



XI

CBSE

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KINETIC THEORY OF GASES
UNIT-09

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KINETIC THEORY OF GASES

CBSE-XI

PHYSICS

Fundamentals of Kinetic Theory:

- Dive into the foundational concepts of kinetic theory, unraveling the connection between macroscopic properties of gases and the motion of individual molecules.

Gas Laws Unveiled:

- Explore the application of kinetic theory in explaining Boyle's law, Charles's law, and Avogadro's law. Understand how molecular motion influences the observed behavior of gases.

Derivation of Gas Laws:

- Delve into the mathematical derivations of gas laws, connecting theoretical concepts to quantitative relationships. Strengthen your problem-solving skills with step-by-step explanations.

Behavior of Real Gases:

- Examine the deviations of real gases from ideal behavior. Understand the factors that contribute to these deviations and their significance in real-world scenarios.

Mean Free Path and Collision Frequency:

- Explore the mean free path and collision frequency of gas molecules. Gain insights into the distances traveled between collisions and the frequency of these collisions.

Concept of Pressure:

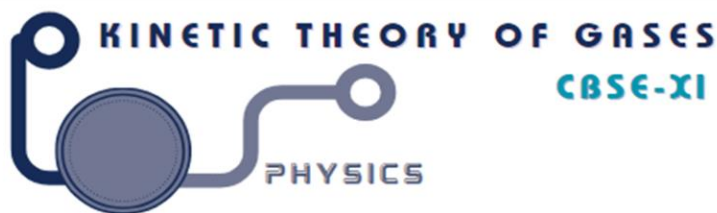
- Unravel the molecular origin of pressure in gases. Connect the microscopic motion of gas molecules to the force exerted on the walls of the container.

Temperature and Kinetic Energy:

- Understand the relationship between temperature and the kinetic energy of gas molecules. Explore how changes in temperature impact the motion of particles.

Specific Heat Capacity:

- Connect kinetic theory to the concept of specific heat capacity. Gain a deeper understanding of how energy is transferred to gases and how it affects their temperature.



Kinetic theory of gases based on the basic Assumptions

- 1.] A gas consists of very large no. of molecules. These molecules are identical, perfectly elastic and hard sphere. small that the volume of a molecule is negligible as compared to the volume of the gas.
- 2.] The molecules do not have any preferred direction of motion but the motion completely random.
- 3.] These molecules travel in straight line and are in free motion most of the time. The time interval of the collision between any two molecules is very small.
- 4.] The molecules do not exert any Force of attraction or repulsion on each other, except during collision.
- 5.] The collision between molecules and the wall of the container is perfectly elastic. It means kinetic energy and linear momentum is conserved in such collision.
- 6.] Molecular density is uniform throughout the gas.
- 7.] The path traveled by a molecule between two successive collisions is called free path and the mean distance traveled by a molecule between two successive collision is called mean free path.
- 8.] The effect of gravity of the motion of the molecule is negligible.
- 9.] The collisions are almost instantaneous that is the time of collision of two molecules is negligible as compared to the time interval between two successive collisions. (Relaxation time, τ).
- 10.] The motion of molecules is governed by Newton's Law Of motion.

Kinetic theory and the concept of gas pressure:

A/ Kinetic theory of gases, the molecules of a gas are in a state of random motion. The molecules not only collide with one another but also with the wall of the containing vessel. Even at ordinary temperature & pressure, the no. of molecular collision with the wall is very large. During each collision, certain momentum is transferred to the wall of the container.
According to Newton's 2nd law of motion, the rate of change of momentum is proportional to force. Therefore, in each collision, the molecules exert a force on the wall of the container (no. of collision / $\text{cm}^3 = 10^{19}$ molecules / second). Thus, the walls of the container experience a steady force. This force per unit area is called pressure of the gas. Hence, the pressure of a gas is due to continuous bombardment of the vessel.

$$\therefore \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Change momentum}}{\text{Area} \times t} = \frac{\text{Change in momentum}}{A \times t}$$

i.e., Pressure is equal to the momentum imparted per second per unit area of the walls of the container by the molecules.

Derivation of expression for gas pressure:

Consider an ideal gas contained in a cubical container EFGHBAD. If each side 'l' and volume 'V'

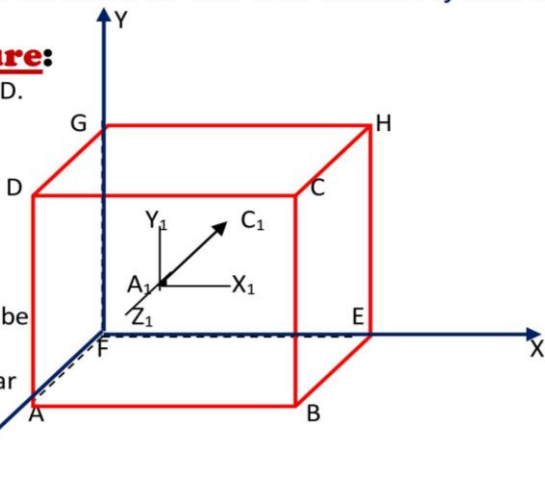
Volume of gas, $V = \text{Volume of the container, } l^3$
i.e., $V = l^3$ ----- [1]

Let, there be 'n' molecules of the gas each having mass 'm'.
Therefore, Total mass of the gas in the container, $M = n \times m$
Again, Let the random velocities of the molecules A_1, A_2, \dots, A_n be C_1, C_2, \dots, C_n respectively.

Let, $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots, (x_n, y_n, z_n)$ be the rectangular component of the velocities C_1, C_2, \dots, C_n along three mutually perpendicular direction OX, OY and OZ, therefore,

$$\left. \begin{aligned} x_1^2 + y_1^2 + z_1^2 &= C_1^2 \\ x_2^2 + y_2^2 + z_2^2 &= C_2^2 \\ \dots &\dots \\ x_n^2 + y_n^2 + z_n^2 &= C_n^2 \end{aligned} \right\} \rightarrow [3]$$

Now the initial momentum of A_1 along OX = Mass \times velocity = $m_1 \times x_1$.



When the molecules strike the wall BECH, it rebound with the same speed towards the face ADGF.

∴ **Momentum** of the molecule A_1 , after collision = $-m_1 x_1$

Change in momentum of the molecule after one collision = $m_1 x_1 - (-m_1 x_1) = 2 m_1 x_1$.

Time taken between the two successive (on the same face) collision = $\frac{\text{Distance}}{\text{Velocity}} = \frac{l+l}{x_1} = \frac{2l}{x_1}$

∴ Time rate of change of momentum due to collision = $\frac{\text{Change in momentum}}{\text{Time taken}} = \frac{2 m_1 x_1}{2l/x_1} = \frac{m_1 x_1^2}{l}$

According to **N's 2nd law of motion** the **rate of change of momentum of a body is equal to the force exerted on it**. Hence the **force** exerted by molecule A, on the wall BCHE will be,

$$f_1 = \frac{m x_1^2}{l}$$

If f_2, f_3, \dots, f_n are the forces exerted by the molecule A_2, \dots, A_n on the wall BECH,

$$f_2 = \frac{m x_2^2}{l}, \dots, f_n = \frac{m x_n^2}{l}$$

Then, **Total force** exerted by all the molecules on wall BECH.

$$\begin{aligned} F_x &= f_1 + f_2 + \dots + f_n \\ &= \frac{m x_1^2}{l} + \frac{m x_2^2}{l} + \dots + \frac{m x_n^2}{l} \\ &= \frac{m}{l} (x_1^2 + x_2^2 + \dots + x_n^2) \end{aligned}$$

∴ **pressure** exerted on the wall BECH is, $P_x = \frac{\text{Force}}{\text{area of the wall}} = \frac{F_x}{l \times l}$

$$= \frac{m}{l^3} (x_1^2 + x_2^2 + \dots + x_n^2)$$

$$= \frac{m}{l^3} (x_1^2 + x_2^2 + \dots + x_n^2)$$

|| y, $P_y = \frac{m}{l^3} (y_1^2 + y_2^2 + \dots + y_n^2)$

and $P_z = \frac{m}{l^3} (z_1^2 + z_2^2 + \dots + z_n^2)$

But, $P_x = P_y = P_z = P$ (say) → $\left\{ \begin{array}{l} \text{molecular density is uniform} \\ \therefore \text{pressure exerted by the gas molecule is the same} \\ \text{in all direction.} \end{array} \right.$

∴ $P_x + P_y + P_z = 3P$

or $P = \frac{P_x + P_y + P_z}{3}$

$$P = \frac{1}{3} \left[\frac{m}{l^3} (x_1^2 + x_2^2 + \dots + x_n^2) + \frac{m}{l^3} (y_1^2 + y_2^2 + \dots + y_n^2) + \frac{m}{l^3} (z_1^2 + z_2^2 + \dots + z_n^2) \right]$$

$$P = \frac{m}{3l^3} \left[(x_1^2 + y_1^2 + z_1^2) + (x_2^2 + y_2^2 + z_2^2) + \dots + (x_n^2 + y_n^2 + z_n^2) \right]$$

$$P = \frac{m}{3V} (C_1^2 + C_2^2 + \dots + C_n^2)$$

$$P = \frac{m \times n}{3V} \left(\frac{C_1^2 + C_2^2 + \dots + C_n^2}{n} \right)$$

$$P = \frac{M}{3V} (C^2)$$

— {A}

(Since, $m \times n = M$ and $C^2 = \left(\frac{C_1^2 + C_2^2 + \dots + C_n^2}{n} \right)$)

Called **Root mean square (r m s) velocity** of the gas.

Defⁿ Of r m s: "It is defined as the square root of the mean squares of the random velocities of the individual molecules of a gas".

again from {A},

$$P = \frac{1}{3} \times \frac{M}{V} C^2$$

$$P = \frac{1}{3} \times \rho C^2 \quad \text{-----} \{B\} \quad \text{(Where density, } \rho = \frac{M}{V})$$

From {B}, $P = \frac{1}{3} \times \rho C^2$

or, $C^2 = \frac{3P}{\rho}$

$$C = \sqrt{\frac{3P}{\rho}}$$

r m s Velocity of the gas at given temperature.

Relation between pressure and Kinetic energy of the gas :

From Kinetic theory of gases, the pressure P exerted by an ideal gas of density ρ and r m s velocity of its gas molecules C is,

$$P = \frac{1}{3} \times \rho C^2$$

Mass per unit volume of the gas = Vol. \times Density = $1 \times \rho = \rho$

\therefore Mean Kinetic energy of Translation per unit Vol. of the gas is,

$$E = \frac{1}{2} \times \rho C^2$$

$$\therefore \frac{P}{E} = \frac{\frac{1}{3} \rho C^2}{\frac{1}{2} \rho C^2} = \frac{2}{3}$$

$$\therefore P = \frac{2}{3} E$$

or,

$$E = \frac{3}{2} P$$

- KE per unit volume of the gas is numerically equal to 3/2 times the pressure exerted by the gas.**
- Pressure exerted by an ideal gas is numerically equal to 2/3 of the mean kinetic energy of per unit vol. of the gas.**

Average kinetic energy per molecule of the gas : Consider, one mole of an ideal at absolute temperature T, of volume V and molecules weight 'M'.

$\therefore M = m \times N$ → Avogadro's no.

m → Mass of each molecule

Pressure exerted by ideal gas is, $P = \frac{1}{3} \times \frac{M}{V} C^2$ → r m s Velocity of the gas molecules.

$$P V = \frac{1}{3} \times M C^2$$

But, $P V = R T$ (Gas Equation)

$$\therefore \frac{1}{3} \times M C^2 = R T$$

$$\frac{1}{2} \times M C^2 = \frac{3}{2} R T$$

Average KE of one mole of the gas,

$$\frac{1}{2} \times M C^2 = \frac{3}{2} R T$$

$$\frac{1}{2} \times m \times N C^2 = \frac{3}{2} R T$$

$$\frac{1}{2} \times m C^2 = \frac{3}{2} \frac{R T}{N}$$

$$\frac{1}{2} \times m C^2 = \frac{3}{2} K T$$

where, $K = \text{Boltzmann's Constant} = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23} \text{ J/Mol/K}$

\therefore , Average Kinetic energy per molecule of a gas, ----- $\frac{1}{2} \times m C^2 = \frac{3}{2} K T$

i.e., Average kinetic energy does not depend upon the mass of the molecules, but depends upon the temperature (T) of the gas.

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◆ **Kinetic interpretation of temperature:** A / Kinetic theory of gases, the pressure P exerted by one mole of an ideal gas is

$$P = \frac{1}{3} \times \frac{M C^2}{V} \quad \text{or,} \quad P V = \frac{1}{3} \times M C^2$$

but, $P V = R T$ (Gas Equation), $\therefore \frac{1}{3} \times M C^2 = R T$ [1]

or, $C^2 = \frac{3RT}{M}$

$$C = \sqrt{\frac{3RT}{M}}$$

as R and M are constant, therefore $C \propto \sqrt{T}$

or, $\sqrt{T} \propto C$ i.e., Square root of absolute temperature of an ideal gas is directly proportional to r m s velocity of its molecule.

Again From [1], $\frac{1}{3} \times M C^2 = R T$

or, $\frac{1}{3} m \times N C^2 = R T$

or, $\frac{1}{3} \times m C^2 = \frac{R T}{N}$

or, $\frac{1}{2} \times m C^2 = \frac{3}{2} K T$

or, $\frac{1}{2} \times m C^2 \propto T$ (as K and 3/2 are constant)

$$KE \propto T$$

\therefore "Average Kinetic energy of per molecule of a gas is directly proportional to the absolute temperature of the gas".

▣ **ABSOLUTE ZERO OF TEMPERATURE** : We know that, $C \propto \sqrt{T}$
 When $T = 0$ then $C = 0$

"Absolute temperature may be defined as that temperature at which root mean square velocity of the gas molecules reduce to zero".

- Molecular motion Ceases at absolute Zero of temperature.
- Absolute Zero for an ideal gas = -273°C .

DERIVATION OF GAS LAWS

▣ **[A] BOYLE'S LAW** : It states that "for a constant temperature, the pressure of a gas is inversely proportional to its volume". i.e., $P \propto 1/V$ or, $V \propto 1/P$ {T remains constant}

proof: A / Kinetic theory of gases, For an ideal gas, $P = \frac{1}{3} \times \frac{M C^2}{V}$
 $P V = \frac{1}{3} \times M C^2$

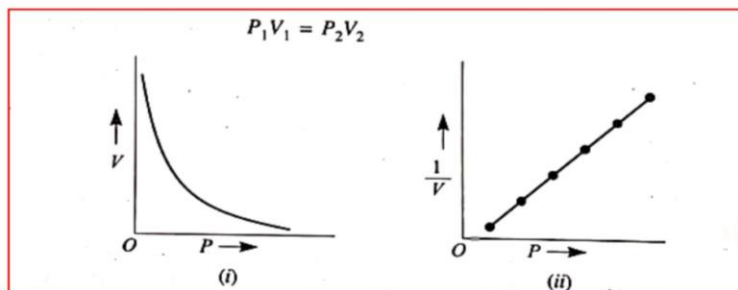
$P V \propto C^2$ (Since $M/3 = \text{Constant}$)

but, $C^2 \propto T$

or, $P V \propto T$

If T is constant, then $P V = \text{Constant}$

$P \propto 1/V$ Boyle's law.



[B] Charles's LAW : It states that "the volume V of a given mass of gas is directly proportional to its absolute temperature T , provided the pressure P remains constant".

i.e., $V \propto T$ [P = Constant]

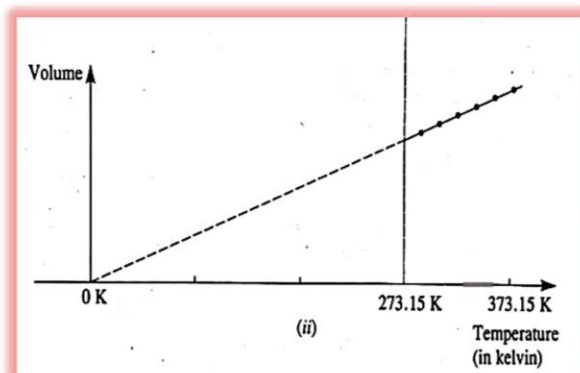
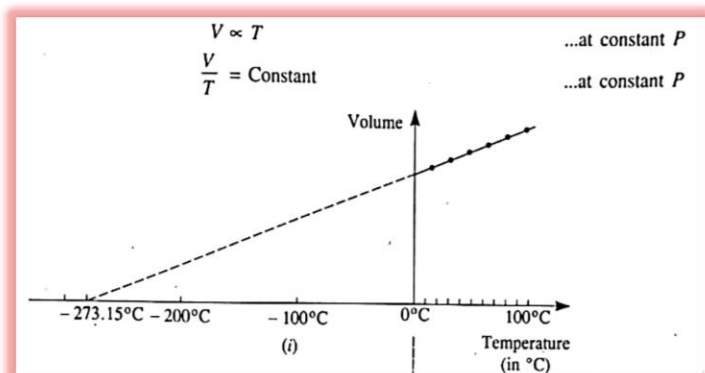
proof: A / Kinetic theory of gases, For an ideal gas, $P = \frac{1}{3} \frac{M C^2}{V}$

$$V = \frac{1}{3} \frac{M C^2}{P}$$

$$V \propto C^2 \quad \{\text{since, } M / P = \text{Constant}\}$$

$$\boxed{V \propto T} \quad \{\text{Since, } C^2 \propto T\}$$

Charles's Law.



Thus if a given mass of a gas has a volume V_1 at $\theta_1^\circ\text{C}$ and is heated at constant pressure to $\theta_2^\circ\text{C}$, its new volume V_2 is given by :

$$\frac{V_2}{V_1} = \frac{273.15 + \theta_2}{273.15 + \theta_1} = \frac{T_2}{T_1}$$

- * This seems to imply that if a gas could be cooled to -273.15°C , it would have zero volume. Although a gas would liquefy before cooling to this temperature (-273.15°C), the graph suggests that there is a limit to how cold objects can be.
- ** This means 0°C becomes 273.15 kelvin (written as $T = 273.15 \text{ K}$) and $100^\circ\text{C} = 373.15 \text{ K}$.

Another statement of Charles' law. The equation of the graph

$$y = mx + c$$

Here

$$c = \text{Intercept on } Y\text{-axis} = \text{Volume at } 0^\circ\text{C} = V_0$$

$$m = \text{Slope of graph} = \frac{V_0}{273.15}$$

$$x = \text{Temperature } \theta_1^\circ\text{C}$$

$$y = \text{Volume } V_1 \text{ of gas at temperature } \theta_1^\circ\text{C}$$

$$\therefore V_1 = \frac{V_0}{273.15} \theta_1 + V_0$$

$$\text{or } V_1 = V_0 + \left(\frac{V_0}{273.15} \right) \theta_1$$

Hence Charles' law can also be stated as under :

Pressure remaining constant, the volume of a given mass of a gas increases or decreases by

$\frac{1}{273.15}$ of its volume at 0°C for each 1°C rise or fall in temperature.

☐☐ **[C] Gay lussac's LAW** : It states that "the pressure P of a given mass of an ideal gas is directly proportional to its absolute temperature T , provided that the Volume V of the gas remains constant".

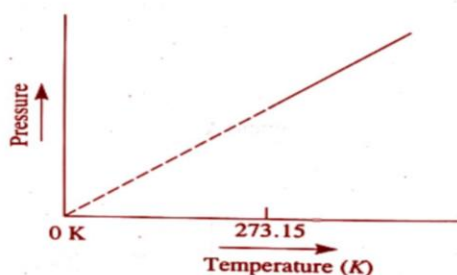
i.e., $P \propto T$ [V = Constant]

proof: A / Kinetic theory of gases, For an ideal gas, $P = \frac{1}{3} \times \frac{M}{V} C^2$

$P \propto C^2$ {since, $M/V = \text{Constant}$ }

$P \propto T$ {Since, $C^2 \propto T$ }

Gay lussac's law.



Thus if a given mass of gas has a pressure P_1 at $\theta_1^\circ\text{C}$ and is heated at constant volume to $\theta_2^\circ\text{C}$, its new pressure P_2 is given by ;

$$\frac{P_2}{P_1} = \frac{273.15 + \theta_2}{273.15 + \theta_1} = \frac{T_2}{T_1}$$

Another statement of Gay Lussac's Law.

Pressure of gas at 0°C ($= 273.15 \text{ K}$) $= P_0$

Slope of the graph, $m = \frac{P_0}{273.15}$

\therefore Pressure at $\theta_1^\circ\text{C}$ is given by ;

$$P_1 = P_0 + m\theta_1 = P_0 + \left(\frac{P_0}{273.15}\right)\theta_1$$

$$\therefore P_1 = P_0 + \left(\frac{P_0}{273.15}\right)\theta_1$$

volume remaining constant, the pressure of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its pressure at 0°C for each 1°C rise or fall in temperature.

☐☐ **[D] Derivation of Avogadro's hypothesis** : It states that "equal volume of all gases at the same temperature & pressure contains equal no. of moles".

proof: Consider two samples A and B of the gases each having volume V , Pressure P and temperature T .

Let, m_1 & m_2 = mass of each molecule of gas A and B respectively.

N_1 & N_2 = No. molecules of gas A and B respectively.

A / Kinetic theory of gases, For an ideal gas, $P = \frac{1}{3} \times \frac{M}{V} C^2$

\therefore **For Gas 'A':** $P_1 = \frac{1}{3} \times \frac{m_1 N_1 C_1^2}{V}$ where C_1 = rms velocity of the molecule of gas A.

For Gas 'B': $P_2 = \frac{1}{3} \times \frac{m_2 N_2 C_2^2}{V}$ where C_2 = rms velocity of the molecule of gas B.

But, $P_1 = P_2$

$$\therefore \frac{1}{3} \times \frac{m_1 N_1 C_1^2}{V} = \frac{1}{3} \times \frac{m_2 N_2 C_2^2}{V}$$

$$m_1 N_1 C_1^2 = m_2 N_2 C_2^2 \text{ ----- [1]}$$

Since temperature of A & B are same. Therefore,
 Average K.E of molecules of A = Average K.E of molecules of B.

$$\frac{1}{2} m_1 C_1^2 = \frac{1}{2} m_2 C_2^2$$

$$m_1 C_1^2 = m_2 C_2^2 \quad \text{----- [2]}$$

From [1] & [2], we get,

$$\boxed{N_1 = N_2} \quad \text{Avogadro's hypothesis.}$$

Grahams law of diffusion of gases : "It states that the rates of diffusion of two gases are inversely proportional to the square of their densities".

A / Kinetic theory of gases, For an ideal gas, $P = \frac{1}{3} \times \rho C^2$
 r m s Velocity of the gas at given temperature.

$$C = \sqrt{\frac{3P}{\rho}}$$

or, $C = \sqrt{\frac{1}{\rho}}$ (as when steady state of diffusion is reached then, $P_1 = P_2 = \text{constant}$)

Since r m s velocity is proportional to the rate of diffusion(r) of the gas, therefore, $r \propto \sqrt{\frac{1}{\rho}}$ i.e., denser the gas, the slower is the rate of diffusion.

Dalton's law of partial pressure : According to this law "the resultant pressure exerted by a mixture of non-interacting gases is equal to the sum of their individual pressure".

proof: Let us consider a no. of gases or vapour mixed together in a vessel. It is assumed that they do not interact with one another in any way. Let $\rho_1, \rho_2, \rho_3, \dots, \rho_n$ be their densities and $C_1, C_2, C_3, \dots, C_n$ be their rms velocity.

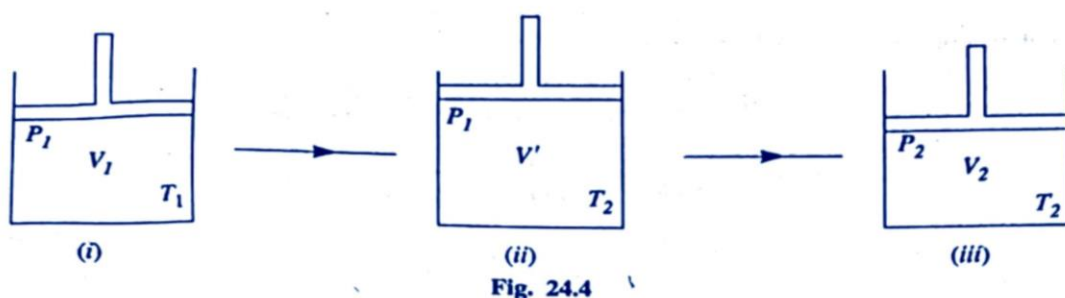
$$\therefore \text{Pressure exerted by the mixture of gases is, } P = \frac{1}{3} \rho_1 C_1^2 + \frac{1}{3} \rho_2 C_2^2 + \dots + \frac{1}{3} \rho_n C_n^2$$

But, $P_1 = \frac{1}{3} \rho_1 C_1^2$; $P_2 = \frac{1}{3} \rho_2 C_2^2$ $P_n = \frac{1}{3} \rho_n C_n^2$

$$\therefore \boxed{P = P_1 + P_2 + \dots + P_n} \quad \text{Dalton's law of partial pressure.}$$

PERFECT GAS EQUATION

The laws of Charles and Boyle can be combined to obtain the general relationship between pressure, volume and temperature of a given mass of gas. This relationship is called perfect gas equation.



In Fig. 24.4 (i), we have the gas occupying a volume V_1 at a pressure P_1 and an absolute temperature T_1 . It is desired to calculate its volume V_2 at an absolute temperature T_2 and pressure P_2 as at Fig. 24.4 (iii). We proceed via Fig. 24.4 (ii), raising the temperature to T_2 while keeping the pressure constant at P_1 . If V' is the volume of the gas in Fig. 24.4 (ii), then by Charles' law,

$$\frac{V'}{V_1} = \frac{T_2}{T_1} \quad \dots(i)$$

We proceed now to Fig. 24.4 (iii), by increasing the pressure to P_2 while keeping the temperature constant at T_2 . By Boyle's law,

$$\frac{V_2}{V'} = \frac{P_1}{P_2} \quad \dots(ii)$$

Eliminating V' between eqs. (i) and (ii), we have,

$$\frac{V_2}{V_1} = \frac{T_2 P_1}{T_1 P_2}$$

or $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

or $\frac{PV}{T} = \text{Constant}$
 or $PV = \text{Constant} \times T$
 or $PV_m = RT$ or $PV = nRT$... (iii)

where V_m is the volume of one mole of a gas and V is the volume occupied by n moles. The constant R is called *universal gas constant* and has the same value for all gases. The experimentally measured value of R is

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

The amount of gas in a sample is measured by the number of moles n given by ;

$$n = \frac{m}{M}$$

where

m = mass of the gas sample

M = atomic or molecular mass of the gas

- ▣ (i) Eq. (iii) is called *perfect (or ideal) gas equation*.
- ▣ (ii) An ideal gas obeys the gas laws and $PV = nRT$ exactly. No such gas exists.
- ▣ (iii) We use the term ideal or perfect because the real gases do not follow eq. (iii). However, perfect gas equation gives reasonable accuracy for many real gases of low density at temperatures well above the temperature at which the gas condenses to a liquid.

▣ ▣ UNIVERSAL GAS CONSTANT

For n moles of a gas, $PV = nRT$

or $R = \frac{PV}{nT} = \frac{\text{pressure} \times \text{volume}}{\text{number of moles} \times \text{temperature}}$

∴ Units of $R = \frac{\text{Nm}^{-2} \times \text{m}^3}{\text{Number of moles} \times \text{K}} = \frac{\text{Nm}}{\text{Number of moles} \times \text{K}}$
 $= \text{J mol}^{-1} \text{K}^{-1}$ (∵ $\text{Nm} = \text{J}$)

▣ ▣ **Numerical value of R .** The fact that constant R has the same value for all gases is a remarkable reflection of the simplicity in nature. From Avogadro's hypothesis, it follows that one mole of all gases, at the same temperature and pressure, occupy equal volumes; at S.T.P. 1 mole of any gas occupies 22.4 litres. Consequently, if V_m is the volume of 1 mole, then the ratio PV_m/T is the same for all gases. So the universal gas constant, which is equal to this ratio, is the *same for all gases*.

At S.T.P., $V_m = 22.4 \text{ litres} = 22.4 \times 10^{-3} \text{ m}^3$
 $P = 760 \text{ mm of Hg} = h \rho g = (760 \times 10^{-3}) \times (13600) \times 9.8$
 $= 1.013 \times 10^5 \text{ N m}^{-2}$
 $T = 273.15 \text{ K}$

∴ $R = \frac{PV_m}{T} = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273.15} = 8.31 \text{ J mol}^{-1} \text{K}^{-1}$

▣ ▣ (i) $R = \frac{PV}{nT} = \frac{\text{Pressure} \times \text{Volume}}{\text{Number of moles} \times \text{Temperature}}$
 $= \frac{\text{Work Done}}{\text{Number of moles} \times \text{Temperature}}$

Therefore, gas constant indicates the work done by (or on) a gas per mole per kelvin.

- ▣ ▣ (ii) If M is the molecular weight of the gas and v is the volume of 1 gram of a gas at pressure P and temperature T , then perfect gas equation for 1 gram of gas is

$$Pv = \frac{R}{M}T$$

or $Pv = rT$

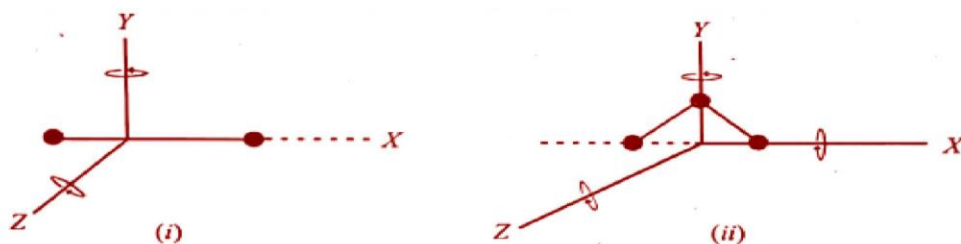


Fig. 24.7

■ (iii) A polyatomic molecule, unless it happens to consist of molecules all in a straight line, has no axis about which its moment of inertia is negligible. It can, therefore, have kinetic energy of rotation about three mutually perpendicular axes as shown in Fig. 24.7 (ii). It has three degrees of rotational freedom. In addition, it has three degrees of translational freedom. Therefore, a polyatomic molecule has *six degrees of freedom*.

In general: The degrees of freedom of the system is given by ;

$$f = 3N - k$$

where

f = degrees of freedom

N = number of particles in the system

k = independent relations among the particles

(i) For a monoatomic gas, $N = 1$ and $k = 0$.

$$\therefore f = 3 \times 1 - 0 = 3$$

Therefore, a monoatomic gas molecule has 3 degrees of freedom.

(ii) For a diatomic gas, $N = 2$, $k = 1$.

$$\therefore f = 3 \times 2 - 1 = 5$$

Therefore, a diatomic gas molecule has 5 degrees of freedom.

(iii) For a triatomic gas, $N = 3$, $k = 3$.

$$\therefore f = 3 \times 3 - 3 = 6$$

Therefore, a triatomic gas molecule has 6 degrees of freedom.

At very high temperatures (above 5000K), a gas molecule possesses vibratory motion also in addition to translatory and rotator motion. However, at ordinary temperature, a gas molecule possesses translational rotational motion only.

□ □ LAW OF EQUIPARTITION OF ENERGY

We know that average kinetic energy of a monoatomic molecule for temperature T is

$$\text{Average K.E. of monoatomic molecule} = \frac{3}{2}kT$$

Since a monoatomic gas has 3 degrees of freedom,

$$\therefore \text{Average K.E. in each degree of freedom} = \frac{1}{2}kT$$

Maxwell assumed that the average kinetic energy of a molecule in each degree of freedom (translational as well as rotational) is the same and is equal to $\frac{1}{2}kT$. This assumption is called *law of equipartition of energy* and may be stated as under :

The average kinetic energy of a molecule in each degree of freedom (translational as well as rotational) is the same and is equal to $\frac{1}{2}kT$.

Experiments show that this is true at room temperature and above. At very low temperatures, when the gas is near liquefaction, it fails. At ordinary temperatures, then, we have,

$$\text{Average K.E. of monoatomic molecule} = 3 \times \frac{1}{2}kT(\text{trans.}) = \frac{3}{2}kT$$

$$\text{Average K.E. of diatomic molecule} = 3 \times \frac{1}{2}kT(\text{trans.}) + 2 \times \frac{1}{2}kT(\text{rot.}) = \frac{5}{2}kT$$

$$\text{Average K.E. of polyatomic molecule} = 3 \times \frac{1}{2}kT(\text{trans.}) + 3 \times \frac{1}{2}kT(\text{rot.}) = \frac{6}{2}kT$$

MEAN FREE PATH

“Mean free path of a molecule in a gas is the average distance traveled by molecule between two successive collision (λ)”

EXPLANATION :The molecule of a gas move with high speed at a given temperature but even than a molecule of the gas takes a very long time to go from one point to another point in the container of the gas. This is because the molecule suffers a no. of collision with other gas molecule surrounding it, As a result of these collision, the path followed by a gas molecule in the container of the gas zig - zag.

During two successive collisions, a molecule in the container of the gas moves in a straight line with constant velocity and the distance traveled by a gas molecule between two successive collision is known as free path.

The distance traveled by a gas molecule between two successive collision is not constant & hence the average distance traveled by a molecule during all collision is to be calculated . This average distance traveled by a gas molecule is known as **Mean free path**.

Let, $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ be the distance traveled by a gas molecule during in collision respectively .
 \therefore Mean free path of a gas molecule.

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n}{n}$$

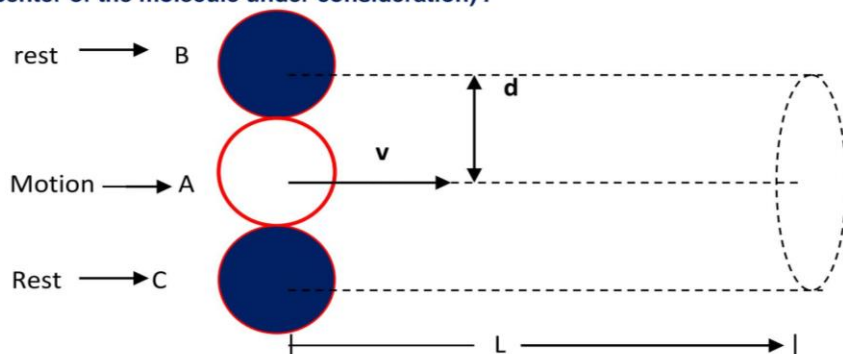
Expression:--

Assumption (i) The molecule of a gas are considered as hard sphere each of diameter 'd' .

(ii) The collision between gas molecule are perfectly elastic .

(iii) All molecule of a gas except the molecule under consideration are at rest.

(iv) A molecule of a gas under consideration collides with all those molecule whose center are at a rest distance 'd'.(from the center of the molecule under consideration) .



Consider a gas in a container having 'n' molecule / unit volume .

Let d = diameter of the molecule A (assumed to be in motion).

The molecule A collides with B & C whose centers are at a distance 'd' from the center of the A. If the molecule moves a distance 'L' then this molecule makes collision with all the molecule lying inside cylinder of volume $\pi d^2 L$.

The no. of molecule in a cylinder of vol. $\pi d^2 L$.

$$= \text{no. of molecule / vol} \times \text{vol. of the cylinder.} = n \times \pi d^2 L = \pi n d^2 L.$$

No. of collision suffered by A

$$= \text{no. of molecule in the cylinder of vol } \pi d^2 L = \pi n d^2 L.$$

Mean free path of molecule is

$$\lambda = \frac{\text{Total distance traveled}}{\text{No. of collision suffered}} = \frac{L}{\pi n d^2 L}$$

$$\lambda = \frac{1}{\pi n d^2}$$

i.e. , λ is inversely proportional to η .

Again, $\lambda = \frac{1}{\pi n d^2}$

is not true as it is our assumption that all the molecules are at rest. Actually, all the molecules are in random motion. So, the chances of a collision by a molecule is greater. Hence mean free path is less than as calculated. It has been estimated that the mean free path.

$$\lambda = \frac{1}{\sqrt{2} \pi n d^2}$$

$$\lambda = \frac{m}{\sqrt{2} \pi n d^2 m}$$

$$\lambda = \frac{m}{\sqrt{2} \pi n d^2 \rho}$$

[as $m n$ = mass per unit volume = density, ρ]

Again, $PV = RT$ or, $P = \frac{RT}{V}$ $= \frac{N}{V} \times \frac{R}{N} T$

$P = nKT$
 $n = \frac{P}{KT}$

Again,

$$\lambda = \frac{KT}{\sqrt{2\pi P} d^2}$$

FACTORS ON WHICH MEAN FREE PATH DEPENDS: --

- ▣ [1] Smaller the no. of molecule per unit volume of the gas, larger will be the mean free path.
- ▣ [2] Smaller the diameter of the molecule, larger will be the Mean free path.
- ▣ [3] Smaller the density of the molecule, larger will be the Mean free path
- ▣ [4] Smaller the pressure of the molecule, larger will be the Mean free path.
- ▣ [5] *Smaller is the temperature, lesser will be the mean free path.*

Kinetic Theory of Gases: In short – For memory Aid:

Kinetic Theory of Gases

The Kinetic Theory of Gases is a fundamental concept in physics that describes the behaviour of gases at the molecular level. It provides a microscopic understanding of gas properties, linking macroscopic observations such as pressure, temperature, and volume to the microscopic actions of gas molecules.

Assumptions of the Kinetic Theory of Gases

1. **Gas Molecules are Point Masses:** Gas molecules are considered as tiny, rigid, and perfectly spherical particles with negligible volume compared to the volume of the gas.
2. **Random Motion:** Gas molecules are in constant, random motion, colliding with each other and with the walls of the container.
3. **Elastic Collisions:** Collisions between gas molecules and between molecules and the container walls are perfectly elastic, meaning there is no loss of kinetic energy.
4. **No Intermolecular Forces:** There are no attractive or repulsive forces between the gas molecules except during collisions.
5. **Kinetic Energy and Temperature:** The average kinetic energy of gas molecules is directly proportional to the absolute temperature (in Kelvin) of the gas. This implies that $KE_{avg} \propto T$.

Key Concepts and Equations

1. **Pressure and Kinetic Energy:** The pressure exerted by a gas in a container is due to the collisions of gas molecules with the container walls. According to the kinetic theory, pressure P is given by:

$$P = \frac{1}{3} \frac{Nm v_{rms}^2}{V}$$

where N is the number of molecules, m is the mass of one molecule, v_{rms} is the root mean square speed of the molecules, and V is the volume of the container.

2. **Root Mean Square Speed (rms speed):** The rms speed is the square root of the average of the squares of the speeds of the gas molecules:

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

where k_B is the Boltzmann constant and T is the absolute temperature.

3. **Maxwell-Boltzmann Distribution:** This distribution describes the spread of speeds among the molecules in a gas. It shows that most molecules have speeds around a certain value, with fewer molecules at very low and very high speeds.
4. **Average Kinetic Energy:** The average kinetic energy of a gas molecule is given by:

$$KE = \frac{3}{2} k_B T$$

5. **Ideal Gas Equation:** The kinetic theory of gases is consistent with the ideal gas law:

$$PV = nRT$$

where P is the pressure, V is the volume, n is the number of moles, R is the universal gas constant, and T is the temperature.

Applications and Implications

1. **Temperature and Molecular Speed:** Higher temperatures correspond to higher average speeds of gas molecules, increasing the kinetic energy and pressure of the gas.
2. **Diffusion and Effusion:** The theory explains the phenomena of diffusion (spreading of gas molecules from high to low concentration) and effusion (escape of gas molecules through a small hole).
3. **Real Gases:** Deviations from the ideal behaviour occur at high pressures and low temperatures due to intermolecular forces and finite molecular volume, leading to modifications like the Van der Waals equation.