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LINEAR PROGRAMMING

BASIC CONCEPTS

1. **Definition:** Linear programming (LP) is an optimisation technique in which a linear function is optimised (*i.e.*, minimised or maximised) subject to certain constraints which are in the form of linear inequalities or/and equations. The function to be optimised is called *objective function*.
2. **Applications of Linear Programming:** Linear programming is used in determining optimum combination of several variables subject to certain constraints or restrictions.
3. **Formation of Linear Programming Problem (LPP):** The basic problem in the formulation of a linear programming problem is to set-up some mathematical model. This can be done by asking the following questions:
   (*a*) What are the unknowns (variables)?
   (*b*) What is the objective?
   (*c*) What are the restrictions?

   For this, let $x_1, x_2, x_3, \dots\dots\dots x_n$ be the variables. Let the objective function to be optimized (*i.e.*, minimised or maximised) be given by Z.

   (*i*) $Z = c_1 x_1 + c_2 x_2 + \dots\dots + c_n x_n$, where $c_i x_i$ $(i = 1, 2, \dots\dots\dots n)$ are constraints.

   (*ii*) Let there be mn constants and let a be a set of constants such that

   $$a_{11} x_1 + a_{12} x_2 + \dots\dots + a_{1n} x_n \ (\le, = \text{or} \ge)\ b_1$$
   $$a_{21} x_1 + a_{22} x_2 + \dots\dots + a_{2n} x_n \ (\le, = \text{or} \ge)\ b_2$$
   $$\dots\dots \quad \dots\dots \quad \dots\dots$$
   $$\dots\dots \quad \dots\dots \quad \dots\dots$$
   $$a_{m1} x_1 + a_{m2} x_2 + \dots\dots + a_{mn} x_n \ (\le, = \text{or} \ge)\ b_m$$

   (*iii*) Finally, let $x_1 \ge 0, x_2 \ge 0, \dots\dots\dots x_n \ge 0$, called non-negative constraints.

   The problem of determining the values of $x_1, x_2, \dots\dots\dots, x_n$ which makes Z, a minimum or maximum and which satisfies (*ii*) and (*iii*) is called the *general linear programming problem.*
4. **General LPP:**
   (*a*) **Decision variables:** The variables $x_1, x_2, x_3, \dots\dots, x_n$ whose values are to be decided, are called *decision variables.*

(*b*) **Objective function:** The linear function $Z = c_1 x_1 + c_2 x_2 + \ldots\ldots + c_n x_n$ which is to be optimized (maximised or minimised) is called the *objective function or preference function* of the general linear programming problem.

(*c*) **Structural constraints:** The inequalities given in (*ii*), are called the *structural constraints* of the general linear programming problem. The structural constraints are generally in the form of inequalities of ≥ type or ≤ type, but occasionally, a structural constraint may be in the form of an equation.

(*d*) **Non-negative constraints:** The set of inequalities (*iii*) is usually known as the set of *non-negative constraints* of the general LPP. These constraints imply that the variables $x_1, x_2, \ldots, x_n$ cannot take negative values.

(*e*) **Feasible solution:** Any solution of a general LPP which satisfies all the constraints, structural and non-negative, of the problem, is called a *feasible solution* to the general LPP.

(*f*) **Optimum solution:** Any feasible solution which optimizes (*i.e.*, minimises or maximises) the objective function of the LPP is called *optimum solution*.

5. **Requirements for Mathematical Formulation of LPP:** Before getting the mathematical form of a linear programming problem, it is important to recognize the problem which can be handled by linear programming problem. For the formulation of a linear programming problem, the problem must satisfy the following requirements:

(*i*) There must be an objective to minimise or maximise something. The objective must be capable of being clearly defined mathematically as a linear function.

(*ii*) There must be alternative sources of action so that the problem of selecting the best course of actions may arise.

(*iii*) The resources must be in economically quantifiable limited supply. This gives the constraints to LPP.

(*iv*) The constraints (restrictions) must be capable of being expressed in the form of linear equations or inequalities.

6. **Solving Linear Programming Problem:** To solve linear programming problems, *Corner Point Method* is adopted. Under this method following steps are performed:

Step I. At first, feasible region is obtained by plotting the graph of given linear constraints and its corner points are obtained by solving the two equations of the lines intersecting at that point.

Step II. The value of objective function $Z = ax + by$ is obtained for each corner point by putting its x and y-coordinate in place of x and y in $Z = ax + by$. Let M and m be largest and smallest value of Z respectively.

Case I: If the feasible region is bounded, then M and m are the maximum and minimum values of Z.

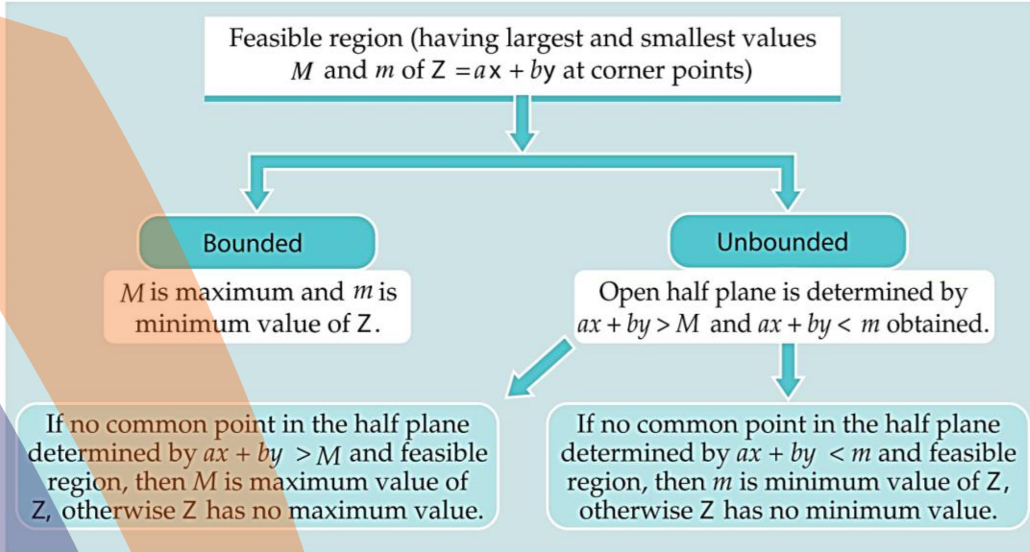
Case II: If the feasible region is unbounded, then we proceed as follows:

Step III. The open half plane determined by $ax + by > M$ and $ax + by < m$ are obtained.

Case I: If there is no common point in the half plane determined by $ax + by > M$ and feasible region, then M is maximum value of Z, otherwise Z has no maximum value.

Case II: If there is no common point in the half plane determined by $ax + by < m$ and feasible region, then m is minimum value of Z, otherwise Z has no minimum value.

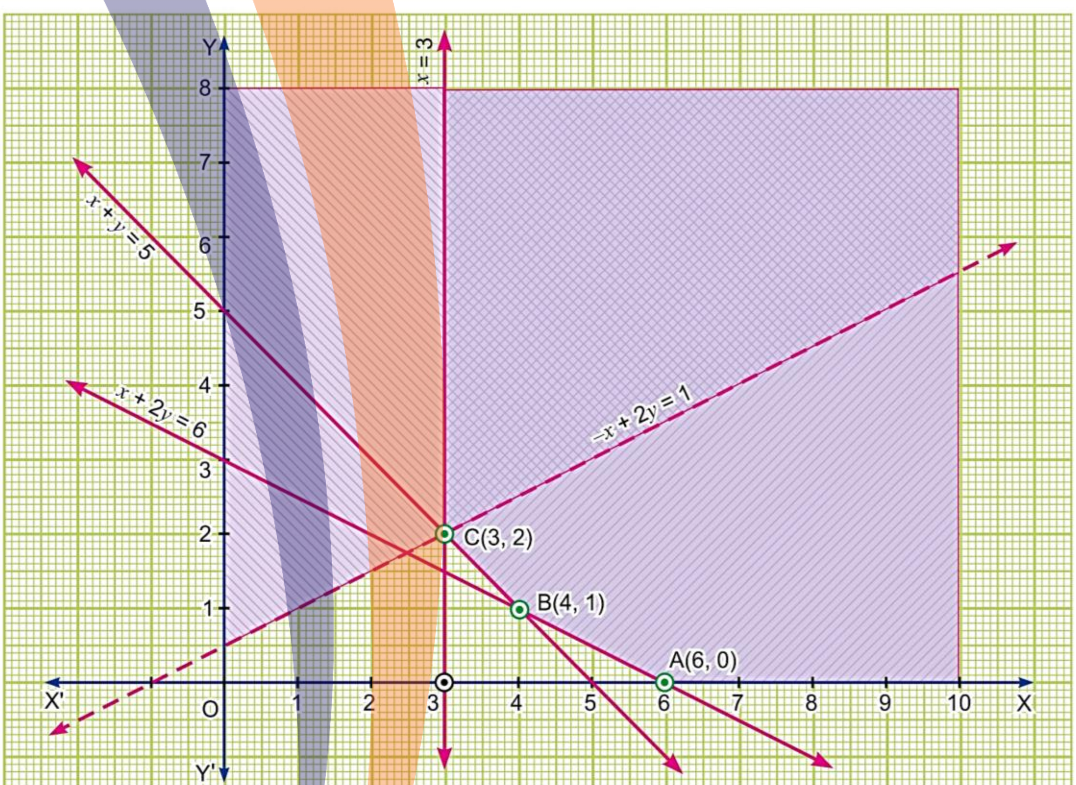
Above facts can be represented by arrow diagram as



Selected NCERT Questions

1. Maximize $Z = -x + 2y$
Subject to $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

Sol. Let us graph the given inequalities



The feasible region is shown shaded which is unbounded.

The coordinates of the corner points of the feasible region are $A(6, 0)$, $B(4, 1)$ and $C(3, 2)$.

Let us evaluate the objective function.

Corner Points	Objective Function $Z = -x + 2y$
A (6, 0)	$Z = -6 + 2 \times 0 = -6 + 0 = -6$
B (4, 1)	$Z = -4 + 2 \times 1 = -4 + 2 = -2$
C (3, 2)	$Z = -3 + 2 \times 2 = -3 + 4 = 1$ ← Maximum

We see that maximum value of Z at (3, 2) is 1.

Since the region is unbounded, so 1 may or may not be the maximum value of Z.

First draw the graph of the inequality $-x + 2y > 1$.

$-x + 2y > 1$ is away from origin.

Since the open half plane of $-x + 2y > 1$ has points in common with the feasible region.

Thus, Z has no maximum value.

2. *A* factory manufactures two types of screws, *A* and *B*. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws *A*, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws *B*. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws *A* at a profit of ₹7 and screws *B* at a profit of ₹10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

Sol. Let x be the number of packages of screws A, y be the number of packages of screws B produced in a day and Z be the total profit of the manufacture in a day.

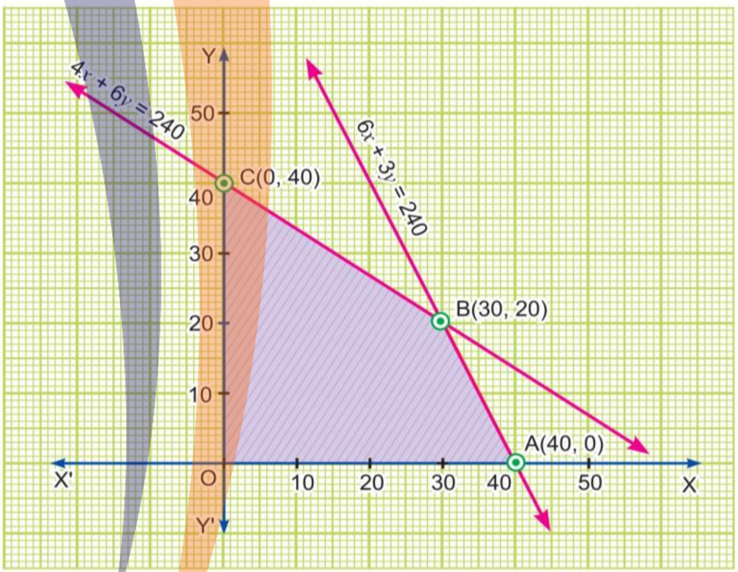
	Automatic Machine	Hand Operated Machine	Profit
Package of Screws A	4 minutes	6 minutes	₹ 7
Package of Screws B	6 minutes	3 minutes	₹ 10
Time Available	4 hours	4 hours	

Thus, the mathematical formulation of the given LPP is

Maximize $Z = 7x + 10y$

Subject to $4x + 6y \le 240$, $6x + 3y \le 240$, $x, y \ge 0$.

Let us draw the graph for system of inequalities representing constraints.



The feasible region is shown (shaded) in figure, which is bounded.

The coordinates of the corner points of the feasible region are $O(0, 0)$, $A(40, 0)$, $B(30, 20)$ and $C(0, 40)$.

Let us evaluate the objective function.

Corner Points	Objective Function $Z = 7x + 10y$
O (0, 0)	$Z = 7 \times 0 + 10 \times 0 = 0$
A (40, 0)	$Z = 7 \times 40 + 10 \times 0 = 280 + 0 = 280$
B (30, 20)	$Z = 7 \times 30 + 10 \times 20 = 210 + 200 = 410$ ← Maximum
C (0, 40)	$Z = 7 \times 0 + 10 \times 40 = 0 + 400 = 400$

Thus, Z is maximum at (30, 20) and maximum value = 410

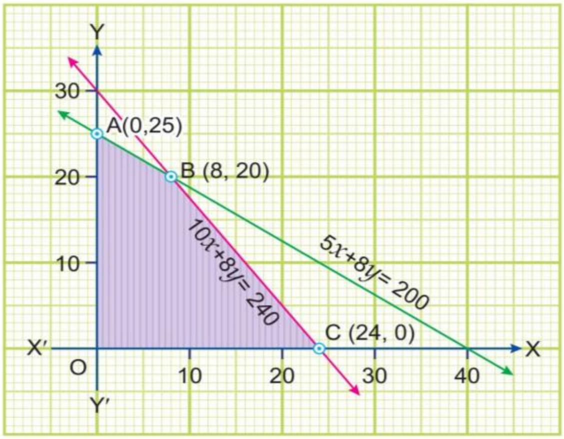
∴ Maximum profit = ₹410 when 30 packages of screws A and 20 packages of screws B are produced in a day.

3. A factory makes two types of items A and B, made of plywood. One piece of item A requires 5 minutes for cutting and 10 minutes for assembling. One piece of item B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The profit on one piece of item A is ₹5 and that on item B is ₹6. How many pieces of each type should the factory make so as to maximise profit? Make it as an LPP and solve it graphically. *[CBSE (F) 2010]*

Sol. Let the factory makes x pieces of item A and y pieces of item B.

Time required by item A (one piece)

cutting = 5 minutes, assembling = 10 minutes



Time required by item B (one piece)

cutting = 8 minutes, assembling = 8 minutes

Total time for

cutting = 3 hours and 20 minutes and

assembling = 4 hours

Profit on one piece

For item A = ₹5, item B = ₹6

Thus, our problem is maximized as

$Z = 5x + 6y$ ...(*i*)

Subject to constraints:

$x \geq 0, y \geq 0$ ...(*ii*)

$5x + 8y \leq 200$ ...(*iii*)

$10x + 8y \leq 240$ ...(*iv*)

From figure, possible points for maximum value of Z are at $O(0, 0)$ $A(0, 25)$, $B(8, 20)$, $C(24, 0)$.

Corner Points	$Z = 5x + 6y$
O (0, 0)	0
A (0, 25)	150
B (8, 20)	40 + 120 = 160 ← Maximum
C (24, 0)	120

Hence, 8 pieces of item A and 20 pieces of item B produce maximum profit of ₹160.

4. **A merchant plans to sell two types of personal computers— a desktop model and a portable model that will cost ₹25000 and ₹40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹70 lakhs and if his profit on the desktop model is ₹4500 and on portable model is ₹5000.** **[*CBSE (AI) 2011*]**

Sol. Let the number of desktop models be x and the number of portable models be y.

Since, the total monthly demand of computers does not exceed 250. Then we have

$$x + y \le 250$$

Also, cost of one desktop computer is ₹ 25000 and one portable computer is ₹ 40000.

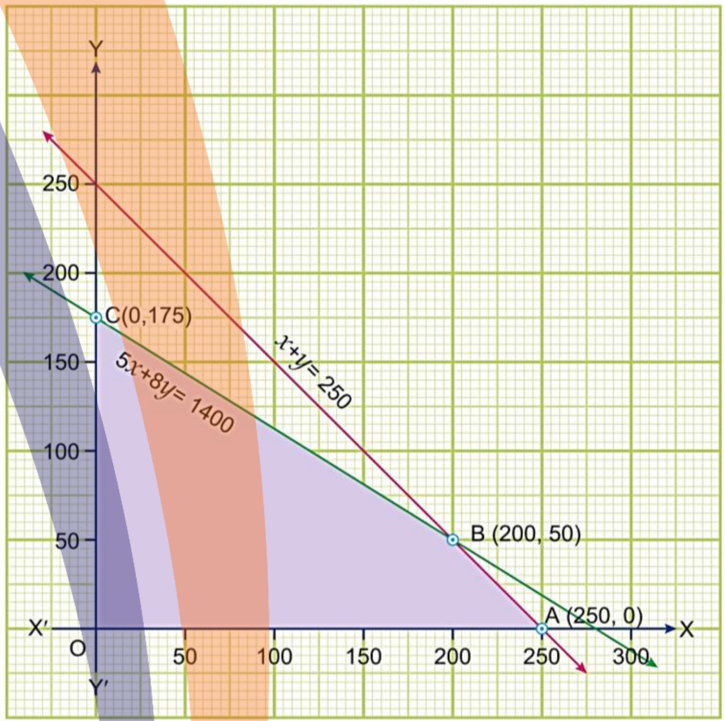
Therefore, the cost of x desktop and y portable computers = ₹ $(25000x + 40000y)$.

We have maximum investment = ₹ 7000000

$\Rightarrow \quad 25000x + 40000y \le 7000000 \quad \Rightarrow \quad 5x + 8y \le 1400$

Now, profit on x desktop and y portable computers is given by

$$Z = 4500x + 5000y$$



Hence our LPP is

Maximise, $Z = 4500x + 5000y$ $\quad\ldots(i)$

Subject to the constraints:

$x + y \le 250$ $\quad\ldots(ii)$

$5x + 8y \le 1400$ $\quad\ldots(iii)$

$x, y \ge 0$ $\quad\ldots(iv)$

Now, the shaded region $OABC$ is the feasible region which is bounded.

The coordinates of corner points are O (0, 0), A (250, 0), B (200, 50), C (0, 175) .

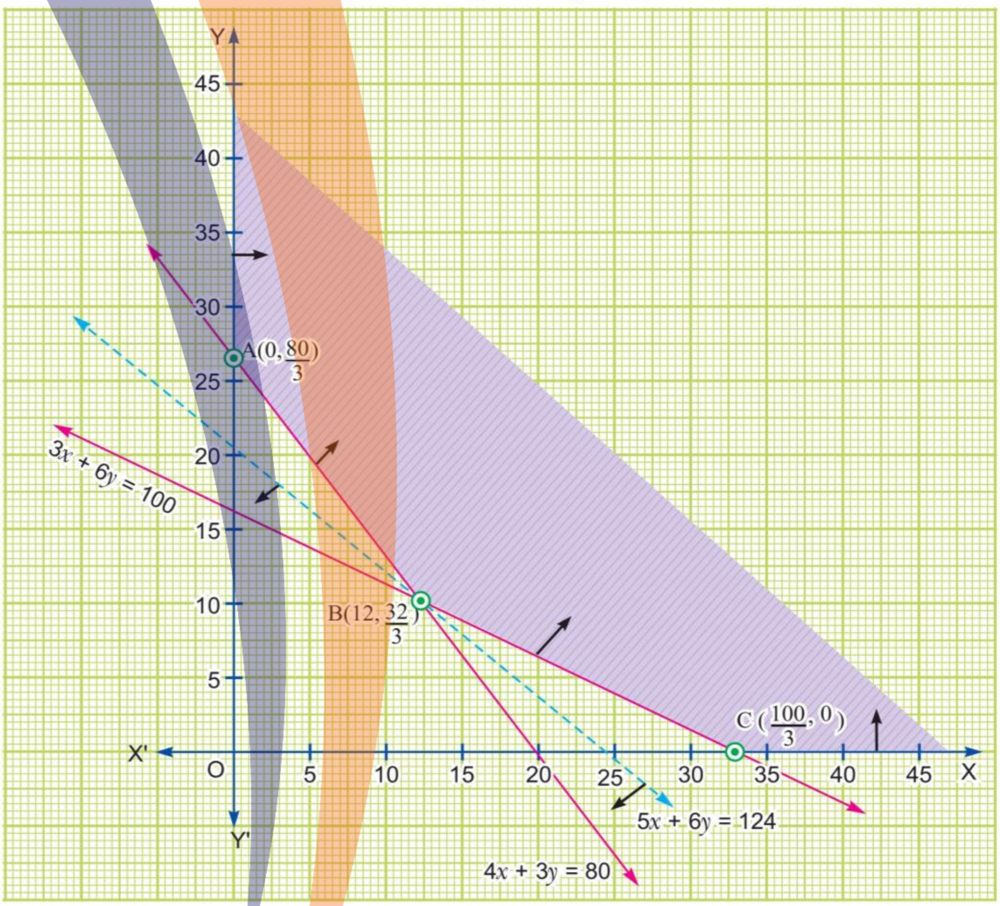
Now, we evaluate Z at each corner point

Corner Points	$Z = 4500x + 5000y$
O (0, 0)	0
A (250, 0)	1125000
B (200, 50)	1150000 ← Maximum
C (0, 175)	875000

Hence, the maximum profit of ₹1150000 is obtained, when he stocks 200 desktop and 50 portable computers.

5. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods, F_1 and F_2 are available costing ₹5 per unit and ₹6 per unit respectively. One unit of food F_1 contains 4 units of vitamin A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Formulate this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement. *[CBSE (South) 2016]*

Sol. Let x units of food F_1 and y units of food F_2 are required to be mixed.



Cost $Z = 5x + 6y$ ...(i) is to be minimised

Subject to following constraints.

$4x + 3y \geq 80$ ...(*ii*)

$3x + 6y \geq 100$ ...(*iii*)

$x \geq 0, y \geq 0$ ...(*iv*)

To solve the LPP graphically, the graph of inequations (*ii*), (*iii*) and (*iv*) is plotted as shown:

The shaded region in the graph is the feasible region of the problem. The corner points are

$$A\left(0, \frac{80}{3}\right), B\left(12, \frac{32}{3}\right), C\left(\frac{100}{3}, 0\right)$$

The value of Z at corner point is given as

Corner Points	$Z = 5x + 6y$	
$A\left(0, \frac{80}{3}\right)$	$5 \times 0 + 6 \times \frac{80}{3} = 160$	
$B\left(12, \frac{32}{3}\right)$	$12 \times 5 + 6 \times \frac{32}{3} = 124$	← Minimum
$C\left(\frac{100}{3}, 0\right)$	$5 \times \frac{100}{3} + 6 \times 0 = 166.6$	

Since, feasible region is unbounded therefore, a graph of $5x + 6y = 124$ is drawn which is shown in figure by dotted line.

Also, since there is no point common in feasible region and region $5x + 6y < 124$.

Hence, the minimum cost is ₹124 and 12 units of F_1 and $\frac{32}{3}$ units of F_2 are required.

6. **Two godowns *A* and *B* have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, *D*, *E* and *F* whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table :**

Transportation cost per quintal (in ₹)		
From/To	***A***	***B***
D	6	4
E	3	2
F	2.50	3

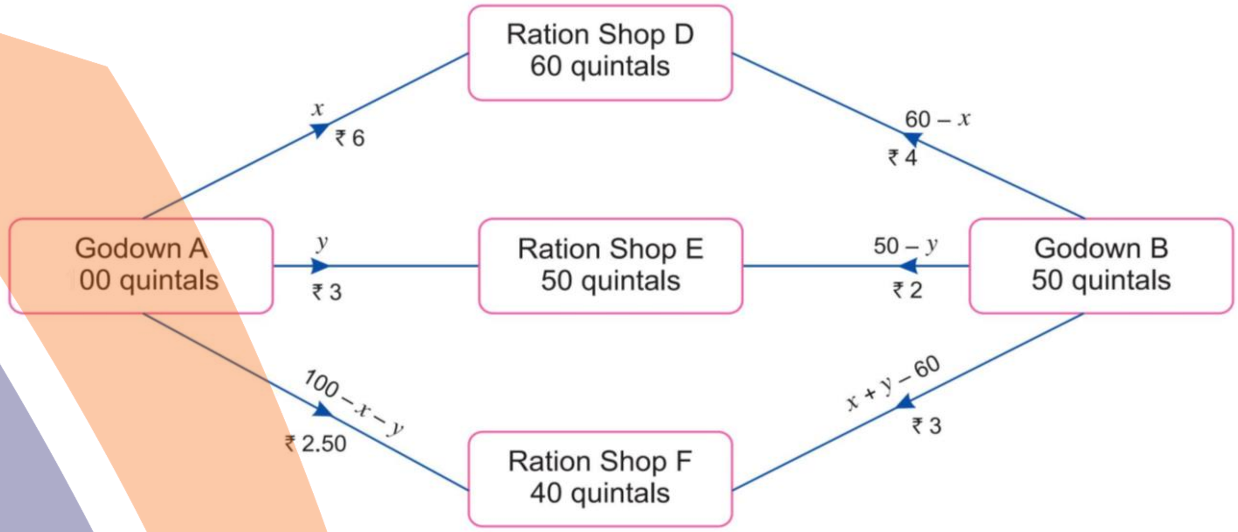
How should the supplier be transported in order that the transportation cost is minimum? What is the minimum cost? **[*HOTS*]**

Sol. Let the godown *A* transport *x* quintals of grain to ration shop *D*, *y* quintals of grain to ration shop *E*.

Since the total capacity of godown *A* is 100 quintals, so the remaining $(100 - x - y)$ quintals of grain can be transported to ration shop *F*.

Now the requirement of ration shop *D* is 60 quintals, out of which *x* quintals are transported from godown *A*. The remaining $(60 - x)$ quintals will be transported from godown *B*.

Also the requirement of ration shop *E* is 50 quintals, out of which *y* quintals are transported from godown *A*. The remaining $(50 - y)$ quintals will be transported from godown *B*.



Since the total capacity of godown B is 50 quintals, so the remaining

$50 - (60 - x + 50 - y) = (x + y - 60)$ quintals can be transported to ration shop F.

Let Z be the total transportation cost then

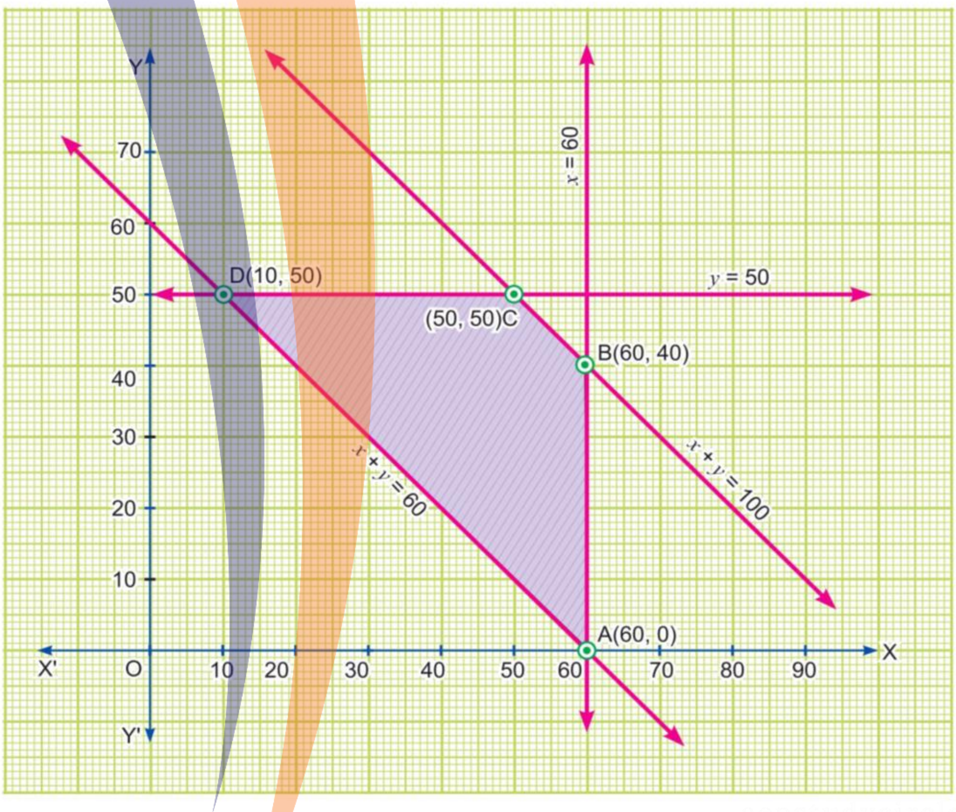
$$Z = 6x + 3y + 2.50(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$$
$$= 6x + 3y + 250 - 2.50x - 2.50y + 240 - 4x + 100 - 2y + 3x + 3y - 180$$
$$= 2.50x + 1.50y + 410$$

Thus, the mathematical formulation of the given LPP is

Minimize $Z = 2.50x + 1.50y + 410$

Subject to $x + y \le 100,\ x + y \ge 60,\ x \le 60,\ y \le 50,\ x, y \ge 0$

Let us graph the system of inequalities representing the constraints.



The feasible region *ABCD* is shown (shaded) in figure which is bounded.

The coordinates of the corner points of the feasible region are *A* (60, 0), *B* (60, 40), *C* (50, 50) and *D* (10, 50). Let us evaluate the objective function.

Corner Points	Objective Function $Z = 2.50x + 1.50y + 410$	
A (60, 0)	Z = 2.50 × 60 + 1.50 × 0 + 410 = 150 + 410 = 560	
B (60, 40)	Z = 2.50 × 60 + 1.50 × 40 + 410 = 150 + 60 + 410 = 620	← Maximum
C (50, 50)	Z = 2.50 × 50 + 1.50 × 50 + 410 =125 + 75 + 410 = 610	
D (10, 50)	Z = 2.50 × 10 + 1.50 × 50 + 410 = 25 + 75 + 410 = 510	← Minimum

Thus, *Z* is minimum at (10, 50) and minimum value = 510.

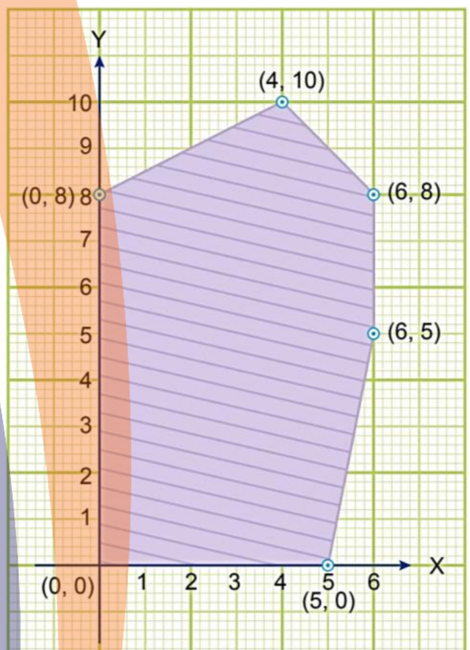
So, the minimum transportation cost is ₹510 when 10 quintals, 50 quintals and 40 quintals are transported from godown *A* and 50 quintals 0 quintal and 0 quintal are transported from godown *B* to ration shops *D*, *E* and *F* respectively.

Multiple Choice Questions [1 mark]

Choose and write the correct option in the following questions.

1. The feasible region for an LPP is shown below: [*NCERT Exemplar, CBSE 2020 (65/3/1)*]

Let $Z = 3x - 4y$ be the objective function. Minimum of *Z* occurs at



(*a*) (0, 0) (*b*) (0, 8) (*c*) (5, 0) (*d*) (4, 10)

2. In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points of which Z_{max} occurs is [*CBSE 2020 (65/4/1)*]

(*a*) 0 (*b*) 2 (*c*) finite (*d*) infinite

3. **Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is**

   (a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

4. **The optimal value of the objective function is attained at the points**

   (a) given by intersection of inequation with y-axis only.

   (b) given by intersection of inequation with x-axis only.

   (c) given by corner points of the feasible region.

   (d) none of these

Answers

1. (b) 2. (d) 3. (b) 4. (c)

Solutions of Selected Multiple Choice Questions

1. Given objective function $Z = 3x - 4y$
   on putting the corner points, we get
   $Z_{min} = -32$ at (0, 8)

3. At (3, 0), $Z_{min} = 3p + q \times 0 = 3p$
   and, at (1, 1), $Z_{min} = p \times 1 + q + 1 = p + q$
   $\therefore\ 3p = p + q$
   $\Rightarrow\ 2p = q\ \Rightarrow\ p = \frac{q}{2}$

Fill in the Blanks [1 mark]

1. The corner points of the feasible region of an LPP are (0, 0), (0, 8), (2, 7), (5, 4) and (6, 0). The maximum profit $P = 3x + 2y$ occurs at the point ____________. *[CBSE 2020, (65/2/1)]*

2. The common region determined by all the linear constraints of a LPP is called the ____________ region.

3. If the feasible region for a LPP is ____________, then the optimal value of the objective function $Z = ax + by$ may or may not exist. *[NCERT Exemplar]*

4. The feasible region for an LPP is always a ____________ polygon.

Answers

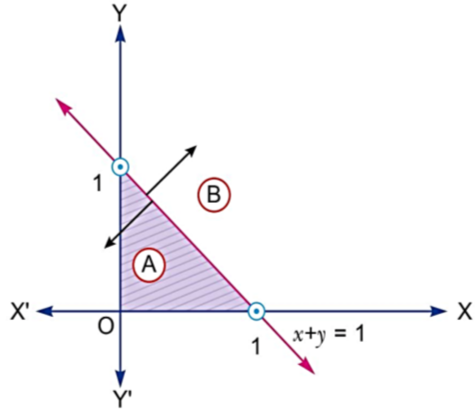
1. (5, 4) 2. feasible 3. unbounded 4. convex

Solutions of Selected Fill in the Blanks

1. We have,
   $P = 3x + 2y$
   $\therefore$ At point (5, 4)
   $P = 3 \times 5 + 2 \times 4 = 15 + 8 = 23$ will be maximum profit.

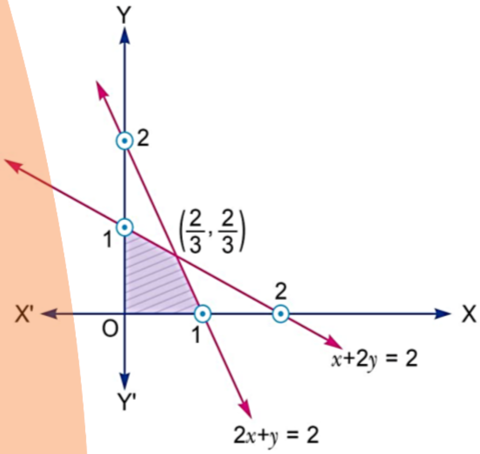
Very Short Answer Questions [1 mark]

1. **In figure, which half plane (A) or (B) is the solution of $x + y > 1$? Justify your answer.**



Sol. Half plane B because (0, 0) does not satisfy $x + y > 1$.

2. **What is the maximum value of objective function $Z = 3x + y$ under given feasible region?**



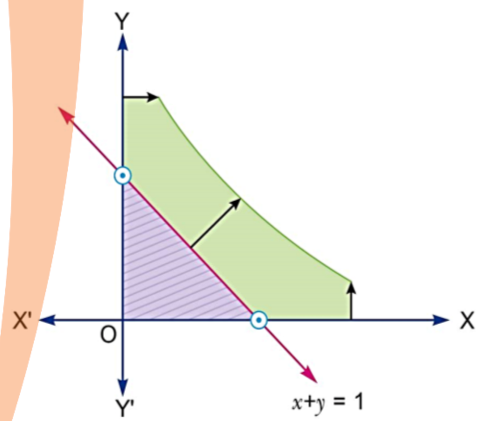
Sol. 3,

$\because$ $Z = 3x + y$ attains maximum value at (1, 0).

$\therefore$ $Z = 3 \times 1 + 0 = 3$

3. **Is feasible region represented by $x + y \geq 1, x \geq 0, y \geq 0$ bounded? Justify your answer.**

Sol. No, feasible region obtained is unbounded as shown in figure.



Short Answer Questions-I [2 marks]

1. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹300. Formulate an LPP for finding how many of each should be produced daily to maximise the profit. It is being given that at least one of each must be produced. [*CBSE Delhi 2017*]

Sol. Let x and y be the number of necklaces and bracelets manufactured by small firm per day. If P be the profit, then objective function is given by

$P = 100x + 300y$ which is to be maximised under the constraints

$$x + y \leq 24$$
$$\frac{1}{2}x + y \leq 16$$
$$x \geq 1, y \geq 1$$

2. Two tailors, A and B, earn ₹300 and ₹400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP. [*CBSE (AI) 2017*]

Sol. Let A and B work for x and y days respectively.

Let Z be the labour cost.

$$Z = 300x + 400y$$

Subject to constraints

$$6x + 10y \geq 60$$
$$4x + 4y \geq 32$$
$$x, y \geq 0$$

3. A company produces two types of goods A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can produce a maximum of 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹40 and that of type B ₹50, formulate LPP to maximize profit. [*CBSE (F) 2017*]

Sol. Let x and y be the number of goods A and goods B respectively. If P be the profit then $P = 40x + 50y$ which is to be maximised under constraints

$$3x + y \leq 9$$
$$x + 2y \leq 8$$
$$x \geq 0, y \geq 0$$

4. A firm has to transport atleast 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ₹400 and each small van is ₹200. Not more than ₹3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost. [*CBSE Delhi (C) 2017*]

Sol. Let the number of large vans and small vans be x and y respectively.

Here transportation cost Z be objective function, then

$Z = 400x + 200y$, which is to be minimized under constraints

$$200x + 80y \geq 1200 \quad \Rightarrow \quad 5x + 2y \geq 30$$
$$400x + 200y \leq 3000 \quad \Rightarrow \quad 2x + y \leq 15$$
$$x \leq y,\ x \geq 0, y \geq 0$$

5. Minimise $Z = 13x - 15y$ subject to the constraints $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0$ and $y \geq 0$. *[NCERT Exemplar]*

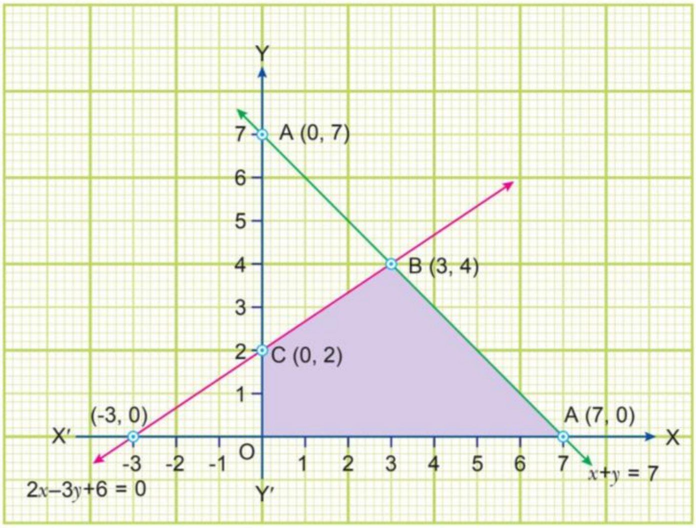
Sol. Minimise $Z = 13x - 15y$ ...(*i*)

Subject to the constraints

$x + y \leq 7$ ...(*ii*)

$2x - 3y + 6 \geq 0$ ...(*iii*)

$x \geq 0, y \geq 0$ ...(*iv*)

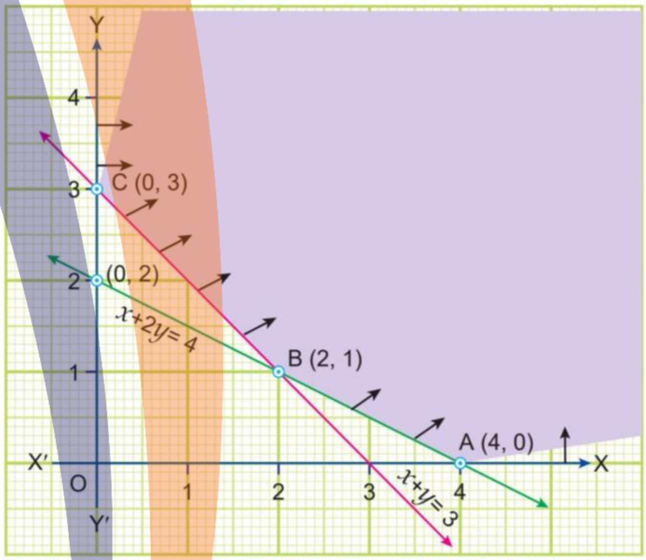


Shaded region shown as *OABC* is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively.

Corner Points	$Z = 13x - 15y$
O (0, 0)	0
A (7, 0)	91
B (3, 4)	–21
C (0, 2)	–30 ← Minimum

Hence, the minimum value of Z is –30 at (0, 2).

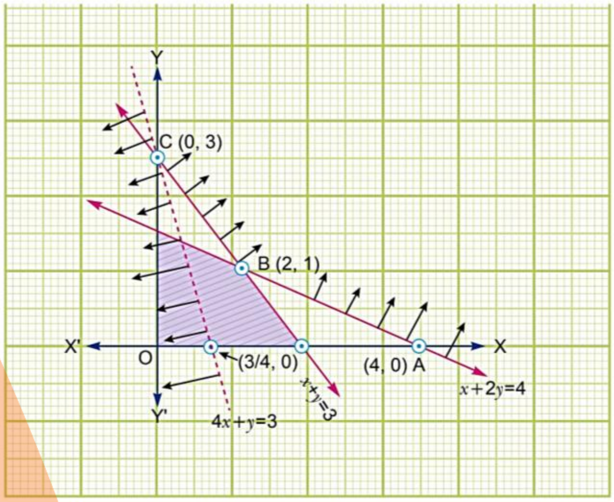
6. The feasible region for a LPP is shown in the following figure. Evaluate $Z = 4x + y$ at each of the corner points of this region. Find the minimum value of Z, if it exists. *[NCERT Exemplar]*



Sol. From the fig, it is clear that feasible region is unbounded with the corner points *A* (4, 0), *B* (2, 1) and *C* (0, 3). [$\because$ $x + 2y = 4$ and $x + y = 3 \Rightarrow y = 1$ and $x = 2$]

Also, we have $Z = 4x + y$

Corner Points	$Z = 4x + y$
A (4, 0)	16
B (2, 1)	9
C (0, 3)	3 ← Minimum



Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that, the region is unbounded, therefore 3 may or may not be the minimum value of Z.

To decide this issue, we graph the inequality $4x + y < 3$ and check wether the resulting open half plane has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph, it is clear that there is no point common with feasible region and hence, Z has minimum value 3 at (0, 3).

Short Answer Questions-II [3 marks]

1. Solve the following LPP graphically:

Minimise $Z = 5x + 7y$

Subject to the constraints

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x, y \geq 0$$

[CBSE (F) 2020, (65/3/1]

Sol. Given constraints are

$$2x + y \geq 8$$
$$x + 2y \geq 10$$

and $x, y \geq 0$

For the graph of $2x + y \geq 8$, we draw the graph of $2x + y = 8$

x	0	4
y	8	0

Now, checking for (0, 0) we have $2 \times 0 + 0 \geq 8 \quad \Rightarrow \quad 0 \geq 8$

$\therefore$ Origin (0, 0) does not satisfy $2x + y \geq 8$

$\therefore$ Region lies away from origin.

For the graph of $x + 2y \geq 10$, we draw the graph of $x + 2y = 10$

x	0	10
y	5	0

Now, checking for origin (0, 0), we have

$0 + 2 \times 0 \geq 10 \quad \Rightarrow \quad 0 \geq 10$

$\therefore$ Origin (0, 0) does not satisfy $x + 2y \geq 10$

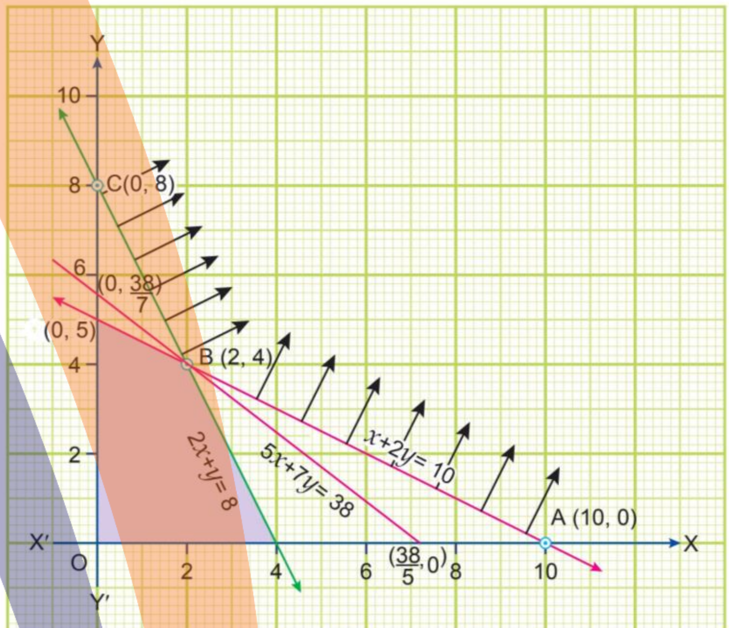
$\therefore$ Region lies away from origin.

Now $x, y \geq 0,$ it means region will lie in first quadrant.

On plotting graph of given inequalities (or constraints)

We get the region (shaded) with corner points

A (10, 0), B(2, 4) and C(0, 8).



Now, the value of Z is evaluated at corner points in the following table.

Corner Points	$Z = 5x + 7y$
A (10, 0)	50
B (2, 4)	38 ← Minimum
C (0, 8)	56

Since, feasible region is ubbounded. Therefore, we have to draw the graph of the inequality.

$5x + 7y < 38$

Since, the graph of this inequality does not have any point common.

So, the minimum value of Z is 38 at (2, 4).

Hence, $Z_{min} = 38$ at (2, 4).

2. Maximise $Z = 8x + 9y$ subject to the constraints given below :

$2x + 3y \leq 6;\ 3x - 2y \leq 6;\ y \leq 1;\ x, y \geq 0$ *[CBSE (F) 2015]*

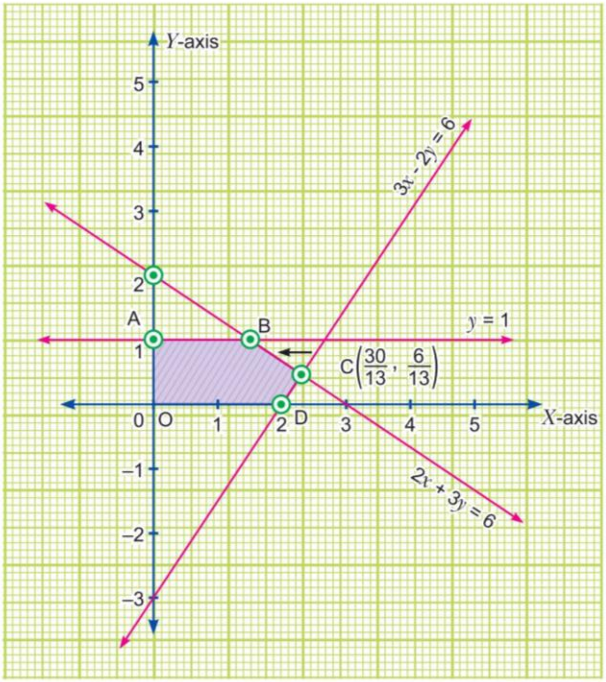
Sol. Given constraints are

$2x + 3y \leq 6$

$3x - 2y \leq 6$

$y \le 1$

$x, y \ge 0$



For graph of $2x + 3y \le 6$

We draw the graph of $2x + 3y = 6$

x	0	3
y	2	0

$2 \times 0 + 3 \times 0 \le 6 \Rightarrow (0,0)$ satisfy the constraints.

Hence, feasible region lie towards origin side of line.

For graph of $3x - 2y \le 6$

We draw the graph of line $3x - 2y = 6$.

x	0	2
y	–3	0

$3 \times 0 - 2 \times 0 \le 6$

$\Rightarrow$ Origin (0, 0) satisfy $3x - 2y = 6$.

Hence, feasible region lie towards origin side of line.

For graph of $y \le 1$

We draw the graph of line $y = 1$, which is parallel to x-axis and meet y-axis at 1.

$0 \le 1 \Rightarrow$ feasible region lie towards origin side of $y = 1$.

Also, $x \ge 0, y \ge 0$ says feasible region is in Ist quadrant.

Therefore, $OABCDO$ is the required feasible region, having corner point $O(0, 0)$, $A(0, 1)$ $B\left(\frac{3}{2}, 1\right)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$, $D(2, 0)$.

Here, feasible region is bounded. Now the value of objective function $Z = 8x + 9y$ is obtained as.

Corner Points	$Z = 8x + 9y$
$O(0, 0)$	0
$A(0, 1)$	9
$B\left(\frac{3}{2}, 1\right)$	21
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	22.6 ← Maximum
$D(2, 0)$	16

Z is maximum when $x = \frac{30}{13}$ and $y = \frac{6}{13}$.

3. Minimize and maximize $Z = 5x + 2y$ subject to the following constraints:

$x - 2y \le 2$, $3x + 2y \le 12$, $-3x + 2y \le 3$, $x \ge 0, y \ge 0$ *[CBSE Panchkula 2015]*

Sol. Here, objective function is

$Z = 5x + 2y$ ...(i)

Subject to the constraints :

$x - 2y \le 2$ ...(ii)

$3x + 2y \leq 12$ ...(*iii*)

$-3x + 2y \leq 3$ ...(*iv*)

$x \geq 0, y \geq 0$ ...(*v*)

Graph for $x - 2y \leq 2$

We draw graph of $x - 2y = 2$ as

x	0	2
y	–1	0

$0 - 2 \times 0 \leq 2$

[By putting $x = y = 0$ in the equation]

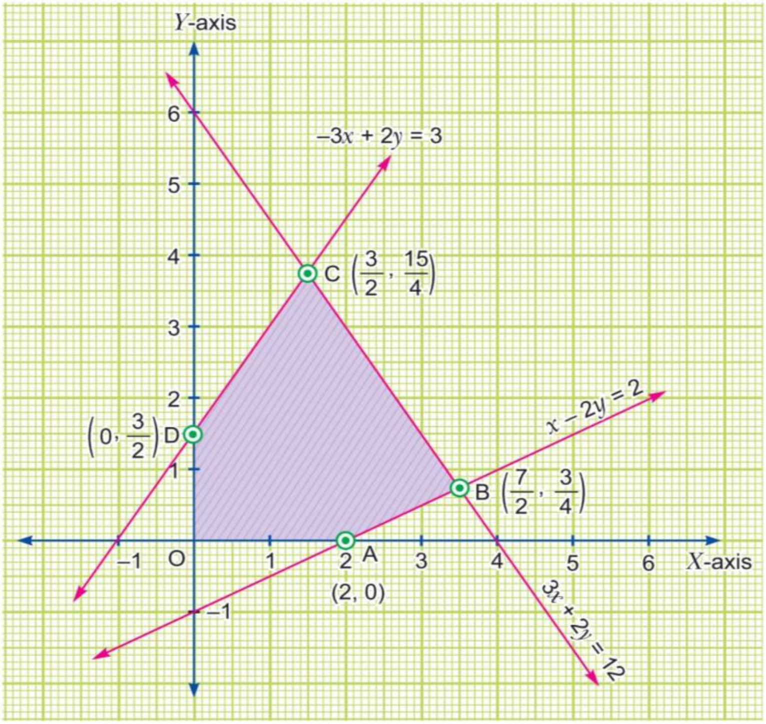
i.e., (0, 0) satisfy (*ii*) $\Rightarrow$ feasible region lie origin side of line $x - 2y = 2$.

Graph for $3x + 2y \leq 12$

We draw the graph of $3x + 2y = 12$.

x	0	4
y	6	0

$3 \times 0 + 2 \times 0 \leq 12$ [By putting $x = y = 0$ in the given equation]



i.e., (0, 0) satisfy (*iii*) $\Rightarrow$ feasible region lie origin side of line $3x + 2y = 12$.

Graph for $-3x + 2y \leq 3$

We draw the graph of $-3x + 2y \leq 3$

x	–1	0
y	0	1.5

$-3 \times 0 + 2 \times 0 \leq 3$ [By putting $x = y = 0$]

i.e., (0, 0) satisfy (*iv*) $\Rightarrow$ feasible region lie origin side of line $-3x + 2y = 3$.

$x \geq 0, y \geq 0 \Rightarrow$ feasible region is in Ist quadrant.

Now, we get shaded region having corner points *O*, *A*, *B*, *C* and *D* as feasible region.

The co-ordinates of *O*, *A*, *B*, *C* and *D* are $O(0, 0)$, $A(2, 0)$, $B\left(\frac{7}{2}, \frac{3}{4}\right)$, $C\left(\frac{3}{2}, \frac{15}{4}\right)$ and $D\left(0, \frac{3}{2}\right)$ respectively. Now, we evaluate Z at the corner points.

Corner Points	$Z = 5x + 2y$	
$O(0, 0)$	0	← Minimum
$A(2, 0)$	10	
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	19	← Maximum
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	15	
$D\left(0, \frac{3}{2}\right)$	3	

Hence, Z is minimum at $x = 0, y = 0$ and minimum value = 0

also Z is maximum at $x = \frac{7}{2}, y = \frac{3}{4}$ and maximum value = 19.

4. **A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type *A* require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit for type *A* souvenirs is ₹100 each and for type *B* souvenirs profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as LPP and then solve it graphically.** *[CBSE 2020 (65/5/1), 2019 (65/5/3)]*

Sol. Let x be the number of souvenirs of type A and y be the number of souvenirs of type B.

	Souvenirs $A(x)$	Souvenirs $B(y)$	Time
Cutting Machine (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$
Profits	100	120	

Maximize $Z = 100x + 120y$ ...(*i*)

Subject to $5x + 8y \le 200$ ...(*ii*)

$10x + 8y \le 240$ ...(*iii*)

i.e., $5x + 4y \le 120$ ...(*iii*)

$x, y \ge 0$ ...(*iv*)

Plotting the constraints

Feasible region is shaded region.

Value of $Z = 100\,x + 120\,y$

At (0, 0), $Z = 0$

At (0, 25) $Z = 3000$

At (24, 0) $Z = 2400$

At (8, 20) $Z = 3200$ ← Maximum

$\therefore$ Maximum profit is ₹3200 at point (8, 20).

So, 8 types A and 20 types B souvenirs should be made to maximise profit.

5. A small firm manufacturers gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

Sol. Total number of rings and chains manufactured per day = 24

Time taken in manufacturing ring = 1 hour

Time taken in manufacturing chain = 30 minutes

Time available per day = 16 hours

Maximum profit on ring = ₹300

Maximum profit on chain = ₹190

Let number of gold rings manufactured per day = x and chains manufactured per day = y

LPP is

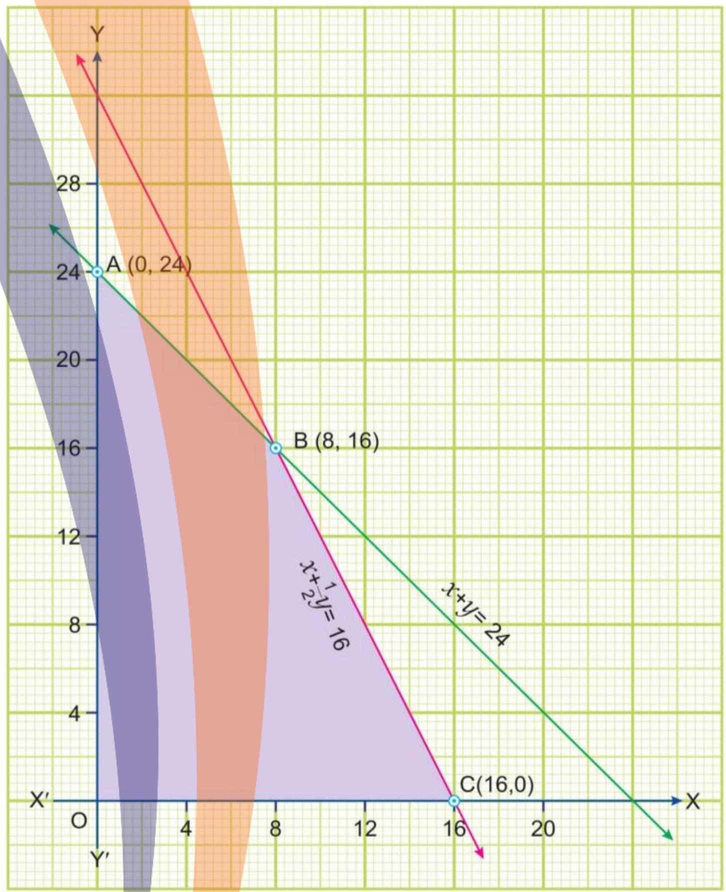
Maximize $Z = 300x + 190y$ ...(*i*)

Subject to constraints $x \geq 0, y \geq 0$ ...(*ii*)

$x + y \leq 24$ ...(*iii*)

$x + \frac{1}{2}y \leq 16$ ...(*iv*)

Possible points for maximum Z are $A(0, 24)$, $B(8, 16)$ and $C(16, 0)$.



Corner Points	$Z = 300x + 190y$
$A(0, 24)$	4560
$B(8, 16)$	5440 ← Maximum
$C(16, 0)$	4800

Z is maximum at (8, 16).

Hence, 8 gold rings and 16 chains must be manufactured per day.

6. A manufacturing company makes two types of teaching aids A and B of mathematics for class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30, respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B. How many pieces of type A and B should be manufactured per week to get a maximum profit? What is the maximum profit per week?

Sol. Let x and y be the number of pieces of type A and B manufactured per week respectively. If Z be the profit then,

Objective function, $Z = 80x + 120y$ ...(*i*)

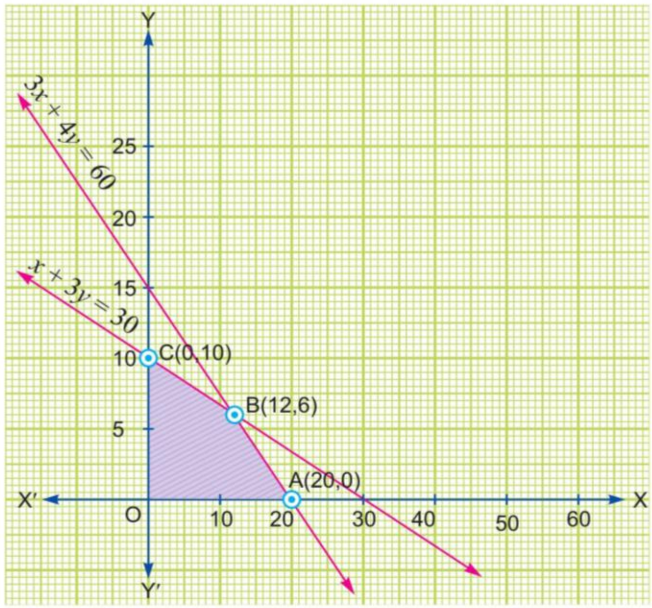
We have to maximize Z, subject to the constraints

$9x + 12y \le 180$

$\Rightarrow$ $3x + 4y \le 60$ ...(*ii*)

$x + 3y \le 30$ ...(*iii*)

$x \ge 0, y \ge 0$ ...(*iv*)



The graph of constraints are drawn and feasible region $OABC$ is obtained, which is bounded having corner points O (0, 0), A (20, 0), B (12, 6) and C (0, 10)

Now the value of objective function is obtained at corner points as

Corner points	$Z = 80x + 120y$	
O (0, 0)	0	
A (20, 0)	1600	
B (12, 6)	1680	← Maximum
C (0, 10)	1200	

Hence, the company will get the maximum profit of ₹1680 by making 12 pieces of type A and 6 pieces of type B of teaching aid.

7. The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients B_1 and B_2. B_1 costs ₹ 5 per kg and B_2 costs ₹ 8 per kg. Strength considerations dictate that the brick should contain not more than 4 kg of B_1 and minimum 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of brick satisfying the above conditions. Formulate this situation as an LPP and solve it graphically.

Sol. Let x kg of B_1 and y kg of B_2 are taken for making brick.

Here, $Z = 5x + 8y$ is the cost which is objective function and is to be maximised subjected to following constraints.

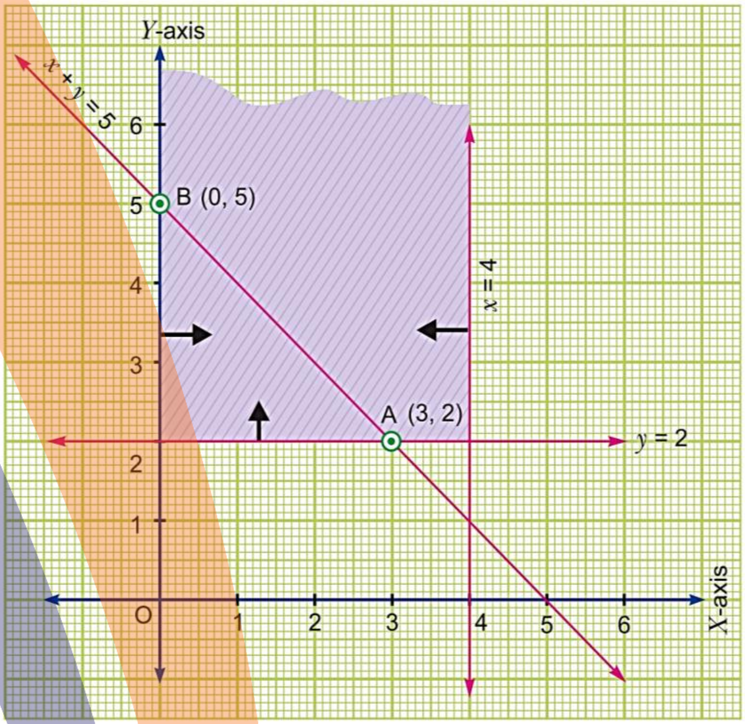
$x + y = 5$ ... (*i*)

$x \leq 4$ ... (*ii*)

$y \geq 2$ ... (*iii*)

$x \geq 0, y \geq 0$ ... (*iv*)

In this case, constraint (*i*) is a line passing through the feasible region determined by constraints (*ii*), (*iii*) and (*iv*).



Therefore, maximum or minimum value of objective function 'Z' exist on end points of line (constraint) (*i*) in feasible region *i.e.*, at A or B.

At $A\ (3, 2)$ $Z = 5 \times 3 + 8 \times 2 = 15 + 16 = 31$

At $B\ (0, 5)$ $Z = 5 \times 0 + 8 \times 5 = 0 + 40 = 40$

Hence, cost of brick is minimum when 3 kg of B_1 and 2 kg of B_2 are taken.

Long Answer Questions [5 marks]

1. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is ₹20 and ₹10 respectively, find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an LPP and solve graphically.

[CBSE Delhi 2011]

Sol. Let the number of tennis rackets and cricket bats manufactured by factory be x and y respectively.

Here, profit on x rackets and y bats is the objective function Z.

$Z = 20x + 10y$ ...(*i*)

We have to maximise Z subject to the constraints:

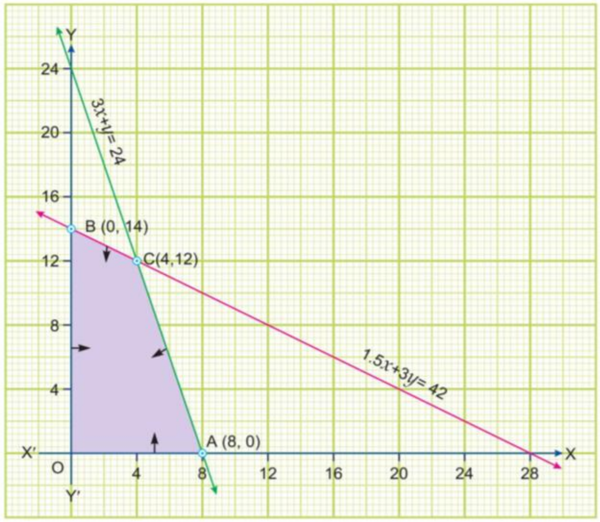
$1.5x + 3y \le 42$ ...(*ii*) [Constraint for machine hour]

$3x + y \le 24$ ...(*iii*) [Constraint for craft man's hour]

$x, y \ge 0$ ...(*iv*) [Non-negative constraints]

Graph of $x = 0$ and $y = 0$ is the y-axis and x-axis respectively.

$\therefore$ Graph of $x \ge, y \ge 0$ is the Ist quadrant.



Graph of $1.5x + 3y = 42$

x	0	28
y	14	0

$\therefore$ Graph for $1.5x + 3y \le 42$ is the part of Ist quadrant which contains the origin.

Graph of $3x + y = 24$

x	0	8
y	24	0

$\therefore$ Graph of $3x + y \le 24$ is the part of Ist quadrant in which origin lie.

Hence, shaded area *OACB* is the feasible region.

For coordinate of *C*, equation $1.5x + 3y = 42$ and $3x + y = 24$ are solved as

$$1.5x + 3y = 42 \quad \text{...}(v)$$

$$3x + y = 24 \quad \text{...}(vi)$$

$$2 \times (v) - (vi) \Rightarrow 3x + 6y = 84$$

$$\underline{{}_{-}3x \pm y = {}_{-}24}$$

$$5y = 60$$

$$\Rightarrow y = 12 \Rightarrow x = 4 \quad \text{(Substituting } y = 12 \text{ in } (vi))$$

Now, value of objective function *Z* at each corner of feasible region is

Corner Points	$Z = 20x + 10y$
O (0, 0)	0
A (8, 0)	$20 \times 8 + 10 \times 0 = 160$
B (0, 14)	$20 \times 0 + 10 \times 14 = 140$
C (4, 12)	$20 \times 4 + 10 \times 12 = 200$ ← Maximum

Therefore, maximum profit is ₹200, when factory make 4 tennis rackets and 12 cricket bats.

2. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5,760 to invest and has space for at the most 20 items. A fan costs him ₹360 and a sewing machine ₹240. He expects to sell a fan at a profit of ₹22 and a sewing machine for a profit of ₹18. Assuming that he can sell all the items that he buys, how should he invest his money to maximise his profit ? Solve it graphically. *[CBSE (AI) 2007, 2009, Delhi 2014]*

OR

A dealer in a rural area wishes to purchase some sewing machines. He has only ₹57,600 to invest and has space for at most 20 items. An electronic machine costs him ₹3,600 and a manually operated machine costs ₹2,400. He can sell an electronic machine at a profit of ₹220 and a manually operated machine at a profit of ₹180. Assuming that he can sell all the machines that he buys, how should he invest his money in order to maximise his profit? Make it as an LPP and solve it graphically. **[*CBSE Bhubaneshwar 2015*]**

Sol. Let the dealer purchases x fans and y sewing machines,then cost of x fans and y sewing machines is given by $360x + 240y$

$\therefore \quad 360x + 240y \leq 5,760 \quad \Rightarrow \quad 3x + 2y \leq 48$

As, he has space for at most 20 items,

$\therefore \quad x + y \leq 20$

Now, profit earned by the dealer on selling x fans and y sewing machines = $22x + 18y$

Hence, our LPP is to

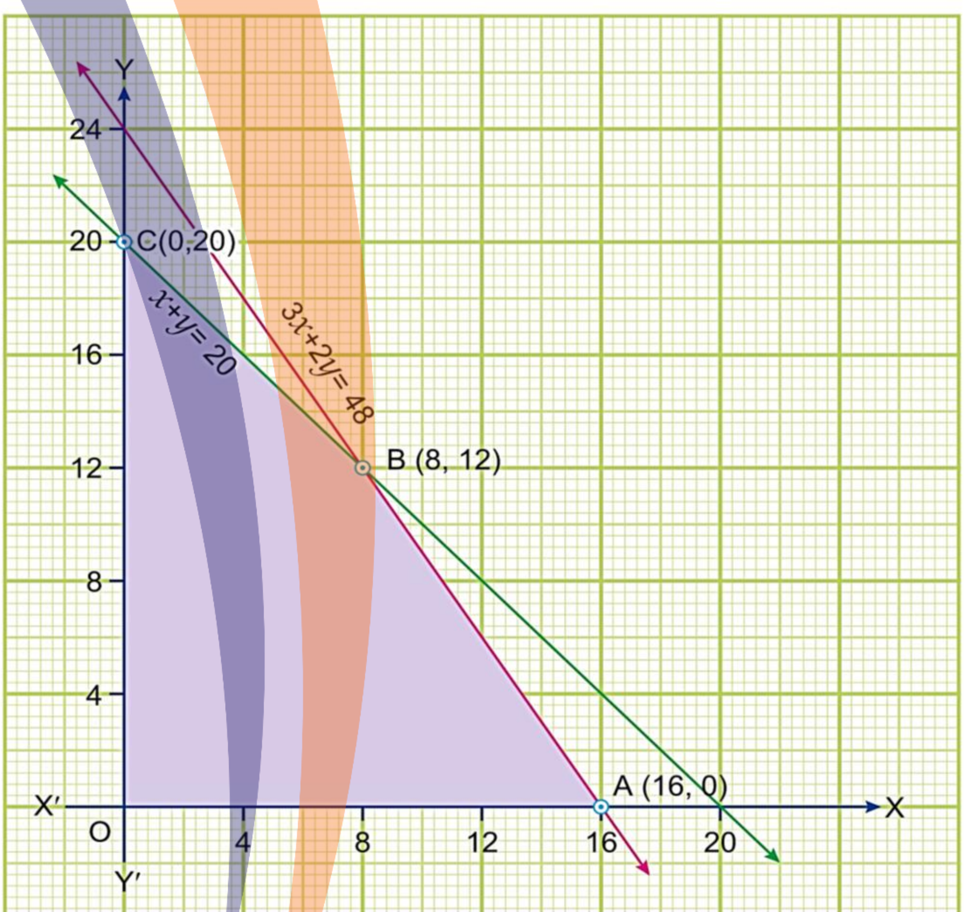
Maximise $Z = 22x + 18y$ ...(*i*)

Subject to the constraints:

$3x + 2y \leq 48$ ...(*ii*)

$x + y \leq 20$ ...(*iii*)

$x, y \geq 0$ ...(*iv*)



Let us evaluate, $Z = 22x + 18y$ at each corner point.

The region satisfying inequalities (*ii*) to (*iv*) is shown (shaded) in the figure.

Corner Points	$Z = 22x + 18y$
O (0, 0)	0
A (16, 0)	352
B (8, 12)	392 ← Maximum
C (0, 20)	360

Thus, maximum value of Z is 392 at B (8, 12).

Hence, the profit is maximum *i.e.*, ₹ 392 when he buys 8 fans and 12 sewing machines.

OR

Solve yourself as above solution. Here $Z = 220x + 180y$ is objective function.

3. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical *A* and 60 units of the chemical *B* go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier *S* had a packet of mix of 4 units of *A* and 2 units of *B* that costs ₹10. The supplier *T* has a packet of mix of 1 unit of *A* and 1 unit of *B* that costs ₹4. How many mix packets mix from *S* and *T* should the company purchase to honour the contract requirement and yet minimize cost? Make a LPP and solve graphically. [*CBSE (F) 2012*] [*HOTS*]

Sol. Let x and y units of packet of mixes be purchased from S and T respectively. If Z is total cost then

$$Z = 10x + 4y \quad ...(i)$$

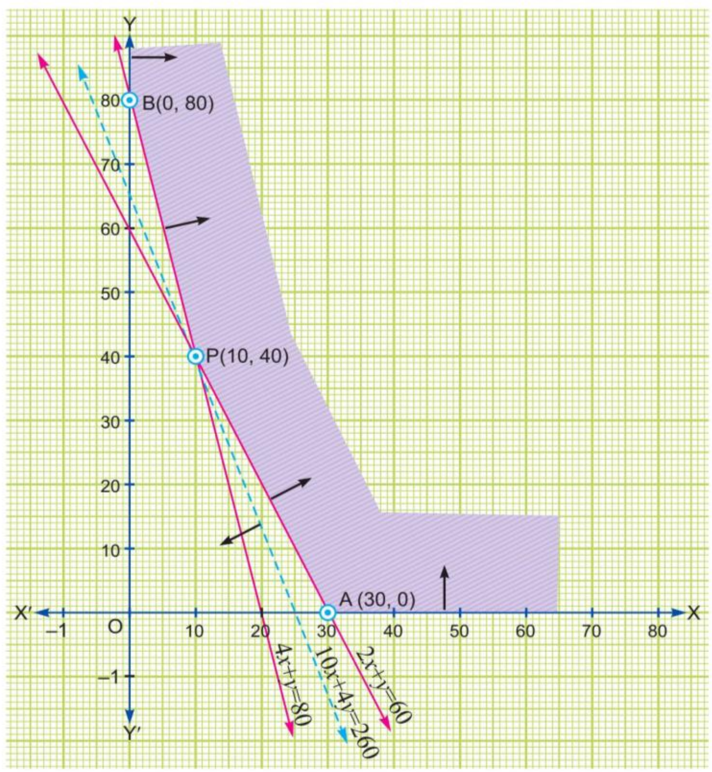
is objective function, which we have to minimize.

Here, constraints are:

$$4x + y \geq 80 \quad ...(ii)$$

$$2x + y \geq 60 \quad ...(iii)$$

Also, $x, y \geq 0 \quad ...(iv)$



On plotting graph of above constraints or inequalities (*ii*), (*iii*) and (*iv*), we get shaded region having corner point A, P, B as feasible region.

For coordinate of P.

Point of intersection of

$$2x + y = 60 \quad ...(v)$$

and $4x + y = 80 \quad ...(vi)$

$(v) - (vi)$

$\Rightarrow \quad 2x + y - 4x - y = 60 - 80$

$\Rightarrow \quad -2x = -20$

$\Rightarrow \quad x = 10$

$\Rightarrow \quad y = 40$

coordinate of $P \equiv (10, 40)$

Now the value of Z is evaluated at corner point in the following table

Corner Points	$Z = 10x + 4y$
A (30, 0)	300
P (10, 40)	260 ← Minimum
B (0, 80)	320

Since, feasible region is unbounded. Therefore we have to draw the graph of the inequality.

$10x + 4y < 260$ ...(*vii*)

Since, the graph of inequality (*vii*) does not have any point common.

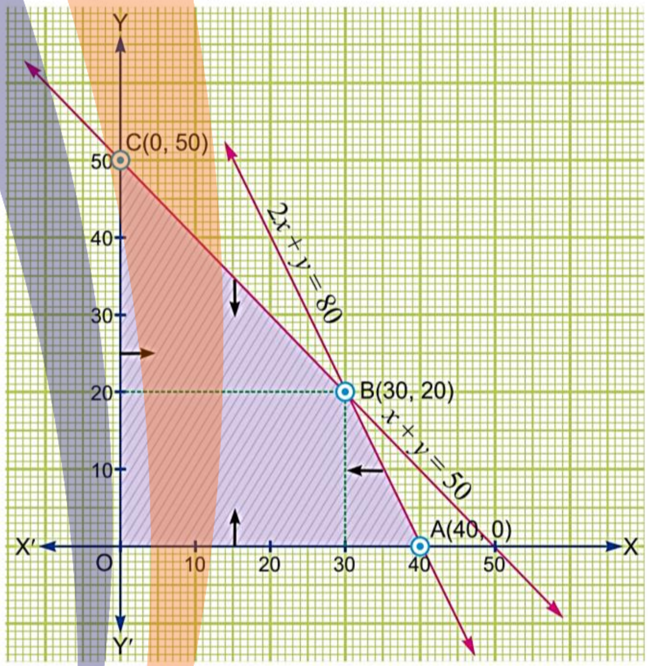
So, the minimum value of Z is 260 at (10, 40).

i.e., minimum cost of each bottle is ₹260 if the company purchases 10 packets of mixes from *S* and 40 packets of mixes from supplier *T*.

4. A cooperative society of farmers has 50 hectares of land to grow two crops *A* and *B*. The profits from crops *A* and *B* per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops *A* and *B* at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. [*CBSE Delhi 2013*]

Sol. Let x and y hectares of land be allocated to crop *A* and *B* respectively. If Z is the profit then

$Z = 10500x + 9000y$ ...(*i*)



We have to maximize Z subject to the constraints:

$x + y \le 50$ ...(*ii*)

$20x + 10y \le 800 \Rightarrow 2x + y \le 80$ ...(*iii*)

$x \ge 0, y \ge 0$ ...(*iv*)

Table for $x + y = 50$

x	0	50
y	50	0

Table for $2x + y = 80$

x	0	40
y	80	0

The graph of system of inequalities (*ii*) to (*iv*) are drawn, which gives feasible region *OABC* with corner points *O* (0, 0), *A* (40, 0), *B* (30, 20) and *C* (0, 50).

Feasible region is bounded.

Now,

Corner points	$Z = 10500x + 9000y$
O (0, 0)	0
A (40, 0)	420000
B (30, 20)	495000 ← Maximum
C (0, 50)	450000

Hence, the co-operative society of farmers will get the maximum profit of ₹ 495000 by allocating 30 hectares for crop *A* and 20 hectares for crop *B*.

5. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods *A* and *B*. To produce one unit of *A*, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of *B*. If *A* and *B* are priced at ₹100 and ₹120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as an LPP and solve graphically. **[*CBSE (AI) 2013*]**

Sol. Let x and y units of goods *A* and *B* are produced respectively.

Let Z be total revenue

Here, $Z = 100x + 120y$ ....(*i*)

Subjects to constraints:

Also $2x + 3y \le 30$ ....(*ii*)

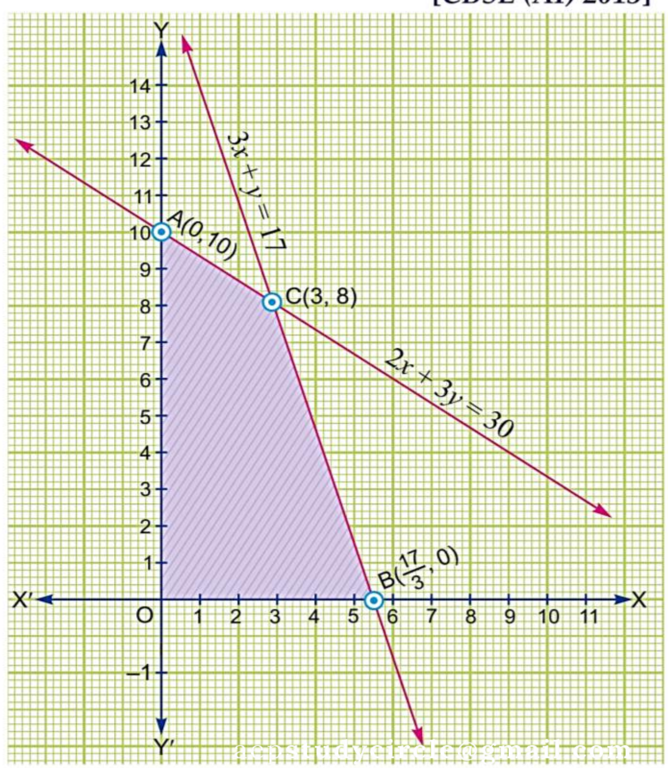
$3x + y \le 17$ ....(*iii*)

$x, y \ge 0$ ....(*iv*)

On plotting graph of above inequalities (*ii*), (*iii*) and (*iv*). We get shaded region as feasible region having corner points *A*, *O*, *B* and *C*.

For coordinate of 'C'

Two equations (*ii*) and (*iii*) are solved and we get coordinate of $C = (3, 8)$



Now, the value of Z is evaluated at corner points as:

Corner points	$Z = 100x + 120y$
$O\ (0, 0)$	0
$A\ (0, 10)$	1200
$B\left(\frac{17}{3}, 0\right)$	$\frac{1700}{3}$
$C\ (3, 8)$	1260 ← Maximum

Therefore, maximum revenue is ₹1, 260 when 3 workers and 8 units capital are used for production.

6. An aeroplane can carry a maximum of 200 passengers. A profit of ₹500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of ₹400 is made on each economy ticket out of which 25% will go for the improvement of facilities provided to economy class passengers. In both cases, the remaining profit goes to the airline's fund. The airline reserves at least 20 seats for executive class. However, at least four times as many passengers prefer to travel by economy class than by the executive class. Determine, how many tickets of each type must be sold in order to maximise the net profit of the airline. Make the above as an LPP and solve graphically. *[CBSE (F) 2013]*

Sol. Let there be x tickets of executive class and y tickets of economy class be sold. Let Z be net profit of the airline.

Here, we have to maximise Z

Now, $\quad Z = 500x \times \frac{80}{100} + 400y \times \frac{75}{100}$

$Z = 400x + 300y \quad \quad ...(i)$

Subject to constraints:

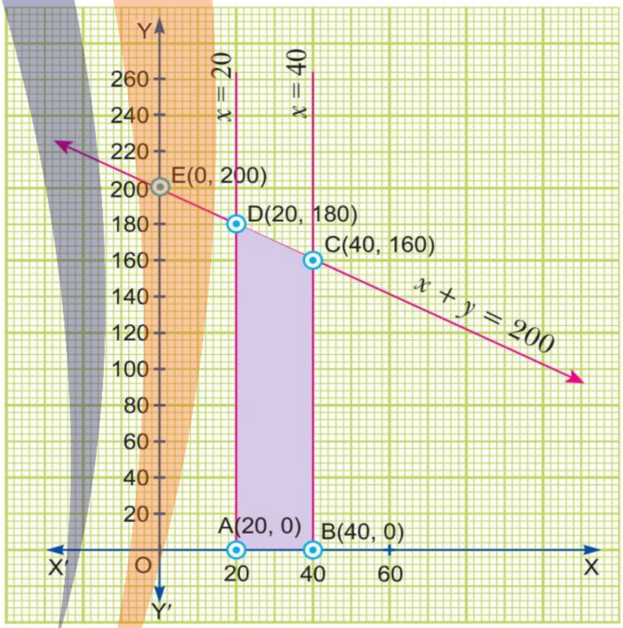
$x \geq 20 \quad \quad ...(ii)$

Also $\quad x + y \leq 200 \quad \quad ...(iii)$

$x + 4x \leq 200 \quad \quad [\because y = 4x]$

$\Rightarrow \quad 5x \leq 200$

$\Rightarrow \quad x \leq 40 \quad \quad ...(iv)$



Shaded region is feasible region having corner points $A(20, 0)$, $B(40, 0)$, $C(40, 160)$, $D(20, 180)$.
Now, value of Z is calculated at corner point as

Corner points	$Z = 400x + 300y$
A (20, 0)	8000
B (40, 0)	16000
C (40, 160)	64000 ← Maximum
E (0, 200)	60000

Hence, 40 tickets of executive class and 160 tickets of economy class should be sold to maximise the net profit of the airlines.

7. A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows :

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise daily output?

Sol. Let the owner buys x machines of type A and y machine of type B.

Then $1000x + 1200y \le 9000$ ...(*i*)

$12x + 8y \le 72$ ...(*ii*)

Objective function is to maximize $Z = 60x + 40y$

From (*i*)

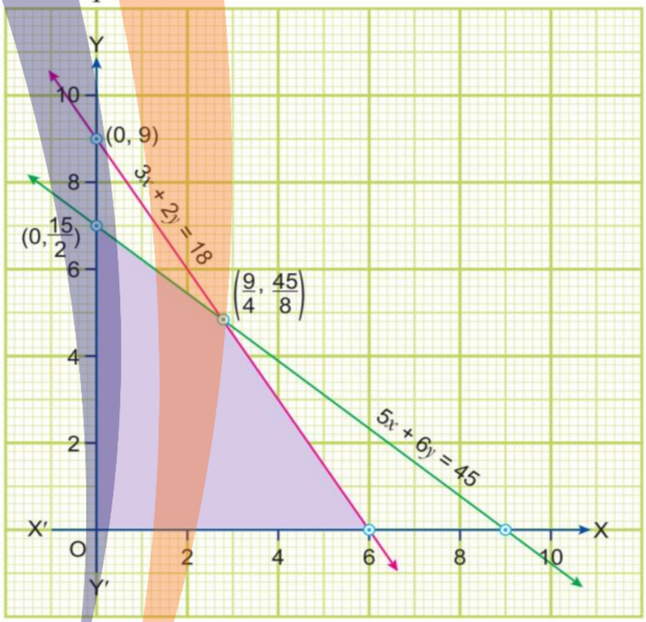
$10x + 12y \le 90$

or $5x + 6y \le 45$ ...(*iii*)

$3x + 2y \le 18$ ...(*iv*) [from (*ii*)]

We plot the graph of inequations shaded region in the feasible solution (*iii*) and (*iv*).

The shaded region in the figure represents the feasible region which is bounded. Let us now evaluate Z at each corner point.



Z at (0, 0) is $60 \times 0 + 40 \times 0 = 0$

Z at $\left(0, \frac{15}{2}\right)$ is $60 \times 0 + 40 \times \frac{15}{2} = 300$

Z at $\left(\frac{9}{4}, \frac{45}{8}\right)$ is $60 \times \frac{9}{4} + 40 \times \frac{45}{8} = 135 + 225 = 360$ ← Maximum

Z at (6, 0) is $60 \times 6 + 0 = 360$ ← Maximum

Therefore, either $x = 6, y = 0$ or $x = \frac{9}{4}, y = \frac{45}{8}$ but second case is not possible as x and y are whole numbers, because number of machines cannot be fraction.

Hence, there must be 6 machines of type A and no machine of type B is required for maximum daily output.

8. A manufacturer produces two models of bikes-model *X* and model *Y*. Model *X* takes a 6 man-hours to make per unit, while model *Y* takes 10 man hours per unit. There is a total of 450 man-hour available per week. Handling and marketing costs are ₹2000 and ₹1000 per unit for models *X* and *Y*, respectively. The total funds available for these purposes are ₹80000 per week. Profits per unit for models *X* and *Y* are ₹1000 and ₹500, respectively. How many bikes of each model should the manufacturer produce, so as to yield a maximum profit? Find the maximum profit. [*NCERT Exemplar*]

Sol. Let the manufacturer produces x number of model X and y number of model Y bikes.

Model X takes a 6 man-hours to make per unit and model Y takes a 10 man-hours to make per unit. There is total of 450 man-hour available per week.

$\therefore \quad 6x + 10y \le 450$

$\Rightarrow \quad 3x + 5y \le 225 \quad ...(i)$

For models X and Y, handling and marketing costs ₹2000 and ₹1000, respectively, total funds available for these purposes are ₹80000 per week.

$\therefore \quad 2000x + 1000y \le 80000$

$\Rightarrow \quad 2x + y \le 80 \quad ...(ii)$

Also, $\quad x \ge 0, y \ge 0$

Here, the profits per unit for models X and Y are ₹1000 and ₹500, respectively.

$\therefore$ Required LPP is

Maximise $\quad Z = 1000x + 500y$

Subject to, $\quad 3x + 5y \le 225$

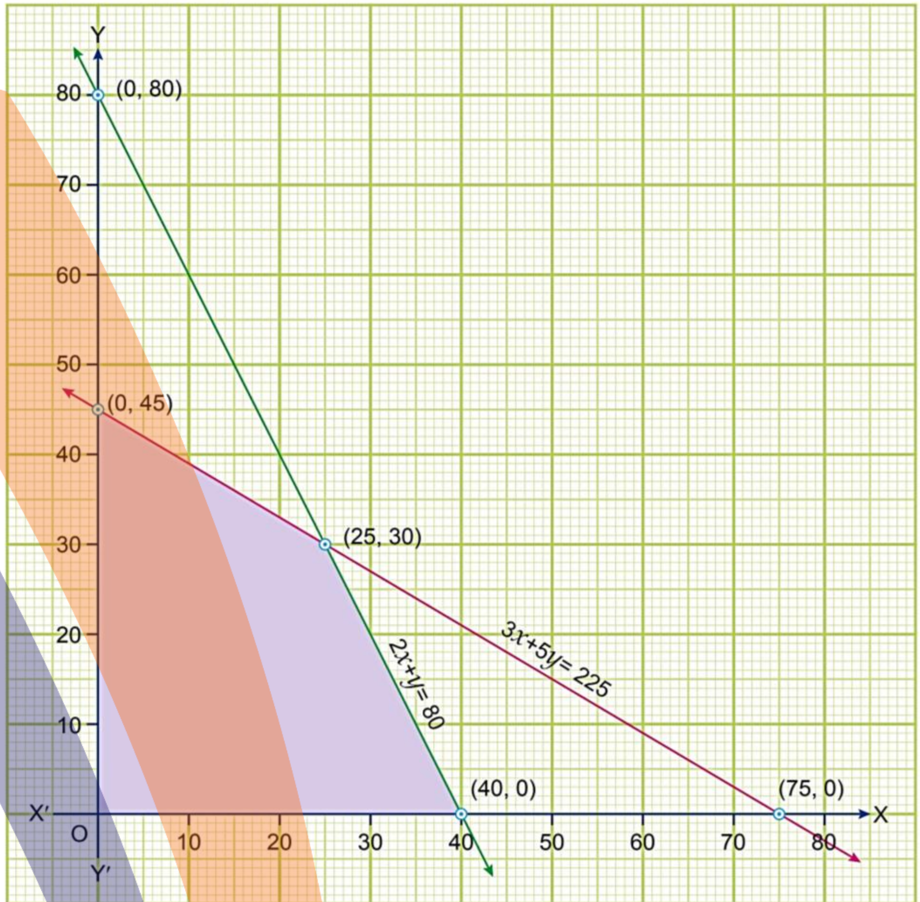
$2x + y \le 80$

$x \ge 0, y \ge 0$

From the shaded feasible region, it is clear that coordinates of corner points are (0, 0), (40, 0), (25, 30) and (0, 45).

On Solving $3x + 5y = 225$ and $2x + y = 80$, we get $x = 25, y = 30$

Corner Points	$Z = 1000x + 500y$	
(0, 0)	0	
(40, 0)	40000	← Maximum
(25, 30)	25000 +15000 = 40000	← Maximum
(0, 45)	22500	



So, the manufacturer should produce 25 bikes of model X and 30 bikes of model Y to get a maximum profit of ₹40000.

Since, in question it is asked that each model bikes should be produced, so the value (40, 0) is ignored.

9. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.

[*CBSE Delhi, 2019*]

Sol. Let x items of model A and y items of model B be made.

$\therefore \quad x, y \geqslant 0$ (number of items can not be negative)

According to question, we have

The making of model A requires 2 hours work by a skilled man and the model B requires 1 hour by a skilled man.

$\therefore \quad 2x + y \leqslant 40$

and, the making of model A requires 2 hours work by a semi skilled man and model B requires 3 hours work by a semi-skilled man.

$\therefore \quad 2x + 3y \leqslant 80$

and, the profit, $Z = 15x + 10y$, which is to be maximised.

Thus, we have the mathematical formulation of the given linear programming problem as

$$Z_{\text{Max.}} = 15x + 10y$$

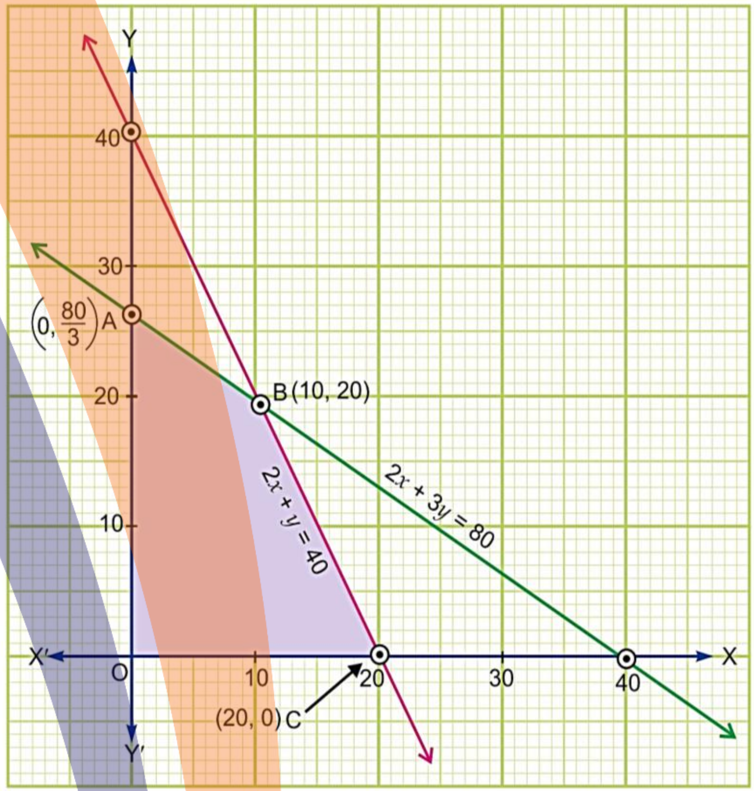
Subject to constraints

$2x + y \le 40$ ... (i)

$2x + 3y \le 80$ ...(ii)

$x,\ y \ge 0$ ...(iii)

The feasible region determined by the system of constraints is $OABC$.



The corner points are $A\left(0, \frac{80}{3}\right), B(10, 20), C(20, 0)$.

Corner points	$Z = 15x + 10y$
$A\left(0, \frac{80}{3}\right)$	$\frac{800}{3}$
$B(10, 20)$	350 ← Maximum
$C(20, 0)$	300

The maximum value of $Z = 350$ which is attained at B (10, 20).

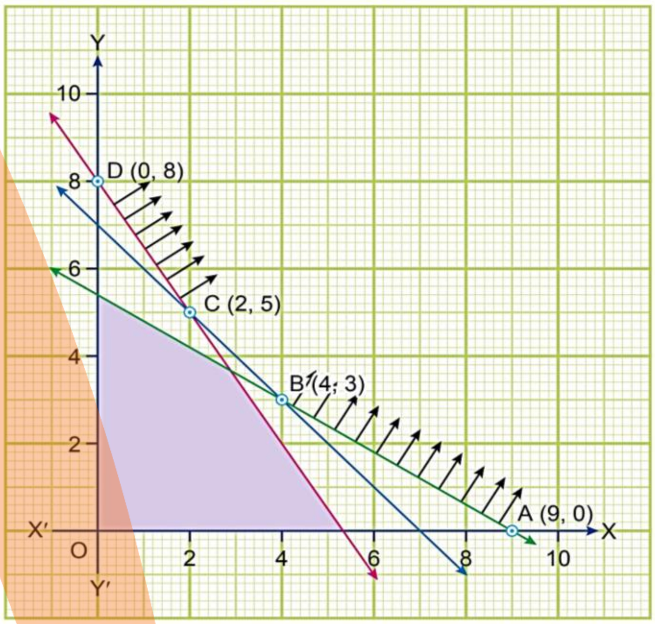
Hence, the maximum profit is ₹350 when 10 units of model A and 20 units of model B are produced.

PROFICIENCY EXERCISE

■ **Objective Type Questions:** **[1 mark each]**

1. **Choose and write the correct option in the following questions.**

(*i*) Feasible region (shaded) for a LPP is shown in the given figure. Minimum of $z = 4x + 3y$ occurs at the point.



(*a*) (0, 8) (*b*) (2, 5) (*c*) (4, 3) (*d*) (9, 0)

(*ii*) The solution set of the inequation $3x + 2y > 3$ is

(*a*) half plane not containing the origin (*b*) half plane containing the origin

(*c*) the point being on the line $3x + 2y = 3$ (*d*) None of these

(*iii*) If the constraints in a linear programming problem are changed

(*a*) solution is not defined (*b*) the objective function has to be modified

(*c*) the problems is to be re-evaluated (*d*) none of these

(*iv*) Which of the following statement is correct?

(*a*) Every LPP admits an optimal solution.

(*b*) Every LPP admits unique optimal solution.

(*c*) If a LPP gives two optimal solutions it has infinite number of solutions.

(*d*) None of these

(*v*) The maximum value of $p = x + 3y$ such that $2x + y \le 20$, $x + 2y \le 20$, $x \ge 0$, $y \ge 0$ is

(*a*) 10 (*b*) 30 (*c*) 60 (*d*) $\frac{80}{3}$

2. **Fill in the blanks.**

(*i*) In a LPP, the linear function which has to be maximised or minimised is called a linear ____________ function.

(*ii*) The maximum value of $Z = 6x + 16y$ satisfying the conditions $x + y \ge 2$, $x \ge 0, y \ge 0$ is ____________.

(*iii*) In a LPP, the inequalities or restrictions on the variables are called ____________.

Very Short Answer Questions: [1 mark each]

3. Determine the maximum value of $Z = 3x + 4y$,
   Subject to the constraints: $x + y \leq 1, x \geq 0, y \geq 0$.
4. If a linear programming problem is $Z_{max} = 3x + 2y$,
   Subject to the constraints: $x + y \leq 2$, find Z_{max}
5. Maximise the function $Z = 11x + 7y$, subject to the constraints: $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.

Short Answer Questions–I: [2 marks each]

6. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of ₹8,000 on each piece of model A and ₹12,000 on each piece of model B. To realise a maximum profit formulate above problem in LPP.
7. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. To get the maximum number of cakes can be made from 5 kg of flour and 1 kg of fat, formulate the problem in LPP.
8. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7.00 per package on bolts. If he operates his machines for at most 12 hours a day, the formulate the problems in LPP to maximise his profit.
9. A merchant plans to sell two types of personal computers a desktop model and a portable model that will cost ₹25,000 and ₹40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. If he does not want to invest more than ₹70 lakhs and if his profit on the desktop model is ₹4500 and on portable mode is ₹5000, then formulate the problems as LPP to get maximum profit.
10. A manufacturing company makes two toys A and B. Each piece of toy A requires 8 labour hours for fabricating and 2 labour hours for finishing. Each piece of toy B requires 16 labour hours for fabricating and 4 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 200 and 50 respectively. The company makes a profit of ₹10,000 on each piece of toy A and ₹14,000 on each piece of toy B. Formulate the problem as LPP to realise a maximum profit.

Short Answer Questions–II: [3 marks each]

11. A cottage manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table:

Item	Machine Time	Craftsman time	Profit (in ₹)
Pedestal lamp	1.5	3	30
Wooden shades	3 hours	1 hours	20

In day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an LPP and solve it graphically.

[*CBSE 2020, (65/2/1)*]

12. A manufacturer has three machines I, II and III installed in his factory. Machine I and II are capable of being operated for atmost 12 hours whereas machine III must be operated for atleast 5 hours a day. He produces only two items M and N each requiring the use of all the three machines.

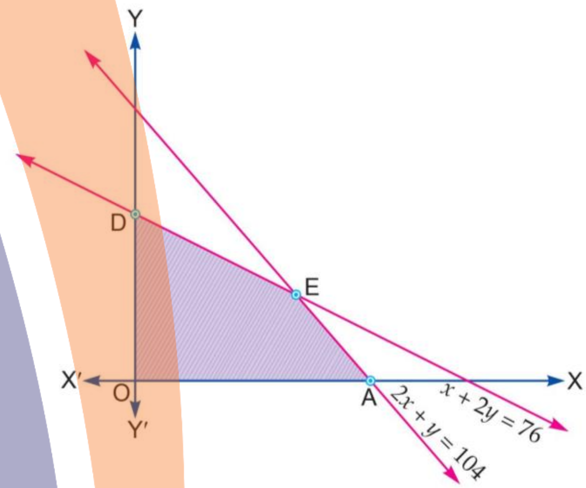
The number of hours required for producing 1 unit of M and N on three machines are given in the following table:

Item	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

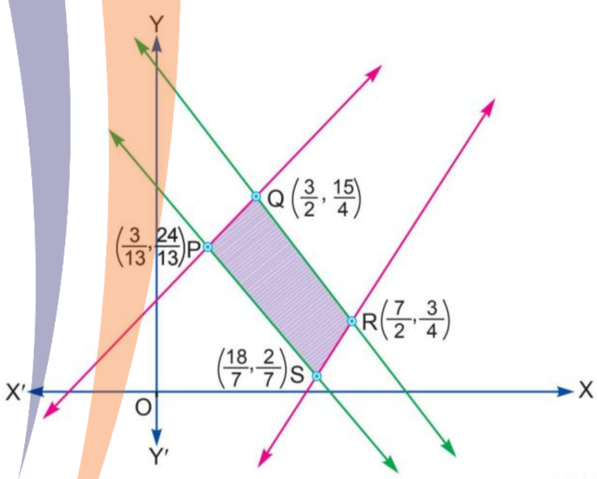
He makes a profit of ₹600 and ₹400 on one unit of items M and N respectively. How many units of each item should he produce so as to maximise his profit assuming that he can sell all the items that he produced. What will be the maximum profit? [CBSE 2020, (65/4/1)]

13. A man rides his motorcycle at the speed of 50 km/hour. He has to spend ₹2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to ₹3 per km. He has at the most ₹120 to spend on petrol and one hour time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.

14. Determine the maximum value of $Z = 3x + 4y$ of the feasible region (shaded) for a LPP is shown in figure.



15. In figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z = x + 2y$.



16. Solve the following linear programming problem graphically:

Minimise $Z = x - 5y + 20$

Subject to constraints: $x - y \geq 0, -x + 2y \geq 2;$

$x \geq 3, y \leq 4, x, y \geq 0$

17. Solve the following LPP:

Maximise $Z = 5x_1 + 7x_2,$

Subject to constraints: $x_1 + x_2 \leq 4,$

$3x_1 + 8x_2 \leq 24,$

$10x_1 + 7x_2 \leq 35,$

$x_1, x_2 \geq 0.$

18. Maximise $Z = x + y$ subject to $x + 4y \leq 8, 2x + 3y \leq 12, 3x + y \leq 9, x \geq 0, y \geq 0.$

19. Solve the following LPP graphically:

Maximise $Z = 1000x + 600y$

Subject to the constraints

$x + y \leq 200$

$x \geq 20$

$y - 4x \geq 0$

$x, y \geq 0$ *[CBSE (F) 2017]*

20. Solve the following LPP graphically:

Maximise $Z = 4x + y$

Subject to following constraints $x + y \leq 50,$

$3x + y \leq 90,$

$x \geq 10$

$x, y \geq 0$ *[CBSE Delhi 2017]*

21. Solve the following linear programming problem graphically:

Maximise $Z = 7x + 10y$

Subject to constraints

$4x + 6y \leq 240$

$6x + 3y \leq 240$

$x \geq 10$

$x \geq 0, y \geq 0$ *[CBSE (AI) 2017]*

22. Fund graphically, the maximum value of $Z = 2x + 5y,$ subject to constraints given below:

$2x + 4y \leq 8,$

$3x + y \leq 6,$

$x + y \leq 4,$

$x \geq 0, y \geq 0$ *[CBSE Delhi 2015]*

23. Solve the following linear programming problem graphically.

Minimise $Z = 3x + 5y$

Subject to the constraints:

$x + 2y \geq 10; \ x + y \geq 6; \ 3x + y \geq 8; \ x, y \geq 0$ *[CBSE Ajmer 2015]*

■ **Long Answer Questions:** **[5 marks each]**

24. (Diet problem) A dietician has to develop a special diet using two foods *P* and *Q*. Each packet (containing 30 g) of food *P* contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin *A*. Each packet of the same quantity of food *Q* contain 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin *A*. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin *A* in the diet? What is the minimum amount of vitamin *A*? *[CBSE Chennai 2015]*

25. One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an LPP and solve it graphically? *[CBSE (AI) 2010]*

26. In order to supplement daily diet, a person wishes to take some *X* and some *Y* tablets. The contents of iron, calcium and vitamins in *X* and *Y* (in miligrams per tablet) are given below:

Tablets	Iron	Calcium	Vitamins
X	6	3	2
Y	2	3	4

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of *X* and *Y* is ₹2 and ₹1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost? *[CBSE (F) 2016]*

27. A company manufactures three kinds of calculators: *A*, *B* and *C* in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind *A*, 4000 of kind *B* and 4,800 of kind *C*. The daily output of factory I is of 50 calculators of kind *A*, 50 calculators of kind B, and 30 calculators of kind *C*. The daily output of factory II is of 40 calculators of Kind *A*, 20 of kind *B* and 40 of kind *C*. The cost per day to run factory I is ₹12,000 and factory II is ₹15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically. *[CBSE Allahabad 2015]*

28. A dealer deals in two items only – item *A* and item *B*. He has ₹50,000 to invest and a space to store at most 60 items. An item *A* costs ₹2,500 and an item B costs ₹500. *A* net profit to him on item *A* is ₹500 and on item *B* ₹150. If he can sell all the items that he purchases, how should he invest his amount to have maximum profit? Formulate an LPP and solve it graphically. *[CBSE Chennai 2015]*

29. The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives ₹225 a day and a woman receives ₹200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum? Formulate an LPP and solve it graphically. *[CBSE Patna 2015]*

30. A retired person wants to invest an amount of ₹50,000. His broker recommends investing in two type of bonds '*A*' and '*B*' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least 20,000 in bond '*A*' and at least 10,000 in bond '*B*'. He also wants to invest at least as much in bond '*A*' as in bond '*B*'. Solve this linear programming problem graphically to maximise his returns. *[CBSE (North) 2016]*

31. A company manufactures two types of cardigans: type *A* and type *B*. It costs ₹360 to make a type *A* cardigan and ₹120 to make a type *B* cardigan. The company can make at most 300 cardigans and spend at most ₹72000 a day. The number of cardigans of type *B* cannot exceed the number of cardigans of type *A* by more than 200. The company makes a profit of ₹100 for each cardigan of type *A* and ₹50 for every cardigan of type *B*.

Formulate this problem as a linear programming problem to maximise the profit of the company. Solve it graphically and find maximum profit. **[*CBSE (East) 2016*]**

32. There are two types of fertilisers '*A*' and '*B*'. '*A*' consists of 12% nitrogen and 5% phosphoric acid whereas '*B*' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If '*A*' costs ₹10 per kg and '*B*' costs ₹8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. **[*CBSE (Central) 2016*]**

33. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at ₹7 profit and that of B at a profit of ₹4. Find the production level per day for maximum profit graphically. **[*CBSE Delhi 2016*]**

34. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine ***A*** and 3 hours on machine ***B*** to produce a package of nuts. It takes 3 hours on machine ***A*** and 1 hour on machine ***B*** to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7 per package on bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? Form the linear programming problem and solve it graphically. **[*CBSE Delhi 2012*]**

35. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin ***A*** and 10 units of vitamin ***C***. Food I contains 2 units/kg of vitamin ***A*** and 1 unit/kg of vitamin ***C*** while Food II contains 1 units/kg of vitamin ***A*** and 2 units/kg of vitamin ***C***. It costs ₹5 per kg to purchase Food I and ₹7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically. **[*CBSE (AI) 2012*]**

Answers

1. (*i*) (*b*) (*ii*) (*a*) (*iii*) (*c*) (*iv*) (*c*) (*v*) (*b*)

2. (*i*) objective (*ii*) 12 (*iii*) linear constraints

3. 4 **4.** 6 **5.** 47

6. $Z = 8000x + 12000y$ is to be maximised under constraints

$9x + 12y \le 180;\ x + 3y \le 30;\ x \ge 0,\ y \ge 0$

7. $Z = x + y$ is to be maximised under the constraints

$2x + y \le 50$

$x + 2y \le 40$

$x, y \ge 0$

where x and y are number of first and second kind of cake and Z the total number of cakes.

8. $Z = 17.5x + 7y$ which is to be maximised under constraints

$x + 3y \le 12;\ 3x + y \le 12;\ x, y \ge 0$

where x nuts and y bolts are produced and Z is the profit.

9. $Z = 4500x + 5000y$ which is to be maximised under constraints
$x + y \leq 250;\ 5x + 8y \leq 1400;\ x, y \geq 0$
10. $Z = 10{,}000x + 14{,}000y$ which is to be maximised under constraints
$8x + 16y \leq 200;\ 2x + 4y \leq 50;\ x \geq 0,\ y \geq 0$
where Z is total profit and x, y are the number of toy A and toy B.
11. The manufacturer should produce 4 pedestal lamps and 12 wooden shades to get maximum profit of 360.
12. Maximum profit is ₹4000 whan a manufacturer produced 4 units of items M and 4 units of item of N.
13. Maximise $Z = x + y$. Subject to constraints: $2x + 3y \leq 120,\ 8x + 5y \leq 400,\ x \geq 0, y \geq 0$.
14. 196
15. Maximum = 9, minimum = $3\frac{1}{7}$
16. At (4, 4), $Z_{min} = 4$
17. $x_1 = \frac{8}{5}, x_2 = \frac{12}{5}; Z_{max} = \frac{124}{5}$
18. $\frac{43}{11}$ at $\left(\frac{28}{11}, \frac{15}{11}\right)$
19. $Z_{max} = 136000$ at $x = 40$ and $y = 160$
20. $Z_{max} = 120$ when $x = 30, y = 0$
21. $Z_{max} = 410$ for $x = 30, y = 20$
22. Maximum value of Z is 10 at $x = 0, y = 2$
23. Minimum value of Z is 26 at (2, 4)
24. 15 packets of food P and 20 packets of food Q, minimum amount of vitamin A is 150 units.
25. 20 cakes of type 1 and 10 cakes of type II to get maximum number of cakes
26. $x = 1, y = 6$ minimum value B
27. Factory I run for 80 days and
Factory II run for 60 days to get minimum cost 2184000
Hint: Objective function $Z = 12000x + 15000y$
Subject to constraints:
$5x + 4y \geq 640 \quad 5x + 2y \geq 400 \quad 3x + 4y \geq 480 \quad x, y \geq 0$
28. Dealer deals in 10 items of A and 50 items of B to get maximum profit ₹12500
Hint: Objective function: $Z = 500x + 150y$
Subject to constraints:
$x + y \leq 60 \quad 5x + y \leq 100 \quad x, y \geq 0$
29. Minimum payroll 6 men and 4 women must be hired minimum cost ₹2150
Hint: Objective function, $Z = 225x + 200y$
Subject to constraints: $x + y \leq 10 \quad 3x + 4y \geq 34 \quad 8x + 5y \geq 68 \quad x, y \geq 0$
30. ₹40000 in bond A and ₹10,000 in bond B for a maximum return of ₹4900.
31. 150 cardigans of type A and 150 of type B for a maximum profit of 22,500.
32. Minimum cost ₹1980, 30 kg of fertiliser A and 210 kg of fertiliser B should be used.
33. Manufacturer will get maximum profit of ₹26 by producing 2 units of A and 3 units of B.
34. Maximum profit is ₹73.5, when 3 package of nuts and 3 package of bolts are produced.
35. Minimum cost of food mixture is ₹38, when 2kg of Food I and 4 kg of food II are mixed.