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PARABOLA C-S

MATHEMATICS

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01

PARABOLA
CONIC SECTION

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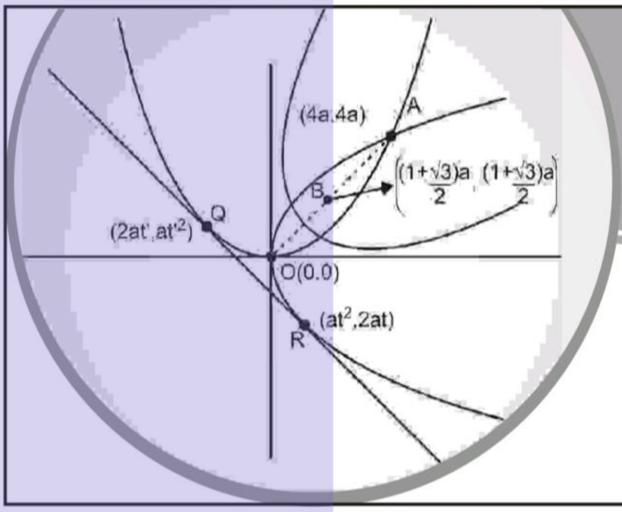


3 PARABOLA

KEY CONCEPTS

1. CONIC SECTIONS

A conic section, or conic is the locus of a point which moves in a plane so that its distance from a fixed point is in a constant ratio to its perpendicular distance from a fixed straight line.

The fixed point is called the Focus.

The fixed straight line is called the Directrix.

The constant ratio is called the Eccentricity denoted by e.

The line passing through the focus and perpendicular to the directrix is called the Axis.

A point of intersection of a conic with its axis is called a Vertex.

2. GENERAL EQUATION OF A CONIC: FOCAL DIRECTRIX PROPERTY

The general equation of a conic with focus (p, q) and directrix $lx + my + n = 0$ is :

$$(l^2 + m^2)[(x-p)^2 + (y-q)^2] = e^2(lx + my + n)^2 \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

3. DISTINGUISHING BETWEEN THE CONIC

The nature of the conic section depends upon the position of the focus S w.r.t. the directrix and also upon the value of the eccentricity e. Two different cases arise.

Case (I) : When The Focus Lies On The Directrix.

In this case $D \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and the general equation of a conic represents a pair of straight lines if :

$e > 1$ the lines will be real and distinct intersecting at S.

$e = 1$ the lines will coincident.

$e < 1$ the lines will be imaginary.

Case (II) : When The Focus Does Not Lie On Directrix.

a parabola	an ellipse	a hyperbola	rectangular hyperbola
$e=1; D\neq 0,$	$0<e<1; D\neq 0;$	$e>1; D\neq 0;$	$e>1; D\neq 0$
$h^2=ab$	$h^2<ab$	$h^2>ab$	$h^2>ab; a+b=0$

4. PARABOLA : DEFINITION

A parabola is the locus of a point which moves in a plane, such that its distance from a fixed point (focus) is equal to its perpendicular distance from a fixed straight line (directrix).

Standard equation of a parabola is $y^2=4ax$. For this parabola:

(i) Vertex is $(0,0)$ (ii) Focus is $(a,0)$ (iii) Axis is $y=0$ (iv) Directrix is $x+a=0$

Focal Distance

The distance of a point on the parabola from the focus is celled the **F**ocal **D**istance **O**f **T**he **P**oint.

Focal Chord

A chord of the parabola, which passes through the focus is called a **F**ocal **C**hord.

Double or Dinate

A chord of the parabola perpendicular to the axis of the symmetry is called a **D**ouble **O**rdinate.

Latus Rectum

A double ordinate passing through the focus or a focal chord perpendicular to the axis of parabola is called the Latus Rectum. For $y^2=4ax$.

Length of the latus rectum $=4a$.

ends of the latus rectum are $L(a,2a)$ and $L'(a,-2a)$.

Note that: (i) Perpendicular distance from focus on directrix = half the latus rectum.

(ii) Vertex is middle point of the focus and the point of intersection of directrix and axis.

(iii Two parabolas are laid to be equal if they have the same latus rectum.

Four standard forms of the parabola are $y^2=4ax$; $y^2=-4ax$; $x^2=4ay$; $x^2=-4ay$

5. POSITION OF A POINT RELATIVE TO PARABOLA

The point $(x_1\ y_1)$ lies outside, on or inside the parabola $y^2=4ax$ according as the expression $y_1^2-4ax_1$ is positive, zero or negative.

6. LINE AND A PARABOLA

The line $y=mx+c$ meets the parabola $y^2=4ax$ in two points real, coincident or imaginary according as $a \lesseqgtr cm \Rightarrow$ condition of tangency is, $c=\dfrac{a}{m}$.

7. Length of the chord intercepted by the parabola on the line $y=mx+c$ is: $\left(\dfrac{4}{m^2}\right)\sqrt{a(1+m^2)(a-mc)}$.

Note: *length of the focal chord making an angle α with the x-axis is* $4a\,\mathrm{Cosec}^2\alpha$.

8. PARAMETRIC REPRESENTATION

The simplest and the best form of representing the co-ordinates of a point on the parabola is $(at^2, 2at)$.

The equations $x = at^2$ and $y = 2at$ together represents the parabola $y^2 = 4ax, t$ being the parameter. The equation of a chord joining t_1 and t_2 is $2x - (t_1 + t_2)y + 2at_1t_2 = 0$.

Note: *If the chord joining* t_1, t_2 and t_3, t_4 *pass through a point* $(c, 0)$ *on the axis, then* $t_1t_2 = t_3t_4 = -c/a$.

9. TANGENTS TO THE PARABOLA $y^2 = 4ax$

(i) $yy_1 = 2a(x + x_1)$ at the point (x_1, y_1) ; **(ii)** $y = mx + \frac{a}{m}(m \neq 0)$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

(iii) $ty = x + at^2$ at $(at^2, 2at)$.

Note: *Point of intersection of the tangents at the point* t_1 *and* t_2 *is* $[at_1t_2, a(t_1 + t_2)]$.

10. NORMALS TO THE PARABOLA $y^2 = 4ax$

(i) $y - y_1 = -\frac{y_1}{2a}(x - x_1)$ at $((x_1, y_1)$;

(ii) $y = mx - 2am - am^3$ at $(am^2 - 2am)$

(iii) $y + tx = 2at + at^3$ at $(at^2, 2at)$

Note: *Point of intersection of normals at* t_1 *and* t_2 *are,* $a(t_1^2 + t_2^2 + t_1t_2 + 2); -at_1t_2(t_1 + t_2)$.

11. THREE VERY IMPORTANT RESULTS :

(a) If t_1 and t_2 are the ends of a focal chord of the parabola $y^2 = 4ax$ then $t_1t_2 = -1$. Hence the co-ordinates at the extremities of a focal chord can be taken as $(at^2, 2at)$ and $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$.

(b) If the normals to the parabola $y^2 = 4ax$ at the point t_1, meets the parabola again at the point t_2, then $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

(c) If the normals to the parabola $y^2 = 4ax$ at the points t_1 and t_2 intersect again on the parabola at the point 't_3' then $t_1t_2 = 2; t_3 = -(t_1 + t_2)$ and the line joining t_1 and t_2 passes through a fixed point $(-2a, 0)$.

General Note:

(i) Length of subtangent at any point $P(x, y)$ on the parabola $y^2 = 4ax$ equals twice the abscissa of the point P. Note that the subtangent is bisected at the vertex.

(ii) Length of subnormal is constant for all points on the parabola and is equal to the semi latus rectum.

(iii) If a family of straight lines can be represented by an equation $\lambda^2 P + \lambda Q + R = 0$ where λ is a parameter and P, Q, R are linear functions of x and y then the family of lines will be tangent to the curve $Q^2 = 4PR$

12. The equation to the pair of tangents which can be drawn from any point $(x_1 y_1)$ to the parabola $y^2 = 4ax$ is given by : $SS_1 = T^2$ where

$S \equiv y^2 - 4ax$; $\quad S_1 = y_1^2 - 4ax_1$; $\quad T \equiv y\,y_1 - 2a(x + x_1)$.

13. DIRECTOR CIRCLE

Locus of the point of intersection of the perpendicular tangents to the parabola $y^2 = 4ax$ is called the **D**irector **C**ircle. It's equation is $x + a = 0$ which is parabola's own directrix.

14. CHORD OF CONTACT

Equation to the chord of contact of tangents drawn from a point $P(x_1, y_1)$ is $yy_1 = 2a(x + x_1)$. Remember that the area of the triangle formed by the tangents from the point (x_1, y_1) and the chord of contact is $(y_1^2 - 4ax_1)^{3/2} \div 2a$. Also note that the chord of contact exists only if the point P is not inside.

15. POLAR AND POLE

(i) Equation of the Polar of the point $P(x_1, y_1)$ w.r.t. the parabola $y^2 = 4ax$ is.

$$y\,y_1 = 2a(x + x_1)$$

(ii) The pole of the line $lx + my + n = 0$ w.r.t. the parabola $y^2 = 4ax$ is $\left(\frac{n}{1}, -\frac{2am}{1}\right)$.

Note: *(i) The polar of the focus of the parabola is the directrix.*

(ii) When the point (x_1, y_1) lies without the parabola the equation of its polar is the same as the equation to the chord of contact of tangents drawn from (x_1, y_1) when (x_1, y_1) is on the parabola the polar is the same as the tangent at the point.

(iii) If the polar of a point P passes through the point Q, then the polar of Q goes through P.

(iv) Two straight lines are said to be conjugated to each other w.r.t. a parabola when the pole of one lies on the other.

(v) Polar of a given point P w.r.t. any Conic is the locus of the harmonic conjugate of P w.r.t. the two points is which any line through P cuts the conic.

16. CHORD WITH A GIVEN MIDDLE POINT

Equation of the chord of the parabola $y^2 = 4ax$ whose middle point is

(x_1, y_1) is $y - y_1 = \dfrac{2a}{y_1}(x - x_1)$. This reduced to $T = S_1$

where $T \equiv y\,y_1 - 2a(x + x_1)$ and $S_1 \equiv y_1^2 - 4ax_1$.

17. DIAMETER

The locus of the middle points of a system of parallel chords of a Parabola is called a Diameter. Equation to the diameter of a parabola is $y = 2a/m$, where m = slope of parallel chords.

Note:

(i) The tangent at the extremity of a diameter of a parabola is parallel to the system of chords it bisects.

(ii) The tangent at the ends of any chords of a parabola meet on the diameter which bisects the chord.

(iii) A line segment from a point P on the parabola and parallel to the system of parallel chords is called the ordinate to the diameter bisecting the system of parallel chords and the chords are called its double ordinate.

18. IMPORTANT HIGHLIGHTS

(a) If the tangent and normal at any point 'P' of the parabola intersect the axis at T and G then $ST = SG = SP$ where 'S' is the focus. In other words the tangent and the normal at a point P on the parabola are the bisectors of the angle between the focal radius and the perpendicular from P on the directrix. From this we conclude that all rays emanating from S will become parallel to the axis of the parabola after reflection.

(b) The portion of a tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus.

(c) The tangents at the extremities of a focal chord intersect at right angles on the **directrix**, and hence a circle on any focal chord as diameter touches the directrix. Also a circle on any focal radii of a point $P(at^2, 2at)$ as diameter touches the tangent at the vertex and intercepts a chord of length $a\sqrt{1+t^2}$ on a normal at the point P.

(d) Any tangent to a parabola and the perpendicular on it from the focus meet on the tangent at the vertex.

(e) If the tangents at P and Q meet in T, then :

(i) TP and TQ subtend equal angles at the focus S.

(ii) $ST^2 = SP.SQ$ and The triangles SPT and STQ are similar.

(f) Tangents and Normals at the extremities of the latus rectum of a parabola $y^2 = 4ax$ constitute a square, their points of intersection being $(-a, 0)$ and $(3a, 0)$.

(g) Semi latus rectum of the parabola $y^2 = 4ax$, is the harmonic mean between segments of any focal chord of the parabola is ; $2a = \frac{2bc}{b+c}$ $i.e., \frac{1}{b} + \frac{1}{c} = \frac{1}{a}$.

(h) The circle circumscribing the triangle formed by any three tangents to a parabola passes through the focus.

(i) The orthocentre of any triangle formed by three tangents to a parabola $y^2 = 4ax$ lies on the directrix and has the co-ordinates $-a, a(t_1 + t_2 + t_3 + t_1t_2t_3)$.

(j) The area of the triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

(k) If normal drawn to a parabola passes through a point $P(h,k)$ then

$k = mh - 2am - am^3$ $i.e., am^3 + m(2a - h) + k = 0$.

Then gives $m_1 + m_2 + m_3 = 0$; $m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$; $m_1 m_2 m_3 = -\frac{k}{a}$.

where $m_1, m_2,$ and m_3 are the slopes of the three concurrent normals. Note that the algebraic sum of the:

- slopes of the three concurrent normals is zero.
- ordinates of the three co-normal points on the parabola is zero.
- Centroid of the Δ formed by three co-normal points lies on the x-axis.

(l) A circle circumscribing the triangle formed by three co-normal points passes through the vertex of the parabola and its equation is, $2(x^2 + y^2) - 2(h + 2a)x - ky = 0$, where (h,k) is the point of concurrence of three normals.

EXERCISE 1

Only One Choice is Correct:

1. Parabolas $(y-\alpha)^2 = 4a(x-\beta)$ and $(y-\alpha)^2 = 4a'(x-\beta')$ will have a common normal (other than the normal passing through vertex of parabola) if :

(a) $\frac{2(a-a')}{\beta'-\beta} < 1$ (b) $\frac{2(a-a')}{\beta-\beta'} < 1$

(c) $\frac{2(a'-a)}{\beta+\beta'} < 1$ (d) $\frac{2(a'-a)}{\beta+\beta'} > 1$

2. If the line $x+y-1=0$ is a tangent to a parabola with focus $(1,2)$ at A and intersects the directrix at B and tangent at vertex at C respectively, then $AC \cdot BC$ is equal to :

(a) 2   (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx+my+n=0$ subtends a right angle at vertex then :

(a) $al+n=0$ (b) $4am+n=0$

(c) $4al+n=0$ (d) $4am-n=0$

4. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then complete set of values of T satisfies :

(a) $T^2 \geq 8$ (b) $T \in (-\infty, -8) \cup (8, \infty)$

(c) $-2 \leq T \leq 2$ (d) $T^2 < 8$

5. If the line $y-\sqrt{3}\,x+3=0$ cuts the parabola $y^2 = x+2$ at A and B, and if $P \equiv (\sqrt{3}, 0)$, then $PA \cdot PB$ is equal to:

(a) $\frac{2(\sqrt{3}+2)}{3}$ (b) $\frac{4\sqrt{3}}{2}$

(c) $\frac{4(2-\sqrt{3})}{3}$ (d) $\frac{4(\sqrt{3}+2)}{3}$

6. $Px+2y=1$ is normal to parabola $y^2 = 4ax$ for :

(a) no value of P (b) exactly one value of P

(c) exactly two values of P (d) exactly three values of P

7. The vertex of a parabola is at $(3,2)$ and its directrix is the line $x-y+1=0$. Then the equation of its latus rectum is :

(a) $x-y=2$ (b) $x-y=3$

(c) $x-y=1$ (d) $x+y=2$

8. Locus of point of intersection of normals drawn at end points of focal chord of a parabola $y^2 = 4ax$ is :

(a) $y^2 = a(x-a)$ (b) $y^2 = a(x-2a)$

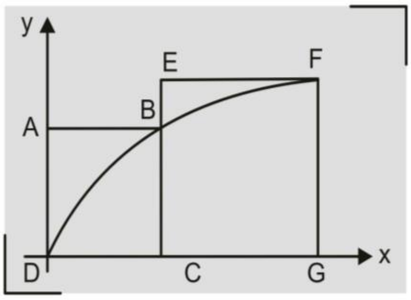
(c) $y^2 = a(x-3a)$ (d) $y^2 = a(x-4a)$

9. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet its axis in P_1 and P_2. If S is the focus of the parabola then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to :

(a) $\frac{4}{a}$ (b) $\frac{2}{a}$

(c) $\frac{1}{a}$ (d) $\frac{1}{4a}$

10. $ABCD$ and $EFGC$ are squares and the curve $y = k\sqrt{x}$ passes through the origin D and the points B and F. The ratio $\frac{FG}{BC}$ is :



(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{3}+1}{2}$

(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{\sqrt{3}+1}{4}$

11. The points of contact Q and R of tangent from the point $P(2,3)$ on the parabola $y^2 = 4x$ are :

(a) $(9,6)$ and $(1,2)$ (b) $(1,2)$ and $(4,4)$

(c) $(4,4)$ and $(9,6)$ (d) $(9,6)$ and $(1/4, 1)$

12. A tangent is drawn to the parabola $y^2 = 4x$ at the point 'P' whose abscissa lies in the interval $[1,4]$. The maximum possible area of the triangle formed by the tangent at 'P', ordinate of the point 'P' and the x-axis is equal to :

(a) 8 (b) 16

(c) 24 (d) 32

13. The set of points (x,y) whose distance from the line $y = 2x+2$ is the same as the distance from $(2,0)$ is a parabola. This parabola is congruent to the parabola in standard form $y = Kx^2$ for some K which is equal to :

(a) $\frac{\sqrt{5}}{12}$ (b) $\frac{\sqrt{5}}{4}$

(c) $\frac{4}{\sqrt{5}}$ (d) $\frac{12}{\sqrt{5}}$

14. If the normal to a parabola $y^2 = 4ax$ at P meets the curve again in Q and if PQ and the normal at Q makes angles α and β respectively with the x-axis then $\tan\alpha(\tan\alpha + \tan\beta)$ has the value equal to :

(a) 0 (b) −2
(c) −1/2 (d) −1

15. Let A and B be two points on a parabola $y^2 = x$ with vertex V such that VA is perpendicular to VB and θ is the angle between the chord VA and the axis of the parabola. The value of $\frac{|VA|}{|VB|}$ is :

(a) $\tan\theta$ (b) $\tan^3\theta$
(c) $\cot^2\theta$ (d) $\cot^3\theta$

16. A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha,0)(\beta,0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is :

(a) $\sqrt{\frac{bc}{a}}$ (b) ac^2
(c) $\frac{b}{a}$ (d) $\sqrt{\frac{c}{a}}$

17. C is the centre of the circle with centre $(0,1)$ and radius unity. P is the parabola $y = ax^2$. The set of values of 'a' for which they meet at a point other than the origin, is :

(a) $a > 0$ (b) $a \in \left(0, \frac{1}{2}\right)$
(c) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \infty\right)$

18. Through the vertex O of the parabola, $y^2 = 4ax$ two chords OP and OQ are drawn and the circles on OP and OQ as diameters intersect in R. If θ_1, θ_2 and ϕ are the angles made with the axis by the tangents at P and Q on the parabola and by OR, then the value of, $\cot\theta_1 + \cot\theta_2$ is equal to:

(a) $-2\tan\phi$ (b) $-2\tan(\pi - \phi)$
(c) 0 (d) $2\cot\phi$

19. Tangents are drawn from the points on the line $x - y + 3 = 0$ to parabola $y^2 = 8x$. Then the variable chords of contact pass through a fixed point whose co-ordinates are :

(a) $(3,2)$ (b) $(2,4)$ (c) $(3,4)$ (d) $(4,1)$

20. The latus rectum of a parabola whose focal chord PSQ is such that $SP = 3$ and $SQ = 2$ is given by:

(a) 24/5 (b) 12/5
(c) 6/5 (d) None of these

21. The equation of the other normal to the parabola $y^2 = 4ax$ which passes through the intersection of those at $(4a, -4a)$ and $(9a, -6a)$ is :

(a) $5x - y + 115a = 0$ (b) $5x + y - 135a = 0$
(c) $5x - y - 115a = 0$ (d) $5x + y + 115 = 0$

22. A common tangent is drawn to the circle $x^2 + y^2 = c^2$ and the parabola $y^2 = 4ax$. If the angle which this tangent makes with the axis of x is $\pi/4$ then the relationship between a and c $(a, c > 0)$ is:

(a) $a = \sqrt{2}\,c$ (b) $c = a\sqrt{2}$

(c) $a = 2c$ (d) $c = 2a$

23. The locus of the foot of the perpendiculars drawn from the vertex on a variable tangent to the parabola $y^2 = 4ax$ is :

(a) $x(x^2 + y^2) + ay^2 = 0$ (b) $y(x^2 + y^2) + ax^2 = 0$

(c) $x(x^2 - y^2) + ay^2 = 0$ (d) None of these

24. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is :

(a) $\frac{A}{2a}$ (b) $\frac{A}{a}$

(c) $\frac{2A}{a}$ (d) $\frac{4A}{a}$

25. The normal chord of a parabola $y^2 = 4ax$ at the point whose ordinate is equal to the abscissa, then angle subtended by normal chord at the focus is :

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}\sqrt{2}$

(c) $\tan^{-1} 2$ (d) $\frac{\pi}{2}$

26. Length of the intercept on the normal at the point $P(at^2, 2at)$ of the parabola $y^2 = 4ax$ made by the circle described on the focal distance of the point P as diameter is :

(a) $a\sqrt{2 + t^2}$ (b) $\frac{a}{2}\sqrt{1 + t^2}$

(c) $2a\sqrt{1 + t^2}$ (d) $a\sqrt{1 + t^2}$

27. In a parabola $y^2 = 4ax$ the angle θ that the latus rectum subtends at the vertex of the parabola is:

(a) dependent on the length of the latus rectum

(b) independent of the latus rectum and lies between $\frac{5\pi}{6}$ and π

(c) independent of the latus rectum and lies between $\frac{3\pi}{4}$ and $\frac{5\pi}{6}$

(d) independent of the latus rectum and lies between $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$

28. The distance between a tangent to the parabola $y^2 = 4Ax\,(A > 0)$ and the parallel normal with gradient 1 is :

(a) $4A$ (b) $2\sqrt{2}A$

(c) $2A$ (d) $\sqrt{2}A$

29. Tangents are drawn from the point $(-1,2)$ on the parabola $y^2 = 4x$. The length, these tangents will intercept on the line $x = 2$ is :

(a) 6 (b) $6\sqrt{2}$

(c) $2\sqrt{6}$ (d) none of these

30. The locus of the middle points of chords of the parabola $y^2 = 4x$, which are of constant length '$2l$' is :

(a) $(4x + y^2)(y^2 - 4) = 4l^2$ (b) $(4y + x^2)(x^2 - 4) = 4l^2$

(c) $(4y - x^2)(x^2 + 4) = 4l^2$ (d) $(4x - y^2)(y^2 + 4) = 4l^2$

31. In a square matrix A of order $3, a_{ii} = m_i + i$ where $i = 1,2,3$ and m_i's are the slopes (in increasing order of their absolute value) of the 3 normals concurrent at the point $(9, -6)$ to the parabola $y^2 = 4x$. Rest all other entries of the matrix are one. The value of det. (A) is equal to :

(a) 37 (b) –6

(c) –4 (d) –9

32. A circle C passes though the points of intersection of the parabola $y + 1 = (x - 4)^2$ and the x-axis. The length of tangent from origin to C is :

(a) 8 (b) 15

(c) $\sqrt{8}$ (d) $\sqrt{15}$

33. For the parabola $y^2 + 4x - 4y = 4$, the straight line $x - y + 3 = 0$ is :

(a) focal chord (b) normal chord

(c) both focal chord and normal chord (d) none of these

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(b)	**7.**	(b)	**8.**	(c)	**9.**	(c)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(a)	**14.**	(b)	**15.**	(d)	**16.**	(d)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(a)
21.	(b)	**22.**	(a)	**23.**	(a)	**24.**	(c)	**25.**	(d)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(d)
31.	(c)	**32.**	(d)	**33.**	(b)														

EXERCISE 2

One or More than One is/are Correct

1. If two distinct chords of a parabola $y^2 = 4ax$ passing through the point $(a, 2a)$ are bisected by line $x + y = 1$, then the length of the latus rectum can not be :

(a) 2 (b) 4

(c) 5 (d) 7

2. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola, then the slope of AB is :

(a) 2 (b) $\sqrt{2}$

(c) $-\sqrt{2}$ (d) None of these

3. Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. One normal is x-axis and the other two normals are perpendicular, then :

(a) $c > \frac{1}{2}$ (b) $0 < c < \frac{1}{2}$

(c) $c = \frac{3}{4}$ (d) $c = \frac{1}{2}$

4. Suppose that a normal drawn at a point $P(at^2, 2at)$ to parabola $y^2 = 4ax$ meets it again at Q. If the length of PQ is minimum, then :

(a) $t = \pm\sqrt{2}$ (b) $t = \pm\sqrt{3}$

(c) $PQ = 6\sqrt{3}$ (d) Q is $(8a, \pm 4\sqrt{2}\, a)$

5. P is a point on the parabola $y^2 = 4x$ and Q is a point on the line $2x + y + 4 = 0$. If the line $x - y + 1 = 0$ is the perpendicular bisector of PQ, then the co-ordinates of P can be :

(a) $(1, -2)$ (b) $(4, 4)$

(c) $(9, -6)$ (d) $(16, 8)$

6. If the tangents to the parabola $y^2 = 4ax$ at (x_1, y_1) and (x_2, y_2) meet at (x_3, y_3), then :

(a) x_1, x_3, x_2 are in A.P. (b) x_1, x_3, x_2 are in G.P.

(c) y_1, y_3, y_2 are in G.P. (d) y_1, y_3, y_2 are in A.P.

7. A variable chord PQ of the parabola $y^2 = 4ax$ is drawn parallel to the line $y = x$. If the parameter of the points P and Q on the parabola be t_1 and t_2 respectively, then :

(a) $t_1 + t_2 = 2$

(b) $t_1 t_2 = \frac{2}{a}$

(c) locus of point of intersection of tangents at P and Q is $y = 2a$

(d) locus of point of intersection of normals at P and Q is $2x - y = 12a$

8. If P_1P_2 and Q_1Q_2, two focal chords of a parabola are at right angles, then :

(a) area of the quadrilateral $P_1Q_1P_2Q_2$ is minimum when the chords are inclined at an angle $\frac{\pi}{4}$ to the axis of the parabola

(b) minimum area is twice the area of the square on the latus rectum of the parabola

(c) minimum area of $P_1Q_1P_2Q_2$ cannot be found

(d) minimum area is thrice the area of the square on the latus rectum of the parabola

9. Let there be two parabolas with the same axis, focus of each being exterior to the other and the latus recta being $4a$ and $4b$. The locus of the middle points of the intercepts between the parabolas made on the lines parallel to the common axis is a :

(a) straight line if $a = b$ (b) parabola if $a \neq b$

(c) parabola $\forall\, a, b \in R$ (d) none of these

10. Let $y^2 = 4ax$ be a parabola and $x^2 - y^2 = a^2$ be a hyperbola. Then number of common tangents is :

(a) 2 for $a < 0$ (b) 1 for $a < 0$

(c) 2 for $a > 0$ (d) 1 for $a > 0$

11. The chord AB of the parabola $y^2 = 4ax$ cuts the axis of the parabola at C. If $A \equiv (at_1^2, 2at_1)$ and $B \equiv (at_2^2, 2at_2)$ and $AC : AB = 1:3$, then :

(a) $t_2 = 2t_1$ (b) $t_2 + 2t_1 = 0$

(c) $t_1 + 2t_2 = 0$ (d) $6t_1^2 = t_2(t_1 + 2t_2)$

12. A line L passing through the focus of the parabola $y^2 = 4(x-1)$ intersects the parabola in two distinct points. If 'm' be the slope of the line L, then :

(a) $m \in (-\infty, 0)$ (b) $m \in [0, \infty)$

(c) $m \in (0, \infty)$ (d) none of these

13. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is :

(a) $\left(\frac{p}{2}, p\right)$ (b) $\left(\frac{p}{2}, -p\right)$

(c) $\left(\frac{-p}{2}, p\right)$ (d) $\left(\frac{-p}{2}, \frac{-p}{2}\right)$

14. Variable circle is described to pass through point (1,0) and tangent to the curve $y = \tan(\tan^{-1} x)$. The locus of the centre of the circle is a parabola whose :

(a) length of the latus rectum is $2\sqrt{2}$

(b) axis of symmetry has the equation $x + y = 1$

(c) vertex has the co-ordinates $(3/4, 1/4)$

(d) none of these

15. The range of α for which the points $(\alpha, 2+\alpha)$ and $\left(\frac{3}{2}\alpha, \alpha^2\right)$ lie on opposite sides of the line $2x+3y=6$ can lie in intervals :

(a) $(-\infty, -2)$ (b) $(-2, 0)$
(c) $(0, 1)$ (d) $(2, 4)$

16. If the point $\left(\sin\theta, \frac{1}{\sqrt{2}}\right)$ lies exterior to both the parabolas $y^2 = |x|$, then θ can belong to :

(a) $\left(0, \frac{\pi}{6}\right)$ (b) $\left(-\frac{\pi}{6}, 0\right)$
(c) $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$ (d) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

17. Equation of a common tangent to the circle, $x^2+y^2=50$ and the parabola, $y^2=40x$ can be :

(a) $x+y-10=0$ (b) $x-y+10=0$
(c) $x+y+10=0$ (d) $x-y-10=0$

18. Let $y^2=4ax$ be a parabola and $x^2+y^2+2\ bx=0$ be a circle. If parabola and circle touch each other externally then :

(a) $a>0,\ b>0$ (b) $a>0,\ b<0$
(c) $a<0,\ b>0$ (d) $a<0,\ b<0$

19. If from the vertex of a parabola $y^2=4ax$ a pair of chords be drawn at right angles to one another and with these chords as adjacent sides a rectangle be made, then the locus of the further angle of the rectangle is :

(a) an equal parabola (b) a parabola with focus at $(8a, 0)$
(c) a parabola with directrix as $x-7a=0$ (d) not a parabola

20. Through a point $P(-2,0)$, tangents PQ and PR are drawn to the parabola $y^2=8x$. Two circles each passing through the focus of the parabola and one touching parabola at Q and other at R are drawn. Which of the following point(s) with respect to the triangle PQR lie(s) on the common chord of the two circles ?

(a) centroid (b) orthocentre
(c) incentre (d) circumcentre

21. If two distinct chords of parabola $y^2=4ax(a>0)$ passing through $(a, 2a)$ are bisected by the line $x+y=1$; then the length of the latus rectum can be:

(a) 1 (b) 4 (c) 3 (d) 2

ANSWERS

1.	(b, c, d)	**2.**	(b, c)	**3.**	(a, c)	**4.**	(a, c, d)	**5.**	(a, c)	**6.**	(b, d)
7.	(a, c, d)	**8.**	(a, b)	**9.**	(a, b)	**10.**	(a, c)	**11.**	(b, d)	**12.**	(a, c)
13.	(a, b)	**14.**	(b, c)	**15.**	(a, c)	**16.**	(a, b, c)	**17.**	(b, c)	**18.**	(a, d)
19.	(a, c)	**20.**	(a, b, c, d)	**21.**	(a, c, d)						

EXERCISE 3

Comprehension: (1)

Let from a point $A(h,k)$, 3 distinct normals can be drawn to parabola $y^2 = 4ax$ and the feet of these normals on parabola be points $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ and $R(at_3^2, 2at_3)$

1. The controid of ΔPAQ has co-ordinates :

(a) $\left(\frac{2}{3}(h-2a), 0\right)$ (b) $\left(\frac{2}{3}(h-3a), 0\right)$

(c) $\left(\frac{2}{3}(2h-a), 0\right)$ (d) $\left(\frac{2}{3}(h-a), 0\right)$

2. If tangents at P and Q to parabola $y^2 = 4ax$ meet on line $x = -a$, then t_1, t_2 are the roots of the equation :

(a) $x^2 - t_3 x + 1 = 0$ (b) $x^2 + t_3 x + 1 = 0$

(c) $x^2 - t_3 x - 1 = 0$ (d) $x^2 + t_3 x - 1 = 0$

3. Let the point A varies such that the points P and Q are the ends of a focal chord then locus of point A is :

(a) $y^2 = a(x-2a)$ (b) $y^2 = a(x-a)$

(c) $y^2 = a(x-3a)$ (d) $y^2 = 3a(x-a)$

Comprehension: (2)

A tangent is drawn at any point P on the parabola $y^2 = 8x$ and on it is taken a point $Q(\alpha, \beta)$ from which pair of tangents QA and QB are drawn to circle $x^2 + y^2 = 4$.

1. The locus of point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 4$ is :

(a) $y^2 - 2x = 0$ (b) $y^2 - x^2 = 4$

(c) $y^2 + 2x = 0$ (d) $y^2 - 2x^2 = 4$

2. The points from which perpendicular tangents can be drawn both to the given circle and the parabola is :

(a) $(4, \pm\sqrt{3})$ (b) $(-1, \sqrt{2})$

(c) $(-\sqrt{2}, -\sqrt{2})$ (d) $(-2, \pm 2)$

3. The locus of circumcentre of ΔAQB if $P \equiv (8,8)$ is :

(a) $x-2y+4=0$ (b) $x+2y-4=0$

(c) $x-2y-4=0$ (d) $x+2y+4=0$

Comprehension: (3)

Let the two parabolas $y^2 = 4ax$ and $x^2 = 4ay, a > 0$ intersect at O and A(O being origin). Parabola P whose directrix is the common tangent to the two parabolas and whose focus is the point which divides OA internally in the ratio $(1+\sqrt{3}):(7-\sqrt{3})$

1. The equation of the common tangent to $y^2 = 4ax$ and $x^2 = 4ay$ is :

(a) $x+y+a=0$ (b) $x+y-a=0$

(c) $x-y+a=0$ (d) $x-y-a=0$

2. The equation of the Parabola P is :

(a) $(x-y)^2 = (2+\sqrt{3})a(x+y-(1+\sqrt{3})a)$

(b) $(x-y)^2 = (2+\sqrt{3})a(2x+2y-(2+\sqrt{3})a)$

(c) $(x-y)^2 = (2+\sqrt{3})a(2x+2y-(1+\sqrt{3})a)$

(d) $(x-y)^2 = (2-\sqrt{3})a(x+y-(1+\sqrt{3})a)$

3. Extremities of latus rectum of P are :

(a) $\left(\frac{a}{2}, \frac{(3+2\sqrt{3})a}{2}\right), \left(\frac{(3+2\sqrt{3})a}{2}, \frac{a}{2}\right)$ (b) $\left(-\frac{a}{2}, \frac{(3-\sqrt{3})a}{2}\right), \left(\frac{(3-\sqrt{3})a}{2}, -\frac{a}{2}\right)$

(c) $\left(\frac{a}{2}, \frac{(3-\sqrt{3})a}{2}\right), \left(\frac{(3-\sqrt{3})a}{2}, \frac{a}{2}\right)$ (d) $\left(-\frac{a}{2}, \frac{(3+2\sqrt{3})a}{2}\right), \left(\frac{(3+2\sqrt{3})a}{2}, -\frac{a}{2}\right)$

Comprehension: (4)

$y = f(x)$ is a parabola of the form $f(x) = x^2 + bx + 1$, b is a constant. The tangent line is drawn at the point where $f(x)$ cuts y-axis, also touches $x^2 + y^2 = r^2 (r > 0)$. It is also given that at least one tangent can be drawn from point P to $y = f(x)$ where P is a point at which $y = |x-\alpha|$ is non differentiable $\forall \alpha \in R$.

1. For maximum value of b, the area of circle is :

(a) $\frac{\pi}{10}$ (b) $\frac{\pi}{5}$

(c) π (d) 5π

2. $\lim\limits_{b \to 0} \dfrac{\sqrt{r_{\max.}} - r}{\sin b} =$

(a) $\dfrac{1}{\sqrt{2}}$ (b) $-\dfrac{1}{\sqrt{2}}$

(c) $\dfrac{1}{2}$ (d) Not exist

3. Locus of vertex of parabola is :

(a) $y = 1 - x^2,\ x \in [-1,1],\ y \in [-1,0]$ (b) $y = 1 - x^2, x \in [-2,2], y \in [0,1]$

(c) $y = 1 - x^2, x \in [-2,2], y \in [-3,1]$ (d) $y = 1 - x^2, x \in [-1,1], y \in [0,1]$

Comprehension: (5)

The limiting value of expression $\dfrac{4x^2 + 2y^2 - 6xy}{6x^2 + \sqrt{2}y - 8xy}$ is A as point (x, y) on curve $x^2 + y^2 = 1$ approaches the position $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ where A is such that $(5A, 0)$ is a point as focus of parabola S having axis parallel to x-axis, vertex at origin.

1. The two common tangents can be drawn to both circle and parabola from external point whose co-ordinates are :

(a) $\left(\dfrac{-4}{\sqrt{15}-1}, 0\right)$ (b) $\left(\dfrac{-4}{\sqrt{17}+1}, 0\right)$

(c) $\left(\dfrac{-4}{\sqrt{17}-1}, 0\right)$ (d) $\left(\dfrac{-4}{\sqrt{15}+1}, 0\right)$

2. Locus of midpoints of chords of parabola, which subtend a right angle at vertex of parabola is :

(a) $y^2 - 4x + 32 = 0$ (b) $y^2 + 4x - 32 = 0$

(c) $y^2 - 32x + 4 = 0$ (d) $y^2 + 32x - 4 = 0$

3. Position of point $\left(\dfrac{1}{5}, 5A\right)$ with respect to circle is :

(a) inside (b) on circle

(c) outside (d) none of these

Comprehension: (6)

Let a tangent to parabola $y^2 = 4ax$ at point $P(at^2, 2at)$, $t \neq 0$ intersects its directrix at point Q. Let 'S' represents the focus of parabola $y^2 = 4ax$ and C represents the circle circumscribing the triangle PQS.

1. The angle between the parabola $y^2 = 4ax$ and the circle C at point P is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

2. If normal to parabola $y^2 = 4ax$ at point P intersects the line joining Q and S at R, then $\frac{(PS)(QR)}{(PQ)(PR)}$ is equal to :

(a) 4 (b) 3

(c) 2 (d) 1

3. Area of circle C is :

(a) $\frac{\pi a^2(1+t^2)^3}{8t^2}$ (b) $\frac{\pi a^2(1+t^2)^3}{4t^4}$

(c) $\frac{\pi a^2(1+t^2)^3}{4t^2}$ (d) $\frac{\pi a^2(1-t^2)^3}{4t^2}$

Comprehension: (7)

Consider one side AB of a square $ABCD$, (read in order) on the line $y = 2x - 17$, and the other two vertices C, D on the parabola $y = x^2$.

1. Minimum intercept of the line CD on y-axis, is :

(a) 3 (b) 4

(c) 2 (d) 6

2. Maximum possible area of the square $ABCD$ can be :

(a) 980 (b) 1160

(c) 1280 (d) 1520

3. The area enclosed by the line CD with minimum y-intercept and the parabola $y = x^2$ is :

(a) $\frac{15}{3}$ (b) $\frac{14}{3}$

(c) $\frac{22}{3}$ (d) $\frac{32}{3}$

Comprehension: (8)

The function f satisfies $f(x)+f(2x+y)+5xy=f(3x-y)+2x^2+1$ for all real numbers x, y. Let a chord to parabola $x^2=4y$, normals to parabola at ends of which satisfy the relation, $m_1m_2=-2$ where m_1, m_2 represent slope of normals, passes through a fixed point 'P' on axis of parabola. Let $y=g(x)$ represent line passing through point P.

1. The value of $f(10)$ is equal to :

(a) -61 (b) -49 (c) -21 (d) -10

2. The minimum area bounded by $y=g(x)$ & $y=f(x)$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

3. The tangent to $y=f(x)$ at $x=0$ has slope equal to :

(a) -1 (b) 0 (c) 1 (d) 2

4. Let $y=g(x)$ intersects $y=f(x)$ at two distinct points A, B, then the slope of $g(x)$ if length of segment AB is 4 units is :

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

Comprehension: (9)

Let $ABCD$ be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of the square $ABCD$. L is a line through A.

1. If P is a point on C_1 and Q in another point on C_2, then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to:

(a) 0.75 (b) 1.25

(c) 1 (d) 0.5

2. A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is:

(a) ellipse (b) hyperbola

(c) parabola (d) parts of straight line

3. A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is:

(a) 1/2 sq. units (b) 2/3 sq. units

(c) 1 sq. units (d) 2 sq. units

Comprehension: (10)

Let curve S_1 be the locus of a point $P(h, k)$ which moves in xy plane such that it always satisfy the relation min $\{x^2 + (h-k)x + (1-h-k)\} =$ max $\{-x^2 + (h+k)x - (1+h+k)\}$. Let S_2 is a curved mirror passing through (8, 6) having the property that all light says emerging from origin, after getting reflected from the mirror becomes parallel to x-axis. Also the area of region bounded between y-axis and S_2 is 8/3.

1. The area of smaller region bounded between S_1 and S_2 is equal to:

(a) 2π (b) $\pi - \frac{8}{3}$ (c) $\pi + \frac{8}{3}$ (d) $2\pi - \frac{8}{3}$

2. If the circle $(x-4)^2 + y^2 = r^2$ internally touches the curve S_2, then $r =$

(a) 5 (b) 4 (c) 3 (d) 2

3. The ratio in which the curve $y = \left[\sin^2 \frac{x}{4} + \cos \frac{x}{4}\right]$, where [.] denote greatest integer function divides the curve S_1 is:

(a) $4\pi + 3\sqrt{3} : 8\pi - 3\sqrt{3}$ (b) $4\pi - 3\sqrt{3} : 8\pi + 3\sqrt{3}$

(c) 1 : 1 (d) $4\pi - \sqrt{3} : 8\pi + \sqrt{3}$

ANSWERS

Comprehension-1:	1.	(a)	2.	(d)	3.	(c)		
Comprehension-2:	1.	(c)	2.	(d)	3.	(a)		
Comprehension-3:	1.	(a)	2.	(c)	3.	(d)		
Comprehension-4:	1.	(b)	2.	(d)	3.	(d)		
Comprehension-5:	1.	(c)	2.	(a)	3.	(c)		
Comprehension-6:	1.	(d)	2.	(d)	3.	(c)		
Comprehension-7:	1.	(a)	2.	(c)	3.	(d)		
Comprehension-8:	1.	(b)	2.	(c)	3.	(b)	4.	(a)
Comprehension-9:	1.	(a)	2.	(c)	3.	(c)		
Comprehension-10:	1.	(d)	2.	(b)	3.	(b)		

EXERCISE 4

Match the Columns:

1. The locus of the middle point of the chords of parabola $y^2 = 4x$ which

	Column-I		Column-II
(a)	are normal to parabola is	**(p)**	$y^4 + 4(1-x)y^2 + 4(1-4x) = 0$
(b)	subtend a constant angle $\frac{\pi}{4}$ at the vertex is	**(q)**	$y^4 + (4-2x)y^2 + 8 = 0$
(c)	are of given length 2	**(r)**	$y^2 = 2(x+2)$
(d)	are such that normals at their extremities meet on same parabola.	**(s)**	$y^4 - 4xy^2 + 4x^2 + 32y^2 - 96x + 64 = 0$

2. Normals of parabola $y^2 = 4x$ at P and Q meets at $R(x_2, 0)$ and tangent at P and Q meets at $T(x_1, 0)$

	Column-I		Column-II
(a)	If $x_2 = 3$, then area of quadrilateral $PTQR$ is	**(p)**	3/2
(b)	If length of tangent PT is $4\sqrt{5}$, then $x_2 =$	**(q)**	6
(c)	The possible values of x_2 so that 3 distinct normals can be drawn to the parabola from point R is/are.	**(r)**	8
(d)	If $x_2 = 4$ and area of circle circumscribing ΔPQR is $k\pi$, then k is equal to	**(s)**	9

3. If $y = x + 1$ is axis of parabola, $y + x = 4$ is tangent of same parabola at its vertex and $y = 2x + 3$ is one of its tangent, then

	Column-I		Column-II
(a)	If equation of directrix of parabola is $ax + by - 29 = 0$, then $a + b =$	**(p)**	9
(b)	If length of latus rectum of parabola is $\frac{a\sqrt{2}}{b}$ where a and b are relatively prime natural numbers, then $a + b =$	**(q)**	18
(c)	Let extremities of latus rectum are (a_1, b_1) and (a_2, b_2), then $[a_1 + b_1 + a_2 + b_2] =$ (where [·] denote greatest integer function)	**(r)**	23
(d)	If equation of parabola is $a(x - y + 1)^2 = b(x + y - 4)$ where a and b are relatively prime natural numbers then $a + b =$	**(s)**	37

4.

	Column-I		Column-II
(a)	The point (8,8) is one extremity of focal chord of parabola $y^2 = 8x$. The length of this focal chord is	**(p)**	1
(b)	The equation $(26x-1)^2 + (26y-3)^2 = k(5x - 12y + 1)^2$ will represent a parabola if k is	**(q)**	4/3
(c)	The length of common chord of curves $y^2 = 4(x+1)$ and $4x^2 + 9y^2 = 36$ is	**(r)**	4
(d)	A focal chord of parabola $y^2 = 4ax$ is of length $4a$. The angle subtended by it at the vertex of the parabola is θ then $\lvert\tan\theta\rvert$ is equal to	**(s)**	25/2

5.

	Column-I		Column-II
(a)	The equation of tangent of the ellipse $\frac{x^2}{2}+y^2=1$ which cuts off equal lengths of intercepts on coordinate axis is $y=\pm x+a$, then a can be equal to	**(p)**	$-\sqrt{3}$
(b)	The normal $y=mx-2am-am^3$ to the parabola $y^2=4ax$ subtends a right angle at the vertex then m can be equal to	**(q)**	$-\sqrt{2}$
(c)	The equation of the common tangent to parabola $y^2=4x$ and $x^2=4y$ is $x+y+\frac{k}{\sqrt{3}}=0$, then k is equal to	**(r)**	$\sqrt{2}$
(d)	An equation of common tangent to parabola $y^2=8x$ and the hyperbola $3x^2-y^2=3$ is $2x+\frac{ky}{\sqrt{2}}+1=0$, then k can be equal to	**(s)**	$\sqrt{3}$

6.

	Column-I		Column-II
(a)	The normal chord at a point $(t^2,2t)$ on the parabola $y^2=4x$ subtends a right angle at the vertex, then t^2 is	**(p)**	4
(b)	The area of the triangle inscribed in the curve $y^2=4x$, whose vertices are $(1,2),(4,4),(16,8)$ is	**(q)**	2
(c)	The number of distinct normal possible from $\left(\frac{11}{4},\frac{1}{4}\right)$ to the parabola $y^2=4x$ is	**(r)**	3
(d)	The normal at $(a,2a)$ on $y^2=4ax$ meets the curve again at $(at^2,2at)$, then the value of $\lvert t-1\rvert$ is	**(s)**	6

7. Normals are drawn from point $(4,1)$ to the parabola $y^2=4x$. The tangents at the feet of normals to the parabola $y^2=4x$ form a triangle ABC.

	Column-I		Column-II
(a)	The distance of focus of parabola $y^2=4x$ from centroid of ΔABC is	**(p)**	$\frac{5}{3}$
(b)	The distance of focus of parabola $y^2=4x$ from orthocentre of ΔABC is	**(q)**	$\frac{\sqrt{10}}{2}$
(c)	The distance of focus of parabola $y^2=4x$ from circumcentre of ΔABC is	**(r)**	$\frac{\sqrt{7}}{2}$
(d)	Area of ΔABC is	**(s)**	$\frac{\sqrt{5}}{2}$
		(t)	$\sqrt{5}$

ANSWERS

1. a → q; b → s; c → p; d → r
2. a → r; b → q; c → q,r,s; d → s
3. a → q; b → r; c → p; d → s
4. a → s; b → r; c → r; d → q
5. a → p,s; b → q,r; c → s; d → q,r
6. a → q; b → s; c → q; d → p
7. a → p; b → t; c → q; d → s

EXERCISE 5

Subjective Problems

1. If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) be three points on parabola $y^2 = 4ax$ and the normals at these points meet in a point, then $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2}$ is equal to

2. A parabola of latus rectum l, touches a fixed equal parabola, the axes of two curves being parallel. The locus of the vertex of moving curve is a parabola of latus rectum kl, then k is equal to.

3. If the normals at the points where the straight line $lx + my = 1$ meet the parabola $y^2 = 4ax$, meet on the normal at the point $\left(k\frac{am^2}{l^2}, \frac{kam}{l}\right)$ of parabola $y^2 = 4ax$, then k is equal to

4. Let PG is the normal at point P to a parabola cuts its axis in G and is produced to Q so that $GQ = 1/2\, PG$. The other normals which pass through Q intersect at an angle of π/k, then $k =$

5. The locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$ which with the tangent at the vertex form a triangle of constant area c^2, is the curve $x^2(y^2 - 4ax) = \lambda c^4$, then $\lambda =$

6. Find the ratio of area of triangle formed by three points on a parabola to the area of triangle formed by tangents at these points.

7. 'O' is the vertex of parabola $y^2 = 4x$ and L is the upper end of latus rectum. If LH is drawn perpendicular to OL meeting x-axis in H, then length of double ordinate through H is $\sqrt{N}$, then $N =$

8. The radius of circle which passes through the focus of parabola $x^2 = 4y$ and touches it at point $(6, 9)$ is $k\sqrt{10}$, then $k =$

9. From the point $(-1, 2)$ tangent lines are drawn to the parabola $y^2 = 4x$. If area of triangle formed by the chord of contact and the tangents is $N\sqrt{2}$, then $N =$

10. Let $K(c, 0)$ be the point which has the property that if any chord PQ of the parabola $y^2 = 4x$ be drawn through it, then $\frac{1}{(PK)^2} + \frac{1}{(QK)^2}$ is the same for all positions of the chord, then $c =$

11. Tangent are drawn at those point on the parabola $y^2 = 16x$ whose ordinate are in the ratio 4 : 1. If the locus of point of intersection of these tangents is $y^2 = kx$, then $[k/3]$ is:
([.] denote greatest integer function)

12. If the focus of parabola $y^2 + 8 = 4x$ coincides with one of the foci of ellipse $3x^2 + by^2 - 12x = 0$, then the reciprocal of eccentricity of ellipse is:

13. In the above problem, $b =$

ANSWERS

1.	0	**2.**	2	**3.**	4	**4.**	2	**5.**	4	**6.**	2	**7.**	80	**8.**	5	**9.**	8	**10.**	2
11.	8	**12.**	2	**13.**	4														

EXERCISE 6

1. Find the equations of the common tangents of the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabola $y^2 = x$. **[REE 1999]**

2. (A) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$ then one of the values of 'k' is:

(a) 1/8 (b) 8 (c) 4 (d) 1/4

(B) If $x + y = k$ is normal to $y^2 = 12x$, then 'k' is: **[IIT-JEE (Screening) 2000]**

(a) 3 (b) 9 (c) –9 (d) –3

3. Find the locus of the points of intersection of tangents drawn at the ends of all normal chords of the parabola $y^2 = 8(x - 1)$. **[REE 2001]**

4. (A) The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is: **[IIT-JEE (Screening) 2001]**

(a) $\sqrt{3}y = 3x + 1$ (b) $\sqrt{3}y = -(x + 3)$

(c) $\sqrt{3}y = x + 3$ (d) $\sqrt{3}y = -(3x + 1)$

(B) The equation of the directrix of the parabola, $y^2 + 4y + 4x + 2 = 0$ is:

[IIT-JEE (Screening) 2001]

(a) $x = -1$ (b) $x = 1$ (c) $x = -3/2$ (d) $x = 3/2$

5. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix: **[IIT-JEE (Screening) 2002]**

(a) $x = -a$ (b) $x = -a/2$ (c) $x = 0$ (d) $x = a/2$

6. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is:

[IIT-JEE (Screening) 2002]

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

7. (A) The slope of the focal chords of the parabola $y^2 = 16x$ which are tangents to the circle $(x - 6)^2 + y^2 = 2$ are: **[IIT-JEE (Screening) 2003]**

(a) ± 2 (b) −1/2, 2 (c) ± 1 (d) −2 , 1/2

(B) Normals are drawn from the point 'P' with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself then find α. **[IIT-JEE 2003]**

8. The angle between the tangents drawn from the points (1, 4) to the parabola $y^2 = 4x$ is:

[IIT-JEE (Screening) 2004]

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

9. Let P be a point on the parabola $y^2 - 2y - 4x + 5 = 0$, such that the tangent on the parabola at P intersects the directrix at point Q. Let R be the point that divides the line segment PQ externally in the ratio $\frac{1}{2} : 1$. Find the locus of R. **[IIT-JEE 2004]**

10. (A) The axis of parabola is along the line $y = x$ and the distance of vertex from origin is $\sqrt{2}$ and that of origin from its focus is $2\sqrt{2}$. If vertex and focus both lie in the 1^{st} quadrant, then the equation of the parabola is: **[IIT-JEE 2006]**

(a) $(x+y)^2 = (x-y-2)$ (b) $(x-y)^2 = (x+y-2)$

(c) $(x-y)^2 = 4(x+y-2)$ (d) $(x-y)^2 = 8(x+y-2)$

(B) The equations of common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are: **[IIT-JEE 2006]**

(a) $y = 4(x-1)$ (b) $y = 0$ (c) $y = -4(x-1)$ (d) $y = -30x - 50$

(C) Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$ which intersect at $(3, 0)$. Then: **[IIT-JEE 2006]**

	Column-I		Column-II
(i)	Area of ΔPQR	**(a)**	2
(ii)	Radius of circumcircle of ΔPQR	**(b)**	5/2
(iii)	Centroid of ΔPQR	**(c)**	(5/2, 0)
(iv)	Circumcentre of ΔPQR	**(d)**	(2/3, 0)

11. Statement-1: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$.

because

Statement-2: A parabola is symmetric about its axis. **[IIT-JEE 2007]**

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

12. Comprehension

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S. [IIT-JEE 2007]

(A) The ratio of the areas of the triangles PQS and PQR is:

(a) $1:\sqrt{2}$ (b) $1:2$ (c) $1:4$ (d) $1:8$

(B) The radius of the circumcircle of the triangle PRS is:

(a) 5 (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

(C) The radius of the incircle of the triangle PQR is:

(a) 4 (b) 3 (c) 8/3 (d) 2

13. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are: **[IIT-JEE 2008]**

(a) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$ (b) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(c) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$ (d) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

14. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose: **[IIT-JEE 2009]**

(a) vertex is $\left(\frac{2a}{3}, 0\right)$ (b) directrix is $x = 0$ (c) latus rectum is $\frac{2a}{3}$ (d) focus is $(a, 0)$

15. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be: **[IIT-JEE 2010]**

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

16. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio $1:3$. Then the locus of P is: **[IIT-JEE 2011]**

(a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$

17. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by: **[IIT-JEE 2011]**

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$ (c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

18. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is: **[IIT-JEE 2011]**

19. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is: **[IIT-JEE 2012]**

20. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$. **[IIT-JEE (Mains) 2013]**

Statement-1: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

because

Statement-2: If the line, $y = mx + \frac{\sqrt{5}}{m}(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true.

21. Comprehension

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

[IIT-JEE (Advance) 2013]

(A) If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(a) $\frac{2}{3}\sqrt{7}$ (b) $\frac{-2}{3}\sqrt{7}$ (c) $\frac{2}{3}\sqrt{5}$ (d) $\frac{-2}{3}\sqrt{5}$

(B) Length of chord PQ is:

(a) $7a$ (b) $5a$ (c) $2a$ (d) $3a$

22. A line $L: y = mx + 3$ meets y-axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x, 0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.
Match Column-I with Column-II and select the correct answer using the code given below the columns : **[IIT-JEE (Advance) 2013]**

	Column-I		Column-II
(a)	$m =$	**(p)**	1/2
(b)	Maximum area of ΔEFG is	**(q)**	4
(c)	$y_0 =$	**(r)**	2
(d)	$y_1 =$	**(s)**	1

ANSWERS

1. $x - 2y + 1 = 0$; $y = mx + \frac{1}{4m}$ where $m = \frac{-5 \pm \sqrt{30}}{10}$ **2.** **(A)** c; **(B)** b
2. $(x+3)y^2 + 32 = 0$ **4.** **(A)** c; **(B)** d **5.** c **6.** d
7. **(A)** c; **(B)** $\alpha = 2$ **8.** b **9.** $2(y-1)^2(x-2) = (3x-4)^2$
10. **(A)** d; **(B)** a, b; **(C)** (i) a, (ii) b, (iii) d, (iv) c **11.** a **12.** **(A)** c; **(B)** b; **(C)** d
13. b, c **14.** a, d **15.** c, d **16.** c
17. a, b, d **18.** 2 **19.** 4 **20.** b
21. **(A)** d; **(B)** b **22.** a → s; b → p; c → q; d → r

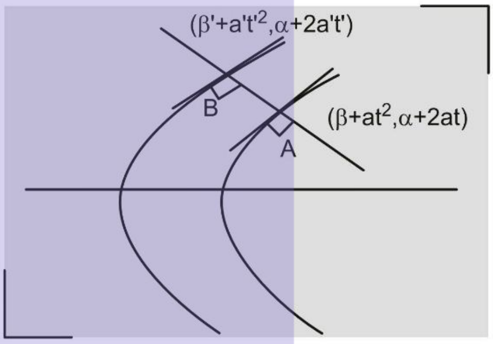
SOLUTIONS 1

Only One Choice is Correct:

1. (a) Eqn. of AB is

$$y-\alpha=-t(x-\beta)+2at+at^3 \quad ...(1)$$

or $$y-\alpha=-t'(x-\beta')+2a't'+a't'^3 \quad ...(2)$$



(1) and (2) are identical

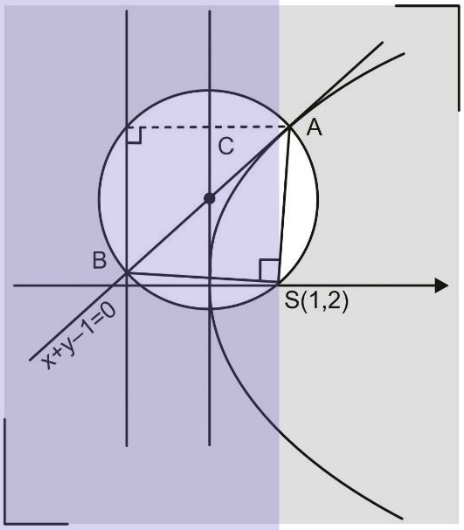
$\Rightarrow$ $t=t'$ and $2at+at^3+\beta t+\alpha$

$$=2a't+a't^3+t\beta'+\alpha$$

$$\Rightarrow t^2=\frac{(\beta'-\beta)+2(a'-a)}{(a-a')} \quad ...(3)$$

$$\Rightarrow \frac{\beta'-\beta}{a-a'}>2 \Rightarrow \frac{2(a-a')}{(\beta'-\beta)}<1$$

2. (a)

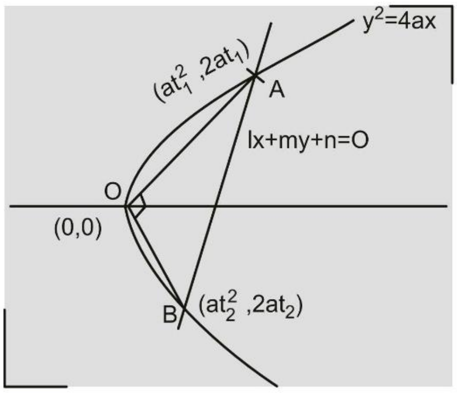


Using power of C

$$(BC)(AC)=(CS)^2=\left(\frac{1+2-1}{\sqrt{2}}\right)^2=2$$

3. (c) $OA\perp OB$

$$\Rightarrow \frac{2}{t_1}\frac{2}{t_2}=-1 \Rightarrow t_1t_2=-4$$



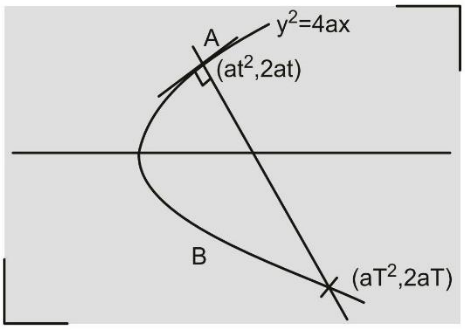
Put $(at^2, 2at)$ in eqn. of AB

$$\Rightarrow lat^2+2mat+n=0 \begin{matrix} t_1 \\ t_2 \end{matrix}$$

$$t_1t_2=-4=\frac{n}{la}$$

$$\Rightarrow n+4la=0$$

4. (a) $T=-t-\dfrac{2}{t}$



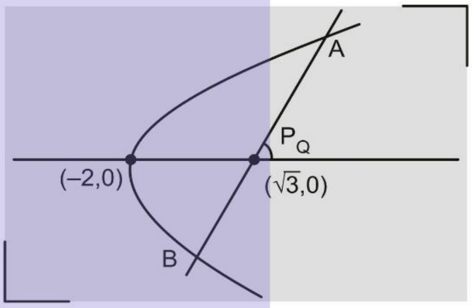
$|T|\geq 2\sqrt{2}$ (applying A.M. $\geq$ G.M.)

$T^2\geq 8$

5. (d) $PA = r_1$, $PB = -r_2$

Put $(\sqrt{3} + r\cos\theta,\ r\sin\theta)$ to $y^2 = x + 2$

$\Rightarrow\ r^2 \sin^2\theta - r\cos\theta - (\sqrt{3}+2) = 0 \begin{smallmatrix} r_1 \\ r_2 \end{smallmatrix}$



$(PA)(PB) = -r_1 r_2 = \dfrac{\sqrt{3}+2}{\sin^2\theta}$

$= (\sqrt{3}+2)(1+\cot^2\theta)$

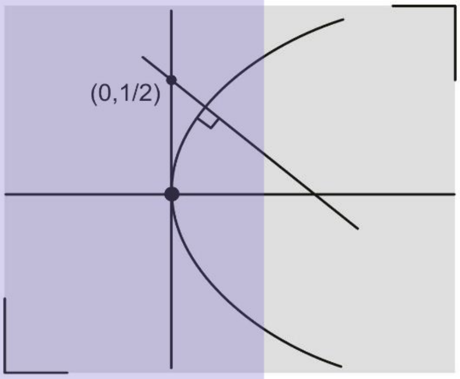
$= (\sqrt{3}+2)\left(1+\dfrac{1}{3}\right)$ $[\because\ \tan\theta = \sqrt{3}]$

$(PA)(PB) = \dfrac{4}{3}(2+\sqrt{3})$

6. (b) Equation of normal to $y^2 = 4ax$

$y = -tx + 2at + at^3$

Put $\left(0, \dfrac{1}{2}\right)$



$2at + at^3 = \dfrac{1}{2}$

$f(t) = 2at^3 + 4at - 1$

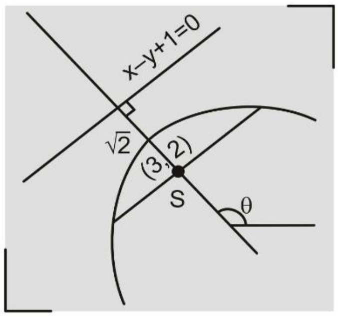
$f'(t) = 6at^2 + 4a = 2a(3t^2+2) \neq 0$

$\Rightarrow$ $f(t)$ can have only one real root.

7. (b) $\tan\theta = -1$

focus, $S \equiv \left(3 - \sqrt{2}\left(-\dfrac{1}{\sqrt{2}}\right), 2 - \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right)\right)$

$S \equiv (4, 1)$



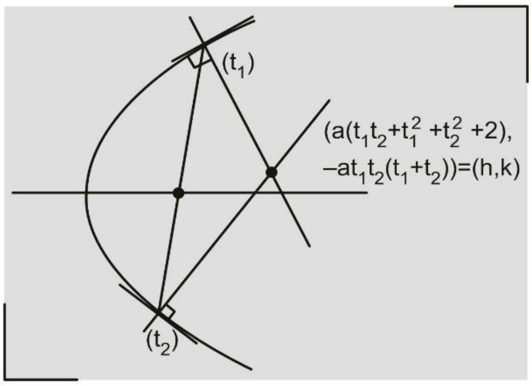
Eqn. of $LR \equiv x - y = 4 - 1 = 3$

$x - y = 3$

8. (c) $t_1 t_2 = -1$

$\Rightarrow$ $k = a(t_1 + t_2)$

$h = a(t_1^2 + t_2^2 + 1)$



$\dfrac{k^2}{a^2} = \left(\dfrac{h}{a} - 1\right) + 2(-1) = \dfrac{h}{a} - 3$

$\Rightarrow$ $y^2 = a(x - 3a)$

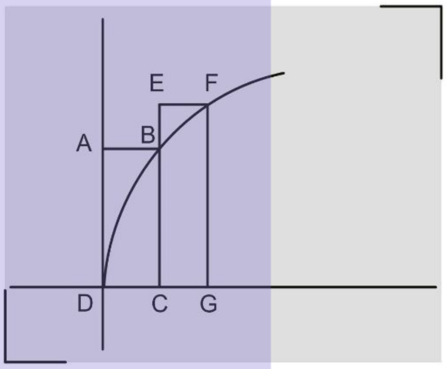
9. (c)

$\dfrac{1}{a+at_1^2} + \dfrac{1}{a+at_2^2} = \dfrac{1}{a+at_1^2} + \dfrac{1}{a + a\left(-\dfrac{1}{t_1}\right)^2}$

$= \dfrac{1}{a}$

10. (a) $y^2 = k^2x \Rightarrow y^2 = 4\left(\frac{k^2}{4}\right)x$

$$B \equiv \left(\frac{k^2}{4}t_1^2, \frac{k^2}{2}t_1\right), F\left(\frac{k^2}{4}t_2^2, \frac{k^2}{2}t_2\right)$$



$$\frac{k^2}{4}t_1^2 = \frac{k^2}{2}t_1 \Rightarrow t_1 = 2$$

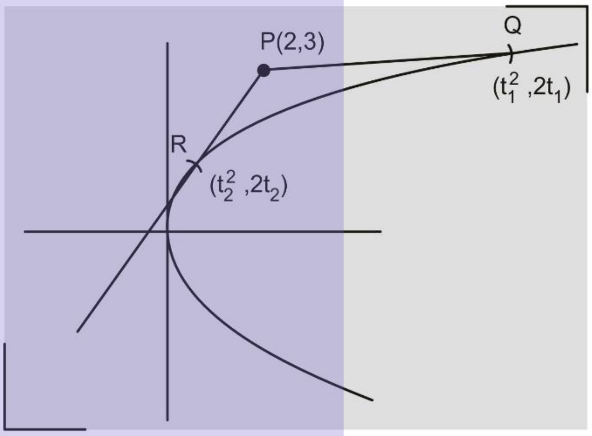
$$\frac{k^2}{2}t_2 = \frac{k^2}{4}(t_2^2 - t_1^2)$$

$$2t_2 = t_2^2 - 4 \Rightarrow t_2^2 - 2t_2 - 4 = 0$$

$$\Rightarrow \quad t_2 = 1 + \sqrt{5}$$

$$\frac{FG}{BC} = \frac{t_2}{t_1} = \frac{1+\sqrt{5}}{2}$$

11. (b) Equation of tangent at $(t^2, 2t)$



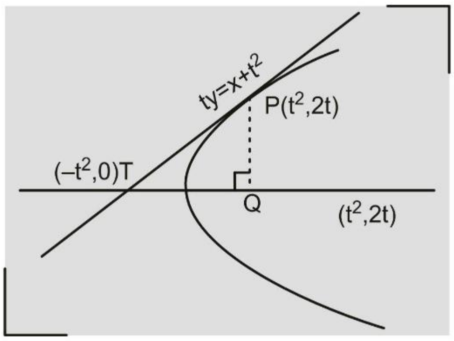
$$yt = x + t^2$$

Put $(2,3) \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, 2 \Rightarrow Q, R \equiv (1,2)$ and $(4,4)$

12. (b) Area of $\Delta PTQ = \left|\frac{1}{2} \times (2t^2)(2t)\right|$

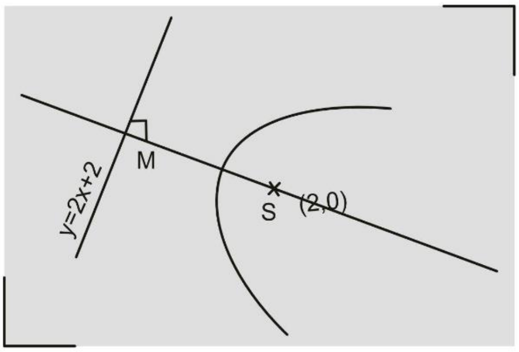
$$\Delta = 2|t^3|$$



$$t^2 \in [1,4] \Rightarrow t \in [-2,-1] \cup [1,2]$$

$$\Delta_{max} = 16 \text{ for } t = \pm 2$$

13. (a) $x^2 = \frac{1}{k}y$



Length of $LR = 2(SM)$

$$= 2\frac{|2(2)+2-0|}{\sqrt{2^2+1}} = \frac{12}{\sqrt{5}}$$

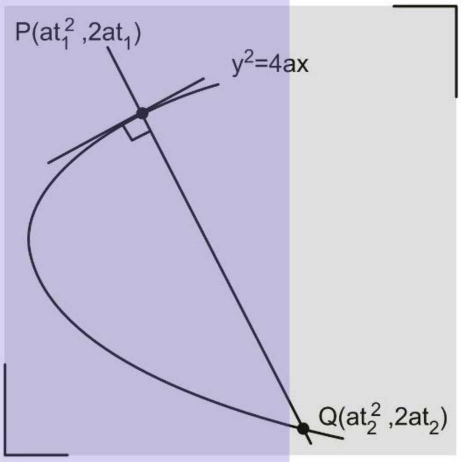
$$\Rightarrow \quad \frac{1}{k} = \frac{12}{\sqrt{5}} \Rightarrow k = \frac{\sqrt{5}}{12}$$

14. (b) $t_2 = -t_1 - \frac{2}{t_1}$

$$(t_1 + t_2)t_1 = -2$$

$$-t_1 = \tan\alpha, -t_2 = \tan\beta$$

$$\Rightarrow \tan\alpha(\tan\alpha + \tan\beta) = -2$$



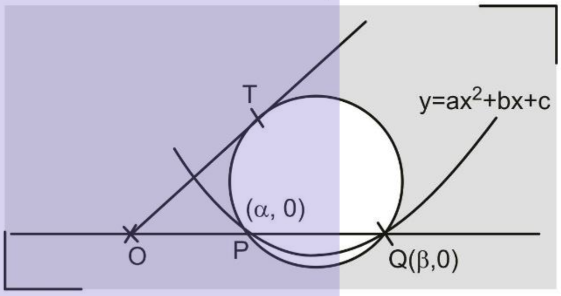
15. (d) $VA \perp VB \Rightarrow \frac{2}{t_1}\frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$

$$\frac{|VA|}{|VB|} = \frac{\sqrt{\frac{1}{4}t_1^2 + \frac{1}{16}t_1^4}}{\sqrt{\frac{1}{4}t_2^2 + \frac{1}{16}t_2^4}} = \frac{|t_1|\sqrt{4+t_1^2}}{|t_2|\sqrt{4+t_2^2}}$$

$$\frac{|t_1|\sqrt{4+t_1^2}}{\left|-\frac{4}{t_1}\right|\sqrt{4+\frac{16}{t_1^2}}} = \frac{|t_1^3|}{8} = \frac{8\cot^3\theta}{8} = \cot^3\theta$$

$[\because \tan\theta = \frac{2}{t_1}]$

16. (d) $ax^2 + bx + c = 0 \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix} \Rightarrow \alpha\beta = \frac{c}{a}$



$$(OT)^2 = (OP)(OQ) = \alpha\beta = \frac{c}{a}$$

$$\Rightarrow \quad OT = \sqrt{\frac{c}{a}}$$

17. (d) Put $\quad x^2 = \frac{y}{a}$

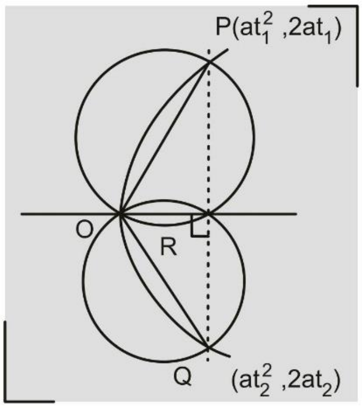
$$\Rightarrow \quad y^2 + \left(\frac{1}{a} - 2\right)y = 0$$

$$\Rightarrow \quad y = 0, \quad y = 2 - \frac{1}{a}$$

$$2 > 2 - \frac{1}{a} > 0 \quad \Rightarrow a > \frac{1}{2}$$

18. (a) $\angle ORP = \frac{\pi}{2} = \angle ORQ$

slope of $PQ = \frac{2}{t_1 + t_2}$



$$\Rightarrow \quad \text{slope of } OR = -\left(\frac{t_1 + t_2}{2}\right) = \tan\phi$$

$$\Rightarrow \quad \frac{1}{t_1} = \tan\theta_1 \quad \frac{1}{t_2} = \tan\theta_2$$

$$\Rightarrow \quad \tan\phi = -\left(\frac{\cot\theta_1 + \cot\theta_2}{2}\right)$$

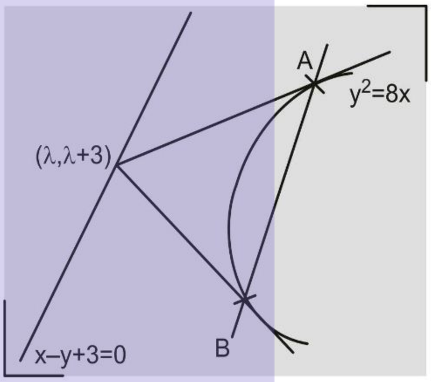
$$\Rightarrow \quad \cot\theta_1 + \cot\theta_2 = -2\tan\phi$$

19. (c) Equation of AB is

$$y(\lambda + 3) = 4(x + \lambda)$$

$$\Rightarrow \quad (3y - 4x) + \lambda(y - 4) = 0$$

represents family of lines passing through $(3, 4)$



20. (a) SP, semi latus rectum, SQ are in H.P.

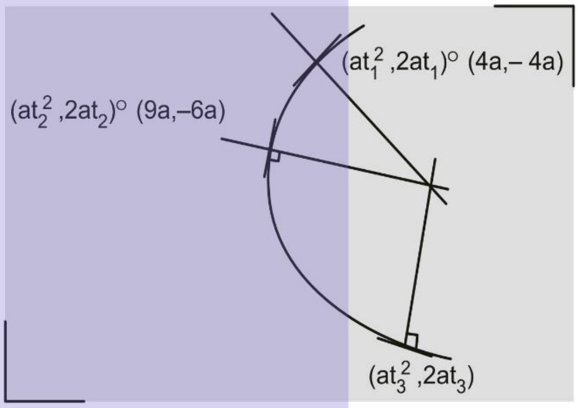
$$\Rightarrow \frac{2}{2a}=\frac{1}{SP}+\frac{1}{SQ}=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$$

$$\Rightarrow a=\frac{6}{5} \quad \Rightarrow \text{Length of } LR=4a=\frac{24}{5}$$

21. (b) $t_1=-2,\ t_2=-3$

$$t_1+t_2+t_3=0$$

$$\Rightarrow t_3=5$$



Equation of normal at $(at_3^2, 2at_3)$

$$y=-t_3x+2at_3+at_3^3$$

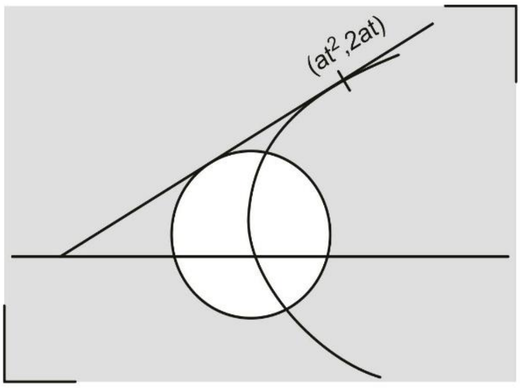
$$y=-5x+10a+125a$$

$$5x+y-135a=0$$

22. (a) Equation of tangent is

$$yt=x+at^2$$

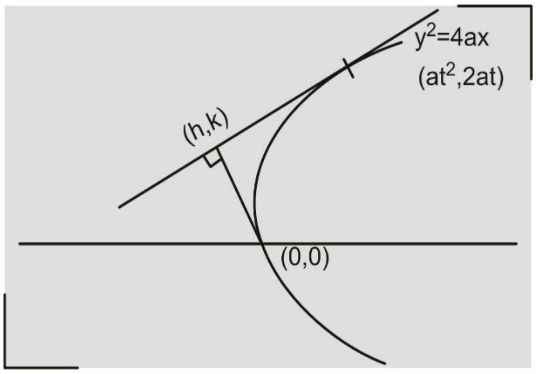
$$\text{slope}=\frac{1}{t}=\tan\frac{\pi}{4}=1 \Rightarrow t=1$$



$$\Rightarrow y=x+a$$

$$\Rightarrow c=\frac{|0+a-0|}{\sqrt{2}}=\frac{a}{\sqrt{2}} \Rightarrow a=\sqrt{2}c$$

23. (a) Equation of tangent 'L' is



$$yt-x=at^2 \qquad \text{...(1)}$$

Also L is given by $hx+ky=h^2+k^2$ ...(2)

(1) and (2) are identical

$$\Rightarrow \frac{t}{k}=-\frac{1}{h}=\frac{at^2}{h^2+k^2} \Rightarrow t=-\frac{k}{h}$$

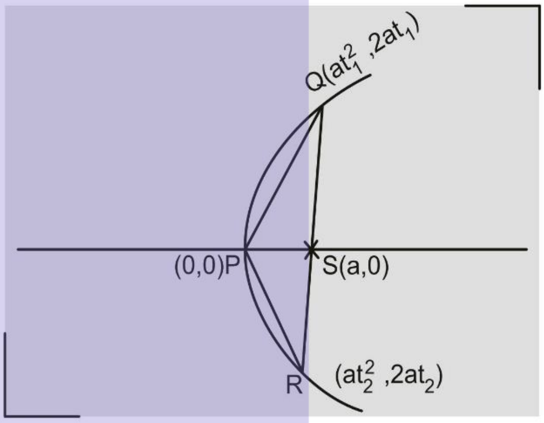
$$\therefore \quad at=\frac{h^2+k^2}{k}=a\left(-\frac{k}{h}\right)$$

$$\Rightarrow x(x^2+y^2)+ay^2=0$$

24. (c) Area = modulus of $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$

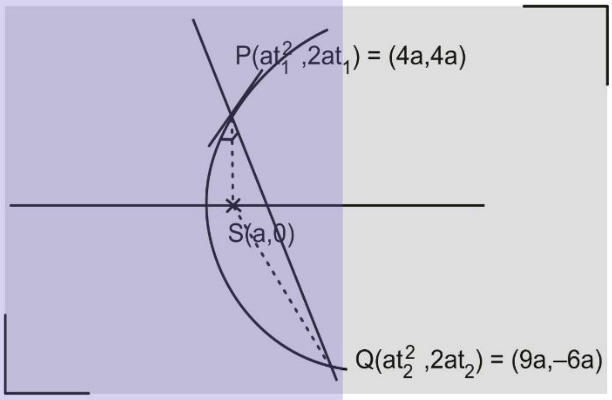
$$=|a^2t_1t_2(t_1-t_2)|$$

$$A=|-a^2(t_1-t_2)|$$



$$|2a(t_1 - t_2)| = \frac{2A}{a}$$

25. (d) $at_1^2 = 2at_1 \Rightarrow t_1 = 2$



$$t_2 = -t_1 - \frac{2}{t_1} = -3$$

$$m_{PS} = \frac{4a-0}{4a-a} = \frac{4}{3}$$

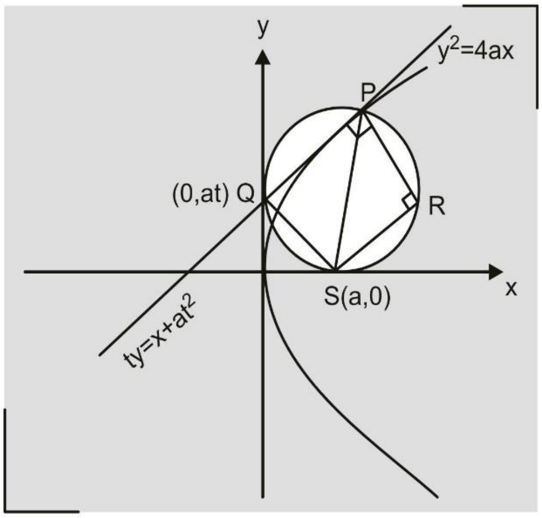
$$m_{QS} = \frac{-6a-0}{9a-a} = -\frac{3}{4}$$

$m_{PS}m_{QS} = -1 \;\Rightarrow PS \perp SQ$

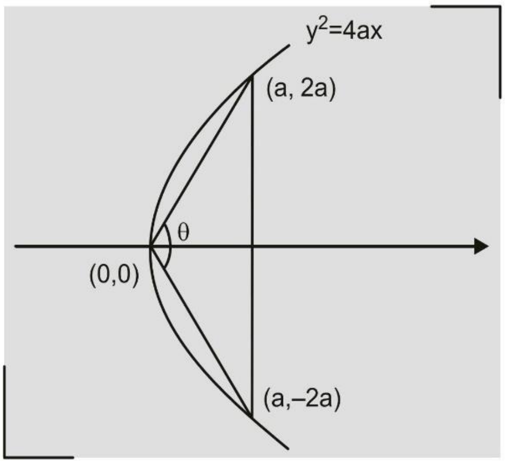
26. (d) $PRSQ$ is a rectangle

$\Rightarrow \quad PR = SQ = \sqrt{a^2 + a^2t^2}$

$PR = a\sqrt{1+t^2}$



27. (d) $\theta = 2\tan^{-1} 2$



$$\sqrt{3} < 2 < \sqrt{2} + 1$$

$$\Rightarrow \quad \frac{\pi}{3} < \tan^{-1} 2 < \frac{3\pi}{8}$$

$$\Rightarrow \quad \frac{2\pi}{3} < \theta < \frac{3\pi}{4}$$

28. (b) $\frac{1}{t_1} = 1 \quad \Rightarrow t_1 = 1$

Equation of tangent at A is

$yt_1 = x + at_1^2$

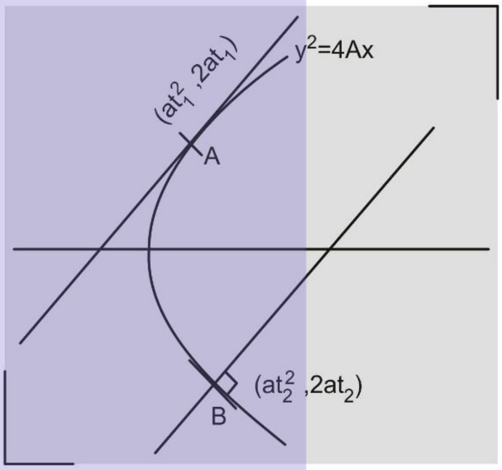
$\Rightarrow \quad y = x + a \qquad ...(1)$

$-t_2 = 1 \Rightarrow t_2 = -1$

Equation of normal at B is

$y = -t_2x + 2at_2 + at_2^3$

$\Rightarrow \quad y = x - a - 2a \Rightarrow y = x - 3a \qquad \ldots(2)$



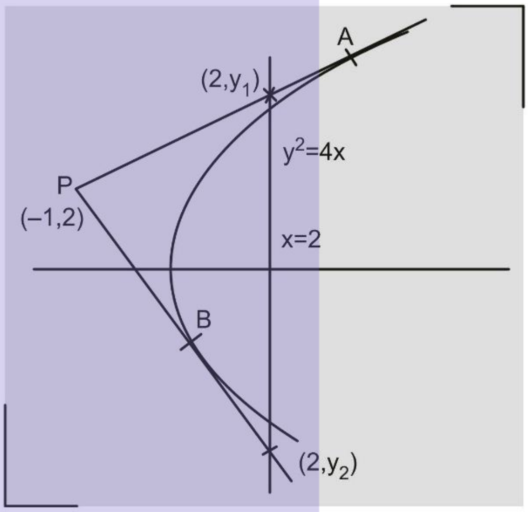
$\perp$ar distance between lines (1) and (2) is

$$\frac{(a+3a)}{\sqrt{1^2+1^2}} = \frac{4a}{\sqrt{2}} = 2\sqrt{2}\,a$$

29. (b) Equation of pair PA and PB is

$$(y(2) - 2(x-1))^2 = (y^2 - 4x)(4+4)$$

Put $\quad x = 2$



$\Rightarrow \quad (y-1)^2 = 2(y^2 - 8)$

$\Rightarrow \quad y^2 + 2y - 17 = 0$

$(y_1 - y_2)^2 = (-2)^2 - 4(-17) = 72$

$\Rightarrow |y_1 - y_2| = 6\sqrt{2}$

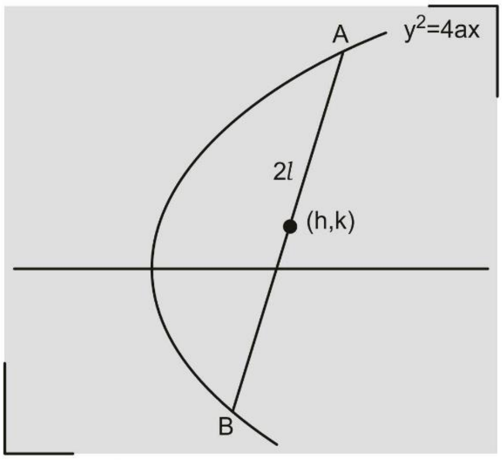
30. (d) Put $(h + r\cos\theta,\ k + r\sin\theta)$ to $y^2 = 4x$

$\Rightarrow k^2 + r^2\sin^2\theta + 2kr\sin\theta = 4h + 4r\cos\theta$

$\Rightarrow \quad r^2\sin^2\theta + (2k\sin\theta - 4\cos\theta)r + (k^2 - 4h) = 0$ (roots l, $-l$)

$l + (-l) = 2k\sin\theta - 4\cos\theta = 0$

$\Rightarrow \quad \tan\theta = \dfrac{2}{k}$

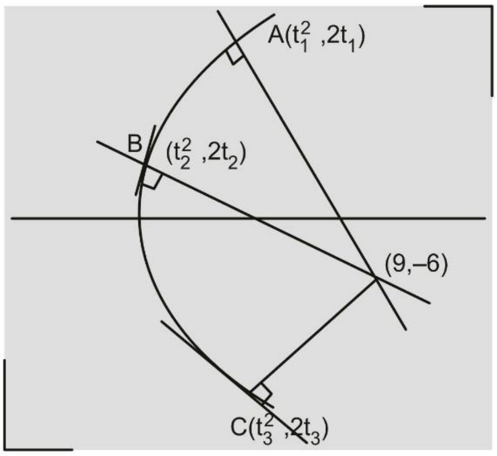


$l(-l) = \dfrac{k^2 - 4h}{\sin^2\theta} = (k^2 - 4h)(1 + \cot^2\theta)$

$\Rightarrow \quad (4x - y^2)\left(1 + \dfrac{y^2}{4}\right) = l^2$

$\Rightarrow \quad (4x - y^2)(4 + y^2) = 4l^2$

31. (c) Equation of normal at $(t^2, 2t)$ is $y = -tx + 2t + t^3$



Put $\quad (9, -6)$

$\Rightarrow t^3 - 7t + 6 = 0 \Rightarrow (t-1)(t-2)(t+3) = 0$

$\Rightarrow t = 1, 2 - 3$

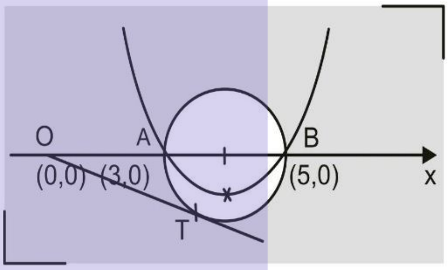
$a_{11} = m_1 + 1 = -1 + 1 = 0$

$a_{22} = m_2 + 2 = -2 + 2 = 0$

$a_{33} = m_3 + 3 = 3 + 3 = 6$

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 6 \end{vmatrix} = -4$$

32. (d)



$(OT)^2 = (OA)(OB)$

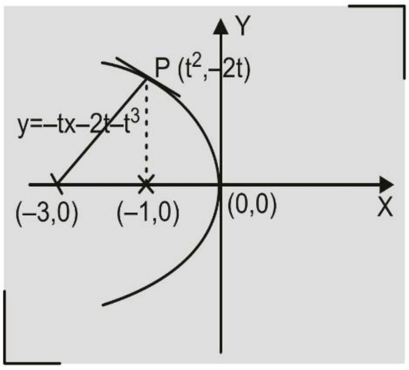
$= 3 \times 5 = 15$

$\Rightarrow \quad OT = \sqrt{15}$

33. (b) $(y-2)^2 = -4(x-2)$

$y - 2 = Y, x - 2 = X$

$Y^2 = -4X$



Line is $X - Y = -3$

$\Rightarrow \quad Y = X + 3 \qquad ...(1)$

$Y = -tX - 2t - t^3 \qquad ...(2)$

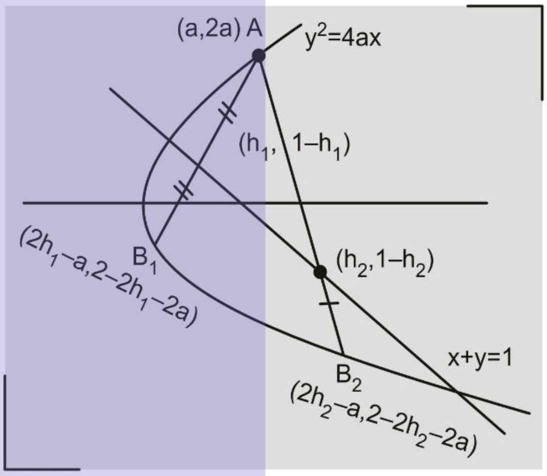
(1) and (2) are identical for $t = -1$

SOLUTIONS 2

One or More than One is/are Correct

1. (b, c, d)

Put $(2h-a, 2-2h-2a)$ in the equation of parabola



$\Rightarrow \quad 4(1-h-a)^2 = 4a(2h-a)$

$\Rightarrow \quad h^2 - 2h + (2a^2 - 2a + 1) = 0 \begin{matrix} h_1 \\ h_2 \end{matrix}$

For two distinct real roots

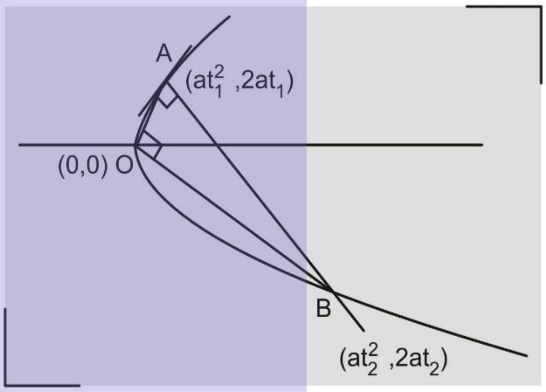
$D > 0 \quad \Rightarrow \quad 4 - 4(2a^2 - 2a + 1) > 0$

$\Rightarrow \quad a\,(a-1) < 0$

$\Rightarrow \quad a \in (0, 1)$

$\Rightarrow \quad$ Length of Latus rectum $\in (0, 4)$

2. (b, c)



$\frac{2}{t_1}\frac{2}{t_2} = -1 \quad \Rightarrow t_1 t_2 = -4$

$t_2 = -t_1 - \frac{2}{t_1} \Rightarrow \frac{-4}{t_1} = -t_1 - \frac{2}{t_1}$

$t_1 = \pm\sqrt{2}$

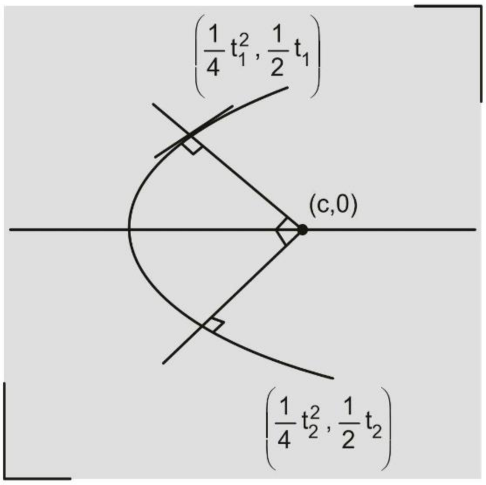
$\therefore \quad$ slope of $AB = -t_1 = \mp\sqrt{2}$

3. (a, c)

Equation of normal at $\left(\frac{1}{4}t^2, \frac{1}{2}t\right)$ is

$y = -tx + \frac{1}{2}t + \frac{1}{4}t^3$

Put $(c, 0)$



$\Rightarrow \quad \frac{1}{4}t^3 + \left(\frac{1}{2} - c\right)t = 0$

$\Rightarrow \quad t^2 - (4c-2) = 0 \begin{matrix} t_1 \\ t_2 \end{matrix}$

$\Rightarrow \quad 4c - 2 > 0 \Rightarrow c > \frac{1}{2}$ and $t_1 t_2 = -1$

$\Rightarrow \quad 4c - 2 = 1 \quad \Rightarrow c = 3/4$

4. (a, c, d)

$$(PQ)^2 = a^2[(t^2 - t'^2)^2 + 4(t - t')^2]$$

$$= a^2[(t + t')^2 - 4tt'][(t + t')^2 + 4]$$

$$t' = -t - \frac{2}{t}$$

$$= a^2\left[\frac{4}{t^2} + 4(t^2 + 2)\right]\left[\frac{4}{t^2} + 4\right]$$

$$= 16a^2\left(t^2 + \frac{1}{t^2} + 2\right)\left(\frac{1}{t^2} + 1\right)$$

$$= 16a^2\left[3 + t^2 + \frac{1}{t^4} + \frac{3}{t^2}\right]$$

$$PQ^2 = 16a^2\left(3 + x + \frac{1}{x^2} + \frac{3}{x}\right) \qquad x > 0$$

$$\frac{d}{dx}(PQ^2) = 16a^2\left(1 - \frac{2}{x^3} - \frac{3}{x^2}\right)$$

$$= 16a^2\frac{(x^3 - 3x - 2)}{x^3}$$

$$= \frac{16a^2(x+1)^2(x-2)}{x^3}$$

PQ^2 is minimum at $t^2 = 2$

$$(PQ)_{\min} = 4a\sqrt{3 + 2 + \frac{1}{4} + \frac{3}{2}} = 6\sqrt{3}\,a$$

$$t' = -\sqrt{2} - \frac{2}{\sqrt{2}} = -2\sqrt{2}$$

$$\Rightarrow \quad Q \equiv (8a, -4\sqrt{2}\,a)$$

or $\quad t' = \sqrt{2} + \dfrac{2}{\sqrt{2}} = 2\sqrt{2}$

$$\Rightarrow \quad Q \equiv (8a, 4\sqrt{2}\,a)$$

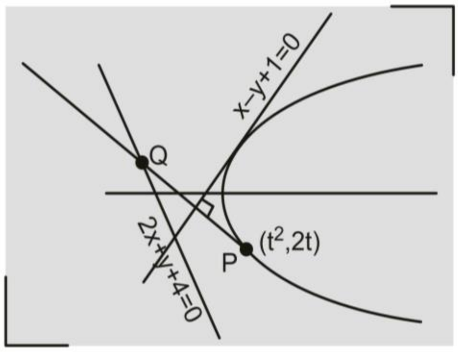
5. (a, c)

Q is image of P w.r.t. $x - y + 1 = 0$

$$\frac{x - t^2}{1} = \frac{y - 2t}{-1} = -2\left(\frac{t^2 - 2t + 1}{1^2 + 1^2}\right)$$

$$Q \equiv (x, y) = (2t - 1, t^2 + 1)$$

Put Q to equation $2x + y + 4 = 0$

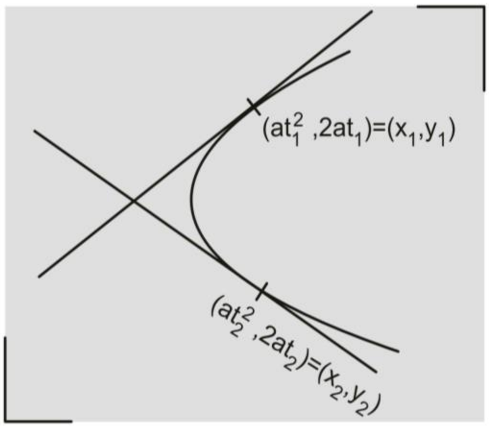


$$\Rightarrow \quad 2(2t - 1) + (t^2 + 1) + 4 = 0$$

$$(t + 3)(t + 1) = 0 \quad \Rightarrow t = -1, -3$$

$$\Rightarrow \quad P \equiv (1, -2), (9, -6)$$

6. (b, d)



$$(at_1^2)(at_2^2) = (at_1t_2)^2$$

$$\Rightarrow \quad x_1x_2 = x_3^2$$

$$\frac{2at_1 + 2at_2}{2} = a(t_1 + t_2)$$

$$\Rightarrow \quad \frac{y_1 + y_2}{2} = y_3$$

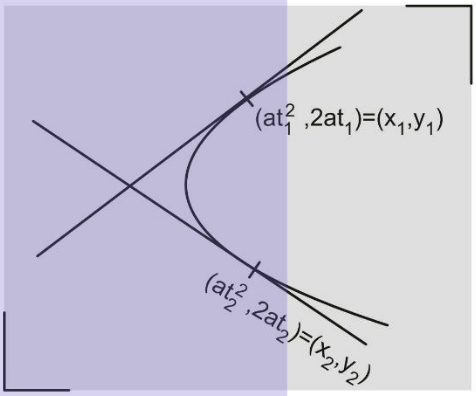
7. (a, c, d)

$$\text{slope} = \frac{2}{t_1 + t_2} = 1 \Rightarrow t_1 + t_2 = 2$$

locus of R is

$$k = a(t_1 + t_2) = 2a$$

$$\Rightarrow \quad y = 2a$$



locus of S is

$h = a((t_1+t_2)^2 - t_1t_2 + 2) = a(6 - t_1 t_2)$

$k = -at_1t_2(t_1+t_2) = -2at_1t_2$

$\therefore \quad 2h - k = 12a$

$2x - y = 12a$

8. (a, b)

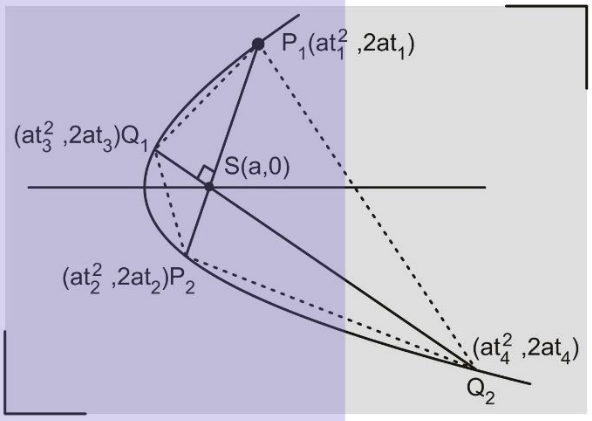
$P_1P_2 = (a + at_1^2) + (a + at_2^2)$

$= a[(t_1+t_2)^2 + 4] \qquad [\because t_1t_2 = -1]$

$Q_1Q_2 = a[t_3+t_4)^2 + 4]$

$$\frac{2}{(t_1+t_2)}\,\frac{2}{(t_3+t_4)} = -1$$

$\Rightarrow \ |(t_1+t_2)(t_3+t_4)| = 4$



Area of $P_1Q_1P_2Q_2, A = \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

$= \frac{1}{2}d_1d_2 = \frac{1}{2}a^2[(t_1+t_2)^2+4][t_3+t_4)^2+4]$

$A \geq \frac{1}{2}a^2 2\sqrt{4(t_1+t_2)^2}\,2\sqrt{4(t_3+t_4)^2}$

$= 8a^2|t_1+t_2|\,|t_3+t_4| = 32a^2$

$A_{min} = 2(4a)^2$ at

$(t_1+t_2)^2 = 4 = (t_3+t_4)^2$

$\Rightarrow \ t_1+t_2 = 2, t_3+t_4 = -2$

or $t_1+t_2 = -2,$

$t_1+t_2 = -2, \ t_3+t_4 = 2$

Slope of $P_1P_2 = 1$, slope of $Q_1Q_2 = -1$

or slope of $P_1P_2 = -1$, slope of $Q_1Q_2 = 1$

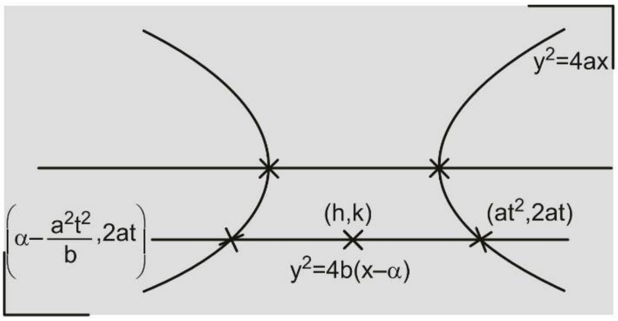
9. (a, b)

$k = 2at$

$$2h = at^2 + \alpha - \frac{a^2t^2}{b}$$

$$2h = \left(a - \frac{a^2}{b}\right)\frac{k^2}{4a^2} + \alpha$$

$$y^2\left(\frac{b-a}{4ab}\right)\frac{k^2}{4a^2} + \alpha = 2x$$

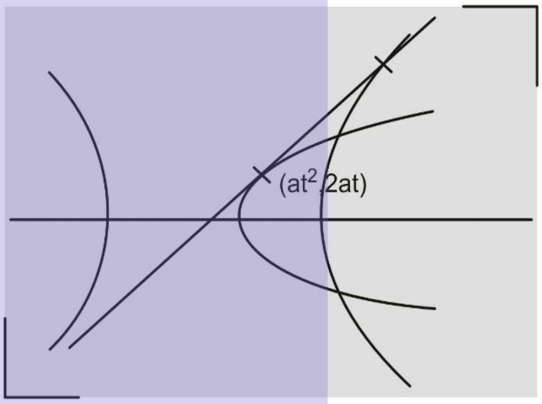


If $a = b, \quad 2x = \alpha,$ straight line

If

$a \neq b, y^2(b-a) + 4ab\alpha = 8abx,$ Parabola

10. (a, c)



Equation of tangent to $y^2 = 4ax$ is

$$yt = x + at^2 \Rightarrow y = \frac{1}{t}x + at \quad ...(1)$$

$$\Rightarrow a^2t^2 = a^2\frac{1}{t^2} - a^2$$

for (1) to represent tangent to $x^2 - y^2 = a^2$

$$\Rightarrow t^4 + t^2 - 1 = 0$$

$$\Rightarrow t^2 = \frac{\sqrt{5}-1}{2} \Rightarrow t = \pm\sqrt{\frac{\sqrt{5}-1}{2}}$$

$\Rightarrow$ There exists two tangents $\forall a \in R - \{0\}$

11. (b, d)

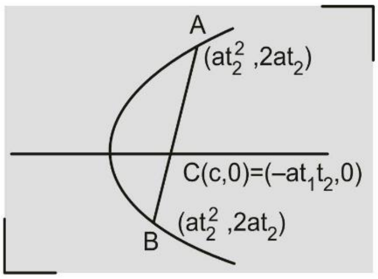
$$t_1 t_2 = -\frac{c}{a} \Rightarrow c = -at_1 t_2$$

$$\frac{AC^2}{CB^2} = \frac{(at_1^2 + at_1t_2)^2 + 4a^2t_1^2}{(at_2^2 + at_1t_2)^2 + 4a^2t_2^2} = \frac{t_1^2}{t_2^2}$$

$$\frac{CB}{AC} = \pm\frac{t_2}{t_1} \Rightarrow \frac{CB}{AC} + 1 = \pm\frac{t_2}{t_1} + 1$$

$$\Rightarrow \frac{AB}{AC} = \frac{t_2 + t_1}{t_1} = 3$$

or $$\frac{AB}{AC} = \frac{t_1 - t_2}{t_1} = 3$$



$$\Rightarrow t_2 + t_1 = 3t_1 \text{ or } t_1 - t_2 = 3t_1$$

$$\Rightarrow 2t_1 - t_2 = 0 \text{ or } 2t_1 + t_2 = 0$$

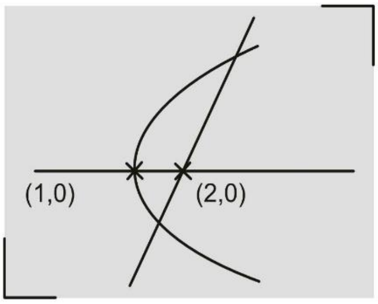
[$\because$ if $\frac{CB}{AC} = -\frac{t_2}{t_1} \Rightarrow t_1$ and t_2 are of opp. sign.]

$$6t_1^2 - 2t_2^2 - t_1t_2 = 0 = 6t_1^2 + 3t_1t_2 - 4t_1t_2 - 2t_2^2$$

$$\Rightarrow (3t_1 - 2t_2)(2t_1 + t_2) = 0$$

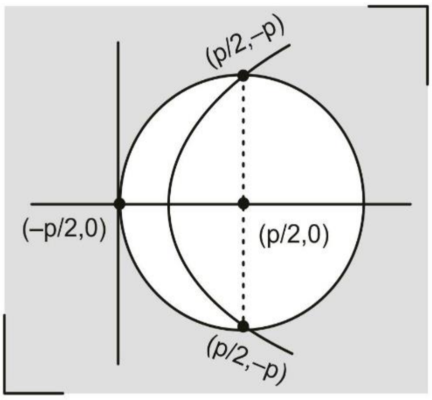
12. (a, c)

$m \neq 0$

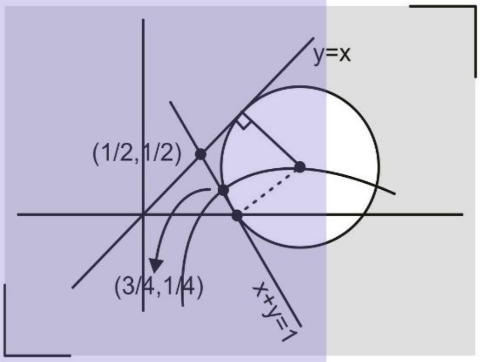


13. (a, b)

AB is latus rectum of parabola

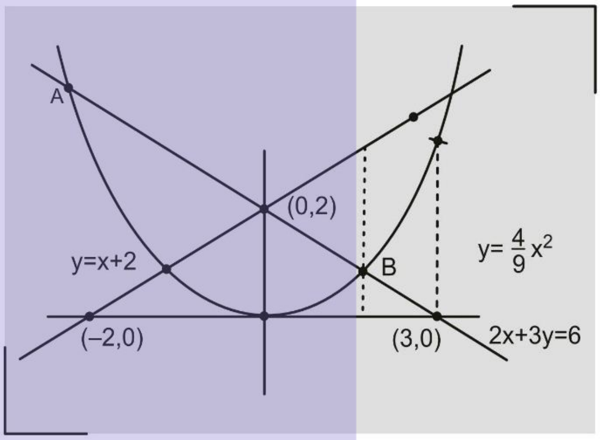


14. (b, c)



$$4a = 2\left(\frac{|0-1|}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$$

15. (a, c)



For point A, B

$$2x + 3\left(\frac{4}{9}x^2\right) = 6$$

$$2x^2 + 3x - 9 = 0$$

$$2x^2 + 6x - 3x - 9 = 0 = (2x-3)(x+3) = 0$$

$$\Rightarrow \quad A \equiv (-3, 4) \text{ and } B \equiv \left(\frac{3}{2}, 1\right)$$

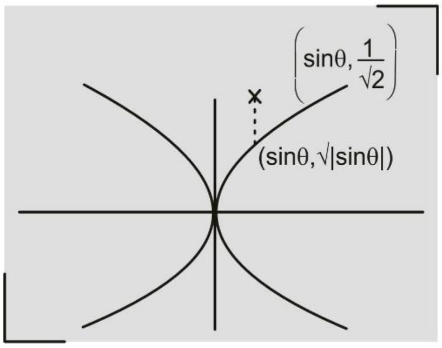
$$\therefore \quad \frac{3}{2}\alpha = -3 \Rightarrow \alpha = -2,$$

$$\frac{3}{2}\alpha = \frac{3}{2} \Rightarrow \alpha = 1$$

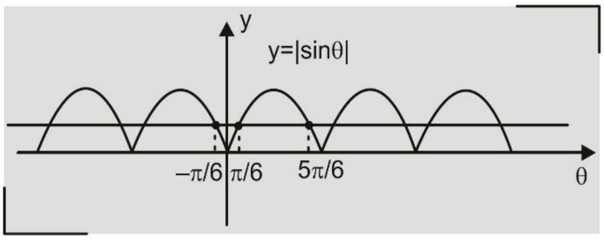
$$\therefore \quad \alpha \in (-\infty, -2) \cup (0, 1)$$

16. (a, b, c)

$$\sqrt{|\sin\theta|} < \frac{1}{\sqrt{2}}$$



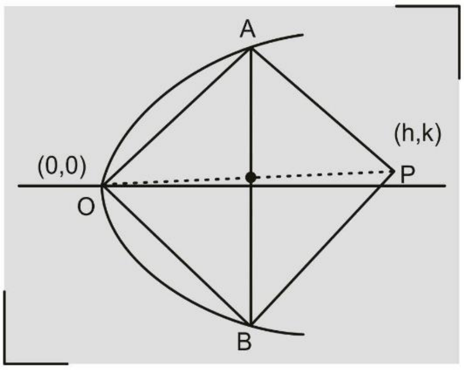
$$\Rightarrow \quad |\sin\theta| < \frac{1}{2}$$



$$\theta \in \left(n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right), n \in I$$

19. (a, c)

Equation of AB is



$$y\left(\frac{k}{2}\right) - 2a\left(x + \frac{h}{2}\right) = \frac{k^2}{4} - 2ah$$

$$2yk - 8ax = k^2 - 4ah$$

Homogenising, we get

$$(k^2 - 4ah)y^2 - 4ax(2yk - 8ax) = 0$$

coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \quad k^2 - 4ah + 32a^2 = 0$$

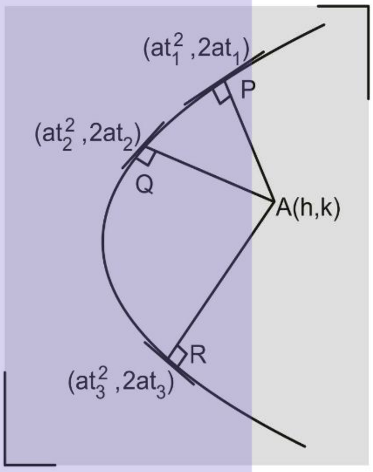
$$\Rightarrow \quad y^2 = 4a(x - 8a)$$

SOLUTIONS 3

Comprehension: (1)

1. (a) $y = -tx + 2at + at^3$ is normal to $y^2 = 4ax$ at $(at^2, 2at)$

Put (h, k)



$\Rightarrow at^3 + (2a-h)t - k = 0 \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix}$

$t_1 + t_2 + t_3 = 0,$

$t_1t_2 + t_2t_3 + t_3t_1 = \dfrac{2a-h}{a}$

$t_1^2 + t_2^2 + t_3^2 = (\Sigma t_1)^2 - 2\Sigma t_1t_2$

$= 0 - \dfrac{2(2a-h)}{a}$

$\therefore$ Centroid

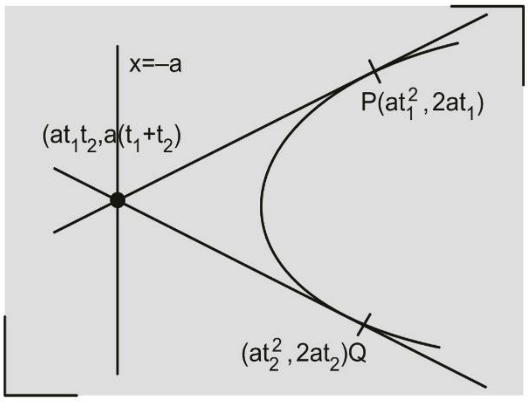
$\equiv \left(\dfrac{a}{3}(t_1^2 + t_2^2 + t_3^2), \dfrac{2a}{3}(t_1 + t_2 + t_3)\right)$

$\equiv \left(\dfrac{a}{3}\dfrac{2}{a}(h-2a), 0\right)$

Centroid $\equiv \left(\dfrac{2}{3}(h-2a), 0\right)$

2. (d) Intersection point

$\equiv (at_1t_2, a(t_1 + t_2)) \equiv (-a, -at_3)$



Quadratic whose roots are t_1, t_2 is

$\Rightarrow x^2 - (t_1 + t_2)x + t_1t_2 = 0$

$\Rightarrow x^2 + t_3x - 1 = 0$

3. (c) $at^3 + (2a-h)t - k = 0 \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix}$

$t_1t_2 = 1 \Rightarrow \quad t_1t_2t_3 = -t_3 = -t_3 = \dfrac{k}{a}$

$\Rightarrow \quad t_3 = -\dfrac{k}{a}$

Put t_3 to the cubic

$a\left(-\dfrac{k^3}{a^3}\right) - \dfrac{k}{a}(2a-h) - k = 0$

$\dfrac{k^2}{a^2} + 2 - \dfrac{h}{a} + 1 = 0$

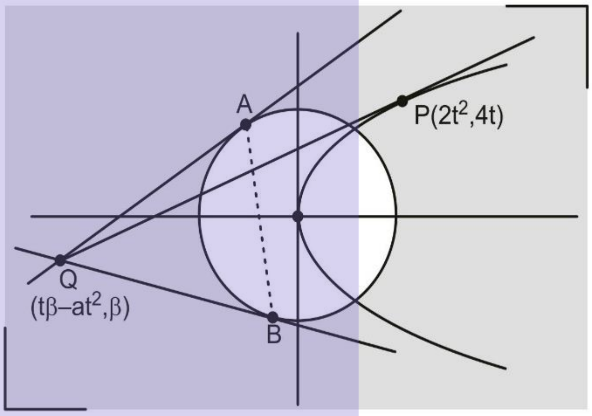
$k^2 = a(h - 3a)$

$\Rightarrow y^2 = a(x - 3a)$

Comprehension: (2)

1. (c) Equation of tangent PQ is $ty = x + 2t^2$

Equation of COC of Q w.r.t. $x^2 + y^2 = 4$ is $x(t\beta - 2t^2) + y\beta = 4$



$$(tx+y)\beta-(4+2t^2x)=0$$

$\therefore$ COC's are concurrent at $\left(-\frac{2}{t^2},\frac{2}{t}\right)$

$\therefore \quad h=-\frac{2}{t^2}$

$k=\frac{2}{t}\Rightarrow k^2=\frac{4}{t^2}$

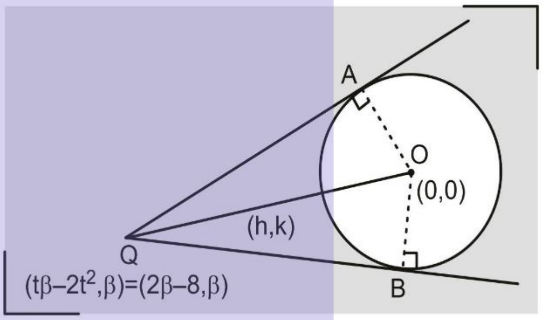
$\Rightarrow \quad \frac{k^2}{h}=-2 \quad\Rightarrow\quad y^2=-2x$

2. (d) Point lies on $x=-2$

and $x^2+y^2=2(4)=8$

$\therefore \quad y^2=4 \quad\Rightarrow\quad y=\pm 2$

3. (a) Circumcentre is midpoint of OQ



$2h=2\beta-8$

$2k=\beta$

$\Rightarrow \quad 4k-2h=8$

$2y=x+4$

Comprehension: (3)

1. (a), 2. (c), 3. (d)

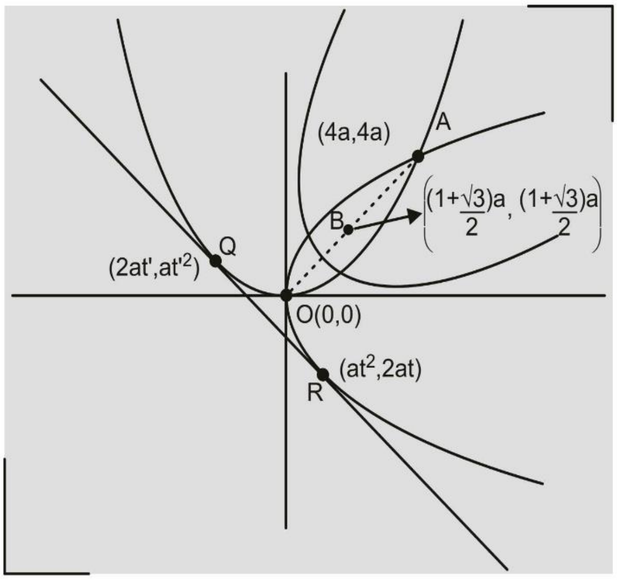
Eqn. of tangent at Q

$xt'-y=at'^2$ ...(1)

Eqn. of tangent at P

$yt-x=at^2$ ...(2)

(1) and (2) are identical



$\Rightarrow \quad -t=-\frac{1}{t'}=\frac{at^2}{at'^2}$

$\Rightarrow \quad t^4=-t\Rightarrow t=-1 \quad (t=0 \text{ rejected})$

$\therefore$ Common tangent is $x+y+a=0$

LR of P is

$$2\left(\frac{\frac{(1+\sqrt{3})a}{2}+\frac{(1+\sqrt{3})a}{2}+a}{\sqrt{2}}\right)=\sqrt{2}(2+\sqrt{3})a$$

Eqn. of parabola P is

$$\frac{(y-x)^2}{2}=\sqrt{2}(2+\sqrt{3})a$$

$$\left(\frac{\left(x+y-\frac{(1+\sqrt{3})a}{2}\right)}{\sqrt{2}}\right)$$

$$(y-x)^2=(2+\sqrt{3})\,a\,(2x+2y-(1+\sqrt{3})a)$$

Extremities of $L.R.$ of P is

$$\left(\frac{(1+\sqrt{3})}{2}a-\frac{1}{\sqrt{2}}\frac{(2+\sqrt{3})a}{\sqrt{2}},\frac{(1+\sqrt{3})a}{2}+\frac{1}{2}\frac{(2+\sqrt{3})a}{\sqrt{2}}\right)$$

$$\left(\frac{(1+\sqrt{3}}{2}a+\frac{1}{\sqrt{2}}\frac{(2+\sqrt{3})a}{\sqrt{2}},\frac{(1+\sqrt{3})a}{2}-\frac{1}{\sqrt{2}}\frac{(2+\sqrt{3})a}{\sqrt{2}}\right)$$

$$\equiv\left(-\frac{a}{2},\frac{(3+2\sqrt{3})a}{2}\right),\left(\frac{(3+2\sqrt{3})a}{2},-\frac{a}{2}\right)$$

Comprehension: (4)

$$y=x^2+bx+1$$

For tangent from all points $(\alpha,0)$ on x-axis

$\therefore\quad x^2+bx+1\geq 0\ \forall\ x\in R$

$\Rightarrow\quad D\leq 0\ \Rightarrow\quad b^2-4\leq 0$

$\Rightarrow\quad b\in[-2,2]$

$\therefore\quad b_{max}=2$

Eqn. of tangent at (0, 1) to $y=x^2+bx+1$

is $y+1=bx+2\Rightarrow y=bx+1$ ...(1)

$\because$ Eqn. (1) represents tangent to

$$x^2+y^2=r^2$$

$$\Rightarrow\quad r=\frac{1}{\sqrt{1+b^2}}$$

1. (b) for $b=2,\ r=\dfrac{1}{\sqrt{5}}$

$$A=\frac{\pi}{5}$$

2. (d) $r=\dfrac{1}{\sqrt{1+b^2}},\quad r_{max}=1$

$$\lim_{b\to 0}\frac{\sqrt{1-\frac{1}{\sqrt{1+b^2}}}}{\sin b}=\lim_{b\to 0}\frac{\sqrt{(1+b^2)^{1/2}-1}}{(1+b^2)^{1/4}\sin b}$$

$$=\lim_{b\to 0}\frac{|b|}{(\sqrt{1+b^2}+1)^{1/2}(1+b^2)^{1/4}\sin b}$$

$LHL=-\dfrac{1}{\sqrt{2}},\ RHL=\dfrac{1}{\sqrt{2}}\Rightarrow$ Limit does not exist

3. (d) Vertex $\equiv\left(-\dfrac{b}{2},1-\dfrac{b^2}{4}\right)=(h,k)$,

$$k=1-h^2,\quad y=1-x^2,$$

$$-\frac{b}{2}\in[-1,1]\Rightarrow\frac{b^2}{4}\in[0,1]$$

$$\Rightarrow\quad 1-\frac{b^2}{4}\in[0,1]$$

$$\Rightarrow\quad x\in[-1,1],\quad y\in[0,1]$$

Comprehension: (5)

$$(x,y)=(\cos\theta,\sin\theta)$$

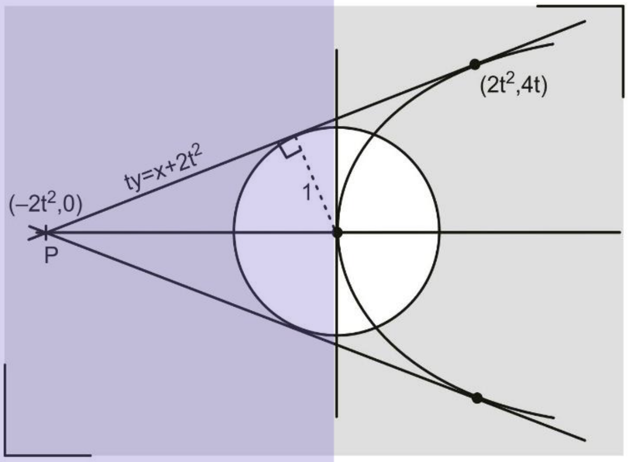
$$A=\lim_{\theta\to\frac{\pi}{4}}\frac{4\cos^2\theta+2\sin^2\theta-6\sin\theta\cos\theta}{6\cos^2\theta+\sqrt{2}\sin\theta-8\sin\theta\cos\theta}$$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{(4\cos\theta+2\sin\theta)(\cos\theta-\sin\theta)}{6\cos\theta(\cos\theta-\sin\theta)+\sqrt{2}\sin\theta(1-\sqrt{2}\cos\theta)}$$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{(4\cos\theta-2\sin\theta)(\cos\theta-\sin\theta)}{6\cos\theta+\sqrt{2}\sin\theta\frac{(1-\sqrt{2}\cos\theta)}{(\cos\theta-\sin\theta)}}$$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{(4\cos\theta-2\sin\theta)}{6\cos\theta-\sqrt{2}\sin\theta\frac{(\cos\theta+\sin\theta)}{(1+\sqrt{2}\cos\theta)}}=\frac{2}{5}$$

1. (c) Perpendicular from $(0,0)=1$



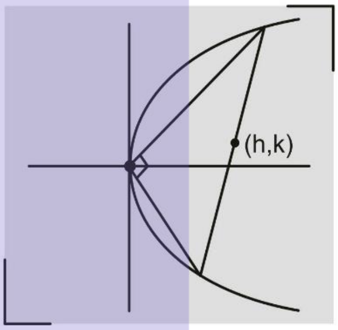
$$\Rightarrow \frac{2t^2}{\sqrt{t^2+1}}=1$$

$$\Rightarrow 4t^4=t^2+1$$

$$\Rightarrow t^2=\frac{1+\sqrt{17}}{8}$$

$$P\equiv\left(-\frac{\sqrt{17}+1}{4},0\right)=\left(-\frac{4}{\sqrt{17}-1},0\right)$$

2. (a) Equation of chord whose midpoint is (h,k) is



$$yk-4(x+h)=k^2-8h$$

$$yk-4x=k^2-4h$$

Homogenising,

$$(k^2-4h)y^2=8x(yk-4x)$$

$$\Rightarrow (k^2-4h)y^2+32x^2-8kxy=0$$

coefficient of x^2 + coefficient of $y^2=0$

$$\Rightarrow k^2-4h+32=0$$

Hence, $y^2-4x+32=0$

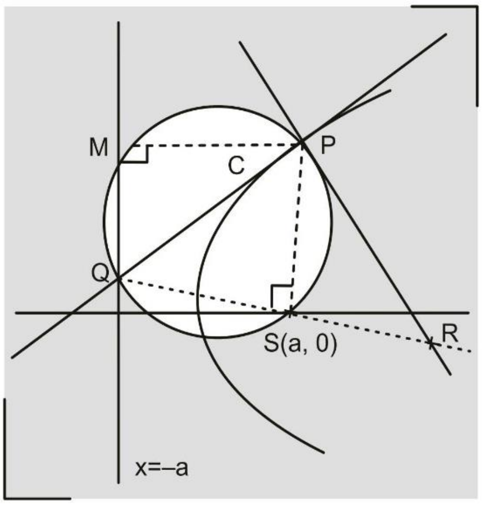
3. (c) Obvious

Comprehension: (6)

1. (d) PQ is diameter of circle

$\Rightarrow$ tangent of parabola is normal to C

$\therefore$ Circle & parabola are orthogonal



2. (d) Area of

$$\Delta PQR=\frac{1}{2}(PS)(QR)=\frac{1}{2}(PQ)(PR)$$

$$\Rightarrow \frac{(PS)(QR)}{(PQ)(PR)}=1$$

3. (c) $(PQ)^2$

$$=(at^2+a)^2+\left(2at-\frac{at^2-a}{t}\right)^2$$

$$=a^2\left[(t^2+1)^2+\frac{(t^2+1)^2}{t^2}\right]=\frac{(t^2+1)^3a^2}{t^2}$$

Area of C $=\frac{\pi}{4}(PQ)^2=\frac{\pi}{4}\frac{(t^2+1)^3}{t^2}a^2$

Comprehension: (7)

1. (a), 2. (c), 3. (d)

Equation of CD is

$$x(t_1+t_2)=2y+\frac{1}{2}t_1t_2.,$$

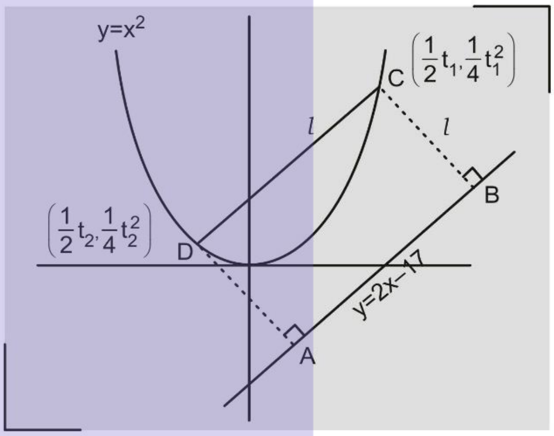
y intercept $= -\frac{1}{4}t_1 t_2$

slope $= \frac{t_1 + t_2}{2} = 2 \Rightarrow t_1 + t_2 = 4$

$$l = \frac{\frac{1}{4}t^2 - 2\left(\frac{1}{2}t\right) + 17}{\sqrt{5}}$$

$\Rightarrow t^2 - 4t + (-4l\sqrt{5} + 68) = 0 \begin{matrix} \nearrow t_1 \\ \searrow t_2 \end{matrix}$ ...(1)

Also $l^2 = \frac{1}{4}(t_1 - t_2)^2 + \frac{1}{16}(t_1^2 - t_2^2)^2$



$\Rightarrow l^2 = [(t_1 + t_2)^2 - 4t_1 t_2]\left[\frac{1}{4} + \frac{1}{16}(t_1 + t_2)^2\right]$

$\Rightarrow l^2 = [16 - 4(-4l\sqrt{5} + 68)]\left[\frac{1}{4} + \frac{1}{16} \times 16\right]$

$= 5[4l\sqrt{5} - 64]$

$\Rightarrow l^2 - 20\sqrt{5}\,l + 320 = (l - 4\sqrt{5})$

$(l - 16\sqrt{5}) = 0$

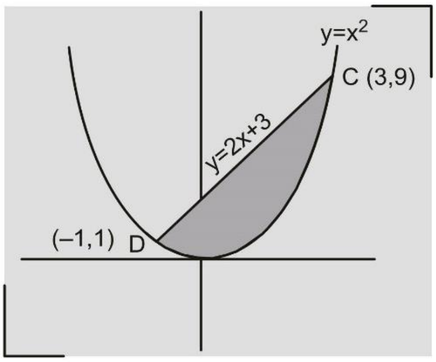
$l = 4\sqrt{5},\ 16\sqrt{5}$

$\Rightarrow A_{max} = l_{max}^2 = (16\sqrt{5})^2 = 1280$

From (1),

$t^2 - 4t - 12 = 0 \begin{matrix} \nearrow t_1 \\ \searrow t_2 \end{matrix}$, If $l = 4\sqrt{5}$

$\Rightarrow$ y intercept of CD is $y = -\frac{1}{4}t_1 t_2 = 3$



$\Rightarrow t_1 = 6,\ t_2 = -2$

$C \equiv (3,9), D(-1,1)$

Area of shaded region

$$= \int_{-1}^{3} (2x + 3 - x^2)\,dx = \frac{32}{3}$$

SOLUTIONS 4

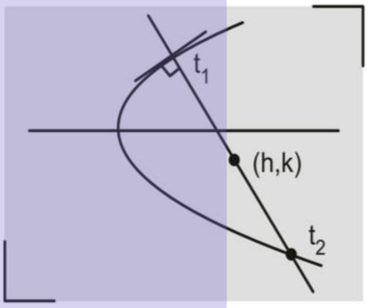
Match the Columns:

1. a-q; b-s; c-p; d-r

(a) $t_2 = -t_1 - \frac{2}{t_1}$

$2(t_1 + t_2) = 2x = -\frac{4}{t_1}$

$\Rightarrow \quad t_1 = \frac{-2}{k}, t_2 = k + \frac{2}{k}$

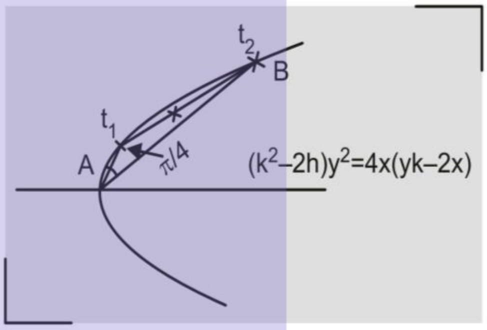


$2h = (t_1^2 + t_2^2) = \frac{4}{k^2} + \left(k^2 + \frac{4}{k^2} + 4\right)$

$= \frac{8}{k^2} + 4 + k^2$

$y^4 + (4 - 2x)y^2 + 8 = 0$

(b)



Eqn. of AB is $T = S_1 \Rightarrow yk - 2x = k^2 - 2h$

Homogenise

$\Rightarrow \quad (k^2 - 2h)y^2 = 4x(yk - 2x)$

$\Rightarrow \quad 8x^2 + (k^2 - 2h)y^2 - 4kxy = 0$

$\Rightarrow (8 + k^2 - 2h)^2 = 4[(2k)^2 - 8(k^2 - 2h)]$

$\Rightarrow \quad (y^2 - 2x + 8)^2 - 16(4x - y^2) = 0$

$\Rightarrow \quad y^4 - 4xy^2 + 4x^2 + 32y^2 - 96x + 64 = 0$

(c) Put $(h + r\cos\theta, k + r\sin\theta)$ in equation of parabola

$k^2 + r^2\sin^2\theta + 2kr\sin\theta$

$-4(h + r\cos\theta) = 0$

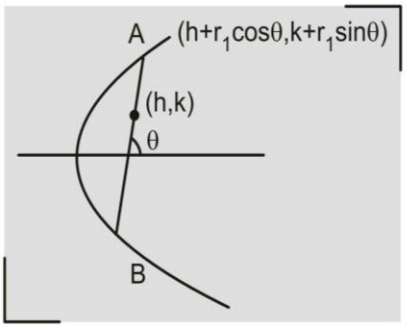
$r^2\sin^2\theta + (2k\sin\theta - 4\cos\theta)r$

$+k^2 - 4h = 0 \begin{cases} 1 \\ -1 \end{cases}$

Sum of roots $= 0$

$\Rightarrow \quad 2k\sin\theta - 4\cos\theta = 0$

$\Rightarrow \quad \tan\theta = \frac{2}{k}$



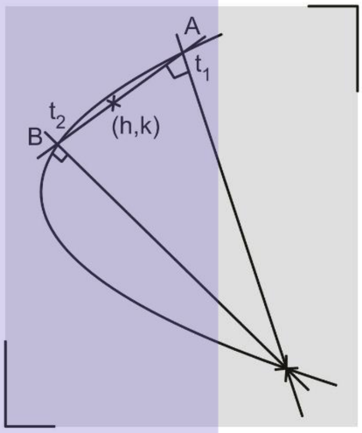
Product of roots $= -1$

$\Rightarrow \quad -1 = (k^2 - 4h)\left(1 + \frac{k^2}{4}\right)$

$-4 = (y^2 + 4)(y^2 - 4x)$

$\Rightarrow \quad y^4 + 4(1 - x)y^2 + 4(1 - 4x) = 0$

(d) Equation of AB is



$yk - 2x = k^2 - 2h$

Put $(t^2, 2t)$

$2t^2 - 2kt + (k^2 - 2h) = 0$

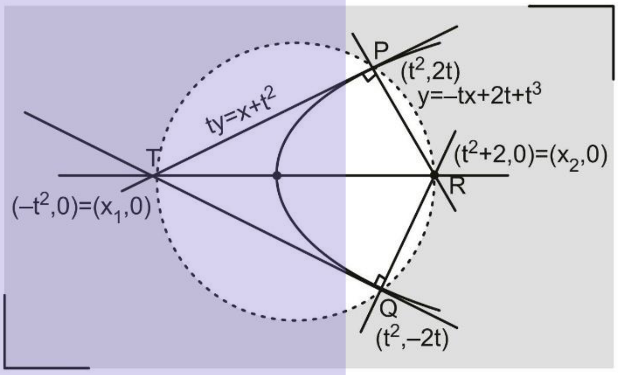
$t_1 t_2 = 2 \Rightarrow \dfrac{k^2 - 2h}{2} = 2$

$y^2 = 2(x + 2)$

2. a- r; b-q; c-q, r, s; d-s

(a) $t^2 + 2 = 3 \;\Rightarrow\; t = \pm 1$

Area of $PTQR = 2\left(\dfrac{1}{2}(3+1)2\right) = 8$



(b) $PT = \sqrt{4t^4 + 4t^2} = 4t\sqrt{1+t^2}$

$4t^2(1+t^2) = 80$

$(t^2+5)(t^2-4) = 0$

$t = \pm 2 \Rightarrow x_2 = 6$

(c) $x_2 = t^2 + 2 > 2$

(d) $x_2 = 4 \Rightarrow t^2 = 2$

TR is diameter, $TR = 6$

Area of circle $= \dfrac{\pi}{4}(TR)^2 = 9\pi$

3. a-q; b-r; c-p; d-s

$O = \left(\dfrac{3}{2}, \dfrac{5}{2}\right),\; Q = \left(\dfrac{1}{3}, \dfrac{11}{3}\right)$

Feet of perpendicular from focus on any tangent lies on tangent at vertex

$\Rightarrow$ focus $S \equiv \left(\dfrac{17}{9}, \dfrac{26}{9}\right)$

foot of directrix

$\equiv \left(2\left(\dfrac{3}{2}\right) - \dfrac{17}{9}, 2\left(\dfrac{5}{2}\right) - \dfrac{26}{9}\right) \equiv \left(\dfrac{10}{9}, \dfrac{19}{9}\right)$

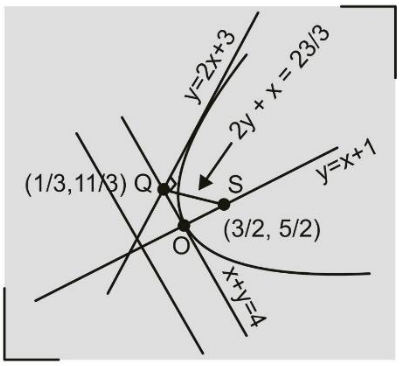
Equation of directrix is

$x + y = \dfrac{29}{9} \Rightarrow 9x + 9y - 29 = 0$

Length of latus rectum

$= 4\sqrt{\left(\dfrac{17}{9} - \dfrac{3}{2}\right)^2 + \left(\dfrac{19}{9} - \dfrac{5}{2}\right)^2}$

Length of $LR = \dfrac{14\sqrt{2}}{9}$ (Here, LR = latus rectum)



Extremities of

$LR \equiv \left(\dfrac{17}{9} + \dfrac{7\sqrt{2}}{9}\left(\cos\dfrac{3\pi}{4}\right), \dfrac{26}{9} + \dfrac{7\sqrt{2}}{9}\left(\sin\dfrac{3\pi}{4}\right)\right)$

$$\left(\frac{17}{9}-\frac{7\sqrt{2}}{9}\left(\cos\frac{3\pi}{4}\right),\frac{26}{9}-\frac{7\sqrt{2}}{9}\left(\sin\frac{3\pi}{4}\right)\right)$$

Extremities of

$$LR \equiv \left(\frac{10}{9},\frac{33}{9}\right) \text{ and } \left(\frac{24}{9},\frac{19}{9}\right)$$

Equation of parabola is

$$\left(\frac{y-x-1}{\sqrt{2}}\right)^2=\frac{14}{9}\sqrt{2}\,\frac{|y+x-4|}{\sqrt{2}}$$

$$\Rightarrow\quad 9(y-x-1)^2=28(y+x-4)$$

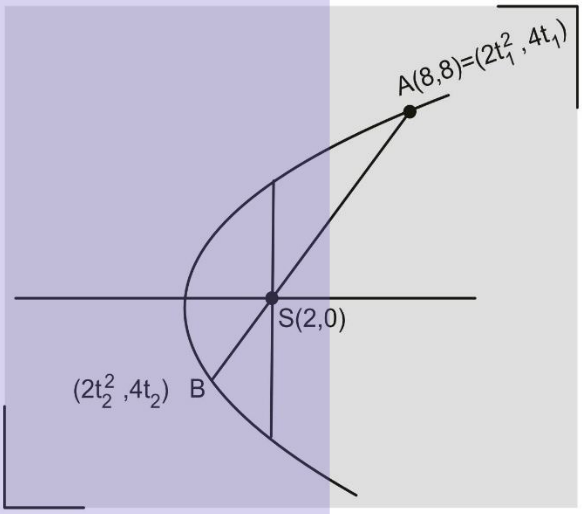
4. a-s; b-r; c-r; d-q

(a) $t_1=2$

Slope of $AB=\dfrac{2}{t_1+t_2}=\dfrac{8}{6}=\dfrac{4}{3}$

$$\Rightarrow\quad t_1+t_2=\frac{3}{2}$$

$$t_2=-\frac{1}{2}$$



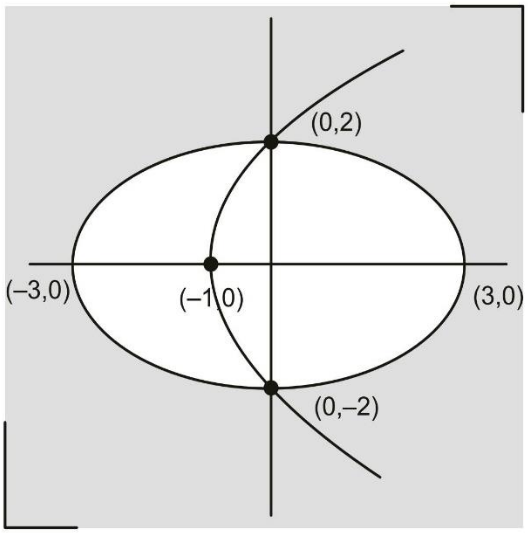
$$AB=AS+SB=(2+2t_1^2)+(2+2t_2^2)$$

$$=4+2\left(4+\frac{1}{4}\right)=\frac{25}{2},\quad AB=\frac{25}{2}$$

(b) $\left(x-\dfrac{1}{26}\right)^2+\left(y-\dfrac{3}{26}\right)^2=\dfrac{k}{4}\left(\dfrac{5x-12y+1}{13}\right)^2$

$$\Rightarrow\quad \frac{k}{4}=1\Rightarrow k=4$$

(c)

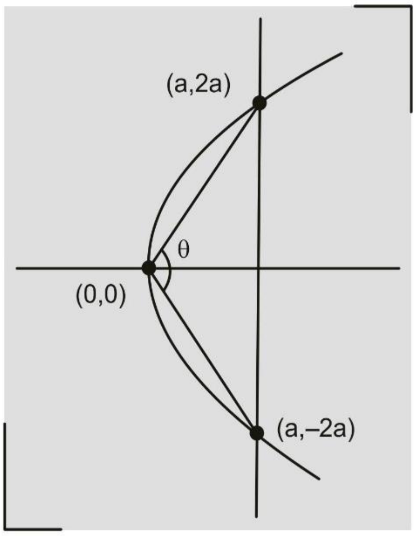


$$4x^2+9\times4(x+1)=36$$

$$\Rightarrow\quad x^2+9x=0\Rightarrow x=0,-9$$

Length of common chord $=4$

(d) $\tan\dfrac{\theta}{2}=2$



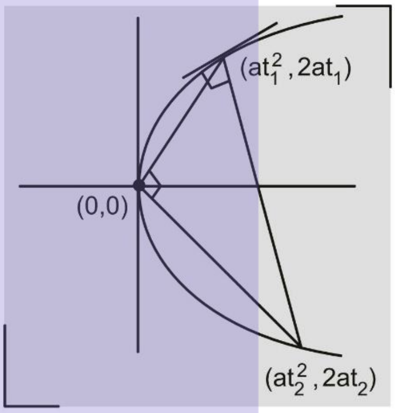
$$|\tan\theta|=\left|\frac{2(2)}{1-(2)^2}\right|=\frac{4}{3}$$

5. a-p,s ; b-q,r ; c-s ; d-q,r

(a) For line to become tangent to ellipse

$$a^2=2+1\Rightarrow a=\pm\sqrt{3}$$

(b)



$$\frac{2}{t_1}\frac{2}{t_2} = -1,\ t_1 t_2 = -4$$

$$t_2 = -t_1 - \frac{2}{t_1} = -\frac{4}{t_1}$$

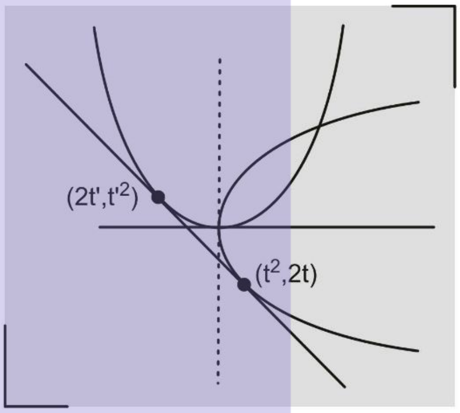
$$m = -t_1 = \pm\sqrt{2}$$

(c) $ty = x + t^2 \quad \Rightarrow \quad ty - x = t^2 \quad ...(1)$

$t'x = y + t'^2 \quad \Rightarrow \quad t'x - y = t'^2 \quad ...(2)$

(1) and (2) are identical

$$\Rightarrow \quad -t = -\frac{1}{t'} = \frac{t^2}{t'^2} = t^4$$



$t^3 = -1 \quad \Rightarrow \quad t = -1$

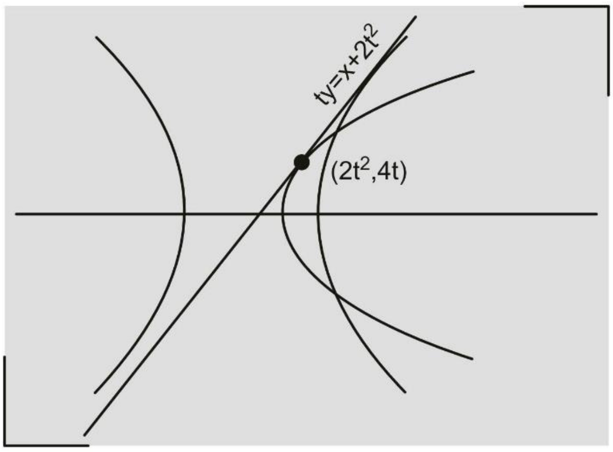
$-y - x = 1$

$x + y + 1 = 0 \Rightarrow k = \sqrt{3}$

(d) $ty = x + 2t^2$

$$y = \frac{1}{t}x + 2t$$

For line to become tangent to hyperbola



$$4t^2 = \frac{1}{t^2} - 3 \quad \Rightarrow \quad 4t^4 + 3t^2 - 1 = 0$$

$$\Rightarrow \quad (4t^2 - 1)(t^2 + 1) = 0$$

$$t = \pm\frac{1}{2}$$

$$\pm\frac{1}{2}y = x + \frac{1}{2} \quad \Rightarrow \quad 2x + 1 = \pm y$$

$$2x \pm y + 1 = 0$$

6. a-q; b-s; c-q; d-p

(a) $t^2 = 2$

(b) $\Delta = |(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

$= 1 \times 2 \times 3 = 6$

$(\because t_1 = 1, t_2 = 2, t_3 = 4)$

(c) Put $\left(\frac{11}{4}, \frac{1}{4}\right)$ to equation

$$y = -tx + 2t + t^3$$

$$\frac{1}{4} = -\frac{11}{4}t + 2t + t^3$$

$$\Rightarrow \quad 4t^3 - 3t - 1 = 0$$

$$\Rightarrow \quad (t - 1)(2t + 1)^2 = 0$$

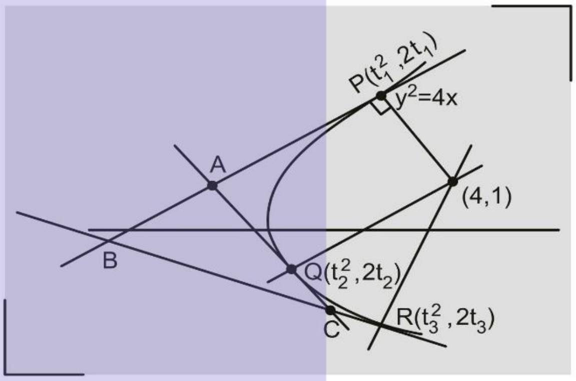
2 distinct normals are drawn through $\left(\frac{11}{4}, \frac{1}{4}\right)$

7. a-p; b-t; c-q; d-s

Equation of normal at $(t^2, 2t)$ is

$$y + tx = 2t + t^3$$

Put $(4,1)$



$$\Rightarrow \quad t^3 - 2t - 1 = 0$$

$$\Rightarrow \quad t = -1, \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}$$

$$\therefore \quad t_1 = \frac{1+\sqrt{5}}{2}, t_2 = \frac{1-\sqrt{5}}{2}, t_3 = -1$$

$$A \equiv (t_1 t_2, t_1 + t_2) \equiv (-1, 1)$$

$$B \equiv (t_1 t_3, t_1 + t_3) \equiv \left(\frac{-1-\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2}\right)$$

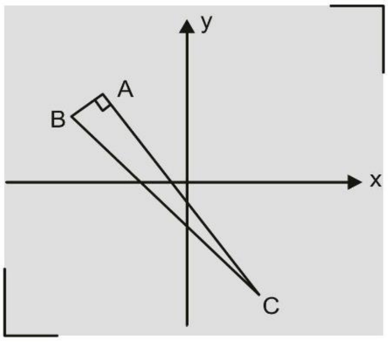
$$C \equiv (t_2 t_3, t_2 + t_3) \equiv \left(\frac{\sqrt{5}-1}{2}, \frac{-\sqrt{5}-1}{2}\right)$$

Centroid of ΔABC, $G \equiv \left(\frac{-2}{3}, 0\right)$

Orthocentre of ΔABC, $H \equiv (-1, 1)$

Circumcentre of ΔABC,

$$O \equiv \left(\frac{-1}{2}, \frac{-1}{2}\right) \equiv \text{midpoint of } BC$$



Focus $S \equiv (1, 0)$

(a) $SG = \frac{5}{3}$

(b) $SH = \sqrt{2^2 + 1^2} = \sqrt{5}$

(c) $SO = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{10}}{2}$

(d) Area of

$$\Delta ABC = \frac{1}{2}(AB)(AC) = \frac{1}{2}\frac{\sqrt{20+8\sqrt{5}}}{2}$$

$$\frac{\sqrt{20-8\sqrt{5}}}{2} = \frac{\sqrt{5}}{2}$$

SOLUTIONS 5

Subjective Problems

1. (0) $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2}$

$= \frac{a(t_1^2 - t_2^2)}{2at_3} + \frac{a(t_2^2 - t_3^2)}{2at_1} + \frac{a(t_3^2 - t_1^2)}{2at_2}$

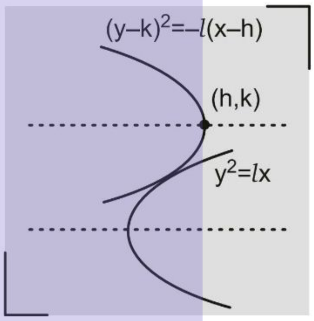
$= \frac{(t_1 - t_2)(-t_3)}{2t_3} + \frac{(t_2 - t_3)(-t_1)}{2t_1} + \frac{(t_3 - t_1)(-t_2)}{2t_2}$

$[\because t_1 + t_2 + t_3 = 0]$

$= -\frac{1}{2}[t_1 - t_2 + t_2 - t_3 + t_3 - t_1]$

$= 0$

2. (2) For intersection of two parabolas



$(y - k)^2 = lh - y^2$

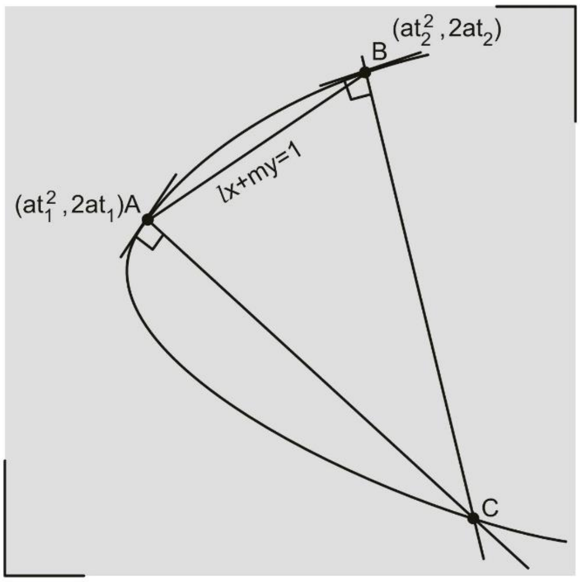
$\Rightarrow \ 2y^2 - 2ky + k^2 - lh = 0$

$D = 0$ ($\because$ curves touches)

$\Rightarrow \ 4k^2 - 8(k^2 - hl) = 0$

$\Rightarrow \ k^2 = 2lh \Rightarrow y^2 = 2lx$

3. (4) Put $(at^2, 2at)$ to equation of AB *i.e.* $lx + my = 1$



$\Rightarrow \ lat^2 + 2mat - 1 = 0 \begin{matrix} t_1 \\ t_2 \end{matrix}$

$\Rightarrow \ t_1 + t_2 = -\frac{2m}{l}$

$\because \ t_1 + t_2 + t_3 = 0$

$[\because A, B, C$ are co-normal points$]$

$\Rightarrow \ t_3 = \frac{2m}{l}$

$\therefore \ C \equiv (at_3^2, 2at_3) = \left(\frac{4m^2a}{l^2}, \frac{4am}{l}\right)$

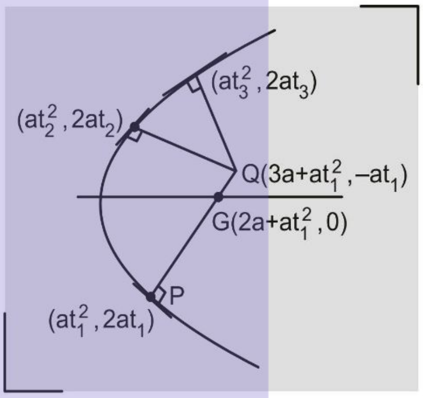
$\Rightarrow \ k = 4$

4. (2) $Q \equiv (a(t_2^2 + t_3^2 + 2 + t_2t_3), -at_2t_3(t_2 + t_3))$

$-at_2t_3(t_2 + t_3) = -at_1$

$\Rightarrow \ -at_2t_3(-t_1) = -at_1$

$[\because t_1, t_2, t_3$ are co-normal points$]$



$\Rightarrow \quad t_2 t_3 = -1$

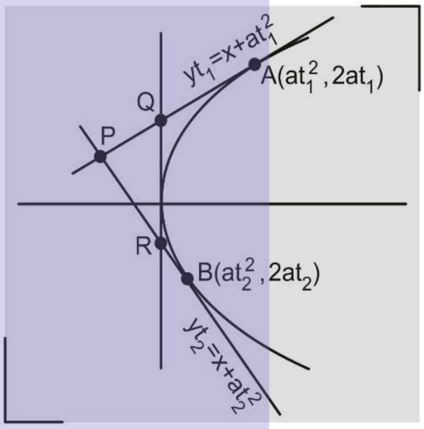
$\therefore \quad (-t_2)(-t_3) = -1$

$\Rightarrow$ normals at t_2 and t_3 are perpendicular.

5. (4) $Q \equiv (0, at_1), R \equiv (0, at_2)$

$P \equiv (at_1 t_2, a(t_1 + t_2)) = (h, k)$

Perpendicular through P to QR bisects QR



$$\text{Area of } \Delta PQR = \frac{1}{2} \times |a(t_1 - t_2) at_1 t_2|$$

$$c^2 = \frac{a^2}{2} |t_1 t_2 (t_1 - t_2)|$$

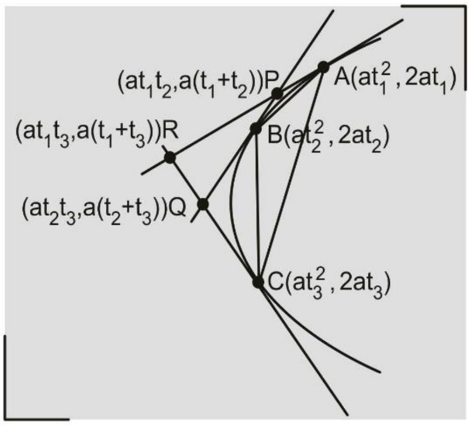
$$\Rightarrow c^4 = \frac{a^4}{4} t_1^2 t_2^2 [(t_1 + t_2)^2 - 4t_1 t_2]$$

$$= \frac{a^4}{4} \frac{h^2}{a^2} \left[\left(\frac{k}{a} \right)^2 - 4 \frac{h}{a} \right]$$

$$4c^4 = h^2 (k^2 - 4ah)$$

$$\Rightarrow x^2 (y^2 - 4ax) = 4c^4 \Rightarrow \lambda = 4$$

6. (2)

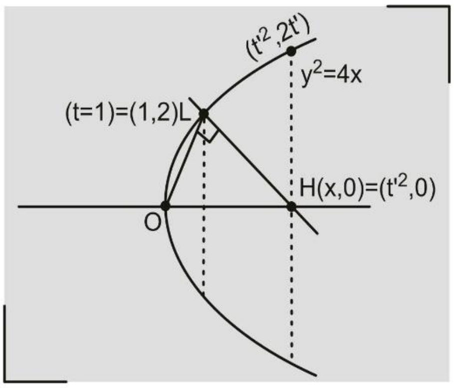


$$\frac{\Delta ABC}{\Delta PQR} = \frac{\left| \frac{1}{2} \begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix} \right|}{\left| \frac{1}{2} \begin{vmatrix} at_1 t_2 & a(t_1 + t_2) & 1 \\ at_2 t_3 & a(t_3 + t_2) & 1 \\ at_3 t_1 & a(t_3 + t_1) & 1 \end{vmatrix} \right|}$$

$$= \frac{a^2 |(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|}{\frac{1}{2} a^2 |(t_1 - t_2)(t_3 - t_2)(t_3 - t_1)|} = 2$$

7. (80) $\dfrac{0-2}{x-1} = -\dfrac{1}{2} \Rightarrow x - 1 = 4 \Rightarrow x = 5$

$t'^2 = 5 \Rightarrow t' = \sqrt{5}$



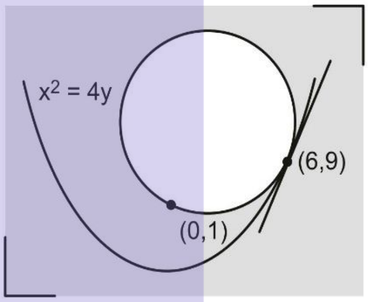
Length of double ordinate

$= 4t' = 4\sqrt{5} = \sqrt{80}$

$N = 80$

8. (5) Equation of tangent to parabola at P is

$2(y+9)=6x \Rightarrow y+9=3x$



Equation of circle is

$(x-6)^2+(y-9)^2+\lambda(y-3x+9)=0$

Put $(0,1)$

$\Rightarrow$ $36+64+\lambda(10)=0$

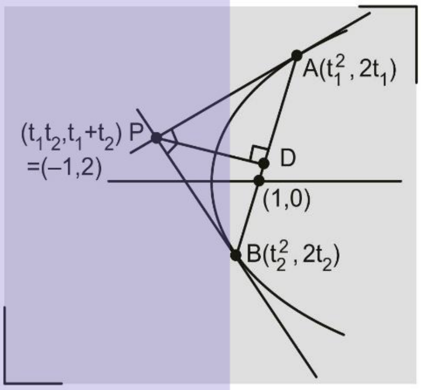
$\Rightarrow$ $\lambda=-10$

$\therefore$ Equation of circle is

$x^2+y^2+18x-28y+27=0$

$\therefore$ radius $=\sqrt{9^2+(14)^2-27}=5\sqrt{10}$

9. (8) Equation of AB is



$2y=2(x-1)$

$\Rightarrow$ $y=x-1$

$PD=\dfrac{|-1-1-2|}{\sqrt{2}}=2\sqrt{2}$

$AB=(1+t_1^2)+(1+t_2^2)=2+t_1^2+t_2^2$

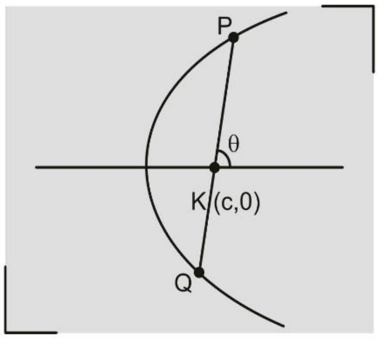
$=(t_1+t_2)^2+4=4+4=8$

Area of $\Delta PAB=\dfrac{1}{2}\times 2\sqrt{2}\times 8=8\sqrt{2}$ $\quad N=8$

10. (2) Put $(c+r\cos\theta, r\sin\theta)$ to the equation $y^2=4x$

$\Rightarrow$ $r^2\sin^2\theta-4\cos\theta\, r-4c=0$ (roots r_1, r_2)

$r_1=PK, r_2=-QK$



$\dfrac{1}{(PK)^2}+\dfrac{1}{(QK)^2}=\dfrac{1}{r_1^2}+\dfrac{1}{r_2^2}$

$=\dfrac{(r_1+r_2)^2-2r_1r_2}{(r_1r_2)^2}$

$=\dfrac{\dfrac{16\cos^2\theta}{\sin^4\theta}+\dfrac{8c}{\sin^2\theta}}{\dfrac{16c^2}{\sin^4\theta}}$

$=\dfrac{16\cos^2\theta+8c\sin^2\theta}{16c^2}$

$8c=16$ for $\dfrac{1}{(PK)^2}+\dfrac{1}{(QK)^2}$ to be independent of θ.

$\Rightarrow$ $c=2$