. The wavelength in the second medium is

(a) $\lambda_1\left(\frac{n_1}{n_2}\right)$ (b) $\lambda_1\left(\frac{n_2}{n_1}\right)$
(c) λ_1 (d) $\lambda_1\left(\frac{n_2 - n_1}{n_1}\right)$

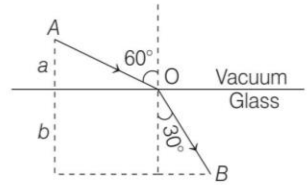
31. Light is incident from a medium X at an angle of incident i and is refracted into a medium Y at angle of refraction r. The graph sin i *versus* sin r is shown in figure. Which of the following conclusions would fit the situation?



1. Speed of light in medium Y is $\sqrt{3}$ times that in medium X.
2. Speed of light in medium Y is $1/\sqrt{3}$ times that in medium X.

(a) 1 only (b) 1 and 2
(c) 2 only (d) Neither 1 nor 2

32. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is **[JEE Main 2019]**



(a) $\frac{2\sqrt{3}}{a} + 2b$ (b) $2a + \frac{2b}{3}$
(c) $2a + 2b$ (d) $2a + \frac{2b}{\sqrt{3}}$

33. A printed page is pressed by a glass of water. The refractive index of the glass and water is 1.5 and 1.33, respectively. If the thickness of the bottom of glass is 1 cm and depth of water is 5 cm, how much the page will appear to be shifted, if viewed from the top? **(JEE Main 2013)**

(a) 1.033 cm (b) 3.581 cm (c) 1.3533 cm (d) 1.90 cm

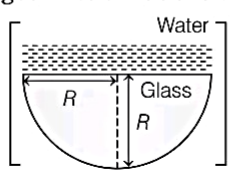
34. A beaker containing liquid is placed on a table underneath a microsope which can be moved along a vertical scale. The microscope is focussed through the liquid, on a mark on the table and the reading on the scale is a. It is next focussed on the upper surface of the liquid and the reading is b. More liquid is added and the observations are repeated, the corresponding reading being c and d. The refractive index of the liquid is

(a) $\dfrac{d-b}{d-c-b+a}$ (b) $\dfrac{b-d}{d-c-b+a}$

(c) $\dfrac{d-c-b+a}{d-b}$ (d) $\dfrac{d-b}{d+c-b-a}$

Total Internal Reflection

35. A ray of light travelling in glass $\left(\mu = \dfrac{3}{2}\right)$ is incident on a horizontal glass air surface at the critical angle θ_C. If thin layer of water $\left(\mu = \dfrac{4}{3}\right)$ is now poured on the glass air surface, the angle at which the ray emerges into air at the water-air surface is

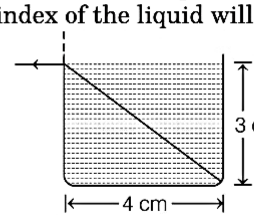


(a) 60° (b) 45° (c) 90° (d) 180°

36. If eye is kept at a depth h inside water of refractive index and viewed outside, then the diameter of the circle through which the outer objects become visible, will be

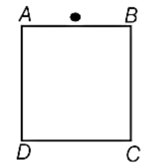
(a) $\dfrac{h}{\sqrt{\mu^2-1}}$ (b) $\dfrac{h}{\sqrt{\mu_2+1}}$ (c) $\dfrac{2h}{\sqrt{\mu^2-1}}$ (d) $\dfrac{h}{\sqrt{\mu^2}}$

37. When the rectangular metal tank is filled to the top with an unknown liquid, as observer with eyes level with the top of the tank can just see the corner E; a ray that refracts towards the observer at the top surface of the liquid is shown. The refractive index of the liquid will be



(a) 1.2 (b) 1.4
(c) 1.6 (d) 1.9

38. A rectangular block of glass $ABCD$ has a refractive index 1.6. A pin is placed midway on the face AB (figure). When observed from the face AD, the pin shall



(a) appear to be near A (b) appear to be near D
(c) not be seen at all (d) Both (a) and (c)

39. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\dfrac{4}{3}$ for this wavelength, will be **[JEE Main 2020]**

(a) 45° (b) 60°
(c) 15° (d) 30°

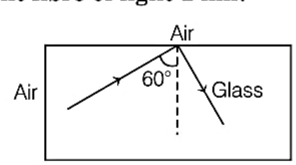
40. When a glass slab is placed on a cross made on a sheet, the cross appears raised by 1 cm. The thickness of the glass is 3 cm. The critical angle for glass is

(a) $\sin^{-1}(0.33)$ (b) $\sin^{-1}(0.5)$
(c) $\sin^{-1}(0.67)$ (d) $\sin^{-1}(\sqrt{3}/2)$

41. Light is incident from a medium into air at two possible angles of incidence (i) 20° and (ii) 40°. In the medium, light travels 3.0 cm in 0.2 ns. The ray will

(a) suffer total internal reflection in both cases (i) and (ii)
(b) suffer total internal reflection in case (ii) only
(c) have partial reflection and partial transmission in case (ii)
(d) have 100% transmission in case (i)

42. A light ray from air is incident as shown in figure at one end of the glass fibre (refractive index $\mu = 1.5$) making an incidence angle of 60° on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fibre of light 1 km?

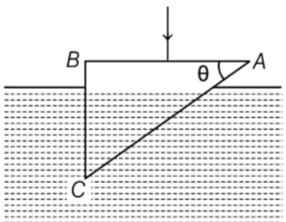


(a) 3.33 μs (b) 5.77 μs
(c) 3.85 μs (d) 6.67 μs

43. A ray of light from a denser medium strikes a rarer medium at angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r', respectively. The critical angle is

(a) $\sin^{-1}(\tan r')$ (b) $\sin^{-1}(\tan r)$
(c) $\tan^{-1}(\tan r')$ (d) $\tan^{-1}(\tan i)$

44. A glass prism ABC (refractive index 1.5), immersed in water (refractive index 4/3). A ray of light is incident normally on face AB. If it is totally reflected at face AC, then



(a) $\sin\theta \geq \frac{8}{9}$ (b) $\sin\theta \geq \frac{2}{3}$
(c) $\sin\theta = \frac{\sqrt{3}}{2}$ (d) $\frac{2}{3} < \sin\theta < \frac{8}{9}$

45. A point source of light is held at a depth h below the surface of water. If C is critical angle of air-water interface, the diameter of circle of light coming from water surface would be

(a) $2h\tan C$ (b) $h\tan C$
(c) $h\sin C$ (d) $h/\sin C$

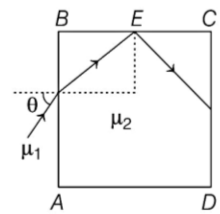
46. You are given four sources of light each one providing a light of a single colour, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statement is correct if the source of yellow light is replaced with that of other lights without changing the angle of incidence? **[NCERT Exemplar]**

(a) The beam of red light would undergo total internal reflection
(b) The beam of red light would bend towards normal while it gets refracted through the second medium
(c) The beam of blue light would undergo total internal reflection
(d) The beam of green light would bend away from the normal as it gets refracted through the second medium

47. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.) **[NCERT]**

(a) 4.6 m^2 (b) 3.2 m
(c) 5.6 m^2 (d) 2.6 m^2

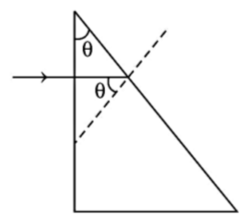
48. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC. Then, θ must satisfy **[JEE Main 2019]**



(a) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$
(c) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (d) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

49. Parallel beam containing light of $\lambda = 400$ nm and 500 nm is incident on a prism as shown in figure. The refractive index μ of the prism is given by the relation,

$$\mu(\lambda) = 1.20 + \frac{0.8\times 10^{-14}}{\lambda^2}$$

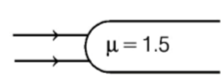


Which of the following statement is correct?

(a) Light of $\lambda = 400$ nm undergoes total internal reflection.
(b) Light of $\lambda = 500$ nm undergoes total internal reflection.
(c) Neither of the two wavelengths undergoes total internal reflection.
(d) Both wavelengths undergoes total internal reflection.

Refraction from Spherical Surfaces and Lenses

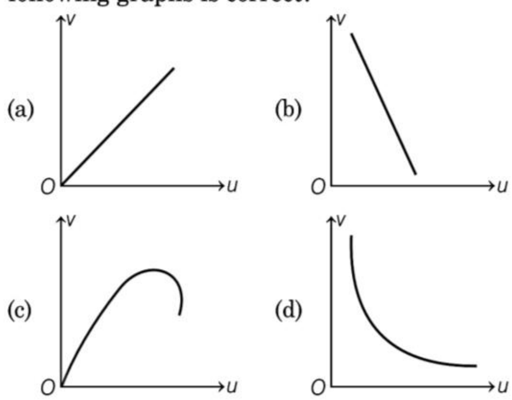
50. Parallel rays of light are falling on convex spherical surface of radius of curvature $R = 20$ cm as shown. Refractive index of the medium is $\mu = 1.5$. After refraction from the spherical surface parallel rays



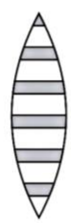
(a) actually meet at some point
(b) appears to meet after extending the refracted rays backwards

(c) meet (or appears to meet) at a distance of 30 cm from the spherical surface
(d) meet (or appears to meet) at a distance of 90 cm from the spherical surface

51. The distance v of the real image formed by a convex lens is measured for various object distance u. A graph is plotted between v and u. Which one of the following graphs is correct?



52. A layered lens as shown in figure is made of two types of transparent materials indicated by different shades. A point object is placed on its axis. The object will form



(a) 1 image (b) 2 images
(c) 3 images (d) 9 images

53. One surface of a lens is convex and the other is concave. If the radii of curvature are r_1 and r_2 respectively, the lens will be convex, if
(a) $r_1 > r_2$ (b) $r_1 = r_2$
(c) $r_1 < r_2$ (d) $r_1 = 1/r_2$

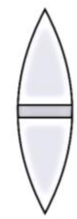
54. A concave lens of focal length 20 cm produces an image half in size of the real object. The distance of the real object is
(a) 20 cm (b) 30 cm (c) 10 cm (d) 60 cm

55. An object 15 cm high is placed 10 cm from the optical centre of a thin lens. Its image is formed 25 cm from the optical centre in the same side of the lens as the object. The height of the image is
(a) 2.5 cm (b) 0.2 cm (c) 16.7 cm (d) 37.5 cm

56. A convex lens of focal length f produces a virtual image n times the size of the object. Then, the distance of the object from the lens is
(a) $(n-1)f$ (b) $(n+1)f$
(c) $\left(\frac{n-1}{n}\right)f$ (d) $\left(\frac{n+1}{n}\right)f$

57. Consider an equiconvex lens of radius of curvature R and focal length f. If $f > R$, the refractive index μ of the material of the lens is greater than
(a) zero but less than 1.5 (b) 1.5 but less than 2.0
(c) one but less than 1.5 (d) None of these

58. Which amongst the following statement is correct regarding the image formed by a convex lens, if the central portion of the lens is wrapped in blank paper as shown in the figure.



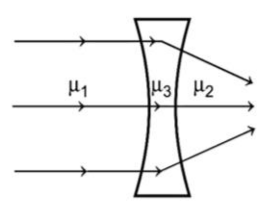
(a) No image will be formed
(b) Full image will be formed but is less bright
(c) Full image will be formed but without the central portion
(d) Two image will be formed, one due to each exposed half

59. A convex lens forms an image of an object placed 20 cm away from it at a distance of 20 cm on the other side of the lens. If the object is moved 5 cm towards the lens, the image will move
(a) 5 cm towards the lens
(b) 5 cm away from the lens
(c) 10 cm towards the lens
(d) 10 cm away from the lens

60. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will **[NCERT Exemplar]**
(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies

61. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image **[NCERT Exemplar]**
(a) moves away from the lens with a uniform speed 5 m/s
(b) moves away from the lens with a uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

62. What is the relation between refractive indices μ_1, μ_2 and μ_3, if the behaviour of light rays is as shown in figure?



(a) $\mu_3 < \mu_2, \mu_3 = \mu_1$ (b) $\mu_2 < \mu_1, \mu_2 = \mu_3$
(c) $\mu_3 < \mu_2 < \mu_1$ (d) $\mu_3 > \mu_2 > \mu_1$

63. A concave lens with unequal radii of curvature made of glass ($\mu_g = 1.5$) has focal length of 40 cm. If it is immersed in a liquid of refractive index $\mu = 2$, then
(a) it behaves like a convex lens of 80 cm focal length
(b) it behaves like a concave lens of 20 cm focal length
(c) its focal length becomes 60 cm
(d) nothing can be said

64. An object is placed 30 cm to the left of a diverging lens whose focal length is of magnitude 20 cm. Which one of the following correctly states the nature and position of the virtual image formed?

	Nature of image	Distance from lens
(a)	inverted enlarged	60 cm to the right
(b)	erect, diminished	12 cm to the left
(c)	inverted, enlarged	60 cm to the left
(d)	erect, diminished	12 cm to the right

65. A 16 cm long image of an object is formed by a convex lens on a screen. On moving the lens towards the screen, without changing the position of the object and the screen, 9 cm long image is formed again on the screen. The size of the object is
(a) 9 cm (b) 11 cm (c) 12 cm (d) 13 cm

66. A thin lens has focal length f_1 and its aperture has diameter d. It forms an image of intensity I. Now, the central part of the aperture upto diameter $\frac{d}{2}$ is blocked by an opaque paper. The focal length and image intensity will be change to
(a) f and $\frac{I}{4}$ (b) f and $\frac{3I}{4}$
(c) $\frac{f}{2}$ and $\frac{I}{2}$ (d) $\frac{3f}{4}$ and $\frac{I}{2}$

67. A convex lens of focal length f is placed somewhere in between an object and a screen. The distance between object and screen is x. If numerical value of magnification produced by lens is m, focal length of lens is
(a) $\frac{mx}{(m+1)^2}$ (b) $\frac{mx}{(m-1)^2}$
(c) $\frac{(m+1)^2}{m}x$ (d) $\frac{(m-1)^2}{m}x$

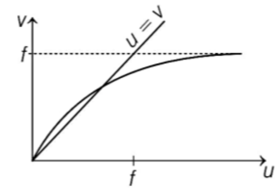
68. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness t is
(a) 15 cm (b) 5 cm (c) 10 cm (d) 20 cm

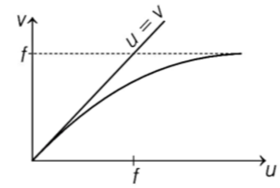
69. The focal lengths for violet, green and red light rays are f_V, f_G and f_R, respectively. Which of the following is the true relationship?
(a) $f_R < f_G < f_V$ (b) $f_V < f_G < f_R$
(c) $f_G < f_R < f_V$ (d) $f_G < f_V < f_R$

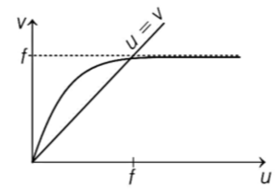
70. A beam of parallel rays is brought to a focus by a plano-convex lens. A thin concave lens of the same focal length is joined to the first lens. The effect of this is
(a) the focal points shifts away from the lens by a small distance
(b) the focus remains undisturbed
(c) the focus shifts to infinity
(d) the focal points shifts towards the lens by a small distance

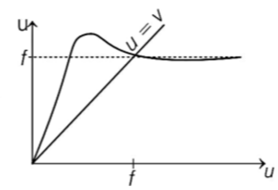
71. The power of a thin convex lens ($_a n_g = 1.5$) is + 5.0 D. When it is placed in a liquid of refractive index $_a n_e$, then it behaves as a concave lens of focal length 100 cm. The refractive index of the liquid $_a n_l$ will be
(a) $5/3$ (b) $4/3$
(c) $\sqrt{3}$ (d) $5/4$

72. For a concave lens of focal length f, the relation between object and image distances u and v respectively, from its pole can best be represented by ($u = v$ is the reference line) **[JEE Main 2020]**

(a)


(b)


(c)


(d)


73. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be **[JEE Main 2019]**
(a) 3.22×10^{-3} m/s towards the lens
(b) 0.92×10^{-3} m/s away from the lens
(c) 2.26×10^{-3} m/s away from the lens
(d) 1.16×10^{-3} m/s towards the lens

74. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be **[JEE Main 2019]**

(a) $f_1 - f_2$ (b) $\frac{R}{\mu_2 - \mu_1}$ (c) $f_1 + f_2$ (d) $\frac{2f_1 f_2}{f_1 + f_2}$

75. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **[JEE Main 2013]**

(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

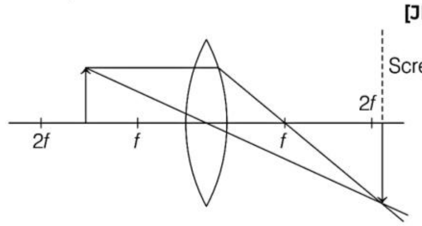
76. A diminished image of an object is to be obtained on a screen 1.0 m away from it. This can be achieved by approximately placing,

(a) a convex mirror of suitable focal length
(b) a convex mirror of suitable focal length
(c) a convex lens of focal length less than 0.25 m
(d) a concave lens of suitable focal length

77. A plano-convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano-concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as **[JEE Main 2019]**

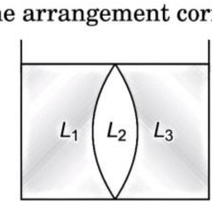
(a) $3\mu_2 - 2\mu_1 = 1$ (b) $2\mu_2 - \mu_1 = 1$
(c) $2\mu_1 - \mu_2 = 1$ (d) $\mu_1 + \mu_2 = 3$

78. Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen? **[JEE Main 2019]**



(a) No change (b) Magnified image
(c) Image disappears (d) Erect real image

79. As shown in figure, the liquids L_1, L_2 and L_3 have refractive indices 1.55, 1.50 and 1.20, respectively. Therefore, the arrangement corresponds to



(a) biconvex lens (b) biconcave lens
(c) concavo-convex lens (d) convexo-concave lens

80. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index? (Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively) **[JEE Main 2021]**

(a) 1 (b) Infinite
(c) $\frac{R_1R_2}{R_1 - R_2}$ (d) Zero

81. The thickness at the centre of a plano-convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is $2 \times 10^8 \text{ ms}^{-1}$. The focal length of the lens is **[JEE Main 2021]**

(a) 0.30 cm (b) 15 cm
(c) 1.5 cm (d) 30 cm

82. A convex lens A of focal length 20 cm and a concave lens B of focal length 56 cm are kept along the same axis with the distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then distance d (in cm) will be

(a) 25 (b) 36
(c) 30 (d) 50

83. A convex lens is placed in contact with a mirror as shown in figure. If the space between them is filled with water, its power will

(a) decrease
(b) increase
(c) remain unchanged
(d) increase or decrease depending on the focal length

84. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is **[JEE Main 2017]**

(a) virtual and at a distance of 40 cm from convergent lens
(b) real and at a distance of 40 cm from the divergent lens
(c) real and at a distance of 6 cm from the convergent lens
(d) real and at a distance of 40 cm from convergent lens

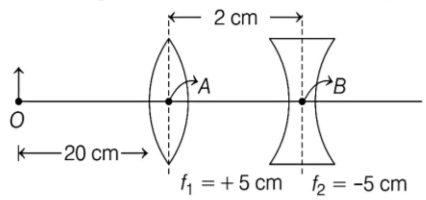
85. Two lenses, one concave and the other convex of same power are placed such that their principal axes coincide. If the separation between the lenses is x, then

(a) real image is formed for $x = 0$ only
(b) real image is formed for all values of x
(c) system will behave like a glass plate for $x = 0$
(d) virtual image is formed for all values of x other than zero

86. Two similar plano-convex lenses are combined together in three different ways as shown in figure. The ratio of focal lengths in three cases will be

(a) 2 : 1 : 1 (b) 1 : 2 : 2 (c) 2 : 2 : 1 (d) 1 : 1 : 1

87. What is the position and nature of image formed by lens combination shown in figure? (where, f_1 and f_2 are focal lengths) **[JEE Main 2019]**



(a) $\frac{20}{3}$ cm from point B at right, real

(b) 70 cm from point B at right, real

(c) 40 cm from point B at right, real

(d) 70 cm from point B at left, virtual

88. A plano-convex lens has a thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. Assume thickness to be negligible.

(a) 85 cm (b) 59 cm (c) 75 cm (d) 7.5 cm

89. A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is **[JEE Main 2020]**

(a) 2.6 m from the mirror, real

(b) 1 m from the mirror, virtual

(c) 1 m from the mirror, real

(d) 2.6 m from the mirror, virtual

90. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now, a glass block (refractive index is 1.5) of 1.5 cm thickness is placed in contact with the light source.

To get the sharp image again, the screen is shifted by a distance d. Then, d is **[JEE Main 2019]**

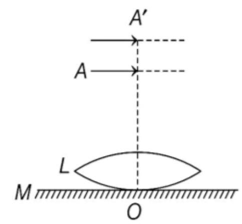
(a) 0

(b) 1.1 cm away from the lens

(c) 0.55 cm away from the lens

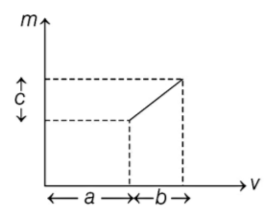
(d) 0.55 cm towards the lens

91. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_l will be **[JEE Main 2019]**



(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) $\frac{4}{3}$ (d) $\frac{3}{2}$

92. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used? **[JEE Main 2019]**



(a) $\frac{b^2c}{a}$ (b) $\frac{b^2}{ac}$ (c) $\frac{a}{c}$ (d) $\frac{b}{c}$

93. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 P is **[JEE Main 2020]**

(a) $2R$ (b) $\frac{R}{2}$ (c) $\frac{3R}{2}$ (d) $\frac{R}{3}$

Refraction and Dispersion Through Prism

94. What is the angle of incidence for an equilateral prism of refractive index $\sqrt{3}$, so that the ray is parallel to the base inside the prism?

(a) 30° (b) 45°

(c) 60° (d) Either 30° or 60°

95. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face if, the refractive index of the material of the prism is 1.524? **[NCERT]**

(a) 16°15′ (b) 29°75′

(c) 45°75′ (d) 58°15′

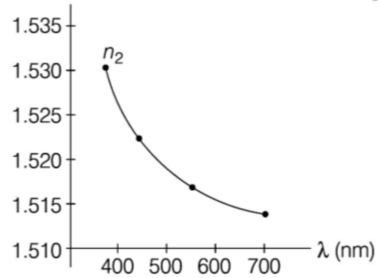
96. The refractive index of a prism for a monochromatic wave is $\sqrt{2}$ and its refracting angle is 60°. For minimum deviation, the angle of incidence will be

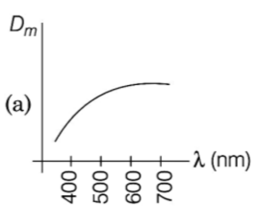
(a) 30° (b) 45°
(c) 60° (d) 75°

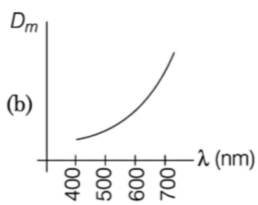
97. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is **[NCERT Exemplar]**

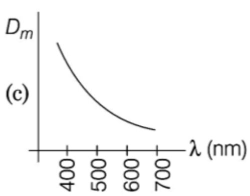
(a) 7.5° (b) 5°
(c) 15° (d) 2.5°

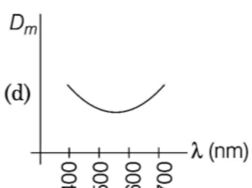
98. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation? **[JEE Main 2019]**



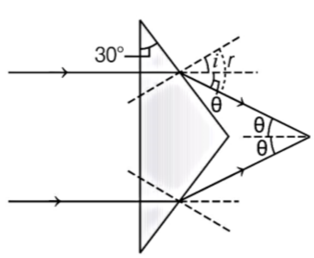
(a)


(b)


(c)


(d)


99. Two parallel light rays are incident at one surface of a prism of refractive index 1.5 as shown in figure. The angle between the emergent rays is nearly



(a) 19° (b) 37° (c) 45° (d) 49°

100. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is **[JEE Main 2019]**

(a) 45° (b) 90° (c) 60° (d) 30°

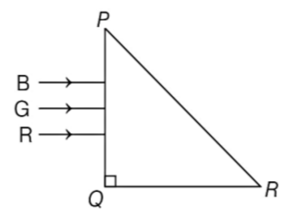
101. The maximum refractive index of a prism which permits the passage of light through it, when the refracting angle of the prism is 90°, is

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{2}$

102. The refractive index of the material of a prism is $\sqrt{2}$ and the angle of prism is 30°. One of its refracting faces is polished, the incident beam of light will retrace back for angle of incidence

(a) 0° (b) 45° (c) 60° (d) 90°

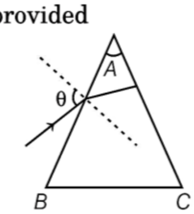
103. Three rays of light, namely red (R), green (G) and Blue (B) are incident on the face PQ of a right angled prism PQ as shown in figure.



The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray (s) emerging out of the face PR is **[JEE Main 2021]**

(a) green (b) red
(c) blue and green (d) blue

104. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided



[JEE Main 2015]

(a) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

105. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 is
(a) 3° (b) 4° (c) 5.33° (d) 2.6°

106. An achromatic convergent doublet of two lenses in contact has a power of + 2 D. The convex lens has power +5D. What is the ratio of the dispersive powers of the convergent and divergent lenses?
(a) 2 : 5 (b) 3 : 5 (c) 5 : 2 (d) 5 : 3

107. It is desired to make a converging achromatic combination of mean focal length 50 cm by using two lenses of materials A and B. If the dispersive powers of A and B are in ratio 1 : 2, the focal lengths of the convex and the concave lenses are respectively
(a) 25 cm and 50 cm (b) 50 cm and 25 cm
(c) 50 cm and 100 cm (d) 100 cm and 50 cm

Optical Instruments

108. A simple microscope consists of a concave lens of power – 10D and a convex lens of power + 20D in contact. If the image is formed at infinity, then the magnifying power when $D = 25$ cm is
(a) 2.5 (b) 3.5 (c) 2.0 (d) 3.0

109. In compound microscope, magnifying power is 95 and the distance of object from objective lens is $\frac{1}{3.8}$ cm. The focal length of objective lens is $\frac{1}{4}$ cm. What is the magnification of eyepiece?
(a) 5 (b) 10 (c) 100 (d) 200

110. The focal lengths of the objective and eyelenses of a microscope are 1.6 cm and 2.5 cm, respectively. The distance between the two lenses is 21.7 cm. If the final image is formed at infinity, the distance between the object and objective lens is
(a) 1.8 cm (b) 1.70 cm
(c) 1.65 cm (d) 1.75 cm

111. The focal length of the objective and the eyepiece of a microscope are 4 mm and 25 mm, respectively. If the final image is formed at infinity and the length of the tube is 16 cm, then the magnifying power of microscope will be
(a) – 337.5 (b) –3.75
(c) 3.375 (d) 33.75

112. An astronomical telescope has a converging eyepiece of focal length 5 cm and objective of focal length 80 cm. When the final image is formed at the least distance of distinct vision (25 cm), the separation between the two lenses is
(a) 75.0 cm (b) 80.0 cm
(c) 84.2 cm (d) 85.0 cm

113. The focal length of objective and eye lens of an astronomical telescope are respectively 2cm and 5cm. Final image is formed at (1) least distance of distinct vision (2) infinity. Magnifying powers in two cases will be
(a) –48, –40 (b) – 40, 48
(c) – 40, + 48 (d) – 48, + 40

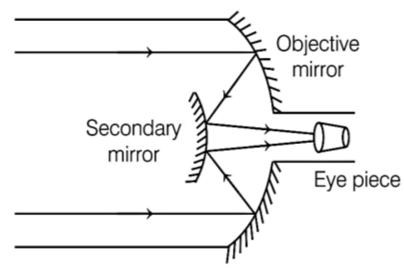
114. The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eyepiece is found to be 20 cm. The focal lengths of the lenses are
(a) 18 cm, 2 cm (b) 11 cm, 9 cm
(c) 10 cm, 10 cm (d) 15 cm, 5 cm

115. The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eyepiece? [JEE Main 2020]
(a) 10 cm (b) 20 cm (c) 30 cm (d) 40 cm

116. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eyepiece should be close to [JEE Main 2020]
(a) 22 mm (b) 2 mm (c) 12 mm (d) 33 mm

117. A telescope has an objective of focal length 50 cm and an eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away. The separation between the objective and eyepiece is
(a) 74 cm (b) 75 cm (c) 60 cm (d) 71 cm

118. A Cassegrain telescope uses two mirrors as shown in figure. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be? [NCERT]



(a) 235 mm away from small mirror
(b) 285 mm away from small mirror
(c) 305 mm away from small mirror
(d) 315 mm away from small mirror

119. A hypermetropic person having near point at a distance of 0.75 m puts on spectacles of power 2.5 D. The near point now is at
(a) 0.75 m (b) 0.83 m
(c) 0.36 cm (d) 0.26 m

ROUND II Mixed Bag

Only One Correct Option

1. A convex lens forms a real image of a point object placed on its principal axis. If the upper half of the lens is painted black, then
(a) the image will be shifted upward
(b) the image will be shifted downward
(c) the intensity of the image will decrease
(d) None of the above

2. A magnifying glass is used, as the object to be viewed can be brougth closer to the eye than the normal near point. This results in
(a) a larger angle to be subtended by the object at the eye and hence viewed in greater detail
(b) the formation of a real image
(c) increase in the field of view
(d) infinite magnification at the near point

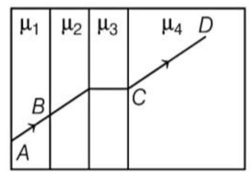
3. Consider an extended object immersed in water contained in a plane trough.
When the object is seen close to the edge of the trough, then the object looked distorted because,
(a) the apparent depth of the points close to the edge are nearer the surface to the water compared to the points away from the edge.
(b) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air
(c) some of the points of the object far away from the edge may not be visible because of total internal reflection
(d) All of the above

4. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm, then
(a) The distance between the objective and the eyepiece is 16.02 m
(b) The angular magnification of the planet is –800
(c) The image of the planet is inverted
(d) All of the above

5. Let the XY-plane be the boundary between two transparent media. Medium 1 with $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}} - 10\,\hat{\mathbf{k}}$ is incident on the plane of separation. The angle of refraction in medium 2 is [AIEEE 2011]
(a) 45° (b) 60°
(c) 75° (d) 30°

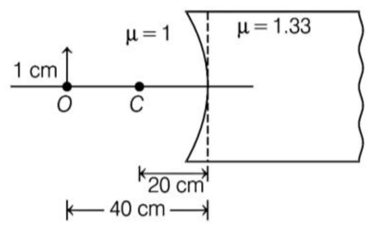
6. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15m/s. The speed of the image of he second car as seen in the mirror of the first one is [AIEEE 2011]
(a) $\frac{1}{15}$ m/s (b) 10 m/s (c) 15 m/s (d) $\frac{1}{10}$ m/s

7. A ray of light passes through four transparent medium with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have



(a) $\mu_1 = \mu_2$ (b) $\mu_1 = \mu_3$ (c) $\mu_3 = \mu_4$ (d) $\mu_3 = \mu_1$

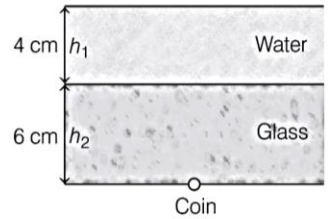
8. For a optical arrangement shown in the figure, find the position and nature of image.



(a) 32 cm (b) 0.6 cm (c) 6 cm (d) 0.5 cm

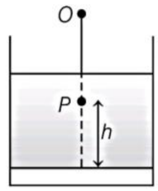
9. A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is
(a) 50° (b) 80°
(c) 100° (d) 40°

10. A 4 cm thick layer of water covers a 6 cm thick glass slab. A coin is placed at the bottom of the slab and is being observed from the air side along the normal to the surface. Find the apparent position of the coin from the surface.



(a) 7.0 cm (b) 8.0 cm
(c) 10 cm (d) 5 cm

11. A plane mirror as placed at the bottom of a tank containing a liquid of refractive index μ and P is a small object at a height h above the mirror. An observer O vertically above P outside the liquid sees P and its image in a mirror. The apparent distance between these two will be

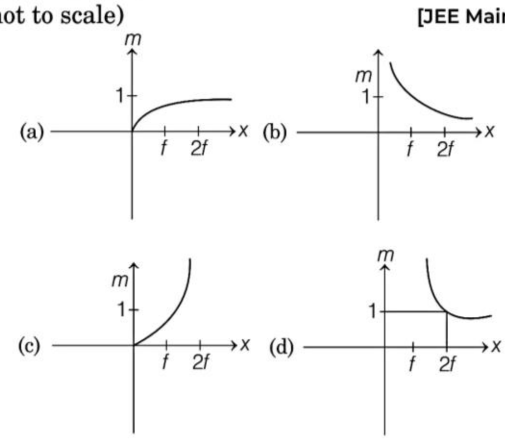


(a) $2\mu h$ (b) $\frac{2h}{\mu}$ (c) $\frac{2h}{\mu - 1}$ (d) $h\left(1+\frac{1}{\mu}\right)$

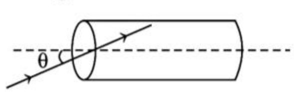
12. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **[AIEEE 2012]**

(a) 7.2 (b) 2.4 (c) 3.2 (d) 5.6

13. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification m *versus* distance of the object from the mirror x is correctly given by (graphs are drawn schematically and are not to scale) **[JEE Main 2020]**



14. A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid point of one end of the rod as shown in the figure.

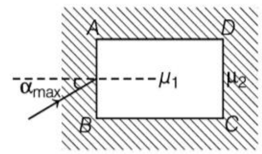


The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

15. A rectangular glass slab $ABCD$ of refractive index μ_1, is immersed in water of refractive index $\mu_2 (\mu_1 > \mu_2)$. A ray of light is incident at the surface AB of the slab as shown in figure. The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other surface CD is given by



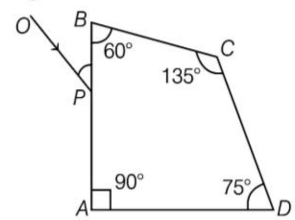
(a) $\sin^{-1}\left[\left(\frac{\mu_1}{\mu_2}\right)\cos\left(\sin^{-1}\frac{\mu_2}{\mu_1}\right)\right]$

(b) $\sin^{-1}\left(\frac{\alpha_2}{\alpha_1}\right)$

(c) $\sin^{-1}\left[\alpha_1 \cdot \cos\left(\sin^{-1}\frac{1}{\alpha_2}\right)\right]$

(d) $\left[\sin^{-1}\frac{\alpha_1}{\alpha_2}\right]$

16. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90°
(d) All of the above

17. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 $(x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is **[JEE Main 2019]**

(a) 5 : 3 (b) 2 : 1 (c) 4 : 3 (d) 3 : 1

18. P is a point on the axis of a concave mirror. The image of P formed by the mirror, coincides with P. A rectangular glass slab of thickness t and refractive index μ is now introduced between P and the mirror. For image of P to coincide with P again, the mirror must be moved

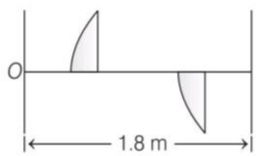
(a) towards P by $(\mu - 1)t$ (b) away from P by $(\mu - 1)t$

(c) towards P by $t\left(1-\frac{1}{\mu}\right)$ (d) away from P $t\left(1-\frac{1}{\mu}\right)$

19. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be **[JEE Main 2019]**

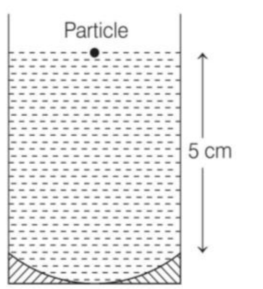
(a) 25 cm (b) 20 cm (c) 10 cm (d) 30 cm

20. A thin plano-convex lens focal length f is split into two halves. One of the halves is shifted along the optical axis and the separation between object and image plane is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and separation between the two halves.



(a) 0.1 m (b) 0.4 m (c) 0.9 m (d) 1 m

21. A concave mirror has radius of curvature of 40 cm and it is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to [Take, refractive index of water = 1.33] **[JEE Main 2019]**

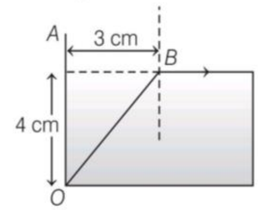


(a) 6.7 cm (b) 13.4 cm
(c) 8.8 cm (d) 11.7 cm

22. For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 D and the least converging power of the eye lens be, hind the cornea is about 20 D. From this rough data, estimate the range of accommodation (*i.e.* the range of converging power of the eye lens) of a normal eye.

(a) 10 D to14 D (b) 20 D to 24 D
(c) 28 D to 32 D (d) 14 D to 18 D

23. A small coin is resting on the bottom of the beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along the surface (figure). How fast is light travelling in the liquid?



(a) 2.4×10^8 m/s (b) 1.8×10^8 m/s
(c) 3.0×10^8 m/s (d) 5.0×10^5 m/s

24. A simple telescope, consisting of an objective of focal length 60 cm and a single eyelens of focal length 60 cm and a single eyelens of focal length 5 cm is focussed on a distant object is such a way that parallel rays come out from the eyelens. If the object subtends an angle 2° at the objective, the angular width of the image will be

(a) 10° (b) 24°
(c) 35° (d) 48°

25. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross-sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly)
[Use the fact that surface area of a spherical cap of height h and radius of curvature R is $2\pi Rh$] **[JEE Main 2020]**

(a) 34% (b) 50% (c) 17% (d) 21%

Numerical Value Questions

26. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be ...... (in degree). **[JEE Main 2021]**

(Round off to the nearest integer)

27. A prism of angle $A = 1°$ has a refractive index $\mu = 1.5$. A good estimate for the minimum angle of deviation (in degree) is close to $N/10$. Value of N is .... **[JEE Main 2020]**

28. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is .... **[JEE Main 2020]**

29. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is .... **[JEE Main 2020]**

30. A compound microscope has an objective and eyepiece as thin lenses of focal lengths 1 cm and 5 cm, respectively. The distance between the objective and the eyepiece is 20 cm. The distance (in cm) at which the object must be placed in front of the objective, if the final image is located at 25 cm from the eyepiece, is numerically ....

31. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eyepiece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eyepiece lens (in cm) is .... **[JEE Main 2020]**

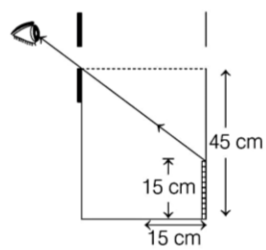
32. A compound microscope consists of an objective lens of focal length 1 cm and an eyepiece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is .... **[JEE Main 2020]**

33. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$ D, where N is an integer.

Then, the value of N is .... **[JEE Main 2020]**

34. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degree) between the reflected and refracted rays at this surface is .... **[JEE Main 2020]**

35. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm, the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where N is an integer, the value of N is .... **[JEE Main 2020]**



Answers

Round I

1. (a)	2. (c)	3. (b)	4. (a)	5. (c)	6. (a)	7. (d)	8. (c)	9. (b)	10. (a)
11. (d)	12. (c)	13. (c)	14. (c)	15. (a)	16. (a)	17. (a)	18. (b)	19. (c)	20. (d)
21. (d)	22. (d)	23. (b)	24. (d)	25. (c)	26. (c)	27. (b)	28. (d)	29. (a)	30. (a)
31. (c)	32. (c)	33. (d)	34. (a)	35. (c)	36. (c)	37. (a)	38. (d)	39. (d)	40. (c)
41. (b)	42. (c)	43. (b)	44. (a)	45. (a)	46. (c)	47. (d)	48. (c)	49. (a)	50. (a)
51. (d)	52. (b)	53. (c)	54. (a)	55. (d)	56. (c)	57. (c)	58. (b)	59. (d)	60. (c)
61. (c)	62. (a)	63. (a)	64. (b)	65. (c)	66. (b)	67. (a)	68. (a)	69. (b)	70. (c)
71. (a)	72. (b)	73. (d)	74. (b)	75. (c)	76. (c)	77. (c)	78. (c)	79. (c)	80. (b)
81. (d)	82. (b)	83. (a)	84. (d)	85. (c)	86. (d)	87. (b)	88. (c)	89. (a)	90. (c)
91. (c)	92. (d)	93. (d)	94. (c)	95. (b)	96. (b)	97. (a)	98. (c)	99. (b)	100. (c)
101. (b)	102. (b)	103. (b)	104 (a)	105. (a)	106. (b)	107. (a)	108. (a)	109. (a)	110. (d)
111. (a)	112. (c)	113. (a)	114. (a)	115. (a)	116. (a)	117. (d)	118. (d)	119. (d)	

Round II

1. (c)	2. (a)	3. (d)	4. (d)	5. (a)	6. (a)	7. (a)	8. (b)	9. (d)	10. (a)
11. (b)	12. (d)	13. (d)	14. (d)	15. (a)	16. (d)	17. (d)	18. (c)	19. (c)	20. (b)
21. (c)	22. (b)	23. (b)	24. (b)	25. (c)	26. 12	27. 5	28. 60	29. 1	30. 1.067
31. 6	32. 50	33. 476	34. 90	35. 158					

Solutions

Round I

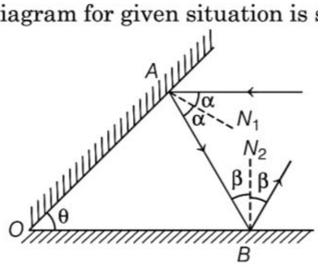
1. The number of images formed, if two plane mirrors are kept at an angle θ is given by $\frac{360°}{\theta}$, if $\frac{360°}{\theta}$ is odd and it is given by $\frac{360°}{\theta} - 1$, if $\frac{360°}{\theta}$ is even.

For θ = 72°, number of images,

$$n = \frac{360°}{72°} = 5$$

As, $\frac{360°}{72°} = 5$ is an odd number.

2. The ray diagram for given situation is shown below.



Let, angle of incidence on mirror OA is α and angle of incidence on mirror OB is β.

$\therefore \quad \angle OAB = 90° - \alpha$

and $\quad \angle OBA = 90° - \beta$

In ΔOAB,

$$\theta + \angle OAB + \angle OBA = 180°$$

$$\Rightarrow \theta + 90° - \alpha + 90° - \beta = 180°$$

$$\Rightarrow \theta = \alpha + \beta$$

$$\Rightarrow \alpha + \beta = \theta \quad \text{...(i)}$$

∴ Total deviation of the ray

$$\delta = (180° - 2\alpha) + (180° - 2\beta)$$

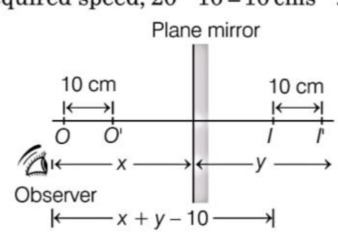
$$= 360° - 2(\alpha + \beta)$$

$$= 360° - 2\theta \quad \text{[from Eq. (i)]}$$

3. The first images is due to reflection from the front surface, *i.e.* in unpolished surface of the mirror. So, only a small fraction is the incident light energy is reflected.

The second image is due to reflection from polished surface. So, a major portion of light is reflected. Thus, the second image is the brightest.

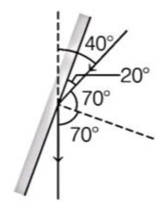
4. It is clear from figure, the distance of image with reference to observer reduces by 10 cm in one second. Thus, required speed, $20 - 10 = 10 \text{ cms}^{-1}$.



5. If the plane mirror is rotated through an angle θ, the reflected ray turns through an angle 2θ in that direction keeping the incident ray fixed.

6. Clearly, $i + r = i + i = 140°$

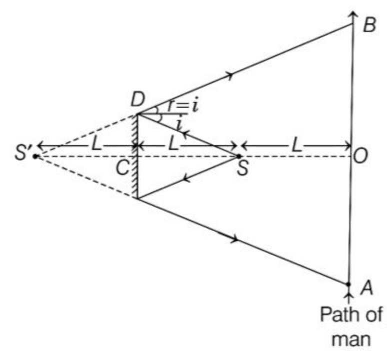
or $\quad i = 70°$



Clearly, the plane mirror makes an angle of 20° with vertical and 70° with horizontal. Thus, β is 70°.

7. As we known in the side mirror, the speed of approaching car would appear to increase, since the distance between the cars decreases.

8. Light from mirror is reflected in a straight line and it is appear to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram below



From ray diagram in similar triangles $\Delta S'CD$ and $\Delta S'OB$, we have

$$\frac{S'C}{S'O} = \frac{CD}{OB}$$

So, $\quad OB = \frac{CD \times S'O}{S'C} = \frac{\frac{d}{2} \times 3L}{L} = \frac{3d}{2}$

Also, $OA = \frac{3d}{2}$

So, distance over which man can see the image S' is

$$\frac{3d}{2} + \frac{3d}{2} = 3d$$

9. We have, $\quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

or $\quad \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$

Now, compare it with equation, $y = mx + c$.
Therefore, graph is a straight line having negative slope.

10. As, $\dfrac{f}{f-u} = \dfrac{1}{4} = \dfrac{f}{f-(-0.5)}$

or $\quad 4f = f + 0.5$ or $3f = 0.5$

or $\quad f = \dfrac{0.5}{3}\,\text{m} = 0.17\ \text{m}$

11. As, $u = f - x_1$ and $v = f - x_2$

Now, $\quad \dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{f-x_1} + \dfrac{1}{f-x_2} = \dfrac{1}{f}$

or $\quad \dfrac{f - x_2 + f - x_1}{(f-x_1)(f-x_2)} = \dfrac{1}{f}$

or $\quad f^2 - fx_2 - fx_1 + x_1x_2 = 2f^2 - f(x_1 + x_2)$

or $\quad f^2 = x_1x_2$

or $\quad f = \sqrt{x_1x_2}$

This is Newton's mirror formula.

12. The image is erect and diminished. So, the mirror is necessarily convex.

13. We have, $\dfrac{1}{-30} + \dfrac{1}{v} = \dfrac{1}{30}$

or $\quad \dfrac{1}{v} = \dfrac{2}{30} = \dfrac{1}{15}$

or $\quad v = 15\ \text{cm}$

The distance is behind the mirror.

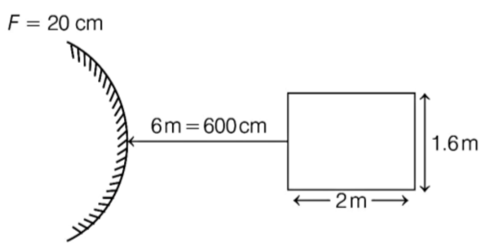
14. We have, $m = \dfrac{f}{f-u}$

$= \dfrac{-16}{-16-(-8)} = \dfrac{-16}{-8} = 2$

15. From the mirror formula,

$$\frac{1}{v} + \frac{1}{(-600)} = \frac{1}{20}$$

The given situation is shown in the figure.

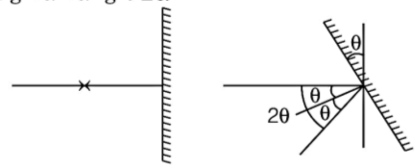


or $\quad \dfrac{1}{v} = \dfrac{1}{20} + \dfrac{1}{600}$

or $\quad \dfrac{1}{v} = \dfrac{31}{600}$

or $\quad v = \dfrac{600}{31}\,\text{cm} = 19.35\ \text{cm}$

16. By keeping the incident ray is fixed, if plane mirror rotates through an angle θ reflected ray rotates through an angle 2θ.



17. As, $m = \dfrac{f}{f-u}$

$\Rightarrow \quad 2 = \dfrac{-0.2}{-0.2-u}$

or $\quad 2 = \dfrac{0.2}{0.2+u}$

or $\quad 0.4 + 2u = 0.2$

or $\quad 2u = 0.2 - 0.4 = -0.2$

or $\quad u = -0.1\ \text{m}$

18. Clearly, the given mirror is a convex mirror.

As $\quad m = \dfrac{f}{f-u}$

$\Rightarrow \quad \dfrac{1}{3} = \dfrac{18}{18-u}$

or $\quad 3 \times 18 = 18 - u$

or $\quad u = -2 \times 18\ \text{cm}$

or $\quad u = -36\ \text{cm}$

19. We have, $\theta = \dfrac{1}{2} \times \dfrac{\pi}{180}$ rad

As, $\quad \dfrac{\text{diameter of image}}{\text{focal length}} = \theta$

or diameter of image

$= \dfrac{1}{2} \times \dfrac{\pi}{180} \times \dfrac{15}{2} \times 100\ \text{cm}$

$= 6.55\ \text{cm}$

20. Using mirror formula,

$$\frac{1}{v} + \frac{1}{(-u)} = \frac{1}{-f}$$

$\Rightarrow \quad v = \dfrac{fu}{f-u}$

$\Rightarrow \quad \dfrac{\delta V}{\delta x} = \dfrac{\delta}{\delta x}\left(\dfrac{fu}{u-f}\right)$

$\Rightarrow \quad \dfrac{\partial V}{\partial x} = \dfrac{f^2}{(u-f)^2} \cdot \dfrac{\partial u}{\partial x}$

$\Rightarrow \quad d_v = \dfrac{f^2}{(u-f)^2} \cdot d_u$

Now, $d_u = b$ (given)

$\therefore \quad d_v = \dfrac{bf^2}{(u-f)^2} = b\left(\dfrac{f}{u-f}\right)^2$

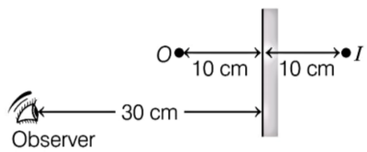
$=$ size of image.

21. Here, $f = \frac{1.6}{2}\text{m} = 0.8\text{ m}, u = -1\text{ m}$

We have, $\frac{1}{v} = \frac{1}{0.8} - \frac{1}{(-1)} = \frac{10}{8} + 1 = \frac{18}{8} = \frac{9}{4}$ or $v = \frac{4}{9}\text{m}$

(Positive sign indicates that the distance is behind the mirror)

22. Clearly, the distance of image from observer is 40 cm.



23. As, magnification, $m = \frac{f}{f-u}$

If $f = -24$ cm and $m = +3$, then

$$3 = \frac{-24}{-24-u}$$

or $-24 - u = -8$

or $u + 24 = 8$

or $u = -16$ cm

Now, if $m = -3$, then

$$-3 = \frac{-24}{-24-u}$$

or $u + 24 = -8$

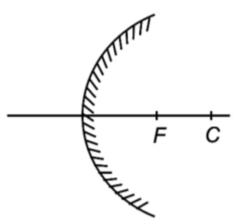
or $u = -32$ cm

24. Given, focal length of convex mirror, $f = +15$ cm (focal length of convex mirror is taken as positive)

Distance of object, $u = -12$ cm

Size of object, $O = 4.5$ cm

Using the mirror formula,



$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{v} - \frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{9}{60}$$

Distance of image from the mirror, $v = 6.7$ cm.

The positive sign shows that the image is formed behind the mirror.

Using the formula of magnification,

$$m = -\frac{v}{u} = \frac{I}{O}$$

$$\frac{-6.7}{-12} = \frac{I}{4.5}$$

Size of image, $I = 2.5$ cm

As I is positive, so image is erect and virtual.

Magnification m is given by

$$m = \frac{I}{O} = \frac{2.5}{4.5} = \frac{25}{45} = \frac{5}{9}$$

As the needle moves away from the mirror, the image also moves away from the mirror (as $u \to \infty, v \to f$) and the size of image goes on decreasing.

25. One image will be real and the other will be virtual. Since, they are of the same size, one will have magnification m and the other $-m$.

$$\frac{1}{u_1} + \frac{1}{u_1 m} = \frac{1}{f}$$

or $$\frac{1}{u_1}\left(1 + \frac{1}{m}\right) = \frac{1}{f} \quad \text{... (i)}$$

and $$\frac{1}{u_2} - \frac{1}{u_2 m} = \frac{1}{f}$$

or $$\frac{1}{u_2}\left(1 - \frac{1}{m}\right) = \frac{1}{f} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{u_1 + u_2}{f} = 2$$

or $$f = \frac{u_1 + u_2}{2}$$

26. Magnification produced by a mirror can also be given as

$$m = \frac{f}{f-u}$$

Substituting the given values, we get

$$5 = \frac{-0.4}{-0.4-u}$$

or $u = -0.32$ m

27. In question figure, PQ is a ray of light passing through focus, and falling on the surface of a concave mirror. On reflection from the mirror, the ray becomes parallel to principal axis of the mirror.

28. Radius of curvature of given concave mirror, $R = -8$ cm

So, focal length, $f = \frac{R}{2} = -4$ cm

Object distance, $u = -10$ cm

Using mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{4} - \frac{1}{(-10)} = -\frac{3}{20}$$

$\Rightarrow \quad v = -6.67$ cm (real)

Also, magnification,

$$m = -\frac{v}{u} = -0.667$$

As, m is negative and less than one, so image is inverted, real and diminished or unmagnified.

29. According to Snell's law,

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin i}{\mu}$$

As μ is negative, $\sin r$ is negative.

Therefore, r is negative.

If incident ray from air (medium 1) incident on those material, the ray refract or bend same side of the normal as in option (a).

30. As, $$n_1 = \frac{c}{v_1} = \frac{v\lambda}{v\lambda_1} = \frac{\lambda}{\lambda_1}$$

and $\quad n_2 = \dfrac{c}{v_2} = \dfrac{\nu\lambda}{\nu\lambda_2} = \dfrac{\lambda}{\lambda_2}$

Now, $\quad \dfrac{n_1}{n_2} = \dfrac{\lambda_2}{\lambda_1}$ or $\lambda_2 = \left(\dfrac{n_1}{n_2}\right)\lambda_1$

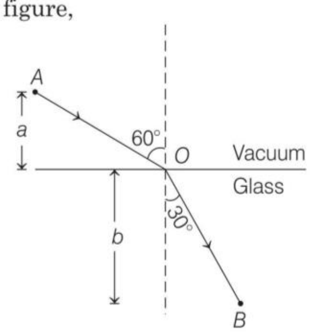
31. We have, $\dfrac{\sin r}{\sin i} = \tan 30° = \dfrac{1}{\sqrt{3}}$

or $\quad \dfrac{\sin i}{\sin r} = \sqrt{3}$

$\Rightarrow \quad \dfrac{\text{Speed of light in medium } X}{\text{Speed of light in medium } Y} = \sqrt{3}$

$\Rightarrow$ Speed of light in medium $Y = \dfrac{1}{\sqrt{3}} \times$ Speed of light in medium X

32. From the figure,



$$AO = \frac{a}{\cos 60°} = 2a$$

and $$BO = \frac{b}{\cos 30°} = \frac{2}{\sqrt{3}}\,b$$

From Snell's law,

$$\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

$\therefore$ Optical path $= AO + \mu\,(BO)$

$$= 2a + \left(\sqrt{3} \times \frac{2}{\sqrt{3}}\,b\right) = 2a + 2b$$

33. Given, $\mu = 1.5,\ t_1 = 5$ cm, $\mu_2 = 1.33$ and $t_2 = 1$ cm

Change in path $= \Delta t_1 + \Delta t_2$

$$= \left(1 - \frac{1}{\mu_1}\right) \times t_1 + \left(1 - \frac{1}{\mu_2}\right) \times t_2$$

$$= \left(1 - \frac{1}{1.5}\right) \times 5 + \left(1 - \frac{1}{1.33}\right) \times 1 \simeq 1.90 \text{ cm}$$

34. We have, $\mu_g \sin \theta_C = \mu_1 \sin 90°$

or $\quad \mu_g \sin \theta_C = 1$

When water is poured, then

$$\mu_w \sin r = \mu_s \sin \theta_C$$

or $\quad \mu_w \sin r = 1$

Again, $\quad \mu_a \sin \theta = \mu_w \sin r$

or $\quad \mu_a \sin \theta = 1$

or $\quad \sin \theta = 1$

or $\quad \theta = 90°$

35. Let r be the radius of circle through which other objects become visible. The rays of light must be incident at critical angle C.

Now, $\quad \sin C = \dfrac{1}{\mu} = \dfrac{r}{\sqrt{r^2 + h^2}}$

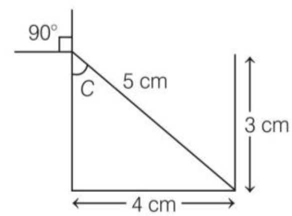
or $\quad \mu^2 r^2 = r^2 + h^2$

or $\quad (\mu^2 - 1) r^2 = h^2$

$\Rightarrow \quad r = \dfrac{h}{\sqrt{\mu^2 - 1}}$

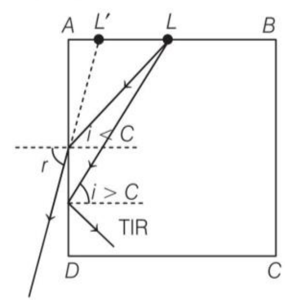
Diameter, $\quad 2r = \dfrac{2h}{\sqrt{\mu^2 - 1}}$

36. Light ray is going from liquid (denser) to air (rarer) and angle of refraction is 90°, so angle of incidence must be equal to critical angle, therefore $\sin C = \dfrac{4}{5}$



Also, $\quad \mu = \dfrac{1}{\sin C} = \dfrac{5}{4} = 1.2$

37. In figure, a pin is held at L, mid-point of AB. When seen from face AD (so long $i < C$) image of L appears to be at L', close to A.



From $\sin C = \dfrac{1}{\mu} = \dfrac{1}{1.6} = 0.625$

$\Rightarrow \quad C = \sin^{-1}(0.625) = 38.7°$

So when angle of incidence becomes greater than $C (= 38.7°)$, the rays starting from L will undergo total internal reflection and pin shall not be seen at all.

38. The real depth = Refractive index (Apparent depth)

In first case, the real depth $h_1 = \mu\,(d - c)$.

Similarly, in the second case, the real depth, $h_2 = \mu\,(d - c)$

Since $h_2 \geq h_1$, the difference of the depths $= h_2 - h_1$

$$= \mu(d - c - b + a)$$

Since, the liquid is added in second case, $h_2 - h_1 = d - b$

$$\mu = \frac{d - b}{d - c - b + a}$$

39. Speed of light in a medium is given by

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \times \frac{1}{\sqrt{\varepsilon_r\mu_r}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

Here, $\varepsilon_r = 3$ and $\mu_r = \frac{4}{3}$

$$\therefore \quad \frac{v}{c} = \frac{1}{\sqrt{\varepsilon_r\mu_r}} = \frac{1}{\sqrt{3 \times \frac{4}{3}}} = \frac{1}{2}$$

So, refractive index of medium w.r.t. vacuum or air,

$$n = \frac{c}{v} = 2$$

Hence, critical angle,

$$i_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\frac{1}{2} = 30°$$

40. We have $3\left(1 - \frac{1}{\mu}\right) = 1$

or $\quad 1 - \frac{1}{\mu} = \frac{1}{3}$

or $\quad \frac{1}{\mu} = 1 - \frac{1}{3} = \frac{2}{3}$ or $\mu = \frac{2}{3}$

Now, $\quad \frac{1}{\sin i_C} = \frac{3}{2}$

or $\quad \sin i_C = \frac{2}{3}$

or $\quad i_C = \sin^{-1}\left(\frac{2}{3}\right)$

or $\quad i_C = \sin^{-1}(0.67)$

41. Speed of light in medium

$$= \frac{3 \times 10^{-2} \times 10}{0.2 \times 10^{-9}}$$

$$= 1.5 \times 10^9 \text{ ms}^{-1}$$

As, $\quad \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

$$\frac{\mu}{1} = \frac{3 \times 10^8}{1.5 \times 10^8} \Rightarrow \mu = 2$$

We have, $\quad \sin C = \frac{1}{2}$

$\Rightarrow \quad C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

Hence, it suffers internal reflection in the case when the angle of incidence is 40°.

42. Given, $\angle i = 60°$ and $x = 1$ km $= 10^3$ m

When the total internal reflection just takes place from lateral surface, $i = C$, i.e. $60° = C$

$\Rightarrow \quad \sin C = \frac{1}{\mu}$

$\Rightarrow \quad \sin 60° = \frac{1}{\mu} = \frac{\sqrt{3}}{2}$

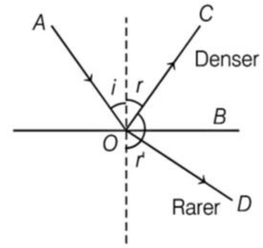
$\Rightarrow \quad \mu = \frac{2}{\sqrt{3}}$

The time taken by light to travel some distance in a medium,

$$t = \frac{\mu x}{c} = \frac{\frac{2}{\sqrt{3}} \times 10^3}{3 \times 10^8} = 3.85\,\mu\text{s}$$

43. At the critical angle, $\sin C = \frac{1}{\mu} = \frac{1}{\sin r'/\sin i} = \frac{\sin i}{\sin r'}$

As is clear as shown in figure



$$\angle COD = 90°$$

$\therefore \quad 90° - r + 90° - r' = 90°$

or $\quad r' = 90° - r$

$\therefore \quad \sin C = \frac{\sin i}{\sin(90° - r)} = \frac{\sin i}{\cos r}$

$$= \frac{\sin i}{\cos i} = \tan i \qquad (\because i = r)$$

$$C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r).$$

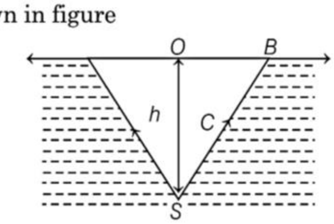
44. For total internal reflection at AC-face

$$\sin i \ge \frac{\mu_w}{\mu_g}$$

$$\sin\theta \ge \frac{4}{3 \times 1.5}$$

$$\sin\theta \ge \frac{8}{9}$$

45. As shown in figure



$$OS = h$$

When angle of incidence is slightly greater than C, light undergoes total internal reflection.

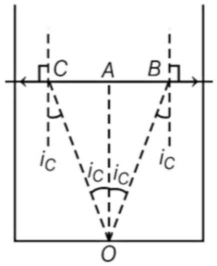
$\therefore$ Diameter of circle of light coming from water surface

$$= 2r = 2(OB) = 2OS\tan C = 2h\tan C$$

46. Here, for yellow light, $r = 90°$ when $i = C$. As, i is kept same, C must be smaller for total internal reflection. From $\mu = \frac{1}{\sin C}$, C will be smaller when μ is larger. Out of given colours, μ is largest for blue colour. Critical angle will be smallest for blue colour. Therefore, blue light would undergo total internal reflection.

47. Let the bulb is placed at point O,

$$AB = AC = r$$



If the light falls at an angle of incidence equal to critical angle i_C, then only a circular area is formed because if angle of incidence is less than the critical angle, it will refract into air and when angle of incidence is greater than critical angle, then it will be reflected back in water.

The source of light is 80 cm below the surface of water, *i.e.* $AO = 80 \text{ cm}, \mu_w = 1.33$

Using the formula for critical angle,

$$\sin i_C = \frac{1}{\mu_w}$$

$$\sin i_C = \frac{1}{1.33} = 0.75$$

$\Rightarrow$ $i_C = 48.6°$

In ΔOAB, $\tan i_C = \frac{AB}{AO}$

$\Rightarrow$ $\tan i_C = \frac{r}{l}$

$\Rightarrow$ $r = l \tan i_C = 80 \tan 48.6$

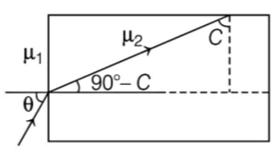
$$r = 80 \times 1.1345 = 90.7 \text{ cm}$$

Area of circular surface of water through which light will emerge,

$$A = \pi r^2$$
$$A = 3.14 \times (90.7)^2 = 25865.36 \text{ cm}^2$$
$$A = 2.58 \text{ m}^2$$
$$\simeq 2.6 \text{ m}^2$$

48. We know that, $\sin C = \frac{\mu_1}{\mu_2}$

$\because$ $\mu_1 \sin\theta = \mu_2 \sin(90° - C)$



$\Rightarrow$ $\sin\theta = \frac{\mu_2\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$

$\Rightarrow$ $\theta = \sin^{-1}\sqrt{\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}}$

For TIR, $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

49. We have, $\mu_1 = 1.20 + \frac{0.8 \times 10^{-14}}{(400 \times 10^{-9})^2}$

or $\mu_1 = 1.25$

or $\sin i_C = \frac{1}{1.25} = 0.8$

$\Rightarrow$ $i_C = 53.13°$

Again, $\mu_2 = 1.20 + \frac{0.8 \times 10^{-14}}{(500 \times 10^{-9})^2}$

$\because$ $\mu_2 = 1.232$

or $\sin i_C = \frac{1}{1.232} = 0.81$

$\Rightarrow$ $i_C = \sin^{-1} 0.81$

$= 54.26°$

Now, $\sin\theta = 0.8$

or $\theta = 53.13°$

This angle is clearly greater than critical angle corresponding to wavelength 400 nm. So, light of 400 nm wavelength undergoes total internal reflection.

50. Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get

$$\frac{1.5}{v} - \frac{1.0}{\infty} = \frac{1.5 - 1.0}{20}$$

or $v = 60 \text{ cm}$

Hence, parallel rays of light actually meet at some point *i.e.* at a distance of 60 cm.

51. From a lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

If $u = \infty$, then $v = f$ and if $u = f$, then $v = \infty$.

Hence, graph between u and v will be a rectangular parabola.

Therefore, option (d) is correct.

52. Since two refractive indices are involved. Therefore, two images will be formed.

53. As, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ (Lens maker formula)

For lens to be concave, $\left(\frac{1}{r_1} - \frac{1}{r_2}\right) > 0$

or $\frac{1}{r_1} > \frac{1}{r_2}$

or $r_1 < r_2$

54. As, $m = \frac{f}{f + u}$

$\Rightarrow$ $\frac{1}{2} = \frac{-20}{-20 + u}$

or $-20 + u = -40$

or $u = -40 + 20$

or $u = -20 \text{ cm}$

55. As, $\frac{I}{O} = \frac{v}{u}$ = magnification

or $$\frac{I}{15} = \frac{-35}{-10}$$

$\therefore$ $$I = 15 \times 2.5 \text{ cm} = 37.5 \text{ cm}$$

56. As, $n = \frac{f}{f+u}$

$\Rightarrow$ $$f + u = \frac{f}{n}$$

$\Rightarrow$ $$u = \frac{f}{n} - f = \left(\frac{1-n}{n}\right)f$$

$\Rightarrow$ $$u = -\left(\frac{n-1}{n}\right)f$$

$\therefore$ $$|u| = \left(\frac{n-1}{n}\right)f$$

57. We have, $\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$ (By lens maker's formula)

or $$f = \frac{R}{2(\mu - 1)}$$

Now, $$f > R$$

$\therefore$ $$\frac{R}{2(\mu-1)} > R$$

or $$\frac{1}{2(\mu-1)} > 1$$

or $$2(\mu - 1) < 1$$

or $$\mu - 1 < \frac{1}{2}$$

or $$\mu < \left(1 + \frac{1}{2}\right)$$

or $$\mu < 1.5$$

58. As the image formed by a large number of rays from the object. So, if the central portion of the lens is wrapped in blank paper, then full image of object will be formed, but due to decrease of number of rays on lens, brightness of image will decrease.

59. Clearly, $2f = 20$ cm or $f = 10$ cm

Now, $u = -15$ cm, $v = ?$

$\because$ $$\frac{1}{v} - \frac{1}{(-15)} = \frac{1}{10}$$

or $$\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$$

or $$\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$$

or $$\frac{1}{v} = \frac{3-2}{30} = \frac{1}{30}$$

or $$v = 30 \text{ cm}$$

The change in image distance is $(30 - 20)$ cm, *i.e.* 10 cm and away from the lens.

60. Here, $\mu = 1.5$

If object lies on its plane side; $R_1 = \infty, R_2 = -20$ cm

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{\infty} + \frac{1}{21}\right) = \frac{1}{40}$$

$\Rightarrow$ $$f = +40 \text{ cm}$$

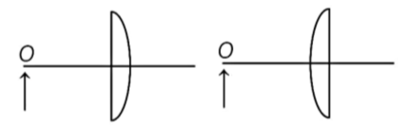
The lens behaves as convex.

If object lies on its curved side; $R_1 = 20$ cm, $R_2 = \infty$

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{\infty}\right) = \frac{1}{40}$$

$\Rightarrow$ $f' = 40$ cm

The lens behaves as convex.



61. When an object approaches a convergent lens from the left of the lens with a uniform speed of 5 m/s, the image moves away from the lens with a non-uniform acceleration. For example, if $f = 20$ m and $v = -50$ m ; -45 m , -40 m and -35 m; we get $v = 33.3$ m ; 36 m ; 40 m and 46.7 m. Clearly, image moves away from the lens with a non-uniform acceleration.

62. The central ray goes undeviated. So, $\mu_1 = \mu_3$.

Also, $\mu_3 < \mu_2$.

63. We have, $-\frac{1}{40} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(from lens maker formula)

$\Rightarrow$ $$\frac{1}{R_1} - \frac{1}{R_2} = -\frac{1}{20}$$

Now, $$\frac{1}{f} = \left(\frac{1.5}{2} - 1\right)\left(-\frac{1}{20}\right)$$

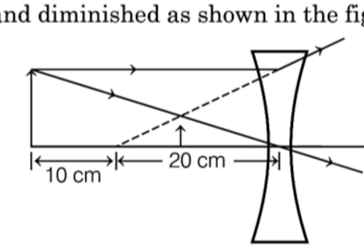
or $$\frac{1}{f} = -\frac{0.5}{2}\left(-\frac{1}{20}\right)$$

or $$\frac{1}{f} = \frac{1}{80}$$

or $$f = 80 \text{ cm}$$

So, it behaves like a convex lens of focal length 80 cm.

64. When an object is placed between $2f$ and f (focal length) of the diverging lens, the image is virtual, erect and diminished as shown in the figure.



To calculate the distance of the image from the lens, we apply

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{-20} = \frac{1}{v} + \frac{1}{30} \qquad \text{(as, } \mu = -30 \text{ cm)}$$

$$\Rightarrow \quad v = -\frac{(20)(30)}{20+30}$$

Thus, distance = –12 cm

(to the left of the diverging lens)

65. As, $y = \sqrt{y_1 \times y_2} = \sqrt{16 \times 9} = 4 \times 3 = 12$ cm

66. As $I \propto A^2$, where I = intensity and A = amplitude

$$\Rightarrow \quad \frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \frac{\pi r^2 - \frac{\pi r^2}{4}}{\pi r^2} = \frac{4\pi r^2 - \pi r^2}{4\pi r^2} = \frac{3\pi r^2}{4\pi r^2} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{3}{4}$$

$\Rightarrow \quad I_2 = \frac{3}{4} I_1$ and the focal length remains unchanged.

67. Here, $x = u + v$ ...(i)

As, $$m = \frac{f}{f+u} = \frac{f-v}{f}$$

and image is real, magnification is negative.

$$\therefore \quad -m = \frac{f}{f+u}, \; u = \frac{-(m+1)f}{m}$$

From $$-m = \frac{f-v}{f}$$

$$\Rightarrow \quad v = (m+1)f$$

Put in Eq. (i), we get

$$x = \frac{-(m+1)}{m} f + (m+1)f$$

Solving, we get

$$f = \frac{mx}{(m+1)^2}$$

68. In the situation given, the image will be formed at infinity, if the object is at focus of the lens, *i.e.* at 20 cm from the lens. Hence, shift in position of object,

$$x = 25 - 20 = \left(1 - \frac{1}{\mu}\right)t$$

$$5 = \left(1 - \frac{1}{1.5}\right)t$$

$$\Rightarrow \quad t = 15 \text{ cm}$$

69. As, $f \propto \frac{1}{\mu - 1}$

$\Rightarrow f \propto \frac{1}{\mu}$

Since, $\mu_V > \mu_G > \mu_R$

$\therefore \quad f_V < f_G < f_R$

70. As, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\Rightarrow \quad f_1 = -f_2$$
$$F = \infty$$

71. We have, from lens maker formula,

$$5 = (1.5 - 1)\left(\frac{2}{R}\right) \qquad \text{...(i)}$$

and $$-1 = \left(\frac{1.5}{n} - 1\right)\left(\frac{1}{R}\right) \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$-5 = \frac{0.5n}{1.5 - n}$$

or $$-7.5 + 5n = 0.5n$$

or $$-7.5 = -4.5n$$

$$\therefore \quad a^n l = n = \frac{75}{45} = \frac{5}{3}$$

72. Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \quad v = \frac{uf}{u+f}$$

Now, considering two cases

Case I If $v = u$

$$\Rightarrow \quad u = \frac{uf}{u+f}$$

$$\Rightarrow \quad u + f = u$$

$$\Rightarrow \quad u = 0$$

So, the curve will pass through origin.

Case II If $u = \infty \;\Rightarrow\; v = f$

So, as u tends towards infinity, v tends towards focus and all other distances will be less than focal length f and it will never intersect $u = v$ line.

Hence, correct option is (b).

73. Lens formula is given as

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad \text{... (i)}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \quad \frac{uf}{u+f} = v \;\Rightarrow\; \frac{v}{u} = \frac{f}{u+f} \qquad \text{...(ii)}$$

Now, by differentiating Eq. (i), we get

$$0 = -\frac{1}{v^2}\cdot\frac{dv}{dt} + \frac{1}{u^2}\cdot\frac{du}{dt}$$

[$\because f$ (focal length of a lens is constant)]

or $$\frac{dv}{dt} = \frac{v^2}{u^2} du/dt$$

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{f}{u+f}\right)^2 \frac{du}{dt} \qquad \text{[using Eq. (ii)]}$$

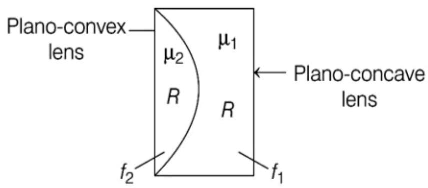
Given, $f = 0.3$ m, $u = -20$ m, $du/dt = 5$ m/s

$$\therefore \quad \frac{dv}{dt} = \left(\frac{0.3}{0.3-20}\right)^2 \times 5 = \left(\frac{3}{197}\right)^2 \times 5$$

$$= 1.16 \times 10^{-3}\ \text{m/s}$$

Thus, the image is moved with a speed of 1.16×10^{-3} m/s towards the lens.

74. Given combination is as shown below



As lenses are in contact, equivalent focal length of combination is

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{(1-\mu_1)}{R}$

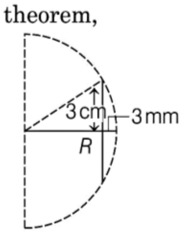
and $\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{(-R)}\right) = \frac{(\mu_2 - 1)}{R}$

$$\therefore \quad \frac{1}{f_{eq}} = \left(\frac{1-\mu_1}{R}\right) + \left(\frac{\mu_2 - 1}{R}\right)$$

$$= \frac{\mu_2 - 1 + 1 - \mu_1}{R} = \frac{\mu_2 - \mu_1}{R}$$

So, $f_{eq} = \frac{R}{\mu_2 - \mu_1}$

75. By Pythagoras theorem,



$$R^2 = (3)^2 + (R - 0.3)^2$$

$$\Rightarrow \quad R \approx 15\ \text{cm}$$

Refractive index of material of lens, $\mu = \frac{c}{v}$

Here, c = speed of light in vacuum $= 3 \times 10^8$ m/s

v = speed of light in material of lens $= 2 \times 10^8$ m/s

$$= \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}$$

From lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $R_1 = R$ and $R_2 = \infty$ (for plane surface)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$\Rightarrow \quad f = 30\ \text{cm}$$

76. Image can be formed on the screen, if it is real. Real image of reduced size can be formed by a concave mirror of a convex lens.

Let $u = 2f + x$, then

mirror equation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{2f + x} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{2f + x} = \frac{f + x}{f(2f + x)}$$

$$\Rightarrow \quad v = \frac{f(2f + x)}{f + x}$$

It is given that, $u + v = 1.0$ m

$$2f + x + \frac{f(2f + x)}{f + x} = (2f + x)\left[1 + \frac{f}{f + x}\right] < 1.0\ \text{m}$$

or $\quad \frac{(2f + x)^2}{f + x} < 1.0$ m

or $\quad (2f + x)^2 < (f + x)$

This will be true only, when $f < 0.25$ m

77. Given, $f_1 = 2f_2$

$$\Rightarrow \quad \frac{1}{|f_1|} = \frac{1}{|2f_2|} \quad \text{...(i)}$$

Using lens maker's formula, we get

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $\quad \frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$

$$\Rightarrow \left|(\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{(-R)}\right)\right| = \left|\frac{1}{2}(\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)\right|$$

[$\because$ using Eq. (i)]

$$\Rightarrow \quad \frac{\mu_1 - 1}{R} = \frac{\mu_2 - 1}{2R}$$

$$\Rightarrow \quad 2\mu_1 - \mu_2 = 1$$

78. When given set up is immersed in water, focal length of lens increases,

Note

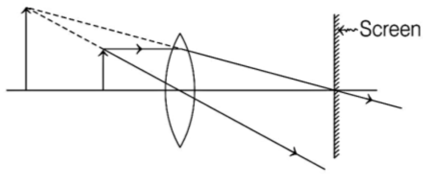
$$\frac{f_{\text{liquid}}}{f_{\text{air}}} = \frac{n_{ga} - 1}{n_{gl} - 1} = \frac{n_{ga} - 1}{\left(\frac{n_{ga}}{n_{la}} - 1\right)}$$

Now, $n_{ga} = \frac{3}{2}$ and $n_{la} = \frac{4}{3}$

$$\therefore f_{\text{liquid}} = f_{\text{air}}\left(\frac{\frac{1}{2}}{\frac{1}{8}}\right) = 4 f_{\text{air}}$$

As, focal length increases, there is no focussing of image on screen.

$\therefore$ Image will disappear



Actually, a virtual image is formed on same side of object.

79. As seen from a relatively rarer medium (L_2 or L_3), the interface L_1 L_2 is concave and L_2 L_3 is convex. The divergence produced by concave surface is much smaller than the convergence due to the convex surface. Hence, the arrangement corresponds to concavo-convex lens.

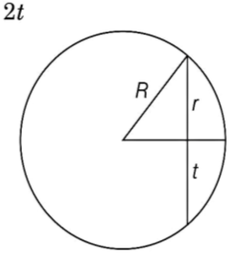
80. $\frac{1}{F} = \left[\frac{\mu_L}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

If $\mu_L = \mu_S \Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$

81.
$$R^2 = r^2 + (R - t)^2$$
$$R^2 = r^2 + R^2 + t^2 - 2Rt$$

Neglecting t^2, we get

$$R = \frac{r^2}{2t}$$



$\therefore$ $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

$\therefore$ $$f = \frac{R}{\mu - 1} = \frac{r^2}{2t(\mu - 1)}$$

$$= \frac{(3 \times 10^{-2})^2}{2 \times 3 \times 10^{-3} \times \left(\frac{3}{2} - 1\right)} = 0.3 \text{ m} = 30 \text{ cm}$$

82. As, power of combination,

$$P = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = 0$$

$\therefore$ $$\frac{1}{f_1} + \frac{1}{f_2} = \frac{d}{f_1 f_2}$$

$\Rightarrow$ $$\frac{1}{20} - \frac{1}{56} = \frac{d}{20(-56)}$$

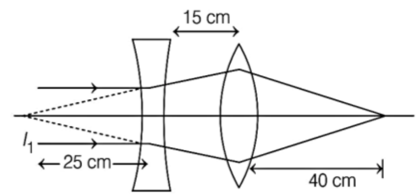
$\Rightarrow$ $$\frac{56 - 20}{20 \times 56} = \frac{d}{-20 \times 56}$$

$\Rightarrow$ $$d = -36 \text{ cm}$$

83. The power of the given system is a combination of the positive power of the convex lens, negative power of the plano-concave lens of water and zero power of the plane mirror. Clearly, the power of the system decreases.

84. Focal length of diverging lens is 25 cm.

As the rays are coming parallel, so the image (I_1) will be formed at the focus of diverging lens, *i.e.* at 25 cm towards left of diverging lens.



Now, the image (I_1) will work as object for converging lens.

For converging lens, distance of object u

(*i.e.* distance of I_1) $= -(25 + 15)$

$= -40$ cm

$f = 20$ cm

$\therefore$ From lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{(-40)}$$

$\Rightarrow$ $$\frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$\Rightarrow$ $$\frac{1}{v} = \frac{1}{90}$$

$\Rightarrow$ $$v = 40 \text{ cm}$$

v is positive, so image will be real and will form at right side of converging lens at 40 cm.

85. As, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} = \frac{1}{f_1} - \frac{1}{f_1} + \frac{x}{f_1^2}$

$\Rightarrow$ $$\frac{1}{f} = \frac{x}{f_1^2}$$

$\Rightarrow$ $f > 0$ for every x.

For $x = 0$, $f = \infty$

Hence, for $x = 0$, system will behave like a glass plate.

86. In every condition, two plano-convex lenses are placed, closed to each other, *i.e.* distance between is assumed to be zero.

i.e. $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Hence, focal length of combination will be same in each cases.

87. For lens A, object distance, $u = -20$ cm

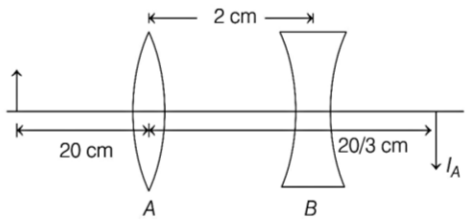
Focal length, $f = +5$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We have, $\frac{1}{v} = \frac{1}{5} - \frac{1}{20}$

$\Rightarrow \quad \frac{1}{v} = \frac{15}{100}$

$\Rightarrow \quad v = \frac{20}{3}$ cm



For lens B, image of A is object for B.

$\therefore \quad u = \frac{20}{3} - 2 = +\frac{14}{3}$ cm

$f = -5$ cm

Now, from lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

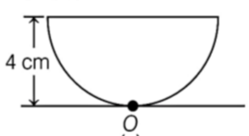
$\Rightarrow \quad \frac{1}{v} = \frac{1}{-5} + \frac{3}{14}$

$\Rightarrow \quad \frac{1}{v} = \frac{1}{70}$

$\Rightarrow \quad v = 70$ cm

Hence, image is on right of lens B and is real in nature.

88. As shown in Fig. (a),



(a)

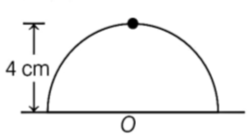
In this case, refraction of the rays starting from t_0 takes place from a plane surface. So, we can use,

$$d_{app} = \frac{d_{actual}}{\mu}$$

or $\quad 3 = \frac{4}{\mu}$

or $\quad \mu = \frac{4}{3}$

As shown in Fig. (b)



(b)

In this case, refraction takes place from a spherical surface. Hence, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}, \text{ we have}$$

$$\frac{1}{(-25/8)} - \frac{4/3}{(-4)} = \frac{1 - 4/3}{-R}$$

or $\quad \frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$

$\therefore \quad R = 25$ cm

Now, to find the focal length, we will use the lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{(-25)}\right) = \frac{1}{75}$$

$\therefore \quad f = 75$ cm

89. Given that, for convex lens,

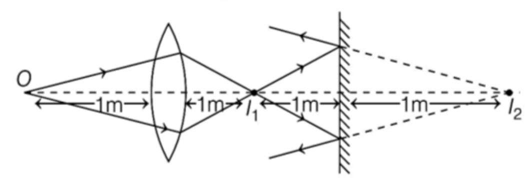
$$u_1 = -1 \text{ m}, f = +0.5 \text{ m}$$

Applying lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$\Rightarrow \quad \frac{1}{v_1} - \frac{1}{(-1)} = \frac{1}{0.5}$

$\Rightarrow \quad v_1 = +1$ m



The image I_1 will behave as an object for plane mirror. So, image will be formed 1 m behind the mirror as shown in the above figure.

Now, the virtual image I_2 will act as an object at a distance of 3m from lens.

For final image I_3,

$$u = -3 \text{ m}, f = +0.5 \text{ m}$$

Again, applying lens formula,

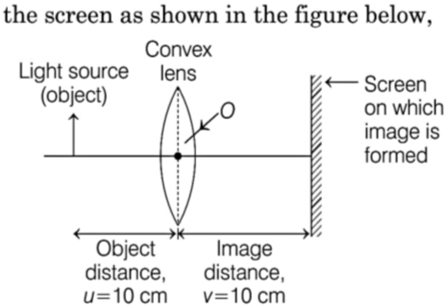
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow \quad \frac{1}{v} - \frac{1}{(-3)} = \frac{1}{0.5}$

$\Rightarrow \quad v = \frac{3}{5} = +0.6$ m

So, total distance of final image from mirror $= 2 + 0.6 = 2.6$ m. The image will be real (but in answer it was given virtual).

90. Initially, when a light source (i.e. an object) is placed at 10 cm from the convex mirror and an image is form on the screen as shown in the figure below,

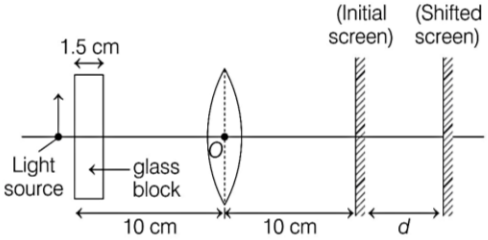


The distance from the optical centre (O) of the lens and $2f$ is $2\times$ focal length (f) = object distance

$$\Rightarrow \quad 2\times f = 10$$

or $$f = 5\text{ cm}$$

Now, when a glass block is placed in contact with the light source *i.e.*, object, then the situation is shown in the figure given below



The shift in the position of the object is given as

$$x = \left(1 - \frac{1}{\mu}\right)t$$

where, μ is the refractive index of the block and t is its thickness.

$$\Rightarrow \quad x = \left(1 - \frac{1}{1.5}\right)1.5 = \left(1 - \frac{2}{3}\right)\frac{3}{2}$$

$$= \frac{1}{3}\times\frac{3}{2} = \frac{1}{2} = 0.5\text{ cm}$$

$\therefore$ The new object distance of the light source in front of the lens will be

$$u' = 10 - 0.5 = 9.5\text{ cm}$$

Since, the focal length of the lens is 5 cm.

Therefore, the image distance of the light source now can be given as,

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} \qquad \text{(using lens formula)}$$

Substituting the values, we get

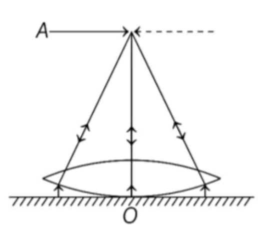
$$\frac{1}{v'} = \frac{1}{5} + \left(\frac{1}{-9.5}\right) = \frac{+9.5 - 5}{47.5} = \frac{4.5}{47.5}$$

or $$v' = 10.55\text{ cm}$$

$\therefore$ The value of $d = v' - v = 10.55 - 10$

$$= 0.55\text{ cm, away from the lens}$$

91. Light from plane mirror is reflected back on it's path, so that image of A coincides with A itself.



This would happen when rays refracted by the convex lens falls normally on the plane mirror, *i.e.* the refracted rays form a beam parallel to principal axis of the lens. Hence, the object would then be considered at the focus of convex lens.

$\therefore$ Focal length of curvature of convex lens,

$$f_1 = 18\text{ cm}$$

With liquid between lens and mirror, image is again coincides with object, so the second measurement is focal length of combination of liquid lens and convex lens.

$$\therefore \quad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}}$$

$$\Rightarrow \quad \frac{1}{18} + \frac{1}{f_2} = \frac{1}{27}$$

$$\Rightarrow \quad f_2 = -54\text{ cm}$$

For convex lens by lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\Rightarrow \quad \frac{1}{18} = 0.5\times\frac{2}{R}$$

$$\Rightarrow \quad R = 18\text{ cm}$$

and for plano-convex liquid lens, we have

$$\frac{1}{f} = (\mu_l - 1)\left(\frac{-1}{R}\right)$$

$$\Rightarrow \quad -\frac{1}{54} = (\mu_l - 1)\left(\frac{-1}{18}\right)$$

$$\Rightarrow \quad \mu_l = 1 + \frac{1}{3} = \frac{4}{3}$$

92. First we find relation of magnification m and focal length f. By lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad 1 - \frac{v}{u} = \frac{v}{f}$$

$$\Rightarrow \quad 1 - m = \frac{v}{f} \qquad \left(\because \frac{v}{u} = m\right)$$

$$\Rightarrow \quad m = 1 - \frac{v}{f} \qquad \text{...(i)}$$

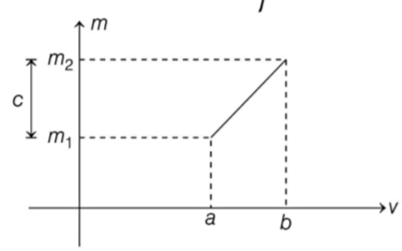
Now, from given graph and from Eq. (i),

At $v = a$, magnification is

$$m_1 = 1 - \frac{a}{f} \qquad \text{...(ii)}$$

At $v = a + b$, magnification is

$$m_2 = 1 - \frac{a+b}{f} \qquad \text{...(iii)}$$



From graph, we can also say that

$$m_2 - m_1 = c \quad \text{...(iv)}$$

So, from Eqs. (ii), (iii) and (iv), we have

$$\left(1 - \frac{(a+b)}{f}\right) - \left(1 - \frac{a}{f}\right) = c$$

$$\Rightarrow \quad \frac{a - (a+b)}{f} = c$$

$$\Rightarrow \quad f = -\frac{b}{c} \text{ or } |f| = \frac{b}{c}$$

93. For double convex lens,

$$R_1 = R_2 = R \quad \text{(given)}$$

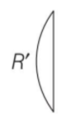
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{(-R)}\right)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

Power, $P = \frac{1}{f} = \frac{2(\mu - 1)}{R}$ ...(i)

For plano-convex lens,

$$R_1 = R', R_2 = \infty$$



$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R'} - \frac{1}{\infty}\right)$$

$$\frac{1}{f'} = \frac{\mu - 1}{R'}$$

Power, $P' = \frac{1}{f'} = \frac{(\mu - 1)}{R'}$

Given, $P' = 1.5P$

$$\Rightarrow \quad 1.5P = \frac{\mu - 1}{R'} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{P}{1.5P} = \frac{\frac{2(\mu-1)}{R}}{\frac{(\mu-1)}{R'}}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{2R'}{R}$$

or $\quad R' = \frac{R}{3}$

94. We have, $\quad \mu = \sqrt{3} = \dfrac{\sin\left(\frac{60^\circ - \delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$

where, μ = refractive index.

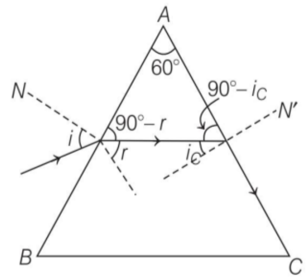
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\sin 60^\circ = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

or $\quad \frac{60^\circ + \delta_m}{2} = 60^\circ$

or $\quad \delta_m = 60^\circ \,;\, i = \frac{A + \delta_m}{2} = \frac{60^\circ + 60^\circ}{2} = 60^\circ$

95. Angle of prism, $\quad A = 60^\circ$



Refractive index of prism, $\mu = 1.524$

Let i be the angle of incidence. The critical angle is i_c because it just suffers total internal refraction, so we use critical angle.

$$\sin i_C = \frac{1}{\mu} = \frac{1}{1.524} = 0.6561$$

$$i_C = 41^\circ$$

For a prism $r_1 + r_2 = A$, here $r_2 = i_c$

$$\therefore \quad r_1 + i_C = A$$

$$r_1 + 41^\circ = 60^\circ$$

$$\Rightarrow \quad r_1 = 19^\circ$$

Using the formula, $\mu = \frac{\sin i_1}{\sin r_1}$

or $\quad \sin i_1 = 1.524 \sin 19^\circ = 1.524 \times 0.3256$

or $\quad i_1 = \sin^{-1}(0.4962)$

$$i_1 = 29^\circ\ 75'$$

Thus, the angle should be 29° 75′.

96. As, $\quad \mu = \dfrac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$

$$\Rightarrow \quad \sqrt{2} = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin\frac{60^\circ}{2}} \quad (\text{as, } \mu = \sqrt{2})$$

$$\frac{1}{\sqrt{2}} = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

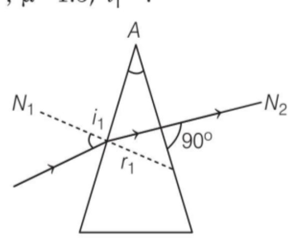
or $\quad \sin 45^\circ = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$

$$\Rightarrow \quad \delta_m = 30^\circ$$

$\therefore$ Angle of incidence,

$$i = \frac{A + \delta_m}{2} = \frac{60 + 30}{2} = 45^\circ$$

97. Here, $A = 5^\circ$, $\mu = 1.5$, $i_1 = ?$



As the ray emerges from the other face of prism normally,

$$i_2 = 0° \quad \therefore \quad r_2 = 0°$$

As, $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A - r_2 = 5 - 0 = 5°$

From $\mu = \frac{\sin i_1}{\sin r_1}$

$\Rightarrow \quad \sin i_1 = \mu \sin r_1$

$= 1.5 \times \sin 5°$

$= 1.5 \times 0.087$

$= 0.1305$

$\Rightarrow \quad i_1 = \sin^{-1}(0.1325) = 7.5°$

98. For a crown glass thin prism, *i.e.* prism with small angle, the angle of minimum deviation is given as

$$D_m = (n-1)A$$

where, A is prism angle and n is refractive index.

$\Rightarrow \quad D_m \propto n \quad \ldots$ (i)

Since, from the given graph, the value of n decrease with the increase in λ. Thus, from relation (i), we can say that, D_m will also decrease with the increase in λ.

Hence, option (c) is correct.

99. Following arguments lead us easily to the right choice.

(i) Angle between any two lines is the same as the angle between their perpendiculars.

$\therefore \quad i = 30°$

(ii) $\frac{1}{1.5} = \frac{\sin 30°}{\sin r}$ (by Snell's law)

or $\sin r = \frac{1.5}{2} = 0.75$

or $r = 48.6°$

(iii) $\theta = r - i = 18.6°$

$\therefore$ Required angle $= 2 \times 18.6 = 37.2°$

100. $n = \sqrt{3}$, prism angle, $A = 60°$

Method 1

Using prism formula, $n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\Rightarrow \quad \sqrt{3} = \frac{\sin\left(\frac{60° + \delta}{2}\right)}{\sin 30°}$

$\Rightarrow \quad \sin\left(\frac{60° + \delta}{2}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\left(\frac{60° + \delta}{2}\right) = \sin 60°$

or $\frac{60° + \delta}{2} = 60°$

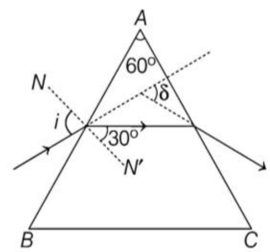
or angle of minimum deviation, $\delta = 60°$

Incident angle, $i = \frac{60 + \delta}{2} = 60°$

Method 2 For minimum deviation, ray should pass symmetrically (*i.e.* parallel to the base of the equilateral prism)

From geometry of given figure, we have

$$r = 30°$$



Using Snell's law, $n = \frac{\sin i}{\sin r}$

$\Rightarrow \quad \sin i = n \sin r = \sqrt{3} \sin 30°$

$\Rightarrow \quad \sin i = \frac{\sqrt{3}}{2}$ or $i = 60°$

101. For passing the ray from prism,

$$\mu \le \operatorname{cosec}\frac{A}{2}$$

$$\mu \le \operatorname{cosec}\left(\frac{90°}{2}\right)$$

$$\mu \le \sqrt{2}$$

$\therefore \quad \mu_{max} = \sqrt{2}$

102. As, $A = r_1 + r_2$

$\therefore \quad 30° = r_1 + 0°$

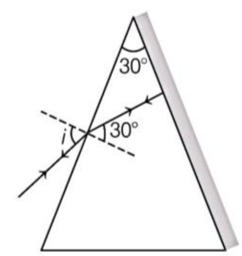
or $r_1 = 30°$

Now, $\frac{\sin i}{\sin 30°} = \sqrt{2}$

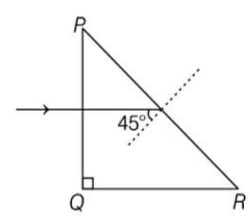
or $\sin i = \sqrt{2} \times \frac{1}{2}$

or $\sin i = \frac{1}{\sqrt{2}}$

or $i = 45°$



103. Assuming that the right angled prism is an isoceles prism, so the other angles will be 45° each. Each incident ray will make an angle of 45° with the normal at face PR.



The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

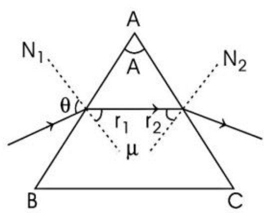
$\Rightarrow \quad \theta_C =$ critical angle

$\Rightarrow \quad \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

If $\theta_C \geq 45°$, then light will pass

$\Rightarrow \quad (\theta_C)\text{ Red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94°$

104. Consider the ray diagram is shown below



A ray of light incident on face AB at an angle θ.

Here, r_1 = angle of refraction on face AB

and r_2 = angle of incidence at face AC.

For transmission of light through face AC,

$$i_{AC} < i_C$$

or $\quad A - r_1 < i_C$

or $\quad \sin(A - r_1) < \sin i_C$

or $\quad \sin(A - r_1) < \frac{1}{\mu}$

or $\quad A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$

or $\quad \sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$

Now, applying Snell's law at the face AB,

$$1 \times \sin\theta = \mu \sin r_1$$

or $\quad \sin r_1 = \frac{\sin\theta}{\mu}$

$\Rightarrow \quad \frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$

or $\quad \theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

105. For dispersion without deviation,

$$\frac{A}{A'} = \frac{(\mu' - 1)}{(\mu - 1)}$$

$\Rightarrow \quad \frac{4}{A_1} = \frac{(1.72 - 1)}{(1.54 - 1)} = \frac{0.72}{0.54}$

$\Rightarrow \quad A_1 = \frac{4 \times 0.54}{0.72} = 3°$

106. The condition for achromatism is

$$\omega_1 P_1 + \omega_2 P_2 = 0$$

$$\omega_1 P_1 = -\omega_2 P_2$$

$\Rightarrow \quad \frac{\omega_1}{\omega_2} = -\frac{P_2}{P_1}$

Now, $\quad P_1 + P_2 = 2\text{D}$

or $\quad 5 + P_2 = 2$

or $\quad P_2 = -3\text{D}$

$\therefore \quad \frac{\omega_1}{\omega_2} = -\frac{-3}{5} = \frac{3}{5}$ (in magnitude)

107. We have, ratio of dispersive powers,

$$\frac{\omega_1}{\omega_2} = \frac{1}{2}$$

Now, $\quad \frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} = -\frac{1}{2}$

or $\quad f_2 = -2f_1$

Now, $\quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ (for the combination of lenses)

$\Rightarrow \quad \frac{1}{50} = \frac{1}{f_1} + \frac{1}{(-2f_1)}$

or $\quad 50f = \frac{-2 + 1}{-2f_1} = \frac{1}{2f_1}$

or $\quad 2f_1 = 50$

or $\quad f_1 = 25$ cm

Again, $\quad f_2 = -2 \times 25\text{ cm} = -50$ cm

108. Power of combination, $P = P_1 + P_2 = +20 - 10 = +10\text{D}$

$\therefore \quad f = \frac{1}{P} = \frac{1}{10}\text{m} = 10$ cm

For image at infinity, $M = \frac{D}{f} = \frac{25}{10} = 2.5$

109. For the objective,

$\therefore \quad \frac{1}{v_o} - \frac{1}{-1/3.8} = \frac{1}{1/4}$

or $\quad \frac{1}{v_o} + 3.8 = 4$

or $\quad \frac{1}{v_o} = 0.2 = \frac{1}{5}$

or $\quad v_o = 5$ cm

Now, $\quad M_o = \frac{5}{-\frac{1}{3.8}} = -19$

Again, $\quad M = M_o \times M_e$

$$-95 = -19 \times M_e$$

or $\quad M_e = \frac{95}{19} = 5$

110. As, $L = v_o + f_e$

$\Rightarrow \quad v_o = L - f_e$

or $\quad v_o = 19.2$ cm

$\therefore$ From lens formula, $\frac{1}{19.2} - \frac{1}{u_o} = \frac{1}{1.6}$

or $\quad -\frac{1}{u_o} = \frac{10}{16} - \frac{10}{192}$

or $$-\frac{1}{u_o}=\frac{120-10}{192}=\frac{110}{192}$$

or $$u_o=-\frac{192}{110}\text{ cm}=-1.75\text{ cm}$$

111. When final image is formed at infinite, then

$$M=\frac{v_o}{u_o}\left(\frac{d}{f_e}\right)=\frac{v_o}{f_o}\left(\frac{d}{f_e}\right)$$

Now, $v_o=16-f_e=16-2.5=13.5$ cm

$\therefore$ $$M=\frac{13.5}{-0.4}\times\frac{25}{2.5}=-337.5$$

112. Separation $=f_o+\dfrac{f_e D}{f_e+D}=80+\dfrac{5\times25}{5+25}=80+\dfrac{125}{30}$

$=84.16$ cm $=84.2$ cm

113. **Case I** As, $M=-\dfrac{f_o}{f_e}\left(1+\dfrac{f_e}{D}\right)$

$$M=-\frac{200}{5}\left(1+\frac{5}{25}\right)$$

$$M=-40\left(1+\frac{1}{5}\right)=-40\times\frac{6}{5}=-48$$

Case II $M=\dfrac{f_o}{f_e}=-\dfrac{200}{5}=-40$

114. As, $M=\dfrac{f_o}{f_e}$

or $9=\dfrac{f_o}{f_e}$ or $f_o=9f_e$

Also, $L=f_o+f_e$

or $20=f_o+f_e$

or $20=9f_e+f_e$

or $20=10f_e$

or $f_e=2$ cm

$\therefore$ $f_o=9\times2$ cm $=18$ cm

115. For a telescope in normal setting,

$f_o+f_e=L$ (length of the tube of telescope)

and $\dfrac{f_o}{f_e}=m$ (magnification)

where, f_o and f_e is the focal length of the objective and eyepiece, respectively.

According to the given values in the question, we have

$$f_o+f_e=60\text{ cm and }\frac{f_o}{f_e}=5\Rightarrow f_o=5f_e$$

Solving the above equations, we have

$f_e=10$ cm

116. In normal setting, magnification obtained by a microscope is given by

$$m=\frac{L\times D}{f_o\times f_e}$$

where, $L=$ tube length $=150$ mm,

$D=$ distance of distinct vision $=25$ cm $=250$ mm,

$f_o=$ focal length of objective $=5$ mm,

$f_e=$ focal length of eyepiece

and $m=$ magnification $=375$.

So, we have

$$375=\frac{150\times250}{5\times f_e}\text{ or }f_e=20\text{ mm}$$

So, focal length of eyepiece required is closest to 22 mm.

117. Here, $f_o=50$ cm, $f_e=5$ cm, $D=25$ cm and $u_o=200$ cm

$\therefore$ Separation between the objective and eyepiece lens is,

$$L=\frac{u_o f_o}{u_o-f_o}+\frac{f_e D}{(f_e+D)}$$

$$=\frac{200\times50}{(200-50)}+\frac{5\times25}{(5+25)}=71\text{ cm}$$

118. Given, distance between objective mirror and another mirror,

$$d=20\text{ mm}$$

Radius of curvature of objective mirror, $R_1=220$ mm

$\therefore$ Focal length of objective mirror, $f_1=\dfrac{220}{2}=110$ mm

Radius of curvature of small mirror, $R_2=140$ mm

$\therefore$ Focal length of small mirror, $f_2=\dfrac{140}{2}=70$ mm

The image of an object placed at infinity, formed by the objective mirror, will act as a virtual object for small mirror.

So, the object distance for small mirror, $u=f_1-d$

i.e. $u=110-20=90$ mm

Using mirror formula, $\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f_2}$

$\Rightarrow$ $$\frac{1}{v}=\frac{1}{f_2}-\frac{1}{u}=\frac{1}{70}-\frac{1}{90}=\frac{2}{630}$$

$\Rightarrow$ $v=315$ mm or $v=31.5$ cm

Thus, the final image is formed at 315 mm away from small mirror.

119. As, $\dfrac{1}{f}=\dfrac{1}{v}-\dfrac{1}{u}$ (lens formula)

$\Rightarrow$ $$2.5=\frac{1}{-0.75}-\frac{1}{u}$$

or $$\frac{1}{u}=-\frac{100}{75}-\frac{25}{10}\text{ or }\frac{1}{u}=-\frac{4}{3}-\frac{5}{2}$$

or $$\frac{1}{u}=\frac{-8-15}{6}=-\frac{23}{6}$$

or $$u=-\frac{6}{23}\text{ m}=-0.26\text{ m}$$

Round II

1. By painting black the upper half of the lens, intensity of image will reduce but its position will not be shifted.

2. A magnifying glass is used as the object to be viewed can be brought closer to the eye than the normal near point. This results in a larger angle to be subtended by the object at the eye and hence viewed in greater detail. Also, it results in the formation of a virtual, erect image.

3. Here, an extended object lies immersed in water contained in a plane trough. When seen from close to the edge of the trough, the object looks distorted on account of refraction of light from denser to rarer medium. Therefore, apparent depths of the points close to the edge and nearer to the surface of water is more compared to points away from the edge.Further, the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air. Again, some of the points of the object, far away from the edge may not be visible because of total internal reflection.

4. Distance between objective and eyepiece = Tube length

$$= f_o + f_e$$
$$= 16 + 0.02 = 16.02 \text{ m}$$

Angular magnification $= \dfrac{f_o}{f_e} = -\dfrac{16}{0.02} = -800$

The final image of the planet is inverted as magnification is negative.

5. As, refractive index for $z > 0$ and $z \le 0$ is different XY-plane should be boundary between two media.

Angle of incidence, $\cos i = \left| \dfrac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \right|$

$$= \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 100^2}} = \frac{1}{2}$$

$\therefore \quad i = 60°$

From Snell's law, $\dfrac{\sin i}{\sin r} = \dfrac{\sqrt{3}}{\sqrt{2}}$

$\Rightarrow \quad \sin r = \dfrac{\sqrt{2}}{\sqrt{3}} \sin 60° = \dfrac{1}{\sqrt{2}}$

$$r = 45°$$

6. As, $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

$\Rightarrow \quad -\dfrac{1}{u^2}\dfrac{du}{dt} - \dfrac{1}{v^2}\dfrac{dv}{dt} = 0$

$\Rightarrow \quad \dfrac{dv}{dt} = \dfrac{-v^2}{u^2}\left(\dfrac{du}{dt}\right)$

But $\quad \dfrac{v}{u} = \dfrac{f}{u-f}$

$\therefore \quad \dfrac{dv}{dt} = -\left(\dfrac{f}{u-f}\right)^2 \left(\dfrac{du}{dt}\right)$

$$= \left(\frac{0.2}{-2.8 - 0.2}\right)^2 \times 15$$
$$= \frac{1}{15} \text{ ms}^{-1}$$

7. As, there is no deflection between mediums 1 and 2. Therefore, $\mu_1 = \mu_2$.

8. According to Cartesian sign convention,

$$u = -40 \text{ cm}, R = -20 \text{ cm}$$

Given, $\quad \mu_1 = 1, \text{ and } \mu_2 = 1.33$

Applying equation for refraction through spherical surface, we get

$$\frac{\mu_2}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$
$$\frac{1.33}{v} - \frac{1}{-40} = \frac{1.33}{-20}$$

After solving $\quad v = -32$ cm

The magnification is, $m = \dfrac{h_2}{h_1} = \dfrac{\mu_1 v}{\mu_2 u}$

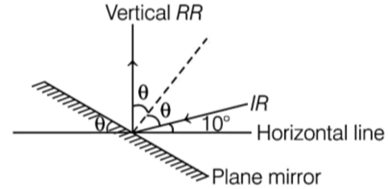
$\therefore \quad \dfrac{h_2}{1} = -\dfrac{1\,(32)}{1.33(-40)}$

or $\quad h_2 = 0.6$ cm

The positive sign shows that the image is erect.

9. From figure, we see that

$$\theta + \theta + 10° = 90°$$



$\Rightarrow \quad 2\theta + 10 = 90$

$\Rightarrow \quad 2\theta = 80$

$\Rightarrow \quad \theta = 40°$

10. Using equation, the total apparent shift,

$$S = h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

or $\quad S = 4\left(1 - \dfrac{1}{4/3}\right) + 6\left(1 - \dfrac{1}{3/2}\right)$

$$= 3.0 \text{ cm}$$

$\therefore$ Apparent position of the coin from the surface,

Thus, $\quad h = h_1 + h_2 - S = 4 + 6 - 3$

$$= 7.0 \text{ cm}$$

11. Image formation by a mirror does not depend on the medium. As, P is at a height h above the mirror, image of P will be at a depth h below the mirror.

If d is depth of liquid in the tank, apparent depth of P,

$$x_1 = \frac{d-h}{\mu}$$

$\therefore$ Apparent distance between P and its image

$$= x_2 - x_1 = \frac{d+h}{\mu} - \frac{d-h}{\mu} = \frac{2h}{\mu}$$

According to Snell's law,

$$\sqrt{3}\sin 45^\circ = \sin r > 1$$

Hence, this is a case of total internal reflection. Thus, the angle of incidence on Q is equal to angle of reflection. This forms 30° as angle of incidence on side AD. By Snell's law

$$\sqrt{3}\sin 30^\circ = \sin r = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \qquad r = 60^\circ$$

Thus, the angle between rays OP and RS is 90°.

17. Magnification for a lens can also be written as

$$m = \left(\frac{f}{f+u}\right)$$

When $m = -2$ (for real image)

$$-2 = \frac{f}{f + x_1}$$

$$\Rightarrow \quad -2f + (-2)\,x_1 = f \quad \text{or} \quad x_1 = -\frac{3f}{2}$$

Similarly, when $m = +2$ (for virtual image),

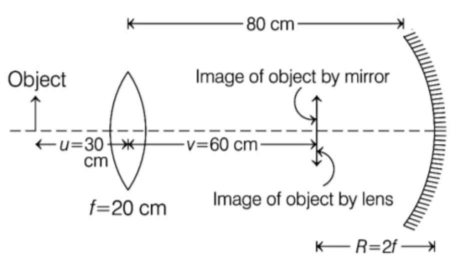
$$+2 = \frac{f}{f + x_2}$$

$$\Rightarrow \qquad 2f + 2x_2 = f \quad \text{or} \quad x_2 = \frac{-f}{2}$$

Now, the ratio of x_1 and $x_2 = \dfrac{\frac{-3f}{2}}{-f/2} = \dfrac{3}{1}$

18. When a slab of thickness, t is introduced between P and the mirror, the apparent position of P shifts towards the mirror by $\left(t - \frac{t}{\mu}\right)$. Hence, the mirror must be moved in the same direction through the same distance.

19. The given situation can be drawn as shown below



For lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Substituting given values, we get

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \quad \Rightarrow \quad v = 60 \text{ cm}$$

So, this image is at a distance of $80 - 60 = 20$ cm from the mirror.

As, the image formed by the mirror coincides with image formed by the lens.

This condition is only possible, if any object that has been placed in front of concave mirror is at centre of curvature, *i.e.* at $2f$.

So, radius of curvature of mirror is $R = 20$ cm.

$\therefore$ Focal length of mirror, $f = \dfrac{R}{2} = 10$ cm

As, for virtual image, the object is to kept between pole and focus of the mirror.

$\therefore$ The maximum distance of the object for which this concave mirror by itself produce a virtual image would be 10 cm.

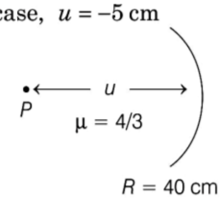
20. This is a modified displacement method problem.

Here, $a = 1.8$ m and $\dfrac{a+d}{a-d} = \dfrac{2}{1}$

Solving, this, we get $\quad d = 0.6$ m

$$\therefore \qquad f = \frac{a^2 - d^2}{4a} = 0.4 \text{ m}$$

21. In the given case, $u = -5$ cm



Focal length,

$$f = \frac{-R}{2} = \frac{-40}{2} = -20 \text{ cm}$$

Now, using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{5} = +\frac{3}{20}$$

$$\Rightarrow \qquad v = +\frac{20}{3} \text{ cm}$$

For the light getting refracted at water surface, this image will act as an object.

So, distance of object,

$$d = 5 \text{ cm} + \frac{20}{3} \text{ cm} = \frac{35}{3} \text{ cm} \quad \text{(below the surface)}$$

Let's assume final image at distance d after refraction.

$$\frac{d'}{d} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \qquad d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3} \text{ cm}\right)\left(\frac{1}{\frac{4}{3}}\right)$$

$$= \frac{35}{3} \times \frac{3}{4} \text{ cm} = \frac{35}{4} \text{ cm}$$

$$= 8.75 \text{ cm} \approx 8.8 \text{ cm}$$

22. Given, the power of cornea = 40 D and least converging power of eye lens = 20 D.

To observe the objects at infinity, the eye uses its least converging power means power is maximum, *i.e.* $40 + 20 = 60$ D.

The distance between cornea = focal length of eye lens

$$f = \frac{100}{P} = \frac{100}{60} = \frac{5}{3} \text{ cm}$$

To focus objects at the near point on the retina

$$u = -25 \text{ cm},\ v = \frac{5}{3} \text{ cm}$$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1 \times 3}{5} + \frac{1}{25} = \frac{15+1}{25} = \frac{16}{25}$$

$$\Rightarrow \quad f = \frac{25}{16} \text{ cm}$$

$$\text{Power of lens} = \frac{1}{f} = \frac{100 \times 16}{25} = 64 \text{ D}$$

$\therefore$ Power of eye lens $= 64 - 40 = 24$ D

Thus, the range of accommodation of the eye lens is 20 D to 24 D.

23. From figure, $OA = 4$ cm, $AB = 3$ cm

$$\therefore \quad OB = \sqrt{4^2 + 3^2} = 5 \text{ cm}$$

$$\text{Now,} \quad \mu = \frac{1}{\sin C} = \frac{1}{AB/OB} = \frac{OB}{AB} = \frac{5}{3}$$

$$\text{From,} \quad \mu = \frac{c}{c/v} = \frac{c}{\mu} = \frac{3 \times 10^8}{5/3} = 1.8 \times 10^8 \text{ m/s}$$

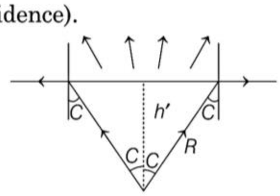
24. As, $m = \frac{f_o}{f_e}$

We know that, $m = \frac{\text{Angle subtended by an image}}{\text{Angle subtended by an object}}$

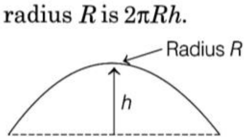
$$\therefore \quad \frac{f_o}{f_e} = \frac{\alpha}{\beta}$$

$$\Rightarrow \quad \alpha = \frac{\beta \times f_o}{f_e} = \frac{6002 \times 60}{5} = 2 \times 12 = 24°$$

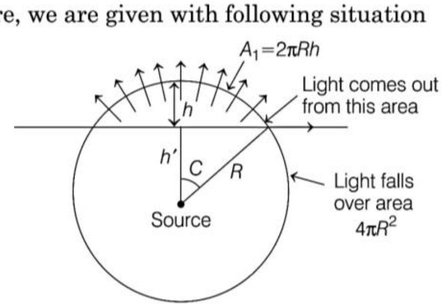
25. Due to total internal reflection, light comes out from a cone of angle $2C$ as shown in the figure (C = angle of critical incidence).



Now, it is given that surface area of spherical cap of height h and radius R is $2\pi Rh$.



Here, we are given with following situation



So, percentage of light that emerges out of surface is given by

% of light coming out

$$= \frac{\text{Area through which light comes out}}{\text{Total area over which light falls}} \times 100$$

$$= \frac{2\pi Rh}{4\pi R^2} \times 100$$

$$= \frac{1}{2} \times \left(\frac{R - h'}{R}\right) \times 100$$

$$\text{Now,} \quad \frac{h'}{R} = \frac{\text{base}}{\text{hypotenuse}} = \cos C$$

$$\text{As,} \quad \sin C = \frac{1}{n} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\Rightarrow \quad \cos C = \frac{\sqrt{7}}{4} = \frac{h'}{R}$$

$\therefore$ % of light coming out

$$= \frac{1}{2} \times \left(\frac{R - h'}{R}\right) \times 100$$

$$= \frac{1}{2} \times \left(1 - \frac{h'}{R}\right) \times 100$$

$$= \frac{1}{2} \times \frac{\sqrt{7}}{4} \times 100$$

$$= 16.9\ \%$$

$$\simeq 17\%$$

Nearest answer is 17 %.

26. $\omega_1 = 0.02;\ \mu_1 = 1.5;\ \omega_2 = 0.03;\ \mu_2 = 1.6$

Achromatic combination,

$$\therefore \quad \theta_{net} = 0$$

$$\theta_1 - \theta_2 = 0$$

$$\theta_1 = \theta_2$$

$$\omega_1 \delta_1 = \omega_2 \delta_2$$

$$\text{and} \quad \delta_{net} = \delta_1 - \delta_2 = 2°$$

$$\delta_1 - \frac{\omega_1 \delta_1}{\omega_2} = 2°$$

$$\delta_1\left(1 - \frac{\omega_1}{\omega_2}\right) = 2°$$

$$\delta_1\left(1 - \frac{2}{3}\right) = 2°$$

$$\delta_1 = 6°$$

$$\delta_1 = (\mu_1 - 1)A_1$$

$$6° = (1.5 - 1)A_1$$

$$A_1 = 12°$$

27. Deviation for small-angled prism is given by

$$\delta = (\mu - 1)\ A$$

Given, $A = 1°,\ \mu = 1.5$

Substituting these values in above equation, we get

$$\delta = (1.5 - 1)1 \Rightarrow \delta = 0.5$$

According to question, $\delta = \frac{N}{10}$

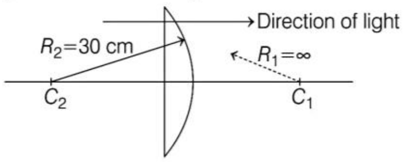
$$\Rightarrow \quad 0.5 = \frac{N}{10} \Rightarrow N = 5$$

Hence, the value of N is 5.

28. Focal length of lens by lens Maker's formula,

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a plano-convex lens,



$$R_1 = \infty, R_2 = -30 \text{ cm}, n_{21} = 1.5$$

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{(-30)}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{2} \times \frac{1}{30} \Rightarrow f = 60 \text{ cm}$$

29. Given, object distance, $u = -30$ cm,

Image distance, $v = -10$ cm

and speed of object, $v_o = 9 \text{ cms}^{-1}$

Let v_i be the speed of image.

Magnification of concave mirror,

$$m = -\frac{v}{u} = -\frac{(-10)}{(-30)} = -\frac{1}{3}$$

Now, $v_i = v_o(m^2) = 9 \times \left(\frac{-1}{3}\right)^2 = 1 \text{ cms}^{-1}$

30. For the eyepiece,

$$v_e = -25 \text{ cm}, f_e = 5 \text{ cm}$$

or $\frac{1}{u_e} = -\frac{1}{25} - \frac{1}{5}$

or $\frac{1}{u_e} = -\frac{-1-5}{25}$

or $u_e = -\frac{25}{6}$

Now, $v_o = L - |u_e| = 20 - \frac{25}{6}$

$$= \frac{120 - 25}{6} \text{ cm} = \frac{95}{6} \text{ cm}$$

Now, $\frac{1}{95/6} - \frac{1}{u_o} = \frac{1}{1}$

or $\frac{1}{u_o} = \frac{6}{95} - 1$

or $u_o = -\frac{95}{89}$ cm

$$\therefore \quad |u_o| = \frac{95}{89} \text{ cm} = 1.067 \text{ cm}$$

31. When the final image is formed at least distance of distinct vision (*i.e.* at 25 cm) from eyepiece of a compound microscope, then the magnification is given by

$$m = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

$$\Rightarrow \quad m = \frac{-L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

$$\Rightarrow \quad -100 = -\frac{20}{1}\left(1 + \frac{25}{f_e}\right)$$

$$\Rightarrow \quad 5 = 1 + \frac{25}{f_e}$$

$$4 = \frac{25}{f_e} \Rightarrow f_e = \frac{25}{4} = 6.25 \simeq 6 \text{ cm}$$

32. Strain on the eye is minimum, so the intermediate image is formed at focal length of eyepiece.

For objective, $v = 5$ cm, $f = 1$ cm

$\therefore$ Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{5} - \frac{1}{u} = \frac{1}{1} \Rightarrow u = \frac{-5}{4} \text{ cm}$$

$$\Rightarrow \quad |u| = \frac{5}{4} \text{ cm} \Rightarrow \frac{n}{40} = \frac{5}{4}$$

$$\Rightarrow \quad n = \frac{5}{4} \times 40 \Rightarrow n = 50$$

33. Using displacement method, the focal length of a lens is given by

$$f = \frac{D^2 - d^2}{4D} \quad \ldots\text{(i)}$$

Given that, distance between object and screen, $D = 100$ cm

Displacement travelled by object, $d = 40$ cm

Substituting these values in Eq. (i), we get

$$f = \frac{(100)^2 - (40)^2}{4(100)} = \frac{10000 - 1600}{400}$$

$$= \frac{8400}{400} = 21 \text{ cm} = 0.21 \text{ m}$$

Now, optical power of lens is

$$P = \frac{1}{f} = \frac{1}{0.21 \text{ m}} = \frac{100}{21 \text{ m}}$$

$$= \frac{100}{21} \text{m}^{-1} = \frac{100}{21} \text{ D} = 4.76 \text{ D} = \left(\frac{476}{100}\right) \text{D}$$

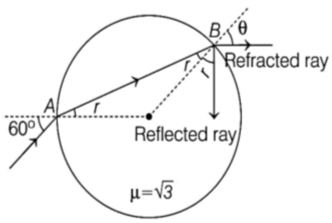
Given that,

$$P = \left(\frac{N}{100}\right) \text{D}$$

On comparing both the equations, we get

$$N = 476$$

34. We are given following situation as shown in figure,



Using Snell's law at point A,

$$n_1 \sin i = n_2 \sin r$$

$$1 \times \sin 60° = \sqrt{3} \times \sin r$$

$$\Rightarrow \quad \sin r = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2} \quad \Rightarrow \quad r = 30°$$

Now, at point B, angle of incidence, $r = 30°$

Again, using Snell's law at point B,

$$\sqrt{3} \sin 30° = 1 \times \sin \theta$$

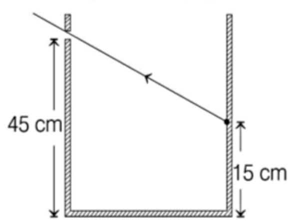
$$\Rightarrow \quad \sin \theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 60°$$

and angle of reflection, $r = 30°$

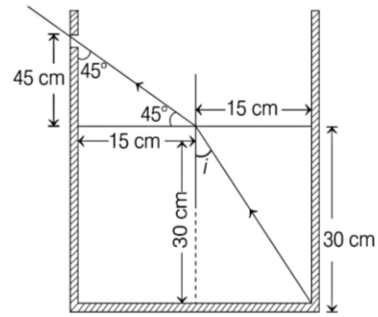
$\therefore$ Angle between reflected ray and refracted ray at point $B = 180° - \theta - r$

$$= 180° - 60° - 30° = 90°$$

35. Initially, when the jar is empty.



Finally, when the jar is filled with liquid.



If i be the angle of incidence, then

$$\tan i = \frac{15}{30} = \frac{1}{2}$$

$$\Rightarrow \quad \sin i = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{5}}$$

According to law of refraction,

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow \quad \mu\left(\frac{1}{\sqrt{5}}\right) = 1 \times \sin 45°$$

$$\Rightarrow \quad \mu = \frac{\sqrt{5}}{\sqrt{2}} = 1.581$$

Also, given that,

$$\frac{N}{100} = \mu$$

$$\Rightarrow \quad N = 100\mu$$

$$= 100 \times 1.581 = 158.1$$

So, nearest integer value of N is 158.