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PHYSICS WAVES

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CONTACT US:

+91-9939586130

+91-9955930311

www.aepstudycircle.com

aepstudycircle@gmail.com

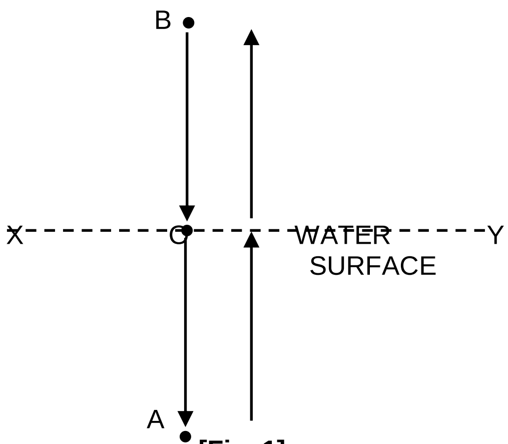
2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

A wave motion is a means of transferring energy and momentum from one point to another without any actual transportation of matter between these points.

EXPLANATION: The most common form of wave motion with which we are familiar is the waves on the surface of water. When we throw a piece of stone on water in a pond, we observe ripples travelling on the surface of water in concentric circles of ever-increasing radius, till they strike the boundary of the pond. When we put a piece of cork on the surface of this water, we observe that the cork piece moves up and down as the wave passes, but the piece does not travel along with the waves. Thus, the particles of the medium certainly oscillate about their mean position, but their displacement away from their original position is not there. The water waves carry energy, but there is no transfer of matter. Similarly, when a drummer beats a drum, the sound is heard at distant points. Obviously, the sound wave carries energy as it vibrates the diaphragm of the ear enabling us to hear. In the process, air particles do vibrate about their mean positions, but they are not transported from one point to the other. Again, when a flag at the top of a flag post flutters in breeze, the ripples travel along the cloth of the flag, but the distance of any spot (like Ashok chakra) on the flag remains unchanged from the four edges of the flag.

Thus, in a wave motion, disturbance travels through some medium, but the medium does not travel along with the disturbance.

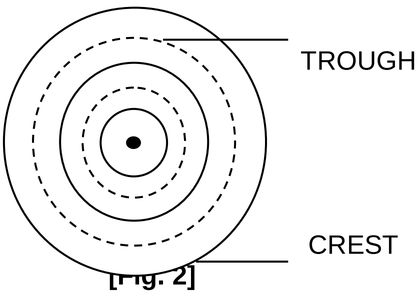
□□**WAVE PROPAGATION:** wave propagation can be understood in terms of two essential properties of the medium, viz. inertia and elasticity. In Fig. 1, XY represents horizontal surface of water. When a stone hits a particle of water at O, the particle moves down to A. During motion of the particle moves down to A. During motion of the particle from O to A, a restoring force develops on account of elasticity of water. Work done against the restoring force in moving the particle from O to A, is stored in the particle at A in the form of potential energy. From A, the particle at A in the form of potential energy. From A, the particle moves towards O, under the action of restoring force. Potential energy of the particle is converted into kinetic energy at O. On account of inertia, therefore, the particle cannot stop at O. It overshoots its mean position O and goes over to B till its entire kinetic energy is converted into potential energy at B. From B, the particle moves to O, under the action of restoring force again and so on. Thus, the particle of water at O executes periodic vibrations due to elasticity and inertial.



[Fig. 1]

This disturbance is communicated to the adjoining particles which also start vibrating simple harmonically about their mean positions. Hence the wave motion travels on the on.

□□□In certain regions, water level is below the normal level XY. These regions are called **troughs**. On either side of trough, there are regions where water is at level slightly higher than the normal level. These are called **crests**. Thus, wave motion travels onwards on the surface of water in the form of crests and troughs as shown in Fig. 2.



[Fig. 2]

Hence, we may define **wave motion as a kind of disturbance which travels through a material medium (having properties of elasticity and inertial) on account of repeated periodic vibrations of the particles of the medium about their mean position, the disturbance being handed on form one particle to the adjoining particle and so on, without any net transport of the medium.**

□□ Such waves which can be produced or propagated only in a material medium are called elastic waves or mechanical waves. Waves on water surface, waves on strings, sound waves etc. are all mechanical waves.

□□□□ There is another kind of waves which can pass even through vacuum. For example, light waves from sun and distant stars reach us after travelling through vacuum. Such waves are called electromagnetic waves or non-mechanical waves. Radio waves, micro waves, X-rays, gamma rays are other examples of electromagnetic waves. In this unit, we shall study only the mechanical waves.

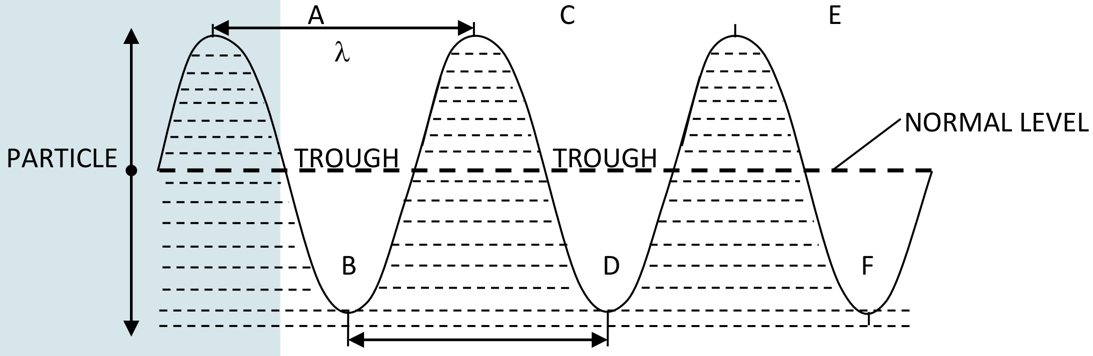
□□ Types of Wave Motion

The mechanical waves are of two types:

1. Transverse wave motion
2. Longitudinal wave motion

□ Tranverse Wave Motion

A transverse wave motion is that wave motion, in which individual particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.



[Fig. 3]

For Example (i) Movement of string of sitar or violin,
(ii) Movement of membrane of a table or dholak,
(iii) Movement of a kink on a rope,

□□□□Strictly speaking, waves set up on the surface of water are a combination of transverse waves and longitudinal waves.
□□□□ light waves and all other electromagnetic waves are also the transverse waves.

●●●<u>A transverse wave travels through a medium in the form of crests and troughs</u>.

●●A crest is a portion of the medium, which is raised temporarily above the normal position of rest of the particles of the medium, when a transverse wave passes through it.

The **center of crest** is the position of maximum displacement in the positive direction (i.e. above the normal level). In fig the points A, C, E are the centers of successive crests.

●●A trough is a portion of the medium, which is depressed temporarily below the normal position of rest of the particles of the medium, when a transverse wave passes through it.

The **center of trough** is the position of maximum displacement in the negative direction ((i.e. below the normal level).In Fig. the points B, D, F are the successive of troughs.

The distance between the consecutive crests or to consecutive troughs is called wave length of the wave. It is represented by λ. Thus, AC = BD = λ.

<u>Formation of propagation of traverse waves</u>

For propagation of transverse waves, the material medium must possess the following four characteristics:

●●(i) Elasticity, so that particles can return to their main position, after having been disturbed.
●●(ii) Inertia, so that particles can store energy and overshoot their mean position.
●●(iii) Minimum friction among the particles of the medium ensures minimum loss of energy so that waves can travel long distance.
●●(iv) Density of the medium is uniform.

●●●<u>To understand the formation of propagation of transverse waves in a medium,</u> let us consider a series of five,equally spaced particles 1, 2, 3, 4, 5 in a straight line. When these particles execute **S.H.M** of equal amplitudes and time periods about their mean positions in the upward and downward direction, we can show that a transverse wave travels to the right. In Fig.4, solid lines represent the mean positions and dotted lines up and down represents the extreme positions, while vibrating.

XY = XY' is the amplitude of vibration. Suppose T is the time period of vibration of each particle, and the disturbance of handed on from one particle to the adjoining particle in T/4 second.

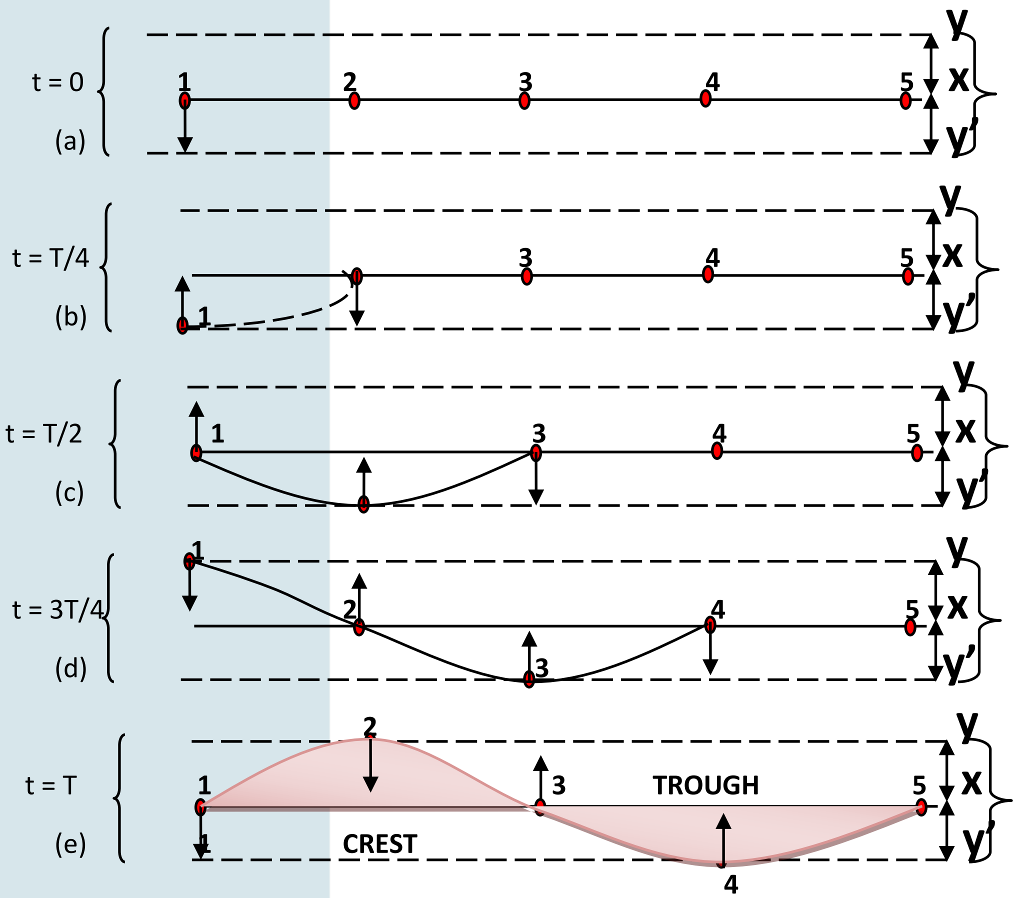
(i) At t = 0, all the particle 1, 2, 3, 4, 5 are at rest at their mean position and the disturbance just reaches particle 1, which starts moving downwards. The other particles 2, 3, 4 and 5 at rest

(ii) At t = T/4, particle 1 completes 1/4th of its vibration and riches the lower extreme position, the disturbance Just riches particle 2, which starts vibrating downwards. The particles 3, 4, 5 are at rest.

(iii) At t = T/2, particle 1 completes 1/2 of its vibration and reaching its lower extreme position, the disturbance Just riches particle 3, which starts vibrating. The particles 4 and 5 are at rest.

(iv) At t = 3T/4, Particle 1 goes to its upper extreme position; particle 2 has completed half its vibration reaching its mean position. Particle 3 has executed. 1/4th of its vibration, reaching its lower extreme position. Disturbance has just reached particles 4 and particle 5 continues to be at rest.

(v) At t = T, particle 1 has just completed one vibration; particle 2 has reached its upper extreme positron; particle 3 has completed half its vibration reaching the mean position. Particle 4 has reached its lower extreme position and disturbance have just reached particle 5, Fig.4.



[Fig. 4]

●●When we join the final positions of particles 1, 2, 3, 4, 5 at t = T, we obtain a sine curve, which is the displacement curve of the transverse wave.

●●The first half of the curve represents a crest, because the medium in this portion lies above the normal Positionof rest of the particles. The second half of the curve represents a trough, because the medium in this portion lies below the normal position of rest of the particles.

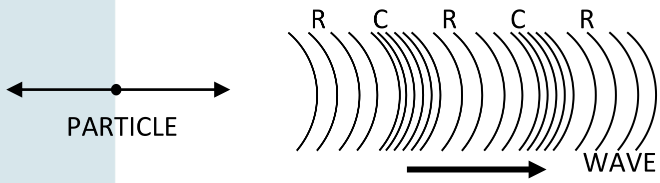
●●Thus, a transverse wave is propagated through a medium in the form of crest and trough. Transverse Waves can be transmitted through solids. They can be set up on the surface of liquids. But they cannot be transmitted inside liquid and gases.

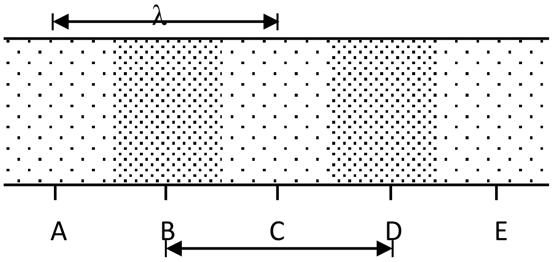
Longitudinal Wave Motion or Pressure Waves

A longitudinal wave motion is that wave motion in which individuals' particles of the medium execute simple harmonic motion about their mean position along the same direction along which the wave is propagated.

For example (i) Sound waves travels through air in the form of longitudinal waves.
(ii) Vibrations of air column in organ pipes are longitudinal.
(iii) Vibration of air column above their surface of water in the tube of a resonance apparatus are longitudinal.

A longitudinal wave travels through a medium in the form of compressions or condensations C and rarefactions (R) as shown in fig. 5.





[Fig. 5]

●●A **compression is a region of the medium in which particles are compressed i.e. particles come closer i.e. distance between the particles the particles become less than the normal distance between them**. Thus, there is a temporary decrease in volume and a consequent increase in density of the medium in the region of compression.

●●A **rarefaction is a region of the medium in which particles are rarefied i.e. particles get farther apart than what they normally are.** Thus, there is a temporary increase in volume and a consequent decrease in density of the medium in the region of rarefaction.

□The distance between the centres of two consecutive compressions and two consecutive rarefactions is the wavelength (λ) of the wave. In Fig. 5, BD = AC = λ

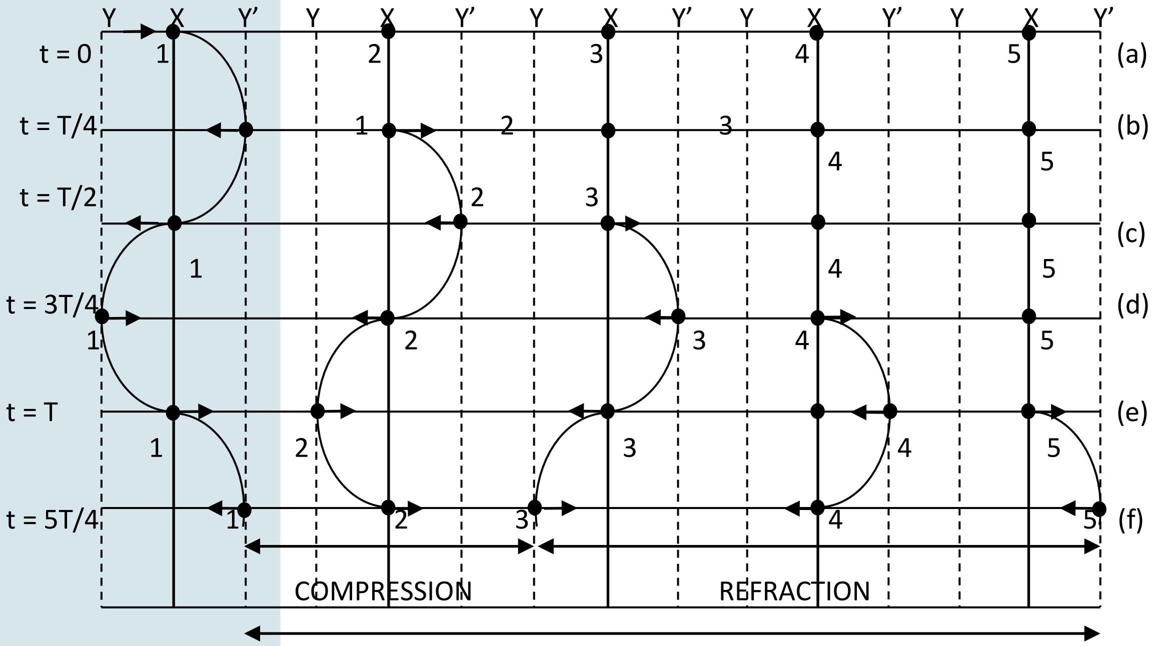
Formation or Propagations of Longitudinal Wave motion

As stated already in the previous articles, for the formation or propagation of longitudinal wave motion, the material medium must possess the properties of elasticity, inertia and minimum friction.

Let us consider a series of five equally spaced particles 1, 2, 3, 4, 5 in one horizontal level. When these particles execute simple harmonic motion of equal amplitudes and equal time period about their mean position along the straight line in which they are place, we can show that a longitudinal wave travels to the right.

In Fig.6, solid lines represent the mean positions and dotted lines around the solid lines represent the extreme positions of the particles while vibrating. XY = XY' is the amplitude of vibration, Suppose T is the time period of vibration of each particle, and the disturbance is handed on from one particle to the adjoining particle in T/4 second.

(i) At t = 0, all the particles 1, 2, 3, 4, 5 are at rest at their mean positions and the disturbance just reaches particle 1, Fig. 6(a).



[Fig. 6]

(ii) At t = T/4, particle 1 completes 1/4th of its vibration and reaches the right extreme position. The disturbance just reaches particles 2, which starts vibrating. The particles 3, 4, 5 are at rest, Fig. 6(b).

(iii) At t = T/2, particle 1 completes 1/2 its vibration returning to the mean position. Particle 2 reaches its right extreme position. The disturbance just reaches particle 3, which starts vibrating. The particles 4 and 5 are at rest, Fig. 6(c).

(iv) At t =3T/4, particle 1 goes to its left extreme position, particle 2 completes 1/2 vibration reaching its mean position, particle 3 has executed 1/4th of its vibration, reaching its right extreme position. Disturbance has just reached particle 4 and particle 5 continues to be at rest, Fig. 6(d).

(v) At t = T, particle 1 has just completed one vibration, 2 has reached its left extreme position, particle 3 has completed half its vibration reaching the mean position. Particle 4 has reached its right extreme position and disturbance has just reached particle 5, Fig. 6(e).

(vi) At t = 5T/4, particle 1 reaches again its right extreme position, particle 2 has completed one vibration returning to its mean position, particle 3 reaches its extreme left position, particle 4 comes to the mean position after completing 1/2 its vibration and particle 5 goes to its right extreme position, Fig. 6 (f).

Join the relative position of particles 1, 2, 3, 4 and 5 at different times as shown in Fig. 6. We find that the distance between the particles 1, 2, 3 is less than their normal distance. Also, the distance between the particles 3, 4, 5 is more than their normal distance.

Particle 1, 2, 3 are said to form a region of compression and particle 3, 4, 5 are said to form a region of rarefaction.

●●Thus, a longitudinal wave is propagated through a medium in the form of compression and rarefaction.
●●The longitudinal waves can be transmitted through solids, liquids and gases.

□□□□□ **A mechanical wave shall be transverse or longitudinal depending on**

(i) Nature of the medium

(ii) Mode of excitation of vibration

For example: In solids, mechanical waves can be either transverse or longitudinal. In strings, mechanical waves are always transverse. In liquids and gases, mechanical waves are always longitudinal. However, transverse waves can be set up on the surface of liquids, as explained already.

Some Important Terms Connected with Wave Motion

1. Wavelength: **Wavelength of a wave is the length of one wave. It is equal to the distance travelled by the wave during the time; any one particle of the medium completes one vibration about its mean position.**

In Fig. 4, when particle 1 completes one vibration about its mean position, the disturbance goes from particle 1 to particle 5. Hence, the distance between particles 1 and 5 is one wavelength (λ). As particles 1 and 5 are vibrating in the same phase*, we may also define wavelength as *the distance between any two nearest particles of the medium, vibrating in the same phase*.

●In transverse wave motion, λ = distance between the centres of two consecutive crests of distance between the centres of two consecutive troughs. Also, wavelength can be taken as the distance in which one crest and one troughs are contained.

●In a longitudinal wave motion, λ = distance between the centres of two consecutive compressions or distance between the centres of two consecutive rarefactions. Also, wavelength can be taken as the distance in which one compression and one rarefaction are contained.

2. Frequency: Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

As one vibration is equivalent to one wavelength, therefore, we may define frequency of a wave as the number of completed wavelengths traversed by the wave in one second. It is represented by *v*.

3. Time period: **Time period of vibration of a particle is defined as the time taken by the particle to complete one vibration about its mean position**. As one vibration is equivalent to one wavelength, therefore, time period of a wave is equal to time taken by the wave to travel a distance equal to one wavelength. It is represented by *T*.

□□□□□□□□□ **Relation between *v* and *T***

By definition,
Time for completing *v* vibrations = 1 sec
Time for completing one vibration
= 1/*v* sec.

i.e.	$T = 1/v$
or	$v = 1/T$
or	$v\,T = 1$

... (1)

Relation between velocity, frequency and wavelength of the wave

Suppose ν = frequency of a wave,
T = time period of the wave,
λ = wavelength of the wave,
υ = velocity of the wave.

By definition, velocity = $\frac{\text{distance}}{\text{time}}$

$$\upsilon = \frac{s}{t} \quad \dots (2)$$

In one complete vibration of the particles, distance travelled, s = λ and time taken, t = T

From (2), $\upsilon = \frac{\lambda}{T} = \frac{1}{T}\lambda$

Using (1), we get $\boxed{\upsilon = \nu\lambda}$

Hence velocity of wave is the product of frequency and wavelength of the wave.
●This relation holds for transverse as well as longitudinal waves.

It should be clearly understood that wave velocity υ is determined only by the elastic and inertialproperties of the medium, therefore, ν is characterised by the source which produces disturbance. Different sources may produce vibrations of different frequencies. Their wavelengths (λ) will differ to keep the product $\nu\lambda = \upsilon$, a constant.

Characteristics of Wave Motion

●●1. Wave motion is a sort of disturbance which travels through a medium.

●●2. A material medium is essential for the propagation of mechanical waves. The medium must possess three properties, viz. elasticity, inertia and minimum friction amongst the particles of the medium.

●●3. When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do not leave their position and move with the disturbance.

●●4. There is a continuous phase difference amongst successive particles of the medium i.e. particle 2 starts vibrating slightly later than particle 1 and so on.

●●5. The velocity of the particles during their vibration is different at different positions.

For example, all the particles cross their mean position with maximum velocity and at extreme positions, their velocity is zero.

●●6. The velocity of wave motion through a particular medium is constant. It depends only on the nature of the medium. The velocity of wave motion does not depend upon its frequency or wavelength or intensity.

●●7. Energy is propagated along with the wave motion without any net transport of the medium.

Sound Waves

The physical cause that produces the sensation of hearing is the vibration of the source.

For example, when we listen to a sitar recital, the sitar wire vibrates. The vibrations are carried by air, as a medium. When these vibrations strike our ear drum, it vibrates. The message is conveyed to our brain and we hear.

Our ear is sensitive only to those vibrations whose frequency lies between 20 hertz to 20,000 hertz. This frequency range is called **audible range**.

□□□**Any vibration whose frequency is greater than 20,000 hertz is called ultrasonic vibration.** It cannot be heard by human ear. The sound waves which have frequencies less than the audible range are called infrasonic waves. They also cannot be heard by human ear.

●●●●Only the vibrations of sound travel through air.

●●The velocity of sound in air at room temperature and normal pressure is roughly332 ms^{-1}, which is approximately 1200 km h^{-1}. This is much greater than the speed of the fastest car. **That is why horn of a motor car approaching us is heard much before the car reaches us.**

●●●●**An object moving with a speed greater than the speed of sound is said to move with a supersonic speed**.

We know that a longitudinal wave motion travels in the form of compressions and rarefactions which involve changes in volume and density of the medium. As air possesses volume elasticity, therefore sound comes to us from the source in the form of longitudinal waves only. As crests and troughs cannot be sustained in air, therefore sound cannot travel trough air in the form of transverse waves.

Both the sound and the light are associated with wave motion. Light waves are transverse electromagnetic waves which can propagate even in free space with a tremendous velocity (= 3×10^8 ms^{-1}). Sound waves are longitudinal mechanical waves which cannot travel in vacuum. For example, two persons on the surface of moon cannot talk to each other, as the moon has no atmosphere through which sound would travel.

Speed of Wave Motion

Wave motion, as we know, can be transverse as well as longitudinal. We give below the mathematical formulae for speeds of transverse and longitudinal waves in different media without their derivations:

●●● **(a) Speed of transverse wave motion**

● (i) The velocity of propagation of a transverse wave on a stretched string is given by

$$\upsilon = \sqrt{T/m} \qquad \dots (4)$$

Where T is tension in the string and m is linear density of the string i.e. mass per unit length of the string.
As tension is force, the dimensions of $\sqrt{T/m}$ are

$$\left(\frac{M^1 L^1 T^{-2}}{M^1 L^{-1}}\right)^{1/2} = [M^0 L^1 T^{-1}],$$

Which are the dimensions of velocity. Hence eqn. (4) is dimensionally current.

●●Velocity of a transverse wave propagating along a string depends only an characteristics of the string (T & m). It does not depend upon frequency of wave.

● (ii) The velocity of transverse waves in a solid is given by

$$\upsilon = \sqrt{\eta/\rho} \quad \dots (5)$$

Where η is modulus of rigidity and ρ is density of the material of the solid.

The dimensions of $\sqrt{\eta/\rho}$ are

$$\left(\frac{M^1 L^{-1} T^{-2}}{M^1 L^{-3}}\right)^{1/2} = [M^0 L^1 T^{-1}],$$

Which are the dimensions of velocity. Therefore, eqn. (5) is also dimensionally correct.

●●● (b) Speed of Longitudinal wave motion

●(i) In a solid medium, the speed of longitudinal waves is given by

$$\upsilon = \sqrt{\left(K + \frac{4}{3}\eta\right)/\rho} \quad \dots (6)$$

Where K is bulk modulus; η is modulus of rigidity and ρ is density of the material of the solid. When the solid is in the form of a long bar, the speed of longitudinal waves through the bar is given by

$$\upsilon = \sqrt{Y/\rho} \quad \dots (7)$$ Where Y is Young's modulus of the material of the bar.

● (ii) In liquids, the velocity of longitudinal waves is given by

$$\upsilon = \sqrt{K/\rho} \quad \dots (8)$$

Where K is bulk modulus and ρ is density of the liquid.

● (iii) In gases, the velocity of longitudinal waves is given by

$$\upsilon = \sqrt{K/\rho} \quad \dots (9)$$

Where K is coefficient of volume elasticity of the gas and ρ is density of the gas.

●●As sound travels through gases in the form of longitudinal waves, therefore, eqn. (9) is also the expression for velocity of sound in a gaseous medium.

.... Equations (6), (7), (8) and (9) are dimensionally correct.

Newton's Formula for velocity of Sound

From purely theoretical considerations, Newton, concluded that velocity of longitudinal waves through any medium, solid, liquid or gas depends upon the elasticity and density of the medium. Newton gave the formula:

$$\upsilon = \sqrt{E/\rho} \quad \dots (10)$$

Where υ = velocity of sound in the medium, E = coefficient of elasticity of the medium,
ρ = density of the (undisturbed) medium.

Newton used this relation to calculate the velocity of sound in a gas. Since a gas has only one type of elasticity, i.e., bulk modulus (K), the velocity of sound in a gas is given by

$$\upsilon = \sqrt{K/\rho} \quad \dots (11)$$

●Sound travels through a gas in the form of compressions and rarefactions. Newton assumed that the changes in pressure and volume of a gas, when sound waves are propagated through it, are isothermal. The amount of heat produced during compression, is lost to the surroundings and similarly the amount of heat lost during rarefaction is gained from the surroundings, so as to keep the temperature constant. Using coefficient of isothermal elasticity, i.e., K_i in (11) Newton's formula becomes:

$$\upsilon = \sqrt{K_i/\rho} \quad \dots (12)$$

●●**Calculation of K_i**: Consider a certain mass of the gas.

Let P = initial pressure of the gas,
V = initial volume of the gas.

Under isothermal conditions

$$PV = \text{constant}$$

Differentiating both sides, we get

$$PdV + VdP = 0$$

$$PdV = -VdP$$

$$P = -\frac{VdP}{dV}$$

$$= -\frac{dP}{dV/V} = K_i \quad \text{[by definition]}$$

Substituting this value in (12), we obtain

$$\upsilon = \sqrt{P/\rho} \quad \dots (13)$$

●● Error in Newton's Formula

Let us calculate the velocity of sound in air at N.T.P.

As P = hdg and

h = 0.76 m of Hg column; d = 13.6×10^3 kg m^{-3}

∴ P = $0.76 \times 13.6 \times 10^3 \times 9.8$ Nm^{-2}

Density of air, $\rho = 1.293$ kg/m^3

From (13),

$$\upsilon = \sqrt{\frac{0.76 \times 13.6 \times 10^3 \times 9.8}{1.293}}$$

$$= 280\ ms^{-1} \quad \ldots (14)$$

The experimental value of the velocity of sound in air at N.T. P. is 332 ms^{-1}.

Difference between the experimental and theoretical values of velocity of sound in air = 332 – 280 = 52 ms^{-1}.

Percentage Error = $\frac{52}{332} \times 100 = 15.7\ \%$

Thus, the value calculated on the basis of Newton's formula was less than the experimental value by about 16%. Such a large error could not be taken as an experimental error.

Newton put forward a number of arguments to explain the above discrepancy but none of them was satisfactory.

●● Laplace's Correction:

Laplace, a French mathematician succeeded in explaining the exact cause of discrepancy between the theoretical and the experimental values of the velocity of sound.

He pointed out that Newton's assumption was wrong.

□According to Laplace, **the changes in pressure and volume of a gas, when sound waves are propagated through it, are not isothermal, but adiabatic.** This is because:

● (i) **Velocity of sound in a gas is quite large**. The pulses of compression and rarefaction, therefore, follow one another so rapidly that there is no lime left for any exchange of heat amongst themselves or with surroundings.

● (ii) **A gas is a bad conductor of heat**. It does not allow the free exchange of heat between compressed layer, rarefied layer and surroundings.

Thus, **no exchange of heat is possible, when a sound wave passes through a gas. Heat produced during compression raises the temperature of the gas and the heat lost during rarefaction reduces the temperature of the gas. Hence the changes are adiabatic and not isothermal.**

Using the coefficient of adiabatic elasticity, i.e., K_a in (12) instead of K_i, we have

$$\upsilon = \sqrt{\frac{K_a}{\rho}} \quad \ldots (15)$$

□□Calculation of 'K_a':

Consider a certain mass of the gas. Let P be the initial pressure and V be the initial volume of the gas. Under adiabatic conditions,

PV^γ = constant ... (16) Where $\gamma = C_p/C_\upsilon$ = ratio of two potential specific heats of the gas

Differentiating both sides of (16), we get

$$P(\gamma V^{\gamma-1} dV) + V^\gamma (dP) = 0$$

Or $\gamma PV^{\gamma-1} dV = -V\gamma (dP)$

Or $\gamma P = -\frac{V^\gamma}{V^{\gamma-1}}\left(\frac{dP}{dV}\right)$

$= -\frac{dP}{dV/V} = K_a$ [by definition]

∴ $K_a = \gamma P$ ... (17)

Corrected formula: Substituting this value of K_a in (15), we get the corrected formula for υ as

$$\upsilon = \sqrt{\gamma P/\rho} \quad \ldots (18)$$ The value of γ depends on nature of the gas.

For air, $\gamma = 1.41$ and from (14),

$\sqrt{P/\rho} = 280$ m/s.

∴ From (18), $\upsilon = \sqrt{\gamma}\sqrt{P/\rho} = \sqrt{1.41} \times 280 = 332.5\ ms^{-1}$

This value agrees fairly well with the experimental value of the velocity of sound in air at N.T.P. Hence the validity of Laplace's correction is established and (18) is the correct relation for the velocity of sound in any gaseous medium.

□□As solids are most elastic and gases are least elastic i.e. $E_s > E_l > E_g$

∴ Velocity of sound in solids is maximum and velocity of sound in gases is minimum. For example:

vel. of sound in air (υ_a) ≈ 332 m/s.

vel. of sound in water (υ_w) ≈ 1500 m/s

vel. of sound in steel (υ_s) ≈ 5900 m/s.

As for sound, $\upsilon_w > \upsilon_a$, therefore, for sound, water is rarer than air. That is why, in travelling from air to water, a beam of sound bends away from normal, while a beam of light bends towards the normal.

● speed of sound in some media.

Medium	Speed of sound
Gases	
1. Air (0°C)	331
2. Air (20°C)	343
3. Helium	365
4. Hydrogen	1284
Liquids	
1. Water (0°C)	1402
2. Water (20°C)	1482
3. Sea water	1522
Solids	
1. Copper	**3560**
2. Steel	**5941**
3. Granite	**6000**
4. Aluminium	**6420**
5. Vulcanized rubber	**54**

●● Factors Affecting Velocity of Sound

The velocity of sound in any gaseous medium is affected by a large number of factors like density, pressure, temperature, humidity and wind velocity etc.

(A) Effect of density: The velocity of sound in a gaseous medium is given by

$$\upsilon = \sqrt{\gamma P/\rho}$$

.................**velocity of sound in a gas is inversely proportional to the square root of density of the gas.**

For example, density of oxygen is 16 times the density of hydrogen.

$$\therefore \quad \frac{\upsilon_H}{\upsilon_O} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

or $\upsilon_H = 4v_O$ Therefore, the velocity of sound in hydrogen is four times the velocity of sound in oxygen.

(B) Effect of Pressure: The formula for velocity of sound in a gas is

$$\upsilon = \sqrt{\gamma P/\rho}$$

According to the standard gas equation for one-gram molecule of a gas, PV = RT, where the letters have their usual meanings.

$\therefore \quad P = RT/V$

$$\therefore \quad \upsilon = \sqrt{\frac{\gamma RT}{\rho \times V}} = \sqrt{\frac{\gamma RT}{M}}$$

Where $\rho \times V = M$, the molecular weight of the gas. For a given gas, R, γ and M is constants. If the temperature T of the gas is kept constant, then υ is constant.

(C) Effect of Temperature: The formula for velocity of sound in a gas is

$$\upsilon = \sqrt{\gamma P/\rho}$$

According to standard gas equation,

PV = RT or $P = \frac{RT}{V}$

$$\therefore \quad \upsilon = \sqrt{\frac{\gamma RT}{\rho \times V}} = \sqrt{\frac{\gamma RT}{M}} \quad \text{... (19)}$$

Where $\rho \times V = M$, the molecular weight of the gas. Clearly, $\upsilon \propto \sqrt{T}$ ... (20)

Hence velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

………. **Sound would travel faster on a hot summer day than on a cold winter day**.

:

Temperature coefficient of velocity of sound in air (α)

The temperature coefficient of velocity of sound is defined as the change in the velocity of sound, when temperature changes by 1°C.

If υ_t = velocity of sound in a gas at t°C.

υ_0 = velocity of sound in the gas at 0_oC,

then, $\alpha = \frac{\upsilon_t - \upsilon_0}{t}$ the **unit of α is m s^{-1} $°C^{-1}$**.

From (20),

$$\frac{\upsilon_t}{\upsilon_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273+0}}$$

$$= \left(1 + \frac{t}{273}\right)^{1/2}$$

Expanding binomially, when t is small, we get

$$\frac{\upsilon_t}{\upsilon_0} = \left(1 + \frac{1}{2} \times \frac{t}{273}\right)$$

∴ $\frac{\upsilon_t}{\upsilon_0} = 1 + \frac{t}{546}$ ... (21)

or $\frac{\upsilon_t}{\upsilon_0} - 1 = \frac{t}{546}$

or $\frac{\upsilon_t - \upsilon_0}{\upsilon_0} = \frac{t}{546}$

or $\frac{\upsilon_t - \upsilon_0}{t} = \frac{t}{546}$

∴ $\alpha = \frac{332}{546} = 0.608$ ms–1 °C^{-1}

[∵ $\upsilon_0 = 332$ ms^{-1}]

Hence **velocity of sound is air increases approximately by 0.61 ms^{-1} for every 1°C rise in temperature**

□□(D) Effect of Humidity: The pressure of water vapours in air changes its density. That is why the velocity of sound changes with humidity of air.

Suppose, ρ_m = density of moist air,
ρ_d = density of dry air,
υ_m = velocity of sound in moist air,
υ_d = velocity of sound in dry air.

Assuming that effect of humidity on γ is negligible.

We get from (18), $\upsilon_m = \sqrt{\frac{\gamma P}{\rho_m}}$

And $\upsilon_d = \sqrt{\frac{\gamma P}{\rho_d}}$

Dividing, we get $\frac{\upsilon_m}{\upsilon_d} = \sqrt{\frac{\rho_d}{\rho_m}}$ ... (22)

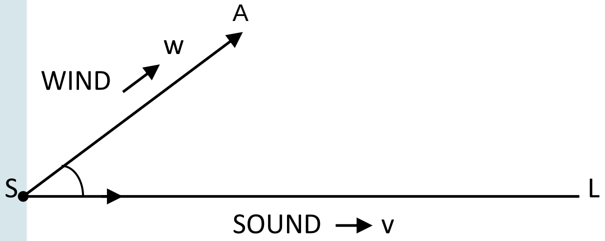
The presence of water vapours reduces the density of air.

i.e. $\rho_m < \rho_d$

therefore, from (22), $\upsilon_m > \upsilon_d$

Hence **velocity of sound in moist air is greater than the velocity of sound in dry air. That is why sound travels faster on a rainy day than on a dry day.**

□□(D) Effect of wind velocity: The velocity of sound in air is affected by the velocity of wind because wind drifts the medium (air) along its direction of motion. The velocity of sound in a particular direction is therefore, the algebraic sum of the velocity of sound and the component of wind velocity in that direction.



[Fig. 7]

Let υ = velocity of sound emitted by a source S in the direction of a listener L.

w = velocity of the wind along SA making an angle θ with the direction of propagation of sound.

Component of wind velocity w along SL

= w cos θ

Since υ and w cos θ act in the same direction (i.e., along SL),

∴ Resultant velocity of sound along

$= \upsilon + w\cos\theta$ … (23)

●● 1. The formula for velocity of sound does not involve frequency or wavelength. Hence sound of any frequency or wavelength travels through a given medium with the same velocity.

●●2. The amplitude normally does not affect the velocity of sound. However, if the amplitude is too large, the velocity of sound increases slightly.

●●3. Velocity of sound in a gas depends also an atomicity of the gas, which determines $\gamma = C_p/C_\upsilon$. For monoatomic gases, $\gamma = 5/3$, For diatomic gases $\gamma = 7/5$ and so on.

●●4. All other factors like phase, loudness, pitch, quality etc. have practically no effect on velocity of sound.

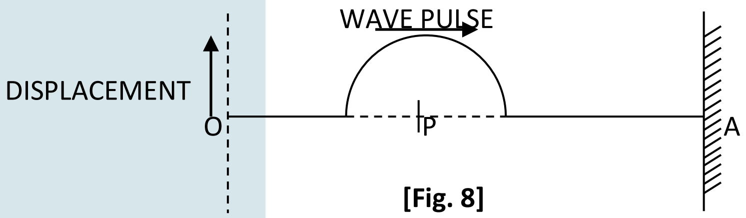
●●5. Kundli's tube is used to measure the velocity of sound in any medium, solid, liquid or gas.

Wave Function

We know that when a quantity y depends on another quantity x ; y is said to be a function of x. We represent it as y = f (x). When a point mass is moving along a straight line, its position x, depends only on time t. Therefore, to describe its motion we need a function of one variable, such as x (t). But when the moving object is not a point object, and has extended dimensions, such as a wave pulse, then to describe its motion, we need functions dependent on two variables such as x and t. Such functions which describe mathematically the motion of a wave pulse are called **wave functions**.

General form of a wave function:

Consider a string OA, fixed at one end A and held taut by pulling the free end O, Fig. 8. Let the point O be treated as the origin and OA be taken as positive direction of X-axis. When free end O of the string is given a sudden jerk perpendicular to OA, a wave pulse is produced at O (x = 0, t = 0). Let this wave pulse travel along OA with a velocity υ.



[Fig. 8]

The displacement (y) at different points along the travelling wave pulse depends on
(i) Distance (x) of the point from the origin,
(ii) Time (t) at which measurement is made.
At instant t = 0, let the displacement be represented by

$$y = f(x) \qquad \ldots (24)$$

……… The functional form of f would be different for waves of different shapes.

If the wave pulse travels without change in its shape, then the displacement at time t, at a distance x from the origin shall be same as at the distance $(x \pm \upsilon t)$ from the origin i.e.,

$$y = (x, t) = f(x \pm \upsilon t) \qquad \ldots (25)$$

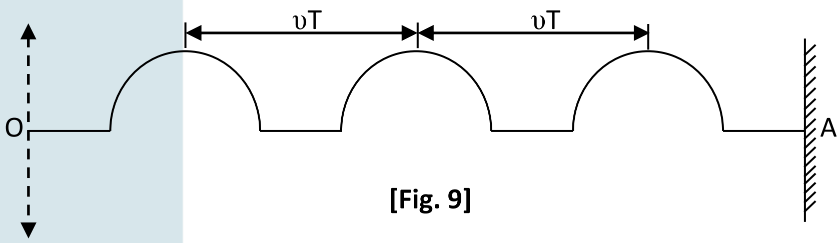
Here, v is the velocity of the wave pulse on the string. F $(x - \upsilon t)$ is for the wave pulse travelling from left to right and f $(x + \upsilon t)$ is for the wave pulse travelling from right to left. We can also represent a travelling wave pulse by the wave function

$$y = (x, t) = f(\upsilon t \pm x) \qquad \ldots (26)$$

Periodic Wave Function

<u>A periodic wave function represents a periodic wave i.e. a wave which repeats itself after a fixed interval of time or after a fixed distance</u>.

let us consider a string OA fixed at end A, Fig. 9. Let the free end O of the string be given periodic jerks at regular intervals of time T, in a direction perpendicular to the length of the string. A wave train of identical wave pulses are found to travel along the length of the string as shown in Fig. 9.



[Fig. 9]

It υ is velocity of wave pulse along the string, then the distance between any two successive wave pulses will be $= \upsilon T = \lambda$, where λ is called periodic length of wavelength of the periodic wave. T is called the period of the wave.

The general form of the wave function is

$$y(x, t) = f(x \pm \upsilon t) \qquad \ldots (27)$$

In a periodic wave, the displacement repeats itself after a fixed periodic length λ i.e.

$$y[(x + n\lambda); t] = y[x, t], \quad \ldots (28)$$

Again, in a periodic wave, the displacement repeats itself after a fixed interval of time T i.e.

$$y[x, (t + mT)] = y[x, t] \quad \ldots (29)$$

Where m is an integer.

The wave function is eqn. (27) which satisfies the periodicity conditions (28) and (29), would represent the periodic wave function.

Harmonic Wave Function

Harmonic wave functions represent harmonic waves. The harmonic waves correspond to periodic waves as simple harmonic motion corresponds to periodic motion. In the last chapter on oscillations, we learnt that the trigonometric functions of sine and cosine represent simple harmonic motion. Therefore, in the general form of wave function, we replace f by sine/cosine, and the argument $(\upsilon t - x)$ by $2\pi/\lambda\,(vt - x)$ to give it the dimensions of angle. Hence a general harmonic wave function can be represented as

$$y(x, t) = r \sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0\right) \quad \ldots (30)$$

Or $$y(x, t) = r \cos\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0\right) \quad \ldots (31)$$

Here, ϕ_0 represents the initial phase.

To check that the harmonic wave function (30) is periodic in x, we calculate y, when x is replaced by $(x + \lambda)$. From (30),

$$y[(x + \lambda), t] = r \sin\left(\frac{2\pi}{\lambda}(\upsilon t - x - \lambda) + \phi_0\right)$$

$$= r \sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) - 2\pi + \phi_0\right)$$

$$= r \sin\left(\left\{\frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0\right\} - 2\pi\right)$$

As $\sin(\theta \pm 2\pi) = \sin\theta$,

$\therefore$ $$y[(x + \lambda), t] = r \sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0\right\}$$

$= y(x, t)$. Therefore, the harmonic wave function is periodic in x.

Let us check if the harmonic wave function is periodic in t. For this, we calculate y when t is replaced by (t + T)

From (30), $y[x, (t + T)]$

$$= r \sin\left(\frac{2\pi}{\lambda}\{\upsilon(t + T) - x\} + \phi_0\right)$$

$$= r \sin \frac{2\pi}{\lambda}\upsilon t + \frac{2\pi}{\lambda}\upsilon T - 2\pi\upsilon T - \frac{2\pi}{\lambda}x + \phi_0$$

$$= r \sin\left(\frac{2\pi}{\lambda}\upsilon t + 2\pi - \frac{2\pi}{\lambda}x + \phi_0\right) \quad (\because \upsilon T = \lambda)$$

$$= r \sin\left(\left\{\frac{2\pi}{\lambda}(vt - x) + \phi_0\right\} + 2\pi\right)$$

$$= r \sin\left(\frac{2\pi}{\lambda}(vt - x) + \phi_0\right) = y(x, t)$$

Hence **$y[x, (t + T)] = y(x, t)$**

$\therefore$ The harmonic wave function represented by eqn. (30) is periodic in t.

Similarly, we can show that the harmonic wave function represented by eqn. (31) is also periodic in x and t.

………… In the harmonic wave function,

y = displacement of particle at time t,

r = maximum displacement of particle which represents amplitude of simple harmonic motion,

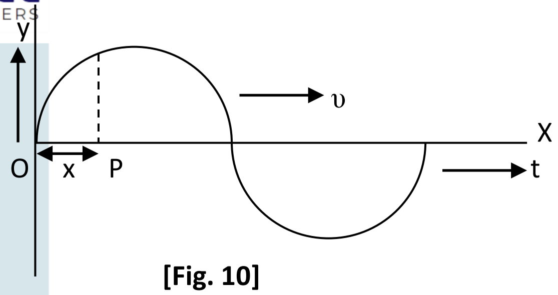
υ = velocity of the wave,

λ = wavelength of the wave,

ϕ_0 = initial phase of the wave.

Equation of a Plane Progressive Simple Harmonic Wave

Suppose a plane simple harmonic wave, starting from the origin O is travelling with a velocity υ along the positive direction of X-axis. The displacement curve of this wave is shown in Fig. 10.



[Fig. 10]

We know that a harmonic wave propagates on account of repeated periodic vibrations of the particles of the medium about their mean position.

If we start counting the time from the instant, particle at the origin O crosses its mean position in the positive direction, then the displacement y of the particle at O, at any time t, can be represented by

$$y(0, t) = r \sin \omega t \qquad \dots (32)$$

Where r is amplitude of S.H.M. and ω is angular frequency of S.H.M. executed by the particle.

We know that in a wave motion, there is a continuous phase difference amongst successive particles of the medium i.e. successive particles start vibrating about their mean position later than the particle at the origin. If ϕ is phase lag of a particle at P w.r.t. the particle at the origin O, then the displacement of this particle at P, at the same time t can be written as

$$y(x, t) = r \sin(\omega t - \phi) \qquad \dots (33)$$

Here, x = OP = distance of P from the origin.

We know that for a distance equal to one wavelength (λ), phase changes by 2 π radian. Therefore, for a distance x, phase change would be

$$\phi = \frac{2\pi}{\lambda} x \qquad \dots (34)$$

From (33), $y(x, t) = r \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$

As $\omega = \frac{2\pi}{T}$, where T is time period of vibration,

$$\therefore y(x, t) = r \sin\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right) \qquad \dots (35)$$

Or $\quad y(x, t) = r \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{T} t - x\right)$

But $\quad \frac{\lambda}{T} = \upsilon$ = velocity of the wave

$$\therefore \quad \boxed{y(x, t) = r \sin \frac{2\pi}{\lambda}(\upsilon t - x)} \qquad \dots (36)$$

This is the equation of a plane progressive simple harmonic wave travelling along positive direction of X-axis.

If such a wave is travelling along negative direction of X-axis, its equation can be written as

$$y(x, t) = r \sin \frac{2\pi}{\lambda}(\upsilon t - x) \qquad \dots (37)$$

□●□●: If ϕ_0 is initial phase of the wave, we may rewrite eqns. (36) and (37) as

$$y(x, t) = r \sin\left[\frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0\right] \qquad \dots (38)$$

$$y(x, t) = r \sin\left[\frac{2\pi}{\lambda}(\upsilon t + x) + \phi_0\right] \qquad \dots (39)$$

We have already proved in H.W.F. that at a given time t, displacement has the same value at a distance $(x + \lambda)$. Therefore, λ is wave length of the wave. Also, we showed in H.W.F. that at a given position x, displacement has the same value at a time (t + T). Therefore, T is time period of the wave.

□□1: It has been established that if in the wave functions, space and time variables appear in the combination of as $(a\,t \pm b\,x)$, the function represents a travelling wave. Negative sign between t and x implies that the wave is travelling along positive x axis and vice-versa.

□□2. Any function of space and time which satisfies the equation $\frac{d^2y}{dx^2} = \frac{1}{\upsilon^2}\frac{d^2y}{dt^2}$ shall represent a wave.

□□3. Periodic functions like $\sin(\omega t \pm k x)$ and $\cos(\omega t \pm k x)$ represent a plane progressive wave.

The periodic functions such as $\tan(\omega t \pm k x)$ are not used in Physics for describing a wave motion.

●● Phase and Phase Difference

It is represented by ϕ.

For a wave travelling along positive direction of X-axis, phase of the wave at position x and time t is given by [using (38)]

$$\phi(x, t) = \frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0 \qquad \ldots (40)$$

At a given position (i.e. for fixed value of x), phase changes with time t,

From (40), $$\boxed{\frac{d\phi}{dt} = \frac{2\pi}{\lambda}\upsilon = \frac{2\pi}{T}} \qquad \ldots (41)$$

Again, at a given time (i.e. for fixed value of t), phase changes with position x,

From (40), $$\boxed{\frac{d\phi}{dx} = -\frac{2\pi}{\lambda}} \qquad \ldots (42)$$

Negative sign indicates the phase lag. It means when a particle is at larger distance from the origin, its phase lag is greater. The reverse is also true.

Phase Difference between any two particles in a wave determines lack of harmony in the vibrating state of two particles i.e. how far one particle leads the other or lags behind the other.

Relation between Particle Velocity and Wave Velocity

The equation of a plane progressive wave travelling with a velocity υ along the positive direction of X-axis is

$$y(x, t) = r\sin\left(\frac{2\pi}{\lambda}(\upsilon t - x) + \phi_0\right) \qquad \ldots (43)$$

If initial phase, $\phi_0 = 0$, then

$$y(x, t) = r\sin\left(\frac{2\pi}{\lambda}(\upsilon t - x)\right) \qquad \ldots (44)$$

At any position x, velocity of particle is the rate of change of displacement of the particle with time. If it is represented by u (x, t), then

$$u(x, t) = \frac{d}{dt}[y(x, t)]$$

$$= \frac{d}{dt}\left(r\sin\left\{\frac{2\pi}{\lambda}(v t - x)\right\}\right)$$

$$u(x, t) = r\,\mathrm{con}\left\{\frac{2\pi}{\lambda}(\upsilon t - x)\right\} \times \frac{2\pi\upsilon}{\lambda} \qquad \ldots (45)$$

Also, $$\frac{d}{dx}[y(x, t)]$$

$$= \frac{d}{dx}\left(r\sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x)\right\}\right)$$

$$= r\cos\left\{\frac{2\pi}{\lambda}(v t - x)\right\}\left(-\frac{2\pi}{\lambda}\right) \qquad \ldots (46)$$

Dividing (45) by (46), we get $$\frac{u(x, t)}{\frac{d}{dx}\{y(x, t)\}} = -\upsilon$$

or $$\boxed{u(x, t) = -\upsilon\frac{d}{dx}\{y(x, t)\}} \qquad \ldots (47)$$

Hence **particle velocity at a given position at a given time is equal to product of wave velocity and negative of slope of the wave curve at the given position and time**.

Particle Acceleration

At any position x, acceleration of particle is the rate of change of velocity of the particle with time. It is represented by a (x, t), then

$$\mathbf{a(x, t)} = \frac{d}{dt}[u(x, t)]$$

$$= \frac{d}{dt}\left(r\cos\frac{2\pi}{\lambda}(\upsilon t - x) \times \frac{2\pi\upsilon}{\lambda}\right)$$

$$a(x, t) = -r\sin\frac{2\pi}{\lambda}(\upsilon t - x) \times \left(\frac{2\pi\upsilon}{\lambda}\right)^2$$

Using (44), $$a(x, t) = -y\frac{4\pi^2\upsilon^2}{\lambda^2}$$

$$= -4\pi^2\left(\frac{v}{\lambda}\right)^2 y = -4\pi^2 n^2 y$$

$a(x, t) = -(2\pi n)^2 y = -\omega^2 y$ ... (48)

Particle acceleration would be maximum at y = r (max.)

∴ $[a(x, t)]_{max} = -\omega^2 r$ ... (49)

Reflection of Waves

In case of sound waves/mechanical waves, the wavelength is very large compared to the wavelength of light waves. Therefore, any hard and plane wooden surface can serve as the reflector of sound waves/mechanical waves.

(A) Reflection of waves from a plane surface

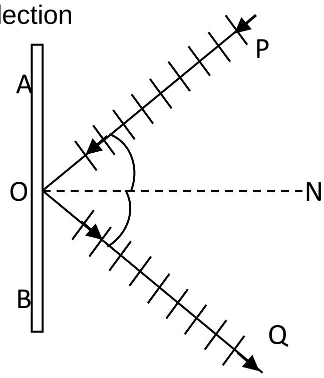
The reflection of water waves can be demonstrated by placing a reflecting surface (i.e. a straight obstacle) in the ripple tank so that the incident plane wave fronts while moving along PO may strike the reflecting surface AB at some angle. The incident wave fronts cannot cross the reflecting surface, but go along OQ after striking the surface. Therefore, the reflection of water waves is said to take place at O.

Here, the waves proceeding along PO are the incident waves and the waves proceeding along OQ are the reflected waves, Fig. 11. If ON is normal to the reflecting surface AB at O, then

∠ PON = I = angle of incidence, and

∠ QON = r = angle of reflection

It is found that, ∠I = ∠r



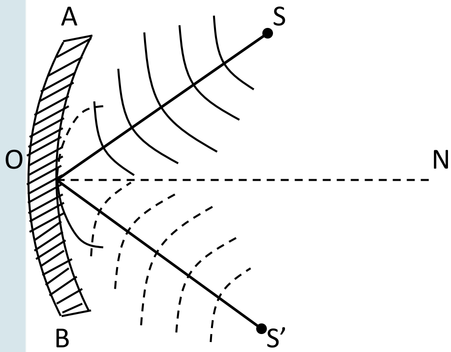
[Fig. 10]

Thus, the water waves are reflected obeying the laws of reflection of light.

(B) Reflection of waves from concave spherical surface

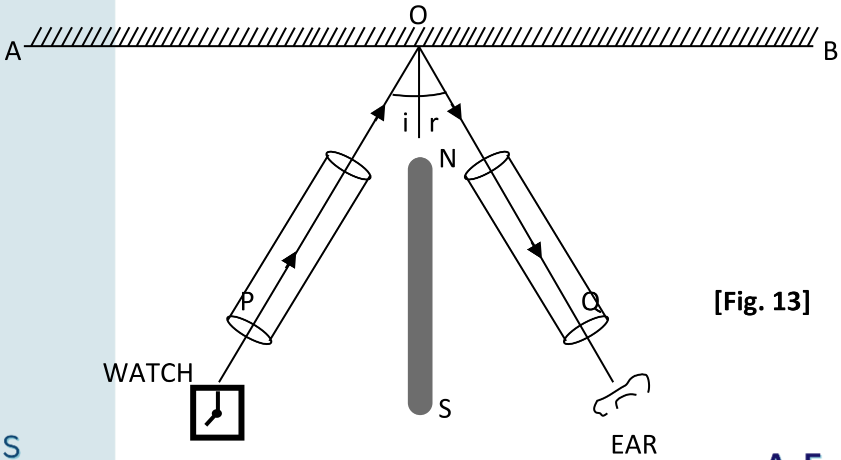
Let AB be a concave reflecting surface and S be a point source of disturbance (i.e. object). The wave fronts go outwards from S. After reflection form the surface AB, they converge to a point S' (image) as shown in Fig. 11. If we could measure the distance between source and reflecting surface (i.e. OS') = υ, say. Then the focal length (f) of the concave surface is found to satisfy the reaction:

$$\frac{1}{f} = \frac{1}{\upsilon} + \frac{1}{u}$$



(C) Reflection of SOUND waves from a plane surface: AB is a plane board fixed vertically.

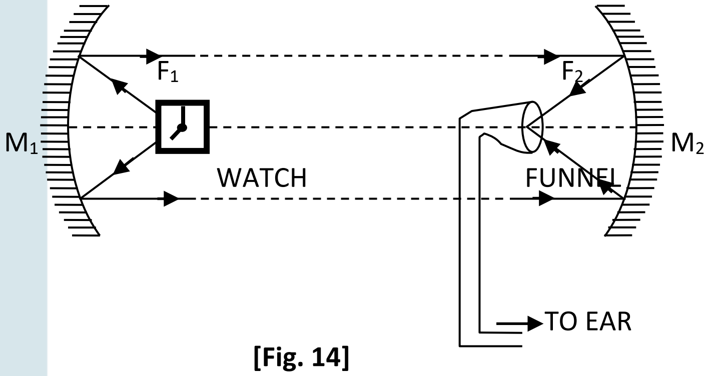
P and Q are two hollow tubes about 1 metre long and 1 cm in diameter. The tubes are placed horizontally close to the board. A watch is held near one end of tube P and the ear is held near the other end of tube Q. A wooden screen S cuts off direct sound from watch to the ear. ON is normal to the surface AB. The positions of tubes P and Q are adjusted till tick-tick of the watch is distinctly heard by the ear. It is found that at this stage, axes of tubes P and Q are inclined equally to the normal On i.e. angle of incidence (i) is equal to angle of reflection (r).



[Fig. 13]

□●□ **(D) Reflection of waves from curved surface:** M_1 and M_2 are two large concave reflectors with foci at F_1 and F_2 respectively. They are held co-axially facing each other as shown. A watch is held at F_1 and F_2 respectively. They are held co-axially facing each other as shown. A watch is held at F_1 and a funnel facing M_2 is held at F_2. The funnel is connected to a rubber tube and ear is held at the free end of the rubber tube.

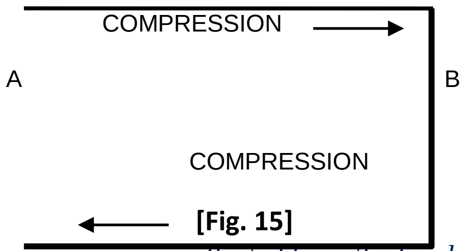
The tick-tick of the watch is heard distinctly only when the funnel is held at F_2. When the funnel is displaced even slightly from F_2, the ticking sound of watch is not audible. Sound waves starting from watch at the focus of M_1, get reflected from M_1, form a wave train parallel to the principal axis, which is reflected from M_2 and collects at F_2. Thus, mechanical waves/sound waves follow the fundamental laws of reflection of light.



[Fig. 14]

□●□ **(E) Reflection of SOUND waves at the end of a closed pipe (denser medium)**

Suppose a train of waves is incident normally at the closed end B of a pipe. As the end is rigid, none of the energy incident upon it can be transmitted forward, so that the layer of air in contact with the end must remain permanently at rest. Hence when a compression reaches the closed end, the only way whereby this layer can free itself from the compression towards the left, Fig. 15. Similarly, a rarefied wave train travelling up the fixed end will come back as a similar wave train.



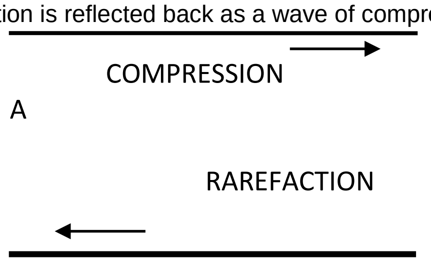
[Fig. 15]

Conclusion: a pulse of compression is reflected from the fixed end (denser medium) as a pulse of compression; and a pulse of rarefaction returns as a pulse of rarefaction. Because on reflection, particle velocity and wave velocity, both are reversed in sign, hence we may conclude that a longitudinal displacement wave suffers a **reversal of phase on reflection at a rigid boundary** (denser medium). Further, as the reflection is almost complete, the intensity and hence amplitude of reflected wave are the same as those of incident wave.

□●□ **(F) Reflection of SOUND waves at the end of an open pipe (rarer medium)**

Suppose a pulse of compression travelling in an open pipe from A to B, arrives at the open-end B. At this end, the pulse of compression encounters much smaller resistance to expansion than it has encountered inside the pipe, as the pulse can now freely spread sideways. In doing so, the pulse more than releases its strain and continues to move forward beyond its normal amplitude (as wave of compression). This causes the air behind to become rarefied. Thus, a wave of

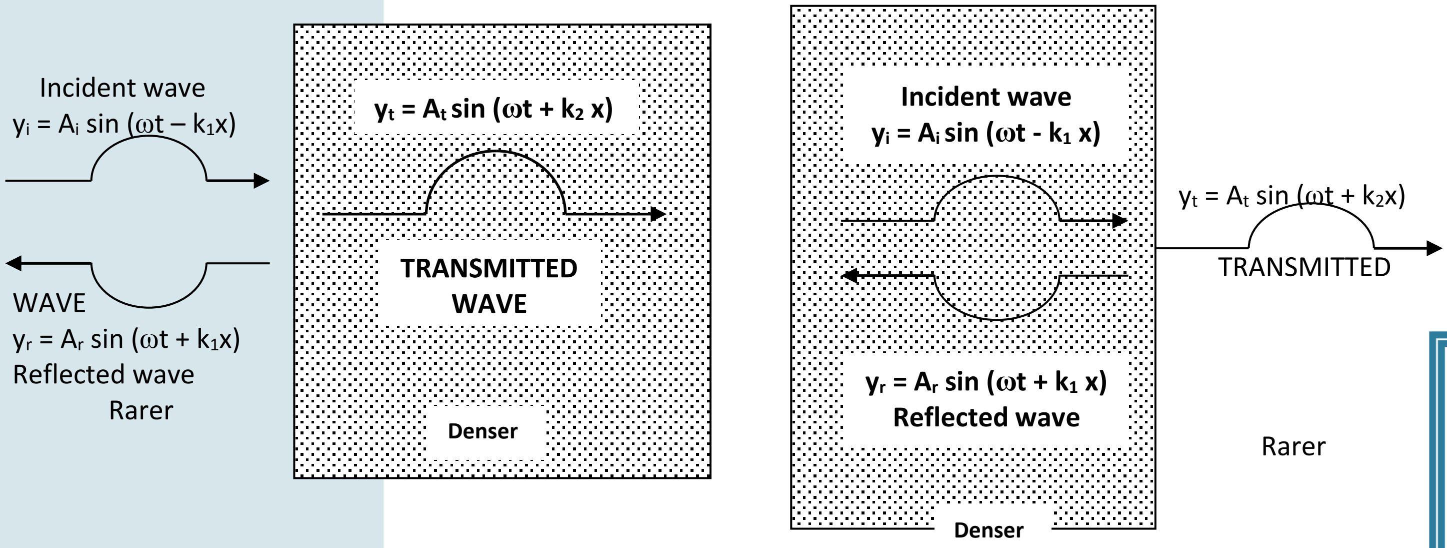
rarefaction travels in the pipe i.e. the phase of compression is reversed, Fig. 16. Similarly, we can show that a wave of rarefaction is reflected back as a wave of compression.



[Fig. 16]

Conclusion: At an open end (rarer medium), a pulse of compression is reflected as a pulse of rarefaction and vice-versa. Now, in the wave reflected from the open end, the particles continue to move in the same direction as in the incident wave, therefore, the phase of particle displacement and particle velocities remains unchanged.

□□□ At the open end, the reflection is partial, because a part of energy of the incident wave will pass out into the open air. Hence intensity and amplitude of reflected wave will be less than those of the incident wave.



[Fig. 17]

Some of the important practical applications of reflection of sound waves are:

- 1. Ear trumpet or hearing aid used by people who are hard of hearing.
- 2. Stethoscope used by doctors is based on the phenomenon of reflection of sound waves.
- 3. Speaking tubes concentrate the sound to well defined directions by multiple reflection of sound waves.
- 4. Reflecting boards are fixed behind the speaker, on the inside wall of large halls/auditoria. Sound waves from the speaker at the focus of the concave reflecting board get reflected from the board and proceed as a concentrated beam.
- 5. Phenomenon of echoes is also based on reflection of sound waves.

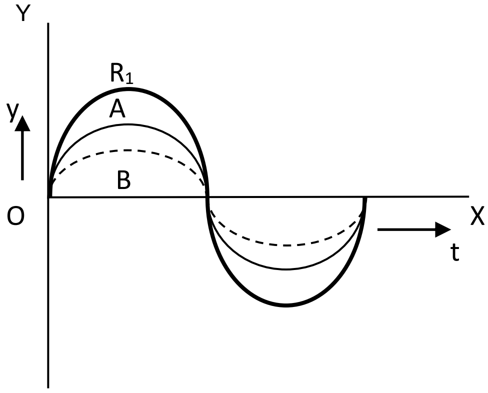
Superposition Principle

The principle of superposition enables us to find the resultant of any number of waves. According to this principle, the displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due to each one of waves at that point at the same time.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$….. are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement y at the same time at the given position would be

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots\ldots$$

For example, in Fig. 18, crest of one wave A falls on crest of the other wave B, and trough falls on trough. Therefore, the amplitude of the resultant wave R_1 is sum of the amplitudes of the two waves, A and B.



One of the most important properties of a wave motion is that is preserve its individuality while travelling through a medium/space. Each wave behaves us if it has nothing to do with other waves.

□□□ **Principle of superposition applies equally well to electromagnetic waves**.

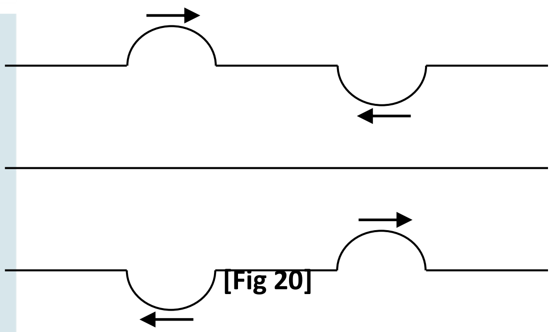
Further, the superposition principle ceases to apply when amplitude of mechanical waves is too large, e.g. in case of shock waves generated in a violent explosion.

For example, (i) Radio waves from different stations having different frequencies cross the antenna. But our T.V/Radio set can pick up any desired frequency.

(ii) In an orchestra, different musical instruments are played simultaneously. We can, however, detect the note or sound produced by any individual instrument.

(iii) When two pulses of equal amplitude on a string approach each other [Fig. 20], then on meeting, they superimpose to produce a resultant pulse of zero amplitude.

After crossing, the two pulses travel independently as shown in Fig.20, as if nothing had happened.



[Fig 20]

□●□●□●**Three important applications of superposition principle are (i) Interference of waves (ii) stationary waves (iii) Beats.**

●● Standing Waves or Stationary Waves

When two sets of progressive wave trains of the same type (i.e. both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves. The resultant waves to not propagate in any direction, nor there is any transfer of energy in the medium. In the stationary waves, there are certain points of the medium, which are permanently at rest i.e. their displacement is zero all throughout. These points are called **Nodes**. Similarly, there are some other points which vibrate about their mean position with largest amplitudes. These points are called **Antinodes**.

Two types of stationary waves:

□●□**1. Longitudinal stationary waves** are formed as a result of superimposition of two identical longitudinal waves travelling in opposite directions. For example, stationary waves produced in organ pipes and in air column of resonance tube apparatus are longitudinal stationary waves.

□●□**2. Transverse stationary waves** are formed as a result of superimposition of two identical transverse waves travelling in opposite directions. For example, stationary waves produced on the vibrating string of a sonometer are transverse stationary waves.

●● Formation fo Stationary Waves In Strings: Graphical Method

The formation of stationary waves is explained in terms of superimposition of a progressive wave on its own reflected wave.

Suppose a transverse progressive wave is travelling along a string form left to right. It is represented by a thin continuous curve in Fig. 21. Let this wave be reflected at the other fixed end (N_5) of the string. This end acts as a denser medium. Therefore, the reflected wave travels on the string form right to left without change of type. It is represented by a thin dotted line curve.

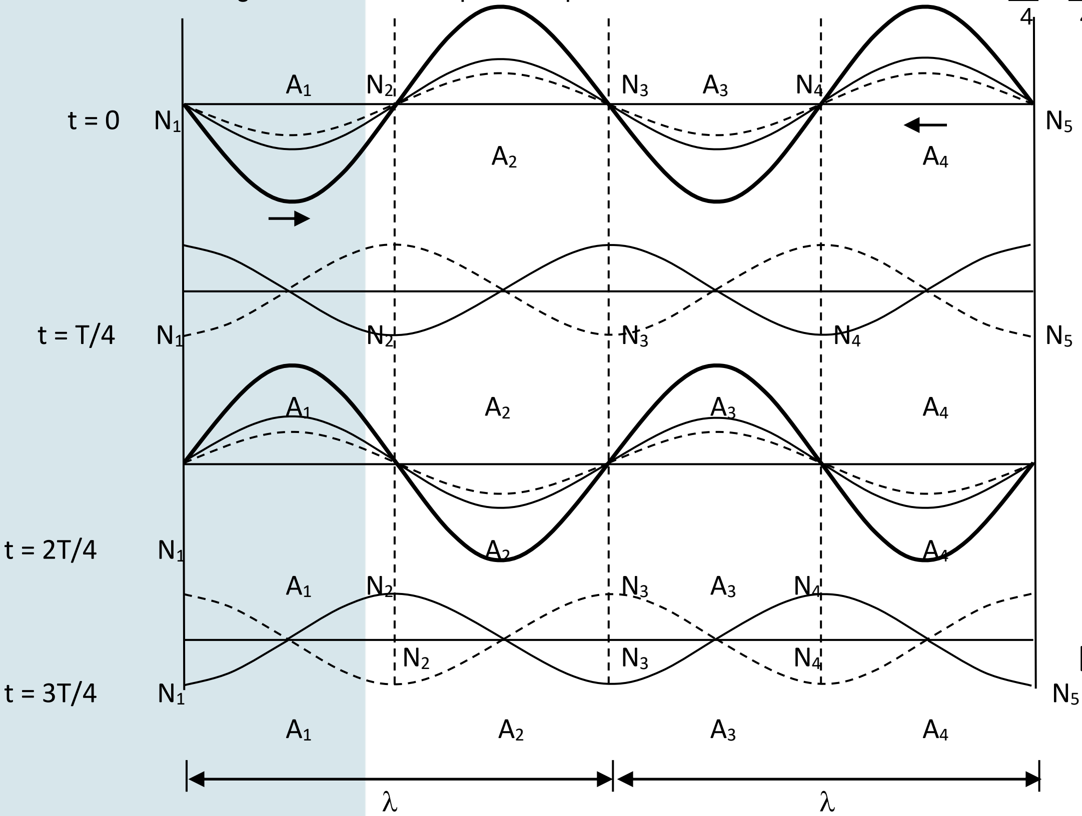
It we assume that there is no loss of energy on reflection, the amplitude of reflected wave would be same as that of the incident wave. The incident and reflected waves travelling in opposite directions superimpose. According to superposition principle, the resultant displacement at any point on the string is equal to vector sum of the individual displacement at that point.

(i) **At t = 0**Fig. 21 shows that crest of one wave falls on crest of the other and trough falls on trough. The resultant displacement at every point is the sum of the individual displacements. The resultant wave is represented by a thick line curve. We observe that displacement of particles at N_1, N_2, N_3, N_4 and N5 is zero. They represented nodes. The displacement of particles at A_1, A_2 and A3 is maximum. These points represent antinodes. Let T be the time period of each wave, and λ be the wavelength of each wave.

(ii) **At t = T/4**The incident wave has travelled a distance = $\lambda/4$ to the right and the reflected wave has travelled an equal distance $\lambda/4$ to the left, on the string, as shown in Fig.21. We find that crest of one wave falls on trough of the other and trough falls on crest. The resultant displacement at each point becomes zero, and is represented by a thick horizontal line. Thus, at t = T/4, all the particles of the medium cross their mean position simultaneously.

(iii) **At t = T/2**The incident wave has travelled a distance = $\lambda/2$ to the right and the reflected wave has travelled an equal distance ($\lambda/2$) to the left, on the string, as shown in Fig. 21. The equivalent path difference between the two waves = $\lambda/2 + \lambda/2 = \lambda$. Therefore, crest of one wave falls on crest of the other and trough falls on trough. The resultant displacement is represented by the thick line curve in Fig. 21.

(iv) At t = 3T/4 The incident wave travels a distance = 3λ/4 to the right and the reflected wave travels the same distance on the string to the left. The equivalent path difference between the waves = $\frac{3\lambda}{4} + \frac{3\lambda}{4} = \frac{3\lambda}{2}$.



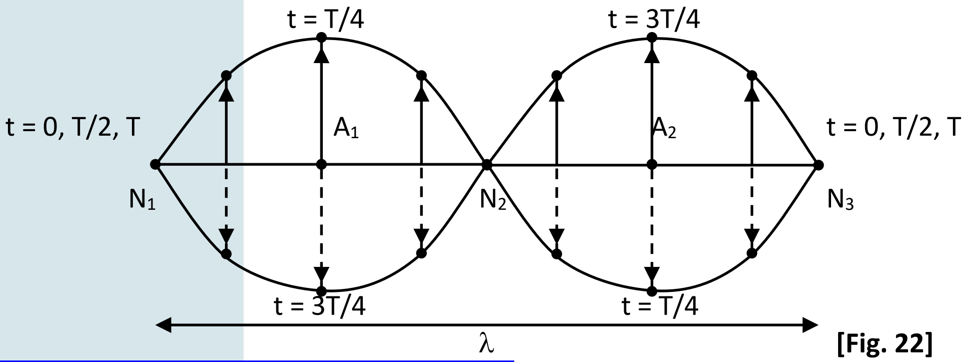
[Fig. 21]

Therefore, crest of one wave falls on trough of the other and vice-versa. The resultant displacement at all points are zero and is represented by thick horizontal line in Fig. 21. Thus, all the particles are passing simultaneously through their mean position, but in a direction opposite to the direction at t = T/4.

(iv) **At t = T,** each of the incident and reflected waves has travelled a distance equal to one full wavelength (λ) to the right and to the left of the string respectively. The equivalent path difference between two waves = λ + λ = 2λ. The situation is same as t = 0.

The entire cycle is repeated time and again. This is how stationary waves are formed on a string.

It may be noted that wavelength of stationary waves is exactly the same as that of either travelling wave. Obviously, the time period and frequency of particle motion in standing waves and progressive waves must be the same, Fig. 22.



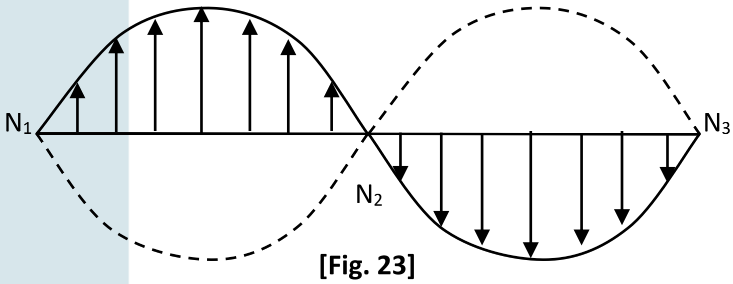
[Fig. 22]

Characteristics of Standing Waves

- **1.** The disturbance is confined to a particular region between the starting point and the reflecting point of the wave.
- **2.** There is no onward motion of the disturbance from one particle to the adjoining particle and so on beyond this particular region.
- **3.** The total energy associated with a stationary wave is twice the energy of each of incident and reflected wave. But there is no flow or transference of energy along the stationary wave.
- **4.** There are certain points in the medium in a standing wave, which are permanently at rest. These are called **nodes**. The distance between two consecutive nodes is λ/2.
- **5.** There are certain other points in the medium in a standing wave, the amplitude of vibration of which is maximum. These are called **antinodes**. The distance between two consecutive antinodes is also λ/2. Antinodes lie in between successive nodes. The distance between a node and adjoining antinodes is λ/4.
- **6.** In a standing wave, the medium splits up into a number of segments. Each segment is vibrating up and down as a whole.

□●**7.** All the particles in one particular segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180°. This is shown in Fig. 23.

□●**8.** All the particles except those at nodes, execute simple harmonic motion about their mean position with the same time period.



[Fig. 23]

□●**9.** The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.

□●**10.** Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

□●**11.** The wavelength and time period of stationary waves are the same as for the component waves.

□●**12.** Velocity of particles while crossing mean position varies from maximum at antinodes to zero at nodes.

□□**1**: The nodes divide the medium into segments (or loops). All the particles of medium in one segment vibrate in the same phase. But these particles are in opposite phase with the particles in the adjacent segment.

□□**2.** As in stationary waves, particles at nodes are permanently at rest, therefore, no energy can be transmitted across them i.e. energy of one region or segment is confined in that region only.

□□**3.** In standing waves, if amplitudes of component waves are not equal, resultant amplitude at nodes will be minimum (but not zero). Therefore, some energy will pass across nodes and waves will be partially standing.

●● Standing Waves in Strings and Normal Modes of Vibration: Analytical Treatment

When a string under tension is set into vibration, transverse harmonic waves propagate along its length. When the length of string is fixed, reflected waves will also exist. The incident and reflected waves will also exist. The incident and reflected waves will superimpose to produce transverse stationary waves in the string.

The string will vibrate in such a way that the clamped points of the string are nodes and the point of plucking is antinodes.

Let a harmonic wave be set up on a string of length L, fixed at the two ends x = 0 and x = L. This wave gets reflected from the two fixed ends of the string continuously and as a result of superimposition of these waves, standing waves are formed on the string.

Let the wave pulse moving on the string from left to right (i.e. along positive direction of x – axis) be represented by $y_1 = r \sin \frac{2\pi}{\lambda}(v t - x)$

Where the symbols have their usual meanings. Here x is the distance from the origin in the direction of the wave (from left to right). It is often convenient to take the origin (x = 0) at the interface (the site of reflection), on the right fixed end of the string. In that case, sign of x is reversed because it is measured from the interface in a direction opposite to the incident wave. The equation of incident wave may, therefore, be written as

$$y_1 = r \sin \frac{2\pi}{\lambda}(v t - x) \qquad \text{... (50)}$$

As there is a phase change of π radian on reflection at the fixed end of the string, therefore, the reflected wave pulse travelling from right to left on the string is represented by

$$y_1 = r \sin\left[\frac{2\pi}{\lambda}(v t - x) + \pi\right]$$

$$= - r \sin \frac{2\pi}{\lambda}(v t - x) \qquad \text{... (51)}$$

According to superposition principle, the resultant displacement y at time t and position x is given by

$$y = y_1 + y_2$$

$$= r \sin \frac{2\pi}{\lambda}(v t + x) - r \sin \frac{2\pi}{\lambda}(v t - x)$$

$$y = r\left[\sin \frac{2\pi}{\lambda}(v t + x) - \sin \frac{2\pi}{\lambda}(v t - x)\right] \qquad \text{... (52)}$$

Using the relation,

$$\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$$

We get

$$\boxed{y = 2 r \cos \frac{2\pi}{\lambda} v t \sin \frac{2\pi}{\lambda} x} \qquad \text{... (53)}$$

As the arguments of trigonometrical functions involved in (53) do not have the form $(v\,t \pm x)$, therefore it does not represent a moving harmonic wave. Rather, it represents a new kind of waves called standing or stationary waves.

At one end of the string, where x = 0

From (53)

$$y = 2\,r \cos \frac{2\,\pi}{\lambda} v\,t \sin \frac{2\,\pi}{\lambda}(0) = 0$$

$(\because \sin 0^o = 0)$

At the other end of the string, where x = L

From (53)

$$y = 2\,r \cos \frac{2\,\pi}{\lambda} v\,t \sin \frac{2\,\pi}{\lambda} L \qquad \text{... (54)}$$

As the other end of the string is fixed,

$\therefore$ y = 0, at this end

For this, from (54)

$$\sin \frac{2\,\pi}{\lambda} L = 0 = \sin n\,\pi,$$

Where n = 0, 1, 2, 3....

$$\therefore \quad \frac{2\,\pi\,L}{\lambda} = n\,\pi \qquad \boxed{\text{or } \lambda = \frac{2L}{n}} \qquad \text{... (55)}$$

Note that n = 1, 2, 3...... correspond to 1st, 2nd, 3rd .... normal modes of vibration of the string.

(i) First normal mode of vibration

Suppose λ_1 is the wavelength of standing waves set up on the string corresponding to n = 1.

From (55), $\lambda_1 = \frac{2L}{1}$ or $L = \frac{\lambda_1}{2}$

The string vibrates as a whole in one segment, as shown in Fig. 24.

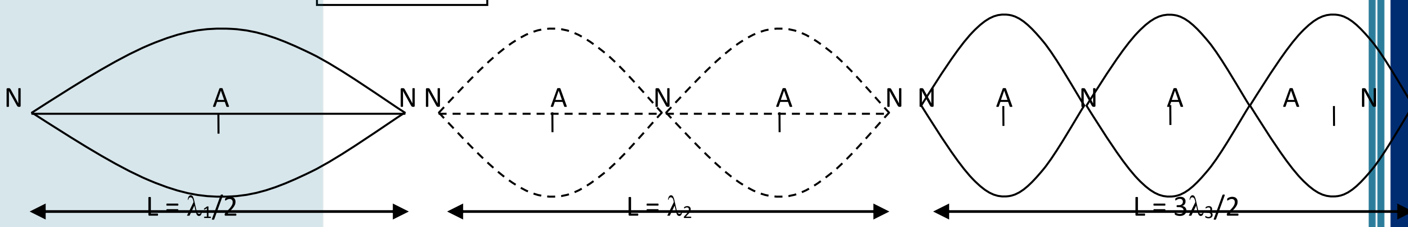
The frequency of vibration is given by

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \qquad \text{... (56)}$$

As $v = \sqrt{T/m}$

Where T is tension in the string and m is mass per unit length of the string.

$$\therefore \text{ from (56), } \quad \boxed{\nu_1 = \frac{1}{2L}\;\frac{T}{m}} \qquad \text{... (57)}$$



[Fig. 24]

This (first) normal mode of vibration is called fundamental mode. The frequency of vibration (ν_1) of string is this mode is minimum and is called *fundamental frequency*. The sound or note so produced is called *fundamental note* or *first harmonic*.

(ii) Second normal mode of vibration

Let λ_2 be the wavelength of standing waves set up on the string corresponding to n = 2.

From (55), $\lambda_2 = \frac{2L}{2} = L$

The string vibrates in two segments of equal length, as shown in Fig. 24. The frequency of vibration is given by➢ $\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \times \frac{v}{2L}$

$$\boxed{\nu_2 = 2\,\nu_1} \qquad \text{... (58)}$$

i.e. frequency of vibration of string becomes twice the fundamental frequency. The note so produced is called second harmonic or first overtone.

(iii) Third normal mode of vibration

Let λ_3 be the wavelength of standing waves set up on the string corresponding to n = 3.

From (55), $\lambda_3 = \frac{2\,L}{3}$ or $L = \frac{3\lambda_3}{2}$

The string vibrates in three segments of equal length, as shown in Fig. 24.

The frequency of vibration is given by

$$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3} = 3\left(\frac{v}{2L}\right)$$

$$\boxed{\nu_3 = 2\,\nu_1} \qquad \text{... (59)}$$

i.e. frequency of vibration of string becomes three times the fundamental frequency. The note so produced is called **third harmonic** or **second overtone**.

In general, the wavelength of nth mode of vibration of string is

$$\lambda_n = \frac{2L}{n} \qquad \text{[From (55)]}$$

The corresponding frequency of vibration would be

$$\nu_n = \frac{v}{\lambda n} = \frac{v}{2L/n} = n\left(\frac{v}{2L}\right)$$

$$\boxed{\nu_n = n\,\nu_1} \qquad \text{... (60)}$$

This frequency is n times the fundamental frequency. The note so produce is nth harmonic or (n – 1) th overtone.

- **Position of Nodes**:

In standing waves, we know that nodes are the positions of zero displacement. They are represented by N. From Fig. 24, we observe that there are two nodes in the first normal mode of vibration, three nodes in the second normal mode, four nodes in the 3rd normal mode of vibration and so on. Therefore, in the nth mode of vibration and so on. Therefore, in the nth mode of vibration, there will be (n + 1) nodes. These nodes are located at the distances:

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} \text{ .... } L.$$

For example, in 1st normal mode n = 1.

The nodes are at $x = 0,\ x = \frac{L}{1} = L$, Fig. 24.

In 2nd normal mode of vibration, n = 2

The nodes are at $x = 0,\ x = \frac{L}{2},\ x = \frac{2L}{2} = L$ These are shown in Fig. 24, and so on.

- **Position of Antinodes:**

In standing waves, antinodes are the position of maximum displacement. They are represented by A.

As is clear from Fig. 24, there will be n antinodes in the nth normal mode of vibration.

As antinodes are located in between nodes, therefore, their position will be given by

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} \text{ .... } \frac{(2n-1)L}{2n}$$

For example, in first normal mode, n = 1, the antinode is at $x = \frac{L}{2\times 1} = \frac{L}{2}$, [Fig 24]

In 2nd normal mode of vibration, n = 2, the antinodes are at $x = \frac{L}{2\times 2} = \frac{L}{4}$ and

$$x = \frac{3L}{2\times 2} = \frac{3L}{4}$$ These are shown in Fig. 24, and so on.

Laws of Vibrations of Strethed Strings

The fundamental frequency of vibration of a stretched string, as deduced above, in eqn. (57) is

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad \text{... (57)}$$

From this equation, we deduce the following three laws:

1. **Law of length**: The fundamental frequency of vibration of a stretched string (ν) is inversely proportional to the length (L) of the string, provided T and m are constants. i.e. $\nu \propto 1/L$.
2. **Law of Tension**: The fundamental frequency of vibration of a stretched string (ν) is directly proportional to the square root of tension (T) in the string, provided L and m are constants. i.e. $\nu \propto \sqrt{T}$
3. **Law of mass**: The fundamental freq. of vibration of a stretched string (ν) is inversely proportional to the square root of mass per unit length (m) of the string, provided L and T constants i.e., $\nu \propto 1/\sqrt{m}$

Two more laws:

Let D be the diameter of the string ρ be the density of material of the string.

$\therefore$ Area of cross section of string = $\pi \frac{D^2}{4}$

Volume of unit length of string = $\pi \frac{D^2}{4} \times 1$

Mass of unit length of string,

$$m = \pi \frac{D^2}{4} \times 1 \times \rho$$

Putting this value of m in (57), we get

$$v = \frac{1}{2L}\sqrt{\frac{T\,4}{\pi D^2 \rho}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

This shows that

$$\boxed{v \propto \frac{1}{D}}$$ ...**law of diameter**

(Provided L, T and ρ are constants)

And $$\boxed{v \propto \frac{1}{\sqrt{\rho}}}$$ ...**law of density**

(Provided L, D and T are constants)

These are the two more laws of vibration of stretched strings.

1: If a string is vibrating in nth harmonic, its frequency will be nv. Number of loops = n. Number of antinodes = n. Number of nodes = (n + 1).

2. In case of vibrations of composite string (i.e. string made up by joining two strings of different lengths, different cross section and densities), having same tension throughout and the joint will be a *node*. The lowest common fundamental frequency of the string will be $v_c = n_1 v_1 = n_2 v_2$.

• Standing Waves in Closed Organ Pipes: Analytical Treatment

Organ pipe are the musical instruments which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves. Organ pipes are of two types:

(i) Closed organ pipes, which are closed at one end,

(ii) Open organ pipes, which are open at both ends.

If closed end of the pipe where reflection occurs were taken as the origin (x = 0), the displacement at position x and time x and time t in the incident wave can be taken as

$$y_1 = r \sin \frac{2\pi}{\lambda}(v\,t + x)$$

The sign of x is reversed because it is measured from the interface in a direction opposite to the incident wave.

The wave reflected from the closed end of the pipe suffers a phase reversal of π. Therefore, the displacement at the same position x and same time t in the reflected wave can be written as

$$y_2 = r\left(\sin \frac{2\pi}{\lambda}(v\,t - x) + \pi = -r \sin \frac{2\pi}{\lambda}(v\,t - x)\right)$$

According to superposition principle, the resultant displacement y at time t and position x is given by ,$y = y_1, y_2$,

$$y_2 = r\left(\sin \frac{2\pi}{\lambda}(v\,t + x) - \sin \frac{2\pi}{\lambda}(v\,t - x)\right)$$

$$y = 2\,r \cos \frac{2\pi}{\lambda} v\,t .\ \sin \frac{2\pi}{\lambda} x \qquad \text{... (61)}$$

This is the equation of longitudinal stationary waves in the pipe.

At the closed end of the pipe, x = 0,

$$\sin \frac{2\pi}{\lambda} x = \sin 0^\circ = 0$$

From (61), y = 0 i.e. a node is formed at the open end of the pipe of length L, x = L, an antinodes is to be formed,

i.e. y = max.

From (61), $y = 2\,r \cos \frac{2\pi}{\lambda} v\,t .\ \sin \frac{2\pi}{\lambda} L$

y will be max, when $\sin \frac{2\pi L}{\lambda}$ = max. = ± 1

$$= \sin (2n - 1)\frac{\pi}{2}$$ Where n = 1, 2, 3.......

∴ $\frac{2\pi L}{\lambda} = (2n-1)\frac{\pi}{2}$

$$\boxed{\lambda = \frac{4L}{(2n-1)}} \quad \text{... (62)}$$

□**(i) First normal mode of vibration**: Let λ_1 be the wavelength of standing waves set up in the pipe corresponding on n = 1.

From (62), $\lambda_1 = \frac{4L}{2\times 1-1} = 4L$ or $L = \frac{\lambda_1}{4}$

This mode of vibration is shown in Fig. 25

The frequency of vibration in this mode is given by $\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

$$\nu_1 = \frac{v}{4L} \quad \text{... (63)}$$

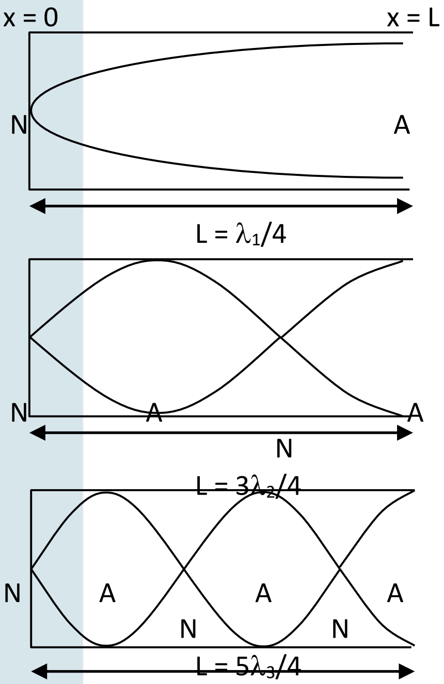
This is the lowest frequency of vibration and is called the fundamental frequency. The note or sound so produced is called fundamental note or first harmonic.

□**(ii) Second normal mode of vibration**

Let λ_2 be the wavelength of standing waves set up in the pipe corresponding to n = 2.

From (62), $\lambda_2 = \frac{4L}{2\times 2-1} = \frac{4L}{3}$

This mode of vibration is shown in Fig. 25. The frequency of vibration in this mode is given by



[Fig. 25]

$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3} = \frac{3v}{4L} = 3\nu_1$

$$\boxed{\nu_2 = 3\nu_1} \quad \text{...(64)}$$

Thus the frequency of vibration in 2[nd] normal mode is thrice the fundamental frequency. The note so produce is called **third harmonic**. It is the first overtone produce in a closed organ pipe.

(iii) Third normal mode of vibration

Let λ_3 be the wavelength of standing waves set up in the pipe corresponding to n = 3.

From (62), $\lambda_1 = \frac{4L}{2\times 3-1} = \frac{4L}{5}$

This mode of vibration is shown in Fig. 25. The frequency of vibration in this mode is given by

$\nu_2 = \frac{v}{\lambda_3} = \frac{v}{4L/5} = 5\frac{v}{4L}$

$$\boxed{\nu_3 = 5\nu_1} \quad \text{... (65)}$$

i.e. the frequency of vibration in the 3[rd] normal mode is five times the fundamental frequency. The note or sound so produced is called **fifth harmonic**. This is the second overtone produced in the closed organ pipe.

Proceeding as above, the frequency of note produced in nth normal mode of vibration of closed organ pipe would be

$$\nu_n = \frac{(2n-1)v}{4L} = (2n-1)\nu_1$$

This is (2n – 1) th harmonic or (n – 1) th overtone.

Position of Nodes is given by

$$x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{2n-1} \ \dots \ \frac{2nL}{(2n-1)}$$

There will be n nodes in nth normal mode of vibration of closed organ pipe.

Position of antinodes, which lie in between nodes, would be given by

$$x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1} \ \dots \ L$$

These positions can be easily verified from Fig. 25.

• Standing Waves in Open Organ Pipes: Analytical Treatment

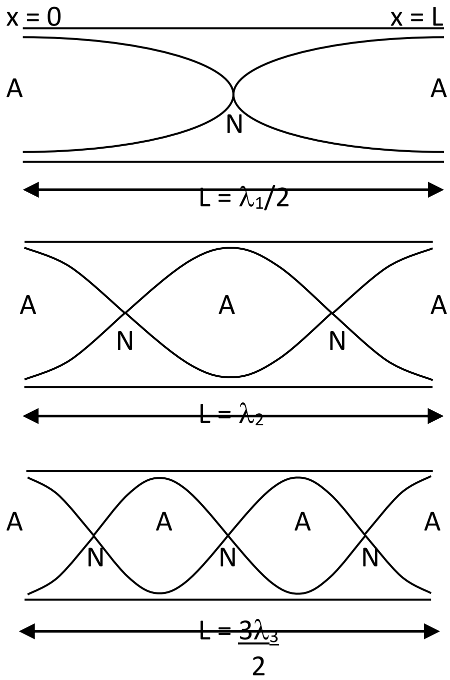
An open organ pipe is open at both ends. Therefore, an antinode is formed at each end. Proceeding as in the case of closed organ-pipe, and setting y = max. at x = 0 and at x = L, we shall obtain

$$\boxed{\lambda = \frac{2L}{N}} \quad \dots (66)$$

Where n = 1, 2, 3.........

(i) First normal mode of vibration

Let λ_1 be the wavelength of stationary waves set up in the open organ pipe corresponding to n = 1



[Fig. 26]

From (66), $\lambda_1 = \frac{2L}{1}$ or $L = \frac{\lambda_1}{2}$

This mode of vibration is shown in Fig. 26. The frequency of vibration in this mode is given by

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

$$\boxed{\nu_1 = \frac{v}{2L}} \quad \dots (67)$$

This is the lowest frequency of vibration and is called the fundamental frequency. The note or sound of this frequency is called fundamental note or first harmonic, Fig. 26.

(ii) Let λ_2 be the wavelength of stationary waves set up in the open organ pipe corresponding to n = 2

From (66), $\lambda_1 = \frac{2L}{2} = L$

This mode of vibration is shown in Fig. 26. The frequency of vibration in this mode is given by

$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$\nu_2 = \frac{2v}{2L} = 2\nu_1 \qquad \text{i.e. } \nu_2 = 2\nu_1 \quad \dots (68)$$

i.e. frequency of vibration in second normal mode is twice the fundamental frequency. The note so produced is called **second harmonic** or **first overtone**.

(iii) Third normal mode of vibration

Let λ_3 be the wavelength of standing waves set up in the open organ pipe corresponding to n = 3.

From (66), $\lambda_3 = \frac{2L}{3}$ or $L = \frac{3\lambda_3}{2}$

This mode of vibration is shown in Fig. 26. The frequency of vibration in this mode is given by

$$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3} = \frac{3v}{2L}$$

$\boxed{\nu_3 = 3\,\nu_1}$ ... (69)

i.e. frequency of vibration in third normal mode is thrice the fundamental frequency. The note so produced is called **third harmonic** or **second overtone**.

In general, the frequency of vibration in nth normal mode of vibration in open organ pipe would be

$\boxed{\nu_n = n\,\nu_1}$ ... (70)

The note so produced would be called nth harmonic or (n – 1) th overtone.

□□ **1**: Comparison of closed and open organ pipes shows that fundamental note in open organ pipe ($\nu_1 = v/2L$) has double the frequency of the fundamental note in closed organ pipe ($\nu_1 = v/4L$). Further, in an open organ pipe, all harmonics are present whereas in a closed organ pipe, only alternate harmonics of frequencies ν_1 3 ν_1, 5 ν_1....... etc. are present. The harmonics of frequencies 2 ν_1, 4 ν_1, 6 ν_1 .... are missing. Hence the overall musical sound produced by an open organ pipe is richer than the musical sound produced by a closed organ pipe.

□□**2**: Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency (ν). Thus first, second, third .... harmonics have frequencies ν, 2ν, 3ν .... respectively.

3: Overtones are the notes/sounds of frequency twice/thrice/four times...... the fundamental frequency (ν) e.g., 2ν, 3ν, 4ν ...... and so on.

Beats

When two sound waves of *slightly different frequencies*, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. *This phenomenon of regular variation in the intensity of sound with time at a particular position, when two sound waves of nearly equal frequencies superimpose on each other is called beats*.

If intensity of sound is maximum at time t = 0, one beat *is said to be formed when intensity becomes maximum again, after becoming minimum once in between*.

The time interval between two successive beats (i.e. two successive maxima of sound) is called ***beat period***. The number of beats produced per second is called ***beat frequency***.

We shall prove that number of beats/sec. i.e. beat frequency is equal to difference in the frequencies of two superimposing component waves.

□□□□□□□□□**Why nearly equal frequencies?**

For the formation of distinct beats, frequencies of two sources of sound should be nearly equal i.e. difference in frequencies of two sources must be small, say less than 10. This can be explained in terms of the property of persistence of hearting. The impression of a sound heard by our ears persists on our mind for 1/10th of a second. If another sound is heard before (1/10) second passes, the impressions of the two sounds mix up and our mind cannot distinguish between the two .

In order to hear distinct beats, time interval between two successive beats must be greater than 1/10 second. Therefore, frequency of beats must be less than 10, i.e. number of beats/secs, which is equal to difference in frequencies of two sources must be less than 10. Hence the two sources should be of nearly equal frequencies.

Formation of Beats

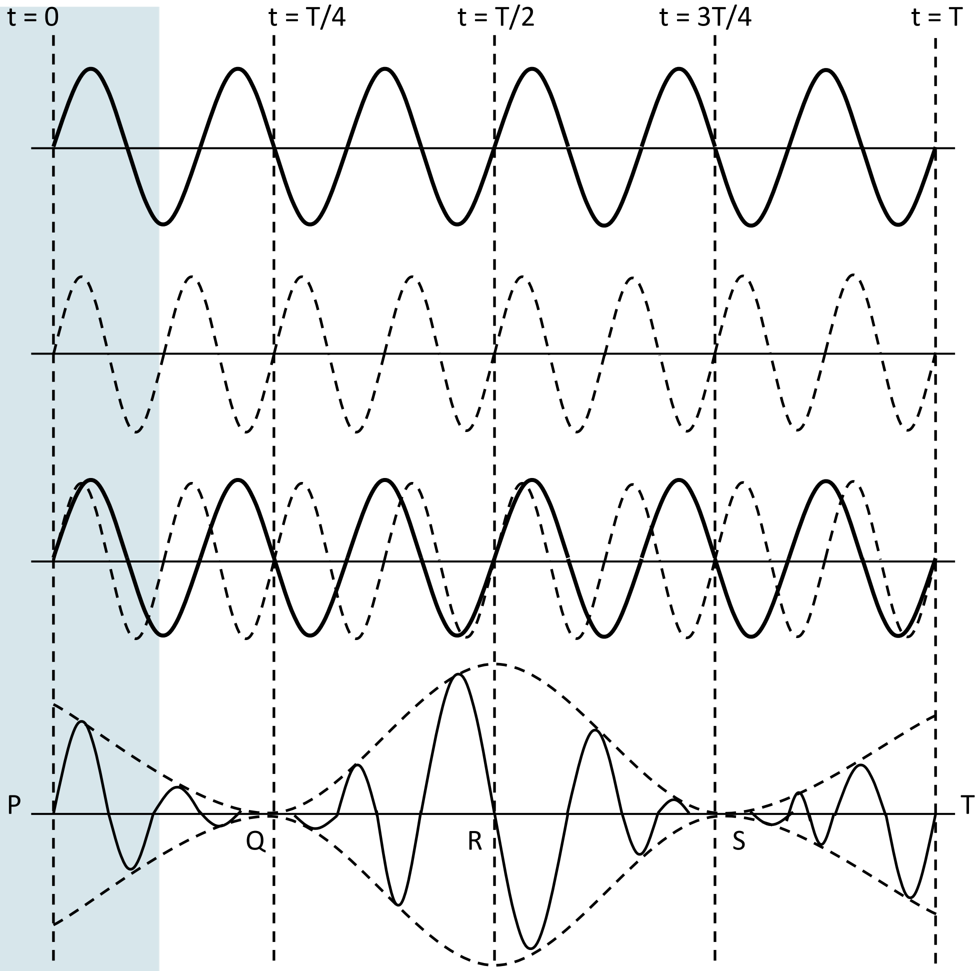
(a) Graphical method

Suppose we have two tuning forks A and B. Let the frequencies of fork A be 6 and frequency of fork B be 8. Let the waves of compression and rarefaction given by the forks A and B be represented by curves (a) and (b) respectively in Fig. 27. In these curves, a crest represents a compression and a trough represents a rarefaction.

Fig. 27, shows super-impose of the two waves from forks A and B and in Fig. 27, we have represented the resultant wave according to the principle of superposition.

(i) In t = ¼ sec.

A completes 6/4 = 1½ vibrations, consisting of compression, rarefaction and a compression. B completes 8/4 = 2 vibrations consisting of compression, rarefaction; compression and rarefaction. Thus a rarefaction due to A would fall on compression due to B, Fig. 27. The resultant amplitude would become minimum and hence intensity of sound would be minimum at Q, Fig. 27.



[Fig. 27]

In t = ½ second

A completes 6/2 = 3 vibrations, consisting of compression rarefaction; compression, rarefaction; compression and rarefaction. B completes 8/2 = 4 vibrations, consisting of compression, rarefaction; compression, rarefaction; compressions, rarefaction; compression and rarefaction. Thus, compression due to A would fall on compression due to B. The resultant amplitude would become maximum and hence resultant intensity of sound would be maximum at R, Fig. 27.

Thus, one beat is formed in ½ second between P and R. Similarly, another beat is formed in the next ½ second between R and T. Hence number of beats per second is equal to two, which is also the difference in frequencies of the two forks A and B.

(b) Analytical Method

Let us consider two wave trains of equal amplitude 'a' and slightly different frequencies v_1 and v_2 travelling in a medium in the same direction. If we count time from the instant the two sound waves are in the same phase, then the displacement y_1, y_2 at time t due to the two waves are given by

$$y_1 = r \sin \omega_1 t$$

$$= r \sin 2\pi v_1 t \qquad \dots (71)$$

$$y_2 = r \sin \omega_2 t = r \sin 2\pi v_2 t \qquad \dots (72)$$

According to superposition principle, the resultant displacement y at the same time t is

$$y = y_1 + y_2 = r \sin 2\pi v_1 t + r \sin 2\pi v_2 t$$

$$y = r [\sin 2\pi v_1 t + \sin 2\pi v_2 t] \qquad \dots (73)$$

Using, $\sin C + \sin D = 2 \cos \frac{C - D}{2} \sin \frac{C + D}{2}$,

We get

$$y = 2 r \cos \pi (v_1 + v_2) t \sin \pi (v_1 + v_2) t$$

$$y = A \sin \pi (v_1 + v_2) t \qquad \dots (74)$$

Where $A = 2 r \cos \pi (v_1 - v_2) t \qquad \dots (75)$

represents the amplitude of the resultant wave given by eqn. (74). Clearly, the resultant amplitude A changes with time. The amplitude A will be maximum, when

$$\cos \pi (v_1 - v_2) t = \max = \pm 1 = \text{con } k\pi$$

$$\therefore \quad \pi (v_1 - v_2) t = k\pi,$$

Where k = 0, 1, 2 …. or $\quad t = \frac{k}{(v_1 - v_2)}$

Hence the amplitude of resultant wave and therefore, resultant intensity of sound will be maximum at times

$$t = 0, \frac{1}{(v_1 - v_2)}, \frac{2}{(v_1 - v_2)}, \frac{3}{(v_1 - v_2)} \ldots$$

Time interval between two successive maxima of sound = $\frac{1}{(v_1 - v_2)} - 0 = \frac{1}{(v_1 - v_2)}$

i.e., frequency of maxima = $(v_1 - v_2)$ ... (76)

Again, ***the amplitude A will be minimum***, when

From (75), $\cos \pi (v_1 - v_2) t$ = minimum = 0

$\therefore \cos \pi (v_1 - v_2) t = \cos (2k + 1) \pi/2$,

Where $k = 0, 1, 2, 3 \ldots$

Or $\pi (v_1 - v_2) t = (2k + 1) \pi/2$

Or $t = \frac{(2k + 1)}{2 (v_1 - v_2)}$ ... (77)

Hence the amplitude of resultant displacement and therefore, resultant intensity of sound will be minimum at times.

- All oscillatory motions are periodic motions but all periodic motions are not oscillatory.