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INTERFRENCE OF LIGHT

NATURE OF LIGHT: INTRODUCTORY CONCEPTS

Various theories about the nature of light have been proposed from time to time.

1. ***Corpuscular theory of light*:** Newton, the great among the greatest, proposed in 1675 A.D. that light consists of tiny particles called corpuscles which are shot out at high speed by a luminous object. This theory could explain the reflection, refraction and rectilinear propagation of light.

2. ***Wave theory of light*:** In 1678, Dutch scientist Christian Huygens suggested that light travels in the form of longitudinal waves just as sound propagates through air. He proposed that light waves propa-gate through an all-pervading hypothetical medium, called ***luminiferous ether***. Later on, the existence of such a medium was discarded due to its contradictory properties. Fresnel and Young showed that light propagates as a transverse wave. This successfully explained the reflection, refraction as well as interference, diffraction and polarization of light waves.

3. ***Electromagnetic nature of light waves*:** In 1873, ***Maxwell*** suggested that light propagates as electric and magnetic field oscillations. These are called electromagnetic waves which require no medium for their propagation. Also, these waves are transverse in nature.

4. ***Planck's quantum theory of light*:** According to Max Planck, light travels in the form of small packets of energy called photons. In 1905, Albert Einstein used this theory to explain photoelectric effect (emission of electrons from a metal surface when light falls on it).

So we see that in phenomena like interference, diffraction and polarisation, light behaves as a wave while in photoelectric effect, it behaves as a particle. De Broglie suggested that ***light has a dual nature***, ***i.e.***, it can behave as particles as well as waves.

WAVEFRONTS AND RAYS

Wavefronts: Suppose a stone is thrown on the surface of still water. Circular patterns of alternate crests and troughs begin to spread out from the point of impact. Clearly, all the particles lying on a crest are in the position of their maximum upward displacement and hence in the same phase. Similarly, all particle lying on a trough are in the position of their maximum downward displacement and, therefore, in the same phase. The locus of all such points oscillating in the same phase is called a wavefront. Thus every crest or a trough is wavefront.

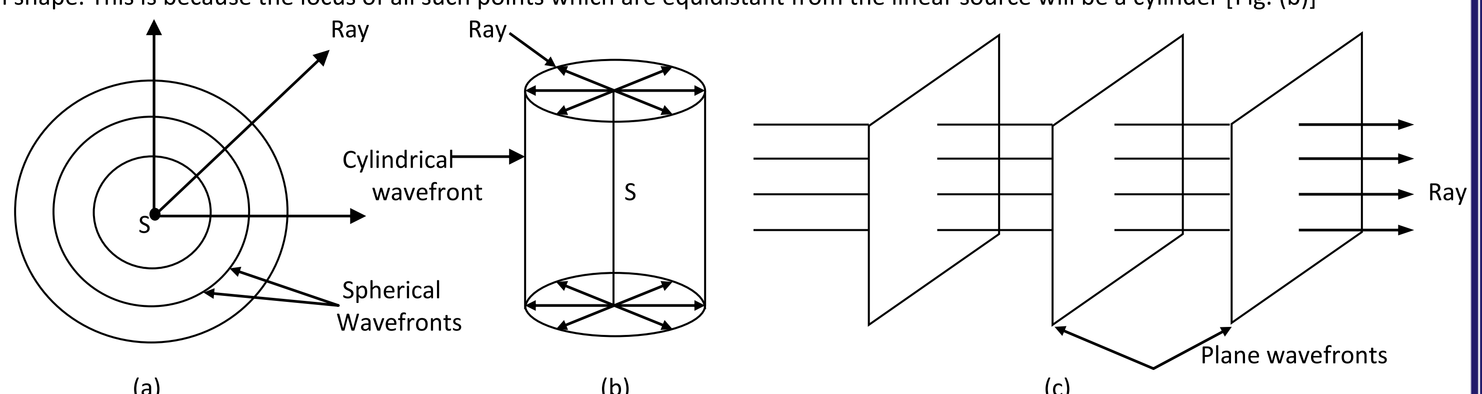
A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase at any instant.

Thus, a wavefront is a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the ***phase speed***.

Different type of wavefronts: The geometrical shape of a wavefront depends on the source of disturbance. Some of the common shapes are:

1. Spherical wavefront: In the case of waves travelling in all directions from a point source, the wavefronts are spherical in shape. This is because all such points which are equidistant from the point source will lie on a sphere [Fig. (a)] and the disturbance starting from the source S will reach all these points simultaneously.

2. Cylindrical wavefront: When the source of light is linear in shape, such as a fine rectangular slit, the wavefront is cylindrical in shape. This is because the locus of all such points which are equidistant from the linear source will be a cylinder [Fig. (b)]



[Different types of wavefronts]

3. Plane wavefront: As a spherical or cylindrical wavefront advances, its curvature decreases progressively. So a small portion of such a wavefront at a large distance from the source will be a plane wavefront [Fig. (c)].

Ray of light: It is seen that whatever is the shape of a wavefront, the disturbance travels outwards along straight lines emerging form the source i.e., **the energy of a wave travels in a direction perpendicular to the wavefront**.

An arrow drawn perpendicular to a wavefront in the direction of propagation of a wave is called a ray.

A ray of light represents the path along which light travels.

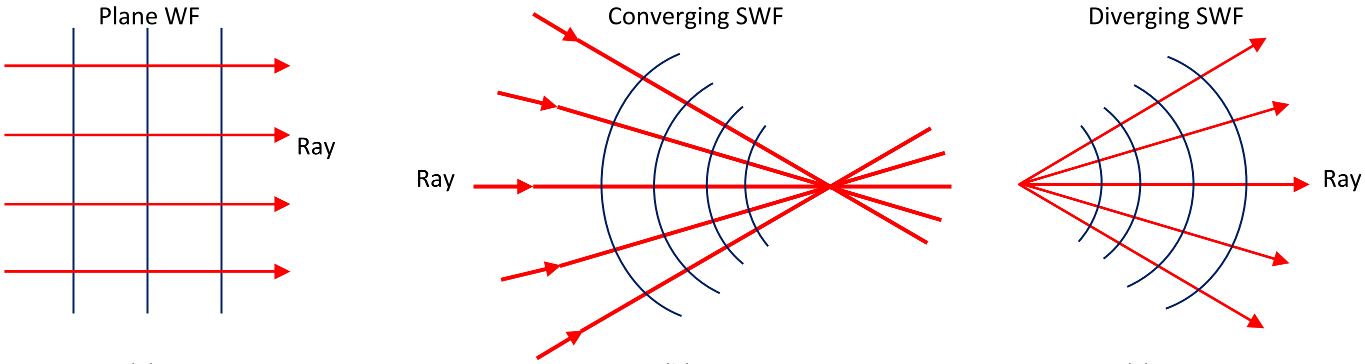
▶ If we measure the separation between a pair of wavefronts along any ray, it is found to be constant.

This illustrates two general principles:

▶▶ 1. Rays are perpendicular to wavefronts.

▶▶ 2. The time taken for light to travel form one wavefront to another is the same along any ray.

In case of a plane wavefront, the rays are parallel [Fig. (a)]. A group of parallel rays is called a beam of light. In case of a spherical wave front, the rays either converge to a point [Fig. (b)] or diverge from a point [Fig. (c)].



[Wavefronts and corresponding rays in three cases (a) plane, (b) converging spherical and (c) diverging spherical]

HUYGEN'S PRINCIPLE OF SECONDARY WAVELETS

Huygens' principle is the basis of wave theory of light.

It tells how a wavefront propagates through a medium. According to Huygens' principle, each point on a wavelength is a source of secondary waves, which add up to give a wavefront at any later time.

This principle is based on the following assumptions:

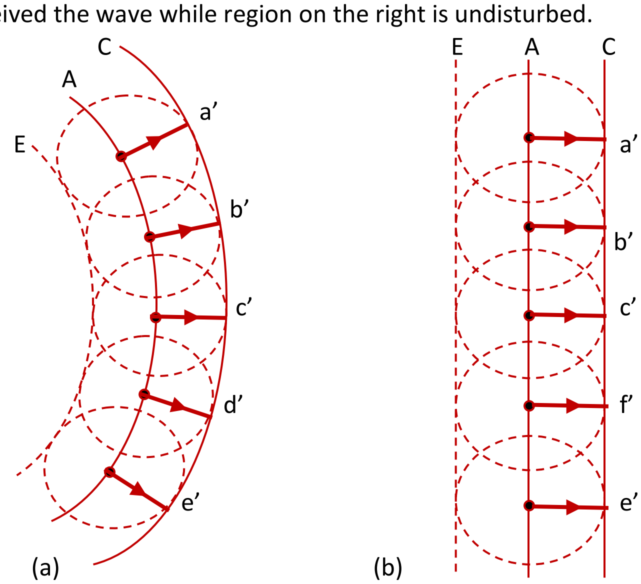
▶ 1. Each point on a wavefront acts as a fresh source of new disturbance, called secondary waves or wavelets.

▶ 2. The secondary wavelets spread out in all directions with the speed of light in the given medium.

▶ 3. The new wavefront at any later time is given by the forward envelope (tangential surface in the forward direction) of the secondary wavelets at that time.

Huygens' construction: It is a geometrical method of locating the new position and shape of a wavefront at any instant from its known position and shape at any other instant. The various steps involved are as follows:

1. Consider a spherical [Fig. (a)] or plane [Fig. (b)] wavefront moving towards right. Let AB be its position at any instant of time. The region on its left has received the wave while region on the right is undisturbed.



[Huygens' geometrical construction for the propagation of (a) spherical, (b) plane wavefront]

2. According to Huygens' principle, each point on AB becomes a source of secondary disturbance, which travels with the same speed c. To find the new wavefront after time t, we draw spheres of radii ct, from each point on AB.

3. The forward envelope or the tangential surface CD of the secondary wavelets gives the new wavefront after time t.

4. The lines aa', bb', cc', etc., are perpendicular to both AB and CD. Along these lines, the energy flows from AB to CD. So, these lines represent the rays. Rays are always normal to wavefronts.

No backward wavefront is possible: There cannot be backward flow of energy during the propagation of a wave. It can be shown mathematically that the amplitude of secondary wavelets is proportional to (1 + cos θ), where θ is the angle between the ray at the point of consideration and the direction of secondary wavelets. For a backward wavefront θ = π, so that 1 + cos θ = 0. Thus the resultant amplitude of all the secondary wavelets at any point on the backward wavefront is zero. In fact, the effects of secondary wavelets cancel out at all points except those lying on the forward envelope. So, a backward wavefront cannot exist.

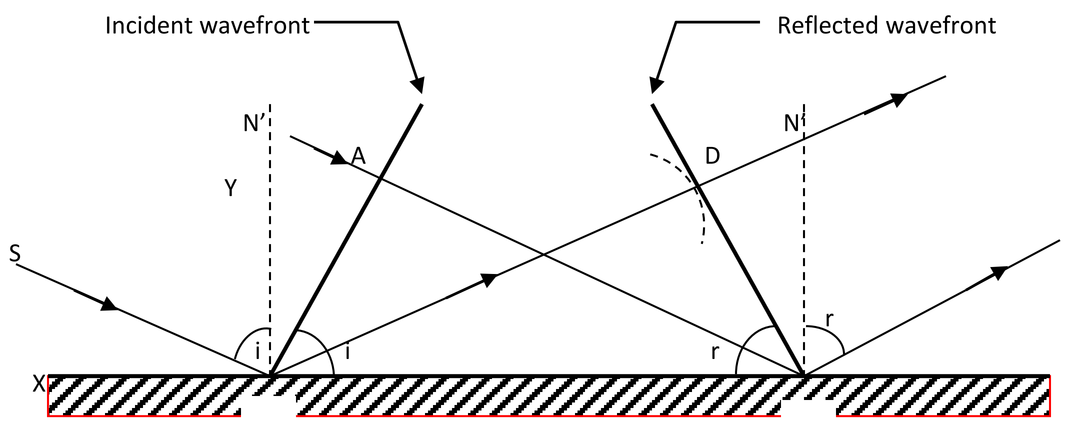
Huygens' principle of secondary wavelets can be used to prove that laws of reflection and refraction.

FOR YOUR KNOWLEDGE...

- ☑ A wavefront is a surface of constant phase.
- ☑ A ray of light is the path along which light travels. It is always normal to the wavefront.
- ☑ In a homogenous and isotropic medium (i.e., a medium having uniform composition and the same properties in all directions), the speed of light is same in all directions), the speed of light is same in all directions and the secondary wavelets are spherical. The rays are then perpendicular to both the wavefronts and the time of travel measured along any ray from one wavefront to the next is always same.
- ☑ Only the front portions of the secondary wavelets add up to give rise to a wavefront in the forward direction. The back portions of the secondary wavelets add up to zero. so no backward wavefront is possible.

REFLECTION ON THE BASIS OF WAVE THEORY

Laws of reflection on the basis of Huygens' wave theory: As shown in Fig. consider a plane wave front AB incident on the plane reflecting surface XY, both the wavefront and the reflecting surface being perpendicular to the plane of paper.



[Wavefronts and corresponding rays for reflection from a plane surface]

First the wavefront touches the reflecting surface at B and then at the successive points towards C. In accordance with Huygens' principle, from each point on BC, secondary wavelets start growing with the speed c. During the time the disturbance from A reaches the point C, the secondary wavelets from B must have spread over a hemisphere of radius BD = AC = ct, where t is the time taken by the disturbance to travel from A to C. The tangent plane CD drawn from the point Cover this hemisphere of radius ct will be the new reflected wavefront.

Let angles of incidence and reflection be i and r respectively. In Δ ABC and Δ DCB, we have

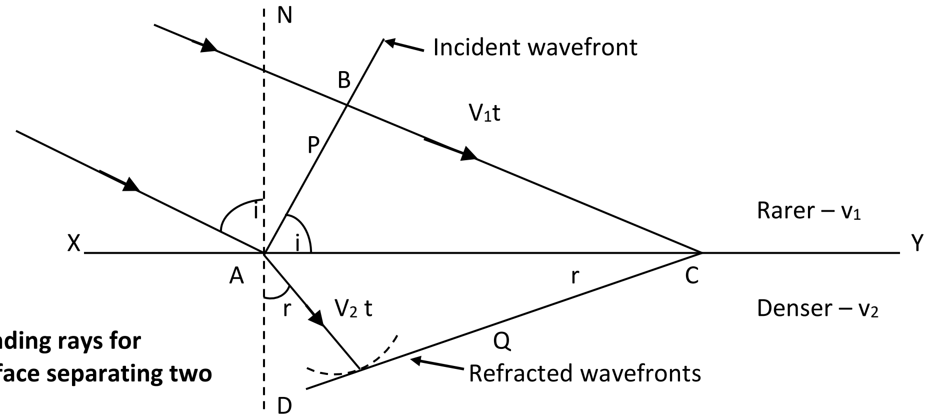
	∠ BAC = ∠CDB	[Each of 90°]
	BC = BC	[Common]
	AC = BD	[Each is equal to vt]
∴	ΔABC ≅ Δ DCB	
Hence	∠ABC = ∠DCB.	
or	i = r	

i.e., the angle of incidence is equal to the angle of reflection. This proves the first law of reflection.

Further, since the incident ray SB, the normal BN and the reflected ray BD are respectively perpendicular to the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD (all of which are perpendicular to the plane of the paper), therefore, they all lie in the plane of the paper, i.e., in the same plane. This proves the second law of reflection.

REFRACTION ON THE BASIS OF WAVE THEORY

Laws of refraction on the basis of Huygens' wave theory: Consider a plane wavefront AB incident on a plane surface XY, separating two media 1 and 2, as shown in Fig. Let v_1 and v_2 be the velocities of light in the two media, with $v_2 < v_1$.



[Wavefronts and corresponding rays for refraction by a plane surface separating two media]

The wavefront first strikes at point A and then at the successive points towards C. According to Huygens' principle, from each point on AC, the secondary wavelets start growing in the second medium with speed v_2. Let the disturbance take time t to travel from B to C, the BC = v_1 t. During the time the disturbance from B reaches the point C, the secondary wavelets from point A must have spread over a hemisphere of radius AD = v_2 t in the second medium. The tangent plane CD drawn from point Cover his hemisphere of radius v_2t will be the new refracted wavefront.

Let the angles of incidence and refraction be i and r respectively.

From right ΔABC, we have

$$\sin \angle BAC = \sin i = \frac{BC}{AC}$$

From right Δ ADC, we have $\sin \angle DCA = \sin r = \frac{AD}{AC}$

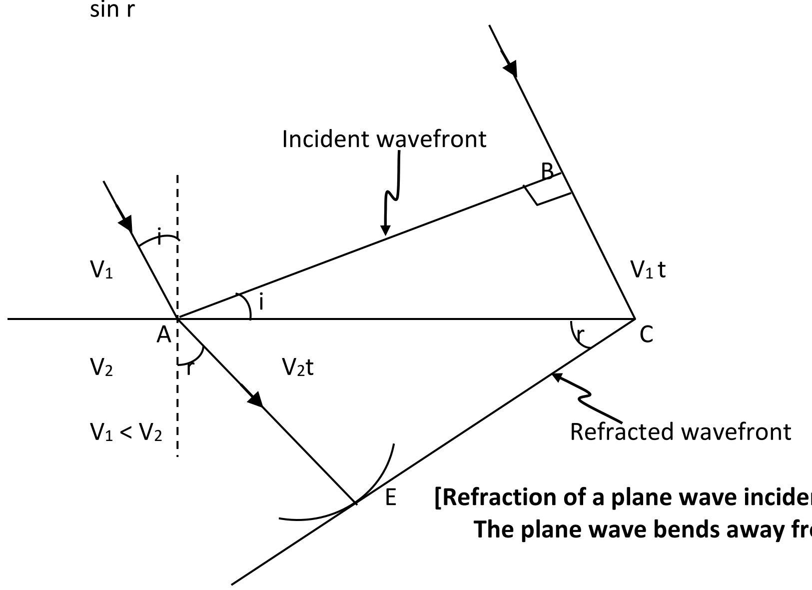
$$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t}$$

or $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2$ (a constant)

This proves Snell's law of refraction. The constant ${}^1\mu_2$ is called the refractive index of the second medium with respect to first medium.

Further, since the incident ray SA, the normal AN and the refracted ray AD are respectively perpendicular to the incident wavefront AB, the dividing surface XY and the refracted wavefront AB, the dividing surface XY and the refracted wavefront CD (all perpendicular to the plane of the paper), therefore, they all lie in the plane of the paper, i.e., in the same plane. This proves another law of refraction.

Refraction at a rarer medium: Fig. shows the refraction of a plane wavefront at a rarer medium i.e., $v_2 > v_1$. The incident and refracted wavefronts are shown in Fig. In this case, the angle of refraction is greater than the angle of incidence. Here also the Snell's law of refraction is valid. That is $\frac{\sin i}{\sin r} = {}^1\mu_2$ [a constant]



[Refraction of a plane wave incident on a rarer medium $v_2 > v_1$. The plane wave bends away from the refracting surface]

EFFECT ON WAVELENGTH, FREQUENCY AND SPEED DURING REFRACTION

Effect on wavelength, frequency and speed during refraction: Consider a source of light at rest in one medium and the observer at rest in another medium. Let there be no relative motion between the two media so that geometry of the source, medium and observer does not change with time. Then the light will take a definite time to travel from the source to the observer.

Suppose the source emits a wavefront after every time interval T and also each wavefront takes time T to travel from the source to the observer. Then the observer will receive ν = 1/T wavefronts per second. Thus, the frequency ν remains the same as light travels from one medium to another. In fact, frequency ν is the characteristic of the source.

As the speeds of light v_1 and v_2 are different in the two media, the wavelength λ_1 and λ_2 will also be different. Using the relation $v = \nu\lambda$, we get

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\nu\lambda_1}{\nu\lambda_2} = \frac{\lambda_1}{\lambda_2}$$

Hence the wavelength in a medium is directly proportional to the phase speed (or wave speed) and inversely proportional to its refractive index.

The refractive index of a medium with respect to vacuum is

$$\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

Since any medium is optically denser than vacuum, so

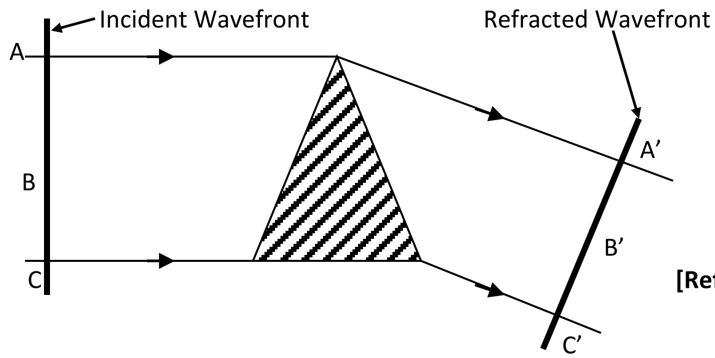
$$\mu > 1$$

Consequently, $c > v$

Thus, the speed of light in an optically rarer medium is greater than that in an optically denser medium.

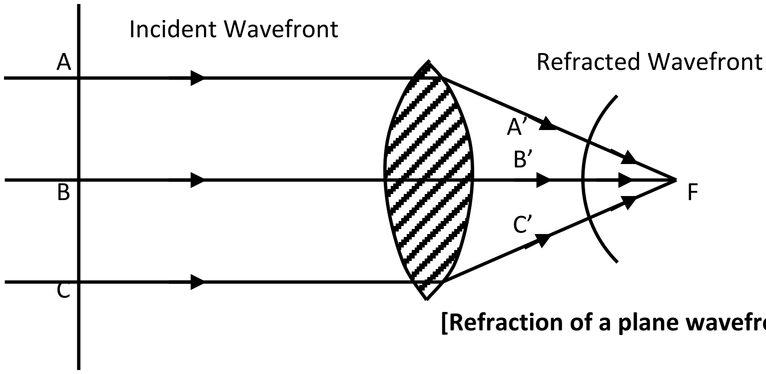
BEHAVIOUR OF A PRISM, LENS AND MIRROR

Behaviour of a prism: Fig. shows the refraction of a plane wavefront through a thin prism. Since the speed of light in glass is smaller than that in air, therefore, the lower part C of the plane wavefront which travels through the greatest thickness of th glass prism is slowed down the most and the upper part A, which travels through the minimum thickness of glass prism, is slowed down the least. This explains the tilting of a plane wavefront after refraction through a glass prism.



[Refraction of a plane wavefront through a prism]

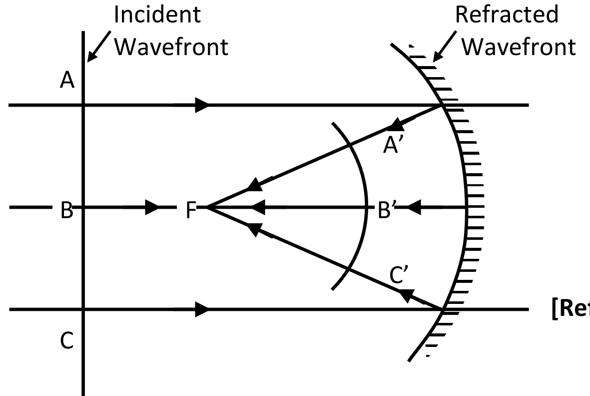
Behaviour of a convex lens: Fig. shows the refraction of a plane wavefront through a convex lens. The central part B of the plane wavefront through a convex lens. The central part B of the plane wavefront travels through the greatest thickness of the lens and is, therefore, slowed down the most. The marginal parts A and C of the wavefront travel through a minimum thickness of the lens and are, therefore, slowed down the least. So, the emerging wavefront is spherical and converges to a focus F.



[Refraction of a plane wavefront through a convex lens]

Behaviour of a concave mirror: Fig. shows the reflection of a plane wavefront from a concave mirror. The central part B of the incident wavefront has to travel the greatest distance before getting reflected, compared to the marginal parts A and C. Therefore, the central portion B' of the reflected wavefront is closer to the mirror than the marginal portions A' and C'.

Hence the reflected wavefront is spherical and converges to a focus.



[Reflection of a plane wavefront from a a concave mirror]

Examples based on Reflection and Refraction of Light Waves

◆ **FORMULA USED**

1. Snell's law, $^1\mu_2 = \frac{\sin i}{\sin r}$
2. $\mu = c = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$
3. Speed of light in vacuum, $c = v\lambda$
4. $\mu = \frac{\lambda}{\lambda'} = \frac{\text{Wavelength in vacuum}}{\text{Wavelength in medium}}$
5. Wavelength in medium, $\lambda' = \frac{\lambda}{\mu}$
6. Optical path (in vacuum) = $\mu \times$ Path in medium
7. Frequency of light remains unchanged during its reflection or refraction.

◆ **UNITS USED**

Speeds of light c and v are in ms^{-1}, wavelengths λ and λ' in metre, frequency v in Hz and refractive index μ has no units.

◆ **CONSTANT USED**

Speed of light in vacuum, $c = 3 \times 10^8\ ms^{-1}$

◆ **CONVERSIONS USED**

$1\ Å = 10^{-8}\ cm = 10^{-10}\ m$, $1\ nm = 10^{-9}\ m$, $1\ \mu m = 10^{-6}\ m$

Q. 1. ***Monochromatic light of wavelength 600 nm incident from air on a glass surface. What are the wavelength, frequency and speed of refracted light? Refractive index of glass 1.5.***

Sol. During refraction, frequency remains unchanged. Both wavelength and speed get changed.

Frequency, $v = \frac{c}{\lambda} = \frac{3 \times 10^8\ ms^{-1}}{600 \times 10^{-9}\ m}$

$= 5 \times 10^{14}$ Hz.

Speed of refracted light,

$v_{glass} = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ ms^{-1}$

Wavelength of refracted light,

$\lambda_{glass} = \frac{v_{glass}}{v}$

$= \frac{2 \times 10^8}{5 \times 10^{14}} = 4 \times 10^{-7}$ m **= 400 nm.**

Q. 2. ***The refractive index of diamond is 2.47 and that of window glass is 1.51. How much faster does light travel in window glass than in diamond?***

Sol. Refractive index of diamond,

$\mu_d = \frac{c}{v_d}$

$\therefore\ v_d = \frac{c}{\mu_d} = \frac{3 \times 10^8}{2.47} = 1.215 \times 10^8\ ms^{-1}$

Refractive index of glass, $\mu_g = \frac{c}{v_g}$

$\therefore\ v_g = \frac{c}{\mu_g} = \frac{3 \times 10^8}{1.51}$

$= 1.987 \times 10^8\ ms^{-1}$

$v_g - v_d = (1.987 - 1.215) \times 10^8$

$= 7.72 \times 10^7\ ms^{-1}$

Thus light travels $7.72 \times 10^7\ ms^{-1}$ faster in window glass than in diamond.

Q. 3. ***Calculate the time which light will take to travel normally through a glass plate of thickness 1 mm. Refractive index of glass is 1.5 and velocity of light is 3×10^8 ms^{-1}.***

Sol. Velocity of light in glass

$$= \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ ms}^{-1}$$

Thickness of glass plate = 1 mm = 10^{-3} m

∴ Time taken by light to pass normally through glass plate,

$$t = \frac{10^{-3}}{2 \times 10^8} = 5 \times 10^{-12} \text{ s.}$$

Q. 4. ***White light consists of waves of wavelengths between 400 nm to 7 nm. What will be the wavelength range if this light goes through water (μ = 1.33)?***

Sol. When λ = 400 nm,

$$\lambda\omega = \frac{\lambda}{\mu} = \frac{400}{1.33} = 300 \text{ nm}$$

When λ = 700 nm,

$$\lambda = \frac{\lambda}{\mu} = \frac{700}{1.33} = 525 \text{ nm.}$$

Thus the wavelength of white light in water varies from 300 nm to 525 nm.

Q. 5. ***The optical path of monochromatic light is the same if it travels 2.0 cm thickness of glass or 2.25 cm thickness of water. If the refractive index of water is 1.33, what is the refractive index of glass?***

Sol. Optical path = $\mu \times$ Path in medium

∵ Optical path for glass = Optical path for water

∴ $\mu_g \times 2.0 = 1.33 \times 2.25$

Or $\mu_g = \dfrac{1.33 \times 2.25}{2.0} = 1.50$

Q. 6. ***The number of waves in a 4 cm thick strip of glass is the same as in 5 cm thick water layer, when the same monochromatic light travels in them. If the refractive index of water is 4/3, what will be that of glass?***

Sol. Number of waves in glass strip = $\dfrac{\text{Thickness of glass strip}}{\text{Wavelength in glass}} = \dfrac{4 \text{ cm}}{\lambda_g}$

Number of waves in water layer = $\dfrac{\text{Thickness of water layer}}{\text{Wavelength in water}} = \dfrac{5 \text{ cm}}{\lambda_\omega}$

As given, $\dfrac{4 \text{ cm}}{\lambda_g} = \dfrac{5 \text{ cm}}{\lambda_\omega}$

∴ $\dfrac{\lambda_g}{\lambda_\omega} = \dfrac{4}{5}$

But wavelength of light in the medium is inversely related to its refractive index, therefore,

$\dfrac{\lambda_g}{\lambda_g} = \dfrac{\mu_\omega}{\lambda_\omega}$ or $\dfrac{4}{5} = \dfrac{\lambda_g}{\lambda_\omega}$ ∴ $\mu_g = \dfrac{5}{4} \times \mu_\omega = \dfrac{5}{4} \times \dfrac{4}{3} = \dfrac{5}{3}$

Q. 7. ***The absolute refractive index of air is 1.0003 and wavelength of yellow light in vacuum is 6000 Å. Find the thickness of air column which will contain one more wavelength of yellow light than in the same thickness of vacuum.***

Sol. Wavelength of yellow light in vacuum,

λ = 6000 Å

Wavelength of yellow light in air, $\lambda' = \dfrac{\lambda}{\mu} = \dfrac{6000}{1.0003}$ Å

Let a thickness t of vacuum contain n waves and the same thickness t of air contain n + 1 waves.

Then $n = \dfrac{t}{\lambda} = \dfrac{t}{6000 \text{ Å}}$ and $n + 1 = \dfrac{t}{\lambda'} = \dfrac{1.0003\, t}{6000 \text{ Å}}$

From the above two equations, we get

$\dfrac{t}{6000 \text{ Å}} + 1 = \dfrac{1.003\, t}{0.0003}$ or t + 6000 Å = 1.0003 t

or $t = \dfrac{6000}{0.0003} = 2 \times 10^7$ Å = 2 mm.

PRINCIPLE OF SUPERPOSITION OF WAVES

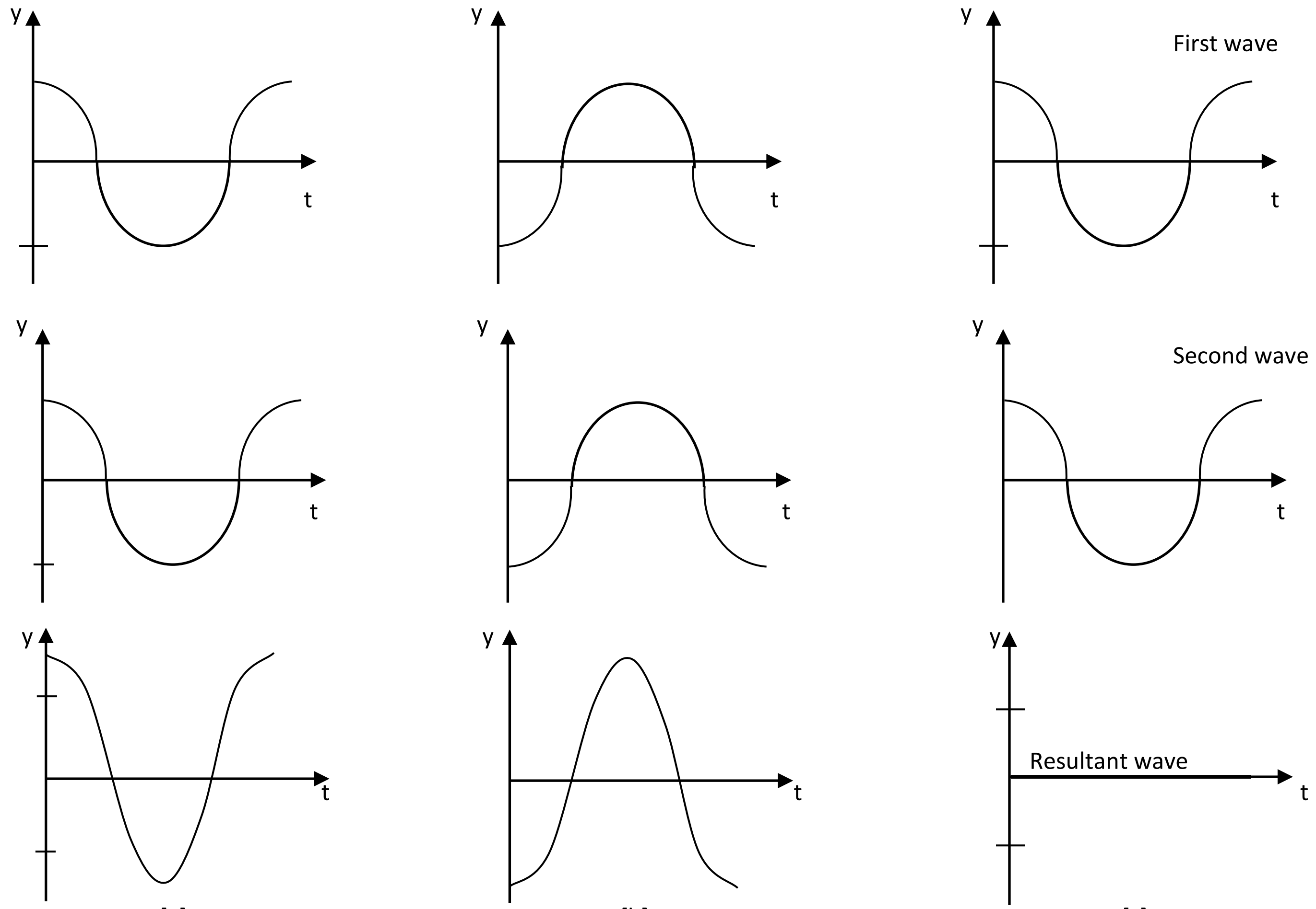
Principle of superposition of waves: When a number of waves travel through a medium simultaneously, each wave travels independently of the others i.e., as if all other waves were absent. An important consequence of this independent behaviour of the waves is that the effects of all these waves get added together. The resultant wave is obtained by the principle of superposition of waves which can be stated as follows:

When a number of waves travelling through a medium superpose on each other, the resultant displacement at any point at a given instant is equal to the vector sum of the displacements due to the individual waves at that point.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ... \vec{y}_n$ are the displacement due to the different waves acting separately, then according to the principle waves acting separately, then according to the principle of superposition, the resultant displacement when all the waves act together is given by the vector sum:

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ... + \vec{y}_n$$

When the two superposing waves are in the same phase i.e., the crest of one falls over the crest of another [Fig. (a)] or the trough of one falls over the trough of another [Fig. (b)], their displacements get added. When the two waves meet in opposite phases i.e., the crest of one falls over the trough of another [Fig. (c)], their displacements get subtracted.



[Illustration of principle of superposition of waves]

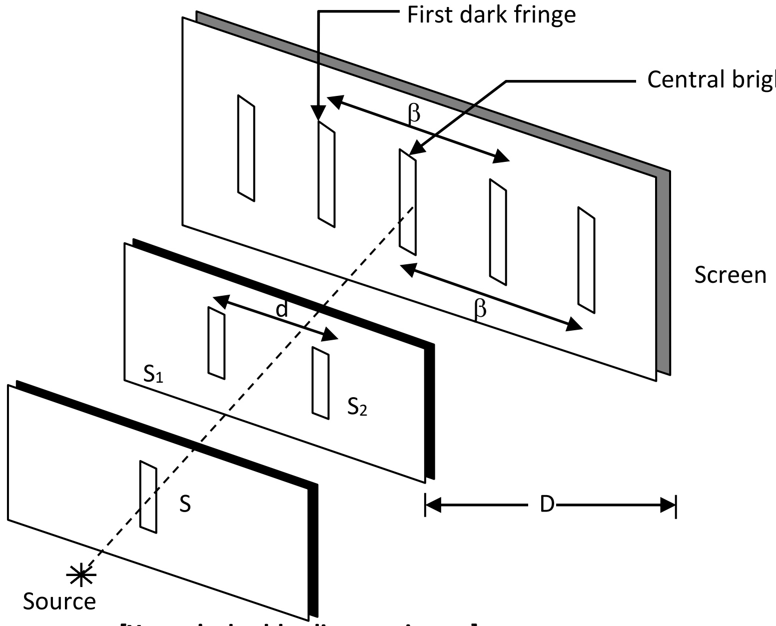
INTERFERENCE OF LIGHT: YOUNG'S DOUBLE SLIT EXPERIMENT

Interference of light: When two light moves of the same frequency and having zero or constant phase difference travelling in the same direction superpose each other, the intensity in the region of superposition gets redistributed, becoming maximum at some points and minimum at others. This phenomenon is called ***interference of light***.

Young's double slit experiment: In 1801, Thomas Young was the first person to demonstrate experimentally the interference of light.

In this experiment, a source of monochromatic light (e.g., a sodium vapour lamp) illuminates a rectangular narrow slit S, about 1 mm wide, as shown in Fig. S_1 and S_2 are two parallel narrow slits which are arranged symmetrically and parallel to the slit S at a distance of about 10 cm from it. The separation between S_1 and S_2 is $\approx$ 2 mm and width of each slit is $\approx$ 0.3 mm. An observation screen is placed at a distance of $\approx$ 2 m from the two slits. Alternate bright and dark bands appear on the observation screen.

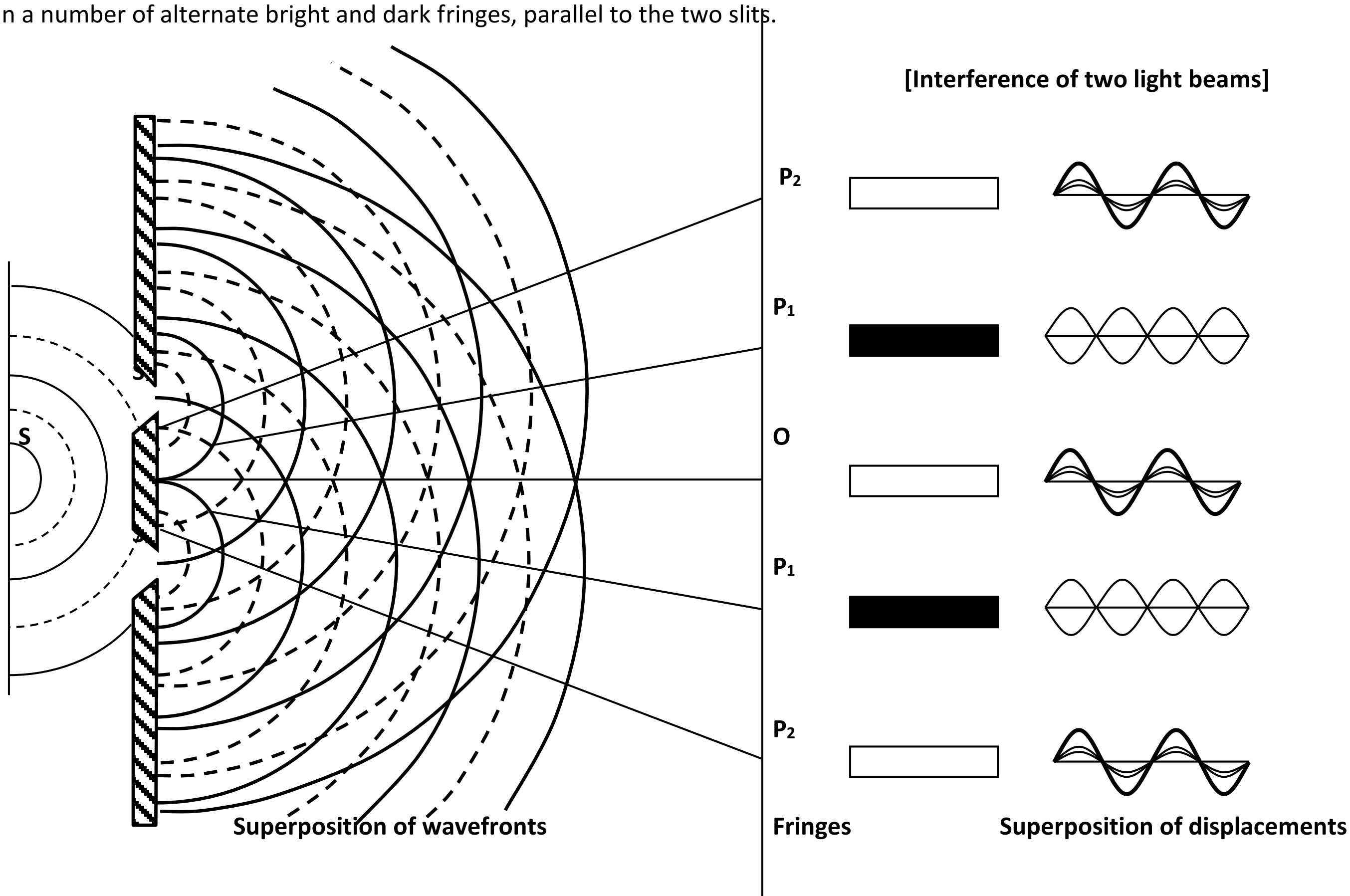
These are called interference fringes. When one of the slits, S_1 or S_2 is closed, bright and dark fringes disappear and the intensity of light becomes uniform.



[Young's double slit experiment]

Experiment: Fig. shows a section of young's experiment in the plane of paper. According to Huygens' principle, cylindrical wavefronts emerge out from slit S, whose sections have been shown by circular arcs. The solid curves represent crests and the dotted curves represent troughs. As $SS_1 = SS_2$, these waves fall on the slits S_1 and S_2 simultaneously so that the waves spreading out from S_1 and S_2 act as two ***coherent sources*** of monochromatic light. Interference takes place between the waves diverging from these sources.

At the lines leading to O, P_2 and P_2', the crest of one wave falls over the crest of other wave or the trough of one wave falls over the crest of other wave or the trough of one wave falls over the trough of other wave, the amplitudes of the two waves get added up and hence the intensity ($I \propto a^2$) becomes maximum. This is called constructive interference. At the lines leading to P_1 and P_1' the crest of one wave falls over the trough of other or the trough of one wave falls over the crest of other wave, the amplitudes of the two waves get subtracted and hence the intensity becomes minimum. This is called destructive interference. So on the observation screen, we obtain a number of alternate bright and dark fringes, parallel to the two slits.



[Interference of two light beams]

CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE.

Expression for intensity at any point in interference pattern: Suppose the displacements of two light waves from two coherent sources S_1 and S_2 at point P on the observation screen at any time t are given by

$$y_1 = a_1 \sin \omega t \quad \text{and} \quad y_2 = a_2 \sin (\omega t + \phi)$$

where a_1 and a_2 are the amplitudes of the two waves, ϕ is the constant phase difference between the two waves. By the super-position principle, the resultant displacement at point P is

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$
$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

or $y = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t$

Put $a_1 + a_2 \cos \phi = A \cos \theta$ ... (1)

and $a_2 \sin \phi = A \sin \theta$ ... (2)

Then $y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t$

or $y = A \sin (\omega t + \theta)$

Thus, the resultant wave is also a harmonic wave of amplitude A and it leads the first harmonic wave by phase angle θ. To determine A, squaring and adding equations (1) and (2), we get

$$A^2 \cos^2 \theta + A^2 \sin^2 \theta = (a_1 + a_2 \cos \phi)^2 + a_2^2 \sin^2 \phi$$

or $A^2 = a_1^2 + a_2^2 (\cos^2 \phi + \sin^2 \phi) + 2a_1a_2 \cos \phi$

or $A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$ ... (3)

But intensity of a wave $\propto$ (amplitude)^2

We write $I = kA^2$, $I_1 = ka_1^2$ and $I_2 = ka_2^2$

where k is proportionality constant. The equation (3) can be written as

$$kA^2 = ka_1^2 + ka_2^2 + 2\sqrt{ka_1}\sqrt{ka_2} \cos \phi$$

or $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$ ... (4)

This equation gives the total intensity at a point where the phase difference is ϕ. Here I_1 and I_2 are the intensities which the two individual sources produce on their own. The total intensity also contains a third term $2\sqrt{I_1 I_2} \cos \phi$. It is called interference term.

Constructive interference: The resultant intensity at the point P will be maximum when

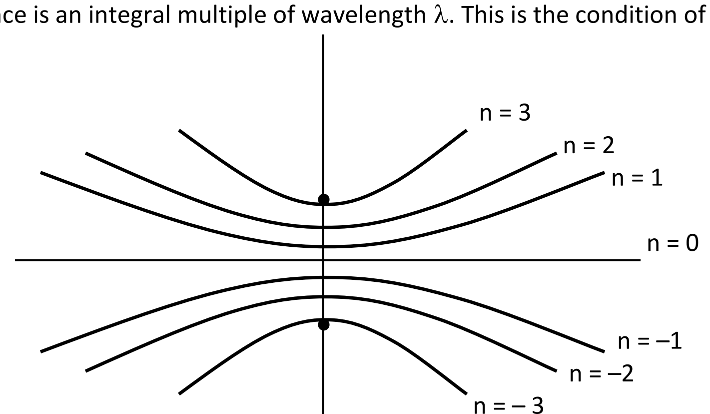
$$\cos \phi = 1 \quad \text{or} \quad \phi = 0, 2\pi, 4\pi, ...$$

Since a phase difference of 2π corresponds to a path difference of λ, therefore, if p is the path difference between the two superposing waves, then

$$\frac{2\pi p}{\lambda} = 0, 2\pi, 4\pi, ...$$

or $p = 0, \lambda, 2\lambda, 3\lambda, ... = n\lambda$

Hence the resultant intensity at a point is maximum when the phase difference between the two superposing waves is an even multiple of π or path difference is an integral multiple of wavelength λ. This is the condition of construction interference.



[Locus of points for which $S_1P - S_2P$ is equal to $0, \pm\lambda, \pm 2\lambda \pm 3\lambda$]

Destructive interference: The resultant intensity at the point P will be minimum when

$$\cos \phi = -1 \quad \text{or} \quad \phi = \pi, 3\pi, 5\pi, ...$$

or $\frac{2\pi p}{\lambda} = \pi, 3\pi, 5\pi, ...$

or $p = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ... = (2n + 1)\frac{\lambda}{2}$

Hence the resultant intensity at a point is minimum when the phase difference between the two superposing waves is an odd multiple of π or the path difference is an odd multiple of $\lambda/2$. This is the condition of destructive interference.

COHERENT & INCOHERENT SOURCES

Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, and called coherent sources.

Two sources of light which do not emit light waves with a constant phase difference are called incoherent sources.

▶ **Need of coherent sources for the production of interference pattern:** When two monochromatic waves of intensity I_1, I_2 and phase difference ϕ meet at a point, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

The last term $2\sqrt{I_1 I_2}\cos\phi$ is called interference term. There are two possibilities:

▶ 1. If $\cos\phi$ remains constant with time, the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1} + \sqrt{I_2})^2$ at points where $\cos\phi$ is +1 and minimum $(\sqrt{I_1} - \sqrt{I_2})^2$ at points where $\cos\phi = -1$. The sources in this case are coherent.

▶ 2. If $\cos\phi$ varies continuously with time assuming both positive and negative values, then the average value of $\cos\phi$ will be zero over time interval of measurement. Then interference term averages to zero. There will be same intensity, $I = I_1 + I_2$ at every point i.e., there will be general illumination on the observation screen. The two sources in the case are incoherent.

Hence **to observe interference, we need to have two sources with the same frequency and with a stable phase difference. Such a pair of sources are called coherent sources**.

▶▶ **Two independent sources cannot be coherent**: This is because of the following reasons:

1. Light is emitted by individual atoms and not by the bulk of matter acting as a whole.
2. Even a tiniest source consists of millions of atoms, and emission of light by them takes place independently.
3. Even an atom emits an unbroken wave of about 10^{-8} second due to its transition from a higher energy state to a lower energy state.

The millions of atoms of a source cannot emit waves in the same phase. The light emitted by the commonly used monochromatic source (a sodium lamp) remains coherent for about 10^{-8} s. After this time, the atoms responsible for emission of light get changed. The phase difference and hence the interference pattern changes 10^8 times in one second. Out eyes cannot see such rapid changes and a uniform illumination is seen on the screen. So two independent light sources cannot produce a sustained interference.

▶▶ **Two coherent sources can be obtained from a single parent source. Some of the methods of producing coherent sources are as follows:**

1. In Young's double slit experiment, the two sources S_1 and S_2 get light from the same source S. Whatever phase changes occur in S_1, the same phase changes occur in S_2. The relative phase difference between S_1 and S_2 remains constant with time. So, they act as coherent sources.

2. In Fresnel's biprism method, two coherent sources are obtained from the same parent source, by refraction.

3. In Lloyd's mirror method, a source and its reflected image act as two coherent sources.

▶▶ **Conditions for obtaining two coherent sources of light:**

▶1. The two sources of light must be obtained from a single source by some method. Then the relative phase difference between the two light waves from the sources will remains constant with time.

▶2. The two sources must give monochromatic light. Otherwise, different colours will produce difference interference patterns and fringes of different colours will overlap.

▶3. The path difference between the waves arriving on the screen from the two sources must not be large. It should not exceed 30 cm. Then the phase difference produced due to path difference will not be constant. There will be general illumination on the screen.

▶▶FOR YOUR KNOWLEDGE...

- To observe interference of light, the two sources of light must be coherent.
- In contrast to light from an ordinary source, the laser light is highly monochromatic and coherent. So two independent laser sources can produce interference fringes and the path difference may be several metres in this case.
- Methods of producing coherent source: There are two general methods of producing coherent sources:
  1. By division of wavefront: In this method, a wavefront is divided into two or more parts by use of slits, mirrors, lenses or prisms. For example, Young's double slit method, Frenel's biprims and Lloyd's mirror.
  2. By division of amplitude: Here the amplitude of the wave is divided into two or more parts by partial reflection of refraction. The divided parts travel along different paths and are made to superpose to produce interference.
  For example, the brilliant colours seen in thin films of transparent materials like soap film, oil film, etc.

Examples based on Amplitude and Intensity at any point on an Interference pattern

◆ **FORMULA USED**

1. Resultant amplitude, $a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$
2. Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$
3. When $I_1 = I_2 = I_0$, $I = 2I_0(1 + \cos\phi) = 4I_0\cos^2\frac{\phi}{2}$

◆ UNITS USED

Amplitudes a, a_1 and a_2 are in metre and intensities I, I_1 and I_2 in watt/m^2.

Q. 1. ***Two plane monochromatic waves propagating in the same direction with amplitudes A and 2A and differing in phase but $\pi/3$ superpose. Calculate the amplitude of the resultant wave.***

Sol. Here $A_1 = A$, $A_2 = 2A$, $\phi = \pi/3$

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$
$$= \sqrt{A^2 + (2A)^2 + 2A \times 2A \cos \pi/3}$$
$$= \sqrt{5A^2 + 4A^2 \times ½} = \sqrt{7A^2} = \sqrt{7}A$$

Q. 2. ***Two sources of intensity I and 4 I are used in an interference experiment. Find the intensity at points where the waves from two sources superimpose with a phase difference* (i) zero (ii) $\pi/2$ and (iii) π.**

Sol. The resultant intensity at a point where phase difference is ϕ is

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

As $I_1 = I$ and $I_2 = 4$ I, therefore

$$I_R = I + 4I + 2\sqrt{I \cdot 4I}\cos\phi = 5I + 4I\cos\phi$$

(i) When $\phi = 0$, $I_R = 5I + 4I\cos 0 = 9I$.

(ii) When $\phi = \frac{\pi}{2}$, $I_R = 5I + 4I\cos\frac{\pi}{2} = 5I$

(iii) When $\phi = \pi$, $I_R = 5I + 4I\cos\pi = 5I - 4I = I$

Q. 3. ***In a Young's double slit experiment, the intensity of light at a point on the screen where the path difference is λ is k units. Find the intensity at a point where the path difference is* (i) $\frac{\lambda}{4}$ (ii) $\frac{\lambda}{3}$ and (iii) $\frac{\lambda}{2}$.**

Sol. Intensity at any point on the screen,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos ⍰$$

Let I_0 be the intensity of either source. Then $I_1 = I_2 = I_0$, and

$$I = 2I_0(1 + \cos\phi) = 4I_0\cos^2\frac{\phi}{2}$$

When $p = \lambda$, $\phi = 2\pi$

$\therefore$ $I = 4I_0\cos^2\frac{\phi}{2} = 4I_0\cos^2\pi = 4I_0 = k$

(i) When $p = \frac{\lambda}{4}$, $\phi = \frac{\pi}{2}$

$\therefore$ $I = 4I_0\cos^2\frac{\pi}{4} = 4I_0 \times \frac{1}{2} = 2I_0 = \frac{k}{2}$

(ii) When $p = \frac{\lambda}{3}$, $\phi = \frac{2\pi}{3}$

$\therefore$ $I = 4I_0\cos^2\frac{\pi}{3} = 4I_0 \times \frac{1}{4} = I_0 = \frac{k}{4}$

(iii) When $p = \frac{\lambda}{2}$, $\phi = \pi$

$\therefore$ $I = 4I_0\cos^2 = \frac{\pi}{2} = 0$

Q. 4. ***Find the ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringes from the centre.***

Sol. If I_0 is the intensity of either source, then intensity at a point is given by

$$I = 2I_0(1 + \cos\phi)$$

At the centre $\phi = 0$, then intensity will be

$$I_1 = 2I_0(1 + \cos 0) = 4I_0$$

The phase difference between two successive fringes is 2π. So the phase difference at a point distant one-quarter of the distance between two fringes from the centre will be $\pi/2$. Hence intensity at this point will be

$$I_2 = 2I_0\left(1 + \cos\frac{\pi}{2}\right) = 2I_0$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2:1$$

Q. 5. ***Find the ratio of intensities of two points P and Q on a screen in a Young's double slit experiment when waves from sources S_1 and S_2 have phase difference of* (i) $\frac{\pi}{3}$ and (ii) $\frac{\pi}{2}$ respectively.**

Sol. As $I = 2I_0(1 + \cos\phi)$

$\therefore$ $I \propto 1 + \cos\phi$

Hence $\frac{I_P}{I_Q} = \frac{1 + \cos\pi/3}{1 + \cos\pi/2} = \frac{1 + ½}{1 + 0} = \frac{3}{2} = 3:2$

Q. 6. ***Find the ratio of intensities at two points in a screen in Young's double slit experiment, when waves from the two slits have path difference of (i) 0 and (ii) $\lambda/4$.***

Sol. Intensity at any point of an interference pattern is given by $I = 2I_0 (1 + \cos\phi)$
where I_0 is the intensity of either wave.

Here $\phi_P = 0$, $\phi_Q = \frac{2\pi p}{\lambda} = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$\therefore$ $\frac{I_P}{I_Q} = \frac{1 + \cos\phi_P}{1 + \cos\phi_Q} = \frac{1 + \cos 0}{1 + \cos \pi/2}$

$= \frac{1+1}{1+0} = \frac{2}{1} = 2 : 1$

Q. 7. ***Find the maximum intensity in case of interference of n identical waves each of intensity I_0, if the interference is (i) coherent and (ii) incoherent.***

Sol. **(i) When the interference is coherent:** When two waves of intensities I_1 and I_2 and having a phase difference ϕ interference, the resultant intensity at any point is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

For instant to be maximum,

$\phi = 0$ or $\cos\phi = 1$

$\therefore$ $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

When n identical waves of each of intensity I_0 interference

$I_{max} = (\sqrt{I_0} + \sqrt{I_0} + \sqrt{I_0} + \ldots\ldots \text{ n terms})^2 = (n\sqrt{I_0})^2$

or $I_{max} = n^2 I_0$

(ii) When interference in incoherent: Here the phase difference ϕ between two waves changes randomly with time. So the average value of $\cos\phi$ over a complete cycle is zero.

Consequently,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \times 0 = I_1 + I_2$$

When n identical waves each of intensity I_0 interfere,

$I_{max} = I_0 + I_0 + I_0 + \ldots\ldots\ldots \text{ n terms} = n I_0$

THEORY OF INTERFERENCE FRINGES: FRINGE WIDTH

Experiment for fringe width in Young's double slit experiment: As shown in fig. suppose a narrow slit S is illuminated by monochromatic light of wavelength λ. S_1 and S_2 are two narrow slits at equal distance from S. Being derived from the same parent sources S_1 and S_2 act as two coherent sources, separated by small distance d. Interference fringes are obtained on a screen placed at distance D from the sources S_1 and S_2.

Consider a point P on the screen at distance x from the centre O. The nature of the interference at the point P depends on path difference,

$$p = S_2P - S_1P$$

From right-angled ΔS_2BP and ΔS_1AP,

$S_2P^2 - S_1P^2 = [S_2B^2 + PB^2] - [S_1A^2 + PA^2]$

$= \left(D^2 + \left(x + \frac{d}{2}\right)^2\right) - \left(D^2 + \left(x - \frac{d}{2}\right)^2\right)$

or $(S_2P - S_1P)(S_2P + S_1P) = 2xd$

or $S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$

In practice, the point P lies very close to O, therefore $S_1P \simeq S_2P \simeq D$. Hence

$p = S_2P - S_1P = \frac{2xd}{2D}$

or $p = \frac{xd}{D}$

◆ **Positions of bright fringes**: For constructive interference, $p = \frac{xd}{D} = n\lambda$

or $x = \frac{nD\lambda}{d}$ where n = 0, 1, 2, 3, ...

Clearly, the positions of various bright fringes are as follows:

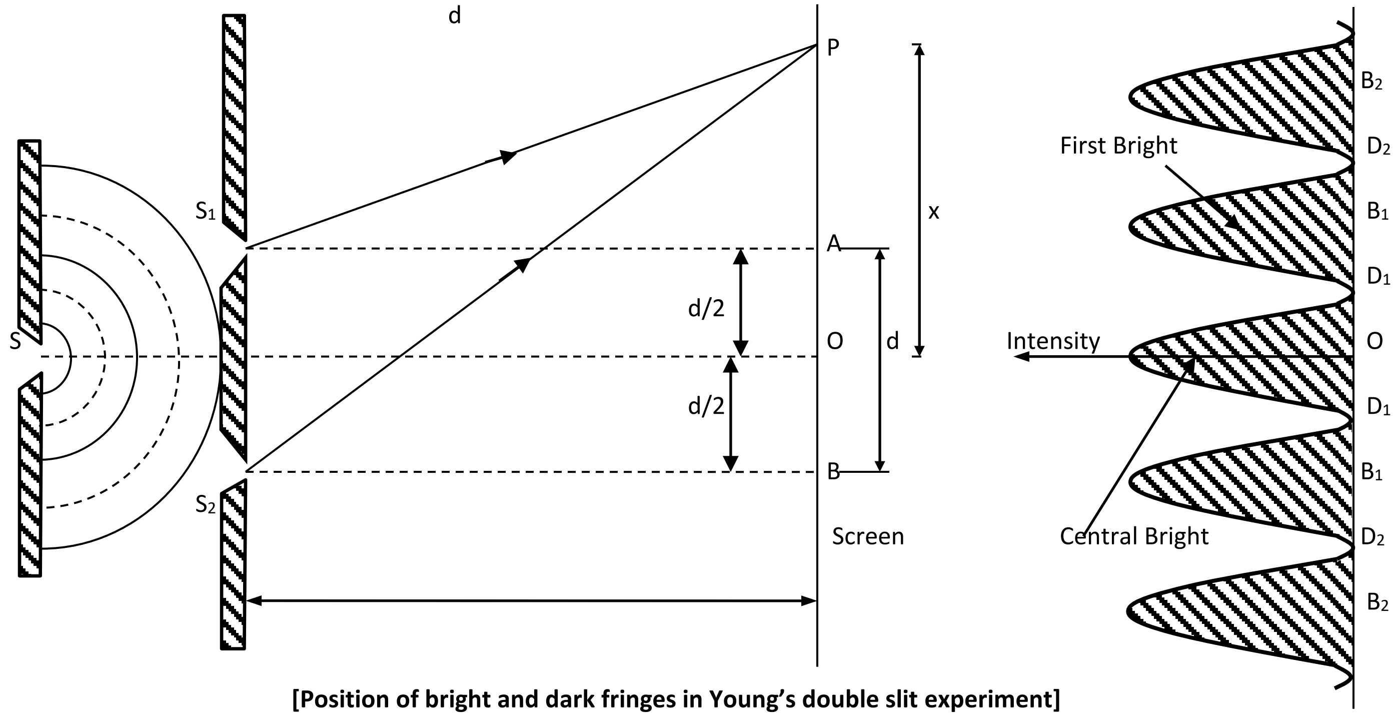
For n = 0, $x_0 = 0$ Central bright fringe

For n = 1, $x_1 = \frac{D\lambda}{d}$ First bright fringes

For n = 2, $x_2 = \frac{2D\lambda}{d}$ Second bright fringe

....

For n = n, $x_n = \frac{nD\lambda}{d}$ nth bright fringe



[Position of bright and dark fringes in Young's double slit experiment]

◆ **Position of dark fringes:** For destructive interference, $p = xd = (2n - 1)\frac{\lambda}{2}$

or $x = (2n - 1) D\lambda$ where n = 1, 2, 3, ...

Clearly, the positions of various dark fringes are as follows:

For n = 1, $x'_1 = \frac{1}{2}\frac{D\lambda}{d}$ First dark fringe

For n = 2, $x'_2 = \frac{3}{2}\frac{D\lambda}{d}$ Second dark fringe

....

For n = n, $x'_n = (2n - 1)\frac{D\lambda}{2d}$ nth dark fringe

Since the central point O is equidistant from S_1 and S_2, the path difference p for it is zero. There will be a bright fringe at the centre O. But as we move from O upwards or downwards, alternate dark and bright fringes are formed.

◆◆ **Fringe width: It is the separation between two successive bright or dark fringes,**

Width of a dark fringe = Separation between two consecutive bright fringes

$= x_n - x_n - 1 = \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d} = \frac{D\lambda}{d}$

Width of a bright fringe = Separation between two consecutive dark fringes = $x'_n - x'_{n-1}$

$= (2n - 1)\frac{D\lambda}{2d} - [2(n - 1) - 1]\frac{D\lambda}{2d} = \frac{D\lambda}{d}$

Clearly, both the bright and dark fringes are of equal width. Hence the expression for the fringe width in Young's double slit experiment can be written as $\beta = \frac{D\lambda}{d}$

- As β is independent of n (the order of fringe), therefore, all the fringes are of equal width.
- In the case of light, λ is extremely small, D should be much larger than d, so that the fringe width β may be appreciable and hence observable.

◆ **Measurement of wavelength:** Young's double slit experiment can be used to determine the wavelength of a monochromatic light. The interference pattern is obtained in the focal plane of a micrometre eyepiece and with its help fringe width β is measured. By measuring the distance d between the two coherent sources and their distance D from the eyepiece, the value of wavelength λ can be calculated as $\lambda = \frac{\beta d}{D}$

►► FOR YOUR KNOWLEDGE...

- In young's double slit experiment, the width of the central bright fringe is equal to the distance between the first dark fringes on the two sides of the central bright fringe. So the width of the central bright fringe is given by

$$\beta_0 = 2x'_1 = 2 \times \frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

- As all the bright and dark fringes are of the same width, the angular width of a fringe is given by

$$\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d}$$

- If Young's double slit apparatus is immersed in a liquid of refractive index μ, the wavelength of light decreases to λ' $(=\lambda/\mu)$ and so the fringe width reduces to

$$\beta' = \frac{D\lambda'}{d} = \frac{D\lambda}{\mu d} = \frac{\beta}{\mu}$$

Examples based on Young's Double Slit Experiment

◆ **FORMULA USED**

1. For a bright fringe, path difference, $p = n\lambda$
2. For a dark fringe, $p = (2n - 1)\frac{\lambda}{2}$, n = 1, 2, 3.......
3. Distance of nth bright fringe from the centre of the screen,
   $x_n = n\frac{D\lambda}{d}$, n = 1, 2, 3, ...
4. Distance of nth dark fringe from the centre of the screen, $x'_n = (2n - 1)\frac{D\lambda}{2d}$
5. Fringe width, $\beta = \frac{D\lambda}{d}$
6. Wavelength of light used, $\lambda = \frac{\beta d}{D}$
7. Angular position of nth bright fringe, $\theta_n = \frac{x_n}{D} = \frac{n\lambda}{d}$
8. Angular position of nth dark fringe,
   $\theta'_n = \frac{x_n'}{D} = (2n - 1)\frac{\lambda}{2d}$

◆ **UNITS USED**

Path difference p, distances x_n, x_n', d and D; wavelength λ and fringes width β are all in metre.

Q. 1. In Young's double experiment, the two parallel slits are made one millimetre apart and a screen is placed one metre away. What is the fringe separation when blue green light of wavelength 500 nm is used?

Sol. Here $d = 1\text{ mm} = 10^{-3}\text{ m}$, $\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{ m}$

Fringe width, $\beta = \frac{D\lambda}{d} = \frac{1 \times 500 \times 10^{-9}}{10^{-3}}\text{ m}$

$= 5 \times 10^{-4}\text{ m} = 0.5\text{ mm}$

Q. 2. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 8.1 mm. A second light produced an interference pattern in which the fringes are separated by 7.2 mm. Calculate the wavelength of the second light.

Sol. Here $\lambda_1 = 630$ nm, $\beta_1 = 8.1$ mm, $\beta_2 = 7.2$ mm, $\lambda_2 = ?$

Fringe width, $\beta = \frac{D\lambda}{d}$

For constant D and d, $\beta \propto \lambda$ or $\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$

$\therefore \quad \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1$

$= \frac{7.2 \text{ mm}}{8.1 \text{ mm}} \times 630 \text{ nm} = 560 \text{ nm}$

Q. 3. ***Yellow light of wavelength 6000 Å produces fringes of width 0.8 mm in Young's double slit experiment. What will be the fringe width if the light source is replaced by another monochromatic source of wavelength 7500 Å and the separation between the slits is doubled?***

Sol. Here $\lambda_1 = 6000$ Å, $\beta_1 = 0.8$ mm, $\lambda_2 = 7500$ Å

Fringe width in first case, $\beta_1 = \frac{D\lambda_1}{d}$

Fringe width in second case, $\beta_2 = \frac{D\lambda_2}{2d}$

$\therefore \quad \frac{\beta_2}{\beta_1} = \frac{D\lambda_2/2d}{D\lambda_1 . d} = \frac{1}{2} . \frac{\lambda_2}{\lambda_1}$

or $\quad \beta_2 = \frac{1}{2} . \frac{\lambda_2}{\lambda_1} . \beta_1$

$= \frac{1}{2} \times \frac{7500 \text{ Å}}{6000 \text{ Å}} \times 0.8 \text{ mm} = 0.5 \text{ mm}$

Q. 4. ***The fringe width in a Young's double slit interference pattern is 2.4×10^{-4} m, when red light of wavelength 6400 Å is used. By how much will it change, if blue light of wavelength 4000 Å is used.***

Sol. Here $\beta_1 = 2.4 \times 10^{-4}$ m, $\lambda_1 = 6400$ Å, $\lambda_2 = 4000$ Å

$\therefore \quad \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{Å_1} = \frac{4000}{6400} = \frac{5}{8}$

or $\quad \beta_2 = \frac{5}{8} \times \beta_1$

$= 5 \times 2.4 \times 10^{-4} = 1.5 \times 10^{-4}$ m

Decrease in fringe width

$= \beta_1 - \beta_2 = (2.4 - 1.5) \times 10^{-4}$

$= 0.9 \times 10^{-4}$ m

Q. 5. ***In a two-slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance D from the slits. If the screen is moved 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between the slits is 10^{-3} m, calculate the wavelength of the light used.***

Sol. The fringe width in the two cases will be

$\beta = \frac{D\lambda}{d}$

and $\quad \beta' = \frac{D'\lambda}{d}$

$\therefore \quad \beta - \beta' = \frac{(D - D')\lambda}{d}$

or wavelength, $\lambda = \frac{(\beta - \beta')d}{D - D'}$

But $\quad D - D' = 5 \times 10^{-2}$ m

and $\quad \beta - \beta' = 3 \times 10^{-5}$ m, $\quad d = 10^{-3}$ m

$\therefore \quad \lambda = \frac{3 \times 10^{-5} \times 10^{-3}}{5 \times 10^{-2}}$

$= 6 \times 10^{-7}$ m = 6000 Å

Q. 6. ***In a Young's double-slit experiment, the separation between slits is 2×10^{-3} m whereas the distance of screen from the slits is 2.5 m. A light of wavelengths in the range of 2000 – 8000 Å is allowed to fall on the slits. Find the wavelength in the visible region that will be present on the screen at 10^{-3} m from the central maximum. Also find the wavelength that will be present at that point of screen in the infrared as well as in the ultraviolet region.***

Sol. The distance of a bright fringe from the central maximum on the screen is given by

$x = n\frac{D\lambda}{d}$, where $n = 0, 1, 2, \ldots\ldots$

Here $n = 0$ corresponds to the central maximum.

For n = 1,

$\lambda = \frac{xd}{nD} = \frac{(10^{-3} \text{ m}) \times (2 \times 10^{-3} \text{ m})}{1 \times (2.5 \text{ m})}$

$= 8 \times 10^{-7}$ m = 8000 Å **(infrared)**.

For n = 2,

$\lambda = \frac{8000 \text{ Å}}{2} = 4000$ Å **(visible)**

For n = 3,

$\lambda = \frac{8000 \text{ Å}}{4} = 2000$ Å **(ultraviolet)**

Q. 7. ***In Young's double slit experiment, using light of wavelength 400 nm, interference fringes of width 'X' are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringes width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the interfering sources in the two arrangements.***

Sol. Fringe width X is same in both cases.

In first case, $\quad X = \frac{D_1 \lambda_1}{d}$

In second case, $\quad X = \frac{D_2 \lambda_2}{d/2}$

$\therefore \quad \frac{D_1 \lambda_1}{d} = \frac{D_2 \lambda_2}{d/2}$

or $\frac{D_1}{D_2} = 2.\frac{\lambda_2}{\lambda_1}$

$= \frac{2 \times 600}{400} = \frac{3}{1} = 3 : 1$

Q. 8. In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. Calculate the fringe width if the entire apparatus is immersed in a liquid medium of refractive index 1.33.

Sol. Here, $\beta = 2.0$ mm, $\mu = 1.33$

Refractive index of liquid,

$\mu = \frac{\text{Wavelength of light in vacuum}}{\text{wavelength of light in liquid}} = \frac{\lambda}{\lambda'}$

or $\lambda' = \frac{\lambda}{\mu}$

Fringe width in air, $\beta = \frac{D\lambda}{d}$

Fringe width in liquid, $\beta' = \frac{D\lambda'}{d} = \frac{D\lambda}{d\mu} = \frac{\beta}{\mu} = \frac{2.0 \text{ mm}}{1.33} = 1.5$ mm.

Q. 9. In Young's double slit experiment the light has a frequency of 6×10^{14} Hz and distance between the centres of adjacent fringes is 0.75 mm. If the screen is 1.5 m away, what is the distance between the slits?

Sol. Here $V = 6 \times 10^{14}$ Hz, $c = 3 \times 10^8$ ms^{-1}

$\beta = 0.75$ mm $= 0.75 \times 1^{-3}$ m

Wavelength, $\lambda = \frac{c}{v} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7}$ m

Distance between the slits, $d = \frac{D\lambda}{\beta} = \frac{1.5 \times 5 \times 10^{-7}}{0.75 \times 10^{-3}} = 10^{-3}$ m = 1 mm

Q. 10. In a Young's double slit experiment, red light of wavelength 6000 Å is used and the nth bright fringe is obtained at a point P on the screen. Keeping the same setting, the source is replaced by green light of 5000 Å and now (n + 1)th bright fringe is obtained at the point P. Calculate the value of n.

Sol. Let x be the distance of point P from the centre of the screen.

When red light ($\lambda = 6000$ Å) is used, nth bright fringe is obtained at point P.

$\therefore x = \frac{n D\lambda}{d} = \frac{nD \times 6000 \times 10^{-10}}{d}$

When green light ($\lambda' = 5000$ Å) is used, (n + 1) th bright fringe is obtained at the same point P.

$\therefore \quad x = \frac{(n+1) D \lambda'}{d} = \frac{(n+1) D \times 5000 \times 10^{-10}}{d}$

Equating the two values of x, we get

$\frac{nD \times 6000 \times 10^{-10}}{d} = \frac{(n+1) D \times 5000 \times 10^{-10}}{d}$

or $6n = 5(n + 1)$

or $n = 5$

Q. 11. Two slits 0.125 mm apart are illuminated by light of wavelength 4500 Å. The screen is one metre away from the plane of the slits. Find the separation between the second bright fringes on both sides of the central maximum.

Sol. Here $d = 0.125$ mm $= 0.125 \times 10^{-3}$ m, $\lambda = 4500$ Å $= 4500 \times 10^{-10}$ m, D = 1 m

Distance of 2nd bright fringe from the central maximum is

$x_2 = \frac{2D\lambda}{d} = \frac{2 \times 1 \times 4500 \times 10^{-10}}{0.125 \times 10^{-3}} = 7.2 \times 10^{-3}$ m

Separation between the second bright fringes on both sides of central maximum is

$2x_2 = 2 \times 7.2 \times 10^{-3} = 14.4 \times 10^{-3}$ m = 14.4 mm.

Q. 12. In Young's double slit experiment, the slits are 0.2 mm apart and the screen is 1.5 m away. It is observed that the distance between the central bright fringe and fourth dark fringe is 1.8 cm. Find the wavelength of light used.

Sol. Here $d = 0.2$ mm $= 0.2 \times 10^{-3}$ m, D = 1.5 m $x_4' = 1.8$ cm $= 1.8 \times 10^{-2}$ m.

The distance of nth dark fringe from the central bright fringe is given by

$x'_n = (2n - 1)\frac{D\lambda}{2d}$,

$\therefore \quad x'_4 = \frac{7}{2} . \frac{D\lambda}{d}$

or $\lambda = \frac{2dx'_4}{7D} = \frac{2 \times 0.2 \times 10^{-3} \times 1.8 \times 10^{-2}}{7 \times 1.5} = 6.86 \times 10^{-7}$ m.

Q. 13. *In Young's double slit experiment, the slits are separated by 0.5 mm and screen is placed 1.0 m away. It is found that the ninth bright fringe is at a distance of 8.835 mm from the second dark fringe. Find the wavelength of light used.*

Sol. The distance of nth bright fringe from the central bright fringe is

$$x_n = \frac{nD\lambda}{d} = n\beta \quad \therefore \quad x_9 = 9\beta$$

The distance of nth dark fringe from the central bright fringe is

$$x_n' = (2n-1)\frac{D\lambda}{2d} = (2n-1)\frac{\beta}{2}$$

$\therefore$ $x_2' = \frac{3}{2}\beta$

But $x_9 - x_2' = 8.835$ mm [Given]

or $9\beta - \frac{3}{2}\beta = 8.835$ mm

or $\frac{15}{2}\beta = 8.835$ mm

or $\beta = \frac{8.835 \times 2}{15}$

$= 1.178$ mm $= 1.178 \times 10^{-3}$ m

Hence, $\lambda = \frac{\beta d}{D} = \frac{1.178 \times 10^{-3} \times 0.5 \times 10^{-3}}{1.0}$ m $= 0.5890 \times 10^{-6}$ m $= 5890$ Å.

Q. 14. *In a Young's double experiment, the slits are 1.5mm apart. When the slits are illuminated by a monochromatic light source and the screen is kept 1 m apart from the slits, width of 10 fringes is measured as 3.93 mm. Calculate the wavelength of light used. What will be the width of 10 fringes when the distance between the slits and the screen is increased by 0.5 m. The source of light used remains the same.*

Sol. In first case:

$d = 1.5$ mm $= 1.5 \times 10^{-3}$ m, $D = 1$ m

Width of 10 fringes= 3.93 mm

$\therefore$ Fringe width,

$\beta = \frac{3.93}{10} = 0.393$ mm $= 0.393 \times 10^{-3}$ m

Wavelength, $\lambda = \frac{\beta d}{D} = \frac{0.393 \times 10^{-3} \times 1.5 \times 10^{-3}}{1}$

$= 8.89 \times 10^{-7}$ m

In second case: $D' = 1 + 0.5 = 1.5$ m,

$d = 1.5 \times 10^{-3}$ m, $\lambda = 5.895 \times 10^{-7}$ m

Width of 10 fringes $= 10\beta' = \frac{10 D' \lambda}{D} = \frac{10 \times 1.5 \times 5.895 \times 10^{-7}}{1.5 \times 10^{-3}} = 5.895 \times 10^{-3}$ m.

Q. 15. *A double slit is illuminated by light of wavelength 6000 Å. The slits are 0.1 cm apart and the screen is placed 1 m away. Calculate (i) the angular position of 10^th^ maximum in radian and (ii) separation of the adjacent minima.*

Sol. (i) The angular position of nth maximum is given by

$$\theta_n = \frac{x_n}{D} = \frac{nD\lambda/d}{D} = \frac{n\lambda}{d}$$

$\therefore$ $\theta_{10} = \frac{10 \times 6000 \times 10^{-10}}{0.1 \times 10^{-2}} = 0.006$ rad

(ii) Separation between two adjacent minima i.e., fringe width,

$\beta = \frac{D\lambda}{d} = \frac{1 \times 6000 \times 10^{-10}}{0.1 \times 10^{-2}} = 6 \times 10^{-4}$ m $= 0.6$ mm.

Q. 16. *Sodium light has two wavelengths $\lambda_1 = 589$ nm and $\lambda_2 = 589.6$ nm. As the path difference increase, when is the visibility of the fringes minimum?*

Sol. The visibility of the fringes will be poorest when the path difference p is an integral multiple of λ_1 and a half integral multiple of λ_2. As p is increased, this happens first when

$$\frac{p}{\lambda_1} - \frac{p}{\lambda_2} = \frac{1}{2} \quad \text{or} \quad p\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{1}{2}$$

or $p = \frac{1}{2}\left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\right)$

Now, $\lambda_1 = 589$ nm $= 589 \times 10^{-9}$ m and $\lambda_2 = 589.6 \text{ nm} = 589.6 \times 10^{-9}$ m

$\therefore \quad p = \frac{1}{2} \cdot \frac{589 \times 10^{-9} \times 589.6 \times 10^{-9}}{(589.6 - 589) \times 10^{-9}}$

$= \frac{1}{2} \cdot \frac{347274.4 \times 10^{-9}}{0.6}$ m $= 289395.31 \times 10^{-6}$ mm $= 0.29$ mm.

Q. 17. ***In a Young's interference experimental arrangement, the incident yellow light is composed of two wavelength 5890 Å and 5895 Å. the distance between the two slits is 10^{-3} m and screen is placed 1 m away. Up to what order can fringes be seen on the screen and how far from the centre of the screen does this occur?***

Sol. The fringes can be seen on the screen up to an order at which bright fringe due to one wavelength coincides with the dark fringe due to another wavelength. That is,

$$n\lambda = \left(n \mp \frac{1}{2}\right)\lambda'$$

But $\lambda = 5890$ Å and $\lambda' = 5895$ Å

$\therefore \quad n \times 5890 = \left(n - \frac{1}{2}\right) 5895$

or $\quad \frac{5895}{2} = (5895 - 5890)\, n = 5\, n$

$\therefore \quad n = \frac{5895}{2 \times 5} = 589.5 \simeq 589$

The distance of 589th bright fringe from the centre is

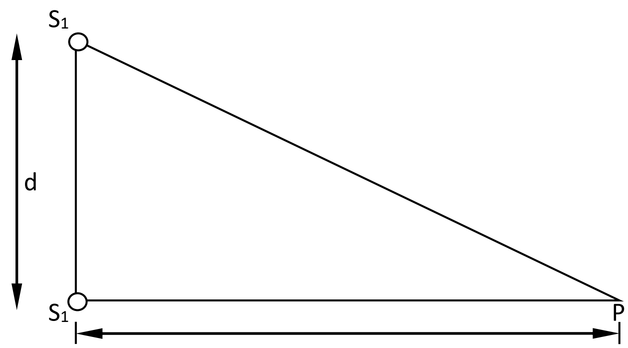
$$x = n\frac{D\lambda}{d}$$

$$= \frac{589 \times 1.0 \times 5890 \times 10^{-10}}{10^{-3}}$$

$= 0.347$ m.

Q. 18. ***Two sources S_1 and S_2 emitting light of wavelength 600 nm are placed 0.1 mm apart. A detector is moved on the line S_1P which is perpendicular to S_1S_2 (i) What would be the minimum and maximum path difference at the detector as it is moved along the line S_1P. (ii) Locate the position of farthest of minimum detected.***

Sol. (i) The situation is shown in Fig. The path difference is minimum when the detector is at large distance from S_1. Then the path difference is near is zero.



The path difference is maximum when the detector lies at point S_1.

$\therefore \quad$ Maximum path difference $= S_1 S_2 = 0.1$ mm.

(ii) The farthest minimum will occur at a point P for which the path difference is $\frac{\lambda}{2}$. Let $S_1P = D$. Then

$p = S_2P - S_1P = \frac{\lambda}{2} \quad$ or $\quad \sqrt{D^2 + d^2} - D = \frac{\lambda}{2}$

or $\quad D^2 + d^2 = \left(D + \frac{\lambda}{2}\right)^2 \quad$ or $\quad d^2 = D\lambda + \frac{\lambda^2}{4}$

or $\quad D = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(0.1 \times 10^{-3})^2}{600 \times 10^{-9}} - \frac{600 \times 10^{-9}}{4}$

$= \frac{1}{60} - 150 \times 10^{-9} \simeq \frac{1}{60}$ m $= 1.7$ cm

CONDITIONS FOR SUSTAINED INTERFERENCE

In order to observe an interference pattern, it is necessary that the positions of maxima and minima do not keep on changing with time, otherwise the maxima and minima of intensity will mix up to produce uniform illumination. The interference pattern, in which the positions of maximum and minima of intensity on the observation screen do not change with time, is called a sustained or permanent interference pattern.

Condition for sustained interference

1. The two sources should continuously emit waves of same frequency or wavelength.

2. The two sources of light should be coherent, i.e., they must vibrate either in the same phase or with a constant phase difference between them.

3. For a better contrast between maxima and minima of intensity, the amplitudes of the interfering waves should be equal.

4. The two sources should be narrow, otherwise interference will occur between waves of different parts of the same source and contrast will be poor.

5. The interfering waves must travel nearly along the same direction.

6. The sources should be monochromatic; otherwise fringes of different colours will overlap just to given a few observable fringes.

7. The interference waves should be in the same state of polarisation.

8. To have sufficient fringe width, the distance between the two coherent sources should be small and the distance between the two sources and the screen should be large.

INTENSITY DISTRIBUTION CURVE FOR INTERFERENCE

Suppose the two interfering waves have the same **amplitude a**.

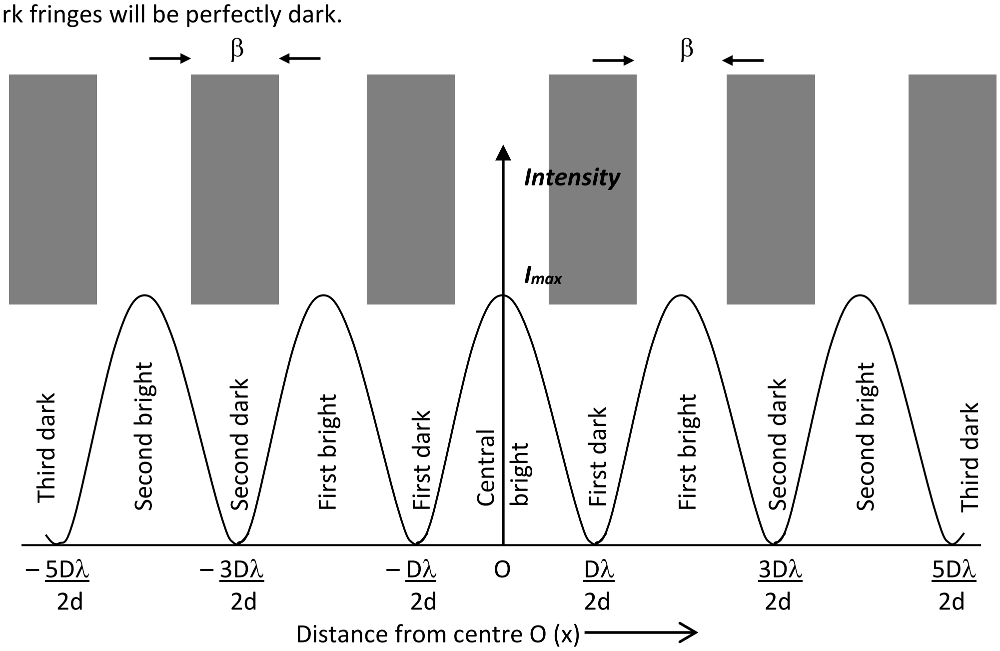
The intensity of a bright fringe will be

$$I_{max} = k\,(a + a)^2 = 4ka^2 = \text{constant}$$

So all bright fringes will have the same maximum intensity. The intensity of a dark fringe will be

$$I_{min} = k\,(a - a)^2 = 0$$

So all dark fringes will be perfectly dark.



[Intensity distribution curve]

On plotting the intensities of bright and dark fringes against distance x from O, we get a curve as shown in Fig. The intensity is maximum at the central point O. Then it becomes zero and maximum alternately on either side of O, depending on x is odd multiple of $\frac{D\lambda}{2d}$ and integral multiple of $\frac{D\lambda}{d}$ respectively.

CONSERVATION OF ENERGY IN INTERFERENCE.

In an interference pattern, the intensities at the points of maxima and minima are such that

$I_{max} \propto (a_1 + a_2)^2$ and $I_{min} \propto (a_1 - a_2)^2$

$\therefore \quad I_{av} \propto \frac{(a_1 + a_2)^2 + (a_1 - a_2)^2}{2}$ or $I_{av} \propto a_1^2 + a_2^2$

If there is no interference between the light waves from the two sources, then intensity at energy point would be same. That is,

$I = I_1 + I_2 \propto a_1^2 + a_2^2$

Which is same as I_{av} in the interference pattern. So there is no violation of the law of conservation of energy in interference. Whatever energy disappears from a dark fringe, an equal energy appears in a bright fringe.

COMPARISON OF INTENSITIES AT MAXIMA AND MINIMA

Let a_1 and a_2 be the amplitudes and I_1 and I_2 be the intensities of light waves from two different sources.

As Intensity $\propto$ Amplitude^2 $\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$

Amplitude at a maximum in interference pattern = $a_1 + a_2$

Amplitude at a minimum in interference pattern = $a_1 - a_2$

Therefore, the ratio of intensities at maxima and minima is

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{[a_1/a_2 + 1]^2}{[a_1/a_2 - 1]^2} = \left(\frac{r + 1}{r - 1}\right)^2$$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = amplitude ratio of the two waves.

- The intensity of light through a slit is proportional to its width, if $\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = r^2$

Examples based on Intensity Ratio at Maxima and Minima of an Interference Pattern

◆ FORMULA USED

1. Intensity of light $\propto$ Width of slit
2. Ratio of slit widths, $\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$
3. Intensity of maxima, $I_{max} \propto (a_1 + a_2)^2$.
4. Intensity of minima, $I_{min} \propto (a_1 - a_2)^2$.
5. Intensity ratio of maxima and minima, $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{r + 1}{r - 1}\right)^2$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = amplitude ratio of the two waves.

◆ UNITS USED

Ratios I_1/I_2. ω_1/ω_2 and I_{max}/I_{min} have no units.

Q. 1. What is the ratio of slit widths if the amplitudes of light waves from them have a ratio of √2 : 1.

Sol. Width ratio,

$$\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \left(\frac{\sqrt{2}}{1}\right)^2 = 2 : 1$$

Q. 2. Two coherent sources have intensities in the ratio 25:16. Find the ratio of intensities of maxima to minima, after interference of light occurs.

Sol. Amplitude ratio,

$$r = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Q. 3. If the two slits in Young's double-slit experiment have width ratio 4 : 1, deduce the ratio of intensity at maxima and minima in the interference pattern.

Sol. Amplitude ratio of the interfering waves,

$$r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{\omega_1}{\omega_2}} = \sqrt{\frac{4}{1}} = 2$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = \frac{9}{1} = 9:1$$

Q. 4. ***In Young's double slit experiment, the ratio of intensity at the maxima and minima in the interference experiment is 25 : 9. What will be the ratio of widths of the two slits?***

Sol. $\frac{I_{max}}{I_{min}} = \frac{25}{9}$ or $\left(\frac{r+1}{r-1}\right)^2 = \frac{25}{9}$

or $\frac{r+1}{r-1} = \frac{5}{3}$ or $3r + 3 = 5r - 5$

or $r = 4 = \frac{a_1}{a_2}$, the amplitude ratio

∴ Width ratio of two slits, $\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{16}{1} = 16 : 1$

Q. 5. ***Two coherent sources of light of intensity ratio β interfere. Prove that in the interference pattern,***

$$\frac{I_{max} - I_{min}}{I_{max} - I_{min}} = \frac{2\sqrt{\beta}}{1 + \beta}$$

Sol. Here $\beta = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$ or $\frac{a_1}{a_2} = \sqrt{\beta}$

As $I_{max} = k\,(a_1 + a_2)^2$ and $I_{min} = k\,(a_1 - a_2)^2$

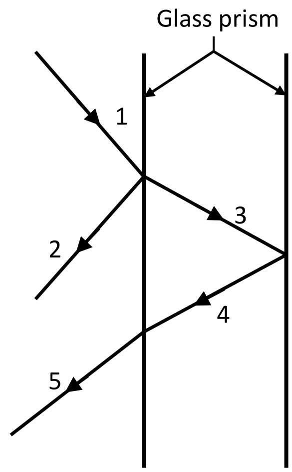
∴ $\frac{I_{max} - I_{min}}{I_{max} - I_{min}} = \frac{k\,(a_1 + a_2)^2 - k\,(a_1 - a_2)^2}{k\,(a_1 + a_2)^2 + k\,(a_1 - a_2)^2}$

$= \frac{(a_1 + a_2)^2 - (a_1 - a_2)^2}{(a_1 + a_2) + (a_1 - a_2)^2}$

$= \frac{4a_1 a_2}{2\,(a_1^2 + a_2^2)} = \frac{2\,(a_1/a_2)}{[a_1^2/a_2^2 + 1]} = \frac{2\sqrt{\beta}}{\beta + 1}$

Q. 6. ***A narrow monochromatic beam of light of intensity I is incident on a glass plate. Another identical glass plate is kept close to first one and parallel to it. Each plate reflects 25 % of the incident light and transmits the remaining. Calculate the ratio of minimum and maximum intensity in the interference pattern formed by two beans obtained after reflection from each plate.***

Sol. Let I be the intensity of beam 1 incident on first glass plate. Each plate reflects 25% of light incident on it and transmits 75%. Therefore,



$I_1 = I$; $I_2 = \frac{25}{100} I = \frac{I}{4}$

$I_3 = \frac{75}{100} I = \frac{3}{4} I$

$I_4 = \frac{25}{100} I_3 = \frac{1}{4} \times \frac{3}{4} I = \frac{3}{16} I$

$I_5 = \frac{75}{100} I_4 = \frac{3}{4} \times \frac{3}{16} I = \frac{9}{64} I$

∴ Amplitude ratio of beams 2 and 5 is

$r = \sqrt{\frac{I_2}{I_5}} = \sqrt{\frac{I \times 64}{4 \quad 9I}} = \frac{4}{3}$

$\frac{I_{min}}{I_{max}} = \left(\frac{r-1}{r+1}\right)^2 = \left(\frac{4/3 - 1}{4/3 + 1}\right)^2 = \frac{1}{49} = 1 : 49$

INTERFERENCE PATTERN WITH WHITE LIGHT

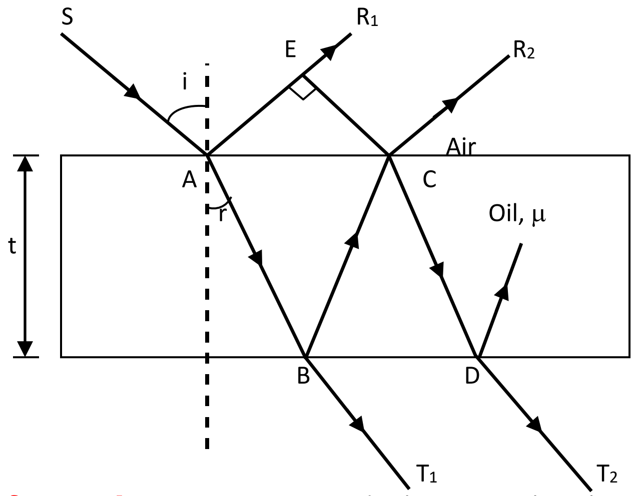
White light consists of colours from violet to red with wavelength range from 4000 Å to 7000 Å. Different component colours of white light produce their own interference pattern. At the centre of the screen, the path difference is zero for all such components. So bright fringes of different colours overlap at the centre. Consequently, the central fringe is white.

Now fringe width $\beta = D\lambda/d$ i.e., $\beta \propto \lambda$. Since the violet colour has the lowest λ, the closest fringe on either side of the central while fringe is violet, while the farthest fringe is red. After a few fringes, the interference pattern is lost due to large overlapping of the fringes and uniform white illumination is seen on the screen.

INTERFERENCE IN THIN FILMS

A thin film means an extremely small thickness of a transparent medium. A soap film or a thin film of oil spread over water, when seen in the reflected white light, shows beautiful colours. This is due to the interference between the light waves reflected by a upper and lower surface of thin films. As they both originate from the same source, they are coherent waves.

As shown in Fig. consider a parallel sided thin film of thickness t and refractive index μ. Suppose a ray SA of monochromatic light is incident on its upper surface. This ray suffers partial reflections and refractions successively at points A, B, C etc; giving a set of parallel reflected rays R_1, R_2, ... and a set of parallel transmitted rays T_1, T_2, ... When these rays are focussed by our eye lens, interference patterns are visible.



[Interference in thin films]

Interference in reflected light: Draw CE perpendicular to AR_1. Then the path difference between two successive reflected rays R_1 and R_2 is

$$p = (AB + BC) \text{ in thin film} - AE \text{ in air}$$
$$= \mu (AB + BC) \text{ in air} - AE \text{ in air}$$

or $p = 2\mu t \cos r$ **[From the geometry of the figure]**

where r is the angle of refraction. As the ray R_1 is reflected by the upper surface of thin film (denser medium), it suffers an extra path difference of $\lambda/2$.

$\therefore$ Net path difference $= 2\mu t \cos r + \frac{\lambda}{2}$

For a bright fringe: $2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

or $2\mu t \cos r = (2n - 1)\frac{\lambda}{2}$

or $2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$, $n = 0, 1, 2, 3, \ldots\ldots$

For a drink fringe: $2\mu t \cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$

or $2\mu t \cos r = 2\lambda$, $n = 0, 1, 2, 3, \ldots$

Interference in transmitted light: As the transmitted rays do not suffer any reflection from the surface of a denser medium, the path difference between any two successive rays will be

$$p = 2\mu t \cos r$$

$\therefore$ For a bright fringe, $2\mu t \cos r = n\lambda$

For a dark fringe, $2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$

Obviously, the conditions for maxima and minima in the reflected system are just opposite to those for the transmitted system. Thus, the reflected and transmitted systems are complimentary, i.e, a film which appears by reflected light, will appear dark by transmitted light and vice versa.

Colours in thin films: When a thin film is seen with monochromatic light, we find alternate bright and dark fringes. But with white light, brilliant colours are seen. This is because the path difference $2\mu t \cos r$ between any two successive rays depends on μ, t and r. For a particular part of the thin film and for a particular position of the eye, t and r are fixed. But μ varies with the wavelength of light. Different constituent of white light has different wavelengths (λ varying from 4000 Å to 7500 Å), so the conditions for maxima and minima for different constituents occur at different points of thin film. For example, if at some place $2\mu t \cos r$ equals 1λ for red (7500 Å), it will be 1.5λ for blue (5000 Å), so that at this place blue colour is best reflected, red colour is not reflected at all and the intermediate colours have intermediate contributions. Clearly, at each place there is a mixture of colours, and the composition of this mixture is different at different places. As a result, the reflected light shows various beautiful shades.

Examples based on Interference in Thin Film

◆ **FORMULA USED**

1. For reflected system of light,
   (i) Maxima: $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$
   (ii) Minima: $2\mu t \cos r = n\lambda$
2. For transmitted system of light,
   (i) Maxima: $2\mu t \cos r = n\lambda$
   (ii) Minima: $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

where n = 0, 1, 2, 3, ...

◆ **UNITS USED**

Thickness t and wavelength λ are in metre, μ and n have no units.

Q. 1. ***White light may be considered to have λ from 4000 Å to 7500 Å. If an oil film has thickness 10^{-6} m, deduce the wavelengths in the visible region for which the reflection along the normal direction will be (i) weak, (ii) strong. Take μ of the oil as 1.40.***

Sol. The condition for dark fringe or weak reflection when seen in reflected light is $2\mu t \cos r = n\lambda$, where n is an integer.

For normal incidence, $r = 0$ and $\cos r = 1$

so that $2\mu t = n\lambda$ or $\lambda = 2\mu t/n$

Substituting the values of μ and t, we get

$$\lambda = \frac{2 \times 1.4 \times 10^{-6}}{n} = \frac{28 \times 10^{-7}}{n}\text{ m}$$

For values of n < 4 or > 7, the values of λ do not lie in the visible range 4000 Å to 7500 Å. But for values of n = 4, 5, 6, 7, the following wavelengths lie in the visible region:

(i) $\lambda = \frac{28 \times 10^{-7}}{4} = 7.0 \times 10^{-7}$ m = 7000 Å

(ii) $\lambda = \frac{28 \times 10^{-7}}{5} = 5.6 \times 10^{-7}$ m = 5600 Å

(iii) $\lambda = \frac{28 \times 10^{-7}}{6} = 4.667 \times 10^{-7}$ m = 4667 Å

(iv) $\lambda = \frac{28 \times 10^{-7}}{7} = 4.0 \times 10^{-7}$ m = 4000 Å

The condition for bright fringe or strong reflection is

$$2\mu t = \frac{(2n+1)\lambda}{2} \quad \text{or} \quad \lambda = \frac{4\mu t}{(2n+1)}$$

Substituting the values of μ and t, we get

$$\lambda = \frac{4 \times 1.4 \times 10^{-6}}{2n+1} = \frac{56 \times 10^{-7}}{2n+1}\text{ m}$$

For values of n < 4 or > 6, the values of λ do not lie in the visible range. But for n = 4, 5, 6 the following wavelengths lie in the visible range:

(i) $\lambda = \frac{56 \times 10^{-7}}{2 \times 4 + 1} = 6.222 \times 10^{-7}$ m = 6222 Å

(ii) $\lambda = \frac{56 \times 10^{-7}}{2 \times 5 + 1} = 5.091 \times 10^{-7}$ m = 5091 Å

(iii) $\lambda = \frac{56 \times 10^{-7}}{2 \times 6 + 1} = 4.308 \times 10^{-7}$ m = 4308 Å

Q. 2. ***In a certain region of a wedge-shaped film, 10 fringes are observed with a light source of wavelength 4358 Å. If the wave-length of the light source is changed to 5893 Å, then how many fringes will be observed in the same region of the film?***

Sol. Let n_1 and n_2 be the number of fringes observed between the points A and B corresponding to wavelengths λ_1 and λ_2. If the thickness of the film changes by Δt between A and B, then for normal incidence

$$2\mu\Delta t = n_1\lambda_1 = n_2\lambda_2$$

$\therefore$ $n_2 = \frac{n_1\lambda_1}{\lambda_2}$

But $n_1 = 10$, $\lambda_1 = 4358$ Å, $\lambda_2 = 5893$ Å

$\therefore$ $n_2 = \frac{10 \times 4358}{5893} = 7.4$ fringes

Q. 3. ***For light of wavelength $\lambda = 6.0 \times 10^{-7}$ m, it is found that in a thin film of air, 9 fringes occur between two points. Deduce the difference of film thickness between these points.***

Sol. The conditions for two maxima at different thicknesses of the thin film (for normal incident) may be written as

$$2\mu t_1 = (2n_1 + 1)\frac{\lambda}{2} \text{ and } 2\mu t_2 = (2n_2 + 1)\frac{\lambda}{2}$$

∴ $2\mu (t_2 - t_1) = (n_2 - n_1)\lambda$

Here $\lambda = 6.0 \times 10^{-7}$, $(n_2 - n_1) = 9$ and for air, $\mu \simeq 1$.

∴ Different of film thickness

$$\Delta t = t_2 - t_1 = \frac{(n_2 - n_1)\lambda}{2\mu} = \frac{9 \times 6.0 \times 10^{-7}}{2 \times 1} \text{ m} = 2.7 \times 10^{-6} \text{ m} = 2.7 \text{ microns.}$$

Q. 4. ***A parallel beam of sodium light of wavelength 5890 Å is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection.***

Sol. Here $\lambda = 5890$ Å $= 5890 \times 10^{-10}$ m, $\mu = 1.5$, $r = 60°$

The condition for minimum thickness corresponding to a dark band is

$2\mu t \cos r = \lambda$

∴ Required thickness, $t = \dfrac{\lambda}{2\mu \cos r} = \dfrac{5890 \times 10^{-10}}{2 \times 1.5 \times \cos 60°}$ m $= 3928.7 \times 10^{-10}$ m $= 3928$ Å

Q. 5. ***A soap film is illuminated by white light incident at an angle of 30°. The reflected light is examined by a spectroscope in which a dark band corresponding to wavelength 6000 Å is found. Calculate the minimum thickness of the film. Given refractive index of film = 4/3***

Sol. Here $i = 30°$, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m, $\mu = 4/3$

From Snell's law, $\sin r = \dfrac{\sin i}{\mu} = \dfrac{1/2}{4/3} = \dfrac{3}{8}$

∴ $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \dfrac{9}{64}} = \sqrt{\dfrac{55}{64}} = 0.927$

The condition for minimum thickness corresponding to a dark band is $2\mu t \cos r = \lambda$

∴ Required thickness, $t = \dfrac{\lambda}{2\mu \cos r} = \dfrac{6 \times 10^{-7}}{2 \times 4/3\ 0.927}$ m $= 2.42 \times 10^{-7}$ m

Q. 6. ***White light reflected at perpendicular incidence from a soap film has, in the visible spectrum, an interference maximum at 6000 Å and a minimum at 4500 Å with no minimum in between. If $\mu = 4/3$ for the film, what is the film thickness?***

Sol. Here $\lambda_1 = 6000$ Å $= 6 \times 10^{-7}$ m, $\lambda_2 = 4500$ Å $= 4.5 \times 10^{-7}$ m, $\mu = 4/3$

For normal incidence, the condition for (n + 1) th maximum is

$$2\mu t = [n + ½]\lambda_1 \quad \text{or} \quad n + ½ = \frac{2\mu t}{\lambda_1} \quad \text{... (i)}$$

The condition for (n + 1)th minimum is

$$2\mu t = (n + 1)\lambda_2 \quad \text{or} \quad n + 1 = \frac{2\mu t}{\lambda_2} \quad \text{... (ii)}$$

Subtracting (i) from (ii), we get

$$½ = 2\mu t\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = 2\mu t\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$$

∴ $t = \dfrac{\lambda_1 \lambda_2}{4\mu(\lambda_1 - \lambda_2)} = \dfrac{6 \times 10^{-7} \times 4.5 \times 10^{-7}}{4 \times 4/3 \times (6 - 4.5)\, 10^{-7}} = \dfrac{27}{8} \times 10^{-5}$ m $= 3.375 \times 10^{-5}$ m

Q. 7. ***A soap film of $\mu = 4/3$ is illuminated by white light incident at an angle of 45°. The transmitted light is examined by spectroscope and bright fringe is found to be for wavelength of 6000 Å. Find the minimum thickness of the film.***

Sol. Here $\mu = 4/3$, $i = 45°$, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m,

As $\mu = \dfrac{\sin i}{\sin r}$ ∴ $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 45°}{4/3} = \dfrac{3}{4\sqrt{2}}$

$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \dfrac{9}{32}} = \sqrt{\dfrac{23}{32}} = 0.8478$

For a bright fringe in transmitted light, $2\mu t \cos r = n\lambda$

For minimum thickness, n = 1

∴ $2\mu t \cos r = \lambda$

or $t = \dfrac{\lambda}{2\mu \cos r} = \dfrac{6 \times 10^{-7}}{2 \times 4/3 \times 0.8478} = 2.6 \times 10^{-7}$ m

Q. 8. ***White light is incident on a soap film at angle of $\sin^{-1} 4/5$ and the reflected light on examination by the spectroscope shows dark bands. the consecutive dark bands correspond to wavelengths 6100 Å and 6000 Å. If the refractive index of the film is 4/3, calculate its thickness.***

Sol. Here $i = \sin^{-1} 4/5 \quad \therefore \quad \sin i = 4/5$

As $\mu = \frac{\sin i}{\sin r} \quad \therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = \frac{3}{5} = 0.6$

$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.6)^2} = 0.8$

For a dark fringe in the reflected light,

$2\,\mu t \cos r = n\lambda \qquad n = 0, 1, 2, 3, \ldots$

Suppose nth and (n + 1) th dark bands correspond to wavelengths 6100 Å and 6000 Å respectively. Then

In first case,

$$2 \times 4/3 \times t \times 0.8 = n \times 6100 \times 10^{-10} \qquad \ldots \text{(i)}$$

In second case,

$$2 \times 4/3 \times t \times 0.8 = (n + 1) \times 6000 \times 10^{-10}$$

$\therefore \quad n \times 6100 \times 10^{-10} = (n + 1) \times 6000 \times 10^{-10}$

or $\quad n = 60$

Putting the value of n in equation (i), we get

$2 \times 4/3 \times t \times 0.8 = 60 \times 6100 \times 10^{-10}$ or $t = 1.716 \times 10^{-5}$ m.

DISPLACEMENT OF INTERFERENCE FRINGES

When a thin transparent sheet of thickness t and refractive index μ is inserted in the path of one of the interfering beams, the extra path difference introduced is

Δp = Length t in transparent medium – Length t in air $= \mu t - t = (\mu - 1)\, t$

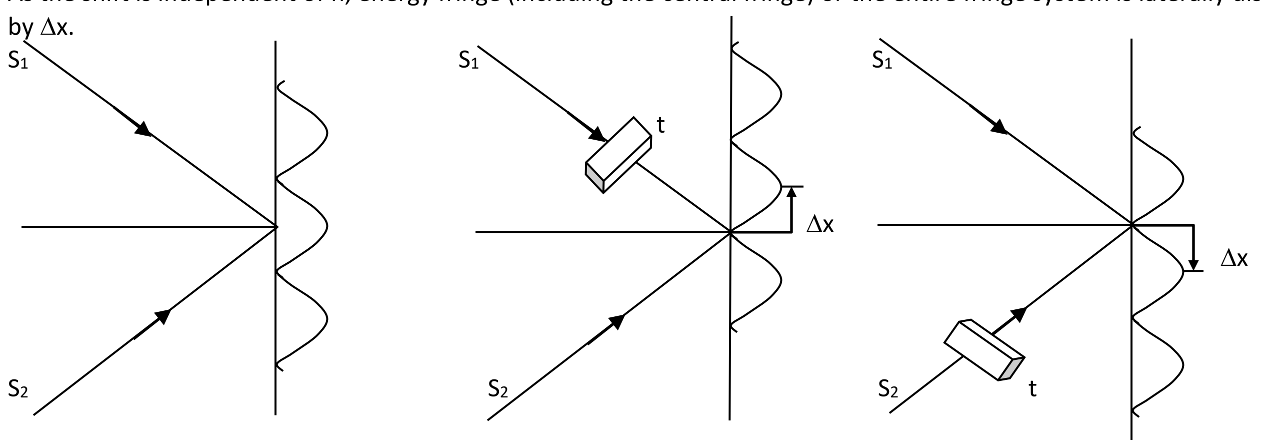
Suppose the present position of the particular fringe is $x = \frac{Dp}{d}$

Then the new position of the same fringe will be $x' = \frac{D}{d}(p + \Delta p)$

Hence the lateral displacement of the particular fringe on the screen is $\Delta x = x' - x = \frac{D\,\Delta p}{d}$

or $\quad \Delta x = \frac{D}{d}(\mu - 1)\, t = \frac{\beta}{\lambda}(\mu - 1)\, t \qquad \left(\beta = \frac{D\lambda}{d} \quad \therefore \quad \frac{D}{d} = \frac{\beta}{\lambda}\right)$

As the shift is independent of n, energy fringe (including the central fringe) or the entire fringe system is laterally displaced by Δx.



[Displacement of interference fringes due to the insertion of a thin glass sheet]

As shown in Fig. the entire fringe system is shifted towards that side in which the thin transparent sheet is introduced. But there is no change in the fringe width.

Examples based on Displacement of Interference Fringes

◆ **FORMULA USED**

1. When a thin transparent sheet of thickness t and refractive index μ is inserted in one of the interfering beams, path difference introduced, $\quad p = (\mu - 1)\, t$

2. Displacement of the central bright fringe,

$$\Delta x = \beta\,(\mu - 1)\, t = D\,(\mu - 1)\, t$$

◆ **UNITS USED** Fringed width β, wavelength λ, thickness t and distances D, d are in metre.

Q. 1. ***Fringes are produced with monochromatic light of wavelength 5.45 × 10^{-5} cm. A thin glass plate of refractive index 1.5 is then placed normally in the path of one of the interference beams and the central bright band of the fringe system is found to move into the position previously occupied by the third bright band from the system. Find the thickness of the glass plate.***

Sol. Extra path difference introduced due to the insertion of glass plate,

p = Length t in glass plate – Length t in air = $\mu t - t = t(\mu - 1)$

As the central bright fringe shifts into the position of third bright fringe, therefore,

$(\mu - 1)\, t = 3\lambda$

∴ Thickness, $t = \dfrac{3\lambda}{\mu - 1} = \dfrac{3 \times 5.45 \times 10^{-5}}{1.5 - 1}$ cm $= 32.7 \times 10^{-5}$ cm.

Q. 2. ***A two slit young's interference experiment is done with monochromatic light of wavelength 6000 Å. The slits are 2 mm apart and fringes are observed on a screen placed 10 cm away from the slits and it is found that the interference pattern shifts by 5 mm, when a transparent plate of thickness 0.5 mm is introduced in the path of one of the slits. What is the refractive index of the transparent plate?***

Sol. Extra path difference introduced due to insertion of glass plate of thickness t,

$p = (\mu - 1)\, t$

∴ By using, $(\mu - 1)\, t = n\lambda = n \cdot \dfrac{\beta d}{D}$ $\left(\because \beta = \dfrac{D\lambda}{d}\right)$

We get the fringe shift, $\Delta x = n\beta = \dfrac{D}{d}(\mu - 1)\, t$

Now $d = 2$ mm $= 2 \times 10^{-3}$ m, $D = 10$ cm $= 0.1$ m, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m, $\Delta x = 5$ mm $= 5 \times 10^{-3}$ m, $t = 0.5$ mm $= 0.5 \times 10^{-3}$ m

∴ $\mu - 1 = \dfrac{\Delta x \cdot d}{D \cdot t} = \dfrac{5 \times 10^{-3} \times 2 \times 10^{-3}}{0.1 \times 0.5 \times 10^{-3}} = 0.2$

Hence $\mu = 1.2$

Q. 3. ***Monochromatic light of wavelength 600 nm is used in a Young's double slit experiment. One of the slits is covered by a transparent sheet of thickness 1.8 × 10^{-5} m made of a material of refractive index 1.6. How many fringes will shift due to the introduction of the sheet?***

Sol. Extra path difference introduced due to insertion of glass plate of thickness t,

$p = (\mu - 1)\, t$

If the insertion of glass plate causes shift of n bright fringes, then $p = (\mu - 1)\, t = n\lambda$

∴ $n = \dfrac{(\mu - 1)\, t}{\lambda} = \dfrac{(1.6 - 1) \times 1.8 \times 10^{-5}}{600 \times 10^{-9}} = 18$

Q. 4. ***A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1.0 mm, and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wave-length in air is 6300 Å. (i) Calculate the fringe-width (ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet of bring the adjacent minimum on the axis.***

Sol. Here $\mu_l = 1.33$, $d = 1.0$ mm $= 10^{-3}$ m; $D = 1.33$ m, $\lambda = 6300$ Å; $\mu_g = 1.53$

(i) The wavelength of light in the liquid is

$\lambda_l = \dfrac{\lambda}{\mu_l} = \dfrac{6300 \times 10^{-10}}{1.33}$ m

Fringe width, $\beta = \dfrac{D\lambda_l}{d} = \dfrac{1.33 \times 6300 \times 10^{-10}}{10^{-3} \times 1.33}$ m $= 6.3 \times 10^{-4}$ m $= 0.63$ mm.

(ii) When one of the slits is covered by a glass sheet of thickness t, the fringe-displacement is given by

$\Delta x = \dfrac{D}{d}({}^{l}\mu_g - 1)\, t = \beta\left(\dfrac{\mu_g}{\mu_l} - 1\right) t$ $\left(\because \beta = \dfrac{D\lambda_1}{d}\right)$

But on covering one of the slits, the adjacent minimum gets shifted to the centre, so the fringe displacement is half the fringe-width i.e., $\Delta x = \beta/2$.

∴ $\dfrac{\beta}{2} = \dfrac{\beta}{\lambda_l}\left(\dfrac{\mu_g}{\mu_l} - 1\right) t$

or $t = \dfrac{\lambda_l}{2\left(\dfrac{\mu_g}{\mu_l} - 1\right)} = \dfrac{(6300/1.33)\text{ Å}}{2\left(\dfrac{1.53}{1.33} - 1\right)} = \dfrac{6300}{2 \times 0.20}$ Å

$= 15750$ Å