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GRAVITATION

REVIEW OF BASIC CONCEPTS

1. Newton's Law of Gravitation

Newton's law of universal gravitation states as follows:

'*Any two particles of matter anywhere in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of the force being along the line joining the particles, i.e.* (Fig. 6.1)

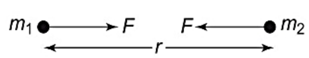


Fig. 6.1

$$F \propto \frac{m_1 m_2}{r^2}$$

where F is the magnitude of the force of attraction between two particles of masses m_1 and m_2 separated by a distance r.

In the form of an equation the law is written as

$$F = \frac{G m_1 m_2}{r^2}$$

where G is a constant called the universal gravitation constant. The value of this constant is to be determined experimentally and is found to be

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.$$

2. Gravitational Force due to Multiple Masses

If a system consists of more than two masses, the gravitational force experienced by a given mass due to all other masses is obtained from the principle of superposition which states that '*the gravitational force experienced by one mass is equal to the vector sum of the gravitational forces exerted on it by all other masses taken one at a time.*'

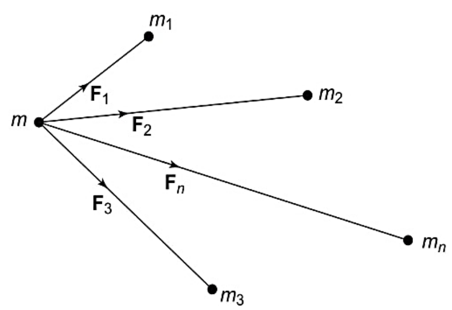


Fig. 6.2

The gravitational force on mass m due to masses m_1, m_2, m_3, ... m_n is given by (Fig. 6.2)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n$$

Note

1. Gravitational force is alway attractive.
2. Gravitational force between two masses does not depend on the medium between them.
3. Gravitational force acts along the straight line joining the centres of the two bodies.

EXAMPLE 1 Two bodies A and B of masses $m_1 = 1$ kg and $m_2 = 16$ kg respectively are placed 1.0 m apart. A third body C of mass $m = 3$ kg is placed on the line joining A and B. At what distance from A should C be placed so that it experiences no gravitational force?

SOLUTION Let x metre be the distance between A and C (Fig. 6.3)

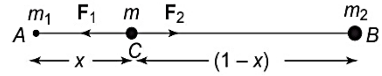


Fig. 6.3

Force exerted by A on C is

$$F_1 = \frac{G\,m_1 m}{x^2} \text{ directed towards } A$$

Force exerted by B on C is

$$F_2 = \frac{G\,m_2 m}{(1-x)^2} \text{ directed towards } B$$

C will experience no force if $F_1 = F_2$, i.e.

$$\frac{G\,m_1 m}{x^2} = \frac{G\,m_2 m}{(1-x)^2}$$

$$\frac{m_2}{m_1} = \frac{(1-x)^2}{x^2}$$

$$16 = \frac{(1-x)^2}{x^2}$$

$$4 = \frac{1-x}{x}$$

which gives $x = 0.2$ m. Note that if body C is placed to the left of body A or to the right of body B, it will experience a finite gravitational force.

EXAMPLE 2 Three bodies, each of mass m, are placed at the vertices of an equilateral triangle of side a as shown in Fig. 6.4(a). Find the magnitude and direction of the force experienced by the body at vertex A.

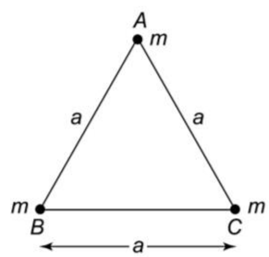


Fig. 6.4(a)

SOLUTION The forces exerted on the body at A by bodies at B and C are shown in Fig. 6.4(b).

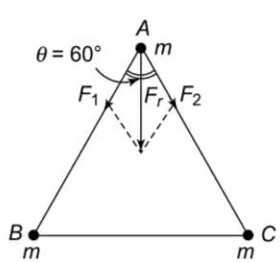


Fig. 6.4(b)

$$F_1 = F_2 = \frac{Gm^2}{a^2} = F \text{ (say)}$$

The magnitude of the resultant force is

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ}$$

$$= \sqrt{3}\,F = \sqrt{3}\,\frac{Gm^2}{a^2},$$

directed vertically downwards.

EXAMPLE 3 A uniform sphere of radius R and mass M exerts a force F on a body of mass m placed at point P at a distance $2R$ from the centre of the sphere. If a spherical cavity of radius $R/2$ is made in the sphere as shown in Fig. 6.5, the force of attraction exerted by the sphere with cavity in it on the same body placed at the same point P will now be

(a) $\frac{3F}{5}$ (b) $\frac{5F}{7}$

(c) $\frac{7F}{9}$ (d) $\frac{F}{2}$

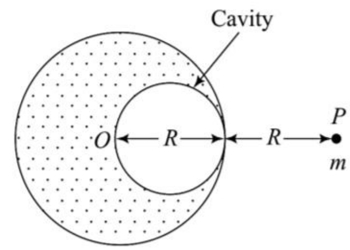


Fig. 6.5

SOLUTION $F = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

Mass per unit volume $= \dfrac{M}{\frac{4\pi}{3}R^3}$

$\therefore$ Mass removed to make the cavity is

$$M' = \frac{M}{\frac{4\pi}{3}R^3} \times \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

The force of attraction the body at P now will be

F' = force due to complete sphere of mass M

– force due to removed part of mass $\frac{M}{8}$

$$= \frac{GmM}{4R^2} - \frac{G\,m \times \frac{M}{8}}{\left(\frac{3R}{2}\right)^2}$$

$$= \frac{GmM}{4R^2} - \frac{GmM}{18R^2} = \frac{7}{36}\frac{GmM}{R^2}$$

or $$F' = \frac{7}{9} \times \frac{GmM}{4R^2} = \frac{7F}{9}$$

EXAMPLE 4 A block of mass m is hung on a spring of spring constant k and of negligible mass. The spring extends by x on the surface of the earth. When taken to a height $h = R/2$, the spring will extend by (R = radius of earth)

(a) x (b) $\frac{2x}{3}$

(c) $\frac{3x}{8}$ (d) $\frac{4x}{9}$

SOLUTION $\frac{GmM}{R^2} = mg$ (M = mass of earth)

But $mg = kx$. Therefore

$$\frac{GmM}{R^2} = kx$$

$$\Rightarrow \quad x = \frac{GMm}{kR^2} \quad \text{(i)}$$

At a height h,

$$\frac{GmM}{(R+h)^2} = mg' = kx'$$

$$\Rightarrow \quad x' = \frac{GmM}{k(R+h)^2} \quad \text{(ii)}$$

From (i) and (ii)

$$x' = x \times \left(\frac{R}{R+h}\right)^2$$

$$= x \times \left(\frac{R}{R+R/2}\right)^2 = \frac{4x}{9}$$

3. Acceleration due to Gravity

Considering the earth as an isolated mass, a force is experienced by a body near it. This force is directed towards the centre of the earth and has a magnitude mg, where g is the acceleration due to gravity.

$$F = mg = \frac{GmM}{R^2}$$

where M is the mass of the earth and R its radius (nearly constant for a body in the vicinity of the earth)

$$\therefore \quad g = \frac{GM}{R^2}$$

All bodies near the surface of the earth fall with the same acceleration which is directed towards the centre of the earth.

4. Variation of g

1. Variation with altitude The acceleration due to gravity of a body at a height h above the surface of the earth is given by

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

where g is the acceleration due to gravity on the surface of the earth. If h is very small compared to R, we can use binomial expansion and retain terms of order h/R. We then get

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Thus, the acceleration due to gravity decreases as the altitude (h) is increased.

2. Variation with depth The acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_d = g\left(1 - \frac{d}{R}\right)$$

This equation shows that the acceleration due to gravity decreases with depth. At the centre of the earth where $d = R$, $g_d = 0$. Thus the acceleration due to gravity is maximum at the surface of the earth, decreases with increase in depth and becomes zero at the centre of the earth.

3. Variation with Latitude Due to the rotation of earth about its axis, the value of g varies with latitude, i.e. from one place to another on the earth's surface. At poles, the effect of rotation on g is negligible. At the equator, the effects of rotation on g is the maximum. In general, the value of acceleration due to gravity at a place decreases with the decrease in the latitude of the place.

The accelration due to gravity at a place on earth where the latitude is ϕ is given by

$$g_\phi = g - \omega^2 R \cos^2 \phi;$$

ω = angular velocity of rotation of earth.

At equator, $\phi = 0 \Rightarrow g_e = g - R\omega^2$ (minimum)

At poles, $\phi = 90° \Rightarrow g_p = g$ (maximum)

$$g_\phi = g - 0.0337 \cos^2 \phi$$

Thus the value of g varies slightly from place to place on earth. Variation of g with altitude and depth is shown in Fig. 6.6.

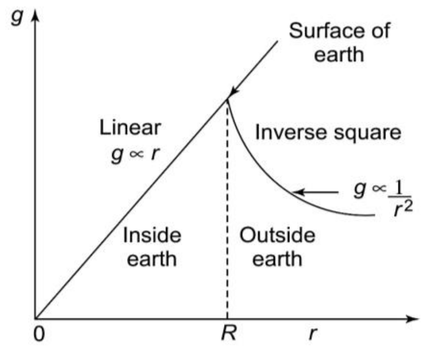


Fig. 6.6 *Variation of g (gravitational acceleration)*

EXAMPLE 5 A body weighs 63 N on the surface of the earth. How much will it weigh at a height equal to half the radius of the earth?

SOLUTION $W = mg = 63$ N

$$W' = mg' = mg\left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad W' = W\left(\frac{R}{R+h}\right)^2$$

$$= 63 \times \left(\frac{R}{R+R/2}\right)^2 = 28 \text{ N}$$

EXAMPLE 6 Assuming the earth to be a sphere of uniform mass density, find the percentage decrease in the weight of a body when it is taken to the end of a tunnel 32 km below the surface of the earth. Radius of earth = 6400 km.

SOLUTION $g' = g\left(1 - \frac{d}{R}\right)$

$$= g\left(1 - \frac{32}{6400}\right) = \frac{199\,g}{200}$$

Decrease in weight $= mg - mg'$

$$= mg\left(1 - \frac{199}{200}\right) = \frac{mg}{200}$$

$$\text{Percentage decrease} = \frac{mg/200}{mg} \times 100 = 0.5\%$$

EXAMPLE 7 At what depth below the surface of the earth will the value of acceleration due to gravity become 90% of its value at the surface? $R = 6.4 \times 10^6$ m.

SOLUTION $g' = 0.9\,g$. $\quad g' = g\left(1 - \frac{d}{R}\right)$

$$\therefore \quad 0.9 = 1 - \frac{d}{R}$$

$$\Rightarrow \quad d = 0.1\,R = 6.4 \times 10^5 \text{ m}$$

5. Gravitational Field Intensity

Just as the region around a magnet has magnetic field and the region around a charge has electric field, the region around a mass has gravitational field.

The gravitational field intensity (or simply gravitational field) at a point is defined as the gravitational force experienced by a unit mass placed at that point.

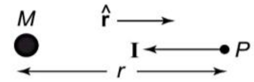


Fig. 6.7

Consider the gravitational field of a body of mass M. To find the strength of the field at a point P at a distance r from M, we place a small mass m at P. The gravitational force exerted m by M is

$$F = \frac{GMm}{r^2}$$

By definition, the gravitational field (intensity) of M at P is given by

$$I = \frac{F}{m} = \frac{GM}{r^2}$$

I is a vector quantity. In vector form

$$\mathbf{I} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from M to P, i.e radially away from M. The negative sign indicates that **I** directed radially inwards towards M.

The SI unit of I is N kg^{-1}.

In three dimensions, if mass M is located at the origin, the magnetic field at a $P\,(x, y, z)$ is given by

$$\mathbf{I} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

where $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ represents the position of point P with respect to mass M at the origin.

For a many body system, the principle of superposition holds for gravitational field (intensities) just as it holds for gravitational forces, i.e.

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \cdots + \mathbf{I}_n$$

where $\mathbf{I}_1, \mathbf{I}_2, \ldots \mathbf{I}_n$ are the gravitational field intensities at a point due to bodies of masses $M_1, M_2, \ldots M_n$.

For continuous mass distributions (i.e rigid bodies), we divide the body into an infinitely large number of infinitesimally small elements. Then the gravitational field intensity is given by

$$\mathbf{I} = \int d\,\mathbf{I}$$

Gravitational Field due to some continuous Mass Distributions

1. Gravitational field due to a circular ring of mass M and radius R at a point at a distance r from the centre and on the axis of the ring is given by

$$I = \frac{GMr}{(R^2 + r^2)^{3/2}}$$

2. Gravitational field due to a thin spherical shell of mass M and radius R at a point P at a distance $r > R$ from the centre of shell,

$$I = \frac{GM}{r^2} \text{ (outside the shell)}$$

Inside the shell, $I = 0$

3. Gravitational field of a solid sphere of mass M and radius R is

$$I = \frac{GM}{r^2} \text{ for } r > R$$

$$I = \frac{GM}{R^2} \text{ for } r = R$$

$$I = \frac{GMr}{R^3} \text{ for } r < R$$

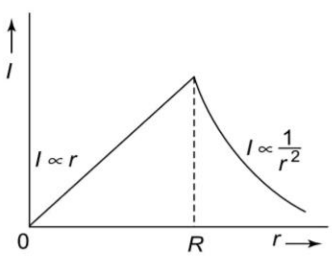


Fig. 6.8

Figure 6.8 shows the variation of the gravitational field of a solid sphere.

6. Gravitational Potential Energy

Gravitational potential energy of a system of two masses M and m held a distance r apart is defined as the amount of work done to bring the masses from infinity to their respective locations along any path and without any acceleration.

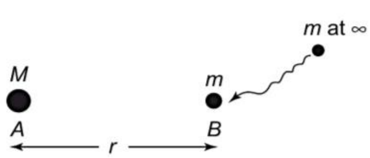


Fig. 6.9

Work done to bring mass M from infinity to A is $W_1 = 0$.
Work done to bring mass m from $r = \infty$ to $r = r$ is

$$W_2 = \int_0^r \mathbf{F} \cdot d\mathbf{r}$$

$$= \int_\infty^r F\, dr \cos\theta$$

Since mass M will attract mass m, angle θ between $\mathbf{F}$ and $d\mathbf{r}$ is zero. Hence

$$W_2 = \int_\infty^r \frac{GMm}{r^2} dr$$

$$= GMm \int_\infty^r r^{-2} dr = -\frac{GMm}{r}$$

Total work $\quad W = W_1 + W_2 = -\frac{GMm}{r}$

$\therefore$ Gravitational potential energy of the system is

$$U = W = -\frac{GMm}{r}$$

The zero of potential energy is assumed to be at $r = \infty$. The negative sign indicates the potential energy is negative for any finite separation between the masses and increases to zero at infinite separation.

Expression for Increase in Gravitational Potential Energy

If the body m is moved away from M, the potential energy of the system increases.

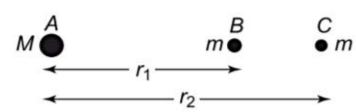


Fig. 6.10

$$\text{P.E. at } B = -\frac{GMm}{r_1}$$

$$\text{P.E. at } C = -\frac{GMm}{r_2}$$

$$\therefore \text{ Increase in P.E.} = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$= GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

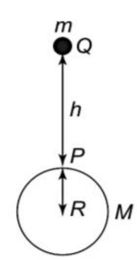


Fig. 6.11

If the body of mass M is the earth, then the increase in gravitational P.E. when a body of mass m is taken from the surface of the earth to a height h above the surface is given by (see Fig. 6.11), R = radius of the earth.

$$\Delta U = \text{P.E. at } Q - \text{P.E at } P$$

$$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GmMh}{R(R+h)}$$

If $h << R$; $\quad \Delta U = \frac{GmMh}{R^2} = mgh \quad \left(\because \frac{GM}{R^2} = g\right)$

7. Gravitational Potential

Gravitational potential at a point P in the gravitational field of a body of M is defined as the amount of work done to bring a unit mass from infinity to that point along any path and without any acceleration, i.e., (Fig. 6.12)

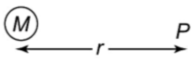


Fig. 6.12

$$V = \frac{W}{m} = -\frac{GMm}{r \times m}$$

$$\Rightarrow \qquad V = -\frac{GM}{r}$$

Potential V is a scalar quantity. Hence the gravitational potential at a point P due to a number of masses $m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots r_n$ respectively from P is given by

$$V = V_1 + V_2 + \cdots + V_n$$

$$= -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \cdots + \frac{m_n}{r_n}\right)$$

The SI unit of V is J kg^{-1}.

Relation between Gravitational Field Intensity (I) and Gravitational Potential (V)

Gravitational field intensity and gravitational potential at a point are related as

$$I = -\frac{dV}{dr}$$

Gravitational Potential due to a Spherical Shell

(i) At a point outside the shell, $V = -\frac{GM}{r}$ (for $r > R$)
where M is the mass and R is the radius of the shell.

(ii) At a point on the surface of the shell, $V = -\frac{GM}{R}$

(iii) At a point inside the shell, $V = -\frac{GM}{R}$ (for $r < R$)

Figure 6.13 shows the variation V with r for a spherical shell.

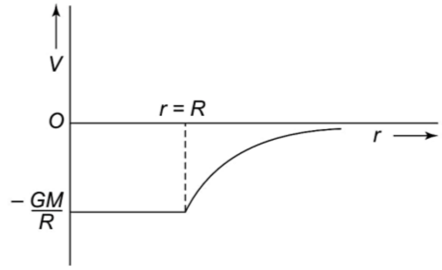


Fig. 6.13

Gravitational Potential due to a Solid Sphere of Mass M and Radius R

(i) For points outside the sphere ($r > R$), $V = -\frac{GM}{r}$

(ii) For points inside the sphere ($r < R$),

$$V = -\frac{3GM}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{6}\right)$$

(ii) At the centre of the sphere ($r = 0$)

$$V = -\frac{3GM}{2R}$$

(iv) On the surface of the sphere ($r = R$)

$$V = -\frac{GM}{R}$$

EXAMPLE 8 Two masses m_1 = 100 kg and m_2 = 8100 kg are held 1 m apart.

(a) At what point on the line joining them is the gravitational field equal to zero? Find the gravitational potential at that point.

(b) Find the gravitational potential energy of the system. Given $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

SOLUTION Gravitational field at P due to m_1 is [Fig. 6.14]

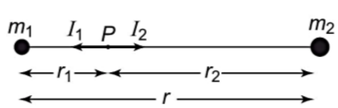


Fig. 6.14

$$I_1 = \frac{Gm_1}{r_1^2} \text{ directed towards } m_1$$

Gravitational field at P due to m_2 is

$$I_2 = \frac{Gm_2}{r_2^2} \text{ directed towards } m_2$$

Gravitational field at P will be zero if

$$I_1 = I_2 \Rightarrow \frac{Gm_1}{r_1^2} = \frac{Gm_2}{r_2^2}$$

or $$\frac{m_2}{m_1} = \left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \quad 81 = \left(\frac{1-r_1}{r_1}\right)^2 \quad (\because r_2 = r - r_1 \text{ and } r = 1 \text{ m})$$

$$\therefore \quad 9 = \frac{1-r_1}{r_1} \Rightarrow r_1 = 0.1 \text{ m}$$

Gravitational potential at P is

$$V = V_1 + V_2 = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$= -6.67 \times 10^{-11} \times \left(\frac{100}{0.1} \times \frac{8100}{0.9}\right)$$

$$= -6.67 \times 10^{-7} \text{ J kg}^{-1}$$

(b) Gravitational potential energy of the system is

$$\text{G.P.E.} = -\frac{G\, m_1\, m_2}{r}$$

$$= -\frac{6.67 \times 10^{-11} \times 100 \times 8100}{1}$$

$$= -5.4 \times 10^{-5} \text{ J}$$

EXAMPLE 9 Three equal masses, each equal to m, are kept at the vertices of an equilateral triangle of side a. Find the gravitational field and gravitational potential at the centroid of the triangle.

SOLUTION Refer to Fig. 6.15.

The centroid O divides the lines AD, BE and CF in the ratio 2:1. Also $AO = BO = CO = r$ (say). Now $AO = \frac{2}{3} AD$

and $$AD = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}\, a}{2}$$

$$\therefore \quad AO = \frac{2}{3} \times \frac{\sqrt{3}\, a}{2} = \frac{a}{\sqrt{3}}, \text{ i.e. } r = \frac{a}{\sqrt{3}}$$

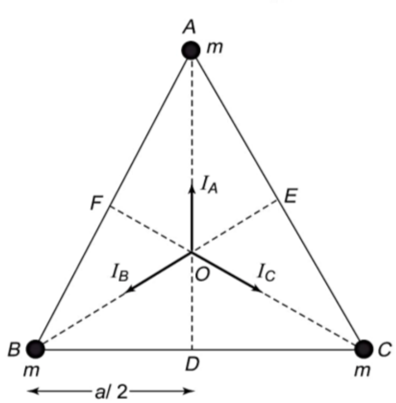


Fig. 6.15

The gravitational fields at O due masses m at A, B and C are $I_A = I_B = I_C = \frac{GM}{r^2} = I$. Their directions are shown in Fig. 6.14. The angle between any two of them is $\theta = 120°$.

The resultant of I_B and I_C is

$$I' = \sqrt{I^2 + I^2 + 2I^2 \cos 120°} = I$$

directed vertically down.

I' will cancel with I_A. Hence the net gravitational field at O is zero.

The gravitational potential at O is

$$V = V_1 + V_2 + V_3$$

$$= -\frac{Gm}{r} - \frac{Gm}{r} - \frac{Gm}{r}$$

$$= -\frac{3Gm}{r} = -3\sqrt{3}\frac{Gm}{a} \quad \left(\because r = \frac{a}{\sqrt{3}}\right)$$

EXAMPLE 10 The gravitational potential at a height h = 1600 km above the surface of the earth is -4.0×10^7 Jkg^{-1}. Assuming the earth to be a sphere of radius R = 6400 km, find the value of the acceleration due to gravity at that height.

SOLUTION Let $r = R + h$. Then

$$V = -\frac{GM}{r}$$

and $$g' = \frac{GM}{r^2}$$

$$\therefore \quad \frac{g'}{|V|} = \frac{GM/r^2}{GM/r} = \frac{1}{r} = \frac{1}{R+h}$$

$$\Rightarrow \quad g' = \frac{|V|}{(R+h)} = \frac{4.0 \times 10^7 \text{ J kg}^{-1}}{(6.4 + 1.6) \times 10^6 \text{ m}}$$

$$= 5.0 \text{ J kg}^{-1} \text{ m}^{-1} = 5.0 \text{ ms}^{-2}$$

EXAMPLE 11 Two particles of masses m_1 and m_2 are initially at rest at an infinite distance apart. They begin to move towards each other due to gravitational attraction. Find the ratio of their accelerations and speeds when the separation between them becomes r.

SOLUTION Since no external force acts on the system, the acceleration of their centre of mass is zero, i.e.

$$a_{CM} = \frac{m_1\, a_1 + m_2 a_2}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$$\Rightarrow \quad m_1 a_1 = -m_2\, a_2$$

$$\Rightarrow \quad \frac{a_1}{a_2} = -\frac{m_2}{m_1}$$

The negative sign indicates that they move in opposite directions.

Let v_1 and v_2 be the speeds of two masses when they are at a distance r. Due to gravitational attraction, they gain speed as they approach each other. Hence their kinetic energy increases and gravitational potential energy (G.P.E) decreases. From the conservation of energy,

$$\text{Loss in G.P.E.} = \text{gain in K.E.}$$

$$(\text{G.P.E.})_i - (\text{G.P.E.})_f = (\text{K.E.})_f - (\text{K.E.})_i$$

$$\Rightarrow \quad 0 - \left(-\frac{Gm_1m_2}{r}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - 0$$

$$\Rightarrow \quad \frac{G\,m_1\,m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{(i)}$$

Since no external force acts, the total momentum of the system is conserved, i.e., $p_i = p_f$ or

$$0 = m_1v_1 - m_2v_2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$v_1 = \left(\frac{2Gm_2^2}{r(m_1+m_2)}\right)^{1/2}$$

and

$$v_2 = \left(\frac{2G\,m_1^2}{r(m_1+m_2)}\right)^{1/2}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

EXAMPLE 12 Infinite number of tiny spheres, each of mass m are placed on the x-axis at distances r, $2r$, $4r$, … etc from origin O. The magnitude of gravitational field intensity at O is

(a) $\dfrac{4Gm}{3r^2}$ (b) $\dfrac{2GM}{3r^2}$

(c) $\dfrac{GM}{r^2}$ (d) infinity

SOLUTION Since gravitational force is attractive, the direction of individual intensities will be towards O. Therefore, their magnitudes simply add up. Thus

$$I = I_1 + I_2 + I_3 + \cdots$$

$$= \frac{Gm}{r^2} + \frac{Gm}{(2r)^2} + \frac{Gm}{(4r)^2} + \cdots$$

$$= \frac{Gm}{r^2}\left(1 + \frac{1}{2^2} + \frac{1}{4^2} + \cdots\right)$$

$$= \frac{Gm}{r^2}\left(\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \cdots\right)$$

$$= \frac{Gm}{r^2} \times \frac{1}{\left(1 - \frac{1}{2^2}\right)} = \frac{4Gm}{3r^2}$$

EXAMPLE 13 In Example 11 above, the gravitational potential at origin O is

(a) $-\dfrac{Gm}{r}$ (b) $-\dfrac{2Gm}{r}$

(c) $-\dfrac{4Gm}{r}$ (d) infinity

SOLUTION Potential is a scalar quantity. Therefore, the gravitational potential at O is

$$V = V_1 + V_2 + V_3 + \cdots$$

$$= -\frac{Gm}{r} - \frac{Gm}{2r} - \frac{Gm}{4r} - \cdots$$

$$= -\frac{Gm}{r}\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)$$

$$= -\frac{Gm}{r}\left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots\right)$$

$$= -\frac{Gm}{r} \times \frac{1}{\left(1 - \frac{1}{2}\right)} = -\frac{2Gm}{r}$$

EXAMPLE 14 The gravitational field due to a mass distribution is given by

$$I = \frac{k}{x^2}$$

along the $+x$ direction, where k is a constant. If the gravitational potential at infinity is taken to be zero, its value at a distance x will be

(a) $\dfrac{k}{x}$ (b) $\dfrac{k}{2x}$

(c) $\dfrac{k}{x^2}$ (d) $\dfrac{k}{2x^2}$

SOLUTION $I = -\dfrac{dV}{dx} \Rightarrow dV = -I\,dx$

$$\therefore \quad V \text{ at } x = -\int_0^x I\,dx = -k\int_0^x \frac{dx}{x^2}$$

$$= +k\left|\frac{1}{x}\right|_0^x = \frac{k}{x}$$

EXAMPLE 15 The gravitational potential at point P whose position coordinate is r (due to a certain mass distribution) is given by

$$V = \frac{a}{\sqrt{b^2 + r^2}}$$

where a and b are constants. The magnitude of the gravitational field at P will be

(a) $\frac{rV}{a}$ (b) $\frac{2rV^2}{3a}$

(c) $\frac{rV^3}{a^2}$ (d) $\frac{2rV^3}{3a^2}$

SOLUTION $E = -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{a}{(b^2 + r^2)^{1/2}}\right]$

$$\Rightarrow \quad E = \frac{ar}{(b^2 + r^2)^{3/2}} \qquad \text{(i)}$$

Now $(b^2 + r^2)^{1/2} = \frac{a}{V}$

$$\therefore \quad (b^2 + r^2)^{3/2} = \frac{a^3}{V^3} \qquad \text{(ii)}$$

Using (ii) in (i) we get

$$E = \frac{rV^3}{a^2}$$

EXAMPLE 16 Two particles, each of mass m, are moving in a circle of radius r under the action of their mutual gravitational force. The speed of each particle round the circle will be proportional to

(a) $\frac{1}{\sqrt{r}}$ (b) $\frac{1}{r}$

(c) $\frac{1}{r^{3/2}}$ (d) $\frac{1}{r^2}$

SOLUTION The centripetal force for circular motion is provided by the gravitational force between the particles. Since the gravitational force is attractive, the particles will be diametrically opposite to each other. If the speed of each particle is v, then for a given particle,

$$\frac{mv^2}{r} = \frac{Gm^2}{(2r)^2}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{r}}$$

So $\quad v \propto \frac{1}{\sqrt{r}}$

EXAMPLE 17 The density of a solid sphere of mass M and radius R varies with distance r from its centre as $\rho = \frac{k}{r}$ where k is a constant. The gravitational field due to this sphere at a distance $2R$ from its centre is given by

(a) $\frac{\pi k G}{2}$ (b) $\frac{\pi^2 k G}{R}$

(c) $\frac{kG}{MR^2}$ (d) $\frac{3}{2}\frac{MG}{k}$

SOLUTION Refer to Fig. 6.16.

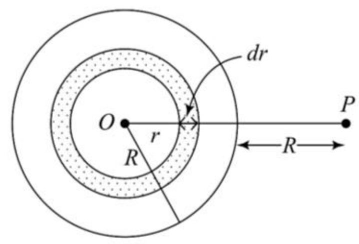


Fig. 6.16

To find gravitational field at point P, we divide the sphere into concentric shells of a very small thickness dr. The mass of each tiny shell can be assumed to be concentrated at centre O. Thus the mass M of the whole sphere can be assumed to be at O. The gravitational field due to this mass M at point P is

$$I = \frac{GM}{(2R)^2} = \frac{GM}{4R^2} \qquad \text{(i)}$$

To find M, consider a shell of radius r and thickness dr. The volume of this shell is

$$dV = (4\pi r^2)\, dr$$

Therefore, the mass of this shell is

$$dM = \rho\, dV = \left(\frac{k}{r}\right) \times (4\pi r^2\, dr)$$
$$= 4\pi k\, r\, dr$$

The mass of the whole sphere is

$$M = \int_0^R dM = 4\pi k \int_0^R r\, dr$$

$$= 4\pi k \frac{R^2}{2} = 2\pi k R^2 \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$I = \frac{G}{4R^2} \times 2\pi k R^2 = \frac{\pi k G}{2}$$

EXAMPLE 18 A solid sphere of mass M and radius R is surrounded by a concentric shell of equal mass M and radius $3R$. The gravitational field at a point P_1 at a distance $r_1 = 2R$ from the centre is I_1 and a point P_2 at distance $r_2 = 4R$ from the centre is I_2. The ratio $\frac{I_2}{I_1}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{9}$ (d) $\frac{9}{25}$

SOLUTION Refer to Fig. 6.17.

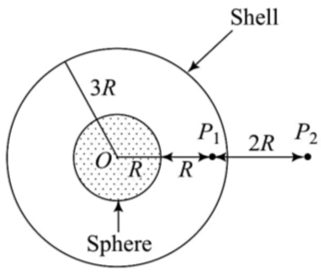


Fig. 6.17

Since point P_1 is inside the shell, the gravitational field at P_1 due to the shell is zero. The field at P_1 due to the solid sphere is

$$I_1 = \frac{GM}{r_1^2} = \frac{GM}{(2R)^2} = \frac{GM}{4R^2}$$

Point P_2 is outside the shell as well as the sphere. The mass M of the sphere and the mass M of the shell can be assumed to be concentrated at O so that the total mass at O is $2M$. The gravitational field due to mass $2M$ at point P_2 is

$$I_2 = \frac{G(2M)}{r_2^2} = \frac{2GM}{(4R)^2} = \frac{GM}{8R^2}$$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{1}{2}$$

8. Escape Velocity

The escape velocity is the minimum velocity with which a body must be projected in order that it may escape the earth's gravitational pull. The magnitude of the escape velocity is given by

$$v_e = \sqrt{\frac{2MG}{R}}$$

where M is the mass of the earth and R its radius. Substituting the known values of G, M and R, we get $v_e = 11.2\ \text{kms}^{-1}$. The escape velocity is independent of the mass of the body. The expression for the escape velocity can be written in terms of g as

$$v_e = \sqrt{2gR}$$

The escape velocity is independent of the mass of the body and the direction of projection.

9. Satellites

A body moving in an orbit around a much larger and massive body is called a satellite. The moon is the natural satellite of the earth.

Orbital Velocity Let us assume that a satellite of mass m goes around the earth in a circular orbit of radius r with a uniform speed v. If the height of the satellite above the earth's surface is h, then $r = (R + h)$, where R is the mean radius of the earth. The centripetal force $\frac{mv^2}{r}$ necessary to keep the satellite in its circular orbit is provided by the gravitational force $\frac{GmM}{r^2}$ between the earth and the satellite. This means that

$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

where M is the mass of the earth. Thus

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

Now the acceleration due to gravity on earth's surface is given by

$$g = \frac{GM}{R^2}$$

or $\quad GM = gR^2$

Substituting for GM we get

$$v = R\sqrt{\frac{g}{(R+h)}}$$

If the satellite is a few hundred kilometres above the earth's surface (say 100 to 300 km), we can replace $(R + h)$ by R. The error involved in this approximation is negligible since the radius of the earth $(R) = 6.4 \times 10^6$ m. Thus, we may write

$$v = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.9 \times 10^3\ \text{ms}^{-1} \simeq 8\ \text{km s}^{-1}$$

Periodic Time The periodic time T of a satellite is the time it takes to complete one revolution and it is given by (since $r = R + h$)

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

If $h \ll R$, we have $v = \sqrt{gR}$. Hence

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \simeq 2\pi\sqrt{\frac{R}{g}}$$

Total Energy of a Satellite

$$\text{Total energy } E = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}mv^2 - \frac{GmM}{r}$$

$$= \frac{GmM}{2r} - \frac{GmM}{r} \qquad \left(\because v = \sqrt{\frac{GM}{r}}\right)$$

$$\Rightarrow \qquad E = -\frac{GmM}{2r} = -\frac{GmM}{2(R+h)}$$

Note

1. $E = \frac{\text{P.E.}}{2}$
2. $E = -\text{(K.E.)}$
3. The total energy is negative which implies that the satellite is bound by the gravitational field of the earth. The binding energy $= \frac{GmM}{2r}$. This energy must be given to the orbiting satellite to escape to infinity.

Angular Momentum

The magnitude of angular momentum of a satellite is given by

$$L = mvr$$

$$= m\sqrt{\frac{GM}{r}}\, r$$

$$\Rightarrow \qquad L = m\sqrt{GMr}$$

Geostationary Satellites A geostationary satellite is a particular type used in communication. A number of communication satellites are launched which remain in fixed positions at a specified height above the equator. They are called *geostationary* or *synchronous satellites* used in international communication.

For a satellite to appear fixed at a position above a certain place on the earth, it must corotate with the earth so that its orbital period around the earth is exactly equal to the rotational period of the earth about its axis of rotation.

This condition is satisfied if the satellite is put in orbit at a height of about 36,000 km above the earth's surface.

10. Kepler's Laws of Planetary Motion

Johannes Kepler formulated three laws which describe planetary motion. They are as follows:

1. *Law of orbits* Each planet revolves about the sun in an elliptical orbit with the sun at one of the focii of the ellipse. The orbit of a planet is shown in Fig. 6.18(a) in which the two focii F_1 and F_2, are far apart. For the planet earth, F_1 and F_2 are very close together. In fact, the orbit of the earth is practically circular.
2. *Law of areas* A line drawn from the sun to the planet (termed the radius) sweeps out equal areas in equal intervals of time. In Fig. 6.18(b) P_1, P_2, P_3 and P_4 represent positions of a planet at different times in its orbit and S, the position of the sun.

   According to Kepler's second law, if the time interval between P_1 and P_2 equals the time interval between P_3 and P_4, then area A_1 must be equal to area A_2. Also the planet has the greater speed in its path from P_1 to P_2 than in its path from P_3 to P_4.
3. *Law of periods* The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. If T_1 represents the period of a planet about the sun, and r_1 its mean distance, then

$$T_1^2 \propto r_1^3$$

If T_2 represents the period of a second planet about the sun, and r_2 its mean distance, then for this planet

$$T_2^2 \propto r_2^3$$

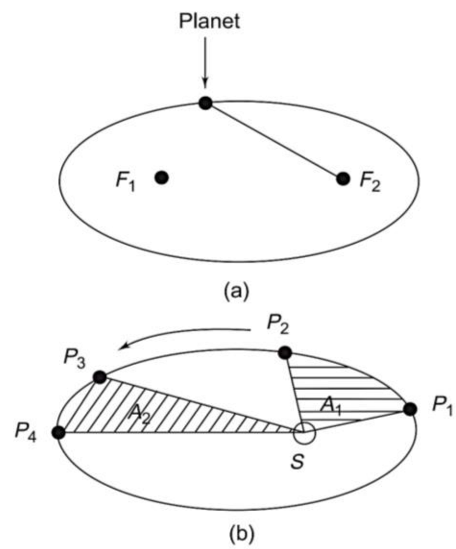


Fig. 6.18

These two relations can be combined since the factor of proportionality is the same for both. Thus

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

EXAMPLE 19 The mass of Jupiter is 318 times that of the earth and its radius is 11.2 times that of the earth. Calculate the escape velocity from Jupiter's surface. Given the escape velocity from earth'surface = 11.2 km s^{-1}.

SOLUTION For Jupiter : $v_J = \sqrt{\frac{2M_J G}{R_J}}$

For Earth : $v_J = \sqrt{\frac{2M_E G}{R_E}}$

$$\therefore \quad \frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}} = \sqrt{318 \times \frac{1}{11.2}} = 5.33$$

$$\therefore \quad v_J = 5.33\, v_E = 5.33 \times 11.2 = 59.7 \text{ km s}^{-1}$$

EXAMPLE 20 Calculate the escape velocity of a body at a height 1600 km above the surface of the earth. Radius of earth = 6400 km.

SOLUTION Work required to move a body of mass m from $r = R + h$ to $r = \infty$ is

$$W = \int_{R+h}^{\infty} F dr = GmM \int_{R+h}^{\infty} \frac{dr}{r^2}$$

$$= \frac{GmM}{R+h}$$

If v_e is the escape velocity, then

$$\frac{1}{2} m v_e^2 = \frac{GmM}{R+h}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2gR^2}{R+h}} \quad \left(\because g = \frac{GM}{R^2}\right)$$

Given $R = 6.4 \times 10^6$ m, and $h = 1.6 \times 10^6$ m and $g = 9.8 \text{ ms}^{-2}$

$$\therefore \quad v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{(6.4 + 1.6)10^6}}$$

$$= 10 \times 10^3 \text{ ms}^{-1} = 10 \text{ km s}^{-1}$$

EXAMPLE 21 A rocket is launched vertically from the surface of the earth with an initial velocity equal to half the escape velocity. Find the maximum height attained by it in terms of R where R is the radius of the earth. Ignore atmospheric resistance.

SOLUTION On the surface of the earth, the total energy of the rocket is

$$E_i = \text{K.E.} + \text{P.E.} = \frac{1}{2} mv^2 - \frac{GmM}{R}$$

At the highest point, $v = 0$. If h is the maximum height attained, the energy of the rocket at height h is

$$E_f = -\frac{GmM}{(R+h)}$$

From conservation of energy, $E_i = E_f$, i.e.,

$$\frac{1}{2} mv^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R} \times \frac{h}{R+h}} = v_e \sqrt{\frac{h}{R+h}}$$

Given $v = \frac{v_e}{2}$. Hence $\frac{v_e}{2} = v_e \sqrt{\frac{R}{R+h}} \quad \Rightarrow \quad h = \frac{R}{3}$

EXAMPLE 22 A body is dropped from a height h equal to $\frac{R}{2}$, where R is the radius of the earth. Show that it will hit the surface of the earth with a speed $v = v_e/\sqrt{3}$, where v_e is the escape velocity from the surface of the earth.

SOLUTION Total energy of the body at height h is

$$E_i = -\frac{GmM}{(R+h)} \qquad \text{(i)}$$

Total energy when it hits the surface of the earth is

$$E_f = \frac{1}{2} mv^2 - \frac{GmM}{R} \qquad \text{(ii)}$$

From conservation of energy, $E_i = E_f$, i.e.

$$-\frac{GmM}{(R+h)} = \frac{1}{2} mv^2 - \frac{GmM}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}} \times \sqrt{\frac{h}{(R+h)}} = v_e \sqrt{\frac{R}{(R+h)}}$$

For $h = \frac{R}{2}$, $v = \frac{v_e}{\sqrt{3}}$.

EXAMPLE 23 Show that the minimum energy required to launch a satellite of mass m from the surface of the earth in a circular orbit at an altitude $h = R$, where R is the radius of the earth is $\frac{3mgR}{4}$ where M is the mass of the earth.

SOLUTION Total energy of the satellite orbiting the earth is

$$E_1 = -\frac{GmM}{2r} = -\frac{GmM}{2(R+h)} = -\frac{GmM}{4R} \qquad (\because h = R)$$

Total energy when the satellite was at rest on the surface of the earth is

$$E_2 = \text{K.E.} + \text{P.E.} = 0 - \frac{GmM}{R} = -\frac{GmM}{R}$$

$\therefore$ Minimum energy required is

$$E_{min} = E_1 - E_2$$

$$= -\frac{GmM}{4R} - \left(-\frac{GmR}{R}\right)$$

$$= \frac{3GmM}{4R} = \frac{3}{4} mgR$$

EXAMPLE 24 A satellite of mass $m = 100$ kg is in a circular orbit at a height $h = R$ above the surface of the earth where R is the radius of the earth. Find

(a) the acceleration due to gravity at any point on the path of the satellite,

(b) the gravitational force on the satellite and

(c) the centripetal force on the satellite.

SOLUTION

(a) $g' = g\left(\dfrac{R}{R+h}\right)^2 = 9.8 \times \left(\dfrac{R}{R+R}\right)^2$

$$= \frac{9.8}{4} = 2.45\ \text{ms}^{-2}$$

(b) Gravitational force on satellite is

$$F_g = mg' = 100 \times 2.45 = 245\ \text{N}$$

(c) Centripetal force

$$F_c = \frac{mv^2}{r} = \frac{GmM}{r^2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$= \frac{GmM}{R^2} \times \frac{R^2}{r^2}$$

$$= mg \times \left(\frac{R}{R+h}\right)^2 = mg' = 245\ \text{N}.$$

Note: Since $F_g = F_c$, the satellite is a freely falling body and is, therefore, weightless.

EXAMPLE 25 A body projected vertically upwards from the surface of the earth with a certain velocity rises to a height of 10 m. How high will it rise if it is projected with the same velocity vertically upwards from a planet whose density is one-third that of the earth and radius half that of earth? Ignore atmospheric resistance.

SOLUTION Since the kinetic energy of the body is the same in both the cases, loss in K.E. = gain in P.E. will be equal, i.e.,

$$mg_p h_p = mg_e h_e$$

$$\Rightarrow \quad h_p = h_e \times \frac{g_e}{g_p}$$

Now $\quad g = \dfrac{GM}{R^2} = \dfrac{G}{R^2} \times \dfrac{4\pi}{3} R^3 \rho = \dfrac{4\pi}{3} GR\rho$

$$\therefore \quad \frac{g_e}{g_p} = \frac{R_e}{R_p} \times \frac{\rho_e}{\rho_p} = 2 \times 3 = 6$$

$$\therefore \quad h_p = 6h_e = 6 \times 10 = 60\ \text{m}$$

EXAMPLE 26 A satellite of mass 2000 kg is orbiting the earth at an altitude $R/2$, where R is the radius of the earth. What extra energy must be given to the satellite to increase its altitude to R? Given $R = 6.4 \times 10^6$ m.

SOLUTION In the first case, $r_1 = R + \dfrac{R}{2} = \dfrac{3R}{2}$ and in the second case, $r_2 = R + R = 2R$.

Required energy $= E_2 - E_1$

$$= -\frac{GmM}{2r_2} - \left(-\frac{GmM}{2r_1}\right)$$

$$= -\frac{GmM}{4R} + \frac{GmM}{3R}$$

$$= \frac{GmM}{12R}$$

$$= \frac{mgR}{12} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$= \frac{2000 \times 9.8 \times (6.4 \times 10^6)}{12}$$

$$= 1.04 \times 10^8\ \text{J}$$

EXAMPLE 27 Two bodies of masses m_1 and m_2 are held at a distance r apart. Show that at the point where the gravitational field due to them is zero, the gravitational potential is given by

$$V = -\frac{G}{r}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

SOLUTION Let the gravitational field be zero at point P (Fig. 6.19). Then

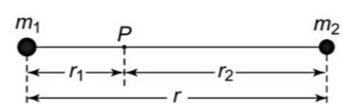


Fig. 6.19

$$\frac{G m_1}{r_1^2} = \frac{G m_2}{r_2^2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$$

$$\Rightarrow \quad \frac{r_1}{r - r_1} = \frac{\sqrt{m_1}}{\sqrt{m_2}} \quad \Rightarrow \quad r_1 = \frac{r\sqrt{m_1}}{\left(\sqrt{m_1} + \sqrt{m_2}\right)} \qquad \text{(i)}$$

Also $\quad r_2 = r - r_1 = r - \dfrac{r\sqrt{m_1}}{\left(\sqrt{m_1} + \sqrt{m_2}\right)}$

$$= \frac{r\sqrt{m_2}}{\left(\sqrt{m_1} + \sqrt{m_2}\right)} \qquad \text{(ii)}$$

Gravitational potential at P is

$$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} \qquad \text{(iii)}$$

Using (i) and (ii) in (iii) and simplifying, we get

$$V = -\frac{G}{r}\left(m_1 + m_2 + 2\sqrt{m_1 m_2}\right)$$

EXAMPLE 28 The distance of a planet from the sun is 10 times that of the earth. Find the period of revolution of the planet around the sun.

SOLUTION From Kepler's law of periods, $T^2 \propto r^3$. Therefore

$$\frac{T_p^2}{T_e^2} = \frac{r_p^3}{r_e^3} = (10)^3 = 1000$$

$$\therefore \quad T_p = T_e \times \sqrt{1000}$$

$$= 1 \text{ year} \times 31.6 = 31.6 \text{ years}$$

EXAMPLE 29 A satellite is revolving in a circular orbit close to the surface of the earth with a speed v. What minimum additional speed must be imparted to it so that it escapes the gravitational pull of the earth? Radius of earth $= 6.4 \times 10^6$ m.

SOLUTION $v = \sqrt{gR}$ and $v_e = \sqrt{2gR}$

$\therefore$ Additional speed required is

$$v_e - v = \left(\sqrt{2} - 1\right)\sqrt{gR}$$

$$= 0.414 \times \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 3.28 \times 10^3 \text{ ms}^{-1}$$

$$= 3.28 \text{ km s}^{-1}$$

EXAMPLE 30 A body of mass m is placed at the centre of a spherical shell of radius R and mass M. Find the gravitational potential on the surface of the shell.

SOLUTION Gravitational potential on the surface of the shell due to body of mass m is

$$V_b = -\frac{Gm}{R}$$

Gravitational potential on the surface of the shell due to shell itself is

$$V_s = -\frac{GM}{R}$$

$$\therefore \quad V = V_b + V_s = -\frac{G}{R}(m + M)$$

EXAMPLE 33 A tunnel is drilled from the surface of the earth to its centre. A body of mass m is dropped into the tunnel. Find the speed with which the body hits the bottom of the tunnel. The mass of earth is M and its radius is R.

SOLUTION Let v be the required speed. Gain in K.E. = loss in P.E. = P.E. at the surface – P.E. at the centre. The potential energy at the centre of the sphere $= -\frac{3}{2}\frac{GmM}{R}$. Hence

$$\frac{1}{2}mv^2 = -\frac{GmM}{R} - \left(-\frac{3}{2}\frac{GmM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

1 SECTION Multiple Choice Questions with One Correct Choice

Level A

1. The acceleration due to gravity g on earth is 9.8 ms^{-2}. What would the value of g for a planet whose size is the same as that of earth but the density in twice that of earth?
   (a) 19.6 ms^{-2} (b) 9.8 ms^{-2}
   (c) 4.9 ms^{-2} (d) 2.45 ms^{-2}

2. If the radius of the earth suddenly decreases to 80% of its present value, the mass of the earth remaining the same, the value of the acceleration due to gravity will
   (a) remain unchanged
   (b) become $(9.8 \times 0.8) \text{ ms}^{-2}$
   (c) increase by 36%
   (d) increase by about 56%

3. The mass of a planet is $1/10^{th}$ that of earth and its diameter is half that of earth. The acceleration due to gravity at the planet will be
   (a) 1.96 ms^{-2} (b) 3.92 ms^{-2}
   (c) 9.8 ms^{-2} (d) 19.6 ms^{-2}

4. The escape velocity of a body projected vertically upwards from the surface of the earth is v. If the body is projected in a direction making an angle θ with the vertical, the escape velocity would be
   (a) v (b) $v \cos \theta$
   (c) $v \sin \theta$ (d) $v \tan \theta$

5. A small planet is revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force between the planet and the star were proportional to $R^{-5/2}$, then T would be proportional to

(a) $R^{3/2}$ (b) $R^{3/5}$
(c) $R^{7/2}$ (d) $R^{7/4}$

6. If g is the acceleration due to gravity on the surface of the earth, the gain in potential energy of an object of mass m raised from the earth's surface to a height equal to the radius R of the earth is

(a) $mgR/4$ (b) $mgR/2$
(c) mgR (d) $2\ mgR$

7. Two satellites of the same mass are orbiting round the earth at heights of R and $4R$ above the earth's surface: R being the radius of the earth. Their kinetic energies are in the ratio of

(a) 4 : 1 (b) 3 : 2
(c) 4 : 3 (d) 5 : 2

8. A satellite is orbiting the earth in a circular orbit of radius r. Its period of revolution varies as

(a) $\sqrt{r}$ (b) r
(c) $r^{3/2}$ (d) r^2

9. If the gravitational force of attraction between any two bodies were to vary as $1/r^3$ instead of $1/r^2$, the period of revolution of a planet round the sun would vary as

(a) $\sqrt{r}$ (b) r
(c) $r^{3/2}$ (d) r^2

10. If both the mass and the radius of the earth decrease by 1%, the value of the acceleration due to gravity will

(a) decrease by 1% (b) increase by 1%
(c) increase by 2% (d) remain unchanged.

11. A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth; R being the radius of the earth. What will be the time period of another satellite at a height 2.5 R from the surface of the earth?

(a) $6\sqrt{2}$ hours (b) $6\sqrt{2.5}$ hours
(c) $6\sqrt{3}$ hours (d) 12 hours

Level B

12. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is given by

(a) $2\left[\dfrac{G(M_1+M_2)^2}{md}\right]^{1/2}$ (b) $2\left[\dfrac{G(M_1+M_2)}{d}\right]^{1/2}$

(c) $2\left[\dfrac{G(M_1-M_2)^2}{md}\right]^{1/2}$ (d) $2\left[\dfrac{G(M_1-M_2)}{d}\right]^{1/2}$

13. The angular momentum of the earth revolving round the sun is proportional to R^n where R is the distance between the earth and the sun. The value of n is

(a) 0.5 (b) 1.0
(c) 1.5 (d) 2.0

14. A satellite is moving around the earth in a stable circular orbit. Which one of the following statements will be wrong for such a satellite?

(a) It is moving at a constant speed.
(b) Its angular momentum remains constant.
(c) It is acted upon by a force directed away from the centre of the earth which counter- balances the gravitational pull of the earth.
(d) It behaves as if it were a freely falling body.

15. Astronauts in a stable orbit around the earth are said to be in a weightless condition. The reason for this is that

(a) the capsule and its contents are falling freely at the same rate
(b) there is no gravitational force acting on them
(c) the gravitational force of the earth balances that of the sun
(d) there is no atmosphere at the height at which they are orbiting.

16. The escape velocity from the earth is v_e. What is the escape velocity from a planet whose radius is twice that of the earth and mean density is the same as that of the earth?

(a) $v_e/2$ (b) v_e
(c) $2\ v_e$ (d) $4\ v_e$

17. Choose the wrong statement. The escape velocity of a body from a planet depends upon

(a) the mass of the body
(b) the mass of the planet
(c) the average radius of the planet
(d) the average density of the planet

18. Choose the wrong statement. The orbital velocity of a body in a stable orbit around a planet depends upon

(a) the average radius of the planet
(b) the height of the body above the planet
(c) the acceleration due to gravity
(d) the mass of the orbiting body

19. Choose the correct statement. In planetary motion
(a) the speed along the orbit remains constant
(b) the angular speed remains constant
(c) the total angular momentum remains constant.
(d) the radius of the orbit remains constant.

20. An object weighs W newton on earth. It is suspended from the lower end of a spring balance whose upper end is fixed to the ceiling of a space capsule in a stable orbit around the earth. The reading of the spring balance will be
(a) W (b) less than W
(c) more than W (d) zero.

21. If M is the mass of the earth, R its radius (assumed spherical) and G the universal gravitational constant, then the amount of work that must be done on a body of mass m so that it completely escapes from the gravity of the earth, is given by
(a) $\frac{GmM}{R}$ (b) $\frac{GmM}{2R}$
(c) $\frac{3GmM}{2R}$ (d) $\frac{3GmM}{4R}$

22. A rocket is fired from the earth to the moon. The distance between the earth and the moon is r and the mass of the earth is 81 times the mass of the moon. The gravitational force on the rocket will be zero, when its distance from the moon is
(a) $\frac{r}{20}$ (b) $\frac{r}{15}$
(c) $\frac{r}{10}$ (d) $\frac{r}{5}$

23. Assuming that the earth is a sphere of radius R, at what altitude will the value of the acceleration due to gravity be half its value at the surface of the earth?
(a) $h = \frac{R}{2}$ (b) $h = \frac{R}{\sqrt{2}}$
(c) $h = \left(\sqrt{2}+1\right)R$ (d) $h = \left(\sqrt{2}-1\right)R$

24. Assuming that the earth is a sphere of uniform mass density, what is the percentage decrease in the weight of a body when taken to the end of a tunnel 32 km below the surface of the earth? Radius of earth = 6400 km.
(a) 0.25% (b) 0.5%
(c) 0.75% (d) 1%

25. An extremely small and dense neutron star of mass M and radius R is rotating at an angular frequency ω. If an object is placed at its equator, it will remain stuck to it due to gravity if
(a) $M > \frac{R\omega}{G}$ (b) $M > \frac{R^2\omega^2}{G}$
(c) $M > \frac{R^3\omega^2}{G}$ (d) $M > \frac{R^2\omega^3}{G}$

26. Two small and heavy spheres, each of mass M, are placed a distance r apart on a horizontal surface. The gravitational field intensity at the mid–point of the line joining the centres of the spheres is
(a) zero (b) $\frac{GM^2}{r^2}$
(c) $\frac{GM^2}{2r^2}$ (d) $\frac{GM^2}{4r^2}$

27. In Q. 26, the gravitational potential at the mid–point of the line joining the centres of the spheres is
(a) zero (b) $-\frac{GM}{r}$
(c) $-\frac{2GM}{r}$ (d) $-\frac{4GM}{r}$

28. Three particles, each of mass m, are placed at the vertices of an equilateral triangle of side a. The gravitational field intensity at the centroid of the triangle is
(a) zero (b) $\frac{Gm^2}{a^2}$
(c) $\frac{2Gm^2}{a^2}$ (d) $\frac{3Gm^2}{a^2}$

29. In Q. 28, the gravitational potential at the centroid is
(a) zero (b) $-3\sqrt{3}\frac{Gm}{a}$
(c) $-2\sqrt{3}\frac{Gm}{a}$ (d) $-\sqrt{3}\frac{Gm}{a}$

30. The distance between the sun and the earth is r and the earth takes time T to make one complete revolution around the sun. Assuming the orbit of the earth around the sun to be circular, the mass of the sun will be proportional to
(a) $\frac{r^2}{T}$ (b) $\frac{r^2}{T^2}$
(c) $\frac{r^3}{T^2}$ (d) $\frac{r^3}{T^3}$

31. A rocket is launched vertically from the surface of the earth of radius R with an initial speed v. If

(a) $\frac{5\,Gm}{4r^2}$ (b) $\frac{4\,Gm}{3r^2}$

(c) $\frac{3\,Gm}{2r^2}$ (d) $\frac{2\,Gm}{r^2}$

45. In Q. 44, the magnitude of the gravitational potential at point O will be

(a) $\frac{Gm}{2r}$ (b) $\frac{Gm}{r}$

(c) $\frac{3\,Gm}{2r}$ (d) $\frac{2\,Gm}{r}$

46. A meteor of mass M breaks up into two parts. The mass of one part is m. For a given separation r the mutual gravitational force between the two parts will be the maximum if

(a) $m = \frac{M}{2}$ (b) $m = \frac{M}{3}$

(c) $m = \frac{M}{\sqrt{2}}$ (d) $m = \frac{M}{2\sqrt{2}}$

47. A body of mass m is raised to a height h above the surface of the earth of mass M and radius R until its gravitational potential energy increases by $\frac{1}{3}$ mgR. The value of h is

(a) $\frac{R}{3}$ (b) $\frac{R}{2}$

(c) $\frac{mR}{(M+m)}$ (d) $\frac{mR}{M}$

48. A satellite of mass m is moving in a circular orbit of radius R above the surface of a planet of mass M and radius R. The amount of work done to shift the satellite to a higher orbit of radius $2R$ is (here g is the acceleration due to gravity on planet's surface)

(a) mgR (b) $\frac{mgR}{6}$

(c) $\frac{mMgR}{(M+m)}$ (d) $\frac{mMgR}{6(M+m)}$

49. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

(a) $\left(\frac{n}{n+1}\right) mgR$ (b) $\left(\frac{n}{n-1}\right) mgR$

(c) $nmgR$ (d) $\frac{mgR}{n}$

50. A body of mass m is dropped from a height nR above the surface of the earth (here R is the radius of the earth). The speed at which the body hits the surface of the earth is

(a) $\sqrt{\frac{2gR}{(n+1)}}$ (b) $\sqrt{\frac{2gR}{(n-1)}}$

(c) $\sqrt{\frac{2gRn}{(n-1)}}$ (d) $\sqrt{\frac{2gRn}{(n+1)}}$

51. Two balls A and B are thrown vertically upwards from the same location on the surface of the earth with velocities $2\sqrt{\frac{gR}{3}}$ and $\sqrt{\frac{2gR}{3}}$ respectively, where R is the radius of the earth and g is the acceleration due to gravity on the surface of the earth. The ratio of the maximum height attained by A to that attained by B is

(a) 2 (b) 4

(c) 8 (d) $4\sqrt{2}$

52. Two solid spheres of radii r and $2r$, made of the same material, are kept in contact. The mutual gravitational force of attraction between them is proportional to

(a) $\frac{1}{r^4}$ (b) $\frac{1}{r^2}$

(c) r^2 (d) r^4

53. A comet is moving in a highly elliptical orbit round the sun. When it is closest to the sun, its distance from the sun is r and its speed is v. When it is farthest from the sun, its distance from the sun is R and its speed will be

(a) $v\left(\frac{r}{R}\right)^{1/2}$ (b) $v\left(\frac{r}{R}\right)$

(c) $v\left(\frac{r}{R}\right)^{3/2}$ (d) $v\left(\frac{r}{R}\right)^2$

54. The value of the acceleration due to gravity at the surface of the earth of radius R is g. It decreases by 10% at a height h above the surface of the earth. The gravitational potential at this height is

(a) $-\frac{gR}{\sqrt{10}}$ (b) $-\frac{2gR}{\sqrt{10}}$

(c) $-\frac{3gR}{\sqrt{10}}$ (d) $-\frac{4gR}{\sqrt{10}}$

55. Two stars of masses m and $2m$ are co-rotating about their centre of mass. Their centres are at a distance r apart. If r is much larger than the sizes of the stars, their common period of revolution is proportional to

(a) r (b) $r^{3/2}$

(c) r^2 (d) r^3

56. In Q. 55 above, the kinetic energies of stars of masses m and $2m$ are in the ratio

(a) $1 : \sqrt{2}$ (b) $\sqrt{2} : 1$
(c) 1 : 2 (d) 2 : 1

57. In Q. 55 above, the angular momenta of the stars of masses m and $2m$ about their centre of mass are in the ratio

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

58. A uniform sphere of mass M and radius R exerts a force F on a small mass m situated at a distance of $2R$ from the centre O of the sphere. A spherical portion of diameter R is cut from the sphere as shown in Fig. 6.20. The force of attraction between the remaining part of the sphere and the mass m will be

(a) $\frac{7F}{9}$ (b) $\frac{2F}{3}$
(c) $\frac{4F}{9}$ (d) $\frac{F}{3}$

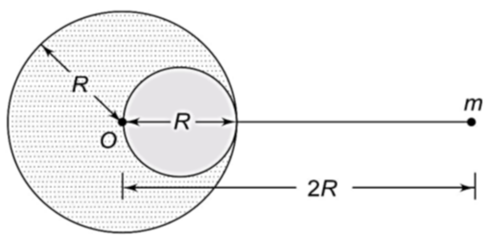


Fig. 6.20

59. The centres of a ring of mass m and a sphere of mass M of equal radius R, are at a distance $\sqrt{8}\,R$ apart as shown in Fig. 6.21. The force of attraction between the ring and the sphere is

(a) $\frac{2\sqrt{2}}{27}\frac{GmM}{R^2}$ (b) $\frac{GmM}{8R^2}$
(c) $\frac{GmM}{9R^2}$ (d) $\frac{\sqrt{2}}{9}\frac{GmM}{9R^2}$

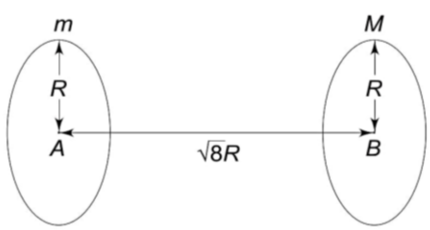


Fig. 6.21

60. A solid sphere of uniform density and radius 4 units is located with its centre at origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A $(-2, 0, 0)$ and B $(2, 0, 0)$ respectively are taken out of the solid sphere leaving behind spherical cavities as shown in Fig. 6.22. Choose the incorrect statement from the following.

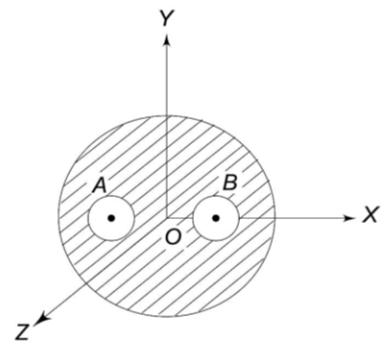


Fig. 6.22

(a) The gravitational force due to this object at the origin is zero.
(b) The gravitational force at point $B(2, 0, 0)$ is zero.
(c) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.
(d) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 4$.

61. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[\frac{2G(m_1 + m_2)}{r}\right]^{1/2}$

(b) $\left[\sqrt{\frac{2G}{r}}\frac{(m_1 + m_2)}{2}\right]^{1/2}$

(c) $\left[\frac{r}{2G(m_1 m_2)}\right]^{1/2}$

(d) $\left(\frac{2G}{r}m_1 m_2\right)^{1/2}$

62. Two objects of masses m and $4m$ are at rest at infinite separation. They move towards each other under mutual gravitational attraction. Then, at a separation r, which of the following is true?

(a) The total energy of the system is not zero.
(b) The force between them is not zero.
(c) The centre of mass of the system is at rest.
(d) All the above are true.

63. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of the second satellite is longer than that of the first by approximately

(a) 0.5% (b) 1.0%
(c) 1.5% (d) 3.0%

64. If the distance between the earth and the sun were half its present value, the number of days in a year would have been

(a) 64.5 (b) 129
(c) 182.5 (d) 730

65. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is

(a) $-E_0$ (b) 1.5 E_0
(c) 2 E_0 (d) E_0

66. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Which of the following statements is correct?

(a) The acceleration of S is always directed towards the centre of the earth.
(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(c) The total mechanical energy of S remains constant.
(d) The linear momentum of S remains constant in magnitude.

67. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$
(c) 4 (d) 2

68. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(a) 4 Mg/k (b) 2 Mg/k
(c) Mg/k (d) $Mg/2k$

69. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometres above the earth's surface (R_{Earth} = 6400 km) will approximately be

(a) (1/2) h (b) 1 h
(c) 2 h (d) 4 h

70. A mass M is divided into two parts xm and $(1 - x)m$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$
(c) 1 (d) 2

71. The height of the point vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the earth)

(a) 8 R (b) 9 R
(c) 10 R (d) 20 R

72. A body is projected up with a velocity equal to $\frac{3}{4}$ of the escape velocity from the surface of the earth. The height it reaches is: (Radius of the earth = R)

(a) $\frac{10R}{9}$ (b) $\frac{9R}{7}$
(c) $\frac{9R}{8}$ (d) $\frac{10R}{3}$

73. Two bodies of masses $M_1 = m$ and $M_2 = 4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$
(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$

74. The radius of the earth is R and g is the acceleration due to gravity on its surface. What should be the angular speed of the earth so that bodies lying on the equator may appear weightless?

(a) $\sqrt{\frac{g}{R}}$ (b) $\sqrt{\frac{2g}{R}}$
(c) $\sqrt{\frac{g}{2R}}$ (d) $2\sqrt{\frac{g}{R}}$

75. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in Fig. 6.23) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

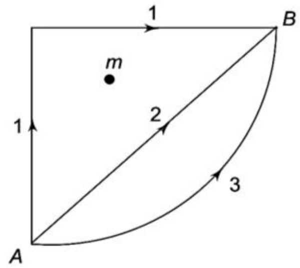


Fig. 6.23

(a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_3 < W_2$ (d) $W_1 < W_2 < W_3$

76. A binary star system consists of two stars of masses M_1 and M_2 revolving in circular orbits of radii R_1 and R_2 respectively. If their respective time periods are T_1 and T_2, then
(a) $T_1 > T_2$ if $R_1 > R_2$ (b) $T_1 > T_2$ if $M_1 > M_2$
(c) $T_1 = T_2$ (d) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$

77. A uniform bar AB of mass M and length L has a particle of mass m held at a distance a from end A as shown in Fig. 6.24. The gravitational force exerted by the bar on mass m in given by
(a) $\frac{GmM}{\left(\frac{L}{2}+a\right)^2}$ (b) $\frac{GmM}{a(L+a)}$
(c) $\frac{GmM}{L(L+a)}$ (d) $\frac{GmM}{L(L+2a)}$

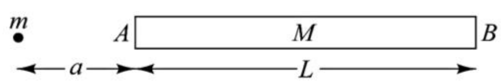


Fig. 6. 24

78. A satellite of mass m is moving in a circular orbit of radius r around the earth of mass M and radius R. The amount of work needed (or energy spent) to shift the satellite into a new orbit of radius $2r$ will be
(a) $\frac{GmM}{2(R+r)}$ (b) $\frac{GmM}{4(R+r)}$
(c) $\frac{GmM}{2r}$ (d) $\frac{GmM}{4r}$

79. A comet of mass m is in an elliptical orbit around the SUN (mass M) a shown in Fig. 6.25. A is the position of the comet when it is closest to the SUN and $\mathbf{v}_1$ is its velocity at A. B is the position of the comet when it is farthest from the sun and $\mathbf{v}_2$ is its velocity at B. The ratio $\frac{v_1}{v_2}$
(a) $\frac{r_1}{r_2}$ (b) $\frac{r_2}{r_1}$
(c) $\sqrt{\frac{r_1}{r_2}}$ (d) $\sqrt{\frac{r_2}{r_1}}$

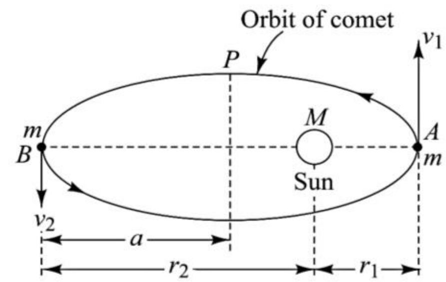


Fig. 6.25

80. If $m << M$ and a is the semi-major axis of the ellipse, the speed v_1 when the comet is at A is given by
(a) $2\sqrt{\frac{GM\ r_2}{r_1 a}}$ (b) $\sqrt{\frac{2GM\ r_2}{r_1 a}}$
(c) $\sqrt{\frac{GM\ r_2}{r_1 a}}$ (d) $\sqrt{\frac{GM\ r_2}{2r_1 a}}$

81. The speed v_2 of the comet at B is given by
(a) $\sqrt{\frac{GM\ r_1}{2r_2 a}}$ (b) $\sqrt{\frac{GM\ r_1}{r_2 a}}$
(c) $\sqrt{\frac{2GM\ r_1}{r_2 a}}$ (d) $2\sqrt{\frac{GM\ r_1}{r_2 a}}$

82. In Q.81 above, the magnitude of the angular momentum of the comet about the centre of the sun when the comet is at A is proportional to
(a) r_1 (b) r_2
(c) $\frac{r_1 r_2}{(r_1+r_2)}$ (d) $\frac{r_1^2}{(r_1+r_2)}$

83. The speed of the comet when it is a point P marked in Fig. 6.25 is
(a) $\sqrt{\frac{GM}{(r_1+r_2)}}$ (b) $\sqrt{\frac{2GM}{(r_1+r_2)}}$
(c) $2\sqrt{\frac{GM}{(r_2-r_1)}}$ (d) $\sqrt{\frac{GM}{(r_2-r_1)}}$

84. The earth has mass M and radius R. An object of mass m is dropped from a distance of $3R$ from the centre of the earth. The object strikes the surface of the earth with a speed $v = \sqrt{kgR}$, where g is acceleration due to gravity or the surface of the earth. The value of k

is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

85. A planet revolves around the sun is an elliptual orbit. The maximum and minimum distances of the planet from the sun are a and b respectively. The time period of revolution of the planet is proportional to

(a) $a^{3/2}$ (b) $b^{3/2}$

(c) $(a + b)^{3/2}$ (d) $(a - b)^{3/2}$

86. A solid sphere of mass M and a ring of mass m have their centres lying on the x–axis separated by a distance $2\sqrt{2}\,r$ where r is the radius of the ring as shown in Fig. 6.26. The gravitational force exerted by the sphere on the ring is

(a) $\frac{2\sqrt{2}\,GMm}{27\,r^2}$ (b) $\frac{GMm}{9\sqrt{2}\,r^2}$

(c) $\frac{GMm}{8r^2}$ (d) $\frac{\sqrt{2}\,GMm}{9r^2}$

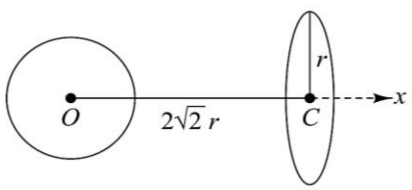


Fig. 6.26

87. There identical spheres A, B and C each of mass M and radius R are placed touching each other as shown in Fig. 6.27. The magnitude of the gravitational force on any sphere due to the other two spheres is $\frac{kGM^2}{R^2}$ where the value of k is

(a) 1 (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}}{4}$ (d) 3

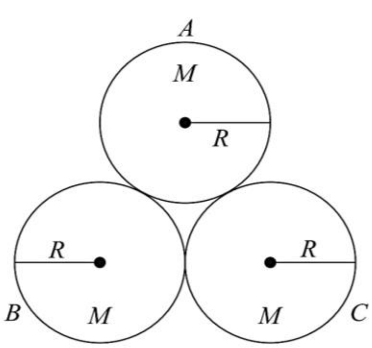


Fig. 6.27

88. A tunnel is dug along the diameter of the earth. An object is held in the tunnel at a distance x from the centre of the earth. The magnitude of the gravitational force on the object is proportional to

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$

(c) x (d) x^2

89. A thin rod of mass M and length L is bent in the form of a circle. The gravitational potential at the centre of the circular rod is

(a) zero (b) $-\frac{GM}{2\pi L}$

(c) $-\frac{GM}{L}$ (d) $-\frac{2\pi GM}{L}$

90. A uniform solid sphere of mass M and radius R has its centre at the origin O of the coordinate system. Two spherical cavities A and B, each of radius $r = R/4$ are made such that their coordinates are $(2r,0,0)$ and $(-2r,0,0)$ respectively as shown in Fig. 6.28. The gravitational field at O due to the remaining part of the sphere is

(a) zero (b) $\frac{7GM}{R^2}$

(c) $\frac{9GM}{R^2}$ (d) $\frac{GM}{R^2}$

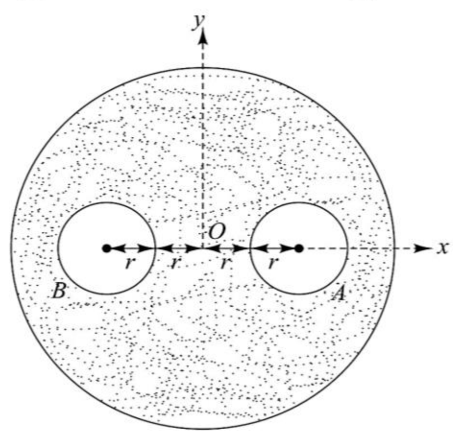


Fig. 6.28

Answers

Level A

1. (a)	2. (d)	3. (b)	4. (a)
5. (d)	6. (b)	7. (d)	8. (c)

9. (d)	10. (b)	11. (a)	

Level B

12. (b)	13. (a)	14. (c)	15. (a)
16. (c)	17. (a)	18. (d)	19. (c)
20. (d)	21. (a)	22. (c)	23. (d)
24. (b)	25. (c)	26. (a)	27. (d)
28. (a)	29. (b)	30. (c)	31. (a)
32. (d)	33. (a)	34. (d)	35. (c)
36. (d)	37. (c)	38. (a)	39. (a)
40. (b)	41. (b)	42. (a)	43. (b)
44. (b)	45. (d)	46. (a)	47. (b)
48. (b)	49. (a)	50. (d)	51. (c)
52. (d)	53. (b)	54. (c)	55. (b)
56. (c)	57. (c)	58. (a)	59. (a)
60. (b)	61. (a)	62. (d)	63. (c)
64. (b)	65. (c)	66. (a)	67. (d)
68. (b)	69. (c)	70. (a)	71. (b)
72. (b)	73. (d)	74. (a)	75. (b)
76. (c)	77. (b)	78. (d)	79. (b)
80. (a)	81. (d)	82. (c)	83. (b)
84. (d)	85. (c)	86. (a)	87. (c)
88. (c)	89. (d)	90. (a)	

Solutions

Level A

1. Volume of earth $(V) = \frac{4\pi}{3} R^3$. Therefore, density of earth is $\rho = \frac{M}{V}$ or $M = V\rho = \frac{4\pi}{3} R^3 \rho$

   Now $g = \frac{GM}{R^2} = \frac{4\pi G\rho R}{3}$. Since R is the same for both planets, if ρ is doubled, the value of g is also doubled. Hence the correct choice is (a).

2. Now $g = \frac{GM}{R^2}$. If R reduces to $R' = 0.8\ R$, the value of g becomes

$$g' = \frac{GM}{R'^2} = \frac{GM}{0.64R^2}$$

$$= \frac{g}{0.64} = \frac{9.8\ \text{ms}^{-2}}{0.64}$$

   Increase in value of $g = \frac{g}{0.64} - g = \frac{0.36g}{0.64}$

   $\therefore$ Percentage increase $= \frac{0.36g}{0.64g} \times 100 = 56.25\%$

   Hence the correct choice is (d).

3. For earth $g = \frac{GM}{R^2}$

   For planet $g' = \frac{GM'}{R'^2}$

$$\therefore \frac{g'}{g} = \frac{M'}{M} \times \frac{R^2}{R'^2} = \frac{1}{10} \times (2)^2 = \frac{2}{5}$$

   Thus $g' = \frac{2g}{5} = \frac{2}{5} \times 9.8 = 3.92\ \text{ms}^{-2}$

   Hence the correct choice is (b).

4. The escape velocity is independent of the direction of projection. Hence the correct choice is (a).

5. Since the gravitational force provides the necessary centripetal force for circular motion, we have

$$\frac{mv^2}{R} \propto R^{-5/2}$$

   or $\frac{mv^2}{R} = kR^{-5/2}$, where k is a constant.

   Therefore $v = \sqrt{\frac{kR^{-3/2}}{m}}$

   Period of revolution $T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{m}{k} \times R^{7/2}}$ or $T \propto R^{7/4}$. Hence the correct choice is (d).

6. If M is the mass of the earth, the gain in potential energy is given by

$$\int_R^{2R} \frac{GmM}{x^2}\,dx = -GmM\left|\frac{1}{x}\right|_R^{2R} = \frac{GmM}{2R}$$

$$= \frac{GM}{2R^2} \cdot mR = \frac{mgR}{2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

   Hence the correct choice is (b). Remember, the expression $PE = mgh$ is an approximate one which is valid if $h << R$.

7. Let a satellite of mass m revolve in an orbit at a height r from the centre of the earth. If the speed of the satellite is v, then

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \text{ or } mv^2 = \frac{GMm}{r}$$

where M is the mass of the earth. The kinetic energy of the satellite is given by

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

The ratio of the kinetic energies of the two satellites is ($\because$ their masses are equal)

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{GMm}{2r_1}\cdot\frac{2r_2}{GMm} = \frac{r_2}{r_1}$$

But $r_1 = R + R = 2R$ and $r_2 = R + 4R = 5R$. Therefore, the ratio $r_2/r_1 = 5/2$. Hence the correct answer is (d).

8. The speed the satellite is given by

$$v = \sqrt{\frac{GM}{r}}$$

Therefore, its period of revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\sqrt{GM}}\cdot r^{3/2}$$

Hence the correct choice is (c).

9. If the gravitational force were to vary as $1/r^3$, the speed of the planet round the sun would be given by

$$v = \sqrt{\frac{GM}{r^2}} \text{ instead of } v = \sqrt{\frac{GM}{r}}$$

Therefore, the period of revolution of the planet would be given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r^2}{\sqrt{GM}}$$

Hence the correct choice is (d).

10. The acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

$$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta M}{M} - \frac{2\Delta R}{R}$$
$$= (-1\%) - (-2 \times 1\%)$$
$$= +1\%$$

i.e. the value of g will increase by 1%. Hence the correct choice is (b).

11. The time period of satellite orbiting at a distance r from the centre of the earth is given by

$$T^2 = \frac{4\pi^2 r^3}{GM^2}$$

where M is the mass of the earth. Therefore, the ratio of the time periods of two satellites at distance r_1 and r_2 respectively from the centre of the earth is

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

or $$T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2}$$

For the geostationary satellite T_1 = 1 day = 24 hours and $r_1 = 6R + R = 7R$.

For the other satellite, $r_2 = 2.5\ R + R = 3.5\ R$. Therefore

$$T_2 = 24 \times \left(\frac{3.5R}{7R}\right)^{3/2} = 24 \times \left(\frac{1}{2}\right)^{3/2}$$
$$= 6\sqrt{2} \text{ hours.}$$

Hence the correct choice is (a).

Level B

12. Let P be a particle of mass m situated midway between the centres of the earth and the moon (Fig. 6.29). The potential energy of particle P due to earth is

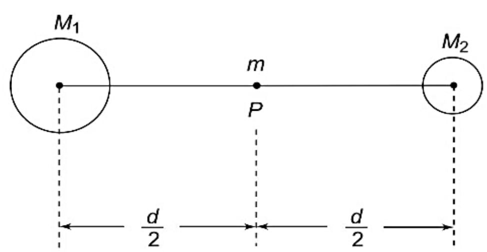


Fig. 6.29

$$-\frac{GmM_1}{r} = -\frac{GmM_1}{d/2} = -\frac{2GM_1m}{d}$$

and that due to moon $= -\dfrac{2GM_2m}{d}$

$\therefore$ Total potential energy $= -\dfrac{2Gm}{d}(M_1 + M_2)$

If the particle P is projected with a velocity v, its kinetic energy $= \frac{1}{2}mv^2$. Therefore, the total initial energy of the particle is

$$E_i = -\frac{2Gm}{d}(M_1 + M_2) + \frac{1}{2}mv^2$$

If the particle is to escape to infinity, its final potential and kinetic energy will be zero. Thus the total energy $E_f = 0$. From the principle of conservation of energy,

$$E_i = E_f$$

or $-\frac{2Gm}{d}(M_1+M_2)+\frac{1}{2}mv^2=0$

which gives $v=2\left[\frac{G(M_1+M_2)}{d}\right]^{\frac{1}{2}}$

Hence the correct choice is (b). Notice that v is independent of the mass m of the particle. This is the minimum value of the velocity for the particle to escape to infinity.

13. We know that

$$\frac{mv^2}{R}=\frac{GMm}{R^2}$$

which gives $v=\sqrt{\frac{GM}{R}}$

Now, angular momentum $L=mvR=m\times\sqrt{\frac{GM}{R}}\times R$

$=m\sqrt{GM}\;R^{1/2}$.

or $L\propto R^{1/2}$. Hence the correct choice is (a).

14. Choices (a), (b) and (d) are all correct. Hence the choice (c) is the correct answer.

15. The correct choice is (a).

16. For earth : $v_e=\sqrt{\frac{2M_eG}{R_e}}$

For planet : $v_p=\sqrt{\frac{2M_pG}{R_p}}$

Therefore, $\frac{v_p}{v_e}=\sqrt{\frac{M_p}{M_e}\cdot\frac{R_e}{R_p}}$ (i)

If ρ_p and ρ_e are the respective average densities of the planet and the earth, then

$$M_P=\frac{4\pi}{3}R_p^3\rho_p$$

and $M_e=\frac{4\pi}{3}R_e^3\rho_e$

Therefore, $\frac{M_p}{M_e}=\frac{R_p^3}{R_e^3}$ $(\because \rho_p=\rho_e)$ (ii)

Using (ii) in (i) we get

$$\frac{v_p}{v_e}=\frac{R_p}{R_e}=2 \qquad (\because R_p=2R_e)$$

or $v_p=2v_e$. Hence the correct choice is (c).

17. Statement (a) is wrong.

18. Statement (d) is wrong.

19. The orbits of planets are elliptical. The speed of the planet and its angular speed (or angular frequency) keep changing. Since no net torque acts on the planet, its angular momentum remains constant. Hence the correct choice is (c).

20. Since the object in the space capsule is in a state of weightlessness (or zero gravity), the reading of the spring balance will be zero. Hence the correct choice is (d).

21. The gravitational force acting on the body is

$$F=\frac{GmM}{r^2}$$

The work done by the body against the gravitational pull of the earth in moving upward through a small distance dr is

$$dW=Fdr=\frac{GmM}{r^2}dr$$

Therefore, the total work done by the body in escaping from the earth, i.e. in moving to an infinite distance, is given by

$$W=\int_R^\infty dW=GmM\int_R^\infty\frac{dr}{r^2}$$

$$=GmM\left|-\frac{1}{r}\right|_R^\infty=\frac{GmM}{R}$$

Hence the correct choice is (a).

22. Let the rocket be at a distance x from the moon when the gravitational force on it is zero. Its distance from earth $=r-x$. Gravitational force on the rocket due to earth is

$$F_e=\frac{GmM_e}{(r-x)^2}$$

where m is the mass of the rocket. Gravitational force on the rocket due to moon is

$$F_m=\frac{GmM_m}{x^2}$$

Since the two forces are in opposite directions, the net force on the rocket will be zero if $F_e=F_m$. Equating the two we get

$$\frac{r-x}{x}=\sqrt{\frac{M_e}{M_m}}=\sqrt{81}=9$$

which gives $x=\frac{r}{10}$. Hence the correct choice is (c).

23. Now

$$\frac{g'}{g}=\frac{R^2}{(R+h)^2}$$

or $\frac{1}{2}=\frac{R^2}{(R+h)^2}$

which gives $h = (\sqrt{2} - 1)R$, which is choice (d).

24. Acceleration due to gravity at depth d is given by

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{32}{6400}\right) = \frac{199g}{200}$$

If m is the mass of the body, its weight on the surface of the earth is mg and at the end of the tunnel it is mg'. Therefore, decrease in weight is

$$mg - mg' = m\left(g - \frac{199g}{200}\right) = \frac{mg}{200}$$

$\therefore$ Percentage decrease $= \dfrac{mg/200}{mg} \times 100 = 0.5\%$

Hence the correct choice is (b).

25. An object of mass m, placed at the equator of the star, will experience two forces: (i) an attractive force due to gravity towards the centre of the star and (ii) an outward centrifugal force due to the rotation of the star. The centrifugal force arises because the object is in a rotating (non–inertial) frame; this force is equal to the inward centripetal force but opposite in direction. Force on object due to gravity is

$$F_g = \frac{GmM}{R^2}$$

Centrifugal force on the object is

$$F_c = mR\omega^2$$

The object will remain stuck to the star and not fly off if

$$F_g > F_c$$

or $$\frac{GmM}{R^2} > mR\omega^2$$

or $$M > \frac{R^3\omega^2}{G}$$

Hence the correct choice is (c).

26. Gravitational field intensity at a point is defined as the gravitational force experienced by a unit mass placed at that point. Since the spheres have the same mass, the gravitational forces exerted by each sphere on a unit mass placed at the mid-point will be equal and opposite. Hence the gravitational field intensity at the mid-point is zero. Thus the correct choice is (a).

27. Gravitational potential at the mid-point is

$$V = -\frac{GM}{r/2} - \frac{GM}{r/2} = -\frac{4MG}{r}$$

Hence the correct choice is (d).

28. Given $AB = BC = AC = a$ (see Fig. 6.30). The perpendiculars from A, B and C on opposite sides meet at the centroid O, which bisect the sides AB, BC and AC. Let $r = AO = BO = CO$. Centroid also divides the lines AD, BE and CF in the ratio 2 : 1, i.e.

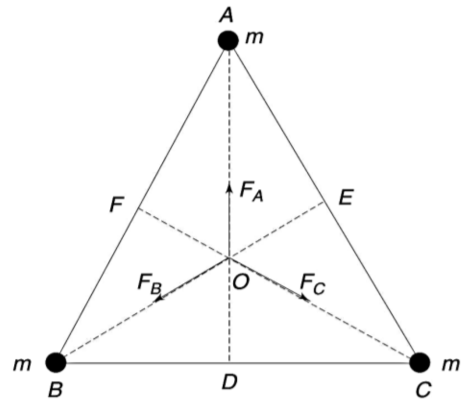


Fig. 6.30

$$AO = \frac{2}{3}AD,\ BO = \frac{2}{3}BE,\ CO = \frac{2}{3}CF.$$

In triangle ABD, $AD = a \sin 60° = \dfrac{\sqrt{3}a}{2}$.

Similarly, $BE = CF = \dfrac{\sqrt{3}a}{2}$.

$$\therefore \quad r = AO = OB = OC = \frac{2}{3} \times \frac{\sqrt{3}a}{2} = \frac{a}{\sqrt{3}}$$

The gravitational field intensity at point O is the net force exerted on a unit mass placed at O due to three equal masses m at vertices A, B and C. Since the three masses are equal and their distances from O are also equal, they exert forces F_A, F_B and F_C of equal magnitude. Their directions are shown in the figure. It follows from symmetry of forces that their resultant at point O is zero. Hence the correct choice is (a).

29. Refer to Fig. 6.25 again. Gravitational potential at O is

$$V = -\frac{Gm}{r} - \frac{Gm}{r} - \frac{Gm}{r}$$

$$= -\frac{3\,Gm}{r} = -\frac{3Gm}{a/\sqrt{3}} = -3\sqrt{3}\frac{Gm}{a}$$

Hence the correct choice is (b).

30. Let m and M be the masses of the earth and the sun respectively and v the speed of the earth in circular orbit. To keep the earth in circular orbit, the gravitational force $\dfrac{GmM}{r^2}$ must balance the

centripetal force $\frac{mv^2}{r}$, i.e.

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

or $$M = \frac{v^2 r}{G}$$

Also $v = \frac{2\pi r}{T}$. Using this, we get $M = \frac{4\pi^2 r^3}{T^2 G}$ or $M \propto \frac{r^3}{T^2}$.

Hence the correct choice is (c).

31. On the surface of the earth, the total energy is

$$\text{KE} + \text{PE} = \frac{1}{2}\, mv^2 - \frac{GmM}{R}$$

where m is the mass of the rocket and M that of earth. At the highest point, $v = 0$ and the energy is entirely potential.

$$\text{PE} = -\frac{GmM}{(R+h)}$$

where h is the maximum height attained. From the law of conservation of energy, we have

$$\frac{1}{2}\, mv^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

which gives $$\frac{R+h}{h} = \frac{2gR}{v^2} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

or $$h = \frac{R}{\left(\frac{2gR}{v^2} - 1\right)}$$

Hence the correct choice is (a).

32. If v_i and v_f are respectively the initial and final speeds of the body, we have, from the law of conservation of energy,

$$\frac{1}{2}\, mv_i^2 - \frac{GmM}{R} = \frac{1}{2}\, mv_f^2 \qquad \text{(i)}$$

where m is the mass of the body and M is the mass of the earth and R its radius. The escape velocity is given by

$$\frac{1}{2}\, mv_e^2 = \frac{GmM}{R} \qquad \text{(ii)}$$

Using (ii) in (i) gives

$$\frac{1}{2}\, mv_i^2 - \frac{1}{2}\, mv_e^2 = \frac{1}{2}\, mv_f^2$$

or $$v_f = (v_i^2 - v_e^2)^{1/2} \qquad \text{(iii)}$$

Given $v_i = 3v_e$. Therefore, $v_f = 2\sqrt{2}\ v_e$. Hence the correct choice is (d).

33. Since the initial velocity of the body is zero, its total energy is

$$E = -\frac{GmM}{r} \qquad \text{(i)}$$

where m is the mass of the body, M the mass of the earth and r its distance from the centre of the earth. When the body reaches the earth, let its velocity be v and its distance from the centre of the earth is the earth's radius R. Therefore, the energy now is

$$E = \frac{1}{2}\, mv^2 - \frac{GmM}{R} \qquad \text{(ii)}$$

Equating (i) and (ii) we get

$$v^2 = 2\, GM\left(\frac{1}{R} - \frac{1}{r}\right)$$

Also $g = \frac{GM}{R^2}$. Therefore $GM = gR^2$. Using this in above equation we get

$$v = R\left[2g\left(\frac{1}{R} - \frac{1}{r}\right)\right]^{1/2}$$

Now $r = 2R$ (given). Therefore

$$v = R\left[2g\left(\frac{1}{R} - \frac{1}{2R}\right)\right]^{1/2} = \sqrt{gR}$$

Hence the correct choice is (a).

34. Distance of the satellite from the centre of the earth $r = R + h$. If v is the speed of the satellite in its orbit, then

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

or $$\frac{1}{2}\, mv^2 = \frac{GmM}{2r}$$

or $$\text{KE} = \frac{GmM}{2(R+h)}$$

Hence the correct choice is (d)

35. Potential energy $= -\frac{GmM}{r} = -\frac{GmM}{(R+h)}$. Hence the correct choice is (c).

36. Total energy of the satellite $= \text{KE} + \text{PE} = \frac{GmM}{2(R+h)} - \frac{GmM}{(R+h)} = -\frac{GmM}{2(R+h)}$. Hence the total energy needed to pull the satellite out of the earth's gravitational

field is $\dfrac{GmM}{2(R+h)}$, which is choice (d).

37. If the earth were to shrink to half its size, the height of the satellite from the surface of the earth would become $\left(h+\dfrac{R}{2}\right)$. Its distance from the centre of the earth would be $\left(h+\dfrac{R}{2}+\dfrac{R}{2}\right) = (h + R)$, the same as the original distance. Hence the kinetic, potential and total energy of the satellite will be the same. Hence the correct choice is (c).

38. For a geostationary satellite, $h = 35870$ km.

$$\therefore \quad \frac{h}{R} = \frac{35870}{6380} = 5.6, \text{ which is choice (a)}$$

39. Consider a satellite of mass m moving with a speed v at an altitude r (measured from the centre of the earth). Then

$$\text{Kinetic energy (KE)} = \frac{1}{2} mv^2$$

$$\text{Gravitational potential energy (PE)} = -\frac{GmM}{r}$$

where M is the mass of the earth.

For a satellite in circular orbit, we have

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \quad \text{or} \quad v^2 = \frac{GM}{r}$$

$$\text{or} \quad \frac{1}{2} mv^2 = \frac{GmM}{2r}$$

$$\text{i.e.} \quad \text{KE} = \frac{GmM}{2r}$$

Thus the KE of a satellite in a circular orbit is numerically half its PE but opposite in sign. The total energy of the satellite in orbit is

$$E = \text{KE} + \text{PE}$$

$$= \frac{GmM}{2r} - \frac{GmM}{r} = -\frac{GmM}{2r}$$

It is given that $r = 2R + R = 3R$, where R is the radius of the earth.

$$\therefore \quad E = -\frac{GmM}{6R}$$

Now PE on the surface of the earth $= -\dfrac{GmM}{R}$

$\therefore$ Minimum energy required

$$(E_{\min}) = -\frac{GmM}{6R} - \left(-\frac{GmM}{R}\right)$$

$$= \frac{5GmM}{6R}$$

Hence the correct choice is (a).

40. The speeds of stars at separation r are negligible. Therefore, their energy is entirely potential at this separation (since KE = 0)

$$E_1 = (\text{PE at } r) = -\frac{Gm_1 m_2}{r} = -\frac{Gm^2}{r}$$

As the stars approach each other under gravitational attraction, they begin to acquire speed and hence kinetic energy at the expense of potential energy. When they eventually collide, the separation between their centres is

$$r = R + R = 2R$$

At $r = 2R$, the total energy is

$$E_2 = \text{PE at } (r = 2R) + \text{KE at } (r = 2R)$$

$$= -\frac{Gm^2}{2R} + \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$\text{or} \quad E_2 = -\frac{Gm^2}{2R} + mv^2$$

From the principle of conservation of energy, $E_1 = E_2$, i.e.

$$-\frac{Gm^2}{r} = -\frac{Gm^2}{2R} + mv^2$$

which gives $v = \sqrt{Gm\left(\dfrac{1}{2R} - \dfrac{1}{r}\right)}$

Hence the correct choice is (b).

41. If M is the mass of the earth, the escape velocity is

$$v_e = \sqrt{\frac{2GM}{R}}$$

For a satellite of mass m and orbital radius r (= its distance from the centre of the earth), the orbital speed v is given by

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$\text{or} \quad v = \sqrt{\frac{GM}{r}}$$

$$\text{But} \quad v = \frac{1}{2} v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{GM}{2R}}$$

$$\therefore \quad \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

or $r = 2R$. Height above earth $= 2R - R = R$. Hence the correct choice is (b).

42. Potential energy of the satellite in its orbit is

$$E_1 = -\frac{GmM}{r} = -\frac{GmM}{2R} \qquad (\because r = 2R)$$

The kinetic energy is zero because the satellite is stopped. Potential energy of the satellite on the surface of the earth is

$$E_2 = -\frac{GmM}{R}$$

$$\therefore \text{ Loss of PE} = E_1 - E_2 = -\frac{GmM}{2R} - \left(-\frac{GmM}{R}\right)$$

$$= \frac{GmM}{2R}$$

This is converted into kinetic energy. If v is the speed with which the satellite hits the surface of the earth, then from the law of conservation of energy, we have

$$\frac{1}{2}mv^2 = \frac{GmM}{2R}$$

or $$v^2 = \frac{GM}{R} = gR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (a).

43. Initial KE $= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_e}{2}\right)^2$

$$= \frac{1}{8}mv_e^2$$

$$= \frac{GmM}{4R} \qquad \left(\because v_e = \sqrt{\frac{2MG}{R}}\right)$$

Initial PE $= -\frac{GmM}{R}$

$\therefore$ Total initial energy $= \frac{GmM}{4R} - \frac{GmM}{R}$

$$= -\frac{3GmM}{4R} \qquad \text{(i)}$$

If the body comes to rest at a height r from the centre of the earth, its final energy will be given be

$$\text{Final energy} = -\frac{GmM}{r} \qquad \text{(ii)}$$

Equating (i) and (ii) we get $r = 4R/3$. Maximum height attained $= r - R = \frac{4R}{3} - R = \frac{R}{3}$. Hence the correct choice is (b).

44. The gravitational field intensity at point O will be

$$I = Gm\left[\frac{1}{r^2} + \frac{1}{(2r)^2} + \frac{1}{(4r)^2} + \frac{1}{(8r)^2} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left[\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots\right]$$

$$= \frac{Gm}{r^2}\left(\frac{1}{1 - \frac{1}{2^2}}\right) = \frac{Gm}{r^2}\left(\frac{1}{1 - \frac{1}{4}}\right) = \frac{4Gm}{3r^2}$$

Hence the correct choice is (b).

45. The gravitational point, in magnitude, at point O is

$$V = Gm\left[\frac{1}{r} + \frac{1}{2r} + \frac{1}{4r} + \frac{1}{8r} + \cdots\right]$$

$$= \frac{Gm}{r}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right]$$

$$= \frac{Gm}{r}\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right]$$

$$= \frac{Gm}{r}\left(\frac{1}{1 - \frac{1}{2}}\right) = \frac{2Gm}{r}$$

Hence the correct choice is (d).

46. Mass of the second part $= M - m$. Gravitational force between the two parts is

$$F = \frac{G(M-m)m}{r^2} = \frac{G}{r^2}(Mm - m^2)$$

F will be maximum if $\frac{dF}{dm} = 0$ and $\frac{d^2F}{dm^2}$ is negative.

Now, $\frac{dF}{dm} = \frac{G}{r^2}(M - 2m)$. Setting $\frac{dF}{dm} = 0$, we get $M - 2m = 0$ or $m = \frac{M}{2}$. Now

$$\frac{d^2F}{dm^2} = -\frac{2G}{r^2}, \text{ which is negative.}$$

Hence the correct choice is (a).

47. PE on the surface of earth $= -\frac{GMm}{R}$

PE at a height h above the surface of earth

$$= -\frac{GMm}{(R+h)}$$

$$\therefore \text{ Increase in PE} = -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GMm\left[\frac{h}{R(R+h)}\right]$$

$$= \frac{gRmh}{(R+h)} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$\therefore$ PE will increase by $\frac{1}{3}mgR$ at a value of h given by

$$\frac{gRmh}{(R+h)} = \frac{1}{3}mgR$$

or $\frac{h}{R+h} = \frac{1}{3}$ or $h = \frac{R}{2}$, which is choice (b).

48. PE at a distance r from the centre of the planet

$$= -\frac{GMm}{r}$$

$$\text{Initial PE} = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$$\text{Final PE} = -\frac{GMm}{R+2R} = -\frac{GMm}{3R}$$

Now, work down = increase in PE

$$= \frac{GMm}{R}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{GMm}{6R}$$

$$= \frac{1}{6}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (b).

49. Change in PE $= \frac{GMm}{R} - \frac{GMm}{(n+1)R} = \left(\frac{n}{n+1}\right)mgR$

Hence the correct choice is (a).

50. From the principle of conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+nR)}$$

which gives $v^2 = \frac{2nR}{(n+1)}\frac{GM}{R^2} = \frac{2nRg}{(n+1)}$

$$\left(\because g = \frac{GM}{R^2}\right)$$

Hence the correct choice is (d).

51. If h is the maximum height attained, then we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

which gives $v^2 = \frac{2ghR}{(R+h)} \qquad \left(\because g = \frac{GM}{R^2}\right)$

For ball A, we have $\frac{4gR}{3} = \frac{2gh_AR}{(R+h_A)}$

$$\Rightarrow \qquad h_A = 4R$$

For ball B, we have $\frac{2gR}{3} = \frac{2gh_BR}{(R+h_B)}$

$$\Rightarrow \qquad h_B = \frac{R}{2}$$

$\therefore \frac{h_A}{h_B} = 8$, which is choice (c).

52. If ρ is the density of the material of each sphere, then the mass of the sphere of radius r is $M_1 = \frac{4\pi}{3}r^3\rho$ and the mass of the sphere of radius $2r$ is

$$M_2 = \frac{4\pi}{3}(2r)^3\rho.$$

Distance between their centres is $d = r + 2r = 3r$.

Now $F = \frac{GM_1M_2}{d^2} = \frac{G\times\left(\frac{4\pi}{3}\right)r^3\rho\times\frac{4\pi}{3}(2r)^3\rho}{9r^2}$

which gives $F \propto r^4$, which is choice (d).

53. The angular momentum of the planet is constant over the entire orbit. Hence $mvr = mVR$ or $V = v\left(\frac{r}{R}\right)$, which is choice (b).

54. $g_h = \frac{Gm}{(R+h)^2}$

Also $g = \frac{Gm}{R^2}$. Thus $\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$. Given

$$g_h = \frac{90g}{100}$$

$$\therefore \frac{R^2}{(R+h)^2} = \frac{9}{10} \text{ or } (R+h) = \frac{\sqrt{10}R}{3}$$

$$\text{Potential energy} = -\frac{GM}{(R+h)} = -\frac{GMR^2}{R^2(R+h)}$$

$$= -\frac{gR^2}{(R+h)} = -\frac{3gR}{\sqrt{10}}$$

Hence the correct choice is (c).

55. The distance x of the star of mass m from the centre of mass is given by

$$\frac{m}{x} = \frac{2m}{(r-x)}$$

which gives $x = \frac{r}{3}$. The orbital speed v_1 of the star of mass $m_1 = m$ is given by (here $m_2 = 2m$)

$$\frac{Gm_1m_2}{r^2} = \frac{m_1v_1^2}{x} = \frac{m_1v_1^2}{r/3}$$

which gives $v_1 = \sqrt{\frac{Gm_2}{3r}} = \sqrt{\frac{2Gm}{3r}}$ (i)

$\therefore$ Time period (T) of $m = \frac{2\pi x}{v_1} = \frac{2\pi r}{3} \times \sqrt{\frac{3r}{2GM}}$

$$= \pi\sqrt{\frac{2}{3Gm}}\,(r)^{3/2}$$

or $T \propto r^{3/2}$, which is choice (b).

56. The orbital speed v_2 of the star of mass $m_2 = 2m$ is given by

$$\frac{G m_1 m_2}{r^2} = \frac{m_2 v_2^2}{(r-x)} \text{ or } \frac{G m(2m)}{r^2} = \frac{2m v_2^2}{2r/3}$$

or $v_2 = \sqrt{\frac{2Gm}{3r}} = v_1$ [see Eq. (i)]

Now $K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m v_1^2$ and $K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(2m) v_2^2$

$\therefore \frac{K_1}{K_2} = \frac{v_1^2}{2v_2^2} = \frac{1}{2}$, since $v_1 = v_2$. Hence the correct choice is (c).

57. Angular momentum of m_1 is $L_1 = m_1 v_1 x = \frac{m v_1 r}{3}$

Angular momentum of m_2 is $L_2 = m_2 v_2 (r - x)$

$$= \frac{2mv_2 \times 2r}{3}$$

$\therefore \frac{L_1}{L_2} = \frac{1}{4}$ (since $v_1 = v_2$), which is choice (c).

58. The force of attraction between the complete sphere and mass m is

$$F = \frac{G m M}{(2R)^2} = \frac{G m M}{4R^2} \quad \text{(i)}$$

Mass of complete sphere is $M = \frac{4\pi}{3} R^3 \rho$. Mass of the cut out portion is $m_0 = \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 \rho$. Thus, $m_0 = \frac{M}{8}$. The distance between the centre of the cut out portion and mass $m = 2R - \frac{R}{2} = \frac{3R}{2}$.

Hence the force of attraction between the cut out portion and mass m is

$$f = \frac{G m_0 m}{(3R/2)^2} = \frac{G(M/8)m}{9R^2/4} = \frac{GmM}{4R^2} \times \frac{2}{9}$$

Using (i), we get $f = \frac{2F}{9}$. Therefore, the force of attraction between the remaining part of the sphere and mass $m = F - f = F - \frac{2F}{9} = \frac{7F}{9}$ which is choice (a).

59. Refer to Fig. 6.31. Let μ be the mass per unit length of the ring. $L = 2\pi R$ is the length of the ring. Consider a small element of length dx of the ring located at C. Then

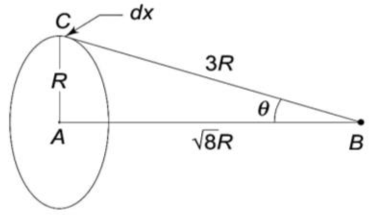


Fig. 6.31

Force along BC is $f = \frac{G M \mu\, dx}{(3R)^2}$. Therefore, force along BA is $dF = f\cos\theta = \frac{G M \mu\, dx}{9R^2}\,\frac{\sqrt{8}R}{3R}$

$$= \frac{\sqrt{8}}{27}\,\frac{GM\,\mu dx}{R^2}$$

$\therefore$ Total force $= \frac{\sqrt{8}}{27}\frac{GM}{R^2}\int \mu\, dx = \frac{\sqrt{8}}{27}\frac{GMm}{R^2}$

because $\int \mu\, dx = \mu \times L = m$, the mass of the ring. Hence the correct choice is (a).

60. The distance of each cavity from the centre O is the same. Since the two cavities are symmetrical with respect to the centre O and the mass of the sphere can be regarded as being concentrated at the centre O, the gravitational force due to the sphere is zero at the centre. Hence choice (a) is correct. For the same reason, the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$ whose radius is 6 units and at all points of the circle $y^2 + z^2 = 4$ whose radius is 2 units. Hence choices (c) and (d) are also correct. But the gravitational force at point B cannot be zero.

61. Initially when the two masses are at an infinite distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P.E. is

$$\text{PE} = -\frac{G m_1 m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If

v_1 and v_2 are their respective velocities when they are a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2} m_1 v_1^2 = \frac{G m_1 m_2}{r} \text{ or } v_1 = \sqrt{\frac{2Gm_2}{r}}$$

and $$\frac{1}{2} m_2 v_2^2 = \frac{G m_1 m_2}{r} \quad \text{or} \quad v_2 = \sqrt{\frac{2Gm_1}{r}}$$

Therefore, their relative velocity of approach is

$$v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}}$$
$$= \sqrt{\frac{2G}{r}(m_2 + m_1)}$$

Hence the correct choice is (a).

62. At a finite separation, the total kinetic energy of the system of two masses and the force between them are both finite. Since the two masses are at rest initially and there is no external force, the centre of mass cannot move. Hence the correct choice is (d).

63. According to Kepler's law of period, $T^2 = kR^3$ where k is a constant. Taking logarithm of both sides, we have

$$2 \log T = \log k + 3 \log R$$

Differentiating, we get

$$2 \frac{\delta T}{T} = 0 + 3 \frac{\delta R}{R}$$

or $$\frac{dT}{T} = \frac{3}{2} \frac{\delta R}{R} = \frac{3}{2} \times \left(\frac{1.01R - R}{R}\right) \times 100$$
$$= 1.5\%$$

Hence the correct choice is (c).

64. According to Kepler's law of period,

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R_1}{R_1/2}\right)^{3/2}$$
$$= (2)^{3/2} = 2\sqrt{2}$$

$$\therefore \quad T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \text{ days}}{2\sqrt{2}} = 129 \text{ days.}$$

65. For a satellite, we have

Kinetic energy $= \dfrac{GmM}{2r}$

Potential energy $= -\dfrac{GmM}{r}$

Total energy $E_0 = \text{KE} + \text{PE} = \dfrac{GmM}{2r} - \dfrac{GmM}{r}$

$$= -\frac{GmM}{2r} = \frac{\text{PE}}{2}$$

or PE $= 2E_0$. Hence the correct choice is (c).

66. For elliptical orbit, the earth is at one focus of the ellipse. For spherical bodies, the gravitational force is central (or radial). Hence statement (a) is correct. The gravitational force exerts no torque on the satellite. Hence the angular momentum of S remains constant in magnitude as well as direction. Hence choice (b) is incorrect. For elliptical orbit, the distance of the satellite from the earth varies periodically. Hence potential energy, kinetic energy and linear momentum vary periodically. Hence choices (c) and (d) are also incorrect.

67. The acceleration due to gravity at a height h above the surface of the earth is given by

$$g_2 = g_1 \left(\frac{R}{R+h}\right)^2$$

where g_1 is the value at the surface of the earth. Now

$$T_2 = 2\pi \sqrt{\frac{l}{g_2}} \text{ and } T_1 = 2\pi \sqrt{\frac{l}{g_1}}$$

$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \frac{R+h}{R}$$
$$= \frac{R+R}{R} = 2 \qquad (\because h = R)$$

Hence the correct choice is (d).

68. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy $= Mgx$. This must be the energy stored in the spring which is given by $\frac{1}{2} kx^2$. Thus

$$\frac{1}{2} kx^2 = Mg\,x \text{ or } x = \frac{2Mg}{k}$$, which is choice (b).

69. For a satellite of mass m moving with a velocity v in a circular orbit of radius r around the earth of mass M, we have

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \text{ or } v = \sqrt{\frac{GM}{r}}$$

Now $$v = \frac{2\pi r}{T}. \text{ Thus } \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

or $$T \propto r^{3/2}.$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \qquad (1)$$

Given r_2 = 6400 km and r_1 = 36000 km. For a geostationary satellite T_1 = 24 h. Using these values in (1), we have get $T_2 = 24 \times \left(\frac{64}{360}\right)^{3/2} = 1.8$ h.

Hence the closest choice is (c).

70. Given $m_1 = xm$ and $m_2 = (1-x)m$. For a separation r between them, the force of attraction is

$$F = \frac{Gm_1 m_2}{r^2} = \frac{G}{r^2}\, xm(1-x)\, m$$

$$= \frac{Gm^2}{r^2}\,(x - x^2)$$

For a given r, F will be maximum if $\frac{dF}{dx} = 0$ and $\frac{d^2F}{dx^2} < 0$, i.e.

$$\frac{d}{dx}\,(x - x^2) = 0 \text{ or } 1 - 2x = 0 \text{ or } x = \frac{1}{2}.$$

Now $\frac{d^2F}{dx^2} = \frac{Gm^2}{r^2}(-2) = -\frac{2Gm^2}{r^2}$, which is negative. Hence the correct choice is (a).

71. $g_h = \frac{gR^2}{(R+h)^2}$. Given $g_h = \frac{g}{100}$. Therefore, we have

$$\frac{gR^2}{(R+h)^2} = \frac{g}{100}$$

or $R + h = 10\,R$ or $h = 9\,R$, which is choice (b).

72. Escape velocity $v_e = \sqrt{\frac{2GM}{R}}$. Velocity of projection $v = \frac{3}{4}\,v_e = \frac{3}{4}\sqrt{\frac{2GM}{R}}$. The total energy of the body when it is projected is

$$E_i = \text{KE} + \text{PE}$$

$$= \frac{1}{2}\,mv^2 - \frac{GmM}{R}$$

$$= \frac{1}{2}m \times \frac{9}{16} \times \frac{2GM}{R} - \frac{GmM}{R}$$

$$= \frac{9}{16}\frac{GmM}{R} - \frac{GmM}{R} = -\frac{7}{16}\frac{GmM}{R}$$

Let h be the maximum height attained by the body. The distance of the body from the centre of the earth is $r = R + h$. At this height, the total energy of the body is

$$E_f = \text{KE} + \text{PE}$$

$$= 0 - \frac{GmM}{r} = -\frac{GmM}{r} = -\frac{GmM}{(R+h)}$$

From the principle of conservation of energy, $E_i = E_f$, i.e.

$$-\frac{7}{16}\frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

or $7\,(R + h) = 16R$ or $7h = 9R$ or $h = \frac{9R}{7}$, which is choice (b).

73. If the gravitational field is zero at a point at a distance x from M_1, then

$$\frac{GM_1}{x^2} = \frac{GM_2}{(r-x)^2}$$

$$\text{or} \quad \frac{x}{(r-x)} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2}$$

which gives $x = \frac{r}{3}$. Therefore, $r - x = \frac{2r}{3}$. The gravitational potential at $x = \frac{r}{3}$ is

$$U = -\frac{GM_1}{x} - \frac{GM_2}{(r-x)}$$

$$= -\frac{Gm}{r/3} - \frac{G(4m)}{2r/3} = -\frac{9Gm}{r}$$

Hence the correct choice is (d).

74. At the equator, the value of g is

$$g' = g - R\omega^2$$

where ω is the angular speed of the earth. For bodies to appear weightless at the equator, $g' = 0$, i.e.

$$g - R\omega^2 = 0$$

which gives $\omega = \sqrt{\frac{g}{R}}$. Hence the correct choice is (a).

75. Gravitational force is conservative. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. Hence the correct choice is (b).

76. In a binary star system, the two stars move under their mutual gravitational force. Therefore, their angular velocities and hence their time periods are equal. Thus the correct choice is (c).

77. Refer to Fig. 6.32.

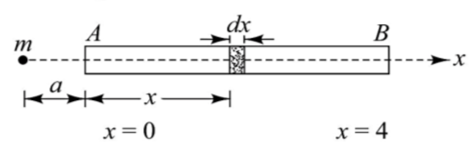


Fig. 6.32

Consider a small element of length dx of the bar at a distance x from end A. The mass of this element $dm = \frac{M}{L}\,dx$. The force exerted by this element on mass m is

$$dF = \frac{G\,m\,dM}{(a+x)^2} = \frac{G\,m\,M\,dx}{L(a+x)^2}$$

The gravitational force exerted by the complete bar on mass m will be

$$F = \int dF = \frac{GmM}{L}\int_0^L \frac{dx}{(a+x)^2}$$

$$= -\frac{GmM}{L}\left|\frac{1}{a+x}\right|_0^L$$

$$= -\frac{GmM}{L}\left[\frac{1}{a+L}-\frac{1}{a}\right]$$

$$= \frac{GmM}{a(a+L)}$$

So the correct choice is (b).

78. Total energy of the satellite when its orbital radius is r is

$$E_1 = \text{K.E.} + \text{P.E.}$$

$$= \frac{GmM}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Total energy of the satellite for orbital radius $2r$ is

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$= \frac{GmM}{2(2r)} - \frac{GMm}{2r} = -\frac{GMm}{4r}$$

The amount of work needed is

$$W = E_2 - E_1$$

$$= -\frac{GMm}{4r} - \left(-\frac{GMm}{2r}\right) = \frac{GMm}{4r}$$

79. The angular momentum of the comet about the centre of the SUN is conserved. If L_1 is the magnitude of the angular momentum when the comet is at A and L_2 that at B, then,

$$L_1 = L_2$$

i.e. $$mv_1r_1 = mv_2r_2$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

So the correct choice is (b).

80. The total energy of the comet of mass m in an elliptical orbit with semi-major axis a around the sun of mass M is given by

$$E = -\frac{GmM}{2a}$$

Since $2a = r_1 + r_2$,

$$E = -\frac{GmM}{(r_1+r_2)}$$

So the kinetic energy of the comet at A is given by
K.E. at A = total energy – P.E. at A

or $$\frac{1}{2}mv_1^2 = -\frac{GmM}{(r_1+r_2)} - \left(-\frac{GmM}{r_1}\right)$$

$$= -\frac{GmM\,r_2}{r_1(r_1+r_2)} = \frac{2GmM\,r_2}{r_1 a}$$

$$\Rightarrow \quad v_1 = 2\sqrt{\frac{GMr_2}{r_1 a}}$$, which is choice (a).

81. The kinetic energy of the comet at B is given by

$$\frac{1}{2}mv_2^2 = -\frac{GmM}{(r_1+r_2)} - \left(-\frac{GmM}{r_2}\right)$$

$$= \frac{GmM\,r_1}{r_2(r_1+r_2)} = \frac{2GmMr}{r_2 a}$$

$$\Rightarrow \quad v_2 = 2\sqrt{\frac{GMr_1}{r_2 a}}$$

So the correct choice is (d).

Note

We notice that

$$\frac{v_1}{v_2} = \frac{2\sqrt{\frac{GMr_2}{r_1 a}}}{2\sqrt{\frac{GMr_1}{r_2 a}}} = \frac{r_2}{r_1}$$

This result we have obtained above by using the principle of conservation of angular momentum.

82. $$L_1 = mv_1r_1$$

$$= m \times 2\sqrt{\frac{GMr_2}{r_1 a}} \times r_1$$

$$= m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}} \qquad \left[\because a = \frac{1}{2}(r_1+r_2)\right]$$

Therefore $L_1 \propto \dfrac{r_1r_2}{(r_1+r_2)}$, which is choice (c).

83. We know that the sum of the distances of any point on ellipse from the two foci = $2a = (r_1 + r_2)$. Since point P is equidistant from the two foci, one of which is at the centre of the SUN, the distance of P from

the centre of the SUN is equal to $a = \frac{1}{2}(r_1 + r_2)$.

Therefore, the kinetic energy of the comet when it is at P is

$$E_p = \text{Total energy at } P - \text{P.E. at } P$$

$$= -\frac{GmM}{(r_1+r_2)} - \left(-\frac{GmM}{\frac{1}{2}(r_1+r_2)}\right)$$

$$= \frac{GmM}{(r_1+r_2)}$$

or $\frac{1}{2}m v_p^2 = \frac{GMm}{r_1+r_2}$

$\Rightarrow \quad v_p = \sqrt{\frac{2GM}{(r_1+r_2)}}$

So the correct choice is (b).

84. Total energy when the object is dropped is

$$E_1 = \text{K.E.} + \text{P.E.}$$

$$= 0 - \frac{GMm}{3R} = -\frac{GMm}{3R}$$

Total energy when the object strickes the earth's surface is

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}mv^2 - \frac{GMm}{R}$$

From conservation of energy, $E_1 = E_2$, i.e.

$$-\frac{GmM}{3R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$v = \sqrt{\frac{\frac{4}{3}GM}{R}}$$

$$= \sqrt{\frac{4}{3}gR} \qquad \left(\because \; g = \frac{GM}{R^2}\right)$$

which gives $k = \frac{4}{3}$. So the correct choice is (d).

85. According to kepler's law, the square of the time period is proportional to the cube of the semi-major axis of the ellipse, i.e. $T^2 \propto \left(\frac{a+b}{2}\right)^3$. So the correct choice is (c).

86. The entire mass of the sphere can be assumed to be concentrated at its centre of mass O. Consider a small element of the ring at A. Let dm be the mass of the element (see Fig. 6.33)

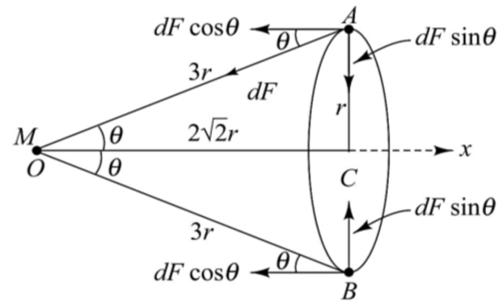


Fig. 6.33

The gravitational force exerted by M on dm is

$$dF = \frac{GMm}{r_1 + r_2} = \frac{GM\,dm}{9r^2}$$

This force can be resolved into x and y components as shown. Now consider another element at B. We find that x ompoments (namely $dF\cos\theta$) add up but y-components (namely of $dF\sin\theta$) cancel each other. Hence the x components of all elements add up and y-components cancel. Hence, the total force exerted by the sphere on the ring is

$$F = \int dF\cos\theta = \int \frac{GM\,dm}{9r^2} \times \frac{2\sqrt{2}}{3}$$

$$= \frac{2\sqrt{2}\,GM}{27r^2}\int dm = \frac{2\sqrt{2}\;GMm}{27r^2}$$

So the correct choice is (a).

87. The mass of a sphere can be assumed to be concentrated at its centre. So the given system can be regarded as system of three particles, each of mass M located at the three vertices of an equilateral triangle of side $2R$ as shown in Fig. 6.34.

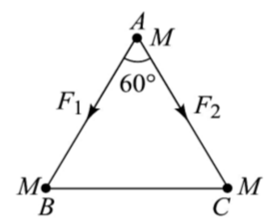


Fig. 6.34

$$AB = BC = AC = 2R.$$

$$F_1 = F_2 = \frac{GM^2}{(2R)^2} = \frac{GM^2}{4R^2} = F\,(\text{say})$$

The resultant force on A has magnitude

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2 \cos 60°}$$

$$= \sqrt{3}F = \frac{\sqrt{3}\,GM^2}{4R^2}$$

So the correct choice is (c).

88. Let M and R respectively be the mass and radius of the earth and let m be the mass of the object (Fig. 6. 35) Gravitational force on the object is

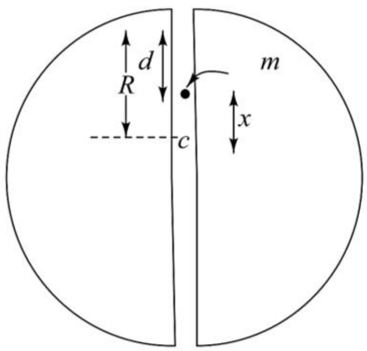


Fig. 6.35

$$F = mg_d$$

$$= mg\left(1 - \frac{d}{R}\right)$$

$$= m \times \frac{MG}{R^2} \times \left(\frac{R-d)}{R}\right)$$

$$= \frac{GmMx}{R^3}$$

So the correct choice is (c).

89. Radius of the circular rod is $R = \frac{L_1}{2\pi}$. Since every point on the circle is at the same distance R from its centre and since potential is a scalar quantity, the gravitation potential at the centre is

$$V = -\frac{GM}{R} = -\frac{2\pi GM}{L}$$, which is choice (d).

90. Mass of complete sphere is $M = \frac{4\pi}{3}R^3\rho$. Mass of each spherical cavity is

$$m = \frac{4\pi}{3}\left(\frac{R}{4}\right)^3 \rho = \frac{M}{64}$$

Gravitational at O due to cavity A is

$$I_A = \frac{GM}{(2r)^2}$$

$$= \frac{GM \times 16}{64 \times 4 \times R^2} = \frac{GM}{16R^2}$$

directed to the left.

Gravitational field at O due to cavity B is

$$I_B = \frac{GM}{16R^2}$$ directed to the right.

If I is gravitational field at O due to the complete sphere, then the gravitational field at O due to the remaining part is

$$I_r = I - I_A + I_B$$

Now $I_A = I_B$ and $I = 0$ (because $x = 0$ see Q. 89 above), $I_r = 0$. So the correct choic is (a).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

A satellite of mass m is revolving in a circular orbit of radius r around the earth of mass M. The speed of the satellite in its orbit is one-fourth the escape velocity from the surface of the earth.

1. The height of the satellite above the surface of the earth is (R = radius of earth)

(a) $2R$ (b) $3R$
(c) 5R (d) $7R$

2. The magnitude of angular momentum of the satellite is

(a) $m\sqrt{GMR}$ (b) $\frac{m}{2}\sqrt{GMR}$
(c) $\frac{m}{2\sqrt{2}}\sqrt{GMR}$ (d) $2m\sqrt{GMR}$

3. If the total energy of the satellite is E, its potential energy is

(a) $-E$ (b) E
(c) $2E$ (d) $-2E$

Solutions

1. $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$ (1)

$v = \frac{v_e}{4} = \frac{1}{4}\sqrt{\frac{2GM}{R}}$ (2)

Equations (1) and (2) give $h = 7R$, which is choice (d)

2. $L = mvR = m\sqrt{\frac{GM}{8R}}\,R = \frac{m}{2\sqrt{2}}\sqrt{GMR}$ $(\because h = 7R)$

The correct choice is (c).

3. The correct choice is (c).

Questions 4 to 6 are based on the following passage.

Passage II

Considering the earth as an isolated mass, a force is experienced by a body at any distance from it. This force is directed towards the centre of the earth and has a magnitude mg, where m is the mass of the body and g is the acceleration due to gravity. The value of the acceleration due to gravity decreases with increase in the height above the surface of the earth and with increase in the depth below the surface of the earth. Even on the surface of the earth, the value of g varies from place to place and decreases with decrease in the latitude of the place.

4. Assuming the earth to be a sphere of uniform mass density, which of the graphs shown in Fig. 6.34 represents the variation g with distance r from the centre of the earth (R = radius of earth)

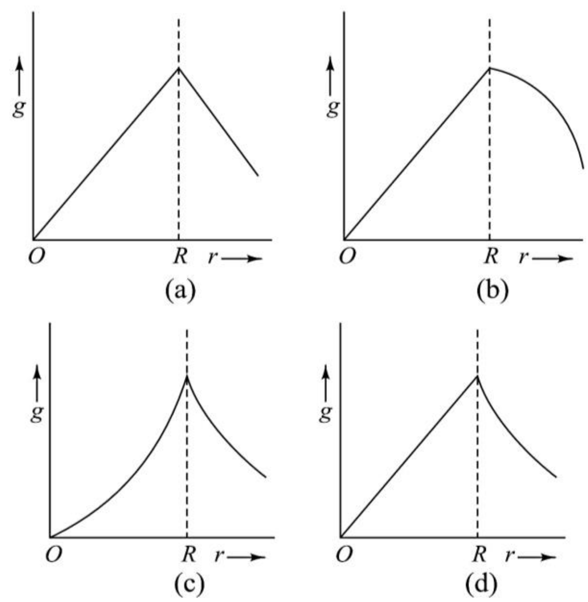


Fig. 6.34

5. Assuming the earth to be a sphere of uniform mass density, the weight of a body when it is taken to the end of a tunnel 32 km below the surface will (radius of earth = 6400 km)
   (a) decrease by 0.5% (b) decrease by 1%
   (c) increase by 0.5% (d) increase by 1%

6. If a tunnel is dug along a diameter of the earth and a body is dropped from one end of the tunnel,
   (a) it will fall and come to rest at the centre of the earth where its weight becomes zero.
   (b) it will emerge from the other end of the tunnel.
   (c) it will execute simple harmonic motion about the centre of the earth.
   (d) it will accelerate till it reaches the centre and decelerate after that eventually coming to rest at the other end of the tunnel.

Solutions

4. At a height h above the surface of the earth,

$$g_h = \frac{g_0 R^2}{(R+h)^2}$$

At a depth d below the surface of the earth

$$g_d = g_0\left(1 - \frac{d}{R}\right)$$

where g_0 = acceleration due to gravity at the surface of the earth. Hence the correct choice is (d).

5. $g = \left(1 - \frac{d}{R}\right)g_0 = \left(1 - \frac{32}{6400}\right)g_0 = \frac{199 g_0}{200}$

$\therefore$ Decrease in weight $= mg_0 - mg$

$$= mg_0\left(1 - \frac{199}{200}\right) = \frac{mg_0}{200}$$

Hence the correct choice is (a).

6. The correct choice is (c).

Questions 7 to 9 are based on the following passage.

Passage III

The escape velocity on a planet or a satellite is the minimum velocity with which a body must be projected from that planet so that it escapes the gravitational pull of the planet and goes into outer space. We obtain the expression for the escape velocity by equating the work required to move the body from the surface of the planet to infinity with the initial kinetic energy given to the body. The escape velocity from a planet of mass M and radius R is given by

$$v_e = \sqrt{\frac{2MG}{R}} = \sqrt{2gR}$$

where g is the acceleration due to gravity on the surface of the planet and G is the gravitation constant.

7. The mass of Jupiter is about 319 times that of the earth and its radius is about 11 times that of the earth. The ratio of the escape velocity on Jupiter to that on earth is
   (a) $\sqrt{29}$ (b) 29
   (c) $\frac{1}{\sqrt{29}}$ (d) $\frac{1}{29}$
8. If R is the radius of the earth and g the acceleration due to gravity on its surface, the escape velocity of a body projected from a satellite orbiting the earth at a height $h = R$ from the surface of the earth will be
   (a) $\sqrt{gR}$ (b) $\sqrt{2gR}$
   (c) $\sqrt{3gR}$ (d) $2\sqrt{gR}$
9. A body is dropped from a height equal to half the radius of the earth. If v_e is the escape velocity on earth and air resistance is neglected, it will strike the surface of the earth with a speed
   (a) $\frac{v_e}{\sqrt{2}}$ (b) $\frac{v_e}{2}$
   (c) $\frac{v_e}{\sqrt{3}}$ (d) $\frac{v_e}{3}$

Solutions

7. $\frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}} = \sqrt{319 \times \frac{1}{11}} = \sqrt{29}$

Hence the correct choice is (a).

8. The escape velocity at a height h is given by

$$v_e' = \sqrt{2g'(R+h)}$$

where g' is the acceleration due to gravity at height h,

$$g' = g\left(\frac{R}{R+h}\right)^2$$

For $h = R$, we get $v_e' = \sqrt{gR}$, which is choice (a).

9. The correct choice is (c). Use conservation of energy, i.e.

Total energy at h (= $R/2$)

= Total energy when the body strikes the earth

$$\Rightarrow \quad -\frac{GmM}{(R+h)} = \frac{1}{2}mv^2 - \frac{GmM}{R}$$

which gives $v = v_e/\sqrt{3}$

3 SECTION Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only *one* choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

A body is projected up with a velocity equal to half the escape velocity from the surface of the earth. If R is the radius of the earth and atmospheric resistance is neglected, it will attain a height $h = R/3$.

Statement-2

The gravitational potential is $-GM/R$ on the surface of the earth and it increases with height; M being the mass of the earth.

2. Statement-1

The total energy (kinetic + potential) of a satellite moving in a circular orbit around the earth is half its potential energy.

Statement-2

The gravitational force obeys the inverse square law of distance.

3. Statement-1

Two bodies of masses $m_1 = m$ and $m_2 = 3m$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach when they are at a separation r is

$$v = \sqrt{\frac{2Gm}{r}}$$

Statement-2

The gain in the kinetic energy of each body equals the loss in its gravitational potential energy.

4. **Statement-1**

An astronaut inside a massive space-ship orbiting around the earth will experience a finite but small gravitational force.

Statement-2

The centripetal force necessary to keep the space-ship in orbit around the earth is provided by the gravitational force between the earth and the space-ship.

5. **Statement-1**

The escape velocity varies with altitude and latitude of the place from where it is projected.

Statement-2

The escape velocity depends on the gravitational potential at the point of projection.

6. **Statement-1**

A comet orbits the sun in a highly elliptical orbit. Its potential energy and kinetic energy both change over the orbit but the total energy remains constant throughout the orbit.

Statement-2

For a comet, only the angular momentum remains constant throughout the orbit.

7. **Statement-1**

The acceleration due to gravity decreases due to rotation of the earth.

Statement-2

A body on the surface of the earth also rotates with it in a circular path. A body in a rotating (non-inertial) frame experiences an outward centrifugal force against the inward force of gravity.

Solutions

1. The correct choice is (b). Use $v_e = \sqrt{\frac{2Gm}{R}}$ and total energy at r (= $R + h$) = total initial energy, i.e. $-\frac{GmM}{r} = \frac{1}{2}mv^2 - \frac{GmM}{R}$

2. The correct choice is (a). The centripetal force needed for circular motion is provided by the gravitational force. Since the gravitational force obeys the inverse square law of distance, the orbital velocity of the satellite is given by

$$v = \sqrt{\frac{GM}{r}}$$

where M = mass of earth and r = orbital radius. Therefore

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{GmM}{2r}$$

where m = mass of the satellite. From the inverse square law of distance, we find that the potential of the satellite is given by

$$\text{Potential energy} = -\frac{GmM}{r}$$

$\therefore$ Total energy E = K.E. + P.E.

$$= \frac{GmM}{2r} + \left(-\frac{GmM}{r}\right)$$

$$= -\frac{GmM}{2r} = \frac{\text{P.E.}}{2}$$

3. The correct choice is (d). Initially when the two masses are at an distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P.E. is

$$\text{P.E} = -\frac{Gm_1m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If v_1 and v_2 are their respective velocities when they are at a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2}m_1v_1^2 = \frac{Gm_1m_2}{r} \text{ or } v_1 = \sqrt{\frac{2Gm_2}{r}}$$

and $\frac{1}{2}m_1v_2^2 = \frac{Gm_1m_2}{r}$ or $v_2 = \sqrt{\frac{2Gm_1}{r}}$

Therefore, their relative velocity of approach is

$$v = v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}}$$

$$= \sqrt{\frac{2G}{r}(m_2 + m_1)}$$

Putting $m_1 = m$ and $m_2 = 3m$, we get $v = 2\sqrt{\frac{2Gm}{r}}$

4. The correct choice is (b). Because the centripetal force equals the gravitational force exerted by the earth on the space-ship, the astronaut does not experience any gravitational force of the earth. The only force of gravity that an astronaut in an orbiting space-ship experiences is that which is due to the gravitational force exerted by the space-ship. Since the space-ship is very massive, this force is finite but very small.
5. The correct choice is (a). The gravitational potential at a point varies with the altitude and latitude of the place.
6. The correct choice is (c).
7. The correct choice is (a).

SECTION 4 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then the satellite will
   (a) continue to move in the original orbit with the same velocity
   (b) move tangentially to the original orbit with the same velocity
   (c) become stationary it its orbit
   (d) move towards the earth **[2002]**
2. The energy required to move a body of mass m from an orbit of radius 2 R to an orbit of radius 3 R is.
   (a) $\frac{GmM}{12R}$ (b) $\frac{GmM}{3R}$
   (c) $\frac{GmM}{8R}$ (d) $\frac{GmM}{6R}$ **[2002]**
3. The escape velocity of a body depends on its mass m as
   (a) m^0 (b) m^1
   (c) m^2 (d) m^3 **[2002]**
4. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is
   (a) $\frac{mgR}{2}$ (b) $2mgR$
   (c) mgR (d) $\frac{mgR}{4}$ **[2002]**
5. An ideal spring with spring-constant k is hung from the ceiling and a block of Mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension of spring is.
   (a) 4 Mg/K (b) 2 Mg/K
   (c) Mg/K (d) $Mg/2\ K$ **[2002]**
6. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometeres above the earth's surface (R_{Earth} = 6400 km) will be approximately be
   (a) $\frac{1}{2}$ h (b) 1 h
   (c) 2 h (d) 4 h **[2002]**
7. The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new period will be
   (a) 10 h (b) 80 h
   (c) 40 h (d) 20 h **[2003]**
8. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
   (a) 2.5 R (b) 4.5 R
   (c) 7.5 R (d) 1.5 R **[2003]**
9. The escape velocity of a body projected vertically upwards from the surface of the earth is 11 km s^{-1}. If the body is projected at angle 45° with the vertical, the escape velocity will be
   (a) $11\sqrt{2}$ km s^{-1} (b) 22 km s^{-1}
   (c) 11 km s^{-1} (d) $\frac{11}{\sqrt{2}}$ km s^{-1} **[2003]**
10. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\frac{gR}{(R-x)}$

(c) $\frac{gR^2}{(R+x)}$ (d) $\left[\frac{gR^2}{(R+x)}\right]^{1/2}$ **[2004]**

11. The time period of an earth satellite in a circular orbit is independent of
(a) the mass of the satellite
(b) the radius of the orbit
(c) both the mass of the satellite and the radius of the orbit
(d) neither the mass of the satellite nor the radius of the orbit **[2004]**

12. If g is the acceleration due to gravity on earth's surface, the gain in potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is
(a) $2\ mgR$ (b) $\frac{1}{2}mgR$
(c) $\frac{1}{4}mgR$ (d) mgR **[2004]**

13. Suppose the gravitational force varies inversely as the n^{th} power of distance, then the time period of a planet in circular orbit of radius R around the sun will be proportional to
(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$
(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$ **[2004]**

14. The average density of the earth
(a) does not depend on the g
(b) is a complex function of g
(c) is directly proportional to g
(d) is inversely proportional to g **[2005]**

15. The change in the value of g at a height h above the surface of the earth is the same as that at a depth d below the surface of the earth. If both h and d are much smaller than the radius of the earth, then which of the following is correct?
(a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$
(c) $d = 2h$ (d) $d = h$ **[2005]**

16. A particle of mass 10 g is kept on the surface of sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere. Take $G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{Kg}^{-2}$
(a) 13.34×10^{-10}J (b) 3.33×10^{-10}J
(c) 6.67×10^{-10}J (d) 6.67×10^{-9}J **[2005]**

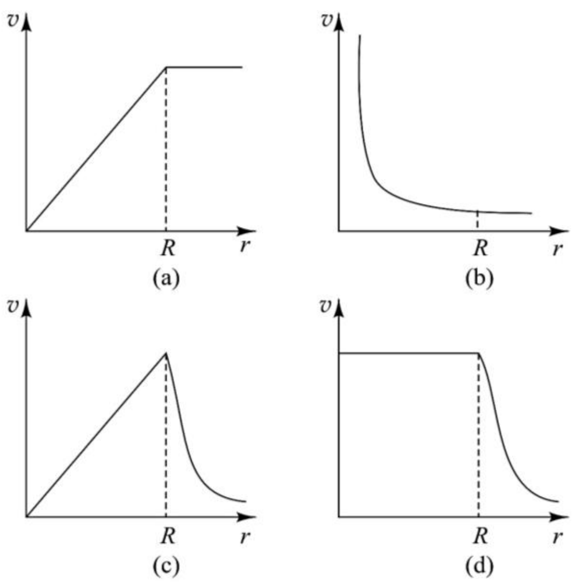
17. If g_E and g_M are the acceleration due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio to $\frac{\text{electonic charge on the moon}}{\text{electronic charge on the earth}}$ be
(a) 1 (b) zero
(c) $\frac{g_E}{g_M}$ (d) $\frac{g_M}{g_E}$ **[2007]**

18. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1:
For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is $4\pi GM$.
And
Statement-2:
If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.
(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not correct explanation for Statement-1.
[2008]

19. A planet in a distant solar system is 10 time more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be
(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}
(c) 11 km s^{-1} (d) 110 km s^{-1} **[2008]**

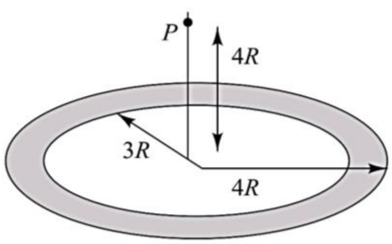
20. The height at which acceleration due to gravity becomes $g/9$ (where g = the acceleration due to gravity on the surface of the earth) in terms or R, the radius of the earth is
(a) $\frac{R}{2}$ (b) $\sqrt{2}R$
(c) $2\ R$ (d) $\frac{r}{\sqrt{2}}$ **[2009]**

21. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$ where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by



[2009]

22. A thin uniform annular disc (see the figure) of mass M has outer radius 4 R and inner radius 3 R. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

[2010]

23. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$

(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$ **[2011]**

24. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be

(a) 6.4×10^{11} Joules (b) 6.4×10^{8} Joules
(c) 6.4×10^{9} Joules (d) 6.4×10^{10} Joules

[2012]

25. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$.

(a) $\frac{2GmM}{3R}$ (b) $\frac{GmM}{2R}$

(c) $\frac{GmM}{3R}$ (d) $\frac{5GmM}{6R}$ **[2013]**

26. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity of Earth is 10 ms^{-2})

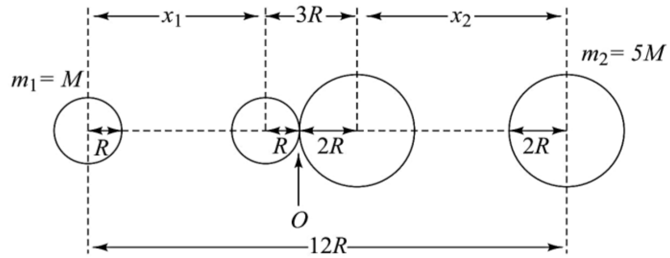
(a) 96 N (b) 108 N
(c) 120 N (d) 150 N **[2014]**

Answers

1. (b)	2. (a)	3. (a)	4. (c)
5. (b)	6. (c)	7. (c)	8. (c)
9. (c)	10. (d)	11. (a)	12. (b)
13. (a)	14. (c)	15. (c)	16. (c)
17. (a)	18. (c)	19. (d)	20. (c)
21. (c)	22. (a)	23. (d)	24. (d)
25. (d)	26. (b)		

Solutions

1. If the gravitational force becomes zero. then there will be no force to provide the necessary centripetal force for the circular motion. Hence the satellite move along the tangent to a point in the orbit when the gravitational force vanishes.
2. Total energy of the body orbiting the earth is
$$E = -\frac{GmM}{2r}$$
where m = mass of body, M = mass of earth and r = radius of the orbit
At $r = 2R$, $\quad E_1 = -\frac{GmM}{4r}$
At $r = 3R$, $\quad E_2 = -\frac{GmM}{6r}$
$\therefore$ Energy required is
$$E = E_2 - E_1 = -\frac{GmM}{6R} - \left(-\frac{GmM}{4R}\right) = \frac{GmM}{12R}$$
So, the correct choice is (a)
3. Escape velocity $v_e = \sqrt{2gR}$ which is independent of the mass of the body. So the correct choice is (a)
4. The minimum kinetic energy required is
$$\text{K. E} = \frac{1}{2}\text{m}v_e^2$$
where v_e is the escape velocity, Now $v_e = \sqrt{2gR}$.
Hence $\quad \text{K. E} = \frac{1}{2} m \times 2gR = mgR$
5. Let x be the extension in the spring when it is loaded with mass M. The change in gravitational potential energy = Mgx. This must be the energy stored in the spring which is given by $\frac{1}{2}kx^2$. Thus $\frac{1}{2}kx^2 = Mgx$ or $x = \frac{2Mg}{k}$
6. $T \propto r^{3/2}$. Therefore,
$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \qquad (1)$$
Given r_2 = 6400 km and r_1 = 36000 km. For a geostationary satellite T_1 = 24 h. Using these values in (1), we have get $T_2 = 24 \times \left(\frac{64}{360}\right)^{3/2}$ =1.8. h.
7. According to Kepler's law of periods $T^2 \propto r^3$. Therefore
$$\frac{T_2^2}{T_1^2} = \left(\frac{r_2}{r_1}\right)^3 = (4)^3 = 64$$
$\therefore \quad T_2 = T_1\sqrt{64} = 8\,T_1 = 8 \times 5\text{ h} = 40\text{ h}$
8. Suppose the two bodies collide at O. If the smaller body $m_1 = M$ moves a distance x_1 to reach O, then the bigger body moves a distance $x_2 = (9R - x_1)$ to reach O.



Since no net external force acts, the acceleration of the centre of mass the system is zero, i.e. $a_{CM} = 0$. Now
$$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$$
Putting $a_{CM} = 0$, $m_1 = M$, and $m_2 = 5\,M$ we get
$$Ma_1 + 5Ma_2 = 0 \quad \Rightarrow \quad a_1 = -5\,a_2$$
The negative sign shows that the two bodies move in opposite directions. If t is the time after which they collide,
$$x_1 = \frac{1}{2}a_1t^2 \quad \text{and} \quad x_2 = \frac{1}{2}a_2t^2$$
Dividing we get
$$x_1 = \frac{a_1}{a_2} = \frac{|5a_2|}{a_2} = 5 \quad \Rightarrow \quad x_1 = 5\,x_2$$
or $\quad x_1 = 5 \times (9R - x_1) \quad \Rightarrow \quad x_1 = 7.5\,R$
9. The escape velocity is independent of the angle of projection. Hence it will remain equal to km s^{-1}.
10. Radius of circular orbit is $r = R + x$. If v is the orbital speed, then
centripetal force = gravitational force
or $\quad \frac{mv^2}{r} = \frac{GmM}{r^2}$
$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{(R+x)}} \qquad \left(\because g = \frac{GM}{r^2}\right)$$
Notice that choices (a), (b) and (c) do not give the dimensions of speed.
11. The time period of the satellite is given by
$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$
where r = radius of orbit and M = mass of earth. Thus the time period is independent of the mass of the satellite.

12. Gain in P.E. $= -\frac{GmM}{(R+h)} - \left(\frac{GmM}{R}\right)$

$$= -\frac{GmM}{2R} + \frac{GmM}{R} \qquad (\because h = R)$$

$$= \frac{GmM}{2R} = \frac{1}{2}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

13. The necessary centripetal force for the circular motion of the planet around the sun is provided by the gravitational force exerted by the sun, i.e.

$$\frac{mv^2}{R} = \frac{GmM}{R^n} \Rightarrow v = \left(\frac{Gm}{R^{(n-1)}}\right)^{1/2}$$

where M = Mass of sun and m = mass of planet. Now the time period is

$$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{(n-1)}}{GM}\right)^{1/2} = \frac{2\pi}{\sqrt{Gm}} \times \left[R^{\frac{(n+1)}{2}}\right]^{1/2}$$

$$T \propto R^{\frac{(n+1)}{2}}$$

14. $\rho = \frac{M}{V} \Rightarrow M = \rho V = \rho \times \frac{4\pi}{3}R^3$

Now $\quad g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4\pi}{3}R^3\rho = \frac{4\pi}{3}G\rho R$

$\Rightarrow \quad \rho = \frac{3g}{4\pi GR}$. Hence $\rho \propto g$

15. $g_h = g\left(1 - \frac{2h}{r}\right) \Rightarrow g - g_h = \frac{2hg}{R}$

$g_d = g\left(1 - \frac{d}{r}\right) \Rightarrow g - g_d = \frac{gd}{R}$

Given $g - g_h = g - g_d \Rightarrow \frac{2hg}{R} = \frac{gd}{R} \Rightarrow d = 2h$

16. Mass of particle m = 10 g = 10^{-2} kg, mass of sphere M = 100 kg and radius of sphere R = 10 cm = 0.1 m. The work needed to take the particle from $r = R$ to $r = \infty$ is given by

$W = \int_R^\infty \mathbf{F}\cdot d\mathbf{r} = \int_R^\infty F dr \cos 180°$ ($\because$ Force and displacement are in opposite directions.)

$$= -\int_R^\infty F dr = -\int_R^\infty \frac{GmM}{r^2} dr$$

$$= -GmM \int_R^\infty r^{-2} dr$$

$$= \frac{GmM}{R} = \frac{6.67\times10^{-11}\times10^{-2}\times100}{0.1} = 6.67\times10^{-10}\,\text{J}$$

17. The charge of an electron is the same anywhere in the universe. Hence t he correct choice is (a).

18. Statement-2 is Gauss's theorem in gravitation. Gauss's theorem holds for any fields which obeys $1/r^2$ dependence. Just as electric field intensity due to a charge Q at a distance r from it is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The gravitational field intensity due to a mass M at at a distance r from it is given by

$$I = \frac{GM}{r^2}$$

The mass M plays the same role in gravitation as charge Q does in electrostatics. Further constant G is analogous to constant $1/4\pi\varepsilon_0$. From Gauss's theorem in electrostatics, electric flus through a closed surface is given by

$$\phi_e = \frac{Q}{\varepsilon_0}$$

where Q is the charge enclosed in the surface.

Replaced Q by M, and ε_0 by $1/4\pi G$, the gravitational flux through a closed surface will be

$$\phi_g = \frac{M}{1/4\pi G} = 4\pi GM$$

where M is the mass enclosed in the surface. Hence statements 1 and 2 are true and Statement 2 is the correct explanation for Statement-1. So the correct choice is (c).

19. $v_e = \sqrt{\frac{GM_e}{R_e}}$ and $v_p = \sqrt{\frac{GM_p}{R_p}}$

$\therefore \quad v_p = v_e = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$

$= (11\ \text{km s}^{-1}) \times \sqrt{10\times10}$

$= 110\ \text{km s}^{-1}$

20. $g_h = g\left(\frac{R}{R+h}\right)^2$

$$\frac{g}{9} = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{1}{3} = \frac{R}{R+h} \Rightarrow h = 2R$$

21. If M is the total mass of the system of particles, the orbital speed of the test mass is

$$v = \sqrt{\frac{GM}{r}}$$

For $r \le R$, $v = \sqrt{\dfrac{GM \times \frac{4\pi}{3} r^3 \rho_0}{r}}$ which gives $v \propto r$,

i.e. v increases linearly with r up to $r = R$. Hence choices (b) and (d) are wrong.

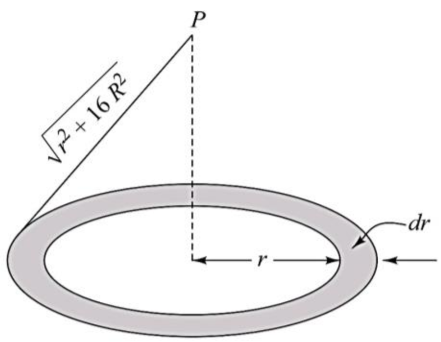
For $r > R$, the whole mass of the system is

$M = \dfrac{4\pi}{3} R^3 \rho_0$, which is constant. Hence for $r > R$.

$$v = \sqrt{\frac{GM}{r}}$$

i.e. $v \propto \dfrac{1}{\sqrt{r}}$. Hence the correct choice is (c).

22. By definition, the work, required to take a unit mass from P to infinity $= -V_p$, where V_p is the gravitational potential at P due to the disc. To find V_p, we divide the disc into small elements, each of thickness dr. Consider one such element at a distance r from the centre of the disc, as shown in the figure.



Mass of the element, $dm = \dfrac{M(2\pi r dr)}{\pi(4R)^2 - \pi(3R)^2}$

or $$dm = \frac{2Mrdr}{7R^2}$$

$$V_p = \int_{3R}^{4R} \frac{Gdm}{\sqrt{r^2 + 16r^2}}$$

$$= -\frac{2MG}{7R^2} \int_{3R}^{4R} \frac{rdr}{(r^2 + 16r^2)^{1/2}}$$

Putting $r^2 + 16 R^2 = x^2$, we get $2\, rdr = 2x\, dx$ or $rdr = x\, dx$.

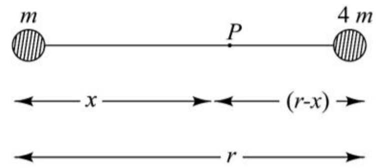
When $r = 3R$, $x = \sqrt{9R^2 + 16R^2} = 5R$

When $r = 4R$, $x = \sqrt{16R^2 + 16R^2} = 4\sqrt{2}\, R$

$$\therefore V_p = -\frac{2MG}{7R^2} \int_{5R}^{4\sqrt{2}R} dx = -\frac{2MG}{7R^2}(4\sqrt{2} - 5)R$$

Hence $-V_p = \dfrac{2GM}{7R}(4\sqrt{2} - 5)$, which is choice (a).

23. Let P be the point where the gravitational field is zero.



Then $$\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\Rightarrow \quad \frac{1}{x^2} = \frac{4}{(r-x)^2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{4}{r-x} \quad \Rightarrow \quad x = \frac{r}{3}$$

Gravitational potential at P is

$$V = V_1 + V_2$$

$$= -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$= -\frac{Gm}{r/3} - \frac{4Gm}{2r/3} = -\frac{9Gm}{r}$$

24. $E = 0 - \left(-\dfrac{GmM}{R}\right)$

$= \dfrac{GmM}{R}$

$= mgr$ $\qquad \left(\because g = \dfrac{GM}{R^2}\right)$

$= 1000 \times 10 \times 6400 \times 10^3$

$= 6.4 \times 10^{10}$ J

25. The speed of the satellite in its circular orbit of radius r is

$v = \sqrt{\dfrac{Gm}{r}} = \sqrt{\dfrac{Gm}{R+h}} = \sqrt{\dfrac{Gm}{3R}}$ $\qquad (\because h = 2R)$

Total energy of the satellite in its circular orbit is

E_1 = K.E + P.E

$= \dfrac{1}{2} mv^2 - \dfrac{GmM}{r}$

$= \dfrac{1}{2} m \times \dfrac{GM}{3R} - \dfrac{GmM}{3R} = -\dfrac{GmM}{6R}$

Total energy when the satellite was on the launching pad is

$$E_2 = \text{K.E} + \text{P.E}$$

$$= 0 - \frac{GmM}{R} = -\frac{GmM}{R}$$

∴ Minimum energy required is

$$E_{\min} = E_1 - E_2 = -\frac{GmM}{6R} - \left(\frac{-GmM}{R}\right) = \frac{5\,GmM}{6R}$$

26. At a depth r below the surface of a planet,

$$g_r = \frac{4\pi G\rho r}{3}$$

If F is the force needed to keep the wire at rest, then

F = weight of the wire = mg_r

$$\therefore \quad F = \int_{\frac{4R}{5}}^{R} (\lambda dr)\left(\frac{4\pi G\rho r}{3}\right)$$

$$= \frac{4\pi G\rho r}{3}\left|\frac{r^2}{2}\right|_{\frac{4R}{5}}^{R}$$

$$\Rightarrow \quad F = \frac{4\pi G\rho r}{3} \times \frac{9R^2}{50} \qquad (1)$$

On the surface of the earth,

$$g_e = \frac{GM_e}{R_e^2} \Rightarrow G = \frac{g_e R_e^2}{M_e} \qquad (2)$$

$$\text{Also} \quad \rho = \frac{M_e}{\frac{4\pi}{3}R_e^3} \qquad (3)$$

$$\text{Given } R = \frac{R_e}{10} \qquad (4)$$

Using (2), (3) and (4) in (1), we get

$$F = \frac{9g_e R_e \lambda}{5\times 10^3}$$

$$= \frac{9\times 10\times(6\times 10^6)\times 10^{-3}}{5\times 10^{+3}} = 108\,\text{N}$$