AEP STUDY CIRCLE
IIT-NEET-CBSE
ACCENTS EDUCATIONAL PROMOTERS

23 YEARS OF LEGACY

CBSE-XII PHYSICS

www.aepstudycircle.com

# NUCLEI

"Embark on a fascinating exploration of the atomic nucleus with our specialized study module for CBSE Class 12 Physics—Nuclei. This comprehensive resource is meticulously crafted to provide an in-depth understanding of nuclear physics, covering key concepts, applications, and real-world significance."

Prepare for CBSE Class 12 Physics success with confidence in Nuclei. Order our Study Module now to embark on a comprehensive learning journey and reinforce your knowledge in this crucial physics topic. Study, practice, and excel in understanding the intricacies of atomic nuclei!

NEET IIT-JEE CBSE

OFFLINE-ONLINE LEARNING ACADEMY

*The Success Destination*

+91-9939586130
+91-7739650505

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

# NUCLEI

CBSE-XII PHYSICS

**Nuclear Structure and Properties:** Dive into the heart of the atom—the nucleus. Explore the structure and properties of nuclei, including nucleons, isotopes, and nuclear forces, unraveling the mysteries of the atomic nucleus.

**Nuclear Reactions:** Understand the principles governing nuclear reactions, from radioactive decay to nuclear fission and fusion. Delve into the energy transformations and applications of these reactions in various scientific and technological domains.

**Radioactivity and Decay:** Explore the phenomenon of radioactivity, examining alpha, beta, and gamma decay processes. Grasp the principles behind half-life and decay constants, essential for understanding the behavior of radioactive substances..

**Nuclear Models:** Examine different nuclear models, including the liquid drop model and the shell model, to comprehend the intricate balance between nuclear forces and the structure of atomic nuclei..

**Nuclear Energy and Applications:** Connect theoretical knowledge to practical applications. Explore the diverse applications of nuclear energy, from power generation to medical imaging and industrial uses.

**Problem-solving Practice:** Hone your problem-solving skills with a variety of practice questions. Covering different difficulty levels, our study module ensures you are well-prepared for the CBSE Class 12 Physics examination.

**Exam-oriented Approach:** Tailored to align with the CBSE examination pattern, our study module ensures you are well-prepared for questions related to Nuclei in the board exams. It emphasizes key points and concepts likely to be tested.

HELPLINE : +91-9939586130 // +91-7739650505
2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

## ATOMIC NUCLEUS: ITS CONSTITUENTS

**Atomic nucleus: Its constituents:** To explain the large angle scattering of α-particles by thin metal foils, **Rutherford in 1911 postulated the existence of a nucleus inside an atom**.

☐**1.** According to Rutherford's planetary model of atom, the entire positive charge and most of the mass of the atom are concentrated in a small volume called the nucleus and a suitable number of electrons revolve around it just as planets revolve around the sun.

☐**2.** From the results of Rutherford scattering experiments, nuclear size is found to be of the order of $10^{-14}$ m whereas the diameter of an atom is of the order of $10^{-10}$ m. Hence most of the atom is empty.

☐**3.** The studies of natural radioactivity revealed that the emissions of α –, β – and γ – particles/radiations have nuclear origin. In a real sense, the γ –rays are not the constituents of nuclei but they are emitted when a nucleus in excited state returns to the ground state.

☐**4.** Researches on artificial radioactivity revealed that many particles like α-particles, protons, neutrons, positrons, β-particles, etc. enter into the constitution of the nuclei in one way or the other.

☐**5.** Finally, the cosmic ray studies established the existence of new fundamental particles, called mesons, which occur in many forms, having different masses and charges.

**Proton-neutron hypothesis of nuclear composition:** (**The discovery of neutrons by Chadwick, led Heisenberg to propose proton-neutron hypothesis)**. According to this hypothesis, protons and neutrons are the main building blocks of the nuclei of all atoms. According to this hypothesis, a nucleus of mass number A and atomic number Z contains Z protons and (A – Z) neutrons. The protons give positive charge to the nucleus, while protons and neutrons together give it mass.
To neutralise, the positive charge of the nucleus, i.e., to make the atom electrically neutral, the number of extra-nuclear electrons is Z.

◙ **Proton:** It is a fundamental particle which may be called the nucleus of hydrogen. It has a positive charge of $1.6 \times 10^{-19}$ C. It has a rest mass of $1.6726 \times 10^{-27}$ kg, which is about 1836 times the rest mass of an electron. A proton has an intrinsic (spin) angular momentum equal to 1/2. It also possesses a magnetic moment much smaller than that of an electron.

◙ **Neutrons:** It is a chargeless fundamental particle having mass slightly greater than that of a proton. Its rest mass is $1.6749 \times 10^{-27}$ kg. It has intrinsic angular momentum equal to that of a proton. In spite of being neutral, a neutron also possesses a small magnetic moment.

*Neutrons and protons are identical particles in the sense that their masses are nearly the same and the force, called nuclear force, does not distinguish them. So, the neutrons and protons have common name, the nucleons. However, as the proton is positively charged and the neutron is electrically neutral, so the electromagnetic force can distinguish the two types of particles.*

### ◙ Composition of an Atomic nucleus:

☐**1. Nucleons:** Protons and neutrons which are present in the nuclei of atoms are collectively known as nucleons.

☐**2. Atomic number:** The number of protons in the nucleus is called the atomic number of the element. It is denoted by Z.

☐**3. Mass number:** The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by A.Hence for a neutral atom, we have the following relation:

◙**Number of protons in an atom = Z**
◙**Number of electrons in an atom = Z**
◙**Number of nucleons in an atom = A**
◙**Number of neutrons in an atom = N = A = Z**

☐**4. Nuclear mass:** The total mass of the protons and neutrons present in a nucleus is called the nuclear mass.

☐**5. Nuclide:** When an atom is talked of which particular reference to its nuclear composition, it is called a nuclide. Thus, a nuclide is a specific nucleus of an atom characterised by its atomic number Z and mass number A. It is symbolically represented as ${}_Z^AX$

Where, X = Chemical symbol of the element,
Z = atomic number, and A = mass number.

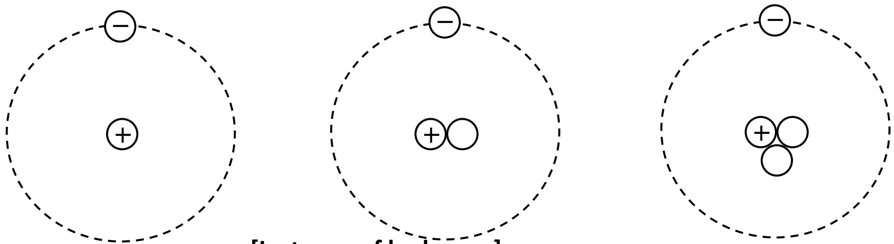
For example, gold nucleus is represented as ${}^{197}_{79}Au$. It contains 197 nucleons, of which 79 are protons and 118 neutrons.

## ISOTOPES, ISOBARS, ISOTONES AND ISOMERS

◈ **Isotopes: The atoms of an element which have the same atomic number but different mass number are called isotopes.** Such atoms contain the same number of protons and electrons but different number neutrons.

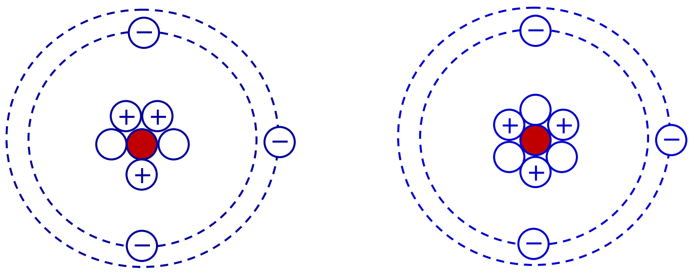
☐☐ ***Because of their similar electronic configuration, isotopes of an element exhibit similar chemical properties and they occupy the same position in the periodic table.***

**Hydrogen has three isotopes**: Hydrogen (protium) ${}_1^1H$ – its nucleus has just one proton; deuterium (${}_1^2H$) – its nucleus has one proton and one neutron; and tritium (${}^3_1H$) – its nucleus has one proton and two neutrons, as shown in Fig.



**[Isotopes of hydrogen]**

Lithium has two isotopes ${}^6_3Li$ and ${}^7_3Li$, as shown in Fig.



**[Isotopes of lithium]**

Gold has 32 isotopes, ranging from A = 173 to 2.4. The different isotopes of an element are found to have different relative abundances. So, the weighted average of the atomic masses of all the isotopes of an element is taken as its average atomic mass. For example, normal chlorine contains 75% of ${}^{35}_{17}Cl$ and 25% of ${}^{37}_{17}Cl$.

$$\therefore \quad \text{Average atomic mass of chlorine} = \frac{35 \times 75 + 37 \times 25}{75 + 25} = 35.50$$

**Isobars: The atoms having the same mass number but different atomic number are called isobars. Such atoms contain different number of protons and elements.**

So, they differ in the chemical properties and occupy different positions in the periodic table. Some examples of isobars are:

1. ${}^3_1H$ and ${}^3_2He$, as both have same A = 3.
2. ${}^{37}_{17}Cl$ and ${}^{37}_{16}S$, as both have same A = 37.
3. ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$, as both have same A = 40.

**Isotones: The nuclides having the same number of neutrons are called isotones**. For example,

1. ${}^{37}_{17}Cl$ and ${}^{39}_{19}K$ are isotones, as both contain the same number of neutrons i.e., for both
   $N = A - Z = 20$
2. ${}^{198}_{80}Hg$ and ${}^{197}_{79}Pu$ are isotones, as for both
   $N = A - Z = 118$

**Isomers: These are the nuclei with same atomic number and same mass number but existing in different energy states**. For example, a nucleus in its ground state and the identical nucleus in metastable excited state are isomers.

## ATOMIC MASSES

**Atomic mass unit:** The mass of the carbon – 12 atom is $1.992678 \times 10^{-26}$ kg, which is very small. Therefore, it is useful to choose a convenient unit for expressing the mass of atoms. This unit is defined by taking mass of carbon – 12 atoms equal to 12 atomic mass units.

**One atomic mass unit is defined as 1/12$^{th}$ of the actual mass of carbon – 12 atoms.**

Atomic mass unit is denoted by amu or just by u.

Thus, $1 \text{ amu} = \frac{1}{12} \times \text{Mass of carbon – 12 atoms}$

$= \frac{1}{12} \times 1.992678 \times 10^{-27}$ kg

or $1 \text{ amu} = 1.660565 \times 10^{-27}$ kg.

We can now express different masses in terms of amu.

Mass of an electron $m_e = 0.00055 \text{ amu} = 9.11 \times 10^{-31}$ kg

Mass of a proton, $m_p = 1.0073 \text{ amu} = 1.6726 \times 10^{-27}$ kg

Mass of a neutron, $m_n = 1.0086 \text{ amu} = 1.6749 \times 10^{-27}$ kg

Mass of a hydrogen atom, $m_H = m_p + m_e = 1.0078$ amu

The atomic masses can be measured accurately by using an instrument called mass spectrometer.

☐**Electron volt: It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt** and is denoted by eV. $1\ eV = 1.602 \times 10^{-19}$ J

It is a convenient unit of energy used commonly in atomic physics. For example, 13.6 eV energy is needed to remove an electron from a hydrogen atom. A bigger unit called million electron volt (MeV) is used for measuring energy changes in nuclear reactions. For example, about 2.2 MeV energy is needed to separate neutron and proton in a deuterium nucleus.

$$1\ MeV = 10^6\ eV = 1.602 \times 10^{-13}\ J$$

☐☐**Relation between amu and MeV: Energy equivalent of amu**. The Einstein's mass-energy equivalence relationship is

$E = mc^2$

This relation shows that the energy content of an object is equal to its mass times the square of the speed of light.

☐ **To determine the energy equivalent of one atomic mass unit**, we take

$m = 1\ amu = 1.66 \times 10^{-27}$ kg

$c = 2.998 \times 10^8\ ms^{-1}$

Then $E = 1.66 \times 10^{-27} \times (2.998 \times 10^8)^2$ J

$$= \frac{1.66 \times 10^{-27} \times (2.998 \times 10^8)^2}{1.602 \times 10^{-19}}\ eV = 931\ MeV$$

∴ **1 amu = 931 MeV.**

## ◆◆ ISOTOPES

**Nuclear size:** Like an atom, a nucleus is not a solid object. Its surface is not a well-defined boundary. Still, we can assign a size to the nucleus.

By performing scattering experiments using high energy probes such as fast-moving protons, neutrons or electrons, nuclear sizes of different elements have been accurately measured. Assuming nuclei to be spherical, their volumes can be estimated. **Experimental observations show that the volume of a nucleus is directly proportional to its mass number.**

If R is the radius of a nucleus having mass number A, then

$$\frac{4}{3}\pi R^3 \propto A$$

or $\quad \mathbf{R \propto A^{1/3}}$

Thus, the radius R of a nucleus is proportional to cube root of its mass number. We can write $\quad R = R_0 A^{1/3}$

Here $R_0$ is a constant, which is of the order of the range of nuclear force. It is believed to be the average nucleon size and is known as nuclear unit radius. The value of $R_0$ depends on the nature of probe particles.

For electrons, $\quad R_0 = 1.2 \times 10^{-15}\ m = 1.2$ fm

## ◆◆NUCLEAR DENSITY

*The density of nuclear matter is the ratio of the mass of a nucleus to its volume.*

**As the volume of a nucleus is directly proportional to its mass number A, so the density of nuclear matter is independent of the size of the nucleus. Thus, the nuclear matter behaves like a liquid of constant density.** Different nuclei are like drops of this liquid, of different sizes but of some density.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

Mass of nucleus = mA

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$$

$$= \frac{4}{3}\pi R_0^3 A$$

$$\therefore \text{Nuclear density} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Clearly, nuclear density is independent of mass number A or the size of the nucleus.

Taking $m = 1.67 \times 10^{-27}$ kg

and $\quad R_0 = 1.2 \times 10^{-15}$ m, we get

$$\rho_{nu} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.142 \times (1.2 \times 10^{-15})^3} = 2.30 \times 10^{17}\ kg\ m^{-3}$$

Thus, the nuclear mass density is of the order $10^{17}\ kg\ m^{-3}$. This density is very large as compared to the density of ordinary matter, say water, for which $\rho = 1.0 \times 10^3\ kg\ m^{-3}$.

**Examples based on Equivalent Energy, Atomic Mass, Nuclear Size and Nuclear Density**

◆ Formulae used 1. Einstein's mass-energy equivalence, $E = mc^2$

2. $1\text{ amu} = \frac{1}{12} \times$ Mass of C-12 atom

3. Nuclear radius, $R = R_0 A^{1/3}$,
Where $R_0 = 1.2 \times 10^{-15}$ m

4. $\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{nu}}{\frac{4}{3}\pi R^3}$

5. Average atomic mass of an element = Weighted average of the masses of all isotopes.

◆ Units used R and $R_0$ are in metre, $\rho$ in $kgm^{-3}$.

◆ Conversions used $1\text{ amu} = 1.66 \times 10^{-27}$ kg = 931 MeV, $1\text{eV} = 1.6 \times 10^{-19}$ J.

**Q. 1.** ***Express 16 mg mass into equivalent energy in eV.***

**Sol.** Here $m = 16\text{ mg} = 16 \times 10^{-6}$ kg, $c = 3 \times 10^8\ ms^{-1}$

∴ Equivalent energy, $E = mc^2 = 16 \times 10^{-6} \times (3 \times 10^8)^2$ J

$= \frac{16 \times 10^{-6} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}}\text{ eV} = 9 \times 10^{30}$ eV

**Q. 2.** ***How many electron volts make up one joule?***

**Sol.** By definition, one electron volt is the energy gained by an electron, when accelerated through a potential difference of 1 volt, therefore $1\text{ eV} = 1.602 \times 10^{-19}\text{ C} \times 1\text{ V} = 1.602 \times 10^{-19}$ J

Hence $1\text{ J} = \frac{1}{1.602 \times 10^{-19}}\text{ eV} = 6.242 \times 10^{18}$ eV

**Q. 3.** ***Taking one atomic mass unit equal to 931 MeV, calculate the mass of $^{12}_{6}C$ atom.***

**Sol.** Given 1 amu = 931 MeV $= 931 \times 1.602 \times 10^{-13}$ J

Using Einstein's mass-energy relationship,

$E = mc^2$ or $m = \frac{E}{C^2}$

∴ $1\text{ amu} = \frac{931 \times 1.602 \times 10^{-13}}{(3 \times 10^8)^2}\text{ kg} = 1.657 \times 10^{-27}$ kg

∴ Mass of $^{12}_{6}C$ atom $= 12\text{ amu} = 12 \times 1.657 \times 10^{-27}\text{ kg} = 1.988 \times 10^{-26}$ kg.

**Q. 4.** ***The natural chlorine is found to be a mixture of two isotopes of masses 34.98 amu and 36.98 amu 24.6 percent respectively. Find the composite atomic mass of natural chlorine.***

**Sol.** The average atomic mass of chlorine is

$m(Cl) = \frac{75.4 \times 34.98 + 24.6 \times 36.98}{100}$ amu

$= \frac{2637.49 + 909.71}{100}\text{ amu} = 35.47$ amu

**Q. 5.** ***The natural boron is composed of two isotopes of $^{10}_{5}B$ and $^{11}_{5}B$. The two isotopes have masses 10.003 amu and 11.009 amu, respectively. Find the relative abundance of each isotope in the natural boron if the atomic mass of natural boron is 10.81 amu.***

**Sol.** Suppose the natural boron contains x% of $^{10}_{5}B$ isotopes and (100 – x) % of $^{11}_{5}B$ isotope. Then

Atomic mass of natural boron = Weighted average of the masses of two isotopes

∴ $10.81 = \frac{x \times 10.003 + (100 - x) \times 11.009}{100}$

or $1081 = -0.996x + 1100.94$

or $x = \frac{19.9}{0.996} = 19.98$

∴ Relative abundance of $^{10}_{5}B$ isotope = 19.98 % ; Relative abundance of $^{11}_{5}B$ isotope = 80.02%

**Q. 6.** ***Calculate the radius of $Ge^{70}$. Given $R_0 = 1.1$ fm.***

**Sol.** Here $A = 70$, $R_0 = 1.1$ fm ∴ $R = R_0 A^{1/3} = 1.1 \times (70)^{1/3} = 1.1 \times 4.12 = 4.53$ fm

**Q. 7.** ***The nuclear mass of $^{56}_{26}Fe$ is 55.85 amu. Calculate its nuclear density.***

**Sol.** Here $m_{Fe} = 55.85\text{ amu} = 55.85 \times 1.66 \times 10^{-27}$ kg

$= 9.27 \times 10^{-26}$ kg

Nuclear radius $= R_0 A^{1/3} = 1.1 \times 10^{-15} \times (56)^{1/3}$ m [∵ A = 56]

$\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{Fe}}{\frac{4}{3}\pi R^3} = \frac{9.27 \times 10^{-26}}{\frac{4\pi}{3} \times (1.1 \times 10^{-15})^3 \times 56} = 2.9 \times 10^{17}\text{ kg m}^{-3}$

**Q. 8.** ***Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. Given that nuclear radius is given by $R = 1.2 \times 10^{-15} A^{1/3}$ metre and mass of a nucleon = $1.67 \times 10^{-27}$ kg.***

**Sol.** Here $R = 1.2 \times 10^{-15} A^{1/3}$ m,

mass of a nucleon, $m = 1.67 \times 10^{-27}$ kg

Nuclear mass = $mA = 1.67 \times 10^{-27} \times A$ kg

Nuclear volume = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (1.2 \times 10^{-15} A^{1/3})^3$ m³

Nuclear density, $\rho_{nu} = \dfrac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi (1.2 \times 10^{-15} A^{1/3})^3} = 2.307 \times 10^{17}$ kg m$^{-3}$

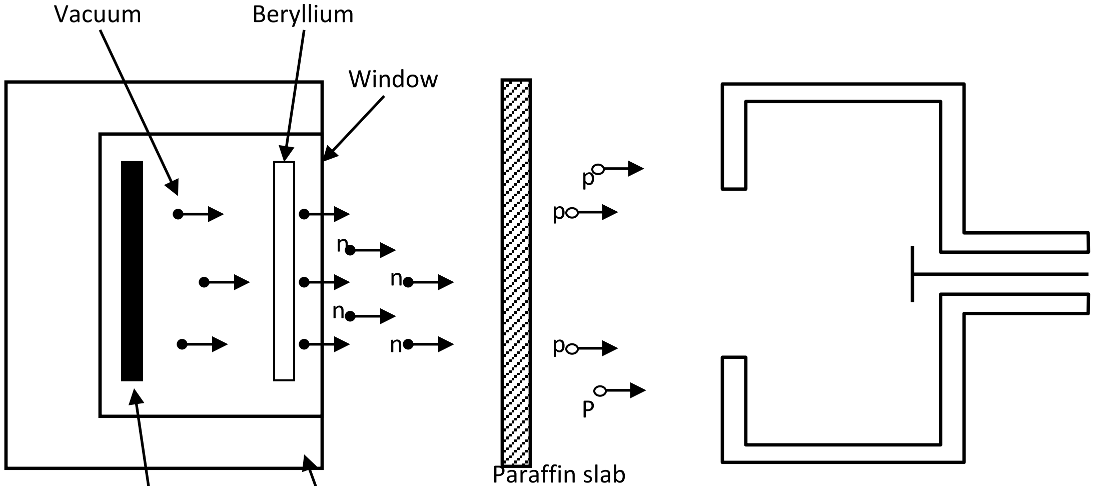
Now, density of water, $\rho_{wat} = 10^3$ kg m$^{-3}$

$\therefore \quad \dfrac{\rho_{nu}}{\rho_{wat}} = \dfrac{2.307 \times 10^{17}}{10^3} = 2.307 \times 10^{14}$

## ◆◆ DISCOVERY OF NEUTRONS.

The neutrons were discovered by James Chadwick in 1932.

In 1932, Chadwick performed an experiment in which $\alpha$-particles from a radioactive Polonium source were used to bombard beryllium nuclei. Highly penetrating rays were found to come out of the beryllium metal, which could not be deflected by electric and magnetic fields. These radiations were used to bombard hydrocarbons like paraffin wax.




**[Experiment set up used by Chadwick to discover neutrons]]**

High energy protons were knocked out from the paraffin wax. The energy of the ejected protons was found to be too high to be accounted for $\gamma$-ray photons. By using the laws of conservation of energy and momentum, Chadwick concluded that the penetrating radiation consisted of neutral particles, each having a mass nearly that of a proton. These particles were called neutrons.

The reaction may be written as: $\quad {}^4_2He + {}^9_4Be \rightarrow {}^1_0n + {}^{12}_6C$

Here ${}^1_0n$ denotes a neutron having zero charge and mass nearly the same as that of a proton.

### ◆ Properties of neutrons:

**☐1. Neutrons is an elementary particle present in the nuclei of all elements except hydrogen.**

**☐2. Neutron has no charge and its mass is slightly more than that of a proton**

**$m_n = 1.0086$ amu $= 1.6749 \times 10^{-27}$ kg**

**☐3. Inside a nucleus, a neutron is stable, But outside a nucleus, it is unstable. A free neutron spontaneously decays into a proton, electron and antineutron (an elementary particle with zero charge and zero rest mass) with a mean life of about 1000 s.**

$${}^1_0n \rightarrow {}^1_1H + {}_{-1}^{0}e + \overline{v} + Q$$

**☐4. Being neutral, they do not interact with electrons. So neutrons have low ionising powers.**

**☐5. Being neutral, neutrons are not repelled or attracted by the nucleus and the electrons of an atom. They can easily penetrate heavy nuclei and induce nuclear reactions.**

**☐6. They induce radioactivity in many elements.**

**☐7. In heavier nuclei, the number of neutrons is more than that of protons. Protons being positively charged, repel each other and in order to maintain stability of the nucleus, more neutrons become necessary for heavier nuclei.**

## NUCLEAR FORCE

The average separation between two nucleons is about $10^{-15}$ m. At this separation, positively charged protons feel strong coulombic repulsion. Also, the gravitational force of attraction between two nucleons is about $10^{-36}$ times smaller than the electrostatic repulsion, it cannot hold the nucleons together. So, there must be some other strong attractive force acting between the nucleons that overcomes the electrostatic repulsion. This strong attractive interaction acting between the nucleons is called nuclear force or strong interaction.

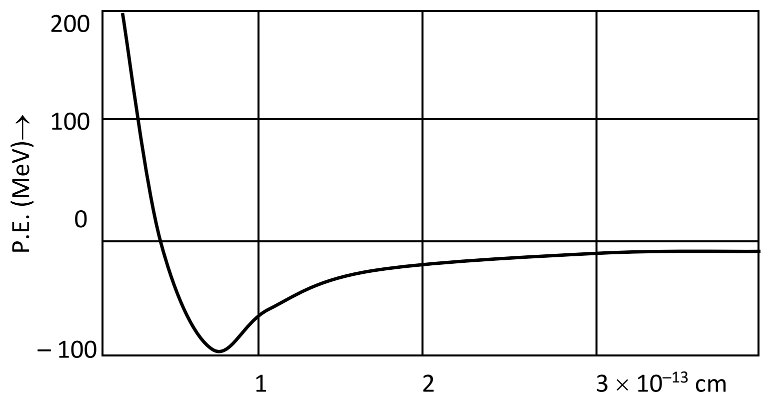
☐**Nuclear force is a strong attractive force that binds the protons and neutrons together inside a tiny nucleus**.

**1. Strongest interaction:** Nuclear force is the strongest interaction known in nature that holds the nucleons together despite the strong electrostatic repulsion between the protons. The relative strength of gravitational, electrostatic, and nuclear forces is

$$F_g : F_e : F_n = 1 : 10^{36} : 10^{38}$$

**2. Short-range force:** Unlike gravitational and electrostatic forces, nuclear force is a short-range force. It operates only up to a very short distance of about 2 – 3 fm from a nucleon.

**3. Variation with distance:** Fig. shows a graph of the potential energy of a pair of nucleons as a function of their separation. The graph indicates that the attractive force between the two nucleons is strongest at a separation $r_0 \simeq 1$ fm. For a separation greater than $r_0$, the force is attractive and for separation less than $r_0$, the force is strongly repulsive. It rapidly decreases with distance and becomes negligibly small at a separation of about 4 fm. It varies inversely not as the square of distance but depends on some higher power of distance.




**[Graph between the potential energy of a pair of nucleons as a function of their separation]**

**4. Charge independent character:** It is seen from experiments that the attractive force between two neutrons (nn-force) is nearly equal to that between two protons (pp-force) or between a proton and a neutron (pn-force). Thus, the nuclear force does not depend on the charge of the particles.

In case of pp-nuclear force, there is a repulsive force between two protons, but this is weak compared to the strong nuclear force.

**5. Saturation effect:** Nuclear forces show saturation effect i.e., a nucleon interacts only with its neighbouring nucleon. This property is supported by the fact that the binding energy per nucleon is same over a wide range of mass numbers.

**6. Spin dependent character**: The nuclear force between two nucleons having parallel spins is stronger than that between two nucleons having antiparallel spins.

**7. Exchange forces:** In 1935, a Japanese physicist H. Yukawa suggested that the nuclear force between two nucleons arises from the constant exchange of particles, called mesons, between them.

**8. Non-central forces:** The nuclear force between two nucleons does not act along the line joining their centres.

## ☐☐ MASS DEFECT AND PACKING FRACTION

**Einstein's mass-energy equivalence:** In his special theory of relativity, Einstein showed that $E = mc^2$

Here c is the speed of light in vacuum and is equal to $3 \times 10^8$ ms$^{-1}$. The above equation expresses equivalence between mass and energy. This equation suggests that even when a particle is at rest (having zero kinetic energy), it still possess an enormous of energy because of its mass. The mass of a particle measured in a frame of reference in which the particle is at rest, is called rest mass and is denoted by $m_0$. Thus the total energy of a particle is sum of

(i) its rest mass energy $m_0c^2$ and

(ii) its kinetic energy T. That is

$$E = m_0c^2 + T$$

Clearly, the mass of a particle is greater when it is in motion than when it is at rest.

As mass and energy are convertible into each other, we cannot define separate laws of conservation of mass and conservation of energy. We need to define a unified law of conservation of mass and energy together.

**The law of conservation of mass-energy states that the sum of the mass-energy of a system of particles is the same before and after their interaction.**

**The most convicting evidence that this principle operates in nature comes from nuclear physics. It is central to our understanding of nuclear energy and harnessing it as a source of power. Using the principle, the Q-value of a nuclear process (decay or reaction) can be expressed also in terms of initial and final masses.**

☐ **Mass defect:** It is found that the mass of a stable nucleus is always less than the sum of the masses of its constituent protons and neutrons in their free state.

**The difference between the rest mass of a nucleus and the sum of the rest masses of its constituent nucleons is called its mass defect.**

Consider the nucleus ${}^{A}_{Z}X$. It has Z protons and (A – Z) neutrons. Therefore, its mass defect will be

$$\Delta m = Zm_p + (A - Z)m_n - m$$

Where $m_p$, $m_n$ and m are the rest masses of a proton, neutron and the nucleus ${}^{A}_{Z}X$ respectively.

☐ **Packing fraction:** The packing fraction of a nucleus is its mass defect per nucleon. Thus

$$\text{P.F. of a nucleus} = \frac{\text{Mass defect}}{\text{Mass number}} = \frac{\Delta m}{A}$$

◆ If P.F. is positive (as in case of nuclei with mass number less than 20 and above 200), then the nucleus is unstable.

◆ If P.F. is negative (as in case of nuclei with mass number between 20 and 200), then it indicates that some mass has been converted into energy which binds the nucleons together and so the nucleus is stable.

**Thus the P.F. is directly related to the availability of nuclear energy and the stability of the nucleus.**

## ☐☐ BINDING ENERGY AND BINDING ENERGY PER NUCLEON

**Binding energy:** An atomic nucleus is a stable structure. Inside it, the protons and neutrons are bound together by means of strong attractive nuclear forces. Thus, a definite amount of work is required to be done to break up the nucleus into its constituent particles and to place them at infinite distance from one another. This work gives a measure of the building energy of the nucleus.

**The binding energy of a nucleus may be defined as the energy required to break up a nucleus into its constituent protons and neutrons and to separate them to such a large distance that they may not interact with each other.**

◆ **The concept of binding energy may also be understood in terms of Einstein's mass energy equivalence. It is seen that the mass of a stable nucleus is always less than the sum of the masses of the constituent protons and neutrons in their free state. This mass difference is called mass defect which accounts for the $\Delta E_b$ energy released when a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass.** Thus

$$\Delta E_b = \Delta m \times c^2$$

So, the binding energy may also be defined as the surplus energy which the nucleons give up the virtue of their attraction when they become bound together to form a nucleus.

◆ **The energy equivalent to the mass defect is radiated in the form of electromagnetic radiation when the nucleons combine to form a nucleus.**

**Expression for binding energy:** The nucleus ${}^{A}_{Z}X$ contains Z protons and (A – Z) neutrons. Its mass defect is

$$\Delta m = Zm_p + (A - Z)\, m_n - m_N \quad \text{... (1)}$$

where $m_N$ is the nuclear mass of ${}^{A}_{Z}X$. From Einstein's mass-energy equivalence, the binding energy of the nucleus is

$$\Delta E_b = \Delta m \times c^2 = [Zm_p + (A - Z)\, m_n - m_N]c^2 \quad \text{... (2)}$$

Now, in an atom the electrons are bound to the nucleus by electrostatic forces. So, they have a binding energy of their own, which from the mass-energy equivalence is given by

$$(\Delta E_b)_e = [(m_N + Zm_e) - m({}^{A}_{Z}X)]c^2 \quad \text{... (3)}$$

where m (${}^{A}_{Z}X$) is the atomic mass. The binding energy of electrons (≈ eV to keV) is negligible compared to the binding energy of nucleons (≈ eV to keV) is negligible compared to the binding energy of nucleons (≈ $10^3$ MeV). It will be a safe approximation to take,

$$(\Delta E_b)_e = 0$$

∴ $m_N + Zm_e - m({}^{A}_{Z}X) = 0$

or $m_N = m({}^{A}_{Z}X) - Zm_e$

**Thus, in terms of atomic mass the equation (2) becomes**

$$\Delta E_b = [Zm_p + (A - Z)\, m_n - m({}^{A}_{Z}X) + Zm_e]c^2$$

$$= [Z(m_p + m_e) + (A - Z)\, m_n - m({}^{A}_{Z}X)]c^2 \quad \text{... (4)}$$

**But $m_p + m_e = m_H$ = mass of a hydrogen atom.**

∴ **The equation (4) can be written in terms of $m_H$ as**

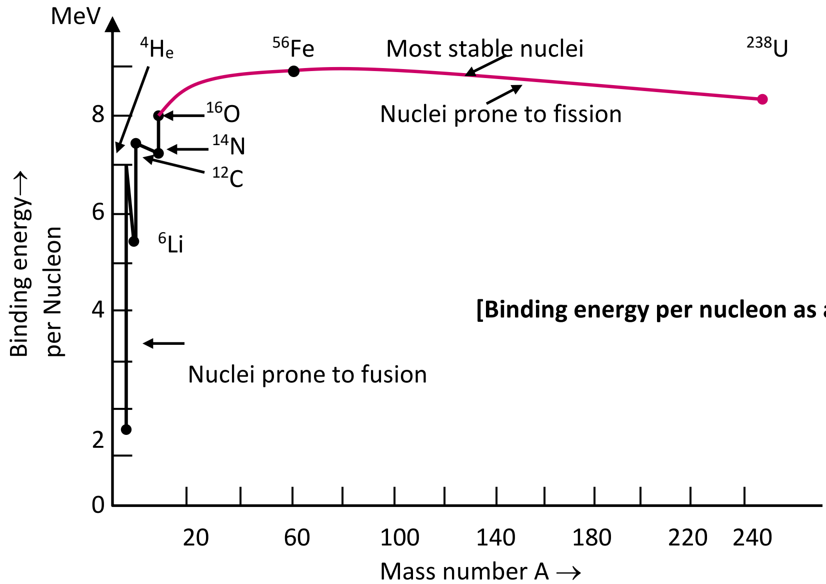
$$\Delta E_b = [Zm_H + (A - Z)\, m_n - m({}^{A}_{Z}X)]\, c^2 \quad \text{... (5)}$$

**Binding energy per nucleon:** The binding energy per nucleon is the average energy required to extract one nucleon from the nucleus. It is obtained by dividing the binding energy of a nucleus by its mass number. The expression for binding energy per nucleon can be written as

$$\Delta E_{bn} = \frac{\Delta E_b}{A} = \frac{[Zm_H + (A - Z)\,m_n - m(^A_ZX)]c^2}{A} \quad ...(6)$$

The binding energy per nucleon gives a measure of the force which binds the nucleons together inside a nucleus.

☐☐ **BINDING ENERGY CURVe** The value of binding energy per nucleon of a nucleus gives a measure of the stability of that nucleus, more stable is the nucleus. Fig. shows the graph of binding energy per nucleon drawn against mass number A.




**[Binding energy per nucleon as a function of mass number A]**

**Important features:**

**1. Except for some nuclei like $^4_2$He, $^{12}_6$C and $^{16}_8$O, the values of binding energy per nucleon lie on or near a smooth curve.**

**2. The B.E./nucleon is small for light nuclei like $^1_1$H, $^2_1$H and $^3_1$H.**

**3. In the mass number range 2 to 20, there are well defined maxima and minima on the curve. The maxima occur for $^4_2$He, $^{12}_6$C and $^{16}_8$O, indicating the higher stability of these nuclei than the neighbouring ones. The minima, corresponding to low stability, occur for $^6_3$Li, $^{10}_5$B and $^{14}_7$N.**

**4. The curve has a broad maximum close to the value 8.5 MeV/nucleon in the mass number range from about 40 to 120. It has a peak value of 8.8 MeV/nucleon for $^{56}_{26}$Fe.**

**5. As the mass number increases further, the B.E./nucleon shows a gradual decrease is due to coulomb repulsion between the protons which makes the heavier nuclei less stable.**

☐ **Importance of binding energy curve**: The binding energy curve can be used to explain the phenomena of nuclear fission and nuclear fusion as follows.

☐**1. Nuclear fission: Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e., heavier nuclei are less stable. When a heavier nucleus splits into the lighter nuclei, the B.E./nucleon changes from about 7.6 MeV to 8.4 MeV. Greater binding energy of the product nuclei results in the liberation of energy. This is what happens in nuclear fission which is the basis of the atom bomb.**

☐**2. Nuclear fusion: The binding energy per nucleon is small for light nuclei, i.e., they are less stable. So, when two light nuclei combine to form a heavier nucleus, the higher binding energy per nucleon of the latter results in the release of energy. This is what happens in a nuclear fusion which is the basis of the hydrogen bomb.**

## Examples based on Binding Energy of a Nucleus

◆ **Formulae used** ◆ **Units used**

1. Mass defect, $\Delta m = [Z\,m_p + (A - Z)\,m_n - m_N]$ Mass defect $\Delta m$ is in amu or kg and B.E. in joule or MeV.
2. $B.E. = (\Delta m)\,c^2$
3. $B.E./nucleon = \frac{B.E.}{A}$
4. Packing fraction $= \frac{\Delta m}{A}$

**Q. 1.** ***Calculate the binding energy of an $\alpha$-particle in MeV. Given:***
***$m_p$ (mass of proton) = 1.007825 amu, $m_n$ (mass of neutron) = 1.008665 amu***
***Mass of He nucleus = 4.002800 amu, 1 amu = 931 MeV.***

**Sol.** An $\alpha$-particle contains 2 protons and 2 neutrons.

Mass of 2 protons = 23 × 1.007825 = 2.015650 amu

Mass of 2 neutrons = 2 × 1.008665 = 2.017330 amu

Total mass = 4.032980 amu

Mass of He nucleus = 4.002800 amu

Mass defect, $\Delta m$ = 0.030180 amu

B.E. of $\alpha$-particle = 0.030180 × 931 = 28.097 MeV.

**Q. 2.** ***Express one atomic mass unit in energy units, first in Joules and then in MeV. Using this, express the mass defect of $^{16}_{8}O$ in MeV.***

**Sol.** We have, $m = 1 \text{ amu} = 1.660565 \times 10^{-27}$ kg

$c = 2.9979 \times 10^{8}\ \text{ms}^{-1}$

$\therefore\quad E = mc^2 = 1.660565 \times 10^{-27} \times (2.9979 \times 10^{8})^2 \text{ J} = 1.4924 \times 10^{-10}$ J

$= \dfrac{1.4924 \times 10^{-10}}{1.602 \times 10^{-13}}$ MeV $\quad [\because 1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}]$ $= 931.5$ MeV

The $^{16}_{8}O$ nucleus contains 8 protons and 8 neutrons.

Mass of 8 protons = 8 × 1.00727 = 8.05816 amu

Mass of 8 neutrons = 8 × 1.00866 = 8.06928 amu

Total mass = 16.12744 amu

Mass of $^{16}_{8}O$ nucleus = 16.99053 amu

Mass defect, $\Delta m$ = 0.13691 amu

$\Delta E_b$ = 0.13691 × 931.5 MeV = 127.5 MeV

**Q. 3.** ***Calculate the binding energy per nucleon of $^{40}_{20}Ca$ nucleus. Given***
***$m(^{40}_{20}Ca)$ = 30.962589 amu, $m_n$ (mass of a neutron) = 1.008665 amu $m_p$ (mass of a proton) = 1.007825 amu***

**Sol.** The nucleus $^{40}_{20}Ca$ contains 20 protons and 20 neutrons.

Mass of 20 protons = 20 × 1.007825 = 20.1565 amu

Mass of 20 neutrons = 20 × 1.008665 = 20.1733 amu

Total mass = 40.3298 amu

Mass $^{40}_{20}Ca$ nucleus = 39.962589 amu

Mass defect, $\Delta m$ = 0.367211 amu

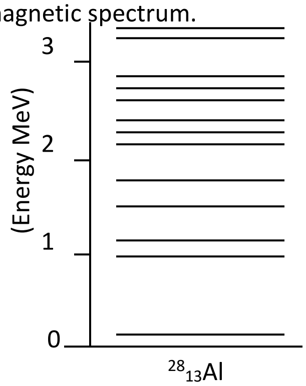
B.E. = 0.367211 × 931 = 341.87 MeV

B.E. per nucleon = $\dfrac{341.87}{40}$ = 8.547 MeV

## □□ NUCLEAR ENERGY LEVELS

The neutrons and protons move inside a nucleus in discrete quantum states with definite energies.

These are nuclear stationary states. The stationary state of lowest energy is called the ground state. If appropriate energy is supplied, a nucleus may be excited from its ground state to the stationary states of higher energy. Fig. shows the energy levels of a low mass nuclide, $^{28}_{13}Al$. These energy levels have energy differences of the order of millions of electronvolt (MeV). So, when a nucleus makes a transition from some higher energy level to a lower energy level, the difference of energy is emitted as a photon in gamma-ray region of the electromagnetic spectrum.




**[Nuclear energy levels $^{28}_{13}Al$ nucleus]**

## □□ NUCLEAR STABILITY AND UNSTABILITY.

Some isotopes of an element may be stable while the others may be unstable.

- A stable nucleus maintains its constitution all the time.
- An unstable nuclide spontaneously emits a particle and transforms itself into a new nuclide.
- The stability of a nuclide is intimately connected to the relative number of neutrons and protons present in that nuclide.

**The neutron-proton graph is called nuclidic chart or Segre chart. This graph reveals the following important features:**

☐**1. The stable nuclides lie on a well-defined narrow band and the unstable nuclides lie above and below this band.**

☐**2. The light stable nuclides tent to lie on the line N = Z. These nuclides have the same numbers of protons and neutrons so that the ratio N/Z = 1.**

☐**3. The ratio N/Z increases for heavier nuclides and becomes 1.6 for heaviest stable nuclides. In heavier nuclei, the proton number becomes large. The proton-proton repulsions are highly effective. So, stability is achieved by having more neutrons than protons because neutrons do not undergo coulombic interactions.**

☐**4. The graph shows that there are no stable nuclei for Z > 83. Thus, the heaviest stable nuclide is $^{209}_{83}Bi$.**

☐**5. The nuclides to the left of the stability region have excess neutrons, while those to the right of the stability region have excess protons. These nuclides are unstable and undergo radioactive disintegration. They are called radioactive nuclides.**

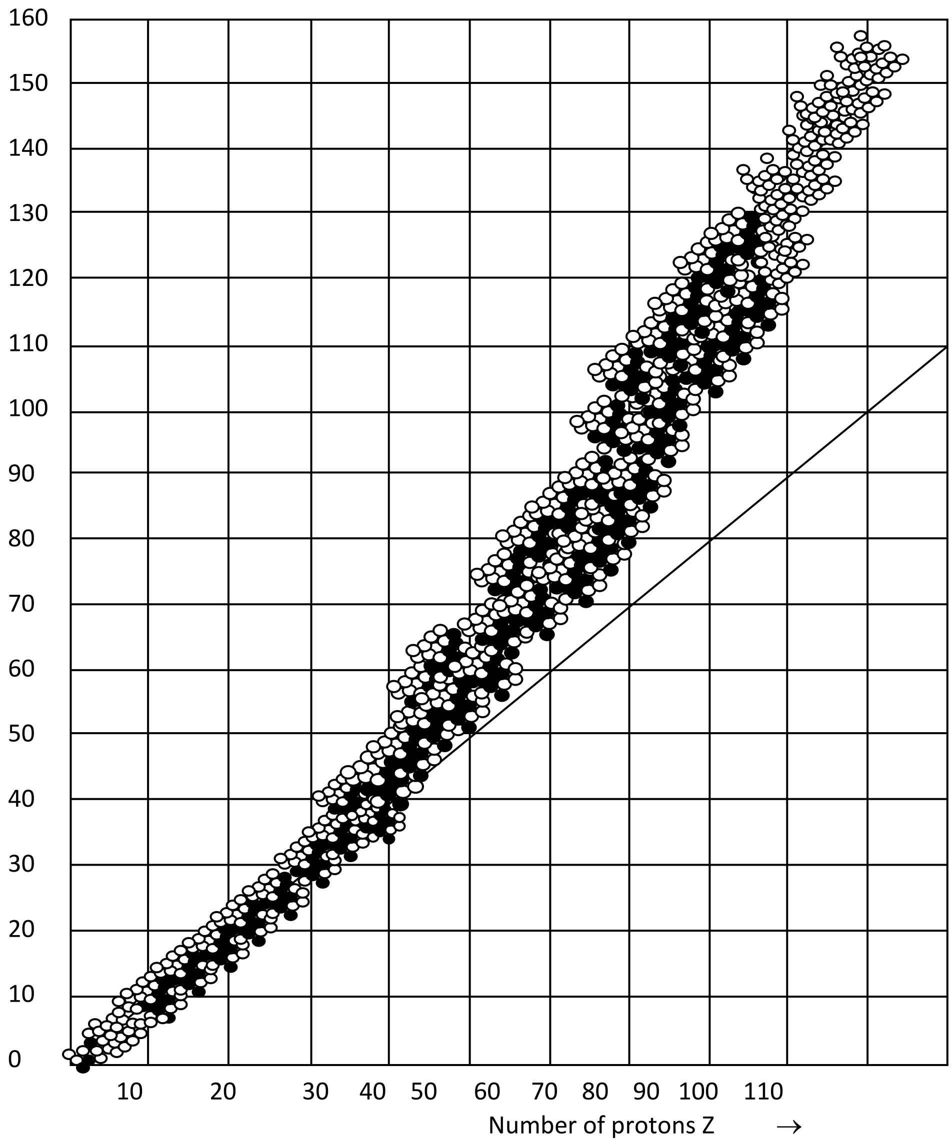
☐**6. The nuclides having even protons and even neutrons are most stable. About 60% of the known nuclides belong to this category.**

☐**7. Only four stable nuclides have both odd Z and odd N:**

**$^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}Bi$, $^{14}_{7}N$**

**These are called odd-odd nuclides. Also, there is no stable nuclide with A = 5 and A = 8.**

Fig. shows the plot of **proton number versus neutron number for the known nuclides**. The black circles represent the stable nuclides while the white circles represent the unstable nuclides.




**[Nuclidic chart]**

Black circles = Stable nuclei
White circles = radioactive nuclei

◆ **To overcome proton-proton coulombic repulsions, heavier nuclei tend to achieve stability by having more neutrons than protons. So, the ratio N/Z increases with A for stable nuclides. But a nucleus with too many neutrons is unstable because not enough of them are paired with protons. This increases the energy and hence, decreases the stability.**

◆ **All known nuclei with atomic numbers ranging from 1 to 117 have isotopes. Some of these have no stable isotopes. Stable isotopes occur for all atomic numbers between Z = 1 (hydrogen) and Z = 83 (bismuth) with the exception of Z = 43 and 61. All the isotopes with Z = 84 to 117 are radioactive. The total number of known isotopes is over 2500. Most of these are radioactive and only 266 isotopes are stable.**

## □□ RADIOACTIVITY

**Radioactivity is the phenomenon of spontaneous disintegration of the nucleus of an atom with the emission of one or more penetrating radiations like α-particles, β-particles or γ-rays.**

A naturally occurring heavy nucleus is unstable. It spontaneously emits a particle, without the stimulus of any outside agency, transforming itself into a different nucleus. Such a nucleus is said to be radioactive and the process of transformation is called radioactive decay. The process is spontaneous in the sense that it occurs by itself. It cannot be initiated, stopped, accelerated or retarded by changing

□**(a) The chemical conditions, or**

□**(b) The physical conditions like temperature, pressure, etc; other than the nuclear bombardment.**

◙ **Discovery:** The phenomenon of radioactivity was discovered accidentally by the French physicist Henry Becquerel in 1896. One day he left some pieces of uranium potassium sulphate wrapped in black paper in a drawer and separated the package from a photographic plate by a piece of silver. When he developed the photographic plate after several hours of exposure, he found, to his surprise, that the plate showed blackening due to some invisible radiations that must have been emitted by the uranium compound and were able to penetrate both the black paper and silver. These radiations were called Becquerel rays. A couple of years later, the husband-and-wife team of Pierre and Marie Curie painstakingly isolated two new elements, radium and polonium. Of these, radium was found to be million times more active than uranium. The substances which spontaneously emit penetrating radiations were called radioactive substances, by the Curie couple. The phenomenon of spontaneous emission of radiations by radioactive substances came to be known as radioactivity. The Nobel Prize for 1903 was shared among the three – Becquerel, Marie Curie and Pierre Curie, for their work on Radioactivity.

**Examples of radioactive substances are**: uranium, polonium, radium, thorium, actinium, etc. It is seen that all naturally occurring elements with atomic number greater than 82 show radioactivity.

## ◆◆ ELECTRICAL NATURE OF BECQUEREL RADIATIONS (**Electrical nature of the radioactive radiations).**

**Rutherford and Villard were the first to analyse the radiation emitted by radium.**

**This radiation was found to consist of three components:**

□**1. A component which could hardly pass through 0.1 cm thick aluminium foil, called α-rays.**

□**2. A component which was stopped by 5 mm thick aluminium sheet, called β-rays.**

□**3. A component which could pass through even 30 cm thickness of an iron piece, called γ-rays.**

**Experimental arrangement**:

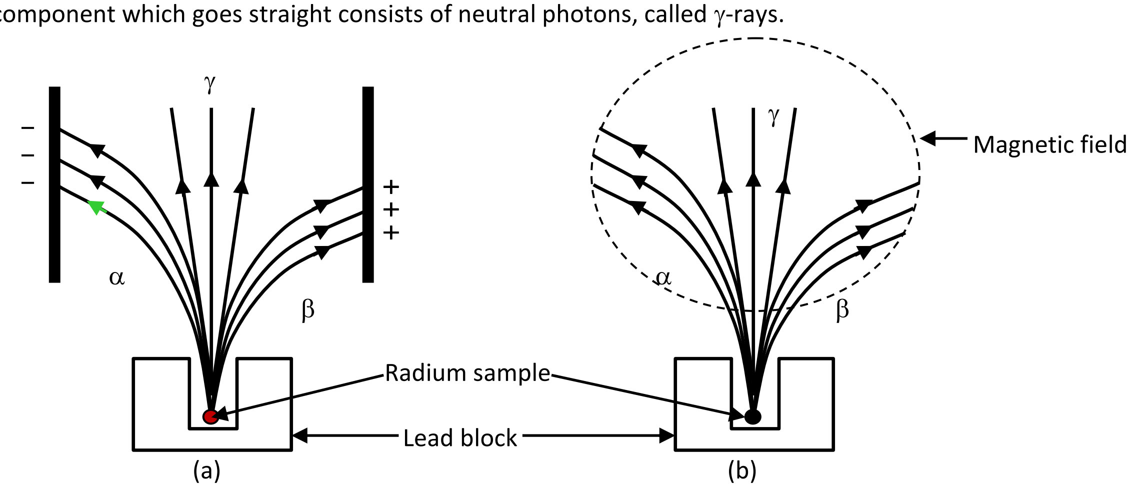
A small hole is drilled in lead block and a piece of radium is placed at its bottom. As the rays entering the walls of the lead block are absorbed before reaching the surface, only a narrow beam of radiation emerges from the hole. The beam is subjected to electric field [Fig. (a)] or magnetic field [Fig (b)].

In both cases, the narrow beam splits into three components:

(i) The component which bends towards the left consists of positively charged particles, called α-rays.

(ii) The components which bends towards right consists of negatively charged particles, called β-rays.

(iii) The component which goes straight consists of neutral photons, called γ-rays.




**[Bending of Becquerel rays in (a) an electric field and (b) a magnetic field]**

## ◆◆◆◆ PROPERTIES OF $\alpha$ –, $\beta$ – AND $\gamma$ – RAYS

### ◆◆Properties of $\alpha$ – rays:

1. These are positively charged particles. These particles have been identified as helium nuclei, i.e., doubly ionised helium atoms.
2. They are deflected by electric and magnetic fields. The directions of deflection indicate that $\alpha$-particles are positively charged particles.
3. Their velocity is of the order of 1/10$^{th}$ of the velocity of light.
4. They excite fluorescence in substances like zinc sulphide and barium platinocyanide.
5. The can affect a photographic plate.
6. They ionise heavily the gases through which they pass. An $\alpha$-particle produces about $10^5$ pairs of ions per cm of its path.
7. They are easily absorbed by matter. They are stopped by an aluminium foil of thickness 0.1 cm or by an ordinary sheet of paper.
8. They are scattered while passing through thin metal.
9. They can cause artificial disintegration of an atom.
10. They produce heating effect when stopped and cause fatal burns on human body.
11. The range of $\alpha$-particle in air, i.e., the distance travelled by $\alpha$-particle through air at S.T.P. before they lose their ionising power, varies from 2.70 cm (for uranium source) to 8.62 cm (for thorium source).

### ◆◆Properties of $\beta$ – rays:

1. They consist of fast-moving electrons of nuclear origin.
2. They are deflected by electric and magnetic fields. The direction of deflection indicates that $\beta$-rays are negatively charged particles.
3. They are emitted with a range of velocities. The maximum velocity depends on the nature of the radioactive source and may be as high as 99% of the speed of light.
4. They excite fluorescence in barium platinocyanide, calcium tungstate, etc.
5. They effect a photographic plate more strongly than $\alpha$-particles.
6. They can ionise a gas but their ionising power is 1/100 times that of $\alpha$-rays.
7. The penetrating power of $\beta$-particles is 100 times that of $\alpha$-particles. They are absorbed by aluminium foil of 5 mm thickness.
8. The range of $\beta$-particles in air is much more than that of $\alpha$-particles.
9. Due to their small masses, $\beta$-particles are easily scattered by atomic nuclei when passed through matter.
10. The emission of a $\beta$-particle is always accompanied by the emission of an elementary particle called neutrino.

### ◆◆Properties of $\gamma$ – rays:

1. They are electromagnetic waves which have wavelength even less than that of X-rays.
2. They are not deflected by electric and magnetic fields, indicating that $\gamma$-particles (photons) do not carry any charge.
3. They travel with the speed of light.
4. They excite fluorescence in certain substances.
5. They affect a photographic plate even more strongly than $\beta$-rays.
6. They ionise gases very slightly. Their ionising power is 1/10000 times that of $\alpha$-rays.
7. Their penetrating power is about 10,000 times that of $\alpha$-rays. They can penetrate a 30 cm thick iron block.
8. Like X-rays, they are diffracted by crystals.
9. They eject $\beta$-particles from substances on which they fall.
10. They show the phenomenon of pair production. When a $\gamma$-ray photon passes close to a nucleus, it gets transformed into an elementary particle and its antiparticle. For example,

$$\underset{\gamma\text{-photon}}{\gamma} \rightarrow \underset{\text{electron}}{{}_0e^-} + \underset{\text{positron}}{{}_0e^+}$$

## ◆◆ COMPARISON BETWEEN THE PROPERTIES OF $\alpha$ – , $\beta$ – AND $\gamma$ – RAYS

| Property | $\alpha$-rays | $\beta$-rays | $\gamma$-rays |
|---|---|---|---|
| 1. Nature | Helium nuclei | Electrons of nuclear origin | High energy |
| 2. Mass | $6.67 \times 10^{-27}$ kg or 4 amu | $9.11 \times 10^{-31}$ kg | Rest mass is zero |
| 3. Charge | + 2e | – e | 0 |
| 4. Deflection by E and B | Deflected towards – ve pole | Deflected towards +ve pole | Nil |
| 5. Speed | $\approx 10^7$ ms$^{-1}$ | $\approx 10^8$ ms$^{-1}$ but variable | $3 \times 10^8$ ms$^{-1}$ |
| 6. Ionising power | $10^4$ times that of $\gamma$-rays | $10^2$ times that of $\gamma$-rays | Minimum |
| 7. Penetrating power | Minimum | $10^2$ times that of $\alpha$-rays | $10^4$ times that of $\gamma$-rays |
| 8. Effect on photo-graphic plate and ZnS phosphor | Strong effect | Less effect | Least effect |

## ◆◆ SODDY-FAJAN'S DISPLACEMENT LAWS

**According to Rutherford-Soddy theory whenever a radioactive disintegration occurs, it does so with the emission of an α or a β-particle. The original nucleus is called parent and the new nucleus formed after disintegration is called daughter**. Rutherford and Soddy used the following **two rules** to infer the nature of daughter nucleus from the parent nucleus and the particle emitted:

1. The algebraic sum of the charges before the disintegration must equal the total electric charge after the disintegration.
2. The sum of the mass numbers of the initial particles must equal the sum of the mass numbers of the final particles.

On the basic of these rules, Soddy and Fajan in 1913, gave simple displacement laws for radioactive transformations, which can be stated as follows:

1. When a radioactive nucleus emits an α-particles, its atomic number decreases by 2 and mass number decreases by 4.

2. When a radioactive nucleus emits a β-particle, its atomic number increases by 1 but mass number remains the same.

3. The emission of a γ-particles does not change the mass number or the atomic number of the radioactive nucleus.

●●●●●●**In addition, the following points about radioactive disintegration may also be noted**:

- ●1. No individual atom can simultaneously emit both α and β-particle.
- ●2. Different atoms of the same element can emit either an α-particle or a β-particle.
- ●3. This emission of a β-particle is usually accompanied by the emission of a γ-rays photon.

Illustrative example: A radioactive nucleus undergoes a series of decays according to the scheme:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, what are these numbers for $A_4$?

**Sol.** The decay processes may be represented as

$$^{180}_{72}A \xrightarrow[\alpha]{^4_2He} {}^{176}_{70}A_1 \xrightarrow[\beta^-]{^0_{-1}e} {}^{176}_{71}A_2$$

$$\xrightarrow[\alpha]{^4_2He} {}^{172}_{69}A_3 \xrightarrow[\gamma]{} {}^{172}_{69}A_4$$

∴ Mass number of $A_4$ = 172 Atomic number of $A_4$ = 69

## □□ RADIOACTIVE DECAY LAW

**According to Rutherford-Soddy theory** ....

**(i)** **The radioactive atoms are unstable and they decay spontaneously to emit α or β-particles along with γ-rays.**
**(ii)** **The disintegration is random. It is purely a matter of chance for any atom to disintegrate first.**
**(iii)** **The disintegration is independent of all physical and chemical conditions and so it can neither be accelerated nor retarded.**

●●●●The above facts show that **it is not possible to predict whether a particular nucleus will decay in a given time interval**.

●●●●By using the concept of probability, the decay behaviour of a collection of a large number of nuclei can be predicted accurately in terms of the radioactive decay law which states:

●●●●**The number of nuclei disintegrating per second of a radioactive sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that instant**.

Expression:

Let $N_0$ = the number of radioactive nuclei present initially at time t = 0 in a sample of radioactive substance.

N = the number of radioactive nuclei present in the sample at any instant t, and

dN = the number of radioactive nuclei which disintegrate in the small-time interval dt.

According to radioactive law, the rate of decay at any instant is proportional to the number of undecayed nuclei, i.e.,

$$-\frac{dN}{dt} \propto N$$

or $$-\frac{dN}{dt} = \lambda N \quad \text{... (1)}$$

where $\lambda$ is a proportionally constant called the decay or disintegration constant. Here the negative sign shows that the number of undecayed nuclei, N decreases with time. The equation (1) can be written as

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating $\int \frac{dN}{B} = -\lambda \int dt$

or $\quad$ $\log_e N = -\lambda t + C$ $\quad$ ... (2)

where C is a constant of integration.

At t = 0, N = $N_0$, therefore from equation (2), we get $\quad \log_e N_0 = C$

Then the equation (2) becomes

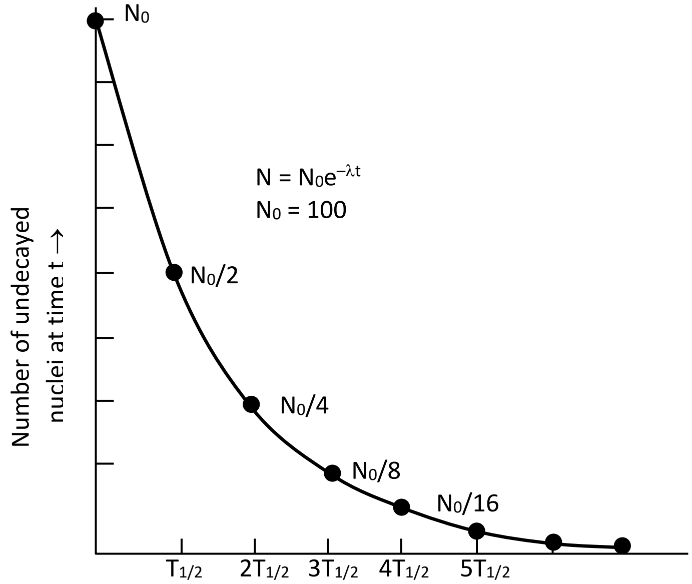
$$\log_e N = -\lambda t + \log_e N_0$$

or $\quad$ $\log_e \frac{N}{N_0} = -\lambda t$

or $\quad$ $\frac{N}{N_0} = e^{-\lambda t}$

or $\quad$ $N = N_0 e^{-\lambda t}$ $\quad$ ... (3)

This equation represents the radioactive decay law. It gives the number of active nuclei left after time t.




**[Decay curve for a radioactive element]**

Fig. shows a graph between the number N of undecayed nuclei and time t.

**●●●●Features**;

**●1. The number of active nuclei in a radioactive sample decrease exponentially with time. The disintegration is fast in the beginning but becomes slower and slower with the passage of time.**

**●2. The larger the value of decay constant $\lambda$, the higher is the rate of disintegration.**

**●3. Irrespective of this nature, a radioactive sample will take infinitely long time to disintegrate completely.**

**Decay of disintegration constant:**

**If in equation (3), $t = \frac{1}{\lambda}$, then**

$$N = N_0 e^{-1} = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368\, N_0$$

**or** $\quad$ $N = \frac{N_0}{e} = 36.8\ \%\ \text{of}\ N_0$ $\quad$ **... (4)**

**The radioactive decay constant may be defined as the reciprocal of the time interval during which the number of active nuclei in a given radioactive sample reduces to 36.8% (or 1/e times) of its initial value.**

**Decay constant may be defined in another way also, we follow:**

**As** $\quad$ $-\frac{dN}{dt} = -\lambda N$

$\therefore$ $\quad$ $\lambda = -\frac{\frac{dN}{dt}}{N}$ $\quad$ **... (5)**

Thus, the **radioactive decay constant may be defined as the ratio of the instantaneous rate of disintegration to the number of active nuclei present in the radioactive sample at the given instant**.

- It gives the probability per unit time for a nucleus of a radioactive substance to decay. The value of $\lambda$ depends on the nature of the radioactive substance.

## HALF-LIFE

**HALF-LIFE : The time interval in which one-half of the radioactive nuclei originally present in radioactive sample disintegrate is called half-life of the radioactive substance. The half-life of a particular radioactive isotope is a characteristic constant of that isotope.**

**It is denoted by $T_{1/2}$.**

### • Relation between half-life and decay constant:

Let $N_0$ = Number of radioactive nuclei present in the radioactive sample initially (at t = 0)

N = Number of radioactive nuclei left at any instant t.

At $t = T_{1/2}$, $N = \frac{N_0}{2}$

Now $N = N_0 e^{-\lambda t}$, where $\lambda$ is the radioactive decay constant.

$$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad \text{or} \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

or, $e^{\lambda T_{1/2}} = 2$

Taking natural logarithm, we get

$$\lambda T_{1/2} \log_e e = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{2.303 \log 2}{\lambda}$$

$$= \frac{2.303 \times 0.3010}{\lambda} \qquad [\because \log_e e = 1]$$

or $$T_{1/2} = \frac{0.693}{\lambda}$$

**Thus, the half-life of a radioactive substance is inversely proportional to its decay constant and is independent of the number $N_0$, the number of radioactive nuclei present initially in the sample.**

**Significance of half-life**: (i) The value of the half-life of a radio isotope gives an ideal of a relative stability of that isotope. An isotope having longer half-life is more stable than the isotope with shorter half-life.

The half-life can be as long as $10^{10}$ years, which is the estimated age of the universe, and can be shorter that $10^{-15}$ s. For example,

$T_{1/2}$ (U – 238) = $4.5 \times 10^9$ years

$T_{1/2}$ (Ra – 226) = 1620 years

$T_{1/2}$ ($R_n$ – 232) = 3.8 days

$T_{1/2}$ (Po – 212) = $3 \times 10^{-7}$ s

The radioactive elements whose half-life is short are not found in observable quantities in nature today. However, they have been seen in nature. Tritium and plutonium belong to this category.

(iii) After one half-life, the number of undecayed nuclei in a given radioactive sample reduces to $N_0/2$, in two half-lives it becomes $N_0/4$, in three half-lives, it becomes $N_0/8$, and so on.

$\therefore$ Number of radioactive nuclei left undecayed after n half-lives

$$= N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

where $t = n \times T_{1/2}$ = total time of n half-lives.

## MEAN LIFE

All the nuclei of a radioactive sample do not disintegrate at the same time. While one nucleus may disintegrate right at the beginning and some other may disintegrate at the end of the process. So, the life time of the different nuclei may vary from zero to infinity.

**The average time for which the nuclei of a radioactive sample exist is called mean life or average life of that sample. It is equal to the ratio of the combined age of all the nuclei to the total number of nuclei present in the given sample. It is denoted by $\tau$.**

$$\text{Mean life} = \frac{\text{Sum of the lives of all the nuclei}}{\text{Total number of nuclei}}$$

☐ **Relation between mean life and decay constant**: Suppose a radioactive sample contains $N_0$ nuclei at time t = 0. After time t, this number reduces to N. Furthermore, suppose dN nuclei disintegrate in time t to t + dt. As dt is small, so the life of each of the dN nuclei can be approximately taken equal to t.

∴ **Total life of dN nuclei = t dN**

**Total life of all the $N_0$ nuclei =** $\int_0^{N_0} t\,dN$

Mean life = $\dfrac{\text{Total life of all the } N_0 \text{ nuclei}}{N_0}$

$$\tau = \frac{1}{N_0}\int_0^{N_0} t\,dn$$

As $N = N_0 e^{-\lambda t}$

∴ $dN = -\lambda N_0 e^{-\lambda t}\,dt$

When $N = N_0$, t = 0 and when N = 0, t = ∞,

Changing the limits of integration in terms of time, we get

$$\tau = \frac{1}{N_0}\int_0^{\infty} t\,\lambda N_0 e^{-\lambda t}\,dt$$

Here we have ignored the negative sign which just tells that N decreases with the passage of time t. Thus

$$\tau = \lambda\int_0^{\infty} t\,e^{-\lambda t}\,dt = \lambda\left\{\left[\frac{te^{-\lambda t}}{-\lambda}\right]_0^{\infty} - \int_0^{\infty}\frac{e^{-\lambda t}\,dt}{-\lambda}\right\}$$

$$= 0 + \frac{\lambda}{\lambda}\int_0^{\infty} e^{-\lambda t}\,dt = \int_0^{\infty} e^{-\lambda t}\,dt = \left(\frac{e^{-\lambda t}}{-\lambda}\right)_0^{\infty}$$

$$= -\frac{1}{\lambda}\left[e^{-\infty} - e^{0}\right] = -\frac{1}{\lambda}[0-1]$$

$$\tau = \frac{1}{\lambda}$$

**Also,** $T_{1/2} = \dfrac{0.693}{\lambda} = 0.693\,\tau$ **or** $\tau = \dfrac{T_{1/2}}{0.693} = 1.44\,T_{1/2}$

## ☐☐ ACTIVITY OF A RADIOACTIVE SUBSTANCE.

☐ **Decay rate or activity of a radioactive sample:** The rate of decay or activity of a sample is defined as the number of radioactive disintegrations taking place per second in the sample.

If a radioactive sample contains N radio nuclei at any time t, then its decay rate or activity R at the same time t will be

$$R = -\frac{dN}{dt}$$

The negative sign shows that the activity of the sample decreases with the passage of time.

According to the radioactive decay law,

$$-\frac{dN}{dt} = \lambda N$$

∴ $R = \lambda N$

As $N = N_0 e^{-\lambda t}$, so we can write

$R = \lambda N_0 e^{-\lambda t}$

or $R = R_0 e^{-\lambda t}$ This is another form of the radioactive decay law. **Here $R_0 = \lambda N_0$, is the decay rate at time t = 0 and R is the decay rate at any subsequent time t. Like N, obviously R also decreases exponentially with time.**

☐ **Units of radioactivity:** The various units of rate of decay or activity of a radioactive substance are as follows:

●•**1. Becquerel (Bq):** This SI unit for activity is Becquerel, named after the discovered of radioactivity, Henry Becquerel. **One becquerel is defined as the decay rate of one disintegration per second.**

●•**2. Curie (Ci):** It is an older practical unit for activity named in honour of Madame Marie Sokolowskis Curie (1867 – 1934). **One curie is the decay rate of $3.7 \times 10^{10}$ disintegrations per second.**

**1 curie = 1 Ci = $3.7 \times 10^{10}$ decays per second= $3.7 \times 10^{10}$ Bq**

●Some other units of activity in common use are

1 m Ci (milli curie) = $3.7 \times 10^7$ Bq

1 μ Ci (micro curie) = $3.7 \times 10^4$ Bq

●●**3. Rutherford (rd):** One Rutherford is the decay rate of $10^6$ disintegrations per second.

1 rd (Rutherford) = $10^6$ decays per second = $10^6$ Bq

1 Ci = $3.7 \times 10^4$ rd

## ●● ALPHA DECAy

**Alpha decay is a process in which an unstable nucleus transforms itself into a new nucleus by emitting an alpha particle (a helium nucleus, ${}^4_2He$).**

Since an α-particle has two protons and two neutrons, so after an α-decay, the parent nucleus is transformed into a daughter nucleus with mass number smaller by 4 and atomic number smaller by 2.

An alpha decay can be expressed by the equation:

$${}^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2He + Q$$

Here Q is the energy released in the process and can be determined from Einstein's mass-energy relation which gives

$Q = [m_X - m_Y - m_{He}]\, c^2$

where $m_X$, $m_Y$ and $m_{He}$ are the masses of the parent nucleus X, daughter nucleus Y and the α-particle respectively. The energy Q is shared by the daughter nucleus X and the α-particle. As the parent nucleus is at rest before its α-decay, the α-particles are emitted with fixed energy. This energy can be determined by applying the laws of conservation of energy and momentum.

For example, uranium-238 on emitting an α-particle changes into thorium-234

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2He + Q$$

Similarly, polonium – 208 is transmuted into lead – 204.

$${}^{208}_{84}PO \rightarrow {}^{204}_{82}Pb + {}^4_2He + Q$$

Generally, the nuclei with mass number 210 or more undergo α-decay. In such nuclei, the long-range repulsive forces between the protons dominate over the short-range nuclear forces which bind the various nucleons together. By emitting α-particles, these nuclei achieve greater stability. An α-particle has a high value of binding energy (≈ 28 MeV). After the emission of an α-particle, the binding energy per nucleon increases and the residual nucleus becomes more stable.

●●**Speed of emitted α-particles:** Consider the alpha decay:

$${}^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2He + Q$$

The speed of the emitted α-particles can be calculated by using the laws of conservation of energy and momentum. Suppose the parent nucleus ${}^A_ZX$ be at rest before decay. Let $v_{He}$ and $v_Y$ be the velocities of α-particle and the daughter nucleus. Applying the law of conservation of momentum, we get

$$m_Y v_Y = m_{He} v_{He} \quad \text{... (1)}$$

As the energy Q released in the decay process appears in the form of kinetic energy of α-particle and the daughter nucleus, so we have

$$\tfrac{1}{2} m_{He} v^2_{He} + \tfrac{1}{2} m_Y v^2_Y = Q$$

Substituting the value of $v_Y$ from Eq. (1), we get

$$\tfrac{1}{2} m_{He} v^2_{He} + \tfrac{1}{2} \frac{m^2_{He} v^2_{He}}{m_Y^2} = Q$$

or $\tfrac{1}{2} m_{He} m_Y v^2_{He} + \tfrac{1}{2} m^2_{He} v^2_{He} = m_Y Q$

or $\tfrac{1}{2} (m_Y + m_{He}) m_{He} v^2_{He} = m_Y Q$

or $K_{He} = \tfrac{1}{2} m_{He} v^2_{He} = \dfrac{m_Y}{m_Y + m_{He}} . Q$

**Now $m_Y = (A - 4)$ amu and $m_{He} \simeq 4$ amu, therefore,**

$$K_{He} = \tfrac{1}{2} m_{He} v^2_{He} \simeq \frac{(A-4)}{A} . Q$$

$$\therefore \quad v_{He} = \sqrt{\frac{2 K_{He}}{m_{He}}} = \sqrt{\frac{2 (A-4) Q}{A\, m_{He}}}$$

For example, in the α-decay of a radon nucleus ${}^{222}_{86}Rn$, we have

$Q = 5.587$ MeV

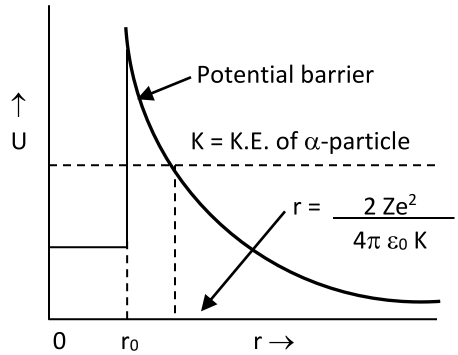
$$\therefore \quad K_{He} = \frac{A-4}{A} Q = \frac{(222-4)}{222} \times 5.587 \text{ MeV}$$

$$= 5.486 \text{ MeV} = 5.486 \times 1.6 \times 10^{-19} \text{ J}$$

$m_{He} = 4$ amu $= 4 \times 1.66 \times 10^{-27}$ kg

Hence $v_{He} = \sqrt{\dfrac{2 \times 5.486 \times 1.6 \times 10^{-19}}{4 \times 1.66 \times 10^{-27}}}\ ms^{-1} = 1.62 \times 10^7\ ms^{-1}$

**Theory of α-decay: Tunnelling effect:** The α-particles emitted by different radioactive nuclei have kinetic energy ranging from 4 to 9 MeV. The nucleus of an α-emitter poses a barrier of height about 25 MeV. Fig. shows a plot of the potential energy U of the system consisting of a α-particle and the residual nucleus. The α-particles are short of about 16 to 25 MeV of energy, needed for the emission. Therefore, classically, we cannot explain the emission of α-particles by radioactive nuclei.




**[Plot of potential energy U of an α-particle as a function of distance r from the centre of the residue nucleus]**

In 1928, Gamow, Congdon and Gurney explained the emission of α-particles in terms of the penetration of the nuclear potential barrier on the basis of quantum theory. **According to this theory:**

- **1. An α-particle may exist as an entity (already formed) inside a nucleus before it escapes from the nucleus.**
- **2. The α-particle is in a state of constant motion inside the nucleus with a speed of about $10^7$ ms$^{-1}$.**
- **3. Quantum mechanically, an α-particles of even insufficient kinetic energy has a small but finite probability p of its crossing the potential barrier.**

As the size of the nucleus ≈ $10^{-14}$ m and speed of α-particle ≈ $10^7$ ms$^{-1}$, the α-particle takes about $10^{-21}$ s to move across the nucleus. Thus α-particle presents itself before the potential barrier $10^{21}$ times in a second. The probability of escape of an α-particle from a nucleus will be $P = p \times v$

As v is large ($10^{21}$ s$^{-1}$), so P is sufficiently large and the α-particle can tunnel through the energy α-decay occurs as a result of barrier tunnelling.

The barrier tunnelling explains why every ${}^{238}_{92}U$ nuclide in a sample of ${}^{238}_{92}U$ atoms does not decay at once, even when its decay process has a positive Q value. Consequently, the half-lives for α-decay of most of the alpha unstable nuclei are very long. For example, the half-life of ${}^{238}_{92}U$ for α-decay is $4.5 \times 10^9$ years.

## BETA DECAY

**Beta decay: The process of spontaneous emission of an electron ($e^-$) or a positron ($e^+$) from a nucleus is called β-decay**.

Like α-decay, β-decay is a spontaneous process, with a definite disintegration energy and half-life. It is also a statistical process, obeyed the law of radioactive decay.

**In beta minus (β⁻) decay, the mass number of the radioactive nucleus remains unchanged but its atomic number increases by one. An electron and a new particle antineutrino ($\bar{v}$) are emitted from the nucleus, as in the decay:**

$${}^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \overline{v} \qquad [T_{1/2} = 2.6 \text{ days}]$$

In general, the beta minus decay may be represented as

$${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + {}^{0}_{-1}e + \overline{v}$$

or

$${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + \beta^- + \overline{v}$$

**The electron emitted from the nucleus is called a beta particle, denoted by β⁻.**

**In beta plus (β⁺) decay, the mass number of the radioactive nucleus remains unchanged but its atomic number decrease by one. A position ($e^+$) and a new particle neutrino (v) are emitted from the nucleus, as in the decay**:

$${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + v \qquad [T_{1/2} = 2.6 \text{ years}]$$

In general, the beta plus decay may be represented as

$${}^{A}_{Z}X \rightarrow {}^{A}{}_{Z-1}Y + {}^{0}_{+1}e + v$$

or

$${}^{A}_{Z}X \rightarrow {}^{A}{}_{Z-1}Y + \beta^+ + v$$

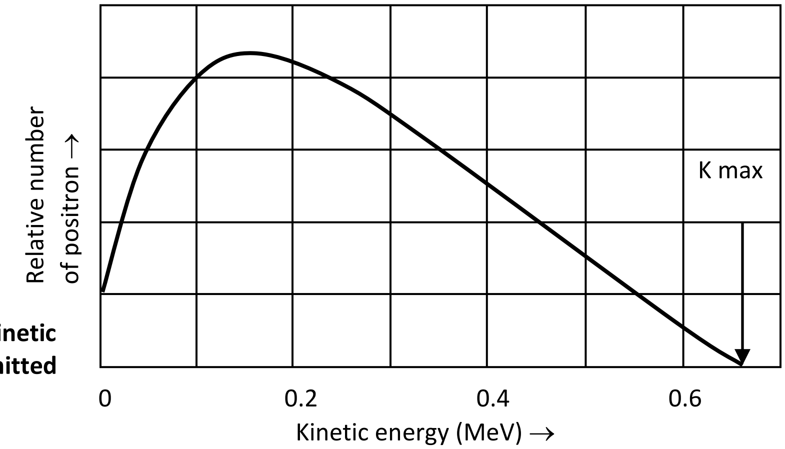
**The positron so emitted is called a beta plus particle (β⁺).** The positron is an antiparticle of electron. It has a positive charge equal in magnitude to the of an electron. Similarly, neutrino and antineutrino are antiparticles of each other. Both are massless, chargeless particles having spins ± ½.

Although a nucleus contains no electrons, positrons and neutrinos, yet can it eject these particles. It is believed that electrons, positrons and neutrinos are created during the process of beta decay.

- If the unstable nucleus has excess neutrons that needed for stability, neutron converts itself into a proton. So, in a beta-minus decay, an electron and an antineutrino are created and emitted from the nucleus via the reaction: $n \rightarrow p + e^- + \overline{v}$
- If the unstable nucleus has excess protons than that needed for stability, a proton converts itself into a neutron. So, in a beta-plus decay, a positron and a neutrino are created and emitted from the nucleus via the reaction: $p \rightarrow n + e^+ + v$

●● **A beta decay process involves the conversion of a neutron into a proton or vice versa. These nucleons have nearly equal masses. That is why the mass number A of a nuclide undergoing beta decay does not change.**

**Continuous energy spectrum for beta rays:** In both α-and β-decays, the disintegration energy Q depends on the nature of the radionuclide. In the α-decay of a particular radionuclide, every emitted α-particle has a definite amount of kinetic energy.




**[The distribution of the kinetic Energies of positrons emitted in the decay of $^{64}_{29}Cu$]**

However, in β-decay, the disintegration energy is shared in all proportions between the three particles: daughter nucleus, electron (or positron) and antineutrino (or neutrino). As a result, the kinetic energy of the electrons (or positrons) is not fixed. Their energy varies from zero to a maximum value $K_{max}$. Thus β-rays have a continuous energy spectrum, as shown in Fig. The maximum kinetic energy or end point energy $K_{max}$ must be equal to disintegration energy Q. When the electron (or positron) has maximum energy, the energy carried by the daughter nucleus and neutrino is nearly zero.

.........................................

◆ **Pauli's neutrino hypothesis:** In a given β-decay reaction, the energy of the electrons is expected to be fixed one. For example, consider the β-decay of $^{210}_{83}Bi$ into $^{210}_{84}Po$:

$$^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + {}^{0}_{-1}e$$

In the above reaction, the electron is expected to come out with a fixed energy of 1.17 MeV because here electron is the only particle that comes out of the nucleus and it should carry whole of the disintegration energy. However, experiments showed that the energy of the emitted electrons varies from zero to a maximum value of 1.17 MeV. Moreover, an electron has spin equal to 1/2, so, to conserve angular momentum, the spin of radio nucleus must change by 1/2. But in actual practice, there is either no change or the spin changes by an integral value.

Thus, the laws of conservation of energy and angular momentum were not found to be obeyed in β-decay. To remove this discrepancy, Pauli in 1930 suggested that an uncharged particle of zero rest mass and spin 1/2 is emitted along with the electron. This particle was named antineutrino (v). The antineutrino can carry away different amounts of energy, leaving the electrons with different energies. Hence the energy distribution of β-rays is continuous.

◆ **Since neutrinos (or antineutrinos) are massless and chargeless, they interact so weakly with matter that it becomes very difficult to detect them. They can penetrate through earth without being absorbed. By ingenious experiments, neutrinos have been detected and their mass and spin of intrinsic angular momentum have been measured**.

◆ **Electrons capture:** Some proton rich nuclei capture one of the atomic electrons (usually from the K shell). A proton in the nucleus combines this electron forming a neutron. A neutrino created in the process is emitted from the nucleus.

$$p + e^- \rightarrow n + v$$

The entire process may be represented as

$$^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + v$$

This process is called electron capture or K-capture. The vacancy created in the K shell is filled by transition of electrons from the outer shells. This results in the emission of characteristic X-rays.

## ●● GAMMA DECAY

**The process of emission of a γ-ray photon during the radioactive disintegration of a nucleus is called gamma decay.**

●As the emitted γ-ray photons have zero rest mass and carry no charge, so in a γ-decay the mass number and atomic number of the nucleus remain unchanged and no new element is formed. A γ-decay can be expressed as

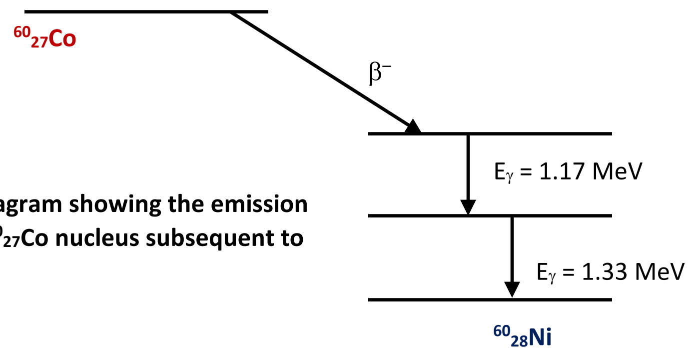
$$^{A}_{Z}X \text{ (Excited state)} \rightarrow {}^{A}_{Z}X \text{ (Ground state)} + \gamma$$

●● A nucleus does not contain photons, yet it can emit photons. These photons are created during the emission process.

**Explanation**: We know that a nucleus can exist in different energy states. After an α or a β-decay, the daughter nucleus is usually left in an excited state. It attains the ground state by single or successive transitions by emitting one or more photons.

As the nuclear states have energies of the order of MeV, therefore, the photos emitted by the nuclei have energy of the order of several MeV. The wavelength of such high energy photons is a fraction of an angstrom. The short wavelength electromagnetic waves emitted by nuclei are called $\gamma$-rays.

An example of $\gamma$-decay is shown through an energy level diagram in Fig. Here an unstable ${}^{60}_{27}Co$ nucleus is transformed via a $\beta$-decay into an excited ${}^{60}_{28}Ni$ nucleus, which in turn reaches the stable ground state by emitting photons of energies 1.17 MeV and 1.33 MeV, in two successive $\gamma$-decay processes.




**[Energy-level diagram showing the emission of $\gamma$-rays by a ${}^{60}_{27}Co$ nucleus subsequent to beta decay]**

Usually, $\gamma$-rays are emitted after $\alpha$- or $\beta$-decay, but there are long lived radioactive nuclei that emit only $\gamma$-rays.

## Examples based on RADIOACTIVITY

◆ Formulae used

1. Displacement laws for radioactive transformations are as follows:

(i) $\alpha$-decay: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

(ii) $\beta$-decay: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{v}$

(iii) $\gamma$-decay: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma$

(Excited state) (Ground state)

2. Radioactive decay law:

(i) $-dN = \lambda N$ (ii) $N = N_0 e^{-\lambda t}$

Where $\lambda$ = decay constant or disintegration constant

3. Half-life: $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$

4. $N = N_0 \left(\frac{1}{2}\right)^n$, where $h = \frac{t}{T_{1/2}}$

5. Mean life: $\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}$

or $T_{1/2} = 0.693\,\tau$

6. Decay rate or activity of a substance:

$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$

7. Time required to reduce the radioactive substance.

$t = \frac{2.303}{\lambda} \log \frac{N_0}{N}$

8. Decay constant: $\lambda = \frac{2.303}{t} \log \frac{N_0}{N}$

◆ Units used

Times t, $T_{1/2}$ and $\tau$ are in second, decay constant $\lambda$ in $s^{-1}$, decay rate in curie or Rutherford.

1 Ci (curie) = $3.70 \times 10^{10}$ disintegration/s

1 rd (Rutherford) = $10^6$ disintegrations/s

**Q. 1.** ***How many $\alpha$ – and $\beta^-$ – particles will be emitted when ${}^{232}_{90}Th$ changes into ${}^{208}_{82}Pb$?***

**Sol.** ${}^{232}_{90}Th \rightarrow {}^{208}_{82}Pb$

Decrease in mass number = 232 – 208 = 24

Number of $\alpha$ – particles emitted due to the above decreases in mass number = $\frac{24}{4} = 6$

Expected decreases in atomic number due to the emission of 6 $\alpha$-particles = $6 \times 2 = 12$

Expected atomic number of the nucleus formed = 90 – 12 = 78

But the atomic number of the nucleus formed = 82

Increase in atomic number = 82 – 78 = 4 Number of $\beta^-$ – particles emitted = 4

Thus 6 $\alpha$ – particles and 4 $\beta^-$ -particles are emitted when ${}^{323}{}_{90}$ The changes into ${}^{208}{}_{82}$ Pb.

**Q. 2.** ***Half-life of a certain radioactive material against $\alpha$-decay is 138 days. After what lapse of time the undecayed fraction of the material will be 6.25%.***

**Sol.** The half-life of polonium is 38 days.

$\therefore$ Amount of undecayed Po left after 138 days = 50 % of initial amount

Amount of undecayed Po left after next 138 days = 25 % of initial amount

Amount of undecayed Po left after next 138 days = 12.5 % of initial amount

Amount of undecayed Po left after next 138 days = 6.25 % of initial amount ∴ Total time lapsed = $138 \times 4 = 552$ days.

**Q. 3.** ***The half-life of radium is 1600 years. After how many years 25% of a radium block remains undecayed?***

**Sol.** Here $N = 25\%$ of $N_0 = \frac{N_0}{4}$

As $N = N_0\left(\frac{1}{2}\right)^n$ ∴ $\frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^n$

or $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$ ∴ $n = 2$

Time of disintegration = Half-life × Number of half-lives = $1600 \times 2 = 3200$ Years

**Q. 4.** ***Find the half-life of a radioactive material if its activity drops to 1/16th of its initial value in 30 years.***

**Sol.** As activity ∝ No. of atoms present

∴ $N = \frac{N_0}{16}$

But $N = N_0\left(\frac{1}{2}\right)^n$

Where n is the number of half lives

∴ $\frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^n$ or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$ or $n = 4$

Half-life period $= \frac{\text{Time of disintegration}}{\text{No. of half-lives}} = \frac{30}{4} = 7.5$ years

**Q. 5.** ***The half-life, of a given radioactive nuclide, is 138.6 days. What is the mean life is this nuclide? After how much time will a given sample of this radioactive nuclide get reduced to only 12.5 % of its initial value?***

**Sol.** Here $T_{1/2} = 138.6$ days

Mean life, $\tau$ $1.44\, T_{1/2} = 1.44 \times 138.6 = 199.58$ days.

Given $\frac{N}{N_0} = 12.5\,\% = \frac{12.5}{100} = \frac{1}{8}$

But $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ ∴ $\frac{1}{8} = \left(\frac{1}{2}\right)^n$

or $\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$

∴ Number of half-lives, $n = 3$ Time taken = Half-life × Number of half-lives = $138.6 \times 3 = 415.8$ days.

**Q. 6.** ***(a). It is observed that only 6.25 % of a given radioactive sample is left undecayed after a period of 16 days. What is the decay constant of this sample, in day⁻¹?***

**Sol.** Here $\frac{N}{N_0} = 6.25\,\%$

or $\left(\frac{1}{2}\right)^n = \frac{6.25}{100} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$ ∴ $n = 4$

$T_{1/2} = \frac{t}{n} = \frac{16 \text{ days}}{4} = 4$ days

Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4} = 0.173\ \text{day}^{-1}$.

**Q. 7.** ***(b) The decay constant, for a given radioactive sample, is 0.3465 day⁻¹. What percentage of this sample will get decayed in a period of 4 years?***

**Sol.** Here $\lambda = 0.3456\ \text{day}^{-1}$, $t = 4$ years

$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = 2$ days

∴ $n = \frac{t}{T_{1/2}} = \frac{4}{2} = 2$

Hence sample left undecayed after a period of 4 years,

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\,\%$

**Q. 8.** ***A sample contains $10^{-2}$ kg each of the two substances A and B with half-lives 4 sec and 8 sec respectively. Their atomic weights are in the ratio of 1: 2. Find the amounts of A and B after an interval of 16 second.***

**Sol.** Let $N_0$ be the initial amount of a radioactive substance. Then the amount left after n half-lives will be

$N = N_0\left(\frac{1}{2}\right)^n$

For A: $n = \frac{t}{T_{1/2}} = \frac{16\text{ s}}{4\text{ s}} = 4$

$\therefore \quad N_A = 10^{-2}\text{ kg}\left(\frac{1}{2}\right)^4 = 6.25 \times 10^{-4}\text{ kg}$

For B: $n = \frac{16\text{ s}}{8\text{ s}} = 2$

$\therefore \quad N_B = 10^{-2}\text{ kg}\left(\frac{1}{2}\right)^2 = 2.5 \times 10^{-3}\text{ kg}$

**Q. 9.** ***A radioactive material is reduced to 1/16 of its original amount in 4 days. How much material should one begin with so that $4 \times 10^{-3}$ kg of the material is left after 6 days.***

**Sol.** As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \quad \therefore \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$

or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$ or $n = 4$

$T_{1/2} = \frac{t}{n} = \frac{4\text{ days}}{4} = 1\text{ day}$

Now, $N = 4 \times 10^{-3}$ kg, $t = 6$ days or $n = 6 \quad \therefore \quad N_0 = N(2)^n = 4 \times 10^{-3} \times 2^6 = 0.256$ kg.

**Q. 10.** ***The half-life of radium is 1500 years. After how many years will one gram of the pure radium (i) reduce to one centigram? (ii) lose one milligram?***

**Sol.** Half-life, $T_{1/2} = 1500$ years; Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1500}\text{ year}^{-1}$

(i) Initial amount, $N_0 = 1$ g

Remaining amount, $N = 1$ centigram $= 0.01$ g

$\therefore$ Required time,

$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303 \times 1500}{0.693}\log\frac{1}{0.01}$ years

$= \frac{2.303 \times 1500 \times 2}{0.693}$ years

$= 9.972 \times 10^3$ years.

(ii) Initial amount, $N_0 = 1$ g

Remaining amount, $N = 1 - 10^{-3} = 0.999$ g

$\therefore$ Required time,

$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303 \times 1500}{0.693}\log\frac{1}{0.999}$ years

$= \frac{2.303 \times 1500 \times 0.0004}{0.693}$ years

$= 1.995$ years.

**Q. 11.** ***The decay constant, for a radionuclide, has a value of 1.386 day$^{-1}$. After how much time will a given sample of this radionuclide get reduced to only 6.25 % of its present number?***

**Sol.** Here $\lambda = 1.386\text{ day}^{-1}$

$N = 6.25\%$ of $N_0 = \frac{6.25}{100}N_0$

As $N = N_0 e^{-\lambda t} \quad \therefore \quad \frac{6.25}{100}N_0 = N_0 e^{-\lambda t}$ or, $e^{\lambda t} = 16$

Taking natural logarithms of both sides, we get, $\lambda t \log_e e = \log_e 16$ or, $\lambda t = 2.303 \log 16 = 2.303 \times 4 \log 2$

or $t = \frac{2.303 \times 4 \times 0.3010}{1.386} = 2$ days

**Q. 12.** ***The disintegration rate of a certain radio-active sample at any instant is 4750 disintegrations per minute. 5 minutes after, the rate becomes 2700 and (ii) half-life of the sample ($\log_{10} 1.760 = 0.2455$).***

**Sol.** As the rate of disintegration is proportional to the number of atoms, so

$\frac{N_0}{N} = \frac{4750}{2700} = 1.76$

As $N = N_0 e^{-t}$

$\therefore \quad \lambda = \frac{2.3026}{t}\log_{10} = \frac{N_0}{N} = \frac{2.3026}{5}\log_{10}(1.76) = \frac{2.3026}{5} \times 0.2455 = 0.1131\text{ min}^{-1}$

(ii) Half-life $= \frac{0.6931}{\lambda} = \frac{0.6931}{0.1131} = 6.13$ minutes

**Q. 13.** ***At a given instant there are 25 % undecayed radio-active nuclei in a sample. After 10 seconds the number of undecayed nuclei reduced to 12.5 %. Calculate (i) mean-life of the nuclei and (ii) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number.***

**Sol.** As the number of undecayed nuclei decreases from 25 % to 12.5 % in 10s, it shows that the half-life of the sample is 10 s i.e., $T_{1/2} = 10$ s Decay constant, $\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{10\text{s}}$

Mean-life, $\tau = \frac{1}{\lambda} = \frac{10\text{ s}}{0.6931} = 14.43$ s

(ii) Further reduction to 6.25 % of reduced number implies becoming $\left(\frac{1}{2}\right)^4$ times i.e., number changes from $\frac{N_0}{8}$ to $\frac{1}{16}\left(\frac{N_0}{8}\right)$.

This process will take a time of 4 half-lives. $\therefore$ Required time $= 10 \times 4 = 40$ s

**Q. 14.** ***The mean lives of a radio-active substance are 1620 years and 405 years for $\alpha$-emission and $\beta$-emission respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by $\alpha$-emission and $\beta$-emission simultaneously. (Loge 4 = 1.386).***

**Sol.** Here $\tau\alpha$ = 1620 yrs. $\tau\beta$ = 405 yr,

$$N = N_0 - \frac{3}{4}N_0 = \frac{N_0}{4},\ t = ?$$

If $\lambda_\alpha$ and $\lambda_\beta$ are the decay constants for $\alpha$-and $\gamma$-emission respectively, then

$$\lambda_\alpha = \frac{1}{\tau_\alpha} = \frac{1}{1620}\ yr^{-1}$$

and $$\lambda_\beta = \frac{1}{\tau_\beta} = \frac{1}{405}\ yr^{-1}$$

Total decay constant, $\lambda = \lambda_\alpha + \lambda_\beta$

$$= \frac{1}{1620} + \frac{1}{405} = \frac{1+4}{1620} = \frac{1}{324}\ yr^{-1}$$

As $N = N_0 e^{-\lambda t}$

$\therefore$ $\frac{1}{4}N_0 = N_0 e^{-\lambda t}$

or $$t = \frac{\log_e 4}{\lambda} = \frac{2.3026 \log 10^4}{1/324} = 449.1\ yr$$

**Q. 15.** ***The half-life of $^{238}_{92}U$ against $\alpha$-decay is $1.5 \times 10^{17}$ s. What is the activity of a sample of $^{238}_{92}U$ having $25 \times 10^{20}$ atoms?***

**Sol.** Here $T_{1/2} = 1.5 \times 10^{17}$ s.

$N = 25 \times 10^{20}$ atoms

$\therefore$ $$R = \lambda N = \frac{0.693}{T_{1/2}} \times N = \frac{0.693 \times 25 \times 10^{20}}{1.5 \times 10^{17}} = 11550 \text{ disintegrations/second}$$

**Q. 16.** ***The half-life of $^{238}_{92}U$ against $\alpha$-decay is $4.5 \times 10^9$ years. Calculate the activity of 1 g sample of $^{238}_{92}U$.***

**Sol.** Here $T_{1/2} = 4.5 \times 10^9$ years $= 4.5 \times 10^9 \times 3.156 \times 10^7$ s,

m = 1 g, M = 238

Number of atoms in 1 g uranium,

$$N = \frac{m}{M} \times \text{Avogadro's number}$$

$$= \frac{1 \times 6.023 \times 10^{23}}{238} \text{ atoms}$$

Activity of the sample, $R = \lambda N = \frac{0.693}{T_{1/2}}.N$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{4.5 \times 3.156 \times 10^{16} \times 238}\ s^{-1} = 1.235 \times 10^4\ Bq.$$

**Q. 17.** ***A radioactive isotope has a half-life of 5 years. How long will it take the activity to reduced to 3.125 %?***

**Sol.** Since activity is proportional to the number of radioactive atoms, therefore

$$\frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

But $$\frac{R}{R_0} = 3.125\ \% = \frac{3.125}{100} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

$\therefore$ $\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5$ or n = 5

Required time, $t = nT_{1/2} = 5 \times 5$ years = 25 years.

**Q. 18.** ***A radioactive sample contains 2.2 mg of pure $^{11}_{6}C$ which has half-life period of 1224 seconds. Calculate: (i) the number of atoms present initially. (ii) the activity when 5 $\mu$g of the sample will be left.***

**Sol.** (i) Number of atoms present in 11 g of the sample $= 6.023 \times 10^{23}$

$\therefore$ Number of atoms present in 2.2 mg of the sample

$$= \frac{6.023 \times 10^{23} \times 2.2 \times 10^{-3}}{11} = 1.2 \times 10^{20} \text{ atoms} = \text{Number of atoms present initially.}$$

(ii) Number of atoms present in 5 $\mu$g of the sample

$$= \frac{6.023 \times 10^{23} \times 5 \times 10^{-6}}{11} = 2.74 \times 10^{17} \text{ atoms}$$

Activity of the sample, $R = \lambda N = \frac{0.693}{T_{1/2}} \times N$

$$= \frac{0.693 \times 2.74 \times 10^{17}}{1224} = 1.55 \times 10^{14} \text{ disintegrations/second.}$$

**Q. 19.** *In an experiment, the activity of 1.2 milligrams of radioactive potassium chloride (chloride of isotope K – 40) was found to be 170 $s^{-1}$. Taking molar mass of K – 40 Cl to be 0.075 kg $mole^{-1}$, find the number of K = 40 atoms in the same and hence find the half-life of K – 40. Avogadro's' number = 6.0 × $10^{23}$ $mole^{-1}$.*

**Sol.** Molar mass of K – 40 Cl,

$M = 0.075 \text{ kg mol}^{-1} = 75 \text{ mol}^{-1}$

Number of molecules present in 1.2 mg of potassium chloride

$N = \frac{m}{M} \times$ Avogadro's number

$= \frac{1.2 \times 10^{-3} \times 6.0 \times 10^{23}}{75} = 9.6 \times 10^{18}$

Given $R = 170 \text{ s}^{-1}$

But $R = \lambda N = \frac{0.693}{T_{1/2}} . N$ ∴ $T_{1/2} = \frac{0.693 N}{R} = \frac{0.693 \times 9.6 \times 10^{18}}{170}$ $= 3.91 \times 10^{16}$ s.

**Q. 20.** *There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 km? Mass of neutron = 1.675 × $10^{-27}$ kg.*

**Sol.** As kinetic energy,

$E_K = ½ m_n v^2$

$v = \sqrt{\frac{2 E_k}{m_n}}$

$= \sqrt{\frac{2 \times 0.327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}}$ $= 2.5 \times 10^3 \text{ ms}^{-1}$

∴ Time taken to traverse a distance of 10 km or $10^4$ m $= \frac{10^4}{2.5 \times 10^3} = 4$ s

Number of half-lives in 4 s $= \frac{4}{700} = \frac{1}{175}$

Now $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{1/175} = 0.996$

∴ Fraction of neutrons decayed $= 1 - \frac{N}{N_0} = 1 - 0.996$ $= 0.004$

**Q. 21.** *Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days the room will be safe for occupation?*

**Sol.** Let t be the time required to reach the permissible level. This means that the activity will drop to 1/50 of its present value time t, i.e., $\frac{R}{R_0} = \frac{1}{50}$

But $R = N\lambda$ and $R_0 = N_0 \lambda$

Hence, $\frac{N}{N_0} = \frac{1}{50}$

As $N = N_0 e^{-\lambda t}$

∴ $\lambda t = \log_e N_0 = \log_e 50$

or $t = \frac{\log_e 50}{\lambda}$

But $\lambda = \frac{\log_e 2}{T_{1/2}}$

∴ $t = T_{1/2} = \frac{\log_e 50}{\text{Log}_e 2} = \frac{T_{1/2} \log_{10} 50}{\text{Log}_{10} 2}$

$= 10 \text{ days} \times \frac{1.6990}{0.3010}$

$= 56.45$ days

**Q. 22.** *A small quantity of solution containing $Na^{24}$ radio nuclide (half-life 15 hours) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 $cm^3$ taken after 5 hours shows an activity of 296 disintegrations per minute. Determine the total volume of blood in the body of the person. (1 curie = 3.7 × $10^{10}$ disintegrations per second.)*

**Sol.** Here $n = \frac{t}{T_{1/2}} = \frac{5}{15} = \frac{1}{3}$

$R_0 = 1.0 \mu \text{ Ci} = 10^{-6} \text{ Ci} = 3.7 \times 10^4$ disintegrations $s^{-1}$

As $N = N_0 \left(\frac{1}{2}\right)^n$ and $R \propto N$

∴ $R = R_0 \left(\frac{1}{2}\right)^n = 3.7 \times 10^4 \left(\frac{1}{2}\right)^{1/3}$

$= 2.94 \times 10^4$ disintegrations $s^{-1}$.

This is the activity of blood after 5 hours.

As 296 disintegrations per sec occur in 1 $cm^3$ of blood, so the volume of the blood showing $2.94 \times 10^4$ disintegrations/sec

$$= (1\ cm^3) \times \frac{60}{296} \times (2.94 \times 10^4) = 5.96 \times 10^3\ cm^3 \simeq 6 \text{ litres}$$

**Q. 23.** ***We are given the following atomic masses:***

***$^{238}_{92}U = 238.05079$ amu $\quad ^{4}_{2}He = 4.00260$ amu***

***$^{234}_{90}Th = 234.04363$ amu $\quad ^{1}_{1}H = 1.00783$ amu $\quad ^{237}_{91}Pa = 237.03121$ amu***

***Here the symbol Pa is for the element protactinium (Z = 91)***

***(a) Calculate the energy released during the $\alpha$-decay of $^{237}_{92}U$. (b) Calculate the kinetic energy of the emitted $\alpha$-particles.***

***(c) Show that $^{238}_{92}U$ cannot spontaneously emit a proton.***

***Sol.*** (a) The equation for the $\alpha$-decay of $^{238}_{92}U$ may be written as

$$^{238}_{92}U \rightarrow \ ^{234}_{90}Th + \ ^{4}_{2}He + Q$$

where Q (called Q-value) represents kinetic energy. Using Einstein's mass-energy equivalence, we get

$$Q = [m_N(^{238}_{92}U) - m_N(^{234}_{90}Th) - m_N(^{4}_{2}He)]\ c^2$$

By adding and subtracting 92 $m_e$ in the bracket, we can write Q in terms of atomic masses as follows:

$$Q = [\{mN(^{238}_{92}U) + 92m_e\} - \{m_N(23490Th) + 90\ m_e\} - \{m_N(^{4}_{2}He) + 2m_e\}]\ c^2$$

$$= [238.05079 - (234.04363 + 4.00260)] \times 931.5 \text{ MeV} \qquad [\because c^2 = 931.5 \text{ MeV/amu}]$$

$$= 0.00456 \times 931.5 \text{ MeV} = 4.25 \text{ MeV}.$$

(b) The kinetic energy of the $\alpha$-particle,

$$E_\alpha = \left(\frac{A-4}{A}\right) Q = \frac{238-4}{238} \times 4.25 \text{ MeV} = 4.18 \text{ MeV}$$

(c) If $^{238}_{92}U$ spontaneously emits a proton, the decay process would be

$$^{238}_{92}U \rightarrow \ ^{237}_{91}Pa + \ ^{1}_{1}H$$

For this process to happen, the Q-value will be

$$Q = [m(^{238}_{92}U) - m(^{237}_{91}Pa) - m(^{1}_{1}H)]c^2 = [238.05079] - 237.05121 - 1.00783]\text{ amu} \times c^2 = -7.68 \text{ MeV}$$

As the Q-value of the process is negative, it cannot occur spontaneously.

## ●● NATURAL AND ARTIFICIAL RADIOACTIVITY

**Natural radioactivity:** The phenomenon of the spontaneous emission of $\alpha$, $\beta$ or $\gamma$-radiations from the nuclei of naturally occurring isotopes is called natural radioactivity.

**Artificial or induced radioactivity:** The phenomenon of inducing radioactivity in certain stable nuclei by bombarding them by suitable high energy particles is called artificial or induced radioactivity.

**Example of induced radioactivity:**

1. When boron is bombarded by $\alpha$-particles, it forms a radioactive isotope of nitrogen which decays into carbon with a half-life of 10.1 minutes.

$$^{10}_{5}B + \ ^{4}_{2}He \rightarrow \ ^{13}_{7}N + \ ^{1}_{0}n$$

$$^{13}_{7}N \rightarrow \ ^{13}_{6}C + \ ^{0}_{1e} + v$$

2. When aluminium is bombarded by $\alpha$-particles, it produces radioactive phosphorus which decays into silicon with a half life of 2.55 minutes.

$$^{27}_{13}Al + \ ^{4}_{2}He \rightarrow \ ^{30}_{15}P + \ ^{1}_{0}n$$

$$^{30}_{15}P \rightarrow \ ^{30}_{14}Si + \ ^{0}_{1}e + v$$

## ●● NUCLEAR REACTION

**A reaction which involves the change of stable nucleus of one element into the nucleus of another element is called nuclear reaction**.

It is usually caused by bombarding the reacting species with suitable high energy particles.

Getting a clue from the spontaneous disintegration of radioactive nuclei, Rutherford thought that it might be possible to penetrate heavy nucleus with a high-speed particle such as $\alpha$-particle and thereby either produce a nucleus with a greater mass number of induce an artificial disintegration of the nucleus.

In 1919, Rutherford succeeded in bringing about the first artificial transmutation by bombarding nitrogen nuclei with $\alpha$-particles which produced an isotope of oxygen and a proton according to the reaction:

$$^{14}_{7}N + \ ^{4}_{2}He \rightarrow \ ^{17}_{8}O + \ ^{1}_{1}H \quad \ldots (1)$$

Such an artificial nuclear transformation is termed as a nuclear reaction. Nuclear reactions can be used for the production of newer nuclei or for energy generation.

### ●● Symbolic representation of a nuclear reaction:

In general, in a nuclear reaction there is a collision between a target nucleus A and bombarding particle or projectile a which produces another nucleus B and an elementary particle b or gamma ray. This reaction may be represented as

$$A + a \rightarrow B + b + Q \quad \ldots (2)$$

● **The factor Q** represents the **energy absorbed or released** in the nuclear reaction. This is called **Q-value or nuclear reaction energy.** It is equal to the **difference between the sum of the rest masses of reacting particles and the sum of the rest masses of the product particles.** (Equal to the total change in kinetic energy of the system}.

The nuclear reaction represented by equation (2) can be expressed in a compact notation devised by Bethe:

A (a, b) B

So, we can represent the reaction of equation (1) as follows:

N ($\alpha$, p) O

● **Exothermic or Exoergic reaction**: **If the initial rest mass is greater than the final rest mass, then the Q-value of the reaction is positive**. Such a nuclear reaction is called exothermic or exoergic reaction. The energy released in the reaction can be harnessed as a source of energy, called nuclear energy. This energy appears in the form of kinetic energy of the product particles.

● **Endothermic or Endoergic reaction**: **If the initial rest mass is less than the final rest mass, the Q-value of the reaction is negative.** Such a reaction is called endothermic or endoergic reaction. This much energy is absorbed in the reaction. For such a reaction to take place, the required energy has to be supplied in the form of kinetic energy of the bombarding particle.

The energy released or absorbed in a nuclear reaction can be calculated from the Einstein's mass-energy equivalence relation:

$E = \Delta m.\ c^2$

□●●●●In any nuclear reaction, the following quantities are conserved:

●1. Momentum: The total momentum of the particles entering into the reaction is equal to the total momentum of the products after the reaction.

●2. Nucleons: The total number of nucleons before and after the reaction remains the same.

●3. Charge: The total charge of the product particles is equal to that of the reactant particles.

●4. Energy: The total energy (kinetic energy + rest mass energy) of the product particles is equal to the total energy of the reactant particles.

## □●●●●A NUCLEAR REACTION VS. A CHEMICAL REACTION

A nuclear reaction differs markedly from a chemical reaction. In a chemical reaction, only the electrons revolving around the nucleus take part in the reaction and no change occurs inside the nucleus whereas in a nuclear reaction, the nucleus itself undergoes a transformation. The energy changes involved in chemical reactions are much smaller than the energy changes involved in nuclear reactions.

## □●●●● TYPES OF NUCLEAR REACTIONS

In nuclear reactions, the bombarding particles or projectiles generally used are $\alpha$-particle ($^4_2$He), proton ($^1_1$H), deuteron ($^2_1$H), neutron ($^1_0$n) and gamma ray photon ($\gamma$). Depending on the nature of the projective particle and the outgoing particle, the various nuclear reactions are classified as follows:

### □●●1. Alpha particle-induced reactions:

(i) $^{10}_5B + ^4_2He \rightarrow ^{13}_6C + ^1_1H$ ($\alpha$, p) reaction

(ii) $^9_4Be + ^4_2He \rightarrow ^{12}_6C + ^1_0n$ ($\alpha$, n) reaction

### □●●2. Deutron-induced reactions:

(i) $^{12}_6 + ^2_1H \rightarrow ^{13}_7N + ^1_0n$ (d, n) reaction

(ii) $^{16}_8O + ^2_1H \rightarrow ^{14}_7N + ^4_2He$ (d, $\alpha$) reaction

(iii) $^7_3Li + ^2_1H \rightarrow ^8_3Li + ^1_1H$ (d, p) reaction

### □●●3. Proton-induced reactions:

(i) $^{14}_7N + ^1_1H \rightarrow ^{15}_8O + \gamma$ (p, $\gamma$) reaction

(ii) $^7_3Li + ^1_1H \rightarrow ^4_2He + ^4_2He$ (p, $\alpha$) reaction

(iii) $^{18}_8O + ^1_1H \rightarrow ^{18}_9F + ^1_0n$ (p, n) reaction

(iv) $^9_4Be + ^1_1H \rightarrow ^8_4Be + ^2_1H$ (p, d) reaction

### □●●4. Neutron-induced reactions:

(i) $^{23}_{11}Na + ^1_0n \rightarrow ^{20}_9F + ^4_2He$ (n, $\alpha$) reaction

(ii) $^{14}_7N + ^1_0n \rightarrow ^{14}_6C + ^1_1H$ (n, p) reaction

(iii) $^{238}_{92}U + ^1_0n \rightarrow ^{239}_{92}U + \gamma$ (n, $\gamma$) reaction

(iv) $^{131}_{52}Te + ^1_0n \rightarrow ^{132}_{53}I + ^0_{-1}e$ (n, $\beta$) reaction

### □●●5. Gamma ray photon-induced reactions:

Such reactions are also called photo-nuclear reactions or photo disintegrations.

$^{31}_{15}P + \gamma \rightarrow ^{30}_{15}P + ^1_0n$ ($\gamma$, n) reaction

$^{25}_{12}Mg + Y \rightarrow ^{24}_{11}Na + ^1_1H$ ($\gamma$, p) reaction

## □•••• ENERGY FROM THE NUCLEUS

The energy released during a nuclear reaction is called nuclear energy. As shown in Fig. the curve of binding energy per nucleon has long flat region in the mass number range from 30 to 170. Here the binding energy per nucleon is almost constant, around 8.5 MeV. However, for A < 30 and A > 170, B.E./nucleon is less than this plateau-value. This indicates that these nuclei are less tightly bound or less stable than the nuclei with mass numbers between 30 and 170. Hence whenever an element with a smaller binding energy is transmuted into an element with a larger binding energy, a tremendous amount of energy is released. This is due to the conversion of some mass into energy in accordance with Einstein's mass-energy relation. The energy released in a nuclear reaction due to the decrease in mass $\Delta m$ is given by

$$Q = -\Delta m.\ c^2$$

This equation indicates that even a very small quantity of matter is capable of releasing very large amount of energy. The nuclear reactions which can be exploited to produce energy are of two broad types:

□1. **Nuclear fission** in which a heavy nucleus splits up into two smaller nuclei, liberating a large amount of energy as in an atom bomb.

□2. **Nuclear fusion** in which two smaller nuclei fuse together to form a larger nucleus, releasing a large amount of energy as in a hydrogen bomb.

## □•••• NUCLEAR FISSION

The phenomenon in which a heavy nucleus (A>230) when excited splits into two smaller nuclei of nearly comparable masses is called nuclear fission.

□□In 1938, German scientists Otto Hahn and Fritz Strassmann found that when uranium is bombarded by slow moving neutrons, a ${}^{235}_{92}U$ nucleus gets excited by capturing a slow-moving neutron and splits into two nearly equal fragments like ${}^{141}_{56}Ba$ and ${}^{92}_{36}Kr$ along with the emission of 3 neutrons.

The nuclear reaction involved can be written as

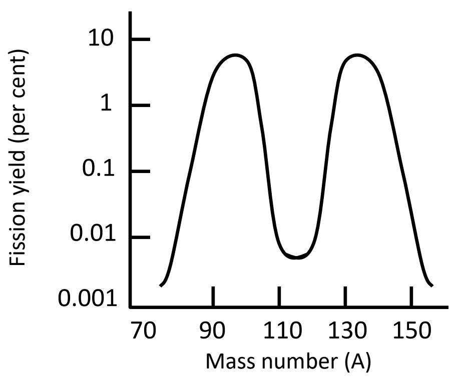
$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q$$

The Q-value of this reaction is about 200 MeV.

□•**Fission products of uranium:** The detailed analysis of the products of fission of ${}^{235}_{92}U$ has revealed that this fission, in general, produces nuclei with mass numbers in the rage 72 (${}_{30}Zn$) to 158 (${}_{63}Eu$). This means that nuclei of different mass numbers can be produced by fission of uranium. For example.

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Kr + 2\,{}^{1}_{0}n$$
$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{95}_{42}Mo + {}^{139}_{57}La + 2\,{}^{1}_{0}n$$
$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2\,{}^{1}_{0}n$$
$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{55}Cs + {}^{90}_{37}Pb + 2\,{}^{1}_{0}n$$

A number of other combinations are formed. The two nuclides have generally unequal mass numbers. If the relative yields of different nuclides are plotted against their mass number; we get a plot of the type shown in Fig.




**[Distribution by mass number of fragments formed due to fission of ${}^{235}U$ nuclei]**

The fission of ${}^{235}U$ yields broadly two groups of nuclei. One of the groups is a 'light group' with mass number range from 85 to 104. The other groups is a 'heavy group' with mass number rage from 139 to 149. The most probable fissions, having about 7% chance are centred around A = 95 and A = 140. The chances of fission with two fragments of equal mass number (A = 117) are extremely small, only about 1.01%.

Generally, the fission nuclides contain an excess of neutrons and are, therefore, highly unstable. They undergo beta decay until they form stable and products.

The decay chain for Xe is

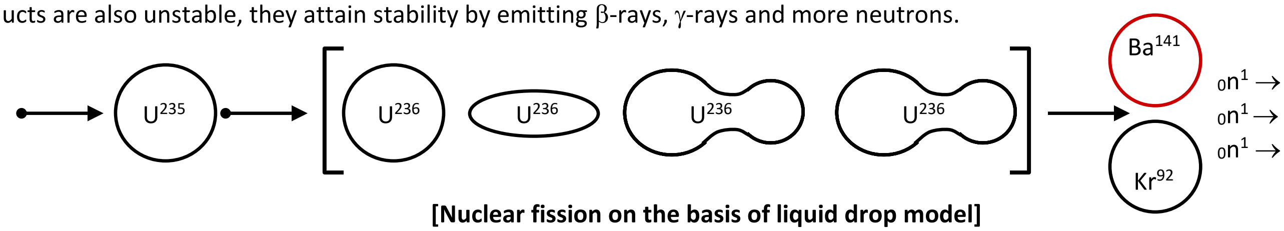
$${}^{140}_{54}Xe \xrightarrow{\beta^-} {}^{140}_{55}Cs \xrightarrow{\beta^-} {}^{140}_{56}Ba \xrightarrow{\beta^-} {}^{140}_{57}La \xrightarrow{\beta^-} {}^{140}_{58}Ce$$

For strontium, the decay chain is

$${}^{94}_{38}Sr \xrightarrow{\beta^-} {}^{94}_{39}Y \xrightarrow{\beta^-} {}^{94}_{40}Zx$$

## □•••• THEORY OF NUCLEAR FISSION

**Explanation of nuclear fission on the basis of liquid drop model:** In 1939, Yakov Frankel, Niels Bohr and John A. Wheeler proposed the liquid drop model to explain fission of nuclei. In this model, the nucleus is assumed as a liquid drop of spherical shape, which is incompressible and has a uniform positive charge. The distance between the positively charged protons tends to split the nucleus. But the forces of surface tension and short-range nuclear forces hold the nucleons together. When a nucleus captures a neutron, its equilibrium is disturbed and it begins to oscillate about its spherical shape due to the energy of the absorbed neutron. When the excitation energy is sufficiently large, the compound nucleus deforms into a dumb-bell shape structure. The oscillations eventually lead to the fission of the nucleus into two fragments, accompanied by emission of neutrons. The nuclei of the fission products are also unstable, they attain stability by emitting β-rays, γ-rays and more neutrons.




**[Nuclear fission on the basis of liquid drop model]**

## □•••• NUCLEAR FISSION AS A SOURCE OF ENERGY

In a nuclear fission, the sum of the masses of the final products is less than the sum of the masses of the reactant components. The difference in masses, called mass defect, is converted into energy according to Einstein's mass-energy relationship ($E = mc^2$).

Thus, an enormous amount of energy is released in a nuclear fission, as can be seen from the following example:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + Q$$

| Initial masses | | Final masses | |
|---|---|---|---|
| $^{235}_{92}U$ | 235.0439 amu | $^{141}_{56}Ba$ | 140.9139 amu |
| $^{1}_{0}n$ | 1.0087 amu | $^{92}_{36}Kr$ | 91.8973 amu |
| | | $3^{1}_{0}n$ | 3.0261 amu |
| | 236.0526 amu | | 235.8373 amu |

Mass defect = 236.0526 – 235.8373 = 0.2153 amu

As 1 amu = 931 MeV

∴ Energy released, Q = 0.2153 × 931 MeV

This energy appears in the form of kinetic energy of the fission products and as γ-rays.

Thus in the fission of a single nucleus of $^{235}_{92}U$, about 200 MeV energy is released which is equivalent to 0.9 MeV/nucleon. The total energy released in the fission of 1 kg of naturally occurring uranium, which contains about $2.56 \times 10^{24}$ atoms of $^{235}_{92}U$ isotopes, will be $200 \times 2.56 \times 10^{24}$ MeV = $10^{14}$ J. This is a very huge amount of energy which is equal to the energy obtained by burning of 3 tons of coal.

## □•••• NUCLEAR CHAIN REACTION

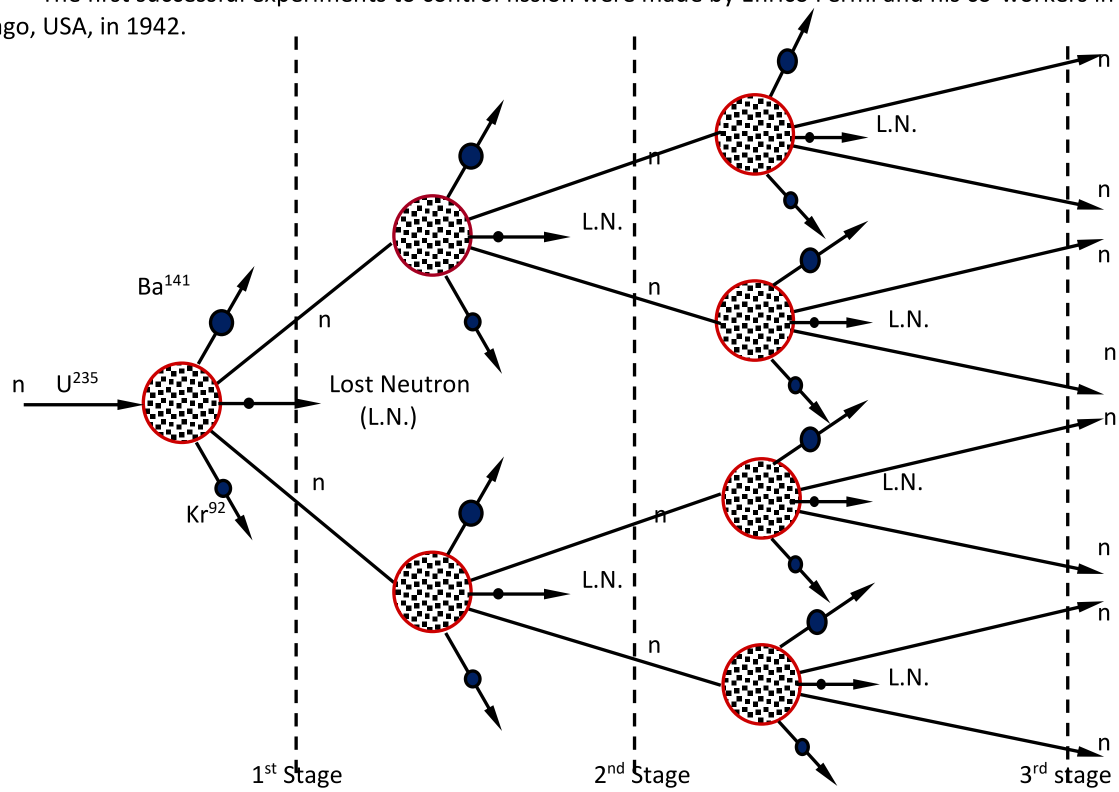
Nuclear fission is a peculiar type of reaction which besides the other fission products, products the same kind of particles that initiate it, viz, neutrons.

When a single $^{235}_{92}U$ nucleus captures a neutron, its fission produced 2.5 neutrons. These freshly produced electrons can further cause the fission of more uranium nuclei, producing still more neutrons, which can further cause the fission of a larger number of nuclei, and so on. The number of fissions taking place at each successive stage goes on increasing at a rapid rate (rather in a geometric progression). Thus, a chain reaction is set up, as illustrated in Fig. Enrico Fermi first suggested the possibility of such a reaction in 1939.

□•**Uncontrolled chain reaction**: **If a chain reaction is started in a fissionable material having mass greater than certain critical mass, then the reaction will accelerate at such a rapid rate that the whole material will explode within a microsecond, liberating a huge amount of energy. Such a chain reaction is called uncontrolled chain reaction. It forms the underlying principle of the atomic bombs.**

□•**Controlled chain reaction**: **The chain reaction can be controlled and maintained steady by absorbing a suitable number of neutrons at each stage of the reaction, so that on an average one neutron remains available for exciting further fission. Such a reaction is called controlled chain reaction. Here the energy released does not get out of control. A nuclear reactor works on the principle of a controlled chain reaction.**

The first successful experiments to control fission were made by Enrico Fermi and his co-workers in university of Chicago, USA, in 1942.




**[A chain reaction in $^{235}_{92}U$]**

## □• THERMAL NEUTRONS AND MODERATOR

**•Thermal neutrons:** The neutrons produced in fission of $^{235}U$ nuclei have average kinetic energy ≈ 2MeV. Such neutrons are called fast neutrons. The $^{235}U$ nuclei have good probability of absorbing slow neutrons of thermal energy ≈ 0.0235 eV, but have poor chance of absorbing fast neutrons. Unless slowed down to thermal energies, the fast neutrons will escape from the fissionable material without causing any fission. The slow-moving neutrons of energies 0.0235 eV are called thermal neutrons. These neutrons have velocities ≈ 2200 $ms^{-1}$, which are the random velocities of atoms and molecules at room temperature.

**•Moderator:** Any substance which is used to slow down fast-moving neutrons to thermal energies (≈ 0.0235 eV) is called a moderator . The commonly used moderators are water, heavy water ($D_2O$), graphite and beryllium.

**•Action of moderator:** Fast neutrons are passed through substances like paraffin, deuterium, or water, which contain large number of hydrogen nuclei or protons. Neutrons and protons have nearly the same mass. When fast moving neutrons are passed through paraffin, they make elastic collisions with protons, which have comparatively much smaller velocities. In few interactions, the velocities of the neutrons get interchanged with those of protons. The final velocities of the neutrons correspond to the random velocities of the atoms or molecules of the moderator at the room temperature. Such neutrons are called thermal neutrons.

About 25 collisions with deuterons (present in heavy water) or 100 collisions with carbon or beryllium are sufficient of slow down a neutron from 2 MeV to thermal energies.

□□□□□**A good moderator has two properties:**

**1. It slows down neutrons by elastic collisions.**

**2. It does remove neutrons from the system by absorbing them.**

## □• DIFFICULTIES IN SUSTAINING A CHAIN REACTION

To ensure high probability of chain reaction, it is desirable to have sufficient concentration of $^{235}U$ in the uranium being used in fission reactions. Natural uranium consists of three isotopes: $^{234}U$, $^{235}U$ and $^{238}U$ with the following compositions:

$^{234}U$ = 0.0058%

$^{235}U$ = 0.715%

and $^{238}U$ = 99.28%

The concentration of $^{235}U$ can be increased by special techniques. This process is called enrichment and the processed uranium is called enriched uranium, which contains about 3% $^{235}U$.

The probability of fission in the enriched uranium is very high. Still there are a number of difficulties due to which a chain reaction dies out.

**•1. Neutron leakage:** Some of the neutrons produced by fission may not interact with other nuclei and escape from the system. But leakage is a surface effect. The fraction of neutrons lost by leakage can be made sufficiently small by making the fissionable system large enough; thereby reducing the surface-to-volume ratio.

**2. Neutron energy:** Neutrons produced in fission are fast neutrons, having kinetic energies of about 2 MeV. Such neutrons have more chances of escaping from the fissionable material, without causing further fission. So, these neutrons are slowed down to thermal energies by mixing suitable moderator with the uranium fuel.

**3. Neutron capture:** When fast neutrons (2 MeV) produced by fission are slowed down in a moderator to thermal energies (≈ 0.04 eV), they pass through a critical energy range 1 – 100 eV. The neutrons of this energy range have good chances of being absorbed by $^{238}U$ nuclei which are present in large numbers (≈99.3%). This resonance capture is non-fissionable and results in the emission of a $\gamma$-ray. In this way, many neutrons get removed from the fission chain. To minimise this non-fissionable capturing, uranium fuel and moderator are not intimately mixed, instead they are placed in alternate columns. The distances between the fuel rods are so adjusted that a neutron coming from one rod is slowed down to thermal energies before it enters the neighbouring rod. This eliminates the possibility of non-fission capture of the neutrons.

## MULTIPLICATION FACTOR AND CRITICAL SIZE

**Whether a chain reaction, once started in a fissionable mass, will remain steady, increase or decrease, depends on a parameter called multiplication factor**. The multiplication factor of a fissionable mass is defined as the ratio of the number of neutrons present at the beginning of a particular generation to the number of neutrons present at the beginning of the previous generation.

Thus
$$k = \frac{\text{Number of neutrons present at the beginning of one generation}}{\text{Number of neutrons present at the beginning of previous generation}}$$

The multiplication factor k gives a measure of the growth rate of the neutrons in a fissionable mass.

- **If k > 1**, the chain reaction grows.
- **If k = 1**, the chain reaction remains steady.
- **If k < 1**, the chain reaction gradually dies out.

**Critical size and critical mass:** Whether the mass of a fissionable material can sustain a chain reaction or not, depends on its multiplication factor. This, in turn, depends on the size of the material. The size of the fissionable material for which the multiplication factor k = 1, is called critical size and its mass is called critical mass. The chain reaction in this case remains steady or sustained.

- **If k > 1**, the neutron population increases exponentially with time and the size of the material is said to be supercritical. The chain reaction builds up at a fast rate and results in an explosion.
- **If k < 1**, the neutrons population decreases exponentially with time and the size of the material is said to be supercritical. The chain reaction gradually comes to an end.

## NUCLEAR REACTOR

It is a device in which a nuclear chain reaction is initiated, maintained and controlled. It works on the principle of controlled chain reaction and provides energy at a constant rate.

**Main parts of a nuclear reactor**:

**1. Nuclear fuel:** It is the material that can be fissioned by neutrons. The isotopes like U-235, Th-232 and Pu-239 can be used as the reactor fuel. A certain mass of the fuel is taken in the form of rods, tightly sealed in aluminium containers. The rods, separated by moderator, are placed in the core of the reactor.

**2. Moderator:** In the fission of uranium, fast neutrons of energy 2 MeV are released. These fast neutrons have more tendency to escape instead of triggering another fission reaction. Also, slow neutrons are more efficient in inducing fission in $^{235}_{92}U$ nuclei than fast neutrons. By the use of a moderator, the fast neutrons are slowed to thermal velocities. Usually, heavy water, graphite and beryllium oxide are used as moderators.

**3. Control rods:** To start, stop or control the chain reaction, rods of neutron absorbing material like cadmium or boron are inserted into the reactor core. The rate of neutron production is controlled by adjusting the depth of control rods.

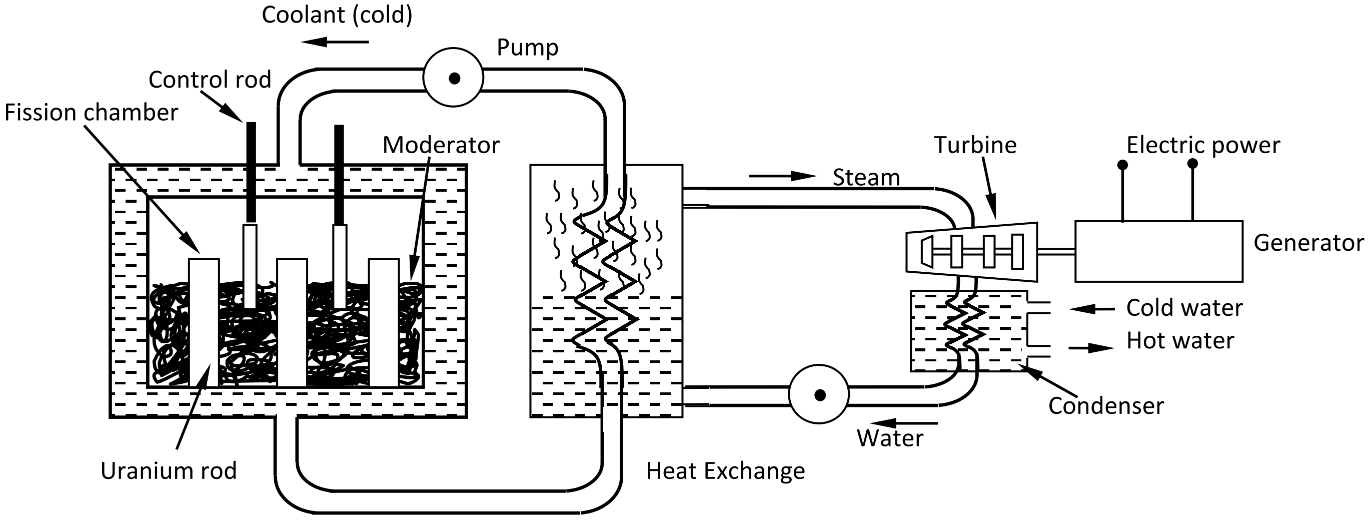
**4. Coolant:** It is the material used to cool the fuel rods and the moderator and is capable of carrying away large amount of heat produced in the fission process. The coolant transfers heat to the working liquid like water and produced steam. The steam drives a turbine which, in turn, runs a generator to generate electric power. The coolant must have high boiling point and high specific heat. Heavy water and liquid sodium are good coolants.

**5. Shielding:** The intense neutrons and gamma radiations produced in nuclear reactor are harmful for human body. To protect the workers from these radiations, the reactor core is surrounded by a thick concrete wall, called the reactor shield.

**Working:** Initially, some neutrons are produced by the action of $\alpha$-particles on polonium or beryllium. They are slowed down and are used to start fission of $^{235}_{92}U$ nuclei. Fast neutrons are released in these fissions which are slowed down to thermal velocities by passing through the moderator. These slow neutrons cause fission of more $^{235}_{92}U$ nuclei and thus the chain reaction builds up. By raising or lowering the control rods, the chain reaction is suitably controlled.

**Uses of nuclear reactor:**

1. In preparation of radio-isotopes, which find extensive use in scientific research, medicine, agriculture and in industry.
2. In the generation of electric power.
3. In the production of fast neutrons which are needed in nuclear bombardment.
4. In producing fissile material like plutonium which is used in atomic bombs.




**[Nuclear reactor]**

## BREEDER REACTOR

**A breeder reactor is one that produces more fissionable nuclei than it consumes.**

Natural uranium contains very little (0.7%) of the fissile $^{235}U$ isotope. It contains mostly (99.3%) of the non-fissile $^{238}U$ isotope. When this isotope is bombarded with fast neutrons, the following nuclear transmutation occurs:

$$^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{239}_{92}U \xrightarrow{-\beta^-} {}^{239}_{92}Np \xrightarrow{-\beta^-} {}^{239}_{94}Pu$$

Thus, the breeder reactor produces fissile plutonium $^{239}_{94}Pu$ from non-fissile uranium. Similarly, naturally more abundant isotope of thorium, $^{238}_{90}Th$, can be used to produce fissile isotope $^{233}_{92}U$ as follows:

$$^{232}_{90}Th + {}^{1}_{0}n \rightarrow {}^{233}_{90}Th \xrightarrow{-\beta^-} {}^{233}_{91}Pa \xrightarrow{-\beta^-} {}^{233}_{92}U$$

In breeder reactors, an alloy of sodium and potassium is used as a coolant.

## NUCLEAR FUSION

**The process in which two light nuclei combine (at extremely high temperature) to form a single heavier nucleus is called nuclear fusion.**

The mass of the heavier nucleus formed is less than the sum of the masses of the combining nuclei. The mass defect is released as energy in accordance with Einstein's mass-energy relation $E = \Delta m \cdot c^2$. For example, two protons combine to form a deuteron and a positron with release of 0.42 MeV energy:

$$^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^+ + \nu + 0.42\ MeV$$

Similarly, two deuterons combine either to form the light isotope of helium and a neutron or a triton and a proton:

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}H + {}^{1}_{0}n + 3.27\ MeV$$

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}n + 4.03\ MeV$$

In all the above reaction, two positively charged particles combine to form a heavier nucleus. The fusing nuclei have to overcome very high electrostatic repulsion between them at extremely small distances. This repulsion prevents the two nuclei from getting close enough to be within the range of their attractive nuclear forces and thus 'fusing'. The height of this coulomb barrier depends on the charges and the radii of the two colliding nuclei. The height of potential barrier is higher for more highly charged nuclei.

To carry nuclear fusion in a bulk material, the temperature of the material has to be raised to $10^6$ K, so that the colliding nuclei have enough energy due to their thermal motion and they can penetrate the coulomb barrier. This process is called thermonuclear fusion.

## NECESSARY CONDITIONS FOR NUCLEAR FUSION

The fusion reactions take place under the conditions of extreme temperature and density due to the following reasons:

1. The high temperature is necessary for the light nuclei to have sufficient kinetic energy so that they can overcome their mutual coulombic repulsions and come closer than the range of nuclear force. That is why a fusion reaction is also called a thermonuclear reaction.

2. High density or pressure increases the frequency of collision of light nuclei and hence increases the rate of fusion.

These conditions exist in the interior of the sun where the temperature is about $2 \times 10^6$ K. Such conditions cannot be easily met in a laboratory.

## FUSION AS A SOURCE OF ENERGY IN SUN AND STARS

The sun has been radiating energy at the rate of $3.8 \times 10^{26}$ Js$^{-1}$ for several billion years without showing any sign of cooling off. A satisfactory explanation for this phenomenon was given by H. Bethe in 1939. Hydrogen nuclei, i.e., protons are most abundant in the body of sun and start. At extremely high temperatures which exist in interior of sun and stars, protons fuse together to form helium nuclei, liberating a huge amount of energy. This fusion takes place via two different cycles:

**1. Proton-proton cycle, and 2. Carbon-nitrogen cycle**.

**1. Proton-proton cycle:** The thermonuclear reactions in a proton-proton cycle take place in the following sequence:

$${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu + 0.42 \text{ MeV}$$
$$e^+ + e^- \rightarrow \gamma + \gamma + 1.02 \text{ MeV}$$
$${}^2_1H + {}^1_1H \rightarrow {}^3_2He + r + 5.49 \text{ MeV}$$
$${}^3_2He + {}^3_2He \rightarrow {}^4_2He + {}^1_1H + {}^1_1H + 12.86 \text{ MeV}$$

For the fourth reaction to occur, the first three reactions must occur twice so that two light helium nuclei (${}^3_2He$) may combine to form a normal helium nucleus (${}^4_2He$). Then the net reaction will be

$$4{}^1_1H + 2e^- \rightarrow {}^4_2He + 2\nu + 6\gamma + 26.7 \text{ MeV}$$

Thus, four protons combine to form one helium nucleus with the liberation of 26.7 MeV of energy.

**2. Carbon-nitrogen cycle:** In this cycle, carbon nuclei successively absorb protons in a series of steps. Finally, they emit $\alpha$-particles and again change into carbon nuclei. The carbon-nitrogen cycle takes place in the following sequence:

$${}^{12}_6C + {}^1_1H \rightarrow {}^{13}_7N + \gamma + 1.93 \text{ MeV}$$
$${}^{13}_7N \rightarrow {}^{13}_6C + e^+ + \nu + 1.20 \text{ MeV}$$
$${}^{13}_6C + {}^1_1H \rightarrow {}^{14}_7N + \gamma + 7.6 \text{ MeV}$$
$${}^{14}_7N + {}^1_1H \rightarrow {}^{15}_8O + \gamma + 7.39 \text{ MeV}$$
$${}^{15}_8O \rightarrow {}^{15}_7N + e^+ + \nu + 1.71 \text{ MeV}$$
$${}^{15}_7N + {}^1_1H \rightarrow {}^{12}_6C + {}^4_2He + 4.99 \text{ MeV}$$

The overall reaction may be represented as

$${}^1_1H + {}^{12}_6C \rightarrow {}^4_2He + {}^{12}_6C + 2e^+ + 2\nu + 3\gamma + 24.8 \text{ MeV}$$

Here again four protons combine to form a helium nucleus, gamma rays, and neutrons and to liberate about 25 MeV of energy. Moreover, carbon is not consumed in the process but acts as a catalyst.

Temperature of the interior of the sun is $2 \times 10^6$ K. Both proton-proton and carbon-nitrogen cycles participate almost equally in the generation of energy in the sun. Start, hotter than the sun, get their energy from the carbon-nitrogen cycle, while those cooler than the sun get their energy from the proton-proton cycle.

## CONTROLLED THERMONUCLEAR REACTIONS

If the energy released in a thermonuclear reaction is controlled in such a manner that a limited amount of energy is produced continuously, it can be used for many useful purposes, particularly for generation of electrical power.

It is very difficult to set up a sustained and controllable source of fusion. The easiest thermonuclear reaction that can be carried on earth is the fusion of two deuterons (d – d reaction) or fusion of a deuteron with a triton (d – t reaction).

$${}^2_1H + {}^2_1H \rightarrow {}^3_2He + {}^1_0n + 3.2 \text{ MeV} \quad (d-d)$$
$${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H + 4.0 \text{ MeV} \quad (d-d)$$
$${}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0n + 17.6 \text{ MeV} \quad (d-t)$$

Deuterium, the source of deuterons for the above reactions, has a very small isotopic abundance about 1 part in 7000, but it is available in plenty in sea-water.

**Essential requirements for a thermonuclear reactor**: The requirements for a successful thermonuclear reactor are as follows:

**(i) High particle density:** The number density of deuterium atoms must be sufficiently high so that the rate of d – d collisions is high enough. At the high temperatures required for fusion, all the electrons get detached from the atoms. We have a mixture of electrons and deuterium nuclei (deuterons) moving at speeds. The overall charge of the system is zero. Such a material consisting of moving charged particles with equal number of positive and negative charges is called plasma.

**(ii) High plasma temperature:** The plasma must be not enough so that the interacting particles may penetrate the coulomb barrier and hence undergo fusion. A plasma ion temperature at 35 keV, corresponding to $4 \times 10^8$ K has been attained in the laboratory.

**(iii) Long confinement time:** To ensure fusion of enough fuel, we need to confine hot plasma at sufficiently high temperature in a small volume for an extended time interval. No solid contained can withstand such high temperatures. At present, two types of techniques are being used to confine hot plasma. In magnetic confinement, also called tokamak design, hot plasma is contained in a toroidal region by specially designed magnetic field. In inertial confinement, lasers are used to confine hot plasma.

The energy produced by fusion is clean and is not accompanied by generation of any radioactive hazardous waste. Moreover, the fuel 'deuterium' used in fusion is available in unlimited quantity in sea-water. Efforts are being made to achieve controlled thermonuclear fusion in laboratory. Once this happens, fusion will become the ultimate source of unlimited and unpolluted energy.

## Examples based on (i) Q-value (ii) Nuclear Fission (iii) Nuclear Fusion

◆ Formulae used

1. Mass defect, $\Delta m$ = Mass of reactant particles – Mass of product particles    2. Q-value = $(\Delta m)\, c^2$
3. Q-value is negative for endothermic reactions and positive for exothermic reactions.

◆ Units used

Mass defect $\Delta m$ is in kg or in amu and Q-value in joule or in MeV. 1 amu = 931 MeV.

**Q. 1.** ***A neutron is absorbed by a $^6_3Li$ nucleus with subsequent emission of an alpha particle. Write the corresponding nuclear reaction. Calculate the energy released in this reaction.***
***Given: m ($^6_3Li$) = 6.015126 amu;***
***m($^4_2He$) = 4.0026044 amu;***
***($^1_0n$) = 1.0086654 amu;***
***M($^3_1H$) = 3.016049 amu.***

**Sol.** $^6_3Li + {}^1_0n \rightarrow {}^4_2He + {}^3_1H + Q$

| **Initial masses** | **Final masses** |
|---|---|
| m ($^6_3Li$) = 6.015126 amu | m ($^4_2He$) = 4.0026044 amu |
| m ($^1_0n$) = 1.0086654 amu | m ($^3_1H$) = 3.016049 amu |

Mass defect, $\Delta m$ = 7.0237914 – 7.0186534
= 0.005138 amu
Energy released, Q = 0.005138 × 931 MeV = 4.78 MeV.

**Q. 2.** ***When a deuteron of mass 2.0141 amu and negligible kinetic energy is absorbed energy is absorbed by a lithium ($^6_3Li$) disintegrates spontaneously into two alpha particles, each of mass 4.0026 amu. Calculate the energy in joules carried by each alpha particle. (1 amu = 1.66 × $10^{-27}$ kg)***

**Sol.** $^6_3Li + {}^2_1H \rightarrow {}^4_2He + {}^4_2He + Q$
m ($^6_3Li$) = 6.0155 amu
m ($^2_1H$) = 2.0141 amu
Total initial mass = 8.0296 amu
Total final mass = 2 m ($^4_2He$) = 2 × 4.0026
= 8.0052 amu
Mass defect, $\Delta m$ = 8.0296 – 8.0052
= 0.0244 amu= 0.0244 × 1.66 × $10^{-27}$ kg

Energy released,
$Q = \Delta m \times c^2$
$= 0.0244 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2$
$= 3.645 \times 10^{-12}$ J
Energy of each $\alpha$-particle = $1.8225 \times 10^{-12}$ J

**Q. 3.** ***The bombardment of lithium with protons gives rise to the following reaction:***
$^7_3Li + {}^1_1H \rightarrow {}^4_2He + {}^4_2He + \text{Energy}$
***The atomic masses of lithium, hydrogen and helium are 7.016 amu, 1.008 amu and 4.004 amu respectively. Find the initial energy of each α-particle (1 amu = 931 MeV).***

**Sol.** In terms of nuclear masses, the Q-value of the reaction is given by
$Q = [m_N(^7_3Li) + m_N(^1_1H) - 2m_N(^4_2He)]\, c^2$
In terms of atomic masses, we can write
$Q = [\{m\,(73Li) - 3m_e\} + \{m\,(_{11}H) - m_e\} - 2\,\{m\,(^4_2He) - 2\,m_e\}]\, c^2$
$= [m\,(^7_3Li) + m\,(^1_1H) - 2m\,(^4_2He)] \times c^2 = [7.016 + 1.008 - 2 \times 4.004] \times 931$ MeV$= 0.06 \times 931 = 14.896$ MeV
Energy of each $\alpha$-particle = $\frac{14.896}{2}$ = 7.448 MeV

**Q. 4.** ***Calculate the disintegration energy Q for the fission of $^{98}_{42}Mo$ into two equal fragments, $^{49}_{21}Sc$. If Q turns out to be positive, explain why this process does not occur spontaneously. Given that:***
***m ($^{98}_{42}Mo$) = 97.90541 amu    m ($^{49}_{21}Sc$) = 48.95002 amu***
***$m_n$ = 1.00867 amu***

**Sol.** The fission of $^{98}_{42}Mo$ can be represented as
$^{98}_{42}Mo + {}^1_0n \rightarrow 2\,{}^{49}_{21}Sc + Q$
The disintegration energy in the fission of $^{98}_{42}Mo$ is given by
$Q = [m\,(^{98}_{42}Mo) + m_n - 2m\,(^{49}_{21}Sc)]\, c^2$
= 997.90541 + 1.00867 – 2 × 48.95002] amu × $c^2$
= [98.91408 – 97.90004] amu × $\frac{931.5 \text{ MeV}}{\text{amu}}$ = 1.01404 × 931.5 = 944.6 MeV.

**Q. 5.** ***If 200 MeV energy is released in the fission of a single nucleus of $^{235}_{92}U$, how many fissions must occur to produce a power of 1 kW?***

**Sol.** Let the number of fissions per second be n. Then,

Energy released per second $= n \times 200$ MeV $= n \times 200 \times 1.6 \times 10^{-13}$ J

Energy required per second = Power × Time = 1 kW × 1 s = 1000 J

$\therefore \quad n \times 200 \times 1.6 \times 10^{-13} = 1000$

or $\quad n = \dfrac{1000}{3.2 \times 10^{-11}} = \dfrac{10}{3.2} \times 10^{13} = 3.125 \times 10^{13}$

**Q. 6.** ***Calculate the energy released by the fission of 1 g of $^{235}_{92}U$ in kWh. Energy per fission is 200 MeV.***

**Sol.** Number of atoms in 1 g of $^{235}_{92}U = \dfrac{\text{Avogadro's number}}{\text{Mass number}} = \dfrac{6.023 \times 10^{23}}{235}$

Energy released per fission = 200 MeV

Energy released by fission of 1 g of $^{235}_{92}U = \dfrac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV} = 5.126 \times 10^{23}$ MeV

$= 5.126 \times 10^{23} \times 1.6 \times 10^{-13}$ J

$= \dfrac{5.126 \times 1.6 \times 10^{10}}{3.6 \times 10^{6}}$ kWh $= 2.278 \times 10^{4}$ kWh. $\quad [\because 1 \text{ kWh} = 3.6 \times 10^{6} \text{ J}]$

**Q. 7.** ***It is estimated that the atomic bomb exploded at Hiroshima released a total energy of $7.6 \times 10^{13}$ J. If on the average 200 MeV energy was released by fission of one $^{235}_{92}U$ atom, calculate.***
***(i) the number of uranium atoms fissioned***
***(ii) the mass of uranium used in the bomb.***

**Sol.** (i) Number of $^{235}_{92}U$ atoms fissioned,

$n = \dfrac{\text{Total energy released}}{\text{Energy released per fission}}$

$= \dfrac{7.6 \times 10^{13}}{200 \times 1.6 \times 10^{-13}} = 2.375 \times 10^{24}$

(ii) Mass of uranium used

$= \dfrac{\text{Mass number}}{\text{Avogadro's number}} \times n = \dfrac{235 \times 2.375 \times 10^{24}}{6.023 \times 10^{23}}$

$= 926.66$ g

**Q. 8.** ***What is the power output of $^{235}_{92}U$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy?***

**Sol.** Mass used in 30 days = 2 kg = 2000 g

Mass used up per second (m) $= \dfrac{2000}{30 \times 24 \times 60 \times 60} = \dfrac{1}{36 \times 36}$ g

No. of atoms in 235 g of $^{235}_{92}U = 6.023 \times 10^{23}$

No. of atoms in $\dfrac{1}{36 \times 36}$ g of $^{235}_{92}U$

$= \dfrac{6.023 \times 10^{23}}{235 \times 36 \times 36} = 1.977 \times 10^{18}$

This is the number of nuclei undergoing fission per second.

Now energy released per fission = 185 MeV

$\therefore$ Total energy released per second $= 1.977 \times 10^{18} \times 185$ MeV $= 1.977 \times 10^{18} \times 185 \times 1.6 \times 10^{-13}$ J $= 5.85 \times 10^{7}$ J

Power output $= 5.85 \times 10^{7}$ Js$^{-1}$ $= 5.85 \times 10^{7}$ W $= 58.5$ MW.

**Q. 9.** ***Under certain circumstances, a nucleus can decay by emitting a particle more massive than an $\alpha$-particle. Consider the following decay processes:***

$$^{235}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$
$$^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

***The Coulomb barrier height for alpha-particle emission is 30.0 MeV. What is the barrier height for $^{14}_{6}C$? The required data is***

***$m(^{223}_{88}Ra) = 223.01850$ amu $\quad m(^{209}_{82}Pb) = 208.98107$ amu***
***$m(^{219}_{86}Rn) = 219.00948$ amu $\quad m(^{14}_{6}C) = 14.00324$ amu***
***$m(^{4}_{2}He) = 4.00260$ amu***

**Sol.** The coulomb barrier height for $\alpha$-decay is equal to the coulomb repulsion between the $\alpha$-particle and the daughter nucleus when they are just touching each other.

The electrostatic potential energy between a particle with charge $Z_1e$ and radius $r_1$, and a nucleus with charge $Z_2e$ and radius $r_2$ is given by $\quad U = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Z_1e \cdot Z_2e}{(r_1 + r_2)}$

where $r_1 = r_0 A_1^{1/3}$ and $r_2 = r_0 A_2^{1/3}$

For the emission of $\alpha$-particle from $^{223}_{88}Ra$,

$Z_1 = 2, \; A_1 = 4$

$Z_2 = 86, \; A_2 = 219$

∴ $U(\alpha) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2e \cdot 86e}{r_0(4^{1/3} + 219^{1/3})}$

For the emission of $^{14}_{6}C$ from $^{223}_{88}Ra$,

$Z_1 = 6, \quad A_1 = 14$

$Z_2 = 82, A_2 = 209$

∴ $U(^{12}_{6}C) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6e.82e}{r_0(14^{1/3} + 209^{1/3})}$

Hence, $\frac{U(^{14}_{6}C)}{U(\alpha)} = \frac{6 \times 82}{2 \times 88} \times \frac{(219^{1/3} + 4^{1/3})}{(209^{1/3} + 6^{1/3})}$

$= \frac{6 \times 82}{2 \times 88} \times \frac{(6.3 + 1.5874)}{(5.935 + 2.410)}$

$= 2.86 \times \frac{7.8874}{8.345} = 2.703$

or $U(^{14}_{6}C) = 2.703\, U(\alpha) = 2.703 \times 30$ MeV

$= 81.09 \approx 81$ MeV

**Q. 10.** ***Two protons, each having a kinetic energy K, are fired at each other. What must K be if the particles are brought to rest by their mutual coulomb repulsion? Assume a proton to be a sphere of radius repulsion? Assume a proton to be a sphere of radius R = 1 fm. Also estimate the temperature at which the protons can overcome this energy barrier.***

**Sol.** The initial mechanical energy, $E_i$ of the two protons before collision is given by

$E_i = 2K$

When the protons stop, their entire energy is the electrostatic potential energy. It is given by

$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \times e}{(R + R)} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2R}$

By conservation of energy,

$2K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2R}$

∴ $K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{4R} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 1 \times 10^{-15}}$ J  [1 fm = $10^{-15}$ m]

$= 5.75 \times 10^{-14}$ J $= \frac{5.75 \times 10^{-14}}{1.6 \times 10^{-16}}$ keV $= 360$ keV $= 400$ keV.

This is approximately the coulomb barrier between two protons.

The temperature T at which protons in a proton gas would have enough energy to overcome the coulomb barrier between them is given by $\frac{3}{2} kT = K_{av}$

or $T = \frac{2K_{av}}{3k}$

Here $K_{av} = 5.75 \times 10^{-14}$ J

Boltzmann constant, $k = 1.38 \times 10^{-23}$ JK$^{-1}$ ∴ $T = \frac{2 \times 5.75 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} \approx 3 \times 10^9$ K

**Q. 11.** ***A star converts all its hydrogen to helium, achieving 100% helium composition. It then converts the helium to carbon via the reaction*** $^4_2He + ^4_2He + ^4_2He \rightarrow ^{12}_6 C + 7.27$ ***MeV***

***The mass of the start is $5.0 \times 10^{32}$ kg, and it generates energy at the rate of $5 \times 10^3$ W. How long will it take to convert all the helium to carbon at this rate?***

**Sol.** Number of nuclei present in 4 g He $= 6 \times 10^{23}$

Number of nuclei present in $5.0 \times 10^{32}$ kg or $5.0 \times 10^{25}$ g of helium

$= \frac{6 \times 10^{23} \times 50 \times 10^{35}}{4} = 7.5 \times 10^{58}$

Energy released in the fusion 3 helium nuclei = 7.27 Mev

Energy released by the fusion of $7.5 \times 10^{58}$ nuclei $= \frac{7.27 \times 7.5 \times 10^{58}}{3}$ MeV

$= 7.27 \times 2.5 \times 10^{58} \times 1.6 \times 10^{-13}$ J $= 2.9 \times 10^{46}$ J

Energy generated per second $= 5 \times 10^{30}$ J

Time taken to convert all helium nuclei into carbon $= \frac{2.9 \times 10^{46}}{5 \times 10^{30}}$ s $= 5.8 \times 10^{15}$ S

$= \frac{5.8 \times 10^{15}}{3.15 \times 10^7}$ years $= 1.8 \times 10^8$ years.

**Q. 12.** ***It is proposed to use the nuclear reaction:***

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He$$

***In a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium fuel will be needed per day? The masses of*** $^{2}_{1}H$ ***and*** $^{4}_{2}He$ ***are 2.0141 amu and 4.0026 amu respectively.***

***Sol.*** $${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + Q$$

Initial mass = 2m ($^{2}_{1}H$) = 2 × 2.0141 = 4.0282 amu

Final mass = m ($^{4}_{2}He$) = 4.0026 amu

Mass defect, $\Delta m$ = 4.0282 – 4.0026 = 0.0256 amu

Energy released Q = 0.0256 × 931 = 23.8336 MeV

Since the efficiency of the reactor is 25%, the energy used per reaction

$$= 23.8336 \times \frac{25}{100} \text{ MeV} = 5.9584 \text{ MeV}$$

$$= 5.9584 \times 1.6 \times 10^{-13} \text{ J} = 9.533 \times 10^{-13} \text{ J}$$

The rating of the reactor is 200 MW

∴ Energy required per day

$$= \text{Power} \times \text{Tie} = 200 \times 10^{6} \times 24 \times 60 \times 60 \quad = 172.8 \times 10^{11} \text{ J}$$

As two deuterium atoms given an energy of $9.533 \times 10^{-13}$ J, so number of deuterium atoms needed per day.

$$= \frac{172.8 \times 10^{11}}{9.533 \times 10^{-13}} = 36.25 \times 10^{24}$$

Now the mass of 1 mole (or $6.02 \times 10^{2}$ atoms) of deuterium is 2.0141 g.

∴ Mass of deuterium needed per day $$= \frac{2.0141 \times 36.25 \times 10^{24}}{6.02 \times 10^{23}} = 121.3 \text{ g}$$

## NUCLEAR FISSION VERSUS NUCLEAR FUSION

| Nuclear fission | Nuclear fusion |
|---|---|
| **1.** Here a heavy nucleus when excited gets split up into two smaller nuclei of nearly comparable masses. | **Here** two lighter nuclei fuse together to form a heavier nucleus. |
| **2.** The conditions of high temperature and pressure are not necessary for its occurrence. It can be carried on the earth. | **The** conditions of extremely high pressure and temperature are necessary for its occurrence. So its cannot be easily carried in a laboratory. |
| **3.** Neutrons are the link particles of this process. | **Protons** are the link particles of this process. |
| **4.** It is a quick process. | **It** occurs in several steps. There is sufficient time gap between initial and final steps. |
| **5.** Here the energy available per nucleon is small, about 0.85 MeV. | **Here** the energy available per nucleon is large, about 6.75 MeV. |
| **6.** The energy obtained from a unit mass of a fissionable material is smaller than that obtained in case of fusion. | **The** energy obtained from a unit mass of a fusible material is large. |
| **7.** It produces very harmful radioactive wastes. | **The** products of fusion are harmless. |
| **8.** The stock of fissionable fusion is limited. | **The** fuel required for fusion is available in plenty. |

## NUCLEAR HOLOCAUST

**Nuclear holocaust:** This is a nuclear era. The discoveries of nuclear fission and nuclear fusion have made available to us vast sources of energy. For example, a single fission of uranium nucleus releases about 200 MeV of energy. Thus 50 kg of such nuclei will release $4 \times 10^{15}$ J. This tremendous amount of energy is equivalent to about 20, 000 tonnes of TNT, which is enough for a super-explosion. The uncontrolled release of large energy is called atomic explosion.

Seeing the enormous destructive power of atomic bombs, there was a mad race between the superpowers for acquiring and stockpiling more and more such bombs. For this a large number of nuclear tests were conducted and are being conducted by many countries. This created new problems like the disposal of radioactive wastes and environmental pollution caused by nuclear radiations. These high energy radiations have very harmful effects on living beings. The radioactive fallout from the increasing use of nuclear fission for generating controlled as well as uncontrolled energy and the actual used of atom bomb poses serious threat to mankind. Such a nuclear holocaust will render this planet unfit for life for all the times.

The radioactive wastes released form nuclear explosion will hang like a cloud in the earth's atmosphere and will absorb all the solar radiation. This will produce a long nuclear winter on the earth. So, we should do our best to see that the possibility of a nuclear holocaust is ruled out. Should we use nuclear energy to improve quality of life on earth or use it to destroy the planet!