

THERMAL PROPERTIES

OF MATTERS

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THERMAL PROPERTIES OF MATTER



Heat Transfer Fundamentals: Delve into the principles of conduction, convection, and radiation. Understand how heat energy moves through solids, liquids, and gases, shaping the thermal behavior of matter.



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Problem-solving Practice: Reinforce your learning through a variety of problem-solving exercises. Apply your newfound knowledge to solve problems related to heat transfer, expansion, and thermodynamic processes.

- Heat:** Heat is a form of energy which produces in us the sensation of hotness or coldness. According to dynamic theory, heat may be regarded as the energy of molecular motion which is equal to the sum total of the kinetic energy possessed by the molecules by virtue of their translational, vibrational and rotational motions.
- Units of heat:** Calorie (cal) is the C.G.S. unit of heat. One calorie is defined as the heat energy required to raise the temperature of one gram of water from 14.5°C to 15.5°C. Like all other forms of energy, the S.I. unit of heat of joule.
 $1 \text{ calorie} = 4.186 \text{ joule.}$
- Joule's mechanical equivalent of heat:** Whenever a given amount of work (W) is converted into heat, always the same amount of heat Q is produced.

$$\text{Thus } W \propto Q \quad \text{or} \quad W = JQ \quad \text{or} \quad J = \frac{W}{Q}$$

The proportionality constant J is called Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.

$$J = 4.2 \times 10^7 \text{ erg Cal}^{-1} = 4.2 \text{ J Cal}^{-1}$$

- Temperature:** It is the degree of hotness of a body. The temperature of a body gives a measure of the average kinetic energy of its molecules.
- Thermometer:** It is device used to measure the temperature of body. It makes use of some measurable property (called thermoelectric property) of a substance which changes linearly with temperature.
- Different temperature scales:**

Temperature Scales	Lower fixed point (Melting point of ice)	Upper fixed point (Boiling point of water)
1. Celsius	0°C	100°C
2. Fahrenheit	32°F	212°F
3. Reaumer	0°R	80°R
4. Kelvin	273.15 K	373.15 K

- Relations between different temperature scales:** If T_C , T_F , T_R and T are the temperatures of a body on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$(i) \frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{373.15 - 273.15}$$

$$\text{or } \frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$$

$$(ii) T_C = \frac{5}{9} (T_F - 32), \quad T_F = \frac{9}{5} T_C + 32$$

$$(iii) T = T_C + 273.15, \quad T_C = T - 273.15$$

$$(iv) T_F = \frac{9}{5} (T - 273.15) + 32 = \frac{9}{5} T - 459.67$$

$$T = \frac{5}{9} T_F + 255.37$$

- Absolute scale of temperature:** The lowest possible temperature of -273.15°C at which a gas is supposed to have zero volume (and zero pressure) and at which entire molecular motion stops is called absolute zero of temperature. The temperature scale which starts with -273.15°C as its zero is called Kelvin scale or absolute scale. The size of degree on Kelvin scale is same as that on Celsius scale.
- Triple point of water:** The triple point of water is the state at which the three phases of water namely ice liquid water and water vapour are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column.
- Constant volume air thermometer:** It is used to measure the pressure of a definite mass of air at different temperatures, the volume of air remaining constant. It is based on the pressure law,

$$\frac{P}{T} = \frac{P_0}{T_0} \quad \text{or} \quad T = T_0 \times \frac{P}{P_0}$$

$$\text{In terms of triple point of water, } T = T_{tr} \times \frac{P}{P_{tr}}$$

- Ideal gas temperature scale:** The ideal gas temperature on the Kelvin scale is defined by the equation

$$T = \lim_{P_{tr} \rightarrow 0} 273.16 \left(\frac{P}{P_{tr}} \right)$$

It is independent of the nature of the gas.

- Platinum resistance thermometer:** It is based on the fact that resistance of a platinum wire varies with temperature. If R_0 and R are the resistances at 0°C and $t^\circ \text{C}$ respectively, then

$$R = R_0 (1 + \alpha t)$$

Here α is the temperature coefficient of resistance of platinum and is defined as the increase in resistance per unit resistance at 0°C for 1°C rise in temperature.

$$\alpha = \frac{R - R_0}{R_0 \times t}$$

Unit of α is $^{\circ}\text{C}^{-1}$

If R_0 and R_{100} are the resistance of a platinum wire at ice point and steam point respectively, then the temperature t_R of a body for which the corresponding resistance is R_t is given by

$$t_R = \frac{R_t - R_0}{R_{100} - R_0} \times 100^{\circ}\text{C}$$

- 13. Thermoelectric thermometer:** It uses thermoelectric e.m.f. (\mathcal{E}) as thermometric property. For the linear part of the thermo e.m.f., the unknown temperature is given by $T_{\mathcal{E}} = \frac{\mathcal{E}_t - \mathcal{E}_0}{\mathcal{E}_{100} - \mathcal{E}_0} \times 100$ degrees
- 14. Linear expansion:** When a solid rod of initial length l is heated through a temperature ΔT , its final (increased) length is given by $l' = l(1 + \alpha\Delta T)$
 where α is coefficient of linear expansion. It is given by $\alpha = \frac{l' - l}{l \times \Delta T}$
 The coefficient of linear expansion of the material of solid rod is defined as the increased in length per unit length per degree rises in temperature.
- 15. Superficial expansion:** When a solid sheet of initial surface area S is heated through a temperature ΔT , its final (increased) surface area is given by $S' = S(1 + \beta\Delta T)$
 where β is coefficient of superficial expansion. It is given by $\beta = \frac{S' - S}{S \times \Delta T}$
 It is defined as the increase in surface area per unit surface area per degree rise in temperature.
- 16. Cubical expansion:** When a solid of initial volume V is heated through a temperature ΔT , its final (increased) volume is given by $V' = V(1 + \gamma\Delta T)$ where γ is coefficient of cubical expansion. It is given by $\gamma = \frac{V' - V}{V \times \Delta T}$
 It is defined as the increased in volume per unit volume per degree rise in temperature.
 The coefficient of cubical expansion of an ideal gas is equal to the reciprocal of its absolute temperature $\gamma = \frac{1}{T}$
- 17. Relation between α , β and γ :** The three coefficients of thermal expansion are related as $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$ or $\beta = 2\alpha$ and $\gamma = 3\alpha$.
- The units of α , β and γ are same viz. $^{\circ}\text{C}^{-1}$ or K^{-1} .
- 18. Coefficient of apparent expansion of a liquid:** It is defined as the apparent increase in volume per unit original volume per degree rise in temperature.

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$
- 19. Coefficient of real expansion of a liquid:** It is defined as the real increase in volume per unit original volume per degree rise in temperature.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

 If γ_g is the coefficient of cubical expansion of glass (material of the container), then $\gamma_r = \gamma_a + \gamma_g$
- 20. Variation of density with temperature:** The density of solid or liquid decreases with the increase of temperature.

$$\rho' = \rho(1 - \gamma\Delta T)$$
- 21. Specific heat:** It may be defined as the amount of heat required to raise the temperature of unit mass of a substance through one degree. If Q heat is needed to raise the temperature of m mass of a substance through ΔT , then specific heat is $c = \frac{Q}{m\Delta T}$
 The cgs unit of specific heat is $\text{Cal g}^{-1} ^{\circ}\text{C}^{-1}$ and the SI unit is $\text{J kg}^{-1} \text{K}^{-1}$. Clearly,
 Heat gained or lost, $Q = mc\Delta T$
- 22. Molar specific heat:** It is defined as the amount of heat required to raise the temperature of 1 mole of the substance through one degree. If Q heat is needed to raise the temperature of n moles of a substance through ΔT , then molar specific heat is $C = \frac{Q}{n\Delta T}$
 The cgs unit of molar specific heat is $\text{Cal mol}^{-1} ^{\circ}\text{C}^{-1}$ and the SI unit is $\text{J mol}^{-1} \text{K}^{-1}$.
- 23. Heat capacity or thermal capacity:** It is defined as the amount of heat required to raise the temperature of a body through one degree.
 Heat capacity = Mass \times Specific heat = mc
 The cgs unit of heat capacity is $\text{Cal}^{\circ} \text{C}^{-1}$ and the S.I. unit is JK^{-1} .

24. **Water equivalent:** The water equivalent of a body is defined as the mass of water which requires the same amount of heat as is required by the given body for the same rise of temperature.
 $w = \text{Mass} \times \text{specific heat} = mc$ □□□□ The cgs unit of water equivalent is g and the SI unit is kg.
25. **Latent heat:** The amount of heat required to change the state of unit mass of a substance at a constant temperature is called its latent heat. It is denoted by L .
26. **Latent heat of fusion:** The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion.
27. **Latent heat of vaporization:** The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point is called latent heat of vaporization.

28. **Principle of calorimetry:** When two bodies at different temperatures are placed in contact with each other, the heat lost by the hot body is equal to the heat gained by the cold body. This is the principle of calorimetry or the principle of mixtures.

$$\text{Heat gained} = \text{Heat lost}$$

29. **Transfer of heat:** The three modes of transfer of heat are conduction, convection and radiation.
30. **Conduction:** It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.
31. **Steady state:** The state of the rod when temperature of every cross-section of the rod becomes constant and there is no further absorption of heat in any part is called steady state.
32. **Factors on which conduction of heat depends:** When two opposite faces of a slab each of area of cross-section A and separated by a distance x are maintained at temperatures T_1 and T_2 ($T_1 > T_2$), then amount of heat that flows in time t ,

$$Q = \frac{KA(T_1 - T_2)t}{x}$$

where K is coefficient of thermal conductivity of the material of the slab between its two faces. The rate of flow of heat through the slab is

$$\frac{dQ}{dt} = -KA \frac{dT}{dx}$$

Here dT/dx is the rate of fall of temperature with distance and is called temperature gradient.

33. **Coefficient of thermal conductivity:** It may be defined as the quantity of heat energy that flows in unit time between the opposite faces of a cube of unit side, the faces being kept at one degree difference of temperature. Its cgs unit is $\text{cal s}^{-1} \text{cm}^{-1} \text{°C}^{-1}$ and SI unit is $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ or $\text{W m}^{-1} \text{K}^{-1}$.

Dimensional formula of K is $[\text{MLT}^{-3} \text{K}^{-1}]$.

Thermal formula of K is $[\text{MLT}^{-3} \text{K}^{-1}]$.

Thermal conductivities of metals are much greater than those for metals. Gases are poor thermal conductors.

34. **Heat current and resistance:** The flow of heat per unit time in conduction is called heat current. The ratio of the temperature difference between the ends of a conductor to the heat current through it is called thermal resistance.

Heat current, $H = \frac{Q}{t} = KA \frac{\Delta T}{\Delta x}$

Thermal resistance, $R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$

SI unit of $R_H = \text{K W}^{-1}$ Dimensions of $R_H = [\text{M}^{-1} \text{L}^{-2} \text{T}^{-3} \text{K}]$

35. **Convection:** It is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of the heated particles of the substance. Fluids are mainly heated through convection. Natural convection arises due to unequal heating and gravity, when more heated and less dense parts rise and the replaced by the cooler parts of the fluid. In forced convection, a material is forced to move by an agency like a pump or a blower.
36. **Radiation:** It is the process by which heat is transmitted from one place to another without heating the intervening medium.
37. **Prevost's theory of heat exchanges:** All bodies emit radiations irrespective of their temperatures. They emit radiations to the surroundings. In the equilibrium state, the exchange of energy between a body and its surroundings occurs in equal amounts.
38. **Thermal radiation:** The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy. Thermal radiations are electromagnetic waves of long wavelength ranging from $1 \mu\text{m}$ to $100 \mu\text{m}$.
39. **Absorptive power or absorptivity (a_λ):** The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy incident on it in the same time within a unit wavelength range round the wavelength λ . It is a dimensionless quantity.
40. **Emissive power (e_λ):** The emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ . Its SI unit is $\text{J s}^{-1} \text{m}^{-2}$ or W m^{-2} .
41. **Emissivity (ϵ):** It is the ratio of the emissive power of body (e) to the emissive power (E) of a black body at the same temperature. It is given by

$$\epsilon = \frac{e}{E}$$

It is a dimensionless quantity having value between 0 and 1. The emissivity of a black body is 1.

- (ii) **Vibratory motion:** That is, the to and fro motion of the molecules about their mean positions. This is common in liquids and gases.
- (iii) **Rotatory motion:** That is, the rotation of the molecules about their axis. This occurs usually at high temperature.

▶ When a body is heated, all these molecular motions become fast. The kinetic energy of a molecule due to each type of motion increases. So, we can regard heat as energy of molecular motion which is equal to the sum total of the kinetic energy possessed by the molecules a body by virtue of their translational, vibrational and rotational motions.

▶▶ **UNIT OF HEAT.**

(i) **Calorie.** *One calorie is the amount of heat required to raise the temperature of 1 gm of water through 1°C.*

Thus if 5 g of water is heated from 50°C to 60°C, then the amount of heat required is $5 \times 10 = 50$ calories. The bigger unit is kilocalorie (kcal). 1 kcal = 1000 calories.

(ii) **Joule.** Since heat is a form of energy, it should be measured in joules—the SI unit of energy. It has been found experimentally that 1 calorie is equal to 4.186 joules i.e.,
 1 calorie = 4.186 joules

▶▶ **JOULE'S MECHANICAL EQUIVALENT OF HEAT**

From experiments, Joule established a relation between the work done and heat produced. He showed that whenever a given amount of work (W) is converted into heat, always the same amount of heat (Q) is produced, thus

$$W \propto Q \quad \text{or} \quad W = JQ$$

$$\text{or} \quad J = \frac{W}{Q}$$

If $Q = 1$, then $J = W$

The proportionality constant J is called Joule's mechanical equivalent of heat. It may be defined as the amount of Work that must be done to produce a unit quantity of heat. $J = 4.186 \text{ J cal}^{-1} = 4.186 \times 10^7 \text{ erg cal}^{-1}$.

** J is not a physical quantity. It just a conversion factor.

■ **CONCEPT OF TEMPERATURE:**

Heat and temperature are closely related to each other but these two are different quantities.

ILLUSTRATION:

- (i) A simple way to define the temperature of a body is as under :
The temperature of a body is its degree of hotness or coldness.
 The concept of temperature springs from the qualitative ideas of "hot" and "cold" based on our sense of touch. A hot oven is said to have a high temperature, whereas a cold tray of ice is said to have a low temperature. However, the bodily sensation of warmth and cold is not reliable. For example, if the hand has been in hot water, lukewarm water will feel cold, whereas if the hand has been in cold water, the same lukewarm water will feel warm.
- (ii) *The temperature of a body is an indicator of the average thermal energy (kinetic energy) of the molecules of the body.*
 The greater the average thermal energy of the molecules of a body, the greater is the temperature of the body. Let us now distinguish between heat and temperature. Suppose the ocean and a glass of water are at the same temperature. Then the average thermal energy of one molecule in the ocean is the same as that of the water molecule in the glass. But the ocean contains much more thermal energy because water molecules are much more in the ocean than in the glass.
- (iii) When two bodies at different temperatures are brought in contact, heat flows from a body at higher temperature to the body at lower temperature till the temperature of the two bodies is the same. Thus temperature of the two bodies decides the direction of heat flow when the two bodies are brought in contact.

▶▶ **CONCLUSION:**

Hence when two bodies are brought in contact, the direction of heat flow determines which body is at higher temperature. Heat will flow from a body at a higher temperature to the body at lower temperature.

Note. Two bodies are said to be in **thermal equilibrium** if no transfer of heat takes when they are brought in contact. Clearly, the two bodies are at the same temperature.

Temperature: Temperature is the degree of hotness or coldness of a body. When two bodies are placed in contact, the heat flows from the body at higher temperature to the body at lower temperature. Thus, temperature may be defined as the thermal state of a body which decides the direction of flow of heat energy from the body to another when they are placed in thermal contact with each other.

▶▶ **Kinetic interpretation of temperature:** The temperature of a body is the measure of the average kinetic energy of its molecules. When a body is heated, its molecules move faster. Their average K.E. increases. This increases the temperature of the body.

In thermodynamics the concept of temperature follows from the zeroth law of thermodynamics.

HEAT VS. TEMPERATURE.

Heat	Temperature
<ol style="list-style-type: none"> Heat is a form of energy which produces in us the sensation of hotness or coldness. It is a cause, when some heat is supplied to a body, its temperature increases. It represents the total kinetic energy of the molecules of a body. Heat flows from high temperature side irrespective of the amounts of heat possessed by the bodies in contact. It is measured in cal, kcal or joule 	<p>Temperature is the degree of hotness or coldness of a body. It is an effect.</p> <p>It represents the average kinetic energy possessed by the molecules of a body. Temperature decides the direction of flow of heat from one body to another.</p> <p>It is measured in °C, °F or K.</p>

03

THERMOMETRY

Thermometry: The branch of physics that deals with the measurement of temperature is called thermometry.

Thermometer: Any device used to measure the temperature of a body is called a thermometer. It is named so because thermo is a Latin word which means heat and meter means a measuring device.

Principle of a thermometer: A thermometer makes used to some measurable property (called thermometric property) of a substance which changes linearly with temperature.

The thermometer properties of different substances and the corresponding thermometers are as follows:

- (i) Length of a liquid column in a capillary (Mercury thermometer).
- (ii) Pressure of a gas at constant volume (Constant pressure gas thermometer).
- (iii) Volume of a gas at constant pressure (Constant pressure gas thermometer).
- (iv) Electrical resistance of a metal wire (Platinum resistance thermometer).
- (v) Thermoelectric e.m.f. (Thermoelectric or thermocouple thermometer).
- (vi) Radiated power (Pyrometers).

□□ **Fixed points on a temperature scale:** To construct a temperature scale, two fixed points (two well defined thermodynamic states) are chosen and are assigned two arbitrary numbers for their temperature. One number fixes the origin of the scale and the other fixes the size of the unit of the scale.

▶ The temperature at which pure ice melts at standard pressure (ice-liquid water equilibrium state) is usually chosen as the lower fixed point.

▶ The temperature at which pure water boils at atmospheric pressure (liquid water-vapour equilibrium state) is chosen as the upper fixed point.

□□ **Thermoelectric scales:** The range between the two fixed temperatures is called fundamental, interval, which when divided into a suitable number of equal divisions forms a thermometric scale.

The commonly used temperature scales are as follows:

□(i) **The Celsius scale:** On this scale, the lower fixed point (ice point) is taken as 0° and the upper fixed point (steam point) as 100° C. The interval between the two fixed points is divided into hundred equal parts (hence the name centigrade) and each part is called 1°C.

□(ii) **The Fahrenheit scale:** On this scale, the lower fixed point is taken as 32°F and the upper fixed point as 212°F. The interval between them is divided into 180 equal parts and each part represents 1°F.

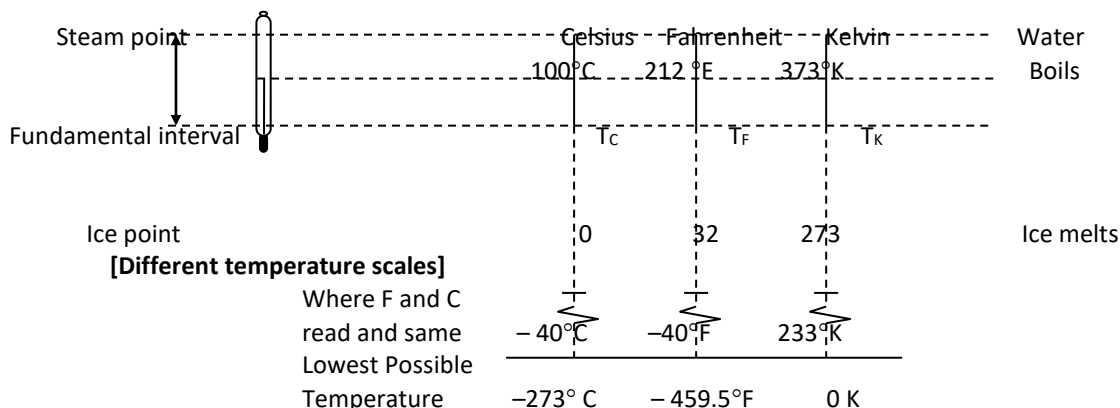
□(iii) **The Reamer scale:** On this scale, the lower fixed point is taken as 0°R and the upper fixed point as 80°R. The interval between them is divided into 80 equal parts and each part represents 1°R.

□(iv) **The Kelvin scale:** On this scale, the lower fixed point is taken as 273.15 K and the upper fixed point is 373 K AND divided into 100 equals parts. The SI unit of temperature is taken (K).

As the fundamental intervals of both Celsius and Kelvin scales have 100 divisions each, so the size of one division on Celsius scale is equal to the size of one division on Kelvin scale.

Conversion of temperature from one scale to another: To convert the temperature from one scale to another, the following relation is used:

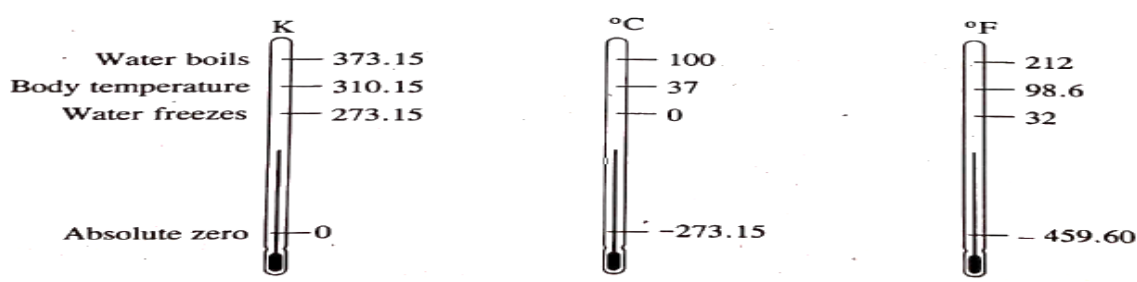
$$\frac{\text{Temperature on one scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}} = \frac{\text{Temperature on other scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}}$$



As shown in the above figure, if the temperature of body is measured as T_C , T_F , T_R and T_K on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T_K - 273.15}{373.15 - 273.15}$$

or $\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273.15}{100}$



ABSOLUTE SCALE OF TEMPERATURE

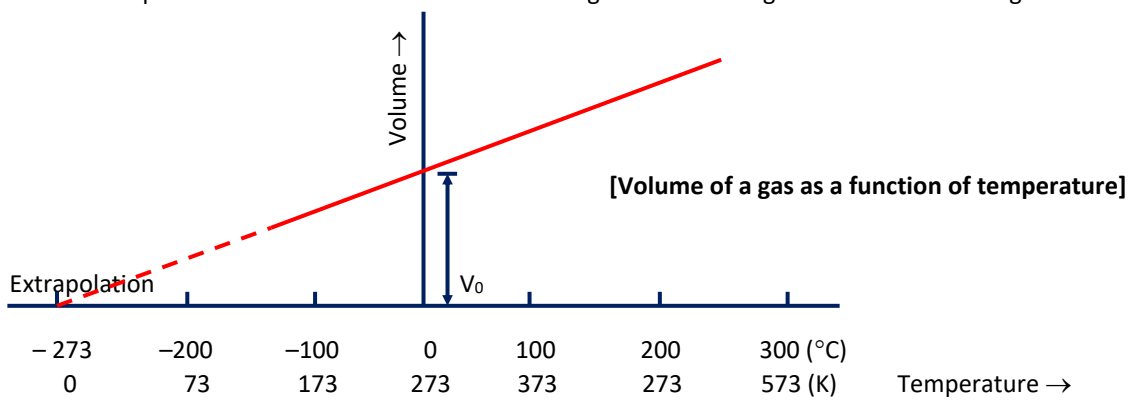
According to Charles' law, if V_t and V_0 are the volumes at $t^\circ\text{C}$ and 0°C respectively of a given mass of a gas at constant pressure P, then

$$V_t = V_0 \left(1 + \frac{t}{273.15} \right)$$

Clearly, the volume of the gas below 0°C will be less than V_0 . For example, volume of the gas at $-t^\circ\text{C}$ is

$$V_{-t} = V_0 \left(1 - \frac{t}{273.15} \right)$$

The decrease of temperature results in the decrease in volume of the gas. This has been shown graphically in Fig. by plotting the volume of a given mass of a gas against temperature at constant pressure. The graph is a straight line. If we extrapolate the straight line, it meets the temperature axis at -273.15°C . Thus, a gas occupies zero or no volume at -273.15°C . Clearly, a temperature below -273.15°C is impossible because then the volume of the gas would be negative which is meaningless.



Moreover, according to the kinetic theory of gases, all molecular motion stops at -273.15°C . Hence the lowest temperature of -273.15°C at which a gas is supposed to have zero volume (and zero pressure) and at which entire molecular motion stops is called the absolute zero of temperature. In practice, all gases condense to liquids and solids before this temperature is reached.

Lord Kelvin suggested new scale of temperature starting with -273.15 as its zero. This scale of thermometer is known as Kelvin scale or absolute scale. The scale of degree of Kelvin scale is same as that on Celsius scale. Therefore,
 $T(K) = t(^{\circ}C) + 273.15$

Thus, ice point ($^{\circ}C$) on absolute scale is 273.15 K and the steam point ($100^{\circ}C$) is 373.15 K. The absolute scale of temperature is also called thermodynamic scale of temperature.

TRIPLE POINT OF WATER

The triple point of water is the state at which the three phases of water namely ice, liquid water and water vapour are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column. Thus, for water,

$$P_{tr} = 0.46 \text{ cm of Hg} \qquad T_{tr} = 273.16 \text{ K or } 0.01^{\circ}C$$

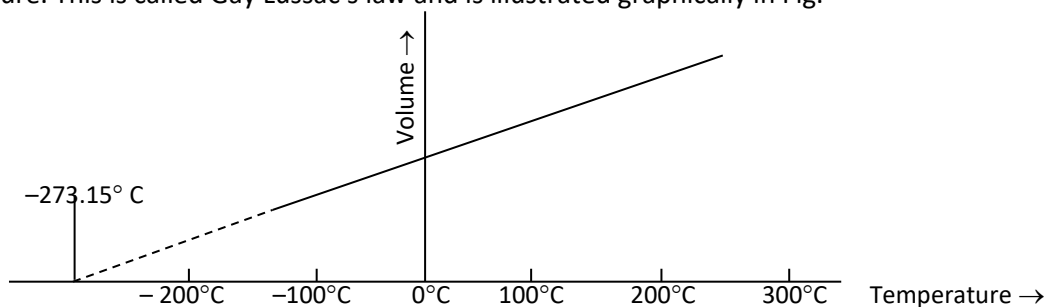
In modern thermometry, the triple point of water is chosen to be one of the fixed points. As it is characterized by a unique temperature and pressure, so it is preferred over the conventional fixed points namely the melting point of ice and boiling point of water. The melting point of ice and boiling point of water both change with pressure. Moreover, the presence of impurities changes their values. But the triple point of water is independent of the external factors.

In the absolute Kelvin scale, the triple point of water is assigned the value 273.16 K. The absolute zero is taken as the other fixed point on this scale.

CONSTANT VOLUME GAS THERMOMETER

A gas thermometer is an ideal thermometer because the increase of volume or pressure of a gas with temperature is independent of the nature of the gas.

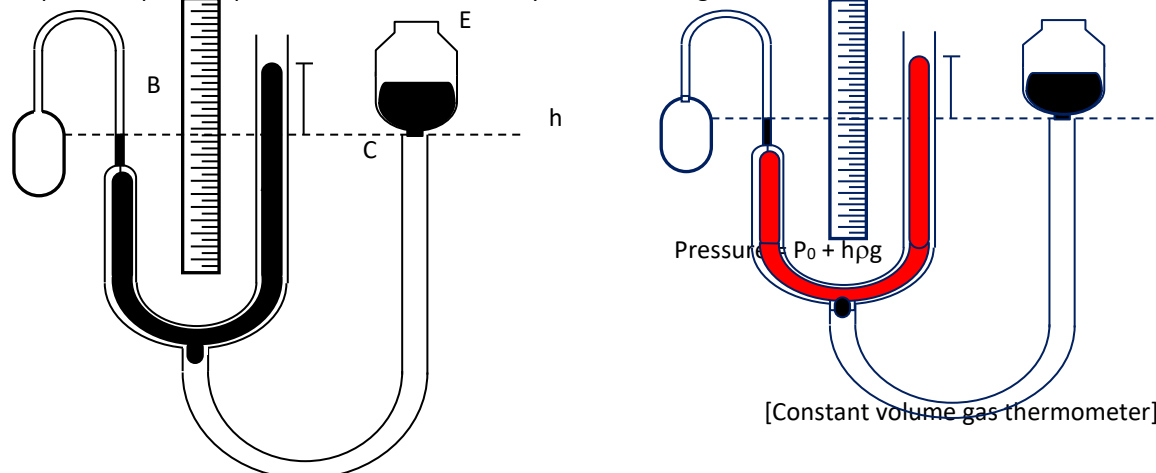
Principle: If the volume is kept constant, the pressure of a given mass of a gas increases linearly with the rise of temperature. This is called Gay Lussac's law and is illustrated graphically in Fig.



[Pressure versus temperature of a low-density gas kept at constant volume]

Construction: Fig. shows a schematic diagram of a constant volume gas thermometer. A fixed mass of gas is enclosed in a bulb A made of glass, quartz or platinum depending on the temperature range required to be measured. The bulb A is connected to a capillary BC which is connected to a manometer CD containing mercury. The end D of the manometer is open to atmosphere. The mercury of the manometer is connected to a mercury reservoir through a rubber tube. The volume of the gas is kept constant by raising or lowering the mercury reservoir F until the mercury level in the left part of the manometer coincides with C.

A vertical scale E measures the difference h of mercury levels in the manometer tube. The pressure of the gas is then the atmospheric pressure plus the pressure due to the mercury column of height h .



Working: The bulb is first immersed in the system (e.g., boiling oil) whose temperature is to be determined. The pressure P of the gas is measured. Next, the bulb is surrounded by water at the triple point, and the level of mercury is again brought to C in the left side of the manometer. The pressure P_{tr} of the gas is measured. According to Gay Lussac's law:

$$\frac{P_{tr}}{T_{tr}} = \frac{P}{T} \quad \text{or} \quad T = T_{tr} \left(\frac{P}{P_{tr}} \right)$$

But $T_{tr} = 273.16 \text{ K}$

$$\therefore T = 273.16 \left(\frac{P}{P_{tr}} \right)$$

Advantages:

- (i) As the expansion of the gas large, so the gas thermometers are very sensitive.
- (ii) The gases expand coefficient of all gases is nearly the same, so thermometers using different gases give same reading.
- (iv) Gas thermometers can be used for a wide range of temperature of about 1600°C can be measured.

Disadvantages:

- (i) The correction has to be applied against the increase in the volume of the glass bulb.
- (ii) The gas in the bulb is not at the same temperature as the gas in the capillary tube. Correction has to be applied for this purpose.
- (iii) The gas is not ideal.
- (iv) the thermometer is large and hence inconvenient to use.
- (v) It is not a direct reading thermometer.
- (vi) It requires a great deal of time to know the unknown temperature.

IDEAL GAS TEMPERATURE

If P and P_{tr} are the pressure of a gas (at constant volume) at constant temperature T and at the triple point (273.16 K) respectively, then from Gay Lussac's law, we have

$$T = 273.16 \left(\frac{P}{P_{tr}} \right)$$

The temperature defined by the above equation depends slightly on the nature of the gas and its pressure. But at low pressures and high temperatures the real gases approach the ideal gas behaviour. So, if the pressure in the bulb of the gas thermometer is taken to be smaller and smaller (i.e., in the limit $P_{tr} \rightarrow 0$), all the different gas thermometers give the same value of temperature for a given system. So, we defined a temperature by the equation

$$T = \lim_{P_{tr} \rightarrow 0} 273.16 \left(\frac{P}{P_{tr}} \right)$$

The temperature defined by the above equation is known as ideal gas temperature on the Kelvin scale and is independent of the nature of the gas. However, it depends on the properties of the gases in general. In thermodynamics, it is possible to define a truly universal absolute temperature scale which does not depend on any property of the thermometric substance and in which the unit of temperature is Kelvin (K). The ideal gas thermometer scale is found to be identical with this absolute scale

OTHER THERMOMETERS

Liquid thermometer: Its working is based on the fact that liquids expand uniformly and regularly on being heated.

Mercury thermometer is the most common liquid thermometer. It consists of a glass capillary tube of uniform bore having a bulb at its one end. The bulb is filled with mercury and the tube is sealed at the top after taking out air from it. The ice point and steam point are respectively marked as its lower and upper fixed points respectively. The use of capillary tube results in an easily observable rise in the level of mercury even for of a small rise in temperature.

If l_0 , l_{100} and l_t be the lengths of mercury column at 0°C , 100°C and an unknown temperature $t^\circ\text{C}$ respectively, then the unknown temperature measured by the mercury thermometer is

$$t = \frac{l_t - l_0}{l_{100} - l_0} \times 100^\circ\text{C}$$

Advantages of using mercury as a thermometric substance as compared to other liquids:

- (i) Mercury has a uniform coefficient of expansion over a wide range of temperature.
- (ii) Mercury is opaque and bright, so its level can be seen easily in a glass tube.
- (iii) It does not stick to the walls of the glass tube.
- (iv) It is a good conductor of heat and so attains the temperature of the hot body quickly.
- (v) It has low specific heat; it absorbs very small amount of heat from the body whose temperature is to be measured.
- (vi) The range of mercury thermometer is quite large because of its low freezing point (-39°C) and high boiling point (357°C).
- (vii) Mercury is non-volatile.

Platinum resistance thermometer: The electric resistance of a metal wire increases linearly with temperature as

$$R_t = R_0 (1 + \alpha t)$$

where α is the temperature coefficient of resistance.

So electric resistance may be used as a thermometric property to define a temperature scale. A platinum wire is often used in this thermometer because platinum has high melting point and its α is constant and large. If R_0 and R_{100} denote the resistance of a platinum wire at ice point and steam point respectively, then temperature t_R of a body for which the corresponding resistance is R_t , is given by

$$t_R = \frac{R_t - R_0}{R_{100} - R_0} \times 100^\circ\text{C}$$

Platinum resistance thermometer can be used in the temperature range -170°C to 200°C . For measuring lower temperatures, a germanium resistance thermometer is used. Germanium is a semiconductor whose resistance increases with the decrease in temperature.

Thermoelectric thermometer: This thermometer is based on **Seebeck effect**: According to this effect, when wires of two different metals (Bi – Sb or Cu – Fe) are joined to form a closed circuit called thermocouple and their junctions are maintained at different temperature, an e.m.f. is produced and a current flow in the circuit. If one junction is at 0°C (cold junction) and the other at t°C (hot junction), then the thermoelectric e.m.f. is given by $\mathcal{E} = at + bt^2$

where a and b are constant for the given pair of metals. To measure an unknown temperature, each individual thermoelectric thermometer is first calibrated by drawing a curve between the temperature of hot junction and the e.m.f. generated.

For the linear part of the thermo e.m.f., the unknown temperature is given by

$$t_x = \frac{\mathcal{E}_t - \mathcal{E}_0}{\mathcal{E}_{100} - \mathcal{E}_0} \times 100 \text{ degrees.}$$

the normal range of a thermoelectric thermometer is – 200° C to 1600° C

Examples based on Measurement of Temperature

Formulae Used

1. If T_C , T_F , T_R and T are the temperature of a body on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{100}$$

$$\text{or } \frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$$

$$2. T_C = \frac{5}{9} (T_F - 32), \quad T_F = \frac{9}{5} T_C + 32$$

$$3. T = T_C + 273.15, \quad T_C = T - 273.15$$

$$4. T_F = \frac{9}{5} (T - 273.15) + 32 = \frac{9}{5} T - 459.67$$

$$\text{or } T = \frac{5}{9} T_F + 255.37$$

$$5. \text{ For a constant volume air thermometer, } T = T_0 \times \frac{P}{P_0}$$

$$\text{In terms of triple point of water, } T = T_r \times \frac{P}{P_{tr}}$$

6. For a platinum resistance thermometer, resistance of platinum at t°C, $R = R_0 (1 + t)$

$$\text{Temperature coefficient of resistance, } \alpha = \frac{R - R_0}{R_0 \times t}$$

Units Used: Pressure is in Pascal (Pa), resistance in ohm (Ω) and temperature coefficient of resistance (α) in $^{\circ}\text{C}^{-1}$.

Q. 1. A faulty thermometer has its fixed points marked as 5° and 95°. Temperature of a body as measured by the faulty thermometer is 59°. Find the correct temperature of the body on Celsius scale.

Sol. $\frac{T_C - 0}{100 - 0} = \frac{\text{Temperature on faulty scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}}$

$$\text{or } \frac{T_C - 0}{100} = \frac{59 - 5}{95 - 5} = \frac{54}{90} \quad \text{or } T_C = 60^{\circ} \text{ C}$$

Q. 2. A thermometer has wrong calibration. It reads the melting point of ice – 10°C. It reads 60°C in place of 50°. Calculate the temperature of boiling point of water on this scale.

Sol. Let θ_1 = Lower fixed point on faulty thermometer θ_2 = Reading on faulty thermometer

n = number of divisions between upper and lower fixed points

$$\text{Now } \frac{C}{100} = \frac{\theta_2 - \theta_1}{n} \quad \dots (1)$$

In first case: $\theta_2 = -10^{\circ} \text{ C}$, $C = 0^{\circ} \text{ C}$

$$\therefore 0 = \frac{-10 - \theta_1}{n} \quad \text{or } \theta_1 = -10^{\circ} \text{ C}$$

In second case, $\theta_2 = 60^{\circ} \text{ C}$, $C = 50^{\circ} \text{ C}$

$$\therefore \frac{50}{100} = \frac{60 - \theta_1}{n}$$

$$\text{or } \frac{1}{2} = \frac{60 - (-10)}{n} = \frac{70}{n} \quad \text{or } n = 140$$

As boiling point of water on Celsius scale is 100° C, so putting C = 100 in equation (1), we get

$$\frac{100}{100} = \frac{\theta_2 - (-10)}{140} \quad \text{or } \theta_2 = 130^{\circ} \text{ C}$$

$$\therefore \text{Boiling point of water on faulty thermometer} = 130^{\circ} \text{ C}$$

Q. 3. Find the temperature for which the readings on Celsius and Fahrenheit scales are identical.

Sol. Let $T_C = T_F = x$

$$\text{As } \frac{T_C - 0}{100} = \frac{T_F - 32}{180}$$

$$\therefore \frac{x}{100} = \frac{x - 32}{180} \quad \text{or}$$

$$180x = 100x - 3200 \quad \text{or} \quad 80x = -3200$$

$$\text{or} \quad x = -40^\circ$$

Hence -40°C and -40°F are identical temperatures.

Q. 4. *The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.*

Sol. For neon: Triple point, $T = 24.57 \text{ K}$

$$\therefore T_C = T (\text{K}) - 273.15 = 24.57 - 273.15 = -248.58^\circ \text{C}$$

$$T_F = \frac{9}{5} T_C + 32 = \frac{9}{5} \times (-248.58) + 32 = -415.44^\circ \text{F}$$

For carbon dioxide: Triple point, $T = 216.55 \text{ K}$

$$\therefore T_C = T (\text{K}) - 273.15 = 216.55 - 273.15 = -56.6^\circ \text{C}$$

$$T_F = \frac{9}{5} T_C + 32 = \frac{9}{5} \times (-56.6) + 32 = -69.88^\circ \text{C}$$

Q. 5. *Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is relation between T_A and T_B ?*

Sol. $\frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$ or $T_A = \frac{4}{7} T_B$

Q. 6. *A constant volume gas thermometer using helium records a pressure of 20.0 kPa at the triple-point of water, and pressure of 14.3 kPa at the temperature of 'dry ice' (solid CO_2). What is the temperature of 'dry ice'?*

Sol. Here $T_{tr} = 273.16 \text{ K}$, $P_{tr} = 20.0 \text{ kPa}$, $P = 14.3 \text{ kPa}$

$$\text{Temperature of dry ice, } T = \frac{P}{P_{tr}} \times T_{tr} = \frac{14.3 \times 273.16}{20} = 195.30 \text{ K}$$

Q. 7. *Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:*

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	$1.250 \times 10^5 \text{ Pa}$	$0.200 \times 10^5 \text{ Pa}$
Normal melting point of sulphur	$1.797 \times 10^5 \text{ Pa}$	$0.287 \times 10^5 \text{ Pa}$

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?

(b) What do you think is the reason for slightly different answers from A and B?

Sol. (i) For pressure thermometer A:

$$T_{tr} = 273 \text{ K}, \quad P_{tr} = 1.250 \times 10^5 \text{ Pa}, \quad P = 1.797 \times 10^5 \text{ Pa}$$

Normal freezing point of sulphur,

$$T = \frac{P}{P_{tr}} \times T_{tr} = \frac{1.797 \times 10^5 \times 273}{1.250 \times 10^5} = 392.46 \text{ K}$$

(ii) For pressure thermometer B:

$$T_{tr} = 273 \text{ K}, \quad P_{tr} = 0.200 \times 10^5 \text{ Pa}, \quad P = 0.287 \times 10^5 \text{ Pa}$$

$$\therefore T = \frac{0.287 \times 10^5 \times 273}{0.200 \times 10^5} = 391.75 \text{ K}$$

(b) The slight difference is due to the fact that oxygen and hydrogen do not behave strictly as ideal gases.

Q. 8. *A constant volume thermometer using helium gas records a pressure of $1.75 \times 10^4 \text{ Pa}$ at normal freezing point of water, and a pressure of $2.39 \times 10^4 \text{ Pa}$ at normal boiling point of water. Obtain from these observations the temperature of absolute zero on the Celsius scale.*

Sol. Let T_1 and T_2 be the absolute temperatures corresponding to normal freezing and normal boiling points (i.e., under 1 atmospheric pressure) of water.

For a constant volume thermometer,

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{2.39 \times 10^4 \text{ Pa}}{1.75 \times 10^4 \text{ Pa}} = \frac{239}{175}$$

Now the temperature t_c on Celsius scale and temperature T on the Kelvin scale are related as

$$T = t_c + T_0$$

where T_0 is a constant. As the normal freezing point and normal boiling point of water are 0°C and 100°C respectively, so from the above equation, we get

$$T_1 = 0 + T_0 \quad \text{and} \quad T_2 = 100 + T_0$$

$$\therefore \frac{T_0 + 100}{T_0} = \frac{239}{175} \quad \text{or} \quad \frac{100}{T_0} = \frac{239}{175} - 1 = \frac{64}{175}$$

$$\text{or} \quad T_0 = \frac{100 \times 175}{64} = 273$$

Hence the absolute zero $T = 0$ as the Celsius scale will be

$$t_c = T - T_0 = 0 - 273 = -273$$

The exact value of absolute zero is -273.15 .

Q. 9. *A platinum wire has resistance of 10Ω at 0°C and 20Ω at 273°C . Find the value of coefficient of resistance.*

Sol. Here $R_0 = 10 \Omega$, $R = 20 \Omega$,

$$\theta = 273^\circ - 0^\circ = 273^\circ \text{C}$$

Temperature coefficient of resistance,

$$\alpha = \frac{R - R_0}{R_0 \times \theta} = \frac{20 - 10}{10 \times 273} = \frac{1}{273} \text{ } ^\circ\text{C}^{-1}$$

- Q. 10.** The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:
 $R = R_0 [1 + 5 \times 10^{-3} (T - T_0)]$

The resistance is 101.6 Ω at the triple point of water, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω ?

Sol. When $T = 273 \text{ K}$, $R = 101.6 \text{ } \Omega$
 $\therefore 101.6 = R_0 [1 + 5 \times 10^{-3} (273 - T_0)]$... (i)

When $T = 600.5 \text{ K}$, $R = 165.5 \text{ } \Omega$
 $\therefore 165.5 = R_0 [1 + 5 \times 10^{-3} (600.5 - T_0)]$... (ii)

Dividing (ii) by (i), we get

$$\frac{165.5}{101.6} = \frac{1 + 5 \times 10^{-3} (600.5 - T_0)}{1 + 5 \times 10^{-3} (273 - T_0)}$$

On solving, $T_0 = -49.3 \text{ K}$
 Substituting in (i), we get $101.6 = R_0 [1 + 5 \times 10^{-3} (273 + 49.3)]$

or $R_0 = \frac{101.6}{1 + 5 \times 10^{-3} \times 322.3} = 38.9 \text{ } \Omega$

For $R = 123.4 \text{ } \Omega$, We have
 $123.4 = 38.9 [1 + 5 \times 10^{-3} (T + 49.3)]$ On solving, $T = 384.8 \text{ K}$

Problems For Practice

- Q. 1.** A faulty thermometer reads 5 $^\circ\text{C}$ in melting ice and 99 $^\circ\text{C}$ in steam. Find the correct temperature in $^\circ\text{F}$ when the faulty thermometer reads 52 $^\circ\text{C}$.

Sol. As the lowest and the highest points on the faulty thermometer are 5 $^\circ\text{C}$ and 99 $^\circ\text{C}$, so

$$\frac{T_C - 5}{99 - 5} = \frac{T_F - 32}{180} \quad \text{or} \quad \frac{52 - 5}{94} = \frac{T_F - 32}{180}$$

or $T_F = 122^\circ \text{ F}$

- Q. 2.** An ungraduated thermometer of uniform bore is attached to a centimeter scale and is found to read 10.3 cm in melting ice, 26.8 cm in boiling water and 6.5 cm in freezing mixture. Calculate the temperature of the freezing mixture.

Sol. Temperature reading of melting ice = 10.3 cm Temperature reading of boiling water = 26.8 cm
 Temperature reading of freezing mixture = 6.5 cm

Let T_C be the temperature of freezing mixture on Celsius scale. Then $\frac{6.5 - 10.3}{26.8 - 10.3} = \frac{T_C - 0}{100 - 0}$

On solving, $T_C = -23.03^\circ \text{ C}$

- Q. 3.** Normal temperature of the human body is 98.4 $^\circ\text{F}$. Find the temperature on Celsius and absolute scale.

Sol. $T_C = \frac{5}{9} (T_F - 32) = \frac{5}{9} (98.4 - 32) = \frac{5}{9} \times 66.4 = 36.88^\circ \text{ C}$

$T = T_C + 273.15 = 36.88 + 273.15 = 310.03 \text{ K}$

- Q. 5.** When a thermometer is taken from the melting ice to a warm liquid, the mercury level rises to 2/5th of the distance between the lower and the upper fixed points. Find the temperature of liquid in $^\circ\text{C}$ and K.

Sol. Let the distance between the lower and upper fixed points be 1 unit. Then

$$\frac{2/5 - 0}{1 - 0} = \frac{T_C - 0}{100} \quad \text{or} \quad T_C = 40^\circ \text{ C}$$

and $T = 40 + 273.15 = 313.15 \text{ K}$

- Q. 6.** The pressure of air in the bulb of constant volume air thermometer is 75 cm of mercury at 0 $^\circ\text{C}$, 100 cm at 100 $^\circ\text{C}$ and 80 cm at the room temperature. Calculate the room temperature.

Sol. $\frac{T_C - 0}{100 - 0} = \frac{P - P_0}{P_{100} - P_0} = \frac{80 - 75}{100 - 75} \quad \therefore T_C = 20^\circ \text{ C}$

- Q. 7.** At what temperature is the Fahrenheit scale reading equal to twice of Celsius scale reading?

Sol. $T_F = \frac{9}{5} T_C + 32$ and $T_F = \frac{1}{2} T_C$
 $\therefore \frac{1}{2} T_C = \frac{9}{5} T_C + 32$ or $T_C = -\frac{320}{13} = -24.6^\circ \text{ C}$

- Q. 8.** At what temperature is the Fahrenheit scale reading equal to half of Celsius scale reading?

Sol. $T_F = \frac{9}{5} T_C + 32$ and $T_F = \frac{1}{2} T_C$
 $\therefore \frac{1}{2} T_C = \frac{9}{5} T_C + 32$ or $T_C = -\frac{320}{13} = -24.6^\circ$

- Q. 10.** The resistance of a resistance thermometer at 19 $^\circ\text{C}$ is 3.50 Ω and at 99 $^\circ\text{C}$ is 3.66 Ω . At what temperature will its resistance be 4.30 Ω ?

Sol. Let R_0 , R_1 and R_2 be the resistances at 0° , $\theta_1^\circ\text{C}$ and $\theta_2^\circ\text{C}$ and α be the temperature coefficient of resistance. Then

$$R_1 = R_0 (1 + \alpha \theta_1)$$

$$R_2 = R_0 (1 + \alpha \theta_2)$$

On dividing and solving, $\alpha = \frac{R_2 - R_1}{R_1 \theta_2 - R_2 \theta_1}$

But $\theta_1 = 19^\circ\text{C}$, $R_1 = 3.50\ \Omega$, $\theta_2 = 99^\circ\text{C}$, $R_2 = 3.66\ \Omega$

$$\therefore \alpha = \frac{3.66 - 3.50}{3.50 \times 99 - 3.66 \times 19} = \frac{0.16}{276.96} \quad \dots (i)$$

Suppose the resistance becomes $4.30\ \Omega$ at $t^\circ\text{C}$. Then

$$\alpha = \frac{4.30 - 3.50}{3.50t - 4.30 \times 19} = \frac{0.80}{3.50t - 81.70} \quad \dots (ii)$$

From (i) and (ii),

$$\frac{0.16}{276.96} = \frac{0.80}{3.50t - 81.70}$$

or $3.50t = 1384.80 + 81.70 = 1466.50$

or $3.50t - 81.70 = 5 \times 276.96$

$\therefore t = 419^\circ\text{C}$

/// THERMAL EXPANSION

Almost all solids, liquids and gases expand on heating. The increase in the size of a body when it is heated is called thermal expansion.

Different types of thermal expansion:

- /// (i) **Linear expansion:** It is the increase in the length of a metal rod on heating.
- /// (ii) **Superficial expansion:** It is the increase in the surface area of a metal sheet on heating.
- /// (iii) **Cubical expansion:** It is the increase in the volume of block on heating.

/// **Cause of thermal expansion:** All solids consist of atoms and molecules. At any given temperature, these atoms and molecules are held at equilibrium distance by forces of attraction. When a solid is heated. The amplitude of vibration of its atoms and molecules increases. The average interacting separation increases. This results in the thermal expansion of the solid.

/// **Coefficient of linear expansion:** Suppose a solid rod of length l is heated through a temperature ΔT and its final (increased) length is l' . It is found from experiments that

(i) Increase in length \propto rise in temperature

i.e., $l' - l \propto \Delta T$

(ii) Increase in length \propto original length

i.e., $l' - l \propto l$

Combining the above two factors, we get

$$l' - l \propto l \Delta T$$

or $l' - l = \alpha l \Delta T$

The proportionality constant α is called coefficient of linear expansion. Its value depends on the nature of the solid. Clearly,

$$l' = l [1 + \alpha \Delta T]$$

and $\alpha = \frac{l' - l}{l \Delta T} = \frac{\Delta l}{l \Delta T}$

or $\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}}$

Hence the **coefficient of superficial expansion of metal sheet is defined as the increase in its surface area per unit original surface area per degree rise in its temperature.**

The unit of α is $^\circ\text{C}^{-1}$ or K^{-1} .

TABLE: Coefficients of linear expansion for some substances

Materials	$\alpha (10^{-5} \text{ K})$
Aluminium	2.5
Brass	1.8
Iron	1.2
Copper	1.7
Silver	1.9
Gold	1.4
Glass (Pyrex)	0.32
Lead	0.29

Table gives the average values of α for some materials in the temperature range $0 - 100^\circ\text{C}$. Clearly, copper expands 5 times more than glass for the same rise in temperature. Generally, metals expand more and have comparatively high values of α .

/// Coefficient of superficial expansion: Suppose a metal sheet of initial surface area S is heated through temperature ΔT and its final surface area becomes S' . Then

$$S' - S \propto \Delta T$$

and $S' - S \propto S$

$$\therefore S' - S \propto S \Delta T$$

or $S' - S = \beta S \Delta T$

The proportionality constant β is called coefficient of superficial expansion and its value depends on the nature of the material. Clearly,

$$S' = S [1 + \beta \Delta T]$$

and $\beta = \frac{S' - S}{S \Delta T} = \frac{\Delta S}{S \Delta T}$

or $\beta = \frac{\text{Increase in surface area}}{\text{Original surface area} \times \text{Rise in temperature}}$

Hence the **coefficient of superficial expansion of metal sheet of defined as the increase in its surface area per unit original surface area per degree rise in its temperature.**

The unit of β is $^{\circ}\text{C}^{-1}$ or K^{-1} .

/// Coefficient of cubical expansion: Suppose a solid block of initial volume V is heated through a temperature ΔT and its final volume is V' .

Then $V' - V \propto \Delta T$

and $V' - V \propto V$

or $V' - V = \gamma V \Delta T$

The proportionality constant γ is called the coefficient of cubical expansion which depends on the nature of the material of the solid. Clearly,

$$V' = V [1 + \gamma \Delta T]$$

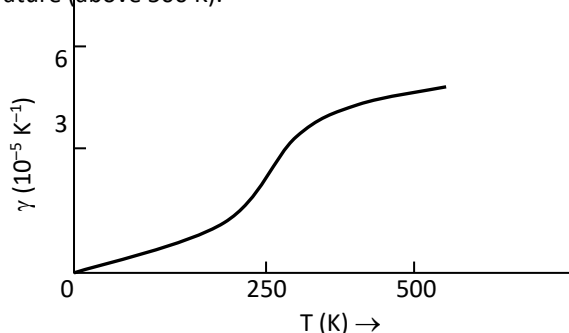
and $\gamma = \frac{V' - V}{V \Delta T} = \frac{\Delta V}{V \Delta T}$

or $\gamma = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$

Hence the **coefficient of cubical expansion of a substance is defined as the increase in volume per unit original volume per degree rise in its temperature.**

- The unit of γ is $^{\circ}\text{C}^{-1}$ or K^{-1} .

Variation of γ with temperature: For a given substance, γ varies with temperature. Fig. shows the variation of the coefficient of cubical expansion of copper with temperature. The value of γ first increases with temperature and then becomes constant at a high temperature (above 500 K).



[Variation of γ of copper with temperature]

TABLE: Coefficients of volume expansion for some substances

Materials	$\gamma (\text{K}^{-1})$	Materials	$\gamma (\text{K}^{-1})$
Aluminium	7×10^{-5}	Hard rubber	2.4×10^{-4}
Brass	6×10^{-5}	Invar	2×10^{-6}
Iron	3.55×10^{-5}	Mercury	18.2×10^{-5}
Paraffin	58.8×10^{-5}	Water	20.7×10^{-5}
Ordinary Glass	2.5×10^{-5}	Ethyl alcohol	110×10^{-5}
Pyrex Glass	1×10^{-5}		

Table gives the average value of γ for some common substances in the temperature range $0 - 100^{\circ}\text{C}$. Solids and liquids have small value of γ . The materials Pyrex glass and invar (an alloy of iron and nickel) have still smaller values of γ . Ethyl alcohol has a higher value of γ than mercury and expands more than mercury for the same rise of temperature.

COEFFICIENT OF CUBICAL EXPANSION OF AN IDEAL GAS

Coefficient of cubical expansion of an ideal gas at constant pressure: For an ideal gas,

$$PV = nRT \quad \dots (i)$$

At constant pressure,

$$P \Delta T = nR \Delta T \quad \dots \text{(ii)} \quad [\because n \text{ and } R, \text{ are constants}]$$

Dividing (ii) by (i), we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T} \quad \text{or} \quad \frac{\Delta V}{V \Delta T} = \frac{1}{T}$$

or $\gamma = \frac{1}{T}$ Hence for an ideal gas, the coefficient of volume expansion decreases with the increase in temperature.

RELATION BETWEEN α , β AND γ

Relation between α , β and γ : Consider a cube of side l . Its original volume is $V = l^3$

Suppose the cube is heated so that its temperature increases by ΔT . Its each side will become

$$l' = l + \Delta l = l + \alpha l \Delta T = l(1 + \alpha \Delta T)$$

The new volume of the cube will be

$$V' = l'^3 = l^3 (1 + \alpha \Delta T)^3 = V (1 + 3 \alpha \Delta T) + 3 \alpha^2 \Delta T^2 + \alpha^3 \Delta T^3$$

As α is small, so the terms containing α^2 and α^3 can be neglected. Then

$$V' = V (1 + 3 \alpha \Delta T)$$

By the definition of the coefficient of cubical expansion,

$$\gamma = \frac{\Delta V}{V \Delta T} = \frac{V' - V}{V \Delta T} = \frac{V(1 + 3\alpha \Delta T) - V}{V \Delta T} = 3\alpha$$

Similarly, it can be proved that $\beta = 2\alpha$

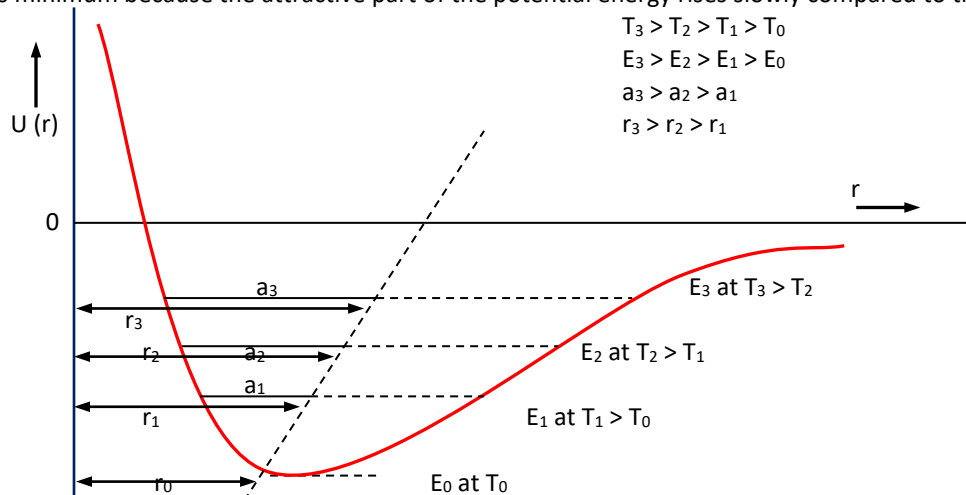
Hence $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$

Knowledge +.....

- The three coefficients of expansion α , β and γ are not constant for a given solid. Their values depend on the temperature range.
- For most of the solids, the value of α lies between 10^{-6} to 10^{-5} K^{-1} in the temperature range θ to 100° C . The value of α is more for ionic solids than that for non-ionic solids.
- The coefficient of linear expansion of a solid rod is independent of the geometrical shape of its cross-section.
- The coefficient of volume expansion of solids and liquids is rather small, particularly very small for Pyrex glass ($1 \times 10^{-5} \text{ K}^{-1}$) and invar (Fe – Ni alloy with $\gamma = 2 \times 10^{-6} \text{ K}^{-1}$).
- For an ideal gas γ varies inversely with temperature i.e., $\gamma = 1/T$. At 0° C or 273 K , $\gamma = 1/273 = 3.7 \times 10^{-3} \text{ K}^{-1}$, which is much larger than that for solids and liquids.
- Water contracts on heating between 0° C and 4° C . This is called anomalous expansion of water. It has the minimum volume and hence the maximum density (1000 kg m^{-3}) at 4° C . Silver iodide also contracts on heating between 80° C to 140° C .

MOLECULAR EXPLANATION OF THERMAL EXPANSION

Molecular explanation of thermal expansion: As shown in Fig. the graph between the potential energy $U(r)$ of two neighbouring atoms in a crystalline solid and their interatomic separation r is an asymmetric parabola. The potential energy curves is asymmetric about its minimum because the attractive part of the potential energy rises slowly compared to the repulsive part.



[P.E. $U(r)$ of two adjacent atoms in a crystalline solid versus interatomic separation r]

At the temperature $T_0 = 0 \text{ K}$, the atoms remain at the equilibrium separation r_0 and their oscillation energy E_0 is minimum. As the temperature increases, the energy of the atoms increases and they start vibrating about their equilibrium positions with the interatomic separation oscillating between its minimum and maximum values: r_{\min} and r_{\max} . The average interatomic separation becomes

$$r = \frac{r_{\min} + r_{\max}}{2}$$

Clearly, as the temperature increases, the amplitude of vibration of the atoms increases. Due to the asymmetry of the P.E. curve, the equilibrium position shifts to the right on the curve (as shown by the dashed inclined line) i.e., the average interatomic separation increases. It is thus in consequence of this increase in the average interatomic separation with temperature that a solid expands when heated.

PRACTICAL APPLICATIONS OF THERMAL EXPANSION

- (i) **A small gap is left between the iron rails of railway tracks:** The two rails are joined by fish plates. If no gap is left between the rails, the rails may bend due to expansion in summer and the train may get derailed.
- (ii) **Space is left between the girders used for supporting bridges:** This allows their expansion during summer. Moreover, the ends of the girders are placed on metal rollers to allow the expansion and contraction to take place easily with the change of season.
- (iii) **The iron ring to be put on the rim of a cart wheel is always of slightly smaller diameter than that of the wheel:** When the iron ring is heated to become red hot, it expands and slips on to the wheel easily. When it is cooled, it contracts and grips the wheel firmly.
- (iv) **Clock pendulums are made of invar:** Invar is an alloy. It has extremely small temperature coefficient not change with the change with the change of season and the clock gives almost correct time.
- (v) **A glass stopper jammed in the neck of a glass bottle can be removed by warming the neck of the bottle:** When the neck of the bottle is slightly warmed; its mouth becomes slightly wider. The stopper becomes loose and comes out easily.
- (vi) **Only platinum wire is used for fusing into glass:** This is because the coefficient of thermal expansion of platinum is almost the same as that of glass.

EXPANSION OF A LIQUID

When a liquid is heated, its container also expands. The observed expansion of the liquid is called apparent expansion which is different from the real expansion of the liquid.

Coefficient of apparent expansion: It is defined as the apparent increase in volume per unit original volume for 1°C rise in temperature. The coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Coefficient of real expansion: It is defined as the real increase in volume per unit original volume for 1°C rise in temperature. The coefficient of real expansion of the liquid is given by

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

It can be proved that

$$\gamma_r = \gamma_a + \gamma_g \quad \text{Where } \gamma_g \text{ is coefficient of cubical expansion of glass (material) of the container.}$$

VARIATION OF DENSITY WITH TEMPERATURE

When a given mass of a solid or a liquid is heated, its volume increases and hence density decreases. If V and V' are the volumes and ρ and ρ' are the densities of a given mass M at temperature T and $T + \Delta T$ respectively, then

$$V' = V (1 + \gamma \Delta T)$$

$$\text{or } \frac{M}{\rho} = \frac{M}{\rho'} (1 + \gamma \Delta T)$$

$$\rho = \rho'$$

$$\text{or } \rho' = \rho (1 + \gamma \Delta T)^{-1}$$

Expanding by Binomial theorem and neglecting the terms containing higher powers of $\gamma \Delta T$, we get

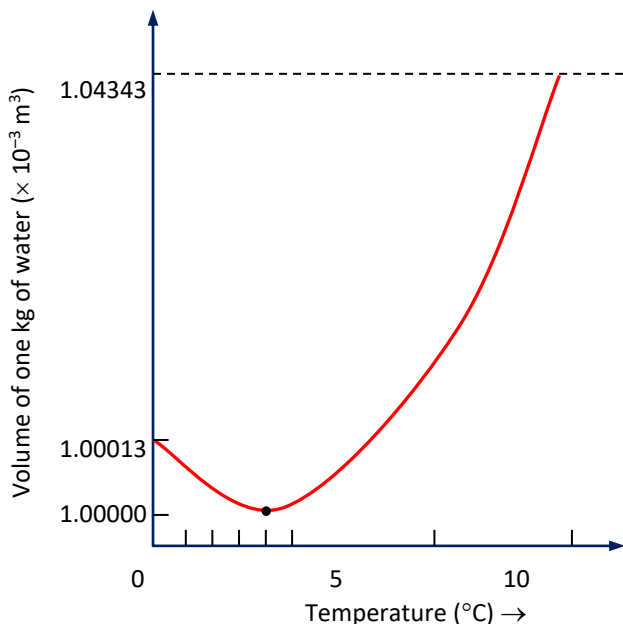
$$\rho' = \rho (1 - \gamma \Delta T)$$

Clearly, **the density of a solid or a liquid decrease with the increase in temperature.**

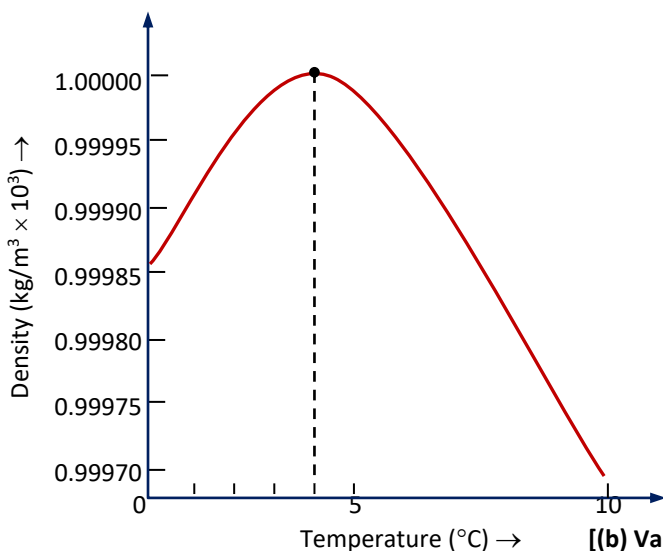
Anomalous expansion of water: Almost all liquids expand on being heated but water behaves in a peculiar manner. When water at 0°C is heated, its volume decreases and, therefore, its density increases, until its temperature reaches 4°C . Above 4°C , the volume increases, and therefore the density decreases. Thus, water at 4°C has the maximum density.

Fig. (a) shows the variation of volume of 1 kg of water as the temperature increases from 0°C to 100°C . Fig. (b) shows the variation of density of water with temperature from 0° to 10°C .

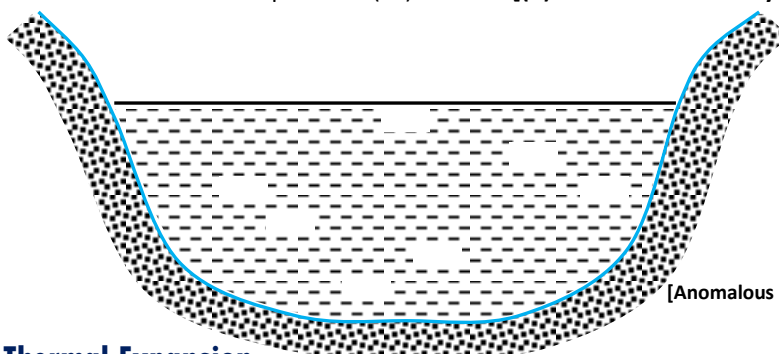
Practical important of anomalous expansion of water: The anomalous expansion of water has a favorable effect on aquatic life. Since the density of water is maximum at 4°C , water at the bottom of the lakes remains at 4°C even if it freezes at the top surface. This allows marine animals to remain alive and move freely near the bottom. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy the entire aquatic animal and plant life.



100
[(a) Thermal expansion of water]



[(b) Variation of density of water with temperature]



[Anomalous expansion of water helps aquatic life]

Examples based on Thermal Expansion

Formulae Used

1. Change in length, $l' - l = l \alpha (T' - T)$ or $\Delta l = l \alpha \Delta T$
2. Coefficient of linear expansion, $\alpha = \frac{\Delta l}{l \Delta T}$
4. Change in surface area, $S' - S = S \beta (t' - t)$ or $\Delta S = S \beta \Delta T$
5. Coefficient of superficial expansion, $\beta = \frac{\Delta S}{S \Delta T}$
7. Change in volume, $V' - V = V \gamma (t' - t)$ or $\Delta V = V \gamma \Delta T$
8. Coefficient of cubical expansion, $\gamma = \frac{\Delta V}{V \Delta T}$

3. Final length, $l' = l (1 + \alpha \Delta T)$

6. Final surface area, $S' = S (1 + \beta \Delta T)$

9. Final volume $V' = V (1 + \gamma \Delta T)$

10. Relation between α , β and γ

$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3} \quad \therefore \quad \beta = 2\alpha \quad \text{and} \quad \gamma = 3\alpha$

11. Final density, $\rho' = \rho (1 - \gamma \Delta T)$

Units Used

Lengths l, l' and Δl are in cm. Surface areas S, S' and ΔS are in cm^2 . Volumes V, V' and ΔV are in cm^3 . Temperatures t, t' and ΔT are in $^\circ\text{C}$ and coefficients α, β and γ are in $^\circ\text{C}^{-1}$.

Q. 1. Show that the coefficient of area expansions, $(\Delta A/A)/\Delta T$ of a rectangular sheet of the solid is twice its linear expansivity, α .

Sol. As shown in Fig. consider a rectangular sheet of the solid material of length a and breadth b . Suppose its temperature increases by ΔT .

$$\text{Increase in length } a, \quad \Delta a = \alpha a \Delta T$$

$$\text{Increase in breadth } b, \quad \Delta b = \alpha b \Delta T$$

$$\Delta A_1 = a (\Delta b)$$

$$\Delta A_3 = (\Delta a) (\Delta b)$$

$$\Delta b$$

$$b$$

$$a$$

$$\Delta a$$

$$\Delta A_2 = b (\Delta a)$$

Total increase in area of the sheet is

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$$

$$\Delta A = a\Delta b + b\Delta a + (\Delta a)(\Delta b) = \alpha ab \Delta T + b\alpha a \Delta T + (\alpha)^2 ab (\Delta T)^2 = \alpha ab \Delta T (2 + \alpha \Delta T) = \alpha A \Delta T (2 + \alpha \Delta T)$$

Since $\alpha \approx 10^{-5} \text{K}^{-1}$, the product $\alpha \Delta T$ for fractional temperature is small in comparison with 2 and may be neglected.

Therefore,
$$\beta = \left(\frac{\Delta A}{A} \right) \frac{1}{\Delta T} \approx 2\alpha$$

Q. 2. Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 66 m long be 3.63 cm at 10°C , then at what temperature will the lines just touch? Coefficient of linear expansion of steel = $11 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$.

Sol. As the rails expand in both directions, so the gap between two rails is filled by the expansion of half-length of each rail. Equivalently, we can take the expansion of one rail in one direction of fill the gap.

$$\therefore l = 66 \text{ m}, \Delta l = 3.63 \times 10^{-2} \text{ m},$$

$$\alpha = 11 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$$

As
$$\Delta l = l \alpha \Delta T$$

$$\therefore \Delta T = \frac{\Delta l}{l \alpha} = \frac{3.63 \times 10^{-2}}{66 \times 11 \times 10^{-6}} = 50^\circ\text{C}$$

$$\text{Final temperature} = \text{Initial temperature} + \Delta T = 10 + 50 = 60^\circ\text{C}$$

Q. 3. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0°C . What is the change in the diameter of the hole when the sheet is heated to 227°C ? Coefficient of linear expansion of copper = $1.70 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.

Sol. When the copper sheet is heated, the diameter of its hold increases in the same manner as the length of a rod.

$$\therefore l = 4.24 \text{ cm}, \quad \alpha = 1.70 \times 10^{-5} \text{ }^\circ\text{C}^{-1},$$

$$\Delta T = 227 - 27 = 200^\circ\text{C}$$

Increase in diameter,

$$\Delta l = l \alpha \Delta T = 4.24 \times 1.70 \times 10^{-5} \times 200 \text{ cm} = 1.44 \times 10^{-2} \text{ cm}$$

Q. 4. A steel tape 1 m long is correctly calibrated for a temperature of 27.0°C . The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C . What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C ? Coefficient of linear-expansion of steel = $1.20 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$?

Sol. Here $t_1 = 27^\circ\text{C}$, $l_1 = 63 \text{ cm}$, $t_2 = 45^\circ\text{C}$, $\alpha = 1.20 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$

Length of the rod on the hot day is

$$l_2 = l_1 [1 + \alpha (t_2 - t_1)] = 63 [1 + 1.20 \times 10^{-5} (45 - 27)] = 63.0136 \text{ cm}$$

As the steel tape has been calibrated for a temperature of 27°C , so length of the steel rod at 27°C = 63 cm.

Q. 5. A large steel wheel is to be fitted on to a shaft of the same material. At 27°C , the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice' (solid carbon dioxide). At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range. $\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{ K}^{-1}$.

Sol. Here $l_1 = 8.70 \text{ cm}$, $l_2 = 8.69 \text{ cm}$,

$$T_1 = 27 + 273 = 300 \text{ K}, T_2 = ?$$

As
$$l_2 - l_1 = \alpha l_1 (T_2 - T_1)$$

$$\therefore T_2 - T_1 = \frac{l_2 - l_1}{\alpha l_1}$$

$$\text{or } T_2 - 300 = \frac{8.69 - 8.70}{1.20 \times 10^{-5} \times 8.70} = -95.8$$

$$\text{or } T_2 = 300 - 95.8 = 204.2 \text{ K} = -68.8^\circ\text{C}$$

Q. 6. A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameters of the rim and the iron ring are 5.243 m and 5.231 m respectively at 27°C. To what temperature should the ring be heated so as to fit the rim of the wheel?

Sol. Here $T_1 = 27^\circ\text{C}$, $l_1 = 5.231\text{ m}$, $l_2 = 5.243\text{ m}$
 As $l_2 - l_1 = \alpha l_1 (T_2 - T_1)$
 $\therefore T_2 - T_1 = \frac{l_2 - l_1}{\alpha l_1} = \frac{5.243 - 5.231}{1.20 \times 10^{-5} \times 5.231} \approx 191^\circ\text{C}$
 or $T_2 = 191 + T_1 = 191 + 27 = 218^\circ\text{C}$

Q. 7. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original lengths are at 40.0°C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand. Coefficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ and that of steel = $1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.

Sol. For brass rod:
 $l = 50\text{ cm}$, $t_1 = 40^\circ\text{C}$, $t_2 = 250^\circ\text{C}$
 $\alpha = 2.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$
 Change in length of brass rod is
 $\Delta l = \alpha l (t_2 - t_1)$
 $= 2.0 \times 10^{-5} \times 50 \times (250 - 40) = 0.21\text{ cm}$
 For steel rod:
 $l = 50\text{ cm}$, $t_1 = 40^\circ\text{C}$, $t_2 = 250^\circ\text{C}$,
 $\alpha = 1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$
 Change in length of steel rod is
 $\Delta l' = \alpha l (t_2 - t_1) = 1.2 \times 10^{-5} \times 50 \times (250 - 40) = 0.13\text{ cm}$
 Change in length of the combined rod at 250°C = $\Delta l + \Delta l' = 0.21 + 0.13 = 0.34\text{ cm}$
 As the rods expand freely, so no thermal stress is development at the junction.

Q. 8. A brass wire 1.8 m long at 27°C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of -39°C, what is the tension developed in the wire, if its diameter is 2.0 mm? Coefficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, Young's modulus of brass = $0.91 \times 10^{11}\text{ Pa}$.

Sol. Here $l = 1.8\text{ m}$, $t_1 = 27^\circ\text{C}$, $t_2 = -39^\circ\text{C}$
 $r = \frac{2.0}{2} = 1.0\text{ mm} = 1.0 \times 10^{-3}\text{ m}$
 $\alpha = 2.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, $Y = 0.91 \times 10^{11}\text{ Pa}$
 As $\Delta l = l \alpha (t_2 - t_1)$ \therefore Strain, $\frac{\Delta l}{l} = \alpha (t_2 - t_1)$
 Stress = Strain \times Young's modulus = $\alpha (t_2 - t_1) \times Y = 2.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1} \times 0.91 \times 10^{11}\text{ Pa}$
 As $\Delta l = l \alpha (t_2 - t_1)$
 \therefore Strain, $\frac{\Delta l}{l} = \alpha (t_2 - t_1)$
 Stress = Strain \times Young's modulus
 $= \alpha (t_2 - t_1) \times Y = 2.0 \times 10^{-5} \times (-39 - 27) \times 0.91 \times 10^{11} = 1.2 \times 10^8 \text{ Nm}^{-2}$ [Numerically]
 \therefore Tension developed in the wire
 $= \text{Stress} \times \text{Area of cross-section} = \text{Stress} \times \pi r^2 = 1.2 \times 10^8 \times 3.14 \times (1.0 \times 10^{-3})^2 = 3.77 \times 10^2 \text{ N}$

Q. 9. A clock with an iron pendulum keeps correct time at 20°C. How much will it loss or gain if temperature changes to 40°C? Coefficient of cubical expansion of iron = $36 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$.

Sol. Time period of simple pendulum,
 $T_{20} = 2\text{ s}$
 Let T_{40} be the time period at 40°C. If l_0 , l_{20} , l_{40} be the lengths of the pendulum at 0°C, 20°C and 40°C respectively, then
 $l_{20} = l_0 (1 + 20 \alpha)$
 $l_{40} = l_0 (1 + 40 \alpha)$
 $T_{20} = 2\pi \sqrt{\frac{l_{20}}{g}} = 2\pi \sqrt{\frac{l_0 (1 + 20 \alpha)}{g}}$
 $T_{40} = 2\pi \sqrt{\frac{l_{40}}{g}} = 2\pi \sqrt{\frac{l_0 (1 + 40 \alpha)}{g}}$
 $\therefore \frac{T_{40}}{T_{20}} = \sqrt{\frac{1 + 40 \alpha}{1 + 20 \alpha}} = (1 + 40 \alpha)^{1/2} (1 + 20 \alpha)^{-1/2}$
 $= [1 + \frac{1}{2} \times 40 \alpha] [1 - \frac{1}{2} \times 20 \alpha]$ [Using Binomial theorem]
 $= (1 + 20 \alpha) (1 - 10 \alpha) = 1 + 10 \alpha$

$$\text{Fractional loss in time} = \frac{T_{40} - T_{20}}{T_{20}} = 10 \alpha$$

$$= 10 \times 1.2 \times 10^{-5} = 1.2 \times 10^{-4}$$

As the temperature increases, time period also increases. The clock runs slow.

$$\text{Time lost in 24 hours} = 1.2 \times 10^{-4} \times 24 \times 3600 = 10.368 \text{ s}$$

Q. 10. A metal ball 0.1 m in radius is heated from 273 to 348 K. Calculate the increases in surface area of the ball. Given coefficient of superficial expansion = 0.000034 K⁻¹.

Sol. Here $r_{273} = 0.1 \text{ m}$, $\Delta T = 348 - 273 = 75 \text{ K}$, $\beta = 0.000034 \text{ K}^{-1}$

$$S_{273} = 4\pi r_{273}^2 = 4\pi (0.1)^2 = \frac{4\pi}{100} \text{ m}^2$$

$$\text{Increase in surface area, } \Delta S = S_{273} \beta \Delta T = \frac{4\pi}{100} \times 0.000037 \times 75$$

$$= 3.206 \times 10^{-4} \text{ m}^2$$

Q. 11. On heating a glass block of 10,000 cm³, from 25 °C to 40 °C, its volume increases by 4 cm³. Calculate coefficient of linear expansion of glass.

Sol. Here $V = 10,000 \text{ cm}^3$

The coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V \cdot \Delta T} = \frac{4}{10,000 \times 15} = 26.67 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

\therefore Coefficient of linear expansion,

$$\alpha = \frac{\gamma}{3} = \frac{26.67 \times 10^{-6}}{3} = 8.89 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

Q. 12. If the volume of a block of metal changes by 0.12 % when it is heated through 20 °C, what is the coefficient of linear expansion of metal?

Sol. Here $\Delta V = 0.12 \% = 0.12$, $\Delta T = 20^\circ \text{C}$

Now $\gamma = \frac{\Delta V}{V \cdot \Delta T} = \frac{0.12}{100 \times 20} = 6.0 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$

$\therefore \alpha = \gamma/3 = 2.0 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$

Q. 13. The coefficient of volume expansion of glycerin is $49 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$. What is the fractional change in its density of a 30 °C rise in temperature?

Sol. Let M be the mass of glycerin, ρ_0 its density at 0°C, ρ_t its density at t°C. Then

$$\gamma = \frac{V_t - V_0}{V_0 \Delta T} = \frac{M/\rho_t - M/\rho_0}{(M/\rho_0) \Delta T} = \frac{1/\rho_t - 1/\rho_0}{(1/\rho_0) \Delta T} = \frac{\rho_0 - \rho_t}{\rho_0 \Delta T}$$

\therefore Fractional change in density, $\frac{\rho_0 - \rho_t}{\rho_0} = \gamma \Delta T = 49 \times 10^{-5} \times 30 = 0.0147$

Q. 14. Density ρ , mass m and volume V are related as $\rho = m/V$. Prove that

$$\gamma = -\frac{1}{\rho} \frac{d\rho}{dT}$$

Sol. Given $\rho = \frac{m}{V} = mV^{-1}$

Differentiating both sides w.r.t. temperature T, we get

$$\frac{d\rho}{dT} = -mV^{-2} \frac{dV}{dT} \quad [\because m = \text{constant}]$$

$$= -\frac{m}{V} \cdot \frac{dV}{V dT} = -\rho \gamma \quad \left(\because \frac{dV}{V \cdot dT} = \gamma \right)$$

$\therefore \gamma = -\frac{1}{\rho} \frac{d\rho}{dT}$

SPECIFIC HEAT

The specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance through one degree. It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is needed to raise the temperature of M mass of a substance through ΔT , then specific heat is given by

$$C = \frac{\Delta Q}{M \times \Delta T}$$

The CGS unit of specific heat is $\text{cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$ and the SI unit is $\text{J kg}^{-1} \text{ K}^{-1}$.

Clearly, the amount of heat required to raise the temperature of M mass of a substance through ΔT is

$$\Delta Q = Mc \Delta T.$$

•**Molar specific heat:** The molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through one degree. It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is required to raise the temperature of n moles of a substance through ΔT , then molar specific heat is given by

$$C = \frac{\Delta Q}{n \Delta T}$$

The CGS unit of molar specific heat is $\text{cal mol}^{-1} \text{ } ^\circ\text{C}^{-1}$ and SI unit is $\text{J mole}^{-1} \text{ K}^{-1}$.

Therefore, the amount of heat required to raise the temperature of n moles of a substance through ΔT is

$$\Delta Q = nC \Delta T$$

•**Heat capacity or thermal capacity:** The heat capacity of a body is defined as the amount of heat required to raise its temperature through one degree.

By definition, the amount of heat required to raise the temperature of unit mass of a body is equal to specific heat c . So, heat required for m mass is $m \times c$.

\therefore Heat capacity = Mass \times Specific heat

or $S = mc$

The CGS unit of heat capacity is $\text{cal } ^\circ\text{C}^{-1}$ and the SI unit is JK^{-1} .

•**Water equivalent:** The water equivalent of a body is defined as the mass of water which requires the same amount heat as is required by the given body for the same rise of temperature.

Water equivalent = Mass \times Specific heat

or $w = mc$

The CGS unit of water equivalent is g and the SI unit is kg .

CALORIMETRY

The branch of physics that deals with the measurement of heat is called calorimetry.

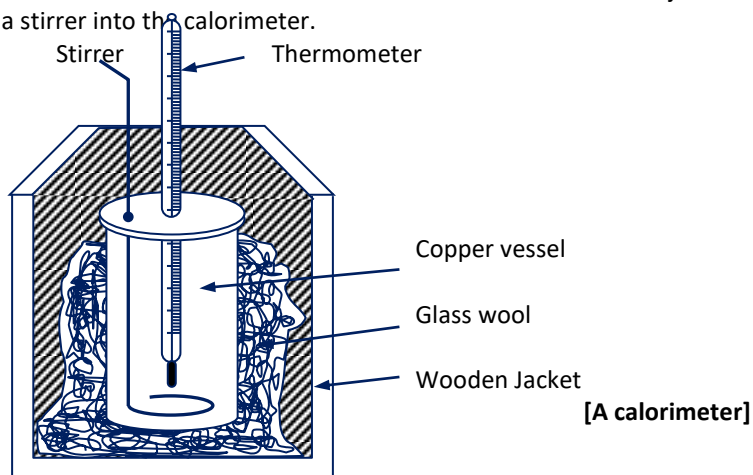
Principle of calorimetry or the law of mixtures: Whenever two bodies at different temperatures are placed in contact with one another, heat flows from the body at higher temperature to the body at lower temperature till the two bodies acquire the same temperature.

The principle of calorimetry states that the heat gained by the cold body must be equal to the heat lost by the hot body, provided there is no exchange of heat with the surroundings.

Heat gained = Heat lost

The principle is a consequence of the law of conservation of energy and useful for solving problems relating to calorimeter.

Calorimeter: It is a device used for measuring the quantities of heat. It consists of a cylindrical vessel of copper provided with a stirrer. The vessel is kept inside a wooden jacket. The space between the calorimeter and the jacket is packed with a heat insulating material like glass wool, etc. Thus, the calorimeter gets thermally isolated from the surroundings. The loss of heat due to radiation is further reduced by polishing the outer surface of the calorimeter and the inner surface of the jacket. The lid is provided with holes for inserting a thermometer and a stirrer into the calorimeter.



When bodies at different temperatures are mixed together in the calorimeter, heat is exchanged between the bodies as well as with the calorimeter. If there is no loss of heat to the surroundings, then according to the principle of calorimetry,

Heat gained by cold bodies = Heat lost by hot bodies.

This equation can be used to determine the specific heat and latent heat of different substances.

CHANGE OF STATE

•**Change of state:** Matter exists in three states solid, liquid and gas. Any state of a substance can be changed into another by heating or cooling it. The transition of a substance from one state to another is called a change of state.

The common changes of states are as follows:

1. Melting of a solid:

Heat

Solid → Liquid

2. Vaporization of a liquid:

Heat

Liquid → Vapour

3. Condensation of vapour:

Cool

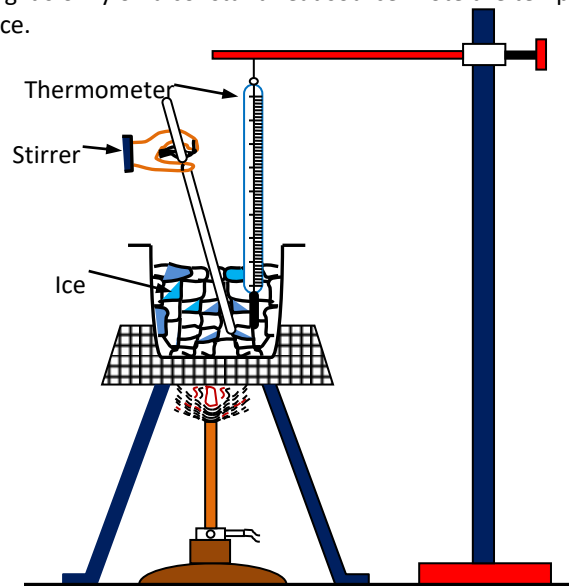
Vapour → Liquid

4. Freezing of a liquid:

Cool

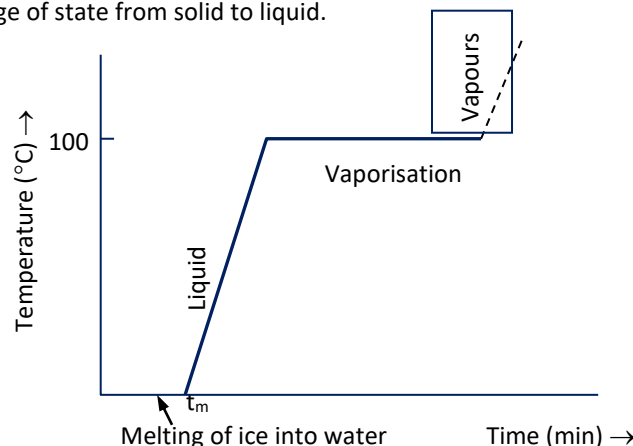
Liquid → Solid

•**Effect of heat on the change of state:** Take some ice cubes in a beaker. Note the temperature of ice. It will be 0°C . As shown in Fig. start heating it slowly on a constant heat source. Note the temperature after every minute. Continuously stir the mixture of water and ice.



[Effect of heat on ice]

Plot a graph between temperature recorded and time. We obtain a curve of the shape shown in Fig. It is found that the temperature does not change as long as there is any ice left in the beaker. Here the heat supplied is being used in changing the state from solid (ice) to liquid (water). The change of solid to liquid is called fusion. It is seen that the temperature remains constant until the entire amount of the solid substance melts. Thus, both the solid and liquid states of the substance coexist in thermal equilibrium during the change of state from solid to liquid.



[A plot of temperature versus time showing the changes in the states of ice on heating (not so scale)]

After the whole of ice gets melted into water and as we continue heating the beaker, we note that the temperature begins to increase till it reaches nearly 100°C when it again becomes steady. The heat supplied is now being used to change state of water from liquid to vapour. The change of state from liquid to vapour is called vaporisation. It is noted that the temperature remains constant until the entire amount of the liquid is converted into vapour. Thus, both the liquid and the vapour state of the substance coexist in thermal equilibrium.

•**Melting point:** The temperature at which the solid and the liquid states of a substance coexist in thermal equilibrium with each other is called its melting point. It is a characteristic of the substance but also depends on pressure. The melting point of a substance at standard atmospheric pressure is called its normal melting point.

•**Boiling point:** The temperature at which the liquid and vapour states of a substance coexist in thermal equilibrium with each other is called its boiling point. The boiling point of a substance at standard atmospheric pressure is called its normal boiling point.

•**Sublimation:** Some substances, on being heated, pass from the solid state to liquid state directly. **The process of transition of a substance from the solid state to the vapour state without passing through the liquid state is called sublimation, and the substance is said to sublime.** Substances like dry ice (solid CO_2), iodine, naphthalene and camphor undergo sublimation when heated. During the sublimation process, the solid and vapour states of a substance coexist in thermal equilibrium with each other.

EFFECT OF PRESSURE ON MELTING AND BOILING POINTS

•**Effect of pressure on melting point:** Under a given pressure; a pure substance melts at definite temperature. However, the melting point changes with the change in pressure. We know that paraffin wax expands on melting. An increase in pressure will make its expansion difficult. We will have to heat it more to melt it. That is, the increase in pressure increases the melting point of wax. ON the other hand, ice contracts on melting. The increase in pressure will help in its contraction. So, we expect at decrease in the melting point of ice as the pressure on it is increased. We can generalize these observations as follows:

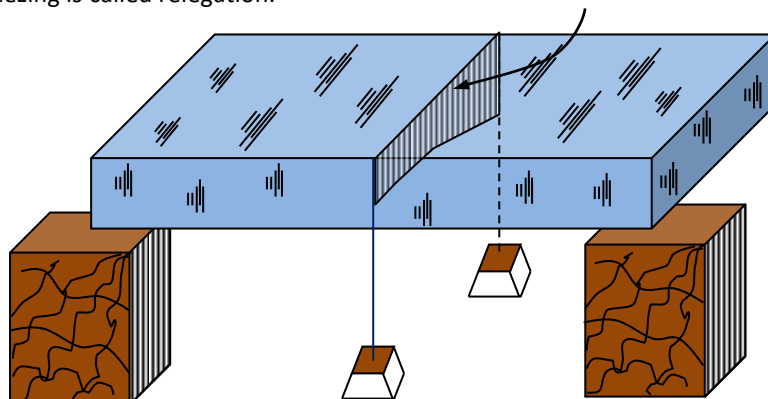
The melting point of those substances which expand on melting (e.g., paraffin wax, phosphorous, Sulphur, etc.) increases with the increase in pressure while the melting point of those substances which contract on melting (e.g., ice, cast iron, bismuth etc.) decreases with increase in pressure.

•**Effect of pressure on freezing point of ice: Regelation:** When two pieces of ice are pressed against one another for few seconds and then released, they get frozen at the surface of contact. As the pressure is increased, the melting point of ice is lowered and ice melts. When pressure is released, the water so formed (at a temperature $< 0^\circ\text{C}$) immediately freezes again. This phenomenon of refreezing is called regelation.

The phenomenon in which ice melts when pressure is increased and again freezes when pressure is removed is called regelation (re = again; gelare = freeze):

•Regelation through the simple experiment.

Take a slab of ice and support it on two wooden blocks. Take a metallic wire and attach two heavy weights, say 5 kg each, at its ends. Put the wire over the slab, as shown in Fig. It is seen that the wire gradually cuts its way through the ice without cutting it into two pieces. Just below the wire, ice melts at a lower temperature due to the increase in pressure. When the wire has passed, water above the wire freezes again. Thus, the wire passes through the slab and the slab does not split. This phenomenon of refreezing is called regelation.



□□ Practical applications of regelation:

1. By pressing snow in our hand, we can transform it into a snow-ball. When snow is pressed, its crystals melt. As the pressure is released, water refreezes forming a snow-ball.
2. When the wheels of cart pass over snow, ice melts due to increase in pressure exerted by the wheels. When pressure is released, water so formed refreezes on the wheels. That is why wheels are covered with ice.
3. Skating is possible due to the formation of water layer below the skates. Water is formed due to the increased of pressure and its acts as a lubricant.
4. The ice of a glacier, pressed against the sides of its valley melts, and in this way adopts itself to the shape of the valley.

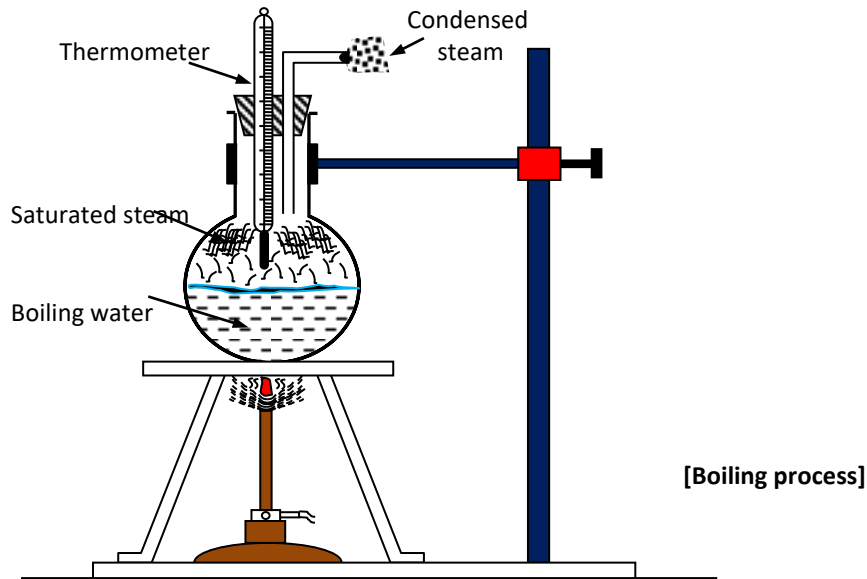
Effect of pressure on the boiling point of a liquid: The boiling point of a liquid increases with the increase in pressure. At 1 atmospheric pressure, the boiling point of water is 100°C . At 2 atmospheric pressures, the boiling point of water is 128°C .

Explanation: To study the effect of pressure on boiling point, take a round-bottom flask and fill it more than half with water. Support it over a burner and fix a thermometer and a steam cutlet through the cork fitted in the neck of the flask, as shown in Fig. Heat the flask over the burner slowly. As the temperature rises, small bubbles of water begin to form at the bottom which rise through water and escape from the surface. At a temperature of about $70^\circ - 80^\circ\text{C}$, bubbles of steam begin to form at the bottom. These bubbles of steam rise to the upper cold layers of water, where they condense and disappear producing of a peculiar noise called singing of the vessel.

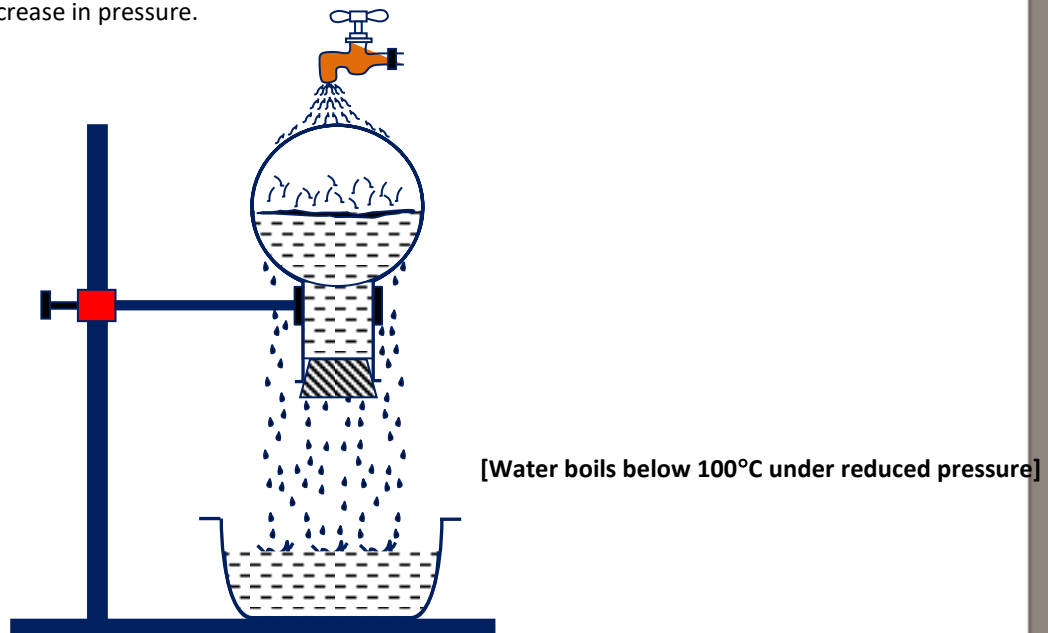
As the temperature of the entire mass of water becomes 100°C , the bubbles of steam begin to escape more rapidly and water gets violently agitated and boiling is said to occur. As the steam comes out of the flask, it condenses as tiny droplets of water, giving a foggy appearance.

If the steam outlet is partly closed, the pressure inside the flask increases and boiling stops. Water now has to be heated to a higher temperature to make it boil again. Thus, the boiling point increases with the increase in pressure.

Remove the burner and allow the water to cool to a temperature below 100°C. If the pressure is reduced by removing the partial covering of the outlet, water begins to boil again. Thus, the boiling point decreases with the decrease in pressure.



Franklin's experiment: Take a round-bottom flask and fill it about half with water. Heat it so that water begins to boil. Continue boiling for some time so that air of the flask escapes to the atmosphere and the empty space is filled with steam. Remove the burner. Close the flask with airtight cork. Hold the flask in the clamp of a stand in an inverted position, as shown in Fig. Allow the water to cool for some time so that boiling stops altogether. Pour cold water on the flask. Due to the condensation of steam, the pressure over the water surface decreases. Water begins to boil again now at a lower temperature. This shows that the boiling point decreases with the decrease in pressure.



□□

Practical Applications:

1. Cooking is difficult at mountains. The atmospheric pressure at mountains is much lower than that at plains, so water starts boiling at a temperature much lower than 100°C. Hence cooking is difficult.
2. The pressure inside a pressure cooker is increased much above the atmospheric pressure by not allowing the steam to escape. This increases the boiling point. Hence the vegetables are cooked inside a pressure cooker in a shorter time.

LATENT HEAT

Latent Heat: When solid changes into liquid or a liquid change into gas, it absorbs heat. But this heat does not show up as an increase in temperature. This heat, used to change the state, is hidden or latent (lying hidden), and is therefore called latent heat.

The amount of heat required changing the state of unit mass of a substance at constant temperature and pressure is called latent heat of the substance.

If m mass of a substance undergoes a change from one state to another, then the amount of heat required for the process

is $Q = mL$

Where L is the latent heat of the substance and is a characteristic of the substance. Its value also depends on the pressure.

Clearly,

$$L = Q/m$$

∴ SI unit of latent heat = J kg^{-1}

CGS unit of latent heat = cal g^{-1}

Latent heat of fusion: The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion or latent heat of melting. It is denoted by L_f .

Latent heat of vaporisation: The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point is called latent heat of vaporisation or latent heat of boiling. It is denoted by L_v .

$$L_f = 3.3 \times 10^5 \text{ J kg}^{-1}$$

That is, $3.3 \times 10^5 \text{ J}$ of heat is needed to melt 1 kg of ice into water at 0°C .

** Once the entire ice melts, the addition of more heat causes the temperature of water to rise. The temperature keeps on increasing till it reaches 100°C . The water starts boiling. The addition of more heat to the boiling water causes vaporisation, without increase in temperature. Obviously, the latent heat of vaporisation of water is,

$$L_v = 22.6 \times 10^5 \text{ J} \text{ That is, } 22.6 \times 10^5 \text{ J of heat is needed to convert 1 kg of water to steam at } 100^\circ\text{C}.$$

Additional heat causes the temperature of steam to rise.

By conservation of energy, when 1 kg of steam condenses to water, it gives up $22.6 \times 10^5 \text{ J}$ of heat. That is why steam burns are more serious than burns from boiling water, even though both are at 100°C .

Table: Temperatures of the change of state and latent heats for various substances at 1 atm pressure

Sub-stance	Melting point ($^\circ\text{C}$)	L_f (10^5 J kg^{-1})	Boiling point ($^\circ\text{C}$)	L_v (10^5 J kg^{-1})
Ethyl alcohol	-114	1.0	78	8.5
Gold	1063	0.645	2660	15.8
Lead	328	0.25	1744	8.67
Mercury	-39	0.12	357	2.7
Nitrogen	-210	0.26	-196	2.0
Oxygen	-219	0.14	-183	2.1
Water	0	3.33	100	22.6

Examples based on Specific Heat and Latent Heat

Formulae Used

- Heat gained or lost, $Q = mc \Delta T$
- According to the principle of calorimetry, Heat gained = Heat lost
- Water equivalent, $w = mc$ (gram)
- Heat capacity = mc ($\text{cal } ^\circ\text{C}^{-1}$)
- Latent heat of vaporisation or fusion, $Q = mL$

Units Used

In C.G.S. system, heat Q is in cal, mass m in gram, specific heat c in $\text{cal g}^{-1} ^\circ\text{C}^{-1}$ and ΔT in $^\circ\text{C}$. In SI, heat Q is in joule, mass m in kg, specific heat c in $\text{J kg}^{-1} \text{K}^{-1}$ and ΔT in K.

Q. 1. A geyser heats water flowing at the rate of 3.0 litres per minute from 27°C to 77°C . If the geyser the fuel if its heat of combustion is $4.0 \times 10^4 \text{ J/g}$?

Sol. Volume of water heated = $3.0 \text{ litre min}^{-1}$
Mass of water heated = 3000 g min^{-1}
Rise in temperature, $\Delta T = 77 - 27 = 50^\circ\text{C}$
Specific heat of water, $c = 4.2 \text{ J g}^{-1}$
Heat absorbed by water,

$$Q = mc \Delta T = 3000 \times 4.2 \times 50 = 63 \times 10^4 \text{ J min}^{-1}$$

Heat of combustion = $4.0 \times 10^4 \text{ Jg}^{-1}$

$$\text{Rate of combustion of fuel} = \frac{63 \times 10^4 \text{ J min}^{-1}}{4.0 \times 10^4 \text{ J g}^{-1}} = 15.75 \text{ g min}^{-1}$$

Q. 2. A 10-kW drilling machine is used to drill a bore in a small aluminum block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminum = $0.91 \text{ Jg}^{-1} \text{C}^{-1}$.

Sol. Power $P = 10 \text{ kW} = 10 \times 10^3 \text{ W}$
Time $t = 2.5 \text{ min} = 2.5 \times 60 \text{ s}$
Total energy used = $Pt = 10 \times 10^3 \times 2.5 \times 60 = 1.5 \times 10^6 \text{ J}$
Energy absorbed by aluminum block,
 $Q = 50\% \text{ of the total energy} = \frac{1.5 \times 10^6}{2} = 0.75 \times 10^6 \text{ J}$

Also, $m = 8.0 \text{ kg} = 8.0 \times 10^3 \text{ g}$, $c = 0.91 \text{ Jg}^{-1} ^\circ\text{C}^{-1}$

$$Q = mc \Delta T$$

$$\therefore \Delta T = \frac{Q}{mc} = \frac{0.75 \times 10^6}{8.0 \times 10^3 \times 0.91} = 103.02^\circ\text{C}$$

- Q. 3.** A sphere of aluminum of 0.047 kg is placed for sufficient time in a vessel containing boiling water, so that the sphere is at 100 °C. It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25 kg of water at 20 °C. The temperature of water rises and attains a steady state at 23 °C. Calculate the specific heat capacity of aluminium.

Sol. Mass of aluminum sphere, $m = 0.047 \text{ kg}$

Fall in temperature of aluminum sphere, $\Delta T = 100 - 23 = 77^\circ\text{C}$

Let specific heat of aluminum = c_{Al}

Heat lost by the aluminum sphere = $mc_{Al} \Delta T = 0.047 \text{ kg} \times c_{Al} \times 77^\circ\text{C}$

Mass of water, $m_1 = 0.25 \text{ kg}$ Mass of calorimeter, $m_2 = 0.14 \text{ kg}$

Initial temperature of water and calorimeter = 20°C

Final temperature of the mixture = 23°C

Rise in temperature of mixture, $\Delta T' = 23 - 20 = 3^\circ\text{C}$

Specific heat of water, $c_w = 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

Specific heat of copper, $c_{Cu} = 0.386 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

Heat gained by water and calorimeter = $m_1 c_w \Delta T' + m_2 c_{Cu} \Delta T' = (0.25 \times 4.18 \times 10^3 + 0.14 \times 0.386 \times 10^3) \times 3 \text{ J}$

In the steady state, Heat lost = Heat gained

or $0.047 \times c_{Al} \times 77 = (0.25 \times 4.18 \times 10^3 + 0.14 \times 0.386 \times 10^3) \times 3$

or $c_{Al} = 0.911 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} = 0.911 \text{ kJ kg}^{-1} \text{ K}^{-1}$

- Q. 4.** In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150 °C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm³ of water at 27 °C. The final temperature is 40 °C. compute the specific heat of the metal.

Sol. Mass of metal block, $m = 0.20 \text{ kg} = 200 \text{ g}$

Fall in temperature of metal block, $\Delta T = 150 - 40 = 110^\circ\text{C}$

Let specific heat of metal block = $c \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$

\therefore Heat lost by metal block = $mc \Delta T = 200 \times c \times 11 \text{ cal}$

Volume of water in calorimeter = 150 cm^3

Mass of water, $m' = 150 \text{ g}$

Water equivalent of calorimeter, $w = 0.025 \text{ kg} = 25 \text{ g}$

Specific heat of water, $c' = 1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$

\therefore Heat gained by water and calorimeter = $(m' + w) c' \Delta T' = (150 + 25) \times 1 \times (40 - 27) \text{ cal} = 175 \times 13 \text{ cal}$

By principle of calorimetry,

Heat lost = Heat gained $\therefore 200 \times c \times 110 = 175 \times 13$

or $c = \frac{175 \times 13}{200 \times 110} = 0.1 \text{ Cal g}^{-1} \text{ }^\circ\text{C}^{-1}$.

- Q. 5.** A child running a temperature of 101 °F is given an antipyrene (i.e., a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98 °F in 20 min, what is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and learnt heat of evaporation of water at that temperature is about 580 Cal g⁻¹.

Sol. Mass of child, $M = 30 \text{ kg} = 30 \times 10^3 \text{ g}$

Fall in temperature,

$$\Delta T = 100 - 98 = 3^\circ\text{F} = 3 \times \frac{5}{9} = \frac{5}{3}^\circ\text{C}$$

Specific heat of human body, $c = \text{specific heat of water} = 1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$

Heat lost by child in the form of evaporation of sweat,

$$Q = Mc \Delta T = 30 \times 10^3 \times 1 \times 5 \times \frac{5}{3} = 50,000 \text{ cal}$$

If M' gram of sweat evaporates from the body of the child, then heat gained by sweat

$$Q = M' L = M' \times 580 \text{ cal} \quad [\because L = 580 \text{ cal g}^{-1}]$$

\therefore Heat gained = Heat lost

$$\therefore M' \times 580 = 50000 \quad \text{or} \quad M' = \frac{50000}{580} = 86.2 \text{ g}$$

Time taken by sweat to evaporate = 20 min

$$\therefore \text{Rate of evaporation of sweat} = \frac{86.2}{20} = 4.31 \text{ g min}^{-1}$$

- Q. 6.** A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500 °C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = 0.39 Jg⁻¹, and heat of fusion of water = 335 Jg⁻¹).

Sol. Mass of copper block,
 $M = 2.5 \text{ kg} = 2.5 \times 10^3 \text{ g}$
 Specific heat of copper, $c = 0.39 \text{ J g}^{-1} \text{ }^\circ\text{C}^{-1}$
 Fall in temperature, $\Delta T = 500 - 0 = 500^\circ \text{C}$

Heat lost by copper block = $mc \Delta T = 2.5 \times 10^3 \times 0.39 \times 500^\circ \text{C}$
 Heat lost by copper block = $mc \Delta T = 2.5 \times 10^3 \times 0.39 \times 500 \text{ J}$

Let mass of ice melted = M gram
 Heat of fusion of ice, $L = 335 \text{ Jg}^{-1}$
 Heat gained by ice = $ML = M \times 335 \text{ J}$
 \therefore Heat gained = Heat lost
 $\therefore M \times 335 = 2.5 \times 10^3 \times 0.39 \times 500$
 or $M = \frac{2.5 \times 10^3 \times 0.39 \times 500}{335}$

$$= 1455.2 \text{ g} = 1.455 \text{ kg}$$

Q. 7. A thermally isolated vessel contains 100 g of water of 0°C when air above the water is pumped out, some of the water freezes and some evaporates at 0°C itself. Calculate the mass of ice formed, if no water is left in the vessel. Latent heat of vaporisation of water at $0^\circ \text{C} = 2.10 \times 10^6 \text{ J kg}^{-1}$ and latent heat of fusion of ice = $3.36 \times 10^5 \text{ J kg}^{-1}$.

Sol. Latent heat of vaporisation of water at 0°C ,
 $L_1 = 2.10 \times 10^6 \text{ J kg}^{-1} = 2.10 \times 10^3 \text{ Jg}^{-1}$
 Latent heat of fusion of ice,
 $L_2 = 3.36 \times 10^5 \text{ J kg}^{-1} = 3.36 \times 10^2 \text{ Jg}^{-1}$
 Let mass of ice formed = m gram
 Then mass of water evaporated = $(100 - m)$ gram
 As no water is left in vessel,
 Heat gained by water in evaporation = Heat lost by water in freezing
 or $(100 - m) L_1 = mL_2$
 or $(100 - m) \times 2.10 \times 10^3 = m \times 3.36 \times 10^2$
 or $m = 86.2 \text{ g}$

Q. 8. When 0.15 kg of ice of 0°C mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C . Calculate the heat of fusion of ice. ($c_{\text{water}} = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$).

Sol. Heat lost by 0.30 kg water when its temp. Falls from 50°C to 6.7°C
 $= mC_{\text{water}} \Delta T = 0.30 \times 4186 \times (50 - 6.7) = 54376.14 \text{ J}$
 Heat required to melt 0.15 kg ice into water at $0^\circ \text{C} = mL_f = 0.15 \times L_f \text{ J}$
 Heat required to raise temperature of 0.15 kg water from 0°C to 6.7°C
 $= mC_{\text{water}} \Delta T = 0.15 \times 4186 \times (6.7 - 0) = 4206.93 \text{ J}$
 By the principle of calorimetry,
 Heat gained = Heat lost
 $0.15 + L_f = 4206.93 = 54376.14$ or $L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$

Q. 9. Calculate the heat required to convert 3 kg of ice at -12°C kept in a calorimeter to steam at 100°C at atmospheric pressure. Given: Specific heat capacity of ice = $2100 \text{ J kg}^{-1} \text{ K}^{-1}$,
 Specific heat capacity of water = $4186 \text{ J kg}^{-1} \text{ K}^{-1}$
 Latent heat of fusion of ice = $3.35 \times 10^5 \text{ J kg}^{-1}$
 and Latent heat of steam = $2.256 \times 10^6 \text{ J kg}^{-1}$

Sol. Mass of the ice, $m = 3 \text{ kg}$
 Specific heat capacity of ice, $C_{\text{ice}} = 2100 \text{ Jkg}^{-1} \text{ K}^{-1}$
 Specific heat capacity of water, $C_{\text{water}} = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$
 Latent heat of fusion of ice, $L_{f, \text{ice}} = 3.35 \times 10^5 \text{ J kg}^{-1}$
 Latent heat of steam, $L_{\text{steam}} = 2.256 \times 10^6 \text{ J kg}^{-1}$
 Heat required to convert ice at -12°C to ice at 0°C ,
 $Q_1 = mC_{\text{ice}} \Delta T_1 = (3 \text{ kg}) (2100 \text{ J kg}^{-1} \text{ K}^{-1}) (0 - (-12))^\circ \text{C} = 75600 \text{ J}$
 Heat required to melt ice at 0°C to water at 0°C
 $Q_2 = mL_{f, \text{ice}} = (3 \text{ kg}) (3.35 \times 10^5 \text{ J kg}^{-1}) = 1005000 \text{ J}$
 Heat required to convert water at 0°C to water at 100°C .
 $Q_3 = mC_w \Delta T_2 = (3 \text{ kg}) (4186 \text{ J kg}^{-1} \text{ K}^{-1}) (100^\circ \text{C}) = 6768000 \text{ J}$
 Total heat required to convert 3 kg of ice at -12°C to steam at 100°C ,
 $Q = Q_1 + Q_2 + Q_3 + Q_4 = 75600 \text{ J} + 1005000 \text{ J} + 1255800 \text{ J} + 6768000 \text{ J} = 9.1 \times 10^6 \text{ J}$

MODES OF TRANSFER OF HEAT:

Heat can be transferred from one place to another by three different methods. These are

- ☞ (i) conduction,
- ☞ (ii) convection and
- ☞ (iii) radiation.

Solids are usually heated by the process of conduction. Liquid and gases are heated by the process of convection. The process of radiation requires no intervening medium.

** We receive heat from the sun by the process of radiation.

** Conduction and convection are slow processes while radiation is a very fast process.

THERMAL CONDUCTION

Thermal conduction is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any flow of matter.

Molecular mechanism of thermal conduction: Solids are heated through conduction. When one end of a metal rod is heated, the molecules at the hot end vibrate with greater amplitude. So, they have greater average kinetic energy. As these molecules collide with the neighbouring molecules of lesser kinetic energy, the energy is shared between them. The kinetic energy of the neighbouring molecules increases. This energy transfer takes place from one layer to the next, without the molecules leaving their average location. This way, heat is passed to the colder end of the rod.

STEADY STATE AND TEMPERATURE GRADIENT

Variable state, steady state and temperature gradient: Consider a metal rod heated at one end A. Heat flows from the hot end A to the cold end B by conduction.

In the process of conduction, each cross-section of the rod receives heat from the adjacent cross-section of the hotter side. A part of this heat is absorbed by the cross-section itself whose temperature increases, and radiation from the sides of cross-section and the rest is conducted to the next cross-section. In this state the temperature of every cross-section of the rod goes on increasing with time. The rod is said to be in the variable state of heat conduction.

Suppose the sides of the rod are covered with some insulating material so that no heat is lost from the sides to the surroundings. After some time, a steady state is reached when the temperature of every cross-section of the rod becomes constant. In this state, no heat is absorbed by the rod. This state of the rod when temperature of every cross-section of the rod becomes constant and there is no further absorption of heat in any part is called steady state.

During steady state,

- (i) the temperatures of two different parts of the rod are different, but the temperature of each part remains constant.
- (ii) every transverse section of the rod is an isothermal surface .
- (iii) the temperature decreases as we move away from the hot end.
- (iv) the quantity of heat flowing per second through every cross-section is constant.

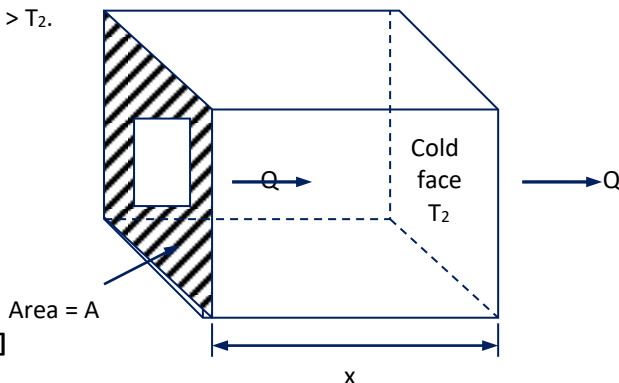
The rate of change of temperature with distance in the direction of flow of heat is called temperature gradient.

If T_1 and T_2 are the temperatures of two isothermal surfaces separated by distance x , then

$$\text{Temperature gradient} = \frac{T_1 - T_2}{x}$$

THERMAL CONDUCTIVITY

As shown in Fig. consider a block of a material of cross-sectional area A and thickness x . Suppose its opposite faces are at temperatures T_1 and T_2 , with $T_1 > T_2$.



[Thermal conductivity]

It is found that the amount of heat Q that flows from hot to cold face during the steady state

- (i) is directly proportional to the cross-sectional area A ,
- (ii) is directly proportional to the temperature difference $(T_1 - T_2)$ between the opposite faces,
- (iii) is directly proportional to time t for which the heat flows,
- (iv) is inversely proportional to thickness x of the block, and
- (v) depends on the nature of the material of the block.

$$\therefore Q \propto \frac{A(T_1 - T_2)t}{x}$$

or
$$Q = \frac{KA(T_1 - T_2)t}{x}$$

The proportionality constant K is called **coefficient of thermal conductivity** of the given material.
 Its value depends on the nature of the material.

If $A = 1$, $T_1 - T_2 = 1$, $t = 1$, $x = 1$, then $Q = K$

Hence the **coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.**

△ If the area of cross-section is not uniform or if the steady state condition is not reached, then we consider a thin layer of the material normal to the direction of heat flow. If A be the area of the cross-section at a place, dx be a small thickness along the direction of heat flow and dT be the temperature difference across this thickness dx, then the rate of flow of heat or heat current H will be

$$H = dQ = -KA \frac{dT}{dx}$$

The quantity **dT/dx is called the temperature gradient**. The negative sign indicates that dT/dx is negative in the direction of flow of heat i.e., temperature decreases along the positive x – direction. Thus, the negative sign in the above equation ensures that K is positive.

Units of dimensions of K: The numerical value of K is

$$K = \frac{Q \cdot x}{A(T_1 - T_2)t}$$

∴ SI unit of K = $\frac{J \cdot m}{m^2 \cdot K \cdot s}$

$$= J s^{-1} m^{-1} K^{-1} \text{ or } W m^{-1} K^{-1}$$

$$\text{CGS unit of K} = \text{cal s}^{-1} \text{cm}^{-1} \text{°C}^{-1}$$

$$\text{Dimensions of K} = \frac{[ML^2 T^{-2}] \cdot [L]}{[L^2] \cdot [K] [T]} = [MLT^{-3} K^{-1}]$$

Solids are better conductors than gases. **Moreover, metals are much better thermal conductors than the non-metals.**

This is because metals have a large number of free electrons which carry heat from hotter to colder regions very fastly. For most of the materials the value of K increases slightly with temperature.

Table: Thermal conductivities of some common materials

Materials	Thermal conductivity in $J s^{-1} m^{-1} K^{-1}$
METALS	
Silver	406
Copper	385
Aluminium	205
Brass	109
Steel	50.2
Lead	34.7
Mercury	8.3
NON-METALS	
Insulating brick	0.15
Concrete	0.8
Body fat	0.20
Felt	0.04
Glass	0.8
Ice	1.6
Rock Wool	0.04
Wood	0.12 – 0.04
Water	0.8
GASES	
Air	0.024
Argon	0.016
Hydrogen	0.14

HEAT CURRENT AND THERMAL RESISTANCE

Heat current and thermal resistance: We know that charge flows in a circuit due to potential difference between its two points. The flow of charge per unit time is called electric current. Similarly, heat flows in a conductor due to temperature difference between its two points. The flow of heat per unit time is called heat current, denoted by H. Thus

$$H = \frac{Q}{t}$$

Its SI unit is $J s^{-1}$ or watt (W).

From Ohm's law, electric resistance is given by

$$R = \frac{V}{I}$$

That is electric resistance is the ratio of the potential difference and the electric current. Similarly, the ratio of the temperature difference between the ends of conductors to the heat current through it is called thermal resistance, denoted by R_H . Thus

$$R_H = \frac{\Delta T}{H}$$

As $Q = KA \frac{\Delta T}{\Delta x} t$

$\therefore H = \frac{Q}{t} = KA \frac{\Delta T}{\Delta x}$

and $R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$

Hence greater the coefficient of thermal conductivity of a material, smaller is the thermal resistance of rod of that material. Units and dimensions of $R_H = \Delta x / KA$

$$\begin{aligned} \text{SI unit of } R_H &= \frac{K}{J s^{-1}} \\ &= \frac{K}{W} = KW^{-1} \end{aligned}$$

$$\text{Dimensions of } R_H = \frac{[K]}{[ML^2 T^{-2}] \cdot [T^{-1}]} = [M^{-1} L^{-2} T^{-3} K]$$

Examples based on Thermal Conductivity

Formulae Used

1. The amount of heat that flows in time t across the opposite faces of a slab of thickness x and cross-section A ,

$$Q = \frac{KA(T_1 - T_2)t}{x}$$

where T_1 and T_2 are the temperatures of hot and cold faces and K is the coefficient of thermal conductivity of the material of the slab.

2. $dQ = -KA \frac{dT}{dx}$

Here dT/dx is the rate of fall of temperature with distance and is called temperature gradient.

Units Used

SI unit of K is $J s^{-1} m^{-1} K^{-1}$ and CGS unit is $cal s^{-1} cm^{-1} ^\circ C^{-1}$.

Q. 1. Calculate the rate of loss of heat through a glass window of area 1000 cm^2 and thickness 0.4 cm when temperature inside is $37^\circ C$ and outside is $-5^\circ C$. Coefficient of thermal conductivity of glass is $2.2 \times 10^{-3} \text{ cal s}^{-1} \text{ cm}^{-1} K^{-1}$.

Sol. Here $A = 1000 \text{ cm}^2$, $x = 0.4 \text{ cm}$.

$$T_1 - T_2 = 37 - (-5) = 42^\circ C,$$

$$K = 2.2 \times 10^{-3} \text{ cal s}^{-1} \text{ cm}^{-1} K^{-1}$$

Rate of loss of heat,

$$\begin{aligned} H &= \frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} \\ &= \frac{2.2 \times 10^{-3} \times 1000 \times 42}{0.4} = 231 \text{ cal s}^{-1} \end{aligned}$$

Q. 2. A 'thermocole' cubical icebox of side 30 cm has a thickness of 5.0 cm . If 4.0 kg of ice are put in the box, estimate the amount of ice remaining after 6 h . The outside temperature is $45^\circ C$ and coefficient of thermal conductivity of thermocole = $0.01 \text{ Js}^{-1} \text{ m}^{-1} ^\circ C^{-1}$. Given heat of fusion of water = $335 \times 10^3 \text{ J kg}^{-1}$.

Sol. Here $A = 6 \times \text{side}^2 = 6 \times 30 \times 30$, $= 5400 \text{ cm}^2 = 0.54 \text{ m}^2$,

$$x = 5 \text{ cm} = 0.05 \text{ m}$$

$$t = 6 \text{ h} = 6 \times 3600 \text{ s}, T_1 - T_2 = 45 - 0 = 45^\circ C$$

$$K = 0.01 \text{ Js}^{-1} \text{ m}^{-1} ^\circ C^{-1},$$

$$L = 335 \times 10^3 \text{ J kg}^{-1}$$

Total heat entering the box through all the six faces,

$$Q = \frac{KA(T_1 - T_2)t}{x} = \frac{0.01 \times 0.54 \times 45 \times 6 \times 3600}{0.05} = 104976 \text{ J}$$

Let $m \text{ kg}$ of ice melt due to this heat. Then,

$$Q = mL$$

$$\text{or } m = \frac{Q}{L} = \frac{104976 \text{ J}}{335 \times 10^3 \text{ J kg}^{-1}} = 0.313 \text{ kg}$$

Mass of ice left after six hours $= 4 - 0.313 = 3.687 \text{ kg}$

Q. 3. A brass boiler has a base are of 0.15 m^2 and thickness 1.0 cm . It boils water at the rate of 6.0 kg min^{-1} , when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = $109 \text{ Js}^{-1} \text{ m}^{-1} ^\circ C^{-1}$ and heat of vaporisation of water = 2256 Jg^{-1} .

Sol. Here $A = 0.15 \text{ m}^2$,

$$x = 1.0 \text{ cm} = 0.01 \text{ m}, \quad K = 109 \text{ Js}^{-1} \text{ m}^{-1} \text{ }^\circ\text{C}^{-1}$$

$$L = 2256 \text{ Jg}^{-1}, \quad T_2 = 100^\circ\text{C}, \quad t = 1 \text{ min} = 60 \text{ s}$$

Let T_1 be the temperature of the part of the flame in contact with boiler. Then amount of heat that flows into water in 1 min,

$$Q = \frac{KA(T_1 - T_2)t}{x} = \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} \text{ J}$$

Mass of water boiled per min = 6 kg = 6000 g

Heat used to boil water,

$$Q = mL = 6000 \text{ g} \times 2256 \text{ J kg}^{-1} = 6000 \times 2256 \text{ J}$$

$$\therefore \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} = 6000 \times 2256$$

$$\text{or } T_1 - 100 = \frac{6000 \times 2256 \times 0.01}{109 \times 0.15 \times 60} = 138^\circ\text{C}$$

$$\text{or } T_1 = 138 + 100 = 238^\circ\text{C}$$

Q. 4. Steam at 100°C is passed into a copper cylinder 10 mm thick and of 200 cm^2 area. Water at 100°C collects at the rate of 150 g min^{-1} . Find the temperature of the outer surface, if the conductivity of copper is $0.8 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ }^\circ\text{C}^{-1}$ and the latent heat of steam is 540 cal g^{-1} .

Sol. Here $T_1 = 100^\circ\text{C}$, $x = 10 \text{ mm} = 1 \text{ cm}$,

$$A = 200 \text{ cm}^2, \quad m = 150 \text{ g}, \quad t = 1 \text{ min} = 60 \text{ s},$$

$$K = 0.8 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ }^\circ\text{C}^{-1}, \quad L = 540 \text{ cal g}^{-1}$$

Heat drawn from steam,

$$Q = mL = 150 \times 540 \text{ cal}$$

$$\text{As } Q = \frac{KA(T_1 - T_2)t}{x}$$

$$\therefore 150 \times 540 = \frac{0.8 \times 200 (100 - T_2) \times 60}{1}$$

$$\text{or } 100 - T_2 = \frac{150 \times 540}{0.8 \times 200 \times 60} = 8.44^\circ\text{C}$$

$$\therefore T_2 = 100 - 8.44 = 91.56^\circ\text{C}$$

Q. 5. A metal rod of length 20 cm and diameter 2 cm is covered with non-conducting substance. One of its ends is maintained at 100°C , while the other end is put in ice at 0°C . It is found that 25 g of ice melts in 5 min. Given latent heat of ice = 80 cal g^{-1} .

Sol. Here length of rod, $x = 20 \text{ cm}$.

$$T_1 = 100^\circ\text{C}, \quad T_2 = 0^\circ\text{C}, \quad t = 5 \text{ min} = 300 \text{ s},$$

Diameter of rod, $d = 2 \text{ cm}$

$$\text{Area of cross-section, } A = \frac{\pi d^2}{4} = \frac{\pi \times (2)^2}{4} = \pi \text{ cm}^2$$

Mass of ice melted, $m = 25 \text{ g}$

Latent heat of ice, $L = 80 \text{ cal g}^{-1}$

Heat used in melting ice,

$$Q = mL = 25 \times 80 = 2000 \text{ cal}$$

$$\text{As } Q = \frac{KA(T_1 - T_2)t}{x}$$

$$\therefore K = \frac{Qx}{A(T_1 - T_2)t} = \frac{2000 \times 20}{\pi \times (100 - 0) \times 300} = \frac{2000 \times 20}{3.142 \times 100 \times 300} = .424 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ }^\circ\text{C}^{-1}$$

Q. 6. A layer of ice 2 cm thick is formed on a pond. The temperature of air is -20°C . Calculate how long it will take for the thickness of ice to increase by 1 mm. Density of ice = 1 g cm^{-3} . Latent heat of ice = 80 cal g^{-1} . Conductivity of ice = $0.008 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ }^\circ\text{C}^{-1}$.

Sol. Let surface area of the ice layer = $A \text{ cm}^2$

Thickness of ice layer to be formed = $1 \text{ mm} = 0.1 \text{ cm}$

$$\therefore \text{Volume of the ice to be formed, } V = A \times 0.1 = 0.1 A \text{ cm}^3$$

$$\text{Mass of the ice to be formed, } M = \text{Volume} \times \text{Density} = 0.1 A \times 1 = 0.1 A \text{ gram}$$

In order to increase the thickness of ice layer, heat has to be conducted from the pond to air through ice layer already formed.

If Q is the amount of heat conducted from the pond to the air, to form 0.1 A gram of ice, then

$$Q = mL = 0.1 A \text{ g} \times 80 \text{ cal g}^{-1} = 8 A \text{ cal}$$

As thickness of layer through which heat is being conducted increases continuously, so we calculate mean thickness x of the layer.

$$x = \frac{\text{Initial thickness} + \text{Final thickness}}{2}$$

$$= \frac{2 \text{ cm} + 2.1 \text{ cm}}{2} = 2.05 \text{ cm}$$

Temperature differences on two sides of ice layer,

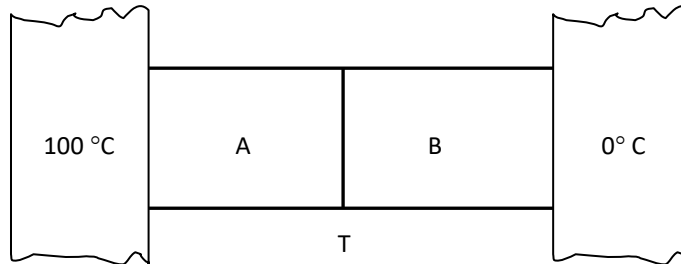
$$T_1 - T_2 = 0 - (-20) = 20^\circ \text{C}$$

Let t be the time taken by the ice layer to increase thickness by 1 mm. Then

$$Q = \frac{KA(T_1 - T_2)t}{x}$$

$$\text{or } t = \frac{Qx}{KA(T_1 - T_2)} = \frac{8A \times 2.05}{0.008 \times A \times 20} = 102.5 \text{ s}$$

- Q. 7.** Two metals cubes A and B of same size are arranged as shown in Fig. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The co-efficient of thermal conductivity of A and B are $300 \text{ W/m}^\circ\text{C}$ and $200 \text{ W/m}^\circ\text{C}$, respectively. After steady state is reached, what will be the temperature T of the interface?



Sol. In the steady-state,
Rate of flow of heat through cube A = Rate of flow of heat through cube B.

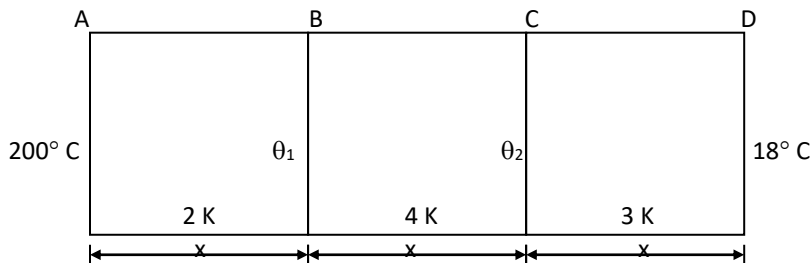
$$\text{or } \frac{K_1 A (100 - T)}{x} = \frac{K_2 A (T - 0)}{x}$$

$$\text{or } \frac{300 A (100 - T)}{x} = \frac{200 A (T - 0)}{x}$$

$$\text{or } 300 - 3T = 2T \quad \text{or } 5T = 300 \quad \therefore T = 60^\circ \text{C}$$

- Q. 8.** Three bars of equal lengths and equal area of cross-section are connected in series. Their thermal conductivities are in the ratio of $2 : 4 : 3$. If the open ends of the first and the last bars are at temperatures 200°C and 18°C respectively in the steady state, calculate the temperature of both the junctions.

Sol. Let θ_1 and θ_2 be the temperature of junctions B and C respectively.



In the steady state, the rate of flow of heat through each bar will be same.

$$\therefore \frac{Q}{t} = \frac{2K \times A (200 - \theta_1)}{x}$$

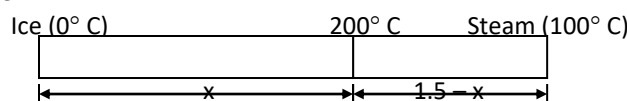
$$= \frac{4K \times A (\theta_1 - \theta_2)}{x} = \frac{3K \times A (\theta_2 - 18)}{x}$$

$$\text{or } 2(200 - \theta_1) = 4(\theta_1 - \theta_2) = 3(\theta_2 - 18)$$

On solving, we get: $\theta_1 = 116^\circ\text{C}$ and $\theta_2 = 74^\circ\text{C}$

- Q. 9.** One end of a copper rod of uniform cross-section and of length 1.5 m is kept in contact with ice and the other end with water at 100°C . At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melted be equal to that of the steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings. Latent heat of fusion of ice = 80 cal g^{-1} . Latent heat of vaporisation of water = 540 cal g^{-1} .

Sol. Let the temperature at 200°C be maintained at a distance x from the end of 0°C . Heat will flow from this point towards both ice and water.



If m is the mass of ice melted = mass of steam produced, then

$$m \times L_{\text{ice}} = \frac{KA(200 - 0)}{x} \quad \dots \text{(i)}$$

$$m \times L_{\text{steam}} = \frac{KA(200 - 100)}{1.5 - x} \quad \dots \text{(ii)}$$

Dividing (i) by (ii), we get

$$\frac{L_{\text{ice}}}{L_{\text{steam}}} = \frac{200 \times 1.5 - x}{x \times 100}$$

or $\frac{80}{540} = \frac{2(1.5 - x)}{x}$

On solving, $x = 1.396 \text{ m}$

Q. 10. What is the temperature of the steel-copper junction in the steady state of the system shown in Fig.? Length of the steel rod = 15.0 cm, length of the copper rod = 10.0 cm, temperature of the furnace = 300 °C, temperature of the other end = 0 °C. The area of cross-section of the steel rod is twice that of the copper rod. Thermal conductivity of steel = 50.2 Js⁻¹ m⁻² °C⁻¹ and of copper = 385 Js⁻¹ m⁻² °C⁻¹.

Sol. Here $K_1 = 50.2 \text{ Js}^{-1} \text{ m}^{-2} \text{ °C}^{-1}$, $K_2 = 385 \text{ Js}^{-1} \text{ m}^{-2} \text{ °C}^{-1}$, $A_1 = 2 A_2$, $x_1 = 15.0 \text{ cm}$, $x_2 = 10.0 \text{ cm}$

Let T be the temperature of the steel-copper junction in the steady state of the system. Then in the steady state, Rate of heat flowing into the system = Rate of heat flowing out of the system

or $\frac{K_1 A_1 (300 - T)}{x_1} = \frac{K_2 A_2 (T - 0)}{x_2}$

or $\frac{300 - T}{T} = \frac{K_2 - A_2 \times x_1}{K_1 A_1 x_2}$

or $= \frac{385 \times 1 \times 15}{50.2 \times 2 \times 10} = 5.75$ or $T = 44.4 \text{ °C}$

Q. 11. An electric heater is used in a room of total wall area 137 m² to maintain a temperature of + 20 °C inside it, when the outside temperature is - 10 °C. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 watt/m/°C respectively.

Sol. Equivalent thermal conductivity of the series combination of three walls is

$$K = \frac{d_1 + d_2 + d_3}{\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}} = \frac{0.025 + 0.01 + 0.25}{\frac{0.025}{0.125} + \frac{0.01}{1.5} + \frac{0.25}{1.0}}$$

$$= \frac{0.285 \times 300}{137} \text{ Wm}^{-1} \text{ °C}^{-1}$$

Rate of flow of heat is $\frac{Q}{t} = K A \frac{\theta_1 - \theta_2}{(d_1 + d_2 + d_3)}$

$= \frac{0.285 \times 300}{137} \times 137 \times \frac{20 - (-10)}{0.285} = 9000 \text{ W}$

This should be equal to the power of the heater.

Q. 12. An iron bar ($L_1 = 0.1 \text{ m}$, $A_1 = 0.02 \text{ m}^2$, $K_1 = 79 \text{ W m}^{-1} \text{ K}^{-1}$) and a brass bar ($L_2 = 0.1 \text{ m}$, $A_2 = 0.02 \text{ m}^2$, $K_2 = 109 \text{ W m}^{-1} \text{ K}^{-1}$) are soldered end of end as shown in Fig. The free ends of the iron bar and brass bar are maintained for the hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar.

Sol. Here $L_1 = L_2 = L = 0.1 \text{ m}$, $A_1 = A_2 = A = 0.02 \text{ m}^2$, $K_1 = 79 \text{ W m}^{-1} \text{ K}^{-1}$, $K_2 = 109 \text{ W m}^{-1} \text{ K}^{-1}$, $T_1 = 373 \text{ K}$, and $T_2 = 273 \text{ K}$

In the steady state,

Heat current through iron bar = Heat current through brass bar

or $H_1 = H_2 = H$ (say)

or $\frac{K_1 A_1 (T_1 - T_0)}{L_1} = \frac{K_2 A_2 (T_0 - T_2)}{L}$

or $\frac{K_1 A (T_1 - T_0)}{L} = \frac{K_2 A (T_0 - T_2)}{L}$

or $K_1 (T_1 - T_0) = K_2 (T_0 - T_2)$

Thus, the junction temperature T_0 of the two bars is

$$T_0 = \frac{(K_1 T_1 + K_2 T_2)}{(K_1 + K_2)}$$

Using this value of T_0 , the heat current through either bar will be

$$H = \frac{K_1 A (T_1 - T_0)}{L} = \frac{K_1 A}{L} \left(T_1 - \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2} \right)$$

$$= \frac{K_1 K_2 A}{K_1 + K_2} \cdot \frac{T_1 - T_2}{L}$$

Thus, the heat current H' through the compound bar of length $L_1 + L_2 = 2L$ and the equivalent thermal conductivity K' , of the compound bar are given by

or $\frac{K' A (T_1 - T_2)}{2L} = \frac{K_1 K_2 A}{K_1 + K_2} \cdot \frac{T_1 - T_2}{L}$

or $K' = \frac{2K_1 K_2}{K_1 + K_2}$

(i) $T_0 = \frac{(K_1 T_1 + K_2 T_2)}{(K_1 + K_2)}$

$$= \frac{(79 \text{ W m}^{-1} \text{ K}^{-1})(373 \text{ K}) + (109 \text{ W m}^{-1} \text{ K}^{-1})(273 \text{ K})}{79 \text{ W m}^{-1} \text{ K}^{-1} + 109 \text{ W m}^{-1} \text{ K}^{-1}} = 315 \text{ K}$$

$$\begin{aligned} \text{(ii) } K' &= \frac{2K_1 K_2}{K_1 + K_2} \\ &= \frac{2 \times (79 \text{ W m}^{-1} \text{ K}^{-1}) \times (109 \text{ W m}^{-1} \text{ K}^{-1})}{79 \text{ W m}^{-1} \text{ K}^{-1} + 109 \text{ W m}^{-1} \text{ K}^{-1}} = 91.6 \text{ m}^{-1} \text{ K}^{-1} \end{aligned}$$

$$\begin{aligned} \text{(iii) } H' = H &= \frac{K' A (T_1 - T_2)}{2L} = \frac{(91.6 \text{ W m}^{-1} \text{ K}^{-1}) \times (0.02 \text{ m}^2) \times (373 \text{ K} - 273 \text{ K})}{2 \times (0.1 \text{ m})} = 916.1 \text{ W} \end{aligned}$$

APPLICATIONS OF CONDUCTIVITY IN DAILY LIFE

(i) In winter, a metallic handle appears colder than the wooden door: In winter, the human body is at a higher temperature than the surrounding objects. As we touch the metallic handle (good conductors), heat flows from our body to the wooden door (bad conductor), so it does not feel that cold as the metallic handle.

(ii) Cooking utensils are provided with wooden handles: Wood is a bad conductor of heat. A wooden handle does not allow heat to be conducted from the hot utensils to the hand. So we can easily hold the hot utensils with the help of wooden handles.

(iii) A new quilt is warmer than an old quilt: A new quilt contains more air in its pores as compared to the old quilt. As air is a bad conductor of heat, it does not allow heat to be conducted away from our body to the surroundings and we feel warmer in it.

(iv) Birds swell their feathers in winter: By doing so the birds enclose air between their feathers and the air of heat from the bodies of the birds to the surroundings and as such they do not feel cold in winter.

(v) Ice is packed in saw dust: Saw dust and air trapped inside it are poor conductors of heat. This prevents the conduction of heat from the surroundings to the ice which may otherwise melt the ice.

(vi) Eskimos make double wall houses of the blocks of ice: Air trapped between the two walls of ice does not allow the heat to be conducted away from the inside of the house to the colder surroundings.

(vii) When a wire gauge is placed over the burning Bunsen's burner, the flame does not go beyond the gauge:

Copper is a very good conductor of heat. The copper gauge absorbs most of the heat. Therefore, the temperature of the gas above the gauge is not high enough to ignite the gas.

(viii) A refrigerator is provided with insulated walls: Generally, fiber glass is used as an insulating material. This is done to minimize the chances of heat flowing into the refrigerator from outside.

CONVECTION

It is the process by which heat flows from the region of higher temperature to the region of lower temperature by the actual movement of the material particles.

Fluids (liquids and gases) are heated mainly by the process of convection in which buoyancy and gravity play an important role. As shown in Fig. when a fluid is heated from below, the hot portion at the bottom expands and becomes less dense. Because of buoyancy, this lighter portion rises up. The denser colder fluid takes its place by moving downwards. Thus, convection current is set up in the fluid. The actual movement of a liquid can be seen by coloring the liquid with potassium permanganate crystals placed at the bottom of the vessel.

Natural convection: If the material moves due to difference in density, the process of heat transfer is called natural or free convection. Natural convection arises due to unequal heating (of fluid) and gravity. Here the more heated and less dense parts of the fluid rise and are replaced by the cooler parts. Natural convection is responsible for the origin of different types of winds in the atmosphere.

Forced convection: If the heated material is forced to move by an agency like a pump or a bowler, the process of heat transfer is called forced convection. Air-conditioning, central heating systems and heating a liquid by brisk stirring are examples of forced convection.