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PHYSICS — SURFACE TENSION

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SURFACE TENSION

UNIT:VII CHAP:02

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SURFACE TENSION

INTRODUCTION TO SURFACE TENSION

What you already know

- Solid mechanics
- Fluid mechanics

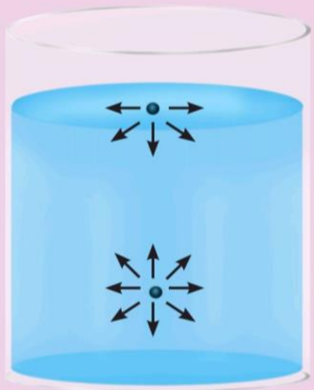
What you will learn

- Surface tension
- Surface energy
- Pressure difference across surfaces of bubble and drop

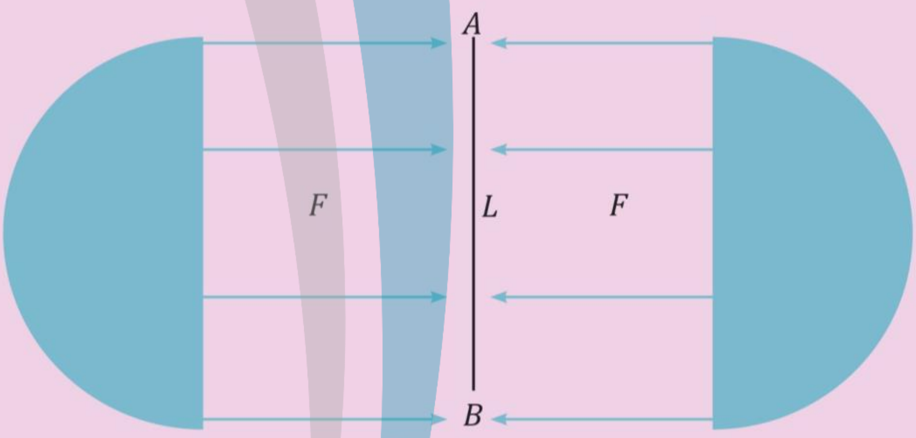
Surface Tension

The property of a liquid at rest by virtue of which its free surface behaves like a stretched membrane under tension and tries to occupy as small area as possible is known as **surface tension**.

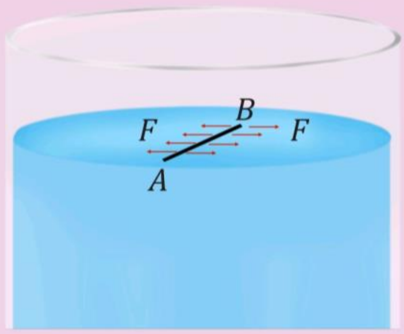
It is present only on the free surface of the liquid because of the unbalanced or asymmetric force on the liquid molecules residing at the surface as shown in the figure.



Consider an imaginary line AB of length L on the free liquid surface. The liquid at the right part of that imaginary line pulls the liquid on the left part by a tensile force F and vice-versa as shown in the figure.



Top view



Side view

Mathematically, surface tension (S) is defined as tensile force (F) per unit length (L).

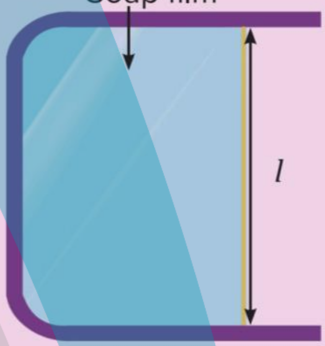
$$\text{Surface tension, } S = \frac{\text{Force}\,(F)}{\text{Length}\,(L)}$$

- Surface tension is a **scalar** quantity.
- The SI unit of surface tension is $\boldsymbol{Nm^{-1}}$.

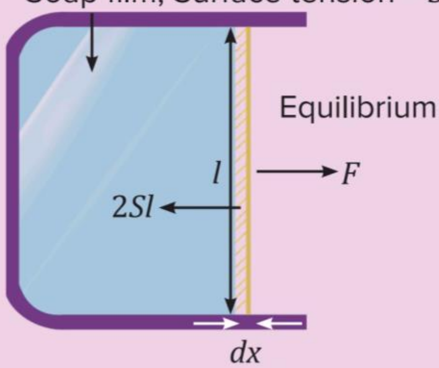
BOARDS MAIN

Work Done by Surface Tension

Consider a system consisting of a U-shaped wire and a slider of length l, which can move frictionlessly on the U-frame. Immerse the system in a soap solution and a soap film will be formed within the system as shown in the figure.



We know that surface tension is force per unit length. Here, the force on the slider due to the surface tension of the soap film is equal to $2Sl$ towards the left. (Factor 2 is arising because of the two faces of the soap film.)



Let a force F be applied quasi-statically on the slider to move it through distance dx.

The work done by the external force is,

$$dW_{ext} = F\,dx$$

$$\Rightarrow dW_{ext} = (2Sl)dx$$

$$\Rightarrow dW_{ext} = S(2l\,dx)$$

$$\Rightarrow dW_{ext} = S\,dA$$

Surface tension is the work done by an external agent that is trying to increase the area of the film per unit area. Mathematically, $S = \dfrac{dW_{ext}}{dA}$

Surface Energy

According to the work-energy theorem, the sum of work done by all forces is equal to the change in kinetic energy. Let W_{ext} be the work done by an external force to stretch a soap film. If the applied force is quasi-static, then the change in kinetic energy of the soap film will be zero.

$W_{all} = \Delta K$, where, ΔK is the change in kinetic energy.

$$\Rightarrow W_{all} = 0$$

$$\Rightarrow W_{ext} + W_{int} = 0$$

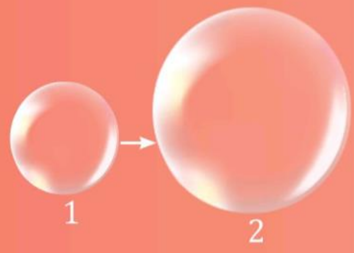
$$\Rightarrow W_{ext} = -W_{int} = \Delta U$$

Where, ΔU is the change in internal energy of the film.

Hence, the work done by the external agent increases the internal energy of the soap film. This change in internal energy is known as **surface energy**.

$$S = \frac{dW_{ext}}{dA} = \frac{dU}{dA}$$

How much work will be done in increasing the radius of a soap bubble from r_1 to r_2? The surface tension of the soap solution is S.



Solution

The change in surface area, when the radius of the bubble increases from r_1 to r_2 is,

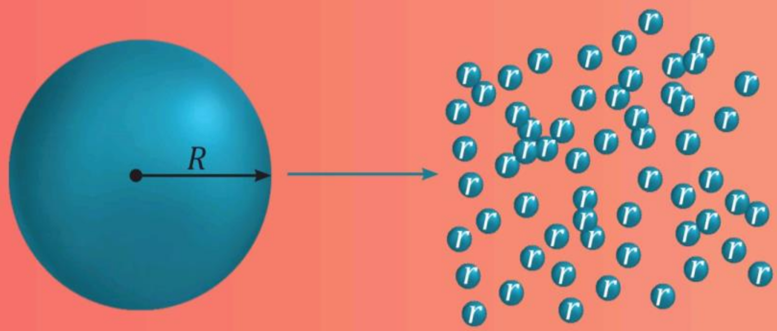
$$\Delta A = 4\pi\left(r_2^2 - r_1^2\right)$$

We know that, $S = \frac{dW_{ext}}{dA}$

Hence, the work done in increasing the radius of the soap bubble from r_1 to r_2 is given by,

$$W_{ext} = S \times (\Delta A) = 4\pi S\left(r_2^2 - r_1^2\right)$$

A water drop of radius R is broken into 1000 equal droplets. Calculate the gain in surface energy. The surface tension of water is S.



Solution

Consider that a big water drop of radius R is broken into 1000 equal droplets of radius r, where $r < R$. The mass of water before and after breaking the drop is conserved. Thus, we can write,

$$\frac{4}{3}\pi R^3 \times \rho = 1000 \times \frac{4}{3}\pi r^3 \times \rho$$

$$\Rightarrow r = \frac{R}{10}$$

Where, ρ is the density of water.

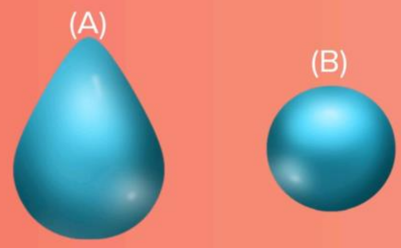
The surface area of the big water drop is, $A_1 = 4\pi R^2$

The total surface area of the droplets is, $A_2 = 1000 \times 4\pi r^2 = 10 \times 4\pi R^2$

The change in surface area is, $\Delta A = A_2 - A_1 = (10 - 1) \times 4\pi R^2 = 36\pi R^2$

Hence, the required work done is, $W_{ext} = S \times \Delta A = 36\pi R^2 S$

Choose the correct representation of a raindrop.



Solution

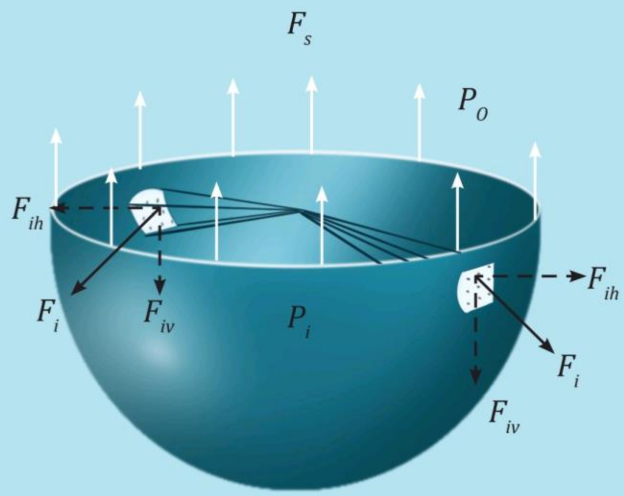
Actually, both (A) and (B) are correct, depending on the environment of the drop. Option (A) is correct for the case where the drop falls under gravity (Example, raindrop on Earth). Option (B) is correct in zero gravity condition (Example, water drop in spacecraft).

In zero gravity, a random volume of water automatically takes the shape of a sphere. Due to the surface tension, the random volume of water tries to minimise its surface area. For a given volume, the surface area of a sphere is the lowest. Thus, the water volume becomes spherical.

BOARDS

Excess Pressure Inside a Drop

The pressure inside the water drop in air is always greater than that of the outside. Consider the half of a drop just like the hemisphere as shown in the figure, with inside pressure P_i, outside pressure P_o, surface tension S, the radius of drop R, and the area of base $A_P = \pi R^2$.

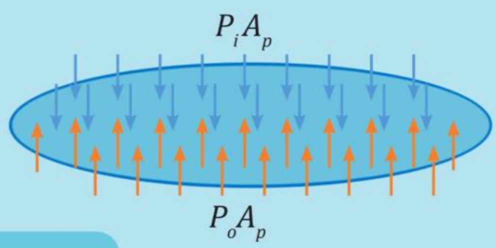


The net force on the drop due to the inside pressure, $F_i = P_i A_P$

The net force on the drop due to the outside pressure, $F_o = P_o A_P$

The force on the drop due to the surface tension, $F_S = S(2\pi R)$

On balancing the forces along the vertical direction, we get the following:



$$P_i A_P = P_o A_P + S(2\pi R)$$

$$\Rightarrow P_i - P_o = \frac{S(2\pi R)}{A_P}$$

$$\Rightarrow P_i - P_o = \frac{S(2\pi R)}{\pi R^2}$$

$$\Rightarrow P_i - P_o = \frac{2S}{R}$$

Therefore, the excess pressure inside a drop is,

$$\Delta P = P_i - P_o = \frac{2S}{R}$$

The excess pressure inside an air bubble in water is $\frac{2S}{R}$.

Excess Pressure Inside a Soap Bubble

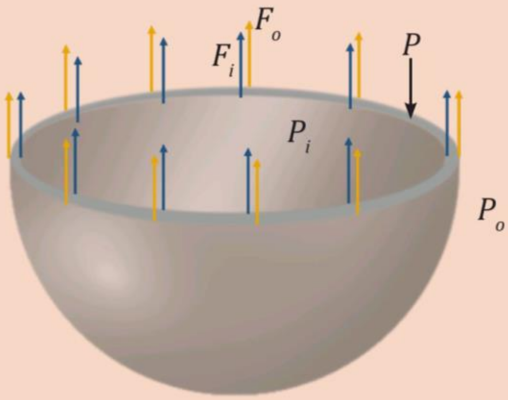
The key difference between a soap bubble and a water drop is that the soap bubble consists of two layers (just like a thick spherical shell), whereas the water drop has only one layer.

Consider that the inside pressure is P_i, outside pressure is P_o, and the pressure in between the two layers is P.

Therefore,

$$P_i - P = \frac{2S}{R} \quad \text{..........}(i)$$

$$P - P_o = \frac{2S}{R} \quad \text{..........}(ii)$$



Adding these two equations, we get,

$$P_i - P_o = \frac{4S}{R}$$

Two separate air bubbles of radii r_1 and r_2 ($r_2 < r_1$), formed of the same liquid (surface tension S), come together to form a double bubble. Find radius R and the sense of curvature of the internal film surface common to both the bubbles.

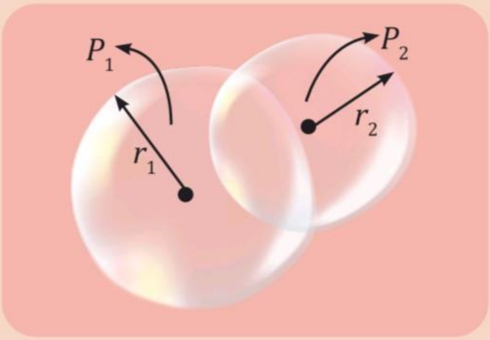
MAIN

Solution

Consider the outside pressure of the bubbles to be P_o.

The pressure inside the bubble of radius r_1 is, $P_1 = P_o + \frac{4S}{r_1}$.

The pressure inside the bubble of radius r_2 is, $P_2 = P_o + \frac{4S}{r_2}$.



Since $r_2 < r_1$, it implies that $P_1 < P_2$. Hence, the internal film surface that is common to both the bubbles will bend in the concave favour to the bubble of radius r_2. So, the excess pressure inside the double bubble is, $P_{excess} = P_2 - P_1$ and $P_{excess} = \frac{4S}{R}$.

Therefore,

$$\frac{4S}{R} = \left(P_o + \frac{4S}{r_2}\right) - \left(P_o + \frac{4S}{r_1}\right)$$

$$\Rightarrow \frac{1}{R} = \frac{1}{r_2} - \frac{1}{r_1}$$

$$\Rightarrow R = \frac{r_1 r_2}{r_1 - r_2}$$

SURFACE TENSION

CAPILLARITY

What you already know

- Surface tension
- Surface energy
- Pressure difference across surfaces of drops and bubbles

What you will learn

- Wettability
- Shape of meniscus
- Contact angle
- Capillary rise and fall

Under an isothermal condition, two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r. The external pressure is P_0. Find the surface tension of the soap in terms of the given parameters.

Solution MAIN

Consider that the number of moles of gas inside the bubble of radius r_1 is n_1 and the number of moles of gas inside the bubble of radius r_2 is n_2.

If n is the number of moles inside the final bubble of radius r, then,

$n = n_1 + n_2$

As the bubbles coalesce isothermally, the temperature in the initial state is equal to the temperature in the final state. Let P be the pressure inside the final bubble, and P_1 and P_2 are the pressure inside the bubble of radius r_1 and r_2, respectively.

According to the ideal gas equation,

$PV = nRT$

We have,

$n = n_1 + n_2$

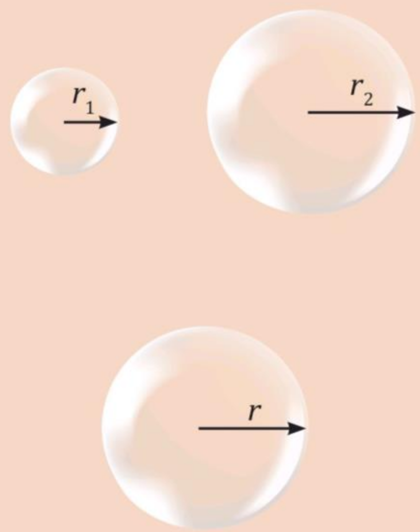
Hence, we get,

$$\frac{PV}{RT} = \frac{P_1V_1}{RT} + \frac{P_2V_2}{RT}$$

$$\Rightarrow PV = P_1V_1 + P_2V_2$$

$$\Rightarrow \left(P_0 + \frac{4S}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_0 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(P_0 + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow P_0\left(r^3 - r_1^3 - r_2^3\right) = 4S\left(r_1^2 + r_2^2 - r^2\right)$$



$$\Rightarrow S = \frac{P_0\left(r^3 - r_1^3 - r_2^3\right)}{4\left(r_1^2 + r_2^2 - r^2\right)}$$

Wettability

Wetting refers to the study of how a liquid deposits on a solid (or liquid) substrate and spreads out. The same liquid can behave differently on different surfaces and the spreading of the liquid is determined by the **contact angle**.

Surface of a liquid

Consider a liquid in a container. The upper surface of the liquid takes a curvature due to the interaction between the liquid and the surface of the container. The upper surface of the liquid is known as the **meniscus**.

Forces governing shapes of the meniscus

There are two forces (adhesive force and cohesive force) responsible for the different shapes of the meniscus.

- **Adhesive force**

  The force of attraction between the molecules of different substances is known as adhesive force $\left(\vec{F}_a\right)$.

- **Cohesive force**

  The force of attraction between the molecules of same substances is known as cohesive force $\left(\vec{F}_c\right)$.

These two forces depend on the nature of the liquid and the solid used. The resultant of $\left(\vec{F}_a\right)$ and $\left(\vec{F}_c\right)$ is defined as $\left(\vec{F}_R\right)$ and the meniscus is such that the resultant force will be perpendicular to the upper surface. Based on the direction of the resultant force, the meniscus can be concave, flat or convex.

Contact angle

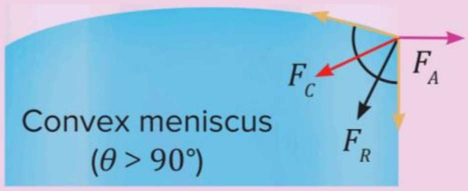
The angle between the tangent planes at the solid surface and the liquid surface at contact is known as the contact angle. The direction of the tangent plane should be taken as follows:

1. Tangent plane to the solid surface should be towards the liquid surface.
2. Tangent plane to the liquid surface should be away from the nearest solid surface.

Shapes of the meniscus

Convex meniscus:

The contact angle, θ, for a convex meniscus is an obtuse angle. It means that $\theta > 90°$.



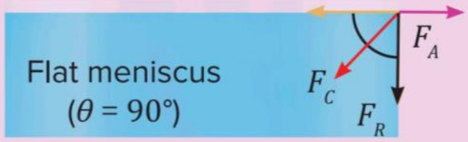
Example:

Mercury (when exposed to air, $\theta \cong 138°$ with glass)

Yellow arrows in the figure represents the directions of tangent plane to solid and liquid surfaces.

Flat meniscus:

The contact angle, θ, for a flat meniscus is 90°.



Example:

For pure water in contact with pure silver, $\theta \cong 90°$.

Concave meniscus:

The contact angle, θ, for a concave meniscus is an acute angle. It means that $\theta < 90°$.

Example:

For ordinary water and glass, $8° < \theta < 16°$.

Lower the contact angle, higher is the wettability.

MAIN BOARDS

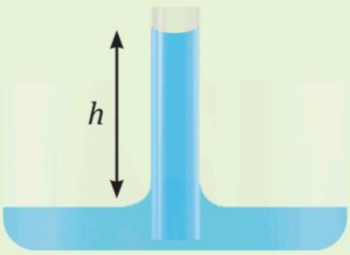
Capillarity

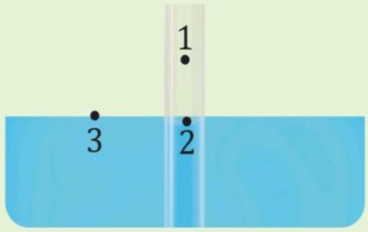
It is the phenomenon of rise or fall of the surface of liquid in a narrow tube relative to the adjacent general level of the liquid, when the tube is held vertically in the liquid.

Expression for capillary rise

Consider the radius of the capillary tube as r and the height of the surface of liquid in the tube from the general level of the liquid as h.

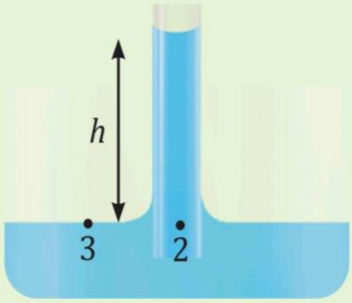
The shape of meniscus for ordinary water and glass is concave and the pressure at the concave side is more than that of the convex side.





Consider the situation when the meniscus inside the tube is at the general level of the liquid. Let the pressure at point 1 be P_0. If R is the radius of curvature of the meniscus, then the pressure just below the meniscus (at point 2) is, $P_2 = P_0 - \frac{2S}{R}$, and the pressure at point 3 is P_0.

Since the pressure at the same level will be equal, the height of the meniscus will rise to h from the general level of the liquid to compensate the deficit in pressure at point 2.



After the rise of the level of meniscus, the pressure at point 2 will be P_0, which is equivalent to the following:

$P_0 = P_2 + \rho g h$

Therefore,

$$P_0 = P_2 + \rho g h$$

$$\Rightarrow P_0 = \left(P_0 - \frac{2S}{R}\right) + \rho g h$$

$$\Rightarrow h = \frac{2S}{\rho g R}$$

From the figure,

$R\cos\theta = r$

Where, θ is the contact angle.

Hence, the expression of h becomes,

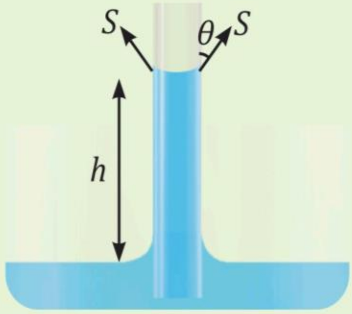
$$h = \frac{2S\cos\theta}{\rho g r}$$

Alternative proof

The force acts on the contact layer of water due to the surface tension in tangential direction. The vertical component of this tensile force because of which the liquid rises is, $S\cos\theta\,(2\pi r)$. The horizontal component vanishes due to cylindrical symmetry of the tube.

This force balances the weight of the liquid that rises up to height h.

If ρ is the density of the liquid, then the weight of the liquid column is, $\rho(\pi r^2 h)g$. Therefore,



$$S\cos\theta(2\pi r) = \rho(\pi r^2 h)g$$

$$\Rightarrow h = \frac{2S\cos\theta}{\rho g r} = \frac{2S}{\rho g R}$$

The height of the liquid that rises in the capillary tube is, $h = \frac{S \quad \theta}{\rho g r} = \frac{S}{\rho g R}$. As the surface tension (S) and density of the liquid (ρ) is constant at a temperature, from the relation written just above, it can be said that $h \propto \frac{1}{R}$. Therefore, the height of the liquid that rises in the capillary tube is inversely proportional to the radius of curvature of the meniscus.

A capillary tube of diameter 2 mm is kept vertical with the lower end in water.

(a) Find the height of water raised in the capillary.
($S_{water} = 0.07\ Nm^{-1}$, $\rho_{water} = 1000\ kg\ m^{-3}$, $g = 10\ ms^{-2}$)
Take contact angle as 0°.

(b) If the length of the capillary tube is half the answer of part (a), find the angle made by the water surface in the capillary tube with the wall.

Solution MAIN

(a) The diameter of the capillary tube is 2 mm, so its radius is, $r = 1\ mm$

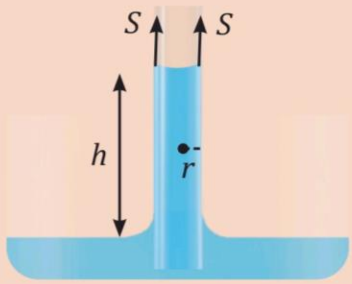
Given quantities are,

$S_{water} = 0.07\ Nm^{-1}$

$\rho_{water} = 1000\ kg\ m^{-3}$

$g = 10\ m^{-2}$

$\theta = 0°$



The height of water raised in the capillary is,

$$h = \frac{2S\cos\theta}{\rho g r}$$

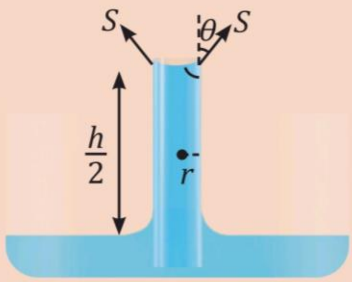
$$\Rightarrow h = \frac{2 \times 0.07 \times 1}{1000 \times 10 \times 0.001}$$

$$\Rightarrow h = 0.014\ m = 14\ mm$$

(b) We have,

$S\cos\theta\,(2\pi r)$ = Weight of the column of liquid in the capillary tube

Now, if ρ is the density of the liquid, then the weight of the column of liquid is, $\rho\left(\pi r^2 \frac{h}{2}\right)g$



Where, h is the height of the column of liquid in the capillary calculated in part (a).

So,

$$S\cos\theta(2\pi r) = \rho\left(\pi r^2 \frac{h}{2}\right)g$$

$$\Rightarrow \cos\theta = \frac{\rho r h g}{4S}$$

$$\Rightarrow \cos\theta = \frac{10^3 \times 10^{-3} \times \left(14 \times 10^{-3}\right) \times 10}{4 \times \left(7 \times 10^{-2}\right)}$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60°$$

If the level of the liquid in the capillary tube falls below h from the general level of the liquid outside of the capillary tube, then the **fall in capillary** is also, $h = \frac{2S\cos\theta}{\rho g r} = \frac{2S}{\rho g R}$ which is **same as the rise in capillary** case.