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ELASTICITY
PROPERTIES OF MATTER

A body is said to be *elastic*, if it regains its original shape and size on the removal of deforming force. The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces is called *elasticity*.

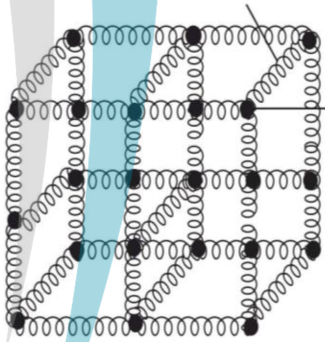
If a body completely gains its natural shape after the removal of the deforming forces it is called *perfectly elastic body*. If a body remains in the deformed state or does not even partially regain its original shape after removal of deforming forces it is called *perfectly inelastic* or *plastic body*.

Elastic Behaviour

The elastic behaviour can be understood by taking the microscopic nature of solids. It is shown in the following figure, in which we can see that the balls represent the atoms or molecules and the springs represent the interatomic or intermolecular forces.

In this system originally, the ball is in the position of its stable equilibrium, if any ball is displaced a little from its equilibrium position, the springs attached to that ball will either be stretched or compressed.

Therefore, the restoring forces are developed in the springs and they will bring the ball back to its natural position. This is known as the elastic behaviour of the solid body.



Stress

When an external force is applied to a body, an internal restoring force is set up at each cross-section of the body which tends to restore the body back to its original state. The restoring force set up inside the body per unit area is known as stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

In SI system, unit of stress is Nm^{-2} or pascal (denoted by Pa) and in CGS system, it is dyne/cm^2. Mainly, there are three types of stress. They are listed as follows.

(i) Normal or Longitudinal Stress

If area of cross-section of a rod is A and a deforming force F is applied along the length of the rod and perpendicular to its cross-section, then in this case, stress produced in the rod is known as normal or longitudinal stress. It is also known as axial stress.

$$\text{Longitudinal stress} = \frac{F_n}{A}$$

Longitudinal stress is of two types

(a) **Tensile stress** A type of longitudinal stress in which length of the rod is increased on application of deforming force over it, the stress produced in rod is called tensile stress.

(b) **Compressive stress** A type of longitudinal stress in which length of the rod is decreased on application of deforming force, the stress produced is called compressive stress.

(ii) Volumetric Stress

In this type of stress, when a force is applied on a body such that it produces a change in volume and density without change in its shape and

(a) at any point, the force is perpendicular to its surface.

(b) at any small area, the magnitude of force is directly proportional to its area.

Then, force per unit area is called volumetric stress.

$$\therefore \quad \text{Volumetric stress} = \frac{F_V}{A}$$

This is the case when small solid body is immersed in fluid.

(iii) Shearing or Tangential Stress

In shearing or tangential stress, the force is applied tangentially or parallel to a surface of a body, which produces a change in shape without change in its volume.

$$\text{Tangential stress} = \frac{F_T}{A}$$

Example 1. *The ratio of radii of two wires of same material is 2 : 1. If these wires are stretched by equal force, what is the ratio of stress produced in them?*

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 5

Sol. (c) Here, $r_1 : r_2 = 2:1, F_1 = F_2 = F$

$$\text{Stress } (S) = \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2}$$

$$\therefore \quad S \propto \frac{1}{r^2}$$

$$\therefore \quad \frac{S_1}{S_2} = \frac{r_2^2}{r_1^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Example 2. *The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit?*

(a) 0.90 mm (b) 1.00 mm
(c) 1.16 mm (d) 1.36 mm **[JEE Main 2019]**

Sol. (c) Let d_{min} be the minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_{min}^2} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

For stress not to exceed elastic limit, we have $\sigma \le 379$ MPa

$$\Rightarrow \quad \frac{4F}{\pi d_{min}^2} \le 379 \times 10^6$$

Here, $F = 400$ N

$$\therefore \quad d_{min}^2 = \frac{1600}{\pi \times 379 \times 10^6}$$

$$\Rightarrow \quad d_{min} = 1.16 \times 10^{-3} \text{ m} = 1.16 \text{ mm}$$

Example 3. *A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi \, ms^{-2}$, what will be the tensile stress that would be developed in the wire?* **[JEE Main 2019]**

(a) $6.2 \times 10^6 Nm^{-2}$ (b) $5.2 \times 10^6 Nm^{-2}$
(c) $3.1 \times 10^6 Nm^{-2}$ (d) $4.8 \times 10^6 Nm^{-2}$

Sol. (c) Given, radius of wire, $r = 2$ mm $= 2 \times 10^{-3}$ m
Weight of load, $m = 4$ kg, $g = 3.1\pi$ ms^{-2}

$$\therefore \quad \text{Tensile stress} = \frac{\text{Force } (F)}{\text{Area } (A)} = \frac{mg}{\pi r^2}$$

$$= \frac{4 \times 3.1 \times \pi}{\pi \times (2 \times 10^{-3})^2}$$

$$= 3.1 \times 10^6 Nm^{-2}$$

Strain

When the size or shape of a body is changed under an external force, the body is said to be strained. The change occurred in the unit size of the body is called strain. Usually, it is denoted by ε.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}} = \frac{\Delta x}{x}$$

Here, Δx is the change (may be in length, in volume etc.) and x is the original value of quantity in which change has occurred. It has no dimension as it is a pure number.

Since, a body may have three types of deformation, *i.e.* in length, in volume or in shape, likewise there are following three types of strains

(i) Longitudinal Strain

It is associated with longitudinal stress. The change in length per unit original length of the body under deformation produced by the external force is known as longitudinal strain.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} \quad \text{or} \quad \varepsilon_l = \frac{\Delta l}{l}$$

It is of two types

(a) **Tensile strain** If on applying a deforming force, there is an increase of Δl in length of a rod, then strain produced in the rod is called tensile strain.

(b) **Compressive strain** If on applying a deforming force, there is decrease of Δl in length of a rod, then strain produced in the rod is called compressive strain.

(ii) Volumetric Strain

It is produced when there's volumetric stress in the body. The change in volume per unit original volume of the body under deformation produced by the external force is known as volumetric strain.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

or $$\varepsilon_V = \frac{\Delta V}{V}$$

(iii) Shearing Strain

This type of strain is produced when a shearing stress is present.

It is defined as the angle in radians through which a plane perpendicular to the fixed surface of the cubical body is turned under the effect of tangential force.

$$\text{Shearing strain, } \tan\phi = \frac{x}{L}$$

or $$\phi = \frac{x}{L} \quad \text{(for very small deformation)}$$

Hooke's Law

It states that for a body within elastic limit, stress applied to a body is proportional to the resulting strain, *i.e.*

$$\text{Stress} \propto \text{Strain} \quad \text{or} \quad \frac{\text{Stress}}{\text{Strain}} = E = \text{constant}$$

where, E is constant and is known as *modulus of elasticity* or *coefficient of elasticity* of the material of the body.

Types of Modulus of Elasticity

Depending on the type of stress applied on a body and resulting strain produced, we have following three modulii of elasticity and they are as follows

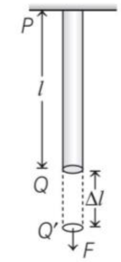
(i) Young's modulus of elasticity
(ii) Bulk modulus of elasticity
(iii) Modulus of rigidity

Young's Modulus of Elasticity (Y)

It is defined as the ratio of normal stress (either tensile or compressive stress) to the longitudinal strain within an elastic limit.

Thus, $$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

Consider a metal wire, PQ of length l, radius r and the uniform area of cross-section A. Let it be suspended from a rigid support at P, a stretching force F be applied normally at the free end Q and let its length increase by $\Delta l (= QQ')$.



Then, longitudinal strain $= \frac{\Delta l}{l}$

$$\text{Normal stress} = \frac{F}{A} = \frac{F}{\pi r^2} \qquad [\because A = \pi r^2]$$

$$\text{Young's modulus } (Y) = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$= \frac{F/\pi r^2}{\Delta l / l} = \frac{Fl}{\pi r^2 \Delta l}$$

Since strain is a dimensionless quantity, therefore the unit of Young's modulus is same as that of stress. Young's modulus has unit of pressure.

Example 4. *A structural steel rod has a radius of 10 mm and a length of 1m. A 100 kN force stretches it along its length and Young's modulus of structural steel is 2×10^{11} Nm^{-2}. The strain on the rod is*

(a) 1% (b) 0.04% (c) 0.08% (d) 0.16%

Sol. (d) We assume that the rod is held by a clamp at one end and the force F is applied at the other end, parallel to the length of the rod. Then,

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2}$$

Given, $F = 100 \text{ kN} = 100 \times 10^3 \text{ N}, r = 10^{-2} \text{ m}$

$$= \frac{F}{A} = \frac{100 \times 10^3 \text{ N}}{3.14 \times (10^{-2})^2} = 3.18 \times 10^8 \text{ Nm}^{-2}$$

Elongation, $$\Delta L = \frac{(F/A)\,L}{Y}$$

$$\Delta L = \frac{(3.18 \times 10^8)(1)}{2 \times 10^{11}} = 1.59 \times 10^{-3} \text{ m}$$

$$\Delta L = 1.59 \text{ mm}$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{1.59 \times 10^{-3} \text{ m}}{1 \text{m}}$$

$$= 1.59 \times 10^{-3} = 0.16\%$$

Example 5. *Young's modulii is of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to* **[JEE Main 2019]**

(a) 1.3 mm (b) 1.5 mm
(c) 1.9 mm (d) 1.7 mm

Sol. (d) When a wire is stretched, the change in length of wire is $\Delta l = \frac{Fl}{\pi r^2 Y}$, where Y is its Young's modulus.

Here, for wires A and B,

$$l_A = 2\text{ m}, l_B = 1.5\text{ m},$$

$$\frac{Y_A}{Y_B} = \frac{7}{4}, r_B = 2\text{ mm} = 2\times10^{-3}\text{ m and } \frac{F_A}{F_B} = 1$$

As, it is given that $\Delta l_A = \Delta l_B$

$$\Rightarrow \quad \frac{F_A l_A}{\pi r_A^2 Y_A} = \frac{F_B l_B}{\pi r_B^2 Y_B}$$

$$\Rightarrow \quad r_A^2 = \frac{F_A}{F_B}\cdot\frac{l_A}{l_B}\cdot\frac{Y_B}{Y_A}\cdot r_B^2 = 1\times\frac{2}{1.5}\times\frac{4}{7}\times4\times10^{-6}\text{ m}$$

$$= 3.04\times10^{-6}\text{ m}$$

$$\Rightarrow \quad r_A = 1.7\times10^{-3}\text{ m}$$

or $\quad r_A = 1.7$ mm

Example 6. *A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm are connected end-to-end. When stretched by a load, the net elongation is found to be 0.70 mm. The load applied (in N) is (Take, $Y_C = 1.1\times10^{11}\,Nm^{-2}, Y_S = 2\times10^{11}\,Nm^{-2}$)* **[NCERT]**

(a) $1.8\times10^2\,N$ (b) $2.5\times10^6\,N$
(c) $3.8\times10^8\,N$ (d) $6.1\times10^{-2}\,N$

Sol. (a) The copper and steel wires are under a tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A.

$$\text{Stress} = \text{Strain}\times\text{Young's modulus}$$

$$\therefore \quad \frac{W}{A} = \left(\frac{\Delta L_C}{L_C}\right)\times Y_C = \left(\frac{\Delta L_S}{L_S}\right)\times Y_S$$

where the subscripts C and S refer to copper and stainless steel, respectively.

$$\frac{\Delta L_C}{\Delta L_S} = \left(\frac{Y_S}{Y_C}\right)\times\left(\frac{L_C}{L_S}\right) \quad \text{...(i)}$$

Given, $\quad L_C = 2.2\text{ m}, L_S = 1.6\text{ m}$

$$\frac{\Delta L_C}{\Delta L_S} = \frac{2\times10^{11}}{1.1\times10^{11}}\times\frac{2.2}{1.6} = 2.5$$

The total elongation is given to be

$$\Delta L_C + \Delta L_S = 7\times10^{-4}\text{ m} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\Delta L_C = 5\times10^{-4}\text{ m},$$

$$\Delta L_S = 2\times10^{-4}\text{ m}$$

$$W = \frac{(A\times Y_C\times\Delta L_C)}{L_C}$$

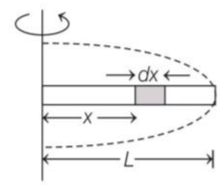
$$= \frac{\pi(1.5\times10^{-3})^2\times(1.1\times10^{11}\times5\times10^{-4})}{2.2}$$

$$= 1.8\times10^2\text{ N}$$

Example 7. *A thin uniform metallic rod of length 0.5 m and radius 0.1 m with an angular velocity 400 rad s^{-1} in a horizontal plane about a vertical axis passing through one of its ends. Elongation in the rod (in m) is (Take, density of material of the rod is 10^4 kgm^{-3} and $Y = 2\times10^{11}\,Nm^{-2}$)* **[NCERT]**

(a) $\frac{10^{-3}}{3}$ (b) $\frac{3}{10^{-3}}$ (c) $\frac{2}{10^2}$ (d) $\frac{10^2}{2}$

Sol. (a) Consider an element of length dx at a distance x from the axis of rotation,



$$dF = dm\,x\omega^2$$

$$= \rho A dx\cdot x\cdot\omega^2$$

$$F = \rho A\omega^2\int_x^L x\,dx$$

$$= \frac{1}{2}\rho A\omega^2(L^2 - x^2)$$

If dy is the elongation in the element of length dx, then

$$\frac{dy}{dx} = \frac{F}{AY}$$

$$dy = \frac{F\,dx}{A\cdot Y}\bigg|_{\Delta l} = \frac{\rho\omega^2}{2Y}\int_0^L (L^2 - x^2)\,dx$$

$$\Delta l = \frac{\rho\omega^2\cdot L^2}{3Y}$$

$$\Delta l = \frac{10^4\times(400)^2\times\left(\frac{1}{2}\right)^3}{3\times2\times10^{11}} = \frac{10^{-3}}{3}\text{ m}$$

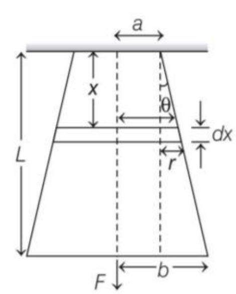
Example 8. *A body of mass 3.14 kg is suspended from one end of a wire of length 10 m. The radius of the wire is changed uniformly from 9.8×10^{-4} m at one end to 5×10^{-4} m at the other end. The change in length of the wire is (Take, $Y = 2\times10^{11}\,Nm^{-2}$)*

(a) $10^{-2}\,m$ (b) $10^{-1}\,m$ (c) $10^{-3}\,m$ (d) $10^{-5}\,m$

Sol. (c) Total change in length of the wire is

$$\tan\theta = \frac{b-a}{L} = \frac{r}{x}$$

$$\Rightarrow \quad r = x\tan\theta$$



Change in dx length of the wire is $dy = \frac{F}{Ry} \cdot \frac{dx}{(a+r)^2}$

or $$dy = \frac{Fdx}{Ry(a + x\tan\theta)^2}$$

So, change in overall length of the wire,

$$\int_0^{\Delta L} dy = \frac{F}{\pi Y}\int_0^L \frac{dx}{(a + x\tan\theta)^2}$$

$$\Delta l = \frac{F}{\pi Y \tan\theta}\left(\frac{1}{a + x\tan\theta}\right)_0^L$$

$$= \frac{FL}{\pi a(a + L\tan\theta)Y}$$

$$= \frac{FL}{\pi abY} \qquad (\because a + L\tan\theta = b)$$

$$\therefore \quad \Delta l = \frac{3.14 \times 9.8 \times 10}{3.14 \times (9.8 \times 10^{-4}) \times 5 \times 10^{-4} \times 2 \times 10^{11}}$$

$$\Delta l = 10^{-3}\ \text{m}$$

Bulk Modulus of Elasticity (K)

It is defined as the ratio of the volumetric stress to the volumetric strain. It is denoted by K. Thus,

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

Suppose a force F acts uniformly over the whole surface of the sphere (shown), decreasing its volume by ΔV. Then,

$$K = \frac{F/A}{-\Delta V/V}$$

or $$K = \frac{-FV}{A\Delta V} = -\frac{pV}{\Delta V}$$

The negative sign indicates that on increasing stress, the volume of the sphere decreases. The units of bulk modulus are Pa or Nm^{-2} in SI system.

Compressibility (C) The reciprocal of the bulk modulus of the material of the body is called the compressibility of the material. Thus,

$$\text{Compressibility } (C) = \frac{1}{K} = \frac{-1}{V}\left(\frac{\Delta V}{\Delta p}\right)$$

Its unit is $N^{-1}m^2$ or Pa^{-1} in SI system.

Note Young's modulus and Bulk modulus for a perfectly rigid body is infinity.

Example 9. *A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to*

(Take, Bulk modulus of metal, $B = 8 \times 10^{10}$ Pa) **[JEE Main 2020]**

(a) 1.67 (b) 0.6 (c) 20 (d) 5

Sol. (a) Bulk modulus, $B = \frac{\Delta p}{\left(-\frac{\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = -\frac{\Delta p}{B}$

$$\frac{\Delta V}{V} \times 100\% = -\frac{\Delta p}{B} \times 100\%$$

$$= \frac{-4 \times 10^9}{8 \times 10^{10}} \times 100\%$$

$$= -\frac{1}{2} \times 10\%$$

$$= -0.5 \times 10\% = -5\%$$

Now, $V = l^3$ (for cube)

$\Rightarrow$ $l = (V)^{1/3}$

$\Rightarrow$ $$\frac{\Delta l}{l} \times 100\% = \frac{1}{3}\left(\frac{\Delta V}{V} \times 100\%\right)$$

$$= \frac{1}{3}(-5\%) = -1.67\ \%$$

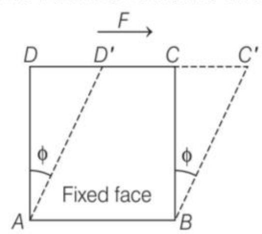
So, length of cube will be decreased by 1.67%.

Modulus of Rigidity

The ratio of tangential stress to shearing strain is known as modulus of rigidity. It is also called *shear modulus*. It is denoted by Greek letter η (eta). Thus,

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

Here, a body (shown) is acted upon by an external force tangential to the surface of the body, the opposite face being kept fixed, its volume remains unchanged. Then,



$$\eta = \frac{F/A}{DD'/AD}$$

Here, $$\tan\phi \approx \phi = \frac{DD'}{AD} \qquad \text{[for small angle } \phi]$$

$$\eta = \frac{F/A}{\phi}$$

or $$\eta = \frac{F}{A\phi}$$

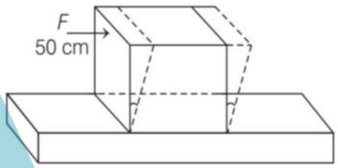
The units of modulus of rigidity are Pa or Nm^{-2} in SI system.

Note Modulus of rigidity (or shear modulus) is involved with solids only. Modulus of rigidity for a solid is generally less than its Young's modulus.

Example 10. *A square lead slab of side 50 cm and thickness 10 cm is subject to a shaping force (on its narrow face) of 9×10^4 N. The lower edge is riveted to the floor. The upper edge is displaced by (Take, G = 5.6 GPa)* **[NCERT]**

(a) 0.30 mm (b) 0.16 mm
(c) 0.28 mm (d) 0.92 mm

Sol. (*b*) The lead slab is fixed and the force is applied parallel to the narrow face. The area of the face parallel to which this force is applied is



$$A = 50\text{ cm} \times 10\text{ cm} = 0.5\text{ m} \times 0.1\text{ m} = 0.05\text{ m}^2$$

$\therefore$ Stress applied $= (9.4 \times 10^4\text{ N}/0.05\text{ m}^2)$

$= 1.80 \times 10^6\text{ Nm}^{-2}$

We know that, shearing strain $= \dfrac{\Delta x}{L} = \dfrac{\text{stress}}{G}$

$\therefore$ Displacement, $\Delta x = \dfrac{\text{Stress} \times L}{G}$

$= \dfrac{(1.8 \times 10^6 \times 0.5)}{5.6 \times 10^9}$

$= 1.6 \times 10^{-4}\text{ m} = 0.16\text{ mm}$

Example 11. *A 5 cm cube has its upper face displaced by 0.2 cm by a tangential force of 8 N. The modulus of rigidity of the material of cube is*

(a) $5 \times 10^4\ Nm^{-2}$

(b) $6 \times 10^4\ Nm^{-2}$

(c) $7 \times 10^4\ Nm^{-2}$

(d) $8 \times 10^4\ Nm^{-2}$

Sol. (*d*) Here, $l = 5\text{ cm} = 5 \times 10^{-2}\text{ m}$,

$\Delta l = 0.2\text{ cm} = 0.2 \times 10^{-2}\text{ m}, F = 8\text{ N}$

Modulus of rigidity, $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

Hence, shearing stress $= \dfrac{F}{A} = \dfrac{F}{l^2} = \dfrac{8}{(5 \times 10^{-2})^2} = 3200\text{ Nm}^{-2}$

Shearing strain $= \dfrac{\Delta l}{l} = \dfrac{0.2}{5} = 0.04$

$\therefore \quad \eta = \dfrac{3200}{0.04} = 80000\text{ Nm}^{-2}$

$= 8 \times 10^4\text{ Nm}^{-2}$

Poisson's Ratio (σ)

The ratio of lateral strain to the longitudinal strain is constant for a given material. This constant is called as Poisson's ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \left(\frac{-\Delta R}{R}\right) \Bigg/ \frac{\Delta l}{l}$$

Here, negative sign shows that if the length increases, then the radius of wire decreases. Poisson's ratio (σ) has no units and dimensions.

Theoretically, $\quad -1 < \sigma = \dfrac{1}{2}$

Practically, $0 < \sigma < \dfrac{1}{2}$ while practically no substance has been found for which σ is negative.

Example 12. *A tension of 20 N is applied to a wire of cross-sectional area 0.01 cm^2. The decrease in cross-sectional area is (Take, Young's modulus of Cu $= 1.1 \times 10^{11}\ Nm^{-2}$ and Poisson's ratio $= 0.32$)*

(a) $1.81 \times 10^{-4}\text{ cm}^2$ (b) $1.16 \times 10^{-6}\text{ cm}^2$

(c) $2.81 \times 10^{-8}\text{ cm}^2$ (d) $5.23 \times 10^{-3}\text{ cm}^2$

Sol. (*b*) As, $\dfrac{\Delta l}{l} = \dfrac{F}{AY} = \dfrac{20}{10^{-6} \times 1.1 \times 10^{11}} = 1.81 \times 10^{-4}$

$\dfrac{\Delta r}{r} = \sigma \times \dfrac{\Delta l}{l} = 0.32 \times 1.81 \times 10^{-4}$

$\dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r} = 2 \times 0.32 \times 1.81 \times 10^{-4}$

$= 1.16 \times 10^{-4}$

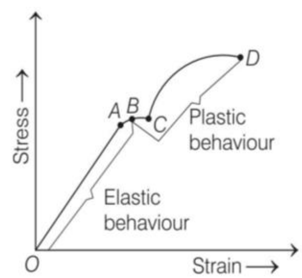
and $\quad \Delta A = A(1.16 \times 10^{-4})$

$= 1.16 \times 10^{-6}\text{ cm}^2 \quad [\because A = 0.01\text{ cm}^2]$

Stress-Strain Relationship

For a small deformation (say < 0.01) the longitudinal stress is proportional to the longitudinal strain. But when the deformation is not small, the relation of stress and strain is complicated.

Figure below shows qualitatively relation between the stress and strain of a stretched metal wire when the load is gradually increased in it.



In figure, A = proportional limit,
B = elastic limit,
C = yield point (lower)
and D = fracture point.

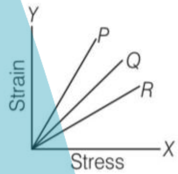
- Point A is *limit of proportionality* and beyond which linear variation of stress and strain ceases. Hooke's law is valid in region OA and it is also called *linear elastic region*.
- Point B is *elastic limit*, *i.e.* the maximum stress upto which a metal wire regains its original shape (length) after removal of applied load. Region AB is called *non-linear elastic region.*
- Point C is called *yield point*, at this point yielding of the wires begins, *i.e.* even if the stretching force is removed the wire does not come back to its original length, some permanent increase in length takes place. The behaviour of wire is now plastic.

- Point D is known as a *fracture point*. The stress corresponding to this point is called breaking stress, region BD represents *plastic region*.

If large deformation takes place between elastic limit and the fracture point, the material is called *ductile*. If it breaks soon after the elastic limit is crossed, it is called *brittle*.

Note Elastic limit is the property of a body whereas elasticity is the property of material of a body.

Example 13. *The strain-stress curves of three wires of different materials are shown in the figure. P, Q and R are the elastic limits of the wires, the figure shows that*



(a) *elasticity of wire P is maximum*
(b) *elasticity of wire Q is maximum*
(c) *elasticity of wire R is maximum*
(d) *None of the above*

Sol. (c) As stress is shown on X-axis and strain on Y-axis.

So, we can say that $y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

So, elasticity of wire P is minimum and of wire R is maximum.

Important Points for Modulus of Elasticity (Y, K and η)

- The value of modulus of elasticity (Y, K and η) is independent of the magnitude of the stress and strain. It depends only on the nature of the material of the body.
- There are three modulii of elasticity, *i.e.* Y, K and η while elastic constants are four, *i.e.* Y, K, η and σ. Poisson's ratio σ is not modulus of elasticity as it is the ratio of two strains and not of stress to strain.

  Elastic constants are found to depend on each other through the relations

  $$Y = 3K(1-2\sigma) \quad \text{and} \quad Y = 2\eta(1+\sigma)$$

  Eliminating σ or Y between these, we get

  $$Y = \frac{9K\eta}{3K+\eta} \quad \text{and} \quad \sigma = \frac{3K-2\eta}{6K+2\eta}$$

- The modulii of elasticity has same dimensional formula and units as that of stress, since strain is dimensionless, *i.e.* the dimensional formula for Y, K or η is $[ML^{-1}T^{-2}]$, while unit is dyne cm^{-2} or Nm^{-2}.
- Greater the value of modulii of elasticity, more elastic is the material. As $Y \propto \left(\frac{1}{\Delta l}\right)$, $K \propto \frac{1}{\Delta V}$ and $\eta \propto \left(\frac{1}{\phi}\right)$.
- The modulii of elasticity Y and η exist only for solids as liquids and gases cannot be deformed along one dimension only and also cannot sustain shear strain. However, K exists for all states of matter, *i.e.* solid, liquid and gas.
- Gases being most compressible are least elastic while solids are most, *i.e.* the bulk modulus of gases is very low while that for liquids and solids is very high. *i.e.*

  $$E_{\text{solid}} > E_{\text{liquid}} > E_{\text{gas}}$$

- If a liquid of density ρ, volume V and Bulk modulus K is compressed, then its density increases.

  As density, $\rho = \frac{M}{V}$

  So, $$\frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V} \quad \text{...(i)}$$

  But by definition of Bulk modulus,

  $$K = -\frac{-V\,\Delta p}{\Delta V}$$

  $$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta p}{K} \quad \text{...(ii)}$$

  From Eqs. (i) and (ii), we get

  $$\frac{\Delta\rho}{\rho} = \frac{\rho' - \rho}{\rho} = \frac{\Delta p}{K}$$

  or $$\rho' = \rho\left(1 + \frac{\Delta p}{K}\right) = \rho\,(1 + C\,\Delta p)$$

Work Done or Potential Energy Stored in a Stretched Wire

When a wire is stretched, work is done against the interatomic forces. This work is stored in the wire in the form of elastic potential energy.

Let us consider a wire of length l and the cross-sectional area A. If a force F acts along the length of the wire and stretches it by x, then

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{x/l} = \frac{Fl}{Ax}$$

$$F = \frac{YA}{l}x$$

So, work done for an additional small increase dx in length,

$$dW = F\,dx = (YA/l)\,x\,dx$$

So, total work done in increasing the length by Δl,

$$W = \int_0^{\Delta l} \frac{YA}{l}\,x\,dx$$

$$W = \frac{1}{2}\frac{YA}{l}(\Delta l)^2$$

$\therefore$ Work done per unit volume,

$$\frac{W}{V} = \frac{1}{2}Y\left(\frac{\Delta l}{l}\right)^2 = \frac{1}{2}Y\,(\text{strain})^2$$

$$\left(\because V = Al \text{ and strain} = \frac{\Delta l}{l}\right)$$

or $$\frac{W}{V} = \frac{1}{2} Y \times \text{Strain} \times \text{Strain}$$

$$\left(\because Y = \frac{\text{Stress}}{\text{Strain}} \quad \text{or } Y \times \text{Strain} = \text{Stress}\right)$$

or $$\frac{W}{V} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

or $$\frac{W}{Al} = \frac{1}{2}\frac{F}{A} \times \frac{\Delta l}{l}$$

or $$W = \frac{1}{2} F \Delta l$$

$$W = \frac{1}{2} \text{Load} \times \text{Elongation}$$

Thus, stored elastic energy is

$$U = \frac{1}{2} \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$U = \frac{1}{2} Y (\text{Strain})^2 \times \text{Volume}$$

$$U = \frac{1}{2} \text{Load} \times \text{Elongation}$$

Example 14. *Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is* **[JEE Main 2020]**

(a) $\sqrt{2} : 1$ (b) $1 : \sqrt{2}$
(c) 2 : 1 (d) 1 : 2

Sol. (a) Elastic potential energy stored in a loaded wire,

$$U = \frac{1}{2} (\text{Stress} \times \text{Strain} \times \text{Volume})$$

$\therefore$ Energy stored per unit volume,

$$u = \frac{U}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right)^2 \times \frac{1}{Y}$$

Here, both wires are of same material and under same load, so the ratio of stored energies per unit volume, for both the wires will be

$$\frac{u_A}{u_B} = \frac{\frac{1}{2Y} \cdot \frac{F^2}{A_A^2}}{\frac{1}{2Y} \cdot \frac{F^2}{A_B^2}} = \frac{A_B^2}{A_A^2}$$

$\Rightarrow$ $$\frac{u_A}{u_B} = \frac{d_B^4}{d_A^4} \qquad \left(\because A = \pi \frac{d^2}{4}\right)$$

Here, $$\frac{u_A}{u_B} = \frac{1}{4}$$

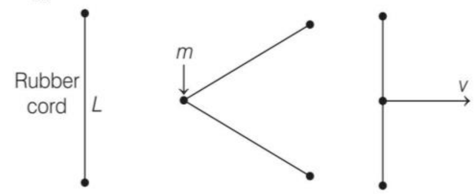
So, $$\frac{d_B^4}{d_A^4} = \frac{1}{4} \quad \text{or} \quad \frac{d_B}{d_A} = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\frac{d_A}{d_B} = \sqrt{2} : 1$$

Example 15. *A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to* **[JEE Main 2019]**

(a) $10^6 Nm^{-2}$ (b) $10^4 Nm^{-2}$ (c) $10^8 Nm^{-2}$ (d) $10^3 Nm^{-2}$

Sol. (a) When rubber cord is stretched, it stores potential energy and when released, this potential energy is given to the stone as kinetic energy.



So, potential energy of stretched cord
= kinetic energy of stone

$$\Rightarrow \quad \frac{1}{2} Y \left(\frac{\Delta L}{L}\right)^2 A \cdot L = \frac{1}{2} m v^2$$

Here, $\Delta L = 20 \text{ cm} = 0.2 \text{ m}$, $L = 42 \text{ cm} = 0.42 \text{ m}$,
$v = 20 \text{ ms}^{-1}$, $m = 0.02 \text{ kg}$
$d = 6 \text{ mm} = 6 \times 10^{-3} \text{m}$

$$\therefore \quad A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{6 \times 10^{-3}}{2}\right)^2$$

$$= \pi (3 \times 10^{-3})^2$$

$$= 9\pi \times 10^{-6} \text{ m}^2$$

On substituting values, we get

$$Y = \frac{mv^2 L}{A(\Delta L)^2} = \frac{0.02 \times (20)^2 \times 0.42}{9\pi \times 10^{-6} \times (0.2)^2}$$

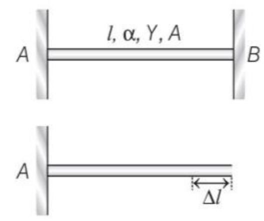
$$\approx 3.0 \times 10^6 \text{Nm}^{-2}$$

So, the closest value of Young's modulus is 10^6 Nm^{-2}.

Thermal Stresses and Strains

When a body is allowed to expand or contract with increasing temperature or decreasing temperature, no stresses are induced in the body.

But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called *thermal stresses* or *temperature stresses*. The corresponding strains are called *thermal strains* or *temperature strains*.



A body having linear dimensions is shown in above figure.

Let the temperature of the rod be increased by an amount t. The length of the rod would increase by an amount Δl, if it is not fixed at two supports. Here,

$$\Delta l = l\alpha t$$

But since the rod is fixed at the supports, a compressive strain will be produced in the rod.

Because at the increased temperature, the natural length of the rod is $l + \Delta l$, while being fixed at two supports, its actual length is l. Hence, thermal strain,

$$\varepsilon = \frac{\Delta l}{l} = \frac{l\alpha t}{l} = \alpha t$$

or $$\varepsilon = \alpha t$$

Therefore, thermal stress,

$$S = Y\varepsilon \qquad (\because \text{Stress} = Y \times \text{Strain})$$

or $$S = Y\alpha t$$

Example 16. *A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume temperature expansion of the material of the rod, is (nearly) equal to* **[JEE Main 2019]**

(a) $9F/(\pi r^2 YT)$

(b) $6F/(\pi r^2 YT)$

(c) $3F/(\pi r^2 YT)$

(d) $F/(3\pi r^2 YT)$

Sol. (c) As length of rod remains unchanged,

Strain caused by compressive forces is equal and opposite to the thermal strain.

Now, compressive strain is obtained by using formula for Young's modulus,

$$Y = \frac{F/A}{\Delta l / l}$$

Compressive strain, $$\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi Y r^2} \qquad \text{...(i)}$$

Also, thermal strain in rod is obtained by using formula for expansion in rod,

$$\Delta l = l\,\alpha\,\Delta T$$

Thermal strain, $$\frac{\Delta l}{l} = \alpha\,\Delta T \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{F}{\pi r^2 Y} = \alpha T \qquad [\because \Delta T = T]$$

$$\Rightarrow \qquad \alpha = \frac{F}{\pi r^2 YT}$$

Hence, coefficient of volume temperature expansion of rod is

$$\gamma = 3\alpha = \frac{3F}{\pi r^2 YT}$$

Example 17. *A rod of length L at room temperature and uniform area of cross-section A, is made of a metal having coefficient of linear expansion α /°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus Y for this metal is* **[JEE Main 2019]**

(a) $\frac{F}{2A\alpha\,\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$

(c) $\frac{2F}{A\alpha\Delta T}$ (d) $\frac{F}{A\alpha\Delta T}$

Sol. (d) If a rod of length L and coefficient of linear expansion $\alpha/^\circ$C, then with the rise in temperature by ΔT K, its change in length is given as

$$\Delta L = L\,\alpha\,\Delta T$$

$$\Rightarrow \qquad \frac{\Delta L}{L} = \alpha\,\Delta T \qquad \text{...(i)}$$

Also, when a rod is subjected to some compressive force (F), then its' Young's modulus is given as

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L}$$

$$\frac{\Delta L}{L} = \frac{F}{YA} \qquad \text{...(ii)}$$

Since, it is given that the length of the rod does not change. So, from Eqs. (i) and (ii), we get

$$\alpha\,\Delta T = \frac{F}{YA}$$

$$\Rightarrow \qquad Y = \frac{F}{A\alpha\,\Delta T}$$

Practice Exercise

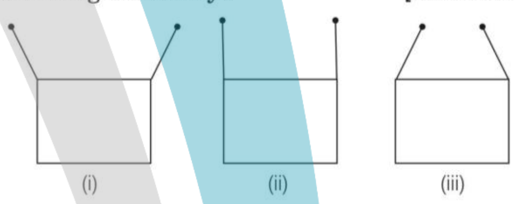
ROUND I Topically Divided Problems

Stress, Strain and Hooke's Law

1. A steel cable with a radius of 1.5 cm supports a chair lift at a ski area. If the maximum stress is not to exceed 10^8 N/m^2, what is the maximum load the cable can support? **[NCERT]**

(a) 7×10^5 N (b) 7×10^6 N
(c) 7×10^4 N (d) 9×10^5 N

2. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports (Fig.) It can be done in one of the following three ways **[NCERT Exemplar]**



The tension in the strings will be
(a) the same in all cases
(b) least in (i)
(c) least in (ii)
(d) least in (iii)

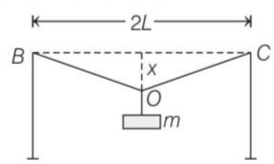
3. Two identical wires of rubber and iron are stretched by the same weight, then the number of atoms in unit volume of iron wire will be
(a) equal to that of rubber
(b) less than that of the rubber
(c) more than that of the rubber
(d) None of the above

4. A force F is required to break a wire of length l and radius r. What force is required to break a wire, of the same material, having twice the length and six times the radius?
(a) F (b) $3F$
(c) $9F$ (d) $36F$

5. A substance breaks down by a stress of 10^6 Nm^{-2}. If the density of the material of the wire is 3×10^3 kg m^{-3}, then the length of the wire of the substance which will break under its own weight when suspended vertically is
(a) 66.6 m (b) 60.0 m
(c) 33.3 m (d) 30.0 m

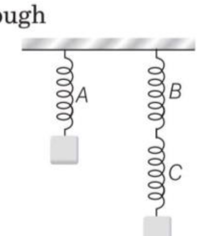
6. A body of mass m = 10 kg is attached to a wire of length 0.3 m. The maximum angular velocity with which it can be rotated in a horizontal circle is (Breaking stress of wire = 4.8×10^7 Nm^{-2} and area of cross-section of a wire = 10^{-6} m^2)
(a) 4 rads^{-1} (b) 8 rads^{-1}
(c) 1 rads^{-1} (d) 2 rads^{-1}

7. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars as shown in figure. A mass m is suspended from the mid-point of the wire. Strain in the wire is **[NCERT Exemplar]**



(a) $\frac{x^2}{2L^2}$ (b) $\frac{x}{L}$ (c) $\frac{x^2}{L}$ (d) $\frac{x^2}{2L}$

8. In the figure, three identical springs are shown. From spring A, a mass of 4 kg is hung and spring shows elongation of 1 cm. But when a weight of 6 kg is hung on B & C as shown in figure, the hook descends through



(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

9. A load suspended by a massless spring produces an extension of x cm, in equilibrium. When it is cut into two unequal parts, the same load produces an extension of 7.5 cm when suspended by the larger part of length 60 cm. When it is suspended by the smaller part, the extension is 5.0 cm. Then
(a) $x = 12.5$
(b) $x = 3.0$
(c) the length of the original spring is 90 cm
(d) the length of the original spring is 80 cm

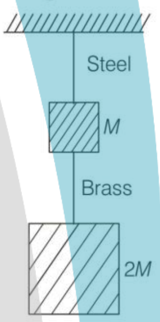
10. The length of an elastic string is a metre when the tension is 4 N and b metre when the tension is 5 N. The length in metre when the tension is 9 N, is
(a) $4a-5b$ (b) $5b-4a$
(c) $9b-9a$ (d) $a+b$

Types of Modulus of Elasticity

11. The ratio of diameters of two wires of same materials is n : 1. The length of each wire is 4 m. On applying the same load, the increase in length of thin wire will be (n > 1)
(a) n^2 times (b) n times
(c) $2n$ times (d) $(2n+1)$ times

12. A rigid bar of mass M is supported symmetrically by three wires each of length l. Those at each end are of copper and the middle one is of iron. The ratio of their diameter, if each is to have the same tension, is equal to [NCERT Exemplar]
(a) Y_{copper}/Y_{iron} (b) $\sqrt{\dfrac{Y_{iron}}{Y_{copper}}}$
(c) $\dfrac{Y_{iron}^2}{Y_{copper}^2}$ (d) $\dfrac{Y_{iron}}{Y_{copper}}$

13. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is [JEE Main 2013]



(a) $\dfrac{3c}{2ab^2}$ (b) $\dfrac{2a^2c}{b}$
(c) $\dfrac{3a}{2b^2c}$ (d) $\dfrac{2ac}{b^2}$

14. In an experiment, brass and steel wires of length 1 m each with areas of cross-section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is

[Take, the Young's modulus for steel and brass as $120\times10^9\,Nm^{-2}$ and $60\times10^9\,Nm^{-2}$, respectively] [JEE Main 2019]
(a) $1.2\times10^6\,Nm^{-2}$ (b) $0.2\times10^6\,Nm^{-2}$
(c) $8\times10^6\,Nm^{-2}$ (d) $4.0\times10^6\,Nm^{-2}$

15. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is $16\times10^{11}\,Nm^{-2}$, the extension of wire over its natural length is [JEE Main 2020]
(a) 0.01 mm (b) 0.04 mm
(c) 0.03 mm (d) 0.02 mm

16. A thick rope of rubber of density $1.5\times10^3\,kgm^{-3}$ and Young's modulus $5\times10^6\,Nm^{-2}$, 8 m in length is hung from the ceiling of a room, the increase in its length due to its own weight is
(a) 9.6×10^{-2} m (b) 19.2×10^{-2} m
(c) 9.6×10^{-3} m (d) 9.6 m

17. A substance breaks down by a stress of $10^6\,Nm^{-2}$. If the density of the material of the wire is $3\times10^3\,kgm^{-3}$, then the length of the wire of that substance which will break under its own weight when suspended vertically is nearly
(a) 3.4 m (b) 34 m (c) 340 m (d) 3400 m

18. A copper wire of negligible mass, 1 m length and cross-sectional area 10^{-6} is kept on a smooth horizontal table with one end fixed. A ball of mass 1 kg is attached to the other end. The wire and the ball are rotated with an angular velocity 20 rad s^{-1}. If the elongation in the wire is 10^{-3} m, then the Young's modulus is
(a) $4\times10^{11}\,Nm^{-2}$ (b) $6\times10^{11}\,Nm^{-2}$
(c) $8\times10^{11}\,Nm^{-2}$ (d) $10\times10^{11}\,Nm^{-2}$

19. Two wires of the same material and length are stretched by the same force. Their masses are in the ratio 3:2. Their elongations are in the ratio
(a) 3 : 2 (b) 9 : 4
(c) 2 : 3 (d) 4 : 9

20. The dimensions of four wires of the same material are given below. In which wire, the increase in length will be maximum?
(a) Length 100 cm, Diameter 1 mm
(b) Length 200 cm, Diameter 2 mm
(c) Length 300 cm, Diameter 3 mm
(d) Length 50 cm, Diameter 0.5 mm

21. Two wires of the same length and same material but radii in the ratio of 1 : 2 are stretched by unequal forces to produce equal elongation. The ratio of the two forces is
(a) 1 : 1 (b) 1 : 2
(c) 2 : 3 (d) 1 : 4

22. Two wires of the same material have lengths in the ratio 1 : 2 and their radii are in the ratio $1:\sqrt{2}$. If they are stretched by applying equal forces, the increase in their lengths will be in the ratio of
(a) $\sqrt{2}:2$ (b) $2:\sqrt{2}$ (c) 1:1 (d) 1:2

23. When a weight of 5 kg is suspended from a copper wire of length 30 m and diameter 0.5 mm, the length of the wire increases by 2.4 cm. If the diameter is doubled, the extension produced is
(a) 1.2 cm (b) 0.6 cm (c) 0.3 cm (d) 0.15 cm

24. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force F, its length increases by l. Another wire of the same material of length 4 L, radius $4r$ is pulled by a force $4F$. The increase in length will be
(a) $\frac{l}{2}$ (b) l
(c) $2l$ (c) $4l$

25. A wire extends by 1 mm when a force is applied. Double the force is applied to another wire of same material and length but half the radius of cross-section. The elongation of the wire (in mm) will be
(a) 8 (b) 4
(c) 2 (d) 1

26. A steel ring of radius r and cross-sectional area A is fitted on a wooden disc of radius $R (R > r)$. If Young's modulus be E, then the force with which the steel ring is expanded is
(a) $AE\frac{R}{r}$ (b) $AE\frac{(R-r)}{r}$
(c) $\frac{E}{A}\left(\frac{R-r}{A}\right)$ (d) $\frac{Er}{AR}$

27. A wire is stretched 1 mm by a force of 1 kN. How far would a wire of the same material and length but of four times that diameter be stretched by the same force ?
(a) $\frac{1}{2}$ mm (b) $\frac{1}{4}$ mm
(c) $\frac{1}{8}$ mm (d) $\frac{1}{16}$ mm

28. Two bars A and B of circular cross-section and of same volume and made of the same material are subjected to tension. If the diameter of A is half that of B and if the force applied to both the rods is the same and it is in the elastic limit, the ratio of extension of A to that of B will be
(a) 16 : 1 (b) 8 : 1
(c) 4 : 1 (d) 2 : 1

29. A steel wire has length 2 m, radius 1 mm and $Y = 2\times10^{11}\,\text{Nm}^{-2}$. A 1 kg sphere is attached to one end of the wire and whirled in a vertical circle with an angular velocity of 2 revolutions per second. When the sphere is at the lowest point of the vertical circle, the elongation of the wire is nearly (Take, $g = 10\ \text{ms}^{-2}$)
(a) 1 mm (b) 2 mm
(c) 0.1 mm (d) 0.01 mm

30. Two wires of equal cross-section but one made of steel and the other of copper are joined end-to-end. When the combination is kept under tension, the elongations in the two wires are found to be equal. What is the ratio of the lengths of the two wires? (Given, Y for steel $= 2\times10^{11}\text{Nm}^{-2}$ and Y for copper $= 1.1\times10^{11}\,\text{Nm}^{-2}$)
(a) 2 : 11 (b) 11 : 2
(c) 20 : 11 (d) 11 : 20

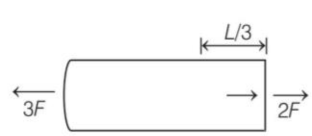
31. The Young's modulus of brass and steel are $10\times10^{10}\,\text{Nm}^{-2}$ and $2\times10^{11}\,\text{Nm}^{-2}$, respectively. A brass wire and a steel wire of the same length are extended by 1 mm under the same force. The radii of the brass and steel wires are R_B and R_S respectively, then
(a) $R_S = \sqrt{2}R_B$ (b) $R_S = \frac{R_B}{\sqrt{2}}$
(c) $R_S = 4R_B$ (d) $R_S = \frac{R_B}{4}$

32. When the tension in a metal wire is T_1, its length is l_1. When the tension is T_2, its length is l_2. The natural length of wire is
(a) $\frac{T_2}{T_1}(l_1 + l_2)$ (b) $T_1 l_1 + T_2 l_2$
(c) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\frac{l_1T_2 + l_2T_1}{T_2 + T_1}$

33. Two wires, one made of copper and other of steel are joined end-to-end (as shown in figure). The area of cross-section of copper wire is twice that of steel wire.

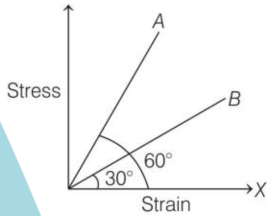
They are placed under compressive force of magnitudes F. The ratio for their lengths such that change in lengths of both wires are same is ($Y_S = 2\times10^{11}\ \text{Nm}^{-2}$ and $Y_C = 1.1\times10^{11}\ \text{Nm}^{-2}$)
(a) 2 : 1 (b) 1.1 : 1
(c) 1 : 2 (d) 2 : 3

34. A uniform slender rod of length L, cross-sectional area A and Young's modulus Y is acted upon by the forces shown in the figure. The elongation of the rod is



(a) $\frac{3FL}{5AY}$ (b) $\frac{2FL}{5FY}$
(c) $\frac{3FL}{8AY}$ (d) $\frac{8FL}{3AY}$

35. The stress *versus* strain graphs for wires of two materials A and B are as shown in the figure. If Y_A and Y_B are the Young's modulus of the materials, then



(a) $Y_B = 2Y_A$ (b) $Y_A = Y_B$
(c) $Y_B = 3Y_A$ (d) $Y_A = 3Y_B$

36. One end of steel wire is fixed to ceiling of an elevator moving up with an acceleration 2 ms^{-2} and a load of 10 kg hangs from other end. Area of cross-section of the wire is 2 cm^2. The longitudinal strain in the wire is (Take, g = 10 ms^{-2} and $Y = 2 \times 10^{11}$ Nm^{-2})
(a) 4×10^{11} (b) 3×10^{-6}
(c) 8×10^{-6} (d) 2×10^{-6}

37. The upper end of a wire of radius 4 mm and length 10 cm is clamped and its other end is twisted through an angle of 30°. Then angle of shear is
(a) 12° (b) 0.12°
(c) 1.2° (d) 0.012°

38. A rubber rope of length 8 m is hung from the ceiling of a room. What is the increase in length of the rope due to its own weight? (Given, Young's modulus of elasticity of rubber = 5×10^6 Nm^{-2}, density of rubber = 1.5×10^3 kgm^{-3} and g = 10 ms^{-2})
(a) 1.5 mm (b) 6 mm
(c) 24 mm (d) 96 mm

39. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$ is

[JEE Main 2018]

(a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$
(c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$

40. If the compressibility of water is σ per unit atmospheric pressure, then the decrease in volume V due to atmospheric pressure p will be
(a) $\sigma\, p/V$ (b) σpV
(c) σ/pV (d) $\sigma V/p$

41. A cube is compressed at 0°C equally from all sides by an external pressure p. By what amount should the temperature be raised to bring it back to the size it had before the external pressure was applied? (Take, K as bulk modulus of elasticity of the material of the cube and α as the coefficient of linear expansion)
(a) $\frac{p}{K\alpha}$ (b) $\frac{p}{3K\alpha}$
(c) $\frac{3\pi\alpha}{p}$ (d) $\frac{K}{3p}$

42. A cube is shifted to a depth of 100 m in a lake. The change in volume is 0.1%. The bulk modulus of the material is nearly
(a) 10 Pa (b) 10^4 Pa
(c) 10^7 Pa (d) 10^9 Pa

43. A cube is subjected to a uniform volume compression. If the side of the cube decreases by 1% the bulk strain is
(a) 0.01 (b) 0.02
(c) 0.03 (d) 0.06

44. A copper bar of length L and area of cross-section A is placed in a chamber at atmospheric pressure. If the chamber is evacuated, the percentage change in its volume will be (Take, compressibility of copper is 8×10^{-12} m^2 N^{-1} and 1 atm = 10^5 Nm^{-2})
(a) 8×10^{-7} (b) 8×10^{-5}
(c) 1.25×10^{-4} (d) 1.25×10^{-5}

45. The compressibility of water is 6×10^{-10} $N^{-1}m^2$. If one litre is subjected to a pressure of 4×10^7 Nm^{-2}. The decrease in its volume is
(a) 2.4 cc (b) 10 cc
(c) 24 cc (d) 15 cc

46. Forces of 100 N each are applied in opposite directions on the upper and lower faces of a cube of side 20 cm. The upper face is shifted parallel to itself by 0.25 cm. If the side of the cube were 10 cm, then the displacement would be
(a) 0.25 cm (b) 0.5 cm
(c) 0.75 cm (d) 1 cm

47. The Young's modulus of the material of a wire is 6×10^{12} Nm^{-2} and there is no transverse strain in it, then its modulus of rigidity will be
(a) 3×10^{12} Nm^{-2} (b) 2×10^{12} Nm^{-2}
(c) 10^{12} Nm^{-2} (d) None of these

48. Equal torsional torques act on two rods x and y having equal length. The diameter of rod y is twice the diameter of rod x. If θ_x and θ_y are the angles of twist, then $\frac{\theta_x}{\theta_y}$ is equal to
(a) 1 (b) 2 (c) 4 (d) 16

from the body are slower due to frictional drag between the layers.

Newton postulated that the shear stress τ between layers for uniform flow of a fluid in parallel layers is proportional to the time rate of strain.

Remember, in the case of solids, shear stress is proportional to the shear strain. In the case of fluids which flow, shear stress is proportional to the strain rate. Suppose Δx is the relative horizontal displacement, in time interval Δt, of a layer of fluid with respect to the neighbouring layer below at a vertical distance Δy, the shear strain is $\frac{\Delta x}{\Delta y}$. The strain rate is $\frac{1}{\Delta t}\left(\frac{\Delta x}{\Delta y}\right)$. In the limit of infinitesimal values of Δx, Δy and Δt, strain rate $= \frac{d}{dy}\left(\frac{dx}{dt}\right) = \frac{dv}{dy}$, where we have interchanged Δt and Δy, without loss of generality and defined the relative velocity of the upper layer as v. $\frac{dv}{dy}$ is the gradient of the velocity profile. According to Newton's postulate,

$$\tau = -\eta \frac{dv}{dy}$$

where η, the coefficient of proportionality is known as the coefficient of viscosity. Since stress $= \frac{\text{Force}}{\text{area}}$, we may also write the equation as

$$F = -\eta A \frac{dv}{dy}$$

The negative sign indicates that the frictional force is opposite to the direction of flow.

The SI unit of viscosity is pascal second (Pa s) and its dimensional formula is $ML^{-1}T^{-1}$. Pa s is also known as *poiseuille* ($P\ell$).

The CGS unit of viscosity is *poise* (P) and its relation to SI unit is

$$10P = 1 \text{ Pa s} = 1\ P\ell;\ 1 \text{ decapoise} = 1 \text{ poiseuille}$$

$$1 \text{ centipoise, cP} = 0.001 \text{ Pa s}$$

At room temperature, water has viscosity of about 1.0×10^{-3} Pa s and in comparison olive oil has viscosity of about 81×10^{-3} Pa s

Popular usage is that water is "thin" whereas oil is "thick".

Stoke's law

Stokes showed that the viscous force F acting on a sphere of radius a, moving with velocity v through a medium whose coefficient of viscosity is η, is given by

$$F = 6\pi\eta av$$

The formula can be derived by dimensional methods.

We assume that the force is proportional to powers of η, a and v and write $F = K\eta^x a^y v^z$, where K is the dimensionless constant.

The above corresponds to its dimensional equivalent, $MLT^{-2} = (ML^{-1}T^{-1})^x L^y (LT^{-1})^z \Rightarrow MLT^{-2} = M^x L^{-x+y+z} T^{-x-z}$

∴ $x = 1, -x + y + z = 1, -x - z = -2$; Solving, $x = 1, y = 1, z = 1$

∴ Stokes law is $F = K\eta av$

Experiments showed that $K = 6\pi$,

∴ $F = 6\pi\eta av$

Stokes law is derived assuming that

(i) the medium is infinite in extent
(ii) the moving body is small in size
(iii) the moving body is rigid and smooth

Terminal velocity

A small solid sphere of radius a and density ρ falls through an infinite column of highly viscous liquid of density σ. The body initially accelerates and after moving down some distance, attains a steady velocity called the *terminal velocity*.

When the sphere is moving with uniform terminal velocity the force balance equation is

$$6\pi\eta av_T + \frac{4}{3}\pi a^3\sigma g = \frac{4}{3}\pi a^3\rho g$$

Here the first term is the viscous force, the second term is the force of buoyancy and the term on the right is the weight of the sphere. Simplifying, we get, for the terminal velocity v_T.

$$v_T = \frac{2}{9}\left(\frac{\rho - \sigma}{\eta}\right)ga^2$$

49. Two rods A and B of the same material and length have their radii r_1 and r_2, respectively. When they are rigidly fixed at one end and twisted by the same couple applied at the other end, the ratio of the angle of twist at the end of A and the angle of twist at the end of B is

(a) $\frac{r_2^4}{r_1^4}$ (b) $\frac{r_1^4}{r_2^4}$
(c) $\frac{r_2^2}{r_1^2}$ (d) $\frac{r_1^2}{r_2^2}$

Thermal Stress, Strain and Poisson's Ratio

50. A 1 m long wire is stretched without tension at 30°C between two rigid supports. What strain will be produced in the wire if the temperature falls to 0°C ? (Given, $\alpha = 12 \times 10^{-6}\ K^{-1}$)

(a) 36×10^{-5} (b) 64×10^{-5}
(c) 0.78 (d) 0.32

51. Two different wires having lengths L_1 and L_2 and respective temperature coefficients of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is **[JEE Main 2020]**

(a) $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$ (b) $2\sqrt{\alpha_1 \alpha_2}$
(c) $\frac{\alpha_1 + \alpha_2}{2}$ (d) $4\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2 L_1}{(L_2+L_1)^2}$

52. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C, it regains its original length of 0.2 m. The value of M is close to

(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m² respectively, $g = 10\ ms^{-2}$) **[JEE Main 2019]**

(a) 9 kg (b) 0.5 kg (c) 1.5 kg (d) 0.9 kg

53. A uniform wire (Young's modulus $2 \times 10^{11}\ Nm^{-2}$) is subjected to longitudinal tensile stress of $5 \times 10^7\ Nm^{-2}$. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to **[JEE Main 2013]**

(a) 1.0×10^{-4} (b) 1.5×10^{-4}
(c) 0.25×10^{-4} (d) 5×10^{-4}

54. The temperature of a rod of length 1 m and area of cross-section 1 cm² is increased from 0°C to 100°C. If the rod is not allowed to increase in length, the force required will be (Take, $\alpha = 10^{-5}$/°C and $Y = 10^{11}$ N/m²)

(a) 10^3 N (b) 10^4 N
(c) 10^5 N (d) 10^9 N

55. For a given material, the Young's modulus is 2.4 times that of modulus of rigidity. Its Poisson's ratio is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

56. When a rubber cord is stretched, the change in volume with respect to change in its linear dimensions is negligible. The Poisson's ratio for rubber is

(a) 1 (b) 0.25 (c) 0.5 (d) 0.75

Potential Energy Stored in Stretched Wire

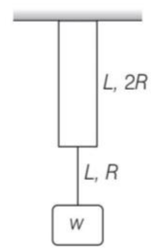
57. Modulus of rigidity of ideal liquids is **[NCERT Exemplar]**

(a) infinity
(b) zero
(c) unity
(d) some finite small non-zero constant value

58. A spring is extended by 30 mm when a force of 1.5 N is applied to it. Calculate the energy stored in the spring when hanging vertically supporting a mass of 0.20 kg, if the spring was relaxed before applying the mass.

(a) 0.01 J (b) 0.02 J (c) 0.04 J (d) 0.08 J

59. Two wires of the same material (Young's modulus Y) and same length L but radii R and $2R$ respectively are joined end-to-end and a weight w is suspended from the combination as shown in the figure. The elastic potential energy in the system is



(a) $\frac{3w^2L}{4\pi R^2 Y}$ (b) $\frac{3w^2L}{8\pi R^2 Y}$ (c) $\frac{5w^2L}{8\pi R^2 Y}$ (d) $\frac{w^2L}{\pi R^2 Y}$

60. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then, the elastic energy stored in the wire is

(a) 0.2 J (b) 10 J
(c) 20 J (d) 0.1 J

61. A height spring extends 40 mm when stretched by a force of 10 N and for tensions upto this value, the extension is proportional to the stretching force. Two such springs are joined end-to-end and the double length spring is stretched 40 mm beyond its natural length. The total strain energy (in joule) stored in the double spring is

(a) 0.05 (b) 0.10 (c) 0.80 (d) 0.40

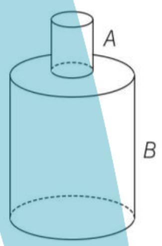
62. The force constant of a wire is k and that of another wire of the same material is $2k$. When both the wires are stretched, then work done is

(a) $W_2 = 1.5\,W_1$ (b) $W_2 = 2\,W_1$
(c) $W_2 = W_1$ (d) $W_2 = 0.5\,W_1$

63. If the shear modulus of a wire material is 5.9×10^{11} dyne cm^{-2}, then the potential energy of a wire of 4×10^{3} cm in diameter and 5 cm long twisted through an angle of 10°, is

(a) 1.253×10^{-12} J (b) 2.00×10^{-12} J
(c) 1.00×10^{-12} J (d) 0.8×10^{-12} J

64. Two cylinders of same material and of same length are joined end-to-end as shown in figure. The upper end of A is rigidly fixed and their radii are in ratio of 1 : 2. If the lower end of B is twisted by an angle θ, the angle of twist of cylinder A is



(a) $\frac{15}{16}\theta$ (b) $\frac{16}{15}\theta$ (c) $\frac{16}{17}\theta$ (d) $\frac{17}{16}\theta$

65. Two wires of the same material and length but diameters in the ratio 1 : 2 are stretched by the same force. The potential energy per unit volume for the two wires when stretched will be in the ratio

(a) 16 : 1 (b) 4 : 1
(c) 2 : 1 (d) 1 : 1

66. If the work done in stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of same material but with double radius of cross-section and half the length by 1 mm is

(a) $\frac{1}{4}$ J (b) 4 J
(c) 8 J (d) 16 J

67. A copper wire 2 m long is stretched by 1 mm. If the energy stored in the stretched wire is converted to heat, calculate the rise in temperature of the wire. (Take, $Y = 12 \times 10^{11}$ dyne cm^{-2}, density of copper = 9 gcm^{-3} and specific heat of copper = 0.1 cal g^{-1}°C^{-1})

(a) 252°C (b) (1/252)°C
(c) 1000°C (d) 2000°C

68. A wire ($Y = 2 \times 10^{11}$ Nm^{-2}) has length 1 m and cross-sectional area 1 mm^2. The work required to increase the length by 2 mm is

(a) 0.4 J (b) 4 J
(c) 40 J (d) 400 J

69. The increase in length on stretching a wire is 0.05%. If its Poisson's ratio is 0.4, the diameter is reduced by

(a) 0.01% (b) 0.02%
(c) 0.03% (d) 0.04%

70. The following data were obtained when a wire was stretched within the elastic region

Force applied to wire = 100 N
Area of cross-section of wire = 10^{-6} m^2
Extension of wire = 2×10^{-3} m

Which of the following deductions can be correctly made from this data?

I. The value of Young's modulus is 10^{11} Nm^{-2}.
II. The strain is 10^{-3}.
III. The energy stored in the wire when the load is applied is 10 J.

(a) I, II and III (b) I and II
(c) Only I (d) Only III

71. When a 4 kg mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 cm. The work required to be done by an external agent in stretching this spring by 5 cm will be

(a) 4.9 J (b) 2.45 J
(c) 0.495 J (d) 0.245 J

72. Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods, then **[NCERT Exemplar]**

(a) Both the rods will elongate but there shall be no perceptible change in shape
(b) the steel rod will elongate and change shape but the rubber rod will only elongate
(c) the steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse.
(d) the steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre

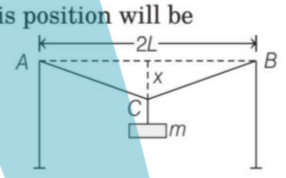
ROUND II Mixed Bag

Only One Correct Option

1. A load of 4.0 kg is suspended from a ceiling through a steel wire of length 2.0 m and radius 2.0 mm. It is found that, the length of the wire increases by 0.031 mm as equilibrium is achieved.
Taking, $g = 3.1\,\pi\ \text{ms}^{-2}$, the Young's modulus of steel is

(a) $2.0 \times 10^{8}\ \text{Nm}^{-2}$ (b) $2.0 \times 10^{9}\ \text{Nm}^{-2}$
(c) $2.0 \times 10^{11}\ \text{Nm}^{-2}$ (d) $2.0 \times 10^{13}\ \text{Nm}^{-2}$

2. A wire of length $2L$ and radius r is stretched between A and B without the application of any tension. If Y is the Young modulus of the wire and it is stretched like ACB, then the tension in the wire in this position will be



(a) $\dfrac{\pi r^2 Y d^3}{2L^2}$ (b) $\dfrac{\pi r^2 Y d^2}{2L^2}$
(c) $\dfrac{\pi^2 r^2 Y \cdot 2L^2}{d^2}$ (d) $\dfrac{\pi r^2 Y \cdot 2L}{d}$

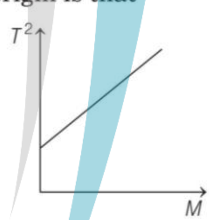
3. One end of a uniform rod of mass m_1, uniform area of cross-section A is suspended from the roof and mass m_2 is suspended from the other end. What is the stress at the mid-point of the rod?

(a) $(m_1 + m_2)g / A$ (b) $(m_1 - m_2)g / A$
(c) $\left[\dfrac{(m_1/2) + m_2}{A}\right]g$ (d) $\left[\dfrac{m_1 + (m_2/2)}{A}\right]g$

4. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains the same, the stress in the leg will change by a factor of **[JEE Main 2017]**

(a) $\dfrac{1}{9}$ (b) 81 (c) $\dfrac{1}{81}$ (d) 9

5. The graph shown was obtained from the experimental measurements of the period of oscillation T for different masses M placed in the scale pan on the lower end of the spring balance. The most likely reason for the line not passing through the origin is that

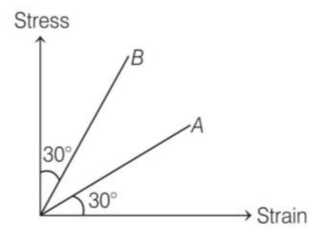


(a) spring did not obey Hooke's law
(b) amplitude of oscillation was too large
(c) clock used needed regulation
(d) mass of the pan was not neglected

6. A rectangular bar 2 cm in breadth and 1 cm in depth and 100 cm in length is supported at its ends and a load of 2 kg is applied at its middle. If Young's modulus of the material of the bar is $20 \times 10^{11}\ \text{dyne cm}^{-2}$, the depression in the bar is

(a) 0.2450 cm (b) 0.3675 cm
(c) 0.1225 cm (d) 0.9800 cm

7. Find the ratio of Young's modulus of wire A to wire B.



(a) 1 : 1 (b) 1 : 1
(c) 1 : 3 (d) 1 : 4

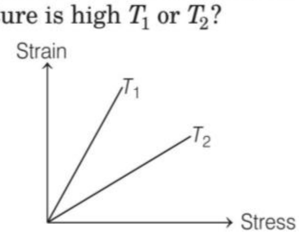
8. If work done in stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of same material, but double the radius and half length by 1 mm is

(a) (1/4) J (b) 4 J
(c) 8 J (d) 16 J

9. A steel wire of length 20 cm and uniform cross-section 1 mm² is tied rigidly at both the ends. The temperature of the wire is altered from 40°C to 20°C. Coefficient of linear expansion of steel is $\alpha = 1.1 \times 10^{-5}\ °\text{C}^{-1}$ and Y for steel is $2.0 \times 10^{11}\ \text{Nm}^2$, the tension in the wire is

(a) 2.2×10^6 N (b) 16 N
(c) 8 N (d) 44 N

10. The stress-strain graph for a metallic wire is shown at two different temperatures T_1 and T_2. Which temperature is high T_1 or T_2?



(a) $T_1 > T_2$ (b) $T_2 > T_1$
(c) $T_1 = T_2$ (d) None of these

11. A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight, then **[NCERT Exemplar]**

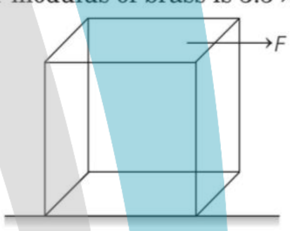
(a) tensile stress at any cross section A of the wire is F/A
(b) tensile stress at any cross section is zero
(c) tensile stress at any cross section A of the wire is $2F/A$
(d) tension at any cross section A of the wire is O

12. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end-to-end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are respectively $1.0\times10^{11}\,Nm^{-2}$ and $2.0\times10^{11}\,Nm^{-2}$, the total extension of the composite wire is **[JEE Main 2013]**

(a) 1.75 mm (b) 2.0 mm (c) 1.50 mm (d) 1.25 mm

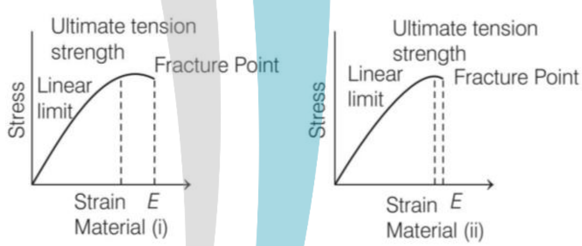
13. Figure shows a 80 cm square brass plate of thickness 0.5 cm. It is fixed at its bottom edge. What tangential force F must be exerted on the upper edge, so that the displacement x of this edge in the direction of force is 0.16 mm?

(The shear modulus of brass is 3.5×10^{10}Pa.)



(a) 2.8×10^{4}N (b) 3.8×10^{-4}N
(c) 5×10^{5}N (d) 4×10^{-5}N

14. The stress-strain graphs for two materials are shown in figure. (assume same scale). **[NCERT Exemplar]**



(a) Material (i) is more elastic than material (ii) and hence material (ii) is more brittle.
(b) Materials (i) and (ii) have the same elasticity and the same brittleness.
(c) Material (ii) is elastic over a larger region of strain as compared to material (i).
(d) Material (ii) is less brittle than material (i).

15. Given, initial stretching force on rod = 100 kN and initial elongation = 1.59×10^{-3} m. What is the increase in elastic potential energy when the stretching force is increased by 200 kN?

(a) 238.5 J (b) 636.0 J
(c) 115.5 J (d) 79.5 J

16. The pressure that has to be applied to the ends of a steel rod of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel Young's modulus is $2\times10^{11}\,Nm^{-2}$ and coefficient of thermal expansion is $1.1\times10^{-5}\,K^{-1}$) **[JEE Main 2014]**

(a) 2.2×10^{8} Pa (b) 2.2×10^{9} Pa
(c) 2.2×10^{7} Pa (d) 2.2×10^{6} Pa

17. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to **[JEE Main 2020]**

(a) 0.06 (b) 2.3
(c) 0.008 (d) 0.8

18. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7×10^{6} Pa. (Take, K for copper = 140×10^{9} Pa) **[NCERT]**

(a) $5\times10^{-7}\,m^3$ (b) $4\times10^{-8}\,m^3$
(c) $5\times10^{-8}\,m^3$ (d) $6\times10^{-8}\,m^3$

19. The bulk modulus of a metal is $8\times10^{9}\,Nm^{-2}$ and its density is 11 gcm^{-2}. The density of this metal under a pressure of 20000 N cm^{-2} will be (in gcm^{-3})

(a) $\frac{440}{39}$ (b) $\frac{431}{39}$
(c) $\frac{451}{39}$ (d) $\frac{40}{39}$

20. The Poisson's ratio of a material is 0.1. If the longitudinal strain of a rod of this material is 10^{-3}, then the percentage change in the volume of the rod will be

(a) 0.008% (b) 0.08%
(c) 0.8% (d) 8%

21. A solid block of silver with density $10.5\times10^{3}\,kg\,m^{-3}$ is subjected to an external pressure of $10^{7}\,Nm^{-2}$. If the bulk modulus of silver is $17\times10^{10}\,Nm^{-2}$, the change in density of silver (in kg m^{-3}) is

(a) 0.61 (b) 1.7
(c) 6.1 (d) 17×10^{3}

22. When a weight w is hung from one end of the wire other end being fixed, the elongation produced in it be l. If this wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be

(a) $4l$ (b) $2l$ (c) l (d) $l/2$

23. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel, if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2, respectively? **[JEE Main 2013]**

(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

24. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes T_M. If the Young's modulus of the material of the wire is Y, then $\frac{1}{Y}$ is equal to (g = gravitational acceleration) **[JEE Main 2015]**

(a) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$
(c) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

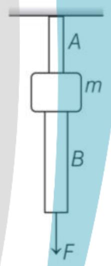
25. The twisting couple per unit twist for a solid cylinder of radius 3 cm is 0.1 N-m. The twisting couple per unit twist, for a hollow cylinder of same material with outer and inner radius 5 cm and 4 cm respectively, will be

(a) 0.1 N-m (b) 0.455 N-m
(c) 0.91 N-m (d) 1.82 N-m

26. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. These at each end are of copper and middle one is of iron. Determine the ratio of their diameters, if each wire is to have the same tension. (Take, Young's modulus of elasticity for copper and steel are 110×10^9 N/m^2 and 190×10^9 N/m^2, respectively.) **[NCERT]**

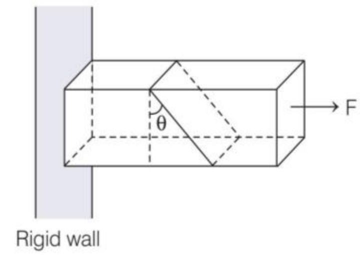
(a) 1 : 1.3 (b) 1.3 : 1 (c) 2.3 : 1.3 (d) 2.3 : 1

27. The wires A and B shown in figure, are made of the same material and have radii r_A and r_B, respectively. A block of mass m is connected between them. When a force F is $mg/3$, one of the wires breaks, then which of the following option is correct?



(a) A will break before B if $r_A > r_B$
(b) A will break before B if $r_A = r_B$
(c) Either A or B will break if $r_A = 2r_B$
(d) The length of A and B must be known to decide which wire will break

28. A uniform rectangular bar of area of cross-section A is fixed at one end and on other end force F is applied as shown in figure. Find the shear stress at a plane through the bar making an angle θ with the vertical as shown in figure.



(a) $\frac{F}{2A}(\cos 2\theta)$ (b) $\frac{F}{2A}$
(c) $\frac{F}{2A}(\sin 2\theta)$ (d) $\frac{F}{2A}\cos\theta$

29. A uniform rod of length L and area of cross-section A is subjected to tensile load F. If σ be Poisson's ratio and Y be the Young's modulus of the material of the rod, then find the volumetric strain produced in rod.

(a) $\frac{F}{AY}(1+2\sigma)$ (b) $\frac{F}{AY}(1-2\sigma)$
(c) Zero (d) None of these

30. A solid sphere of radius r made of a material of Bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid. When a mass m is placed on the piston to compress the liquid, the fractional change in the radius of the sphere $(\Delta r/r)$ is

(a) Ka/mg (b) $Ka/3mg$
(c) $mg/3Ka$ (d) mg/Ka

31. A pendulum clock loses 12 s a day, if the temperature is 40°C and gains 4 s in a day, if the temperature is 20°C. The temperature at which the clock will show correct time and the coefficient of linear expansion (α) of the metal of the pendulum shaft are, respectively **[JEE Main 2016]**

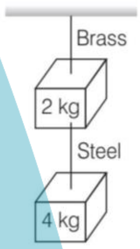
(a) 25°C, $\alpha = 1.85\times10^{-5}$ /°C
(b) 60°C, $\alpha = 1.85\times10^{-4}$ /°C
(c) 30°C, $\alpha = 1.85\times10^{-3}$ /°C
(d) 55°C, $\alpha = 1.85\times10^{-2}$ /°C

32. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area 4.0×10^{-5}m^2 under a given load. The ratio of the Young's modulus of steel to that of copper is **[NCERT]**

(a) 1.2 :1 (b) 1.8 : 1
(c) 1.5 : 1 (d) 1.19 : 1

33. The Poisson's ratio of a material is 0.4. If a force is applied to a wire of this material, there is a decrease of cross-sectional area by 2%. The percentage increase in its length is
(a) 3% (b) 2.5%
(c) 1% (d) 0.5%

34. The ratio of lengths, radii and Young's modulus of steel and brass wires shown in the figure are a, b and c, respectively. The ratio between the increase in length of brass and steel wires would be



(a) $\frac{b^2 a}{2c}$ (b) $\frac{bc}{2a^2}$
(c) $\frac{ba^2}{2c}$ (d) $\frac{c}{2b^2 a}$

35. A wire of cross-sectional area A is stretched horizontally between two clamps loaded at a distance $2l$ metres from each other. A weight w kg is suspended from the mid-point of the wire. The strain produced in the wire, (if the vertical distance through which the mid-point of the wire moves down $x < l$) will be
(a) x^2/l^2
(b) $2x^2/l^2$
(c) $x^2/2l^2$
(d) $x/2l$

36. A boy's catapult is made of rubber cord 42 cm long and 6 mm in diameter. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the cross-section of the cord in stretched position. The stress in the rubber cord is
(a) 1.8×10^6 Nm^{-2}
(b) 1.4×10^6 Nm^{-2}
(c) 2.4×10^5 Nm^{-2}
(d) 1.8×10^5 Nm^{-2}

37. One litre of a gas is maintained at pressure 72 cm of mercury. It is compressed isothermally, so that its volume becomes 900 cm^3. The value of stress and strain will be respectively
(a) 0.106 Nm^{-2} and 0.1 (b) 1.106 Nm^{-2} and 0.1
(c) 106.62 Nm^{-2} and 0.1 (d) 10662.4 Nm^{-2} and 0.1

Numerical Value Questions

38. Wires A and B are made from the same material. A has twice the diameter and three times the length of B. If the elastic limits are not reached, when each is stretched by the same tension, the ratio of energy stored in A to that of B will be ....

39. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that, both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of $a:b$, then a/b can be expressed as $1/x$, where x is .... **[JEE Main 2021]**

40. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency (in Hz) of the transverse vibrations in the wire is (Young's modulus of wire, $Y = 9 \times 10^{10}$ Nm^{-2}), (to the nearest integer) .... **[JEE Main 2020]**

41. A body of mass $m = 10$ kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (breaking stress of wire = 4.8×10^7 Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) .... **[JEE Main 2020]**

42. A solid ball 3 cm in diameter, is submerged in a lake to a depth where the pressure is 10^3 kgfm^{-2}. If Bulk modulus of the material of the ball is 10^7 dyne cm^{-2}, then the change in the volume of ball (in cm^3) will be ....

43. A rubber cube of each side 7 cm has one side fixed, while a tangential force equal to the weight of 300 kgf is applied to the opposite face. If the modulus of rigidity for rubber is 2×10^7 dyne cm^{-2} and $g = 10$ ms^{-2}, then the distance (in cm) through which the strain side moves will be ....

Answers

Round I

1. (c)	2. (c)	3. (c)	4. (d)	5. (c)	6. (a)	7. (a)	8. (c)	9. (a)	10. (b)
11. (a)	12. (b)	13. (c)	14. (c)	15. (c)	16. (a)	17. (b)	18. (a)	19. (c)	20. (d)
21. (d)	22. (c)	23. (b)	24. (b)	25. (a)	26. (b)	27. (d)	28. (a)	29. (a)	30. (c)
31. (b)	32. (c)	33. (b)	34. (d)	35. (d)	36. (b)	37. (c)	38. (d)	39. (c)	40. (b)
41. (b)	42. (d)	43. (c)	44. (b)	45. (c)	46. (b)	47. (a)	48. (d)	49. (a)	50. (a)
51. (a)	52. (a)	53. (c)	54. (b)	55. (b)	56. (c)	57. (b)	58. (c)	59. (c)	60. (d)
61. (b)	62. (b)	63. (a)	64. (c)	65. (a)	66. (d)	67. (b)	68. (a)	69. (b)	70. (b)
71. (b)	72. (d)								

Round II

1. (c)	2. (b)	3. (c)	4. (d)	5. (d)	6. (c)	7. (c)	8. (d)	9. (d)	10. (a)
11. (a)	12. (d)	13. (a)	14. (c)	15. (b)	16. (a)	17. (a)	18. (c)	19. (a)	20. (b)
21. (a)	22. (c)	23. (b)	24. (a)	25. (b)	26. (b)	27. (a)	28. (c)	29. (b)	30. (c)
31. (a)	32. (b)	33. (b)	34. (d)	35. (c)	36. (b)	37. (d)	38. 3 : 4	39. 32	40. 35
41. 4	42. 0.1386	43. 2.1							

Solutions

Round I

1. Given, radius of steel cable, $r = 1.5\text{ cm} = 1.5\times 10^{-2}\text{ m}$

$$\text{Maximum stress} = 10^8\text{ N/m}^2$$

Area of cross-section of steel cable, $A = \pi r^2$

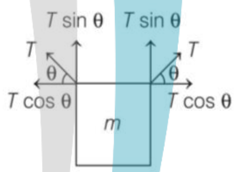
$$= 3.14\times(1.5\times10^{-2})^2\text{ m}^2$$
$$= 3.14\times 2.25\times10^{-4}\text{ m}^2$$

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross-section}}$$

Maximum force = Maximum stress × Area of cross-section

$$= 10^8\times(3.14\times2.25\times10^{-4})\text{ N}$$
$$= 7.065\times10^4\text{ N}$$
$$= 7.1\times10^4\text{ N}\simeq 7\times10^4\text{ N}$$

2. Let m be the mass of rectangular frame and θ be the angle which the tension T in the string make with the horizontal.



Therefore, $2T\sin\theta = mg$

or $$T = \frac{mg}{2\sin\theta}\text{ or } T\propto\frac{1}{\sin\theta}$$

T is least if $\sin\theta$ has maximum value, *i.e.* $\sin\theta = 1 = 90^\circ$ or $\theta = 90^\circ$. i.e. in Fig. (ii).

3. As, rubber is being more stretched as compare to the iron under the action of same weight. Therefore, the number of atoms in unit volume of iron will be more than that of rubber.

4. Breaking force does not depend upon length. Breaking force = Breaking stress × Area of cross-section. For a given material, breaking stress is constant.

$\because$ Breaking force $\propto$ Area of cross-section

$$\Rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{\pi(6r)^2}{\pi r^2} = 36$$

or $$F_2 = 36F_1 = 36F$$

5. Length, $L = \dfrac{\text{Breaking stress}}{\rho g}$

$$L = \frac{10^6}{3\times10^3\times10} = 33.3\text{ m}$$

6. Breaking strength = Tension in the wire = $mr\omega^2$ (centrifugal force)

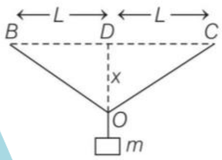
$$\Rightarrow 4.8\times10^7\times10^{-6} = 10\times0.3\times\omega^2$$
$$\omega^2 = \frac{48}{0.3\times10} = 16$$
$$\omega = 4\text{ rads}^{-1}$$

7. According to the figure,

Increase in length $= BO + OC - BC$

$$\Delta L = 2\,BO - 2L$$
$$= 2\,(L^2 + x^2)^{1/2} - 2\,L$$

or $\Delta L = 2L\left[1+\frac{x^2}{L^2}\right]^{1/2} - 2L$



Using binomial theorem, $(1+x)^n = 1 + nx + \ldots +$ higher terms.

$$\Rightarrow \quad \Delta L = 2L\left[1+\frac{x^2}{2L^2}\right] - 2L$$

$$= 2L + 2L \times \frac{x^2}{2L^2} - 2L = \frac{x^2}{L}$$

$$\therefore \quad \text{Strain} = \frac{\Delta L}{2L} = \frac{x^2/L}{2L} = \frac{x^2}{2L^2}$$

8. As, $x = \frac{F}{k}$

If spring constant is k for the first case, it is $\frac{k}{2}$ for second case.

For first case, $1 = \frac{4}{k}$ ...(i)

For second case, $x' = \frac{6}{k/2} = \frac{12}{k}$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$x' = \frac{12/k}{4/k} = 3 \text{ cm}$$

9. Assume original length of spring $= l$

As, $mg = kx$

$\Rightarrow \quad k_1(60) = k_2(l-60) = kl$

According to question,

$\therefore \quad \frac{mg}{k_1} = 7.5$

and $\frac{mg}{k_2} = 5$

$\therefore \quad k_1 = \frac{kl}{60},\ k_2 = \frac{kl}{(l-60)}$

$$\frac{k_1}{k_2} = \frac{5.0}{7.5} = \frac{(l-60)}{60}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{(l-60)}{60}$$

$\therefore \quad l = 100$ cm

and $kx = k_1 \times 7.5$,

$$kx = \left(\frac{5k}{3}\right) \times 7.5$$

$\therefore \quad x = 12.5$ cm

10. As, $T_1 = K(l - l_1)$ ...(i)

and $T_2 = K(l - l_2)$ ...(ii)

So, $\frac{T_1}{T_2} = \frac{l - l_1}{(l - l_2)}$

$\therefore \quad T_1 l - T_1 l_2 = T_2 l - T_2 l_1$

$(T_1 - T_2)\, l = T_1 l_2 - T_2 l_1$

$l = \frac{T_1 l_2 - T_2 l_1}{(T_1 - T_2)} = \frac{(4b - 5a)}{(4-5)}$ ...(i)

$l = (5a - 4b)$ ...(ii)

So from Eqs. (i) and (ii), we get

$$k = \frac{1}{b-a}$$

So, length of wire when tension is 9 N, is given by

$9 = kl'$ (l' = change in length)

$$9 = \frac{1}{(b-a)} \times l'$$

$$l' = 9b - 9a$$

Hence, final length $= l + l'$

$= 5a - 4b + 9b - 9a$

$l_0 = 5b - 4a$

11. As, $Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{Fl}{A\,\Delta l}$

or $Y = \frac{F \times 4}{\pi D^2 \times \Delta l}$

or $\Delta l \propto \frac{1}{D^2}$

or $\frac{\Delta L_2}{\Delta L_1} = \frac{D_1^2}{D_2^2} = \frac{n^2}{1}$ $\left\{\because \frac{D_2}{D_1} = \frac{n}{1}\right\}$

On applying the same load, the increase in length of thin wire will be n^2 times.

12. As, $Y = \frac{mgL}{\pi r^2 l}$ (Y = Young's modulus)

$\Rightarrow \quad Y = \frac{4mgL}{\pi(2r)^2 l} = \frac{4\,mgL}{\pi\,(d)^2 l_1}$

So under same tension,

$\Rightarrow \quad Y \propto \frac{1}{d^2}$

$\Rightarrow \quad d \propto \sqrt{\frac{1}{Y}}$

Then, $d_{\text{copper}} \propto \sqrt{\frac{1}{Y_{\text{copper}}}}$

and $d_{\text{iron}} \propto \sqrt{\frac{1}{Y_{\text{iron}}}}$

So, $\frac{d_{\text{copper}}}{d_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$

13. Given $\frac{l_1}{l_2} = a,\ \frac{r_1}{r_2} = b,\ \frac{y_1}{y_2} = c$

Let Young's modulus of steel be Y_1 and that of brass be Y_2

$$Y_1 = \frac{F_1 l_1}{A_1 \Delta l_1} \quad \text{...(i)}$$

and $$Y_2 = \frac{F_2 l_2}{A_2 \Delta l_2} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii) we get

$$\frac{Y_1}{Y_2} = \frac{F_1 A_2 l_1 \Delta l_2}{F_2 A_1 l_2 \Delta l_1} = \frac{F_1}{F_2} \cdot \frac{\pi r_2^2}{\pi r_1^2} \frac{l_1}{l_2} \cdot \frac{\Delta l_2}{\Delta l_1}$$

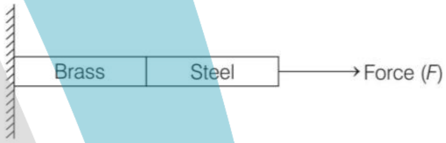
From the figure,

Force on steel wire, $F_1 = 2M + M = 3M$

Force on brass wire, $F_2 = 2M$

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{3a}{2b^2 c}$$

14. In given experiment, a composite wire is stretched by a force F.



Net elongation in the wire = Elongation in brass wire + Elongation in steel wire ...(i)

Now, Young's modulus of a wire of cross-section (A) when some force (F) is applied, $Y = \frac{Fl}{A\Delta l}$

We have,

$$\Delta l = \text{elongation} = \frac{Fl}{AY}$$

So, from Eq. (i), we have

$$\Delta l_{net} = \Delta l_{brass} + \Delta l_{steel}$$

$$\Rightarrow \quad \Delta l_{net} = \left(\frac{Fl}{AY}\right)_{brass} + \left(\frac{Fl}{AY}\right)_{steel}$$

As wires are connected in series and they are of same area of cross-section, length and subjected to same force, so

$$\Delta l_{net} = \frac{F}{A}\left(\frac{l}{Y_{brass}} + \frac{l}{Y_{steel}}\right) \quad \text{...(ii)}$$

Here,

$$\Delta l_{net} = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ m}$$

and $l = 1$ m

$Y_{brass} = 60 \times 10^9 \text{Nm}^{-2}$, $Y_{steel} = 120 \times 10^9 \text{Nm}^{-2}$

On putting the values in Eq (ii), we have

$$0.2 \times 10^{-3} = \frac{F}{A}\left(\frac{1}{60 \times 10^9} + \frac{1}{120 \times 10^9}\right)$$

$$\Rightarrow \quad \text{Stress} = \frac{F}{A} = 8 \times 10^6 \text{Nm}^{-2}$$

15. Speed of transverse wave over a string,

$$v = \sqrt{\frac{T}{\mu}} \quad \text{...(i)}$$

where, T = tension or force on string

and $\mu = \frac{m}{l}$ = mass per unit length.

Also, Young's modulus of string,

$$Y = \frac{Tl}{A\Delta l} \Rightarrow T = \frac{YA\Delta l}{l} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$v^2 = \frac{YA\Delta l}{\mu l} \text{ or } \Delta l = \frac{mv^2}{YA} \quad \text{...(iii)}$$

Here, $m = 6 \text{ g} = 6 \times 10^{-3}$ kg, $l = 60$ cm $= 60 \times 10^{-2}$ m, $A = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$,

$Y = 16 \times 10^{11} \text{ Nm}^{-2}$ and $v = 90 \text{ ms}^{-1}$

Substituting these given values in Eq. (iii), we get

$$\Delta l = \frac{6 \times 10^{-3} \times (90)^2}{16 \times 10^{11} \times 1 \times 10^{-6}} = 3.03 \times 10^{-5} \text{ m}$$

$$\approx 30 \times 10^{-6} \text{ m} = 0.03 \text{ mm}$$

16. If A is the area of cross-section and l is the length of rope, then the mass of rope, $m = Al\rho$. As, the weight of the rope acts at the mid-point of the rope.

So, $$Y = \frac{mg}{A} \times \frac{(l/2)}{\Delta l}$$

$$\Delta l = \frac{mgl}{2AY} = \frac{Al\rho gl}{2AY} = \frac{g\rho l^2}{2Y}$$

or $$\Delta l = \frac{9.8 \times 1.5 \times 10^3 \times 8^2}{2 \times 5 \times 10^6}$$

$$= 9.6 \times 10^{-2} \text{m}$$

17. From the question, $10^6 = \frac{LAdg}{A}$

$$\therefore \quad L = \frac{10^6}{3 \times 10^3 \times 9.8} \text{ m}$$

$$= \frac{1000}{3 \times 9.8} = 34.01 \text{ m} \simeq 34 \text{ m}$$

18. As, $$Y = \frac{Fl}{A\Delta l} = \frac{(ml\omega^2)l}{A\Delta l}$$

or $$Y = \frac{ml^2\omega^2}{A\Delta l}$$

or $$Y = \frac{1 \times 1 \times 1 \times 20 \times 20}{10^{-6} \times 10^{-3}}$$

$$= 4 \times 10^{11} \text{ Nm}^{-2}$$

19. As, $Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l \propto \frac{1}{A}$

Again, $m = Al\rho, m \propto A$

$$\therefore \quad \Delta l \propto \frac{1}{m}$$

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{m_2}{m_1} = \frac{2}{3}$$

20. As, $$Y = \frac{F}{A}\frac{L}{\Delta l}$$

$$\Rightarrow \quad \Delta l \propto \frac{L}{A} \propto \frac{L}{\pi d^2}$$

$\therefore \quad \Delta l \propto \frac{L}{d^2}$

The ratio of $\frac{L}{d^2}$ is maximum for case (d).

21. As, $Y = \frac{Fl}{A\Delta l}$

In the given problem, Y, l and Δl are constants.

$\therefore \quad F \propto A$

or $\quad F \propto \pi r^2 \quad (\because \text{area} = \pi r^2)$

or $\quad F \propto r^2$

or $\quad \frac{F_1}{F_2} = \frac{r_1^2}{r_2^2} = \frac{1}{4}$

22. As, $Y = \frac{Fl}{\pi r^2 \Delta l}$ or $\Delta l = \frac{F}{\pi r^2 Y}$

$\Rightarrow \quad \Delta l \propto \frac{1}{r^2}$ and $\Delta l' \propto \frac{2l}{(\sqrt{2}r)^2}$

$\therefore \quad \Delta l' \propto \frac{1}{r^2}$

Again, $\quad \frac{\Delta l}{\Delta l'} = 1$

23. As, $\quad Y = \frac{Mg \times 4 \times l}{\pi D^2 \times \Delta l} \Rightarrow \Delta l \propto \frac{1}{D^2}$

When D is doubled, Δl becomes one-fourth, *i.e.*

$\frac{1}{4} \times 2.4$ cm, *i.e.* 0.6 cm

24. As, $Y = \frac{FL}{\pi r^2 l}$ (Here, l = change in length)

or $\quad l = \frac{FL}{\pi r^2 Y}$ or $l \propto \frac{FL}{r^2}$

$\Rightarrow \quad \frac{l_1}{l_2} = \frac{\frac{FL}{r^2}}{\frac{(4F) \times (4L)}{(4r)^2}}$

or $\quad \frac{l_1}{l_2} = \frac{F \times L}{r^2} \times \frac{(4r)^2}{4F \times 4L}$

or $\quad l_1 = l_2 = l$

So, l remains unchanged.

25. As, $Y = \frac{Fl}{A\Delta l}$

$\Rightarrow \quad \Delta l \propto \frac{F}{r^2}$

$\Rightarrow \quad \frac{\Delta l_2}{\Delta l_1} = \frac{F_2}{F_1} \times \frac{r_1^2}{r_2^2}$

or $\quad \frac{\Delta l_2}{\Delta l_1} = 2 \times 2 \times 2 = 8$

or $\quad \Delta l_2 = 8\Delta l_1 = 8 \times 1 \text{ mm} = 8 \text{ mm}$

26. Initial length (circumference) of the ring $= 2\pi r$

Final length (circumference) of the ring $= 2\pi R$

Change in length $= 2\pi R - 2\pi r$

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$$

Now Young's modulus,

$$E = \frac{F/A}{l/L} = \frac{F/A}{(R-r)/r}$$

$\therefore \quad F = AE\left(\frac{R-r}{r}\right)$

27. As, $Y = \frac{Fl}{A\Delta l}$

where, Y, l and F are constants.

$\Rightarrow \quad \Delta l \propto \frac{1}{D^2}$

$\Rightarrow \quad \frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = \frac{1}{16}$

$\therefore \quad \Delta l_2 = \frac{1}{16}$ mm $\quad (\because \Delta l_1 = 1 \text{ mm})$

28. We have, $\quad Y = \frac{F}{A} \times \frac{l}{\Delta l} \quad$ ...(i)

and $\quad V = Al$

or $\quad l = \frac{V}{A} \quad$ ...(ii)

$\therefore \quad Y = \frac{FV}{A^2 \Delta l} \quad$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad \Delta l \propto \frac{1}{A^2}$

or $\quad \Delta l \propto \frac{1}{d^4}$

$\Rightarrow \quad \frac{\Delta l_A}{\Delta l_B} = \frac{d_B^4}{d_A^4} = \frac{1^4}{\left(\frac{1}{2}\right)^4} = 16$

29. As, $Y = \frac{(mg + ml\omega^2)\, l}{\pi r^2 \Delta l}$

(because tension at lowest point $T = (mg + m\omega^2 l)$)

or $\quad \Delta l = \frac{m(g + l\omega^2)\, l}{\pi r^2 Y}$

or $\quad \Delta l = \frac{1\,(10 + 2 \times 4\pi^2 \times 4)^2}{\pi\,(1 \times 10^{-3})^2 \times 2 \times 10^{11}}$

or $\quad \Delta l = \frac{(20 + 64 \times 9.88)7}{2 \times 22 \times 10^5}$

$\quad = \frac{4566.24}{44 \times 10^5} \times 10^3 \text{ mm} = 1 \text{ mm}$

30. As, $\quad Y = \frac{\text{stress}}{\text{strain}}$

or $\quad \text{strain} = \frac{\text{stress}}{Y}$

or $\quad \frac{\Delta L}{L} = \frac{\text{stress}}{Y}$

Since, cross-section are equal and same tension exists in both wires, therefore the stresses developed are equal. Also, ΔL is given to be the same for both the wires.

$\therefore \quad L \propto Y$

$\Rightarrow \quad \dfrac{L_s}{L_{Cu}} = \dfrac{Y_s}{Y_{Cu}} = \dfrac{2 \times 10^{11}}{1.1 \times 10^{11}} = \dfrac{20}{11}$

31. As, $\quad Y = \dfrac{F}{\pi R^2} \times \dfrac{l}{\Delta l}$

F, l and Δl are constants.

$\Rightarrow \quad R^2 \propto \dfrac{1}{Y}$

$\Rightarrow \quad \dfrac{R_S^2}{R_B^2} = \dfrac{Y_B}{Y_S} = \dfrac{10^{11}}{2 \times 10^{11}} = \dfrac{1}{2}$

$\therefore \quad \dfrac{R_S}{R_B} = \dfrac{1}{\sqrt{2}}$

or $\quad R_S = \dfrac{R_B}{\sqrt{2}}$

32. We have, $Y = \dfrac{Fl}{A\Delta l}$

where, Y, l and A are constants.

$\therefore \quad \dfrac{F}{\Delta l} = \text{constant} \Rightarrow \Delta l \propto F$

Now, $l_1 - l \propto T_1$ and $l_2 - l \propto T_2$

Dividing, we get $\dfrac{l_1 - l}{l_2 - l} = \dfrac{T_1}{T_2}$

or $\quad l_1T_2 - lT_2 = l_2T_1 - lT_1$

or $\quad l(T_1 - T_2) = l_2T_1 - l_1T_2$

or $\quad l = \dfrac{l_2T_1 - l_1T_2}{T_1 - T_2}$

or $\quad l = \dfrac{l_1T_2 - l_2T_1}{T_2 - T_1}$

33. As, $Y_S = \dfrac{FL_S}{A_S \Delta L_S}$

and $\quad Y_C = \dfrac{FL_C}{A_C \Delta L_C}$

$\therefore \quad \dfrac{L_C}{L_S} = \dfrac{\dfrac{Y_C A_C \Delta L_C}{F}}{\dfrac{Y_S A_S \Delta L_S}{F}} = \left(\dfrac{Y_C}{Y_S}\right)\left(\dfrac{A_C}{A_S}\right)\left(\dfrac{\Delta L_C}{\Delta L_S}\right) \quad \ldots\text{(i)}$

Here, $\quad \dfrac{A_C}{A_S} = 2, \dfrac{\Delta L_C}{\Delta L_S} = 1, \dfrac{Y_C}{Y_S} = \dfrac{1.1}{2}$

Putting the value of ratios in Eq. (i), we get

$\therefore \quad \dfrac{L_C}{L_S} = \dfrac{1.1}{2} \times 2 \times 1 = 1.1$

Hence, $L_C : L_S = 1.1 : 1$

34. Net elongation of the rod is

$$l = \frac{3F\left(\frac{2L}{3}\right)}{AY} + \frac{2F\left(\frac{L}{3}\right)}{AY}$$

$$l = \frac{8FL}{3AY}$$

35. As, $\dfrac{Y_A}{Y_B} = \dfrac{\tan\theta_A}{\tan\theta_B} = \dfrac{\tan 60^\circ}{\tan 30^\circ} = \dfrac{\sqrt{3}}{1/\sqrt{3}} = 3 \Rightarrow Y_A = 3Y_B$

36. Here, $\quad T = m(g + a_0) = 10(10 + 2) = 120\text{ N}$

$\therefore \quad \text{Stress} = \dfrac{T}{A}$

$= \dfrac{120}{2 \times 10^{-4}} = 60 \times 10^4 \text{ Nm}^{-2}$

and $\quad Y = \dfrac{\text{stress}}{\text{strain}}$

$\therefore \quad \text{Strain} = \dfrac{\text{Stress}}{Y}$

$= \dfrac{60 \times 10^4}{2 \times 10^{11}} = 30 \times 10^{-7} = 3 \times 10^{-6}$

37. As, $\phi = \dfrac{r\theta}{l}$; so, $\phi = \dfrac{0.4 \times 30^\circ}{10} = 1.2^\circ$

38. As, $Y = \dfrac{Mg}{A} \times \dfrac{L/2}{\Delta L}$

(Length is taken as $\dfrac{L}{2}$ because weight acts at centre of gravity (CG))

Now, $\quad M = AL\rho$

(For the purpose of calculation of mass, the whole of geometrical length L is to be considered.)

$\therefore \quad Y = \dfrac{Al\rho gL}{2A\,\Delta L}$

or $\quad \Delta L = \dfrac{\rho g L^2}{2Y}$

$= \dfrac{1.5 \times 10^3 \times 10 \times 8 \times 8}{2 \times 5 \times 10^6}$

$= 9.6 \times 10^{-2}$ m

$= 9.6 \times 10^{-2} \times 10^3$ mm

$= 96$ mm

39. $\therefore$ Bulk modulus, $K = \dfrac{\text{Volumetric stress}}{\text{Volumetric strain}} = \dfrac{\Delta p}{\dfrac{\Delta V}{V}}$

$\Rightarrow \quad K = \dfrac{mg}{a\left(\dfrac{3\Delta r}{r}\right)} \quad \left[\because V = \dfrac{4}{3}\pi r^3, \text{ so } \dfrac{\Delta V}{V} = \dfrac{3\Delta r}{r}\right]$

$\Rightarrow \quad \dfrac{\Delta r}{r} = \dfrac{mg}{3aK} = \dfrac{dr}{r}$

40. As, $K = \dfrac{p}{\dfrac{\Delta V}{V}}$ (Here, K = bulk modulus of elasticity)

or $\quad \dfrac{1}{K} = \dfrac{\Delta V/V}{p}$

or $\sigma = \dfrac{\Delta V}{pV}$

or $\Delta V = \sigma p V$

41. As, $K = \dfrac{pV}{\Delta V} = \dfrac{pV}{3V\alpha\Delta T} = \dfrac{p}{3\alpha\Delta T}$

$\Rightarrow \quad \Delta T = \dfrac{p}{3K\alpha}$

42. 10 m column of water exerts nearly 1 atmospheric pressure. So, 100 m column of water exerts nearly 10 atmospheric pressure, *i.e.* 10×10^5 Pa or 10^6 Pa.

$\Rightarrow \quad 10^6 = K\dfrac{\Delta V}{V}$ or $10^6 = K\dfrac{0.1}{100}$

$\Rightarrow \quad K = 10^9$ Pa

43. Let, L be the length of each side of cube, then initial volume $= L^3$. When each side decrease by 1%, then

New length, $L' = L - \dfrac{1}{100} = \dfrac{99L}{100}$

New volume $= L'^3 = \left(\dfrac{99L}{100}\right)^3$,

$\therefore$ Change in volume,

$$\Delta V = L^3 - \left(\frac{99L}{100}\right)^3$$

$$= L^3\left[1 - \left(1 - \frac{3}{100} + \ldots\right)\right] = L^3\left[\frac{3}{100}\right] = \frac{3L^3}{100}$$

$\therefore$ Bulk strain $= \dfrac{\Delta V}{V} = \dfrac{3L^3/100}{L^3} = 0.03$

44. As, $\dfrac{1}{K} = \dfrac{\Delta V/V}{\Delta p}$ or $\dfrac{\Delta V}{V} = \Delta p\left[\dfrac{1}{K}\right]$

or $\dfrac{\Delta V}{V} \times 100 = 10^5 \times 8 \times 10^{-12} \times 100 = 8 \times 10^{-5}$

45. Bulk modulus, $K = -\dfrac{p}{\dfrac{\Delta V}{V}}$

Negative sign shows that an increase in pressure and a decrease in volume occur.

Compressibility, $C = \dfrac{1}{K} = \dfrac{-dV}{pV}$

Decrease in volume, $\Delta V = pVC$

$= 4 \times 10^7 \times 1 \times 6 \times 10^{-10}$

$= 24 \times 10^{-3}$ L

$= 24 \times 10^{-3} \times 10^3$ cm^3 = 24 cc

46. As, $\eta = \dfrac{Fl}{A\Delta l} = \dfrac{Fl}{l^2\Delta l} = \dfrac{F}{l\,\Delta l}$ or $\Delta l \propto \dfrac{1}{l}$ (for, F = constant)

If l is halved, then Δl is doubled, *i.e.* $0.25 \times 2 = 0.5$ cm.

47. As, $\eta = \dfrac{Y}{2(1+\sigma)}$

Now, $\sigma = 0$

then, $\eta = \dfrac{Y}{2} = \dfrac{6 \times 10^{12}}{2} = 3 \times 10^{12}$ Nm^{-2}

48. As, $\tau_x = \dfrac{\pi\eta r^4}{2l}\theta_x$

and $\tau_y = \dfrac{\pi\eta(2r)^4}{2l}\theta_y$

Since, $\tau_x = \tau_y$

$\therefore \quad \theta_x = 16\theta_y$

or $\dfrac{\theta_x}{\theta_y} = 16$

49. As, torque, $\tau = \dfrac{\pi\eta r^4}{2l}\theta$

In the given problem, $r^4\theta$ = constant

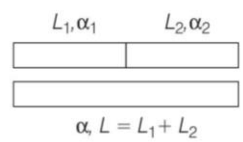
$\Rightarrow \quad \theta \propto \dfrac{1}{r^4}$

$\Rightarrow \quad \dfrac{\theta_A}{\theta_B} = \dfrac{r_2^4}{r_1^4}$

50. Strain = Fractional change in length

$= \dfrac{\Delta l}{l} = \dfrac{l\,\alpha t}{l} = \alpha t = 12 \times 10^{-6} \times 30 = 36 \times 10^{-5}$

51. Let α be the effective temperature coefficient of linear expansion.



Change in length of equivalent wire = Sum of change in length of each wire

$$\Delta L = \Delta L_1 + \Delta L_2$$

$\Rightarrow \quad L\alpha\Delta T = L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$\because \quad L = L_1 + L_2$

$\therefore \quad (L_1 + L_2)\,\alpha\,\Delta T = \Delta T(L_1\alpha_1 + L_2\alpha_2)$

$\Rightarrow \quad \alpha = \dfrac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$

52. Given, $T_1 = 40°$ C and $T_2 = 20°$ C

$\Rightarrow \quad \Delta T = T_1 - T_2 = (40 - 20)°\text{C} = 20°$ C

Also, Young's modulus,

$$Y = 10^{11}\ \text{N/m}^2$$

Coefficient of linear expansion,

$$\alpha = 10^{-5}/°\text{C}$$

Area of the brass wire, $A = \pi \times (10^{-3})^2$ m^2

Now, contraction in the wire due to fall in temperature is

$$\Delta l = l\,\alpha\Delta T$$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \alpha\Delta T \quad \ldots\text{(i)}$

We know that, Young's modulus is defined as

$$Y = \frac{Mgl}{A\Delta l}$$

$\Rightarrow \quad M = \dfrac{YA\Delta l}{gl} \quad \ldots\text{(ii)}$

Using Eq. (i), we get

$$M = \frac{YA}{g} \times \alpha \Delta T$$

$$= \frac{10^{11} \times 22 \times 10^{-6} \times 10^{-5} \times 20}{7 \times 10}$$

$$\Rightarrow \quad M = \frac{22 \times 20}{7 \times 10} = \frac{44}{7} = 6.28 \text{ kg}$$

which is closest to 9, so option (a) is nearly correct.

53. Given, $Y = 2 \times 10^{11}$ N/m^2

$$\text{Stress} = 5 \times 10^7 \text{ N/m}^2$$

As, $\frac{\text{stress}}{\text{strain}} = Y$

$$\Rightarrow \quad \text{Strain} = \frac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}$$

It is symmetric strain.
Now, strain of 2.5×10^{-4} is equivalent.

As, $\frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)$

$$\therefore \quad \frac{2.5 \times 10^{-4}}{3} = \frac{\Delta r}{r} = 0.75 \times 10^{-4}$$

$\therefore$ Fraction decrease in radius

$$= (1.00 - 0.75)\, 10^{-4} = 0.25 \times 10^{-4}$$

54. F = force developed

$$= YA\,\alpha\,(\Delta\theta)$$
$$= 10^{11} \times 10^{-4} \times 10^{-5} \times 100 = 10^4 \text{ N}$$

55. As, $\eta = \frac{Y}{2(1+\sigma)}$

or $\eta = \frac{2.4\eta}{2(1+\sigma)}$

or $1 + \sigma = 1.2$

or $\sigma = 0.2$

56. As volume, $V = \pi r^2 l$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta(\pi r^2 l)}{\pi r^2 l}$$

or $\frac{\Delta V}{V} = \frac{r^2 \Delta l + 2rl\Delta r}{r^2 l}$

or $\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{2\Delta r}{r}$

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{-2\Delta r}{r} \qquad \left(\because \frac{\Delta V}{V} \approx 0\right)$$

Now, Poisson's ratio,

$$\sigma = -\frac{\Delta r / r}{\frac{\Delta l}{l}} = -\frac{\Delta r / r}{-2\frac{\Delta r}{r}} = 0.5$$

57. Modulus of rigidity of ideal liquids is zero, because a liquid at rest begins to move under the effect to tangential force.

58. $K = \frac{1.5 \text{ N}}{30 \times 10^{-3} \text{ m}} = 50 \text{ Nm}^{-1}$ (as, $mg = kx$)

$$\therefore \quad l = \frac{0.2 \times 10}{50} \text{ m} = 0.04 \text{ m}$$

Now, energy stored $= \frac{1}{2} \times 0.20 \times 10 \times 0.04 \text{ J} = 0.04 \text{ J}$

59. As, $k_1 = \frac{Y\pi(2R)^2}{L}$ and $k_2 = \frac{Y\pi(R)^2}{L}$

Since, $k_1 x_1 = k_2 x_2 = w$

Elastic potential energy of the system,

$$U = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$

$$= \frac{1}{2}k_1\left(\frac{w}{k_1}\right)^2 + \frac{1}{2}k_2\left(\frac{w}{k_2}\right)^2$$

$$= \frac{1}{2}w^2\left\{\frac{1}{k_1} + \frac{1}{k_2}\right\} \qquad \text{...(i)}$$

Now, $\frac{1}{k_1} + \frac{1}{k_2} = \frac{L}{4Y\pi R^2} + \frac{L}{Y\pi R^2} \qquad \text{...(ii)}$

$$\therefore \quad U = \frac{1}{2}w^2\left(\frac{5L}{4Y\pi R^2}\right)$$

$$= \frac{5w^2 L}{8\pi Y R^2} \qquad \text{[from Eqs. (i) and (ii)]}$$

60. Elastic energy stored in the wire,

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l} \times Al = \frac{1}{2}F\Delta l$$

or $U = \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1 \text{ J}$

61. As, $mg = kx$

$$k = \frac{10 \text{ N}}{40 \times 10^{-3} \text{ m}} = \frac{1000}{4} \text{ Nm}^{-1} = 250 \text{ Nm}^{-1}$$

Spring constant of combination of two such springs

$$= \frac{250}{2} \text{ Nm}^{-1} = 125 \text{ Nm}^{-1}$$

$$\text{Energy} = \frac{1}{2} \times 125 \times (40 \times 10^{-3})^2 \text{ J} = 0.1 \text{ J}$$

62. Work done in stretching the wire,

$$W = \frac{1}{2} \times \text{force constant} \times x^2$$

For first wire, $W_1 = \frac{1}{2} \times kx^2 = \frac{1}{2}kx^2$

For second wire, $W_2 = \frac{1}{2} \times 2k \times x^2 = kx^2$

Hence, $W_2 = 2W_1$

63. To twist the wire through the angle $d\theta$, it is necessary to do the work

$$dW = \tau d\theta$$

and $\theta = 10' = \frac{10}{60} \times \frac{\pi}{180} = \frac{\pi}{1080}$ rad

$\therefore$ $W = \int dW = \int_0^\theta \tau\, d\theta = \int_0^\theta \frac{\eta\pi r^4\theta\, d\theta}{2l} = \frac{\eta\pi r^4\theta}{4l}$

or $W = \frac{5.9\times10^{11}\times10^{-5}\times\pi(2\times10^{-5})^4\pi^2}{10^{-4}\times4\times5\times10^{-2}\times(1080)^2}$

or $W = 1.253\times10^{-12}$ J

64. For cylinder A, $\tau = \frac{\pi\eta r^4}{2l}\theta'$

For cylinder B, $\tau = \frac{\pi\eta(2r)^4(\theta-\theta')}{2l}$

Since, torque acting both of the cylinders will be equal, so

$$\frac{\pi\eta r^4\theta'}{2l} = \frac{n\eta(2r)^4(\theta-\theta')}{2l} \text{ (for equal torque)}$$

$\Rightarrow$ $\theta' = \frac{16}{17}\theta$

65. As, energy density $= \frac{1}{2}\times\text{stress}\times\text{strain}$

$$= \frac{1}{2}\times\text{stress}\times\frac{\text{stress}}{Y} = \frac{(\text{stress})^2}{2Y} \propto \frac{1}{D^4}$$

$$\left(\because \text{stress} = \frac{\text{force}}{\text{area}} = \frac{4F}{\pi D^2}\right)$$

Now, $\frac{u_A}{u_B} = \frac{D_B^4}{D_A^4} = (2)^4 = 16$

(where, u_A and u_B are energy densities)

Hence, $u_A : u_B = 16:1$

66. As, $W = \frac{1}{2}F\Delta l$ (where, Δl = extension)

$\Rightarrow$ $W = \frac{1}{2}\times\frac{Y\pi r^2\Delta l}{l}\Delta l$ $\left(\because F = \frac{Y\pi r^2\Delta l}{l}\right)$

or $W = \frac{Y\pi r^2\Delta l^2}{2l}$

$\Rightarrow$ $W \propto \frac{r^2}{l}$ and $W' \propto \frac{(2r')^2 2}{l}$

$\Rightarrow$ $\frac{W'}{W} = 8$

or $W' = 8\times2\text{ J} = 16\text{ J}$

67. As, $E = \frac{1}{2}\frac{YA\Delta l^2}{l}$

But $m = Ald$

or $A = \frac{m}{ld}$

$\therefore$ $E = \frac{Ym\Delta l^2}{2l^2d}$

E in calorie $= \frac{Ym\Delta l^2}{2l^2dJ}$

Now, $mS\theta = \frac{Ym\Delta l^2}{2l^2dJ}$

or $\theta = \frac{Y\Delta l^2}{2l^2d\,JS}$

or $\theta = \frac{12\times10^{11}\times10^{-1}\times10^{-3}\times10^{-3}}{2\times2\times2\times9\times10^3\times4.2\times0.1\times10^3}$

$= \frac{12\times10^5}{72\times42\times10^5}$

$= \frac{1}{252}\,^\circ$C

68. As, work done $= \frac{1}{2}F\Delta l = \frac{1}{2}\frac{YA\Delta l^2}{l}$ $\left(\because F = \frac{YA\Delta l}{l}\right)$

$$= \frac{2\times10^{11}\times10^{-6}(2\times10^{-3})^2}{2\times1} = 0.4\text{ J}$$

69. As, $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

or Lateral strain $= \sigma\times$ Longitudinal strain

$= 0.4\times\frac{0.05}{100} = \frac{0.02}{100}$

So, percentage reduction in diameter is 0.02.

70. As, stress $= \frac{100\text{ N}}{10^{-6}\text{ m}^2} = 10^8\text{ Nm}^{-2}$ and

Strain $= \frac{2\times10^{-3}}{2} = 10^{-3}$

$\therefore$ Young modulus, $Y = \frac{10^8}{10^{-3}}\text{ Nm}^{-2} = 10^{11}\text{ Nm}^{-2}$

Thus, energy stored $= \frac{1}{2}\times100\times2\times10^{-3}$ J

$= 10^{-1}\text{ J} = 0.1\text{ J}$

71. As, $mg = kx$

$k = \frac{4\times9.8}{2\times10^{-2}}$ or $k = 19.6\times10^2\text{ Nm}^{-1}$

$\therefore$ Work done $= \frac{1}{2}\times19.6\times10^2\times(5\times10^{-2})^2\text{ J} = 2.45\text{ J}$

72. When a mass M is attached to each of the free ends at the centre of rods, the steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre.

Round II

1. As, $Y = \frac{MgL}{\pi r^2\times l} = \frac{4\times(3.1\,\pi)\times(2.0)}{\pi\times(2\times10^{-3})^2\times(0.031\times10^{-3})}$

$= 2\times10^{11}\text{ Nm}^{-2}$

2. As, $T = \frac{YAl}{L}$ (where, T = force)

Increase in length of one segment of wire,

$$l = \left(L + \frac{1}{2}\frac{d^2}{L}\right) - L = \frac{1}{2}\frac{d^2}{L}$$

So, $T = \frac{Y\pi r^2\cdot d^2}{2L^2}$

3. As, stress = (weight due to mass m_2 + half of the weight of rod)/area

$$= (m_2 g + m_1 g/2)/A = [(m_1/2) + m_2]g/A$$

4. Stress $= \dfrac{\text{Weight}}{\text{Area}}$

Volume will become (9^3) times.

So weight = volume × density × g will also become $(9)^3$ times.

Area of cross-section will become $(9)^2$ times.

$$= \frac{9^3 \times W_0}{9^2 \times A_0} = 9\left(\frac{W_0}{A_0}\right)$$

Hence, the stress increases by a factor of 9.

5. When no weight is placed in pan and T^2 shows some value, it means the pan is not weightless and hence, the mass of the pan cannot be neglected.

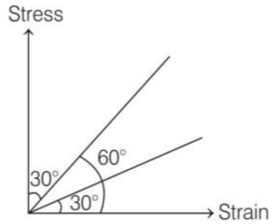
6. Here, $W = 2 \times 1000 \times 980$ dyne; $l = 100$ cm, $b = 2$ cm, $d = 1$ cm, $Y = 20 \times 10^{11}$ dyne cm^{-2},

Now, $\delta = \dfrac{Wl^3}{4Ybd^3}$

$$= \frac{(2 \times 1000 \times 980) \times (100)^3}{4 \times (20 \times 10^{11}) \times 2 \times (1)^3}$$

$= 0.1225$ cm

7. The graph is given as follows



As, $Y = \tan\theta$

$$\therefore \quad \frac{Y_A}{Y_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \quad \left\{\because \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ and } \tan 60^\circ = \sqrt{3}\right\}$$

8. As, work done $= \dfrac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$

$$2 = \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 AL = \frac{YA\Delta L^2}{2L}$$

New work done, $W' = \dfrac{Y(4A)\Delta L^2}{2(L/2)}$

$$= 8\left[\frac{YA\,\Delta L^2}{2L}\right] = 8 \times 2 = 16 \text{ J}$$

9. Increase in length due to rise in temperature,

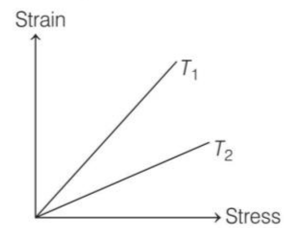
$$\Delta L = \alpha L\,\Delta T$$

As, $Y = \dfrac{FL}{A\Delta L}$

So, $F = \dfrac{YA\Delta L}{L} = \dfrac{YA \times \alpha L \Delta T}{L} = YA\alpha\Delta T$

$\therefore \quad F = 2 \times 10^{11} \times 10^{-6} \times 1.1 \times 10^{-5} \times 20 = 44$ N

10. The slope of stress-strain curve with strain axis gives the value of Young's modulus.



In the above graph, strain is taken along Y-axis. Therefore, the slope of graph at temperature T_1 is less than the slope of graph at temperature T_2.

Now as we know with increase in temperature, the value of modulus of elasticity decreases. Therefore, temperature T_1 is greater than temperature T_2.

11. Tensile stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A}$

Tension = Applied force = F

12. Here, $Y_c = 1 \times 10^{11}$ N/m^2

$Y_s = 2 \times 10^{11}$ N/m^2

$l_c = 1.0$ m, $l_2 = 0.5$ m and $\Delta l_c = 1 \times 10^{-3}$ m

As, $(\text{strain})_c = \dfrac{\text{stress}}{Y_c}$

$\Rightarrow \quad 1 \times 10^{-3} = \dfrac{\text{stress}}{1 \times 10^{11}} \Rightarrow \text{stress} = 10^8$ N/m^2

Now, $Y_s = \dfrac{\text{stress}}{\text{strain}}$

$\Rightarrow \quad \text{strain} = \dfrac{10^8}{2 \times 10^{11}} = 0.5 \times 10^{-3}$

or $\quad \dfrac{\Delta l_s}{1/2} = 0.5 \times 10^{-3} \Rightarrow \Delta l_s = 0.25 \times 10^{-3}$

$\therefore \quad \Delta l = \Delta l_c + \Delta l_s$

$= 1 + 0.25 = 1.25$ mm

13. Shearing strain $= \dfrac{x}{h} = \dfrac{1.6 \times 10^{-4}\text{ m}}{0.8\text{ m}} = 2.0 \times 10^{-4}$

Stress = (Shearing strain) (Shear modulus)

$= (2.0 \times 10^{-4}) \times (3.5 \times 10^{10}$ Pa)

$= 7.0 \times 10^6$ Pa

$\therefore \quad F = \text{Stress} \times \text{Area}$

$= 7.0 \times 10^6$ Pa $\times 0.8$ m $\times 0.005$ m

$= 2.8 \times 10^4$ N

14. From graphs, it is clear that ultimate strength of material (ii) is greater than that of material (i). Therefore, the elastic behaviour of material (ii) is elastic over a larger region of strain as compared to material (i).

If the fracture point of a material is closer to ultimate strength point, then the material is a brittle material.

Therefore, the material (ii) is more brittle than material (i).

15. Initial elastic potential energy,

$$U_1 = \frac{1}{2}F\Delta l = \frac{1}{2}\times(100\times1000)\times(1.59\times10^{-3}) = 79.5 \text{ J}$$

Let, Δl_1 be the elongation in the rod when stretching force is increased by, 200 N.

Since $\Delta l = \frac{F}{\pi r^2}\times\frac{l}{Y}$, so $\Delta l \propto F$

$$\therefore \quad \frac{\Delta l_1}{\Delta l} = 3\Delta l = 3\times1.59\times10^{-3} \text{ m} = 4.77\times10^{-3} \text{ m}$$

Final elastic potential energy,

$$U_1 = \frac{1}{2}F_1\Delta l_1 = \frac{1}{2}\times(300\times10^3)\times(4.77\times10^{-3}) = 715.5 \text{ J}$$

Increase in elastic potential energy

$$= 715.5 - 79.5 = 636.0 \text{ J}$$

16. If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

According to Hooke's law,

$$\text{Young's modulus } (Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

So, $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel rod (L) = 10cm,

Temperature $(\theta) = 100°\text{C}$

As length is constant,

$$\therefore \quad \text{strain} = \frac{\Delta L}{L} = \alpha\Delta\theta$$

Now, pressure = stress = $Y\times$ strain

[Given, $Y = 2\times10^{11}\text{N/m}^2$ and $\alpha = 1.1 \times10^{-5}\text{K}^{-1}$]

$$= 2\times10^{11}\times1.1\times10^{-5}\times100 = 2.2\times10^{8} \text{ Pa}$$

17. $$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Area}\times\text{Length}}$$

$$\Rightarrow \quad \rho = \frac{M}{AL}$$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

But $\Delta M = 0$, as mass remains constant.

$$\therefore \quad \frac{\Delta\rho}{\rho} = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$$

$$= 2\alpha\Delta T + \alpha\Delta T$$

[$\because$ coefficient of expansion of area, $\beta = 2\alpha$]

$$= 3\alpha\Delta T \quad \ldots\text{(i)}$$

Now, given that,

$$\frac{\Delta L}{L}\times100 = \alpha\Delta T = 0.02\%$$

So, from Eq (i), we get

$$\frac{\Delta\rho}{\rho}\times100 = 3\times0.02\% = 0.06\%$$

18. Given, each side of cube (l) = 10 cm = 0.1 m

Hydraulic pressure $(p) = 7\times10^6$ Pa

Bulk modulus for copper $(K) = 140\times10^9$ Pa

Volume contraction (ΔV) = ?

Volume of the cube $(V) = l^3 = (0.1)^3 = 1\times10^{-3} \text{ m}^3$

$$\text{Bulk modulus for copper } (K) = \frac{p}{\Delta V/V} = \frac{pV}{\Delta V}$$

or $$\Delta V = \frac{pV}{K}$$

$$\Delta V = \frac{7\times10^6\times1\times10^{-3}}{140\times10^9} = \frac{1}{20}\times10^{-6} \text{ m}^3$$

$$= 0.05\times10^{-6} \text{ m}^3 = 5\times10^{-8} \text{ m}^3$$

19. Here, $p = 20000 \text{ Ncm}^{-2} = 2\times10^8 \text{ Nm}^{-2}$

As, $$k = \frac{pV}{\Delta V}$$

or $$\Delta V = \frac{pV}{k} = \frac{2\times10^8\times V}{8\times10^9} = \frac{V}{40}$$

New volume of the metal,

$$V' = V - \Delta V = V - \frac{V}{40} = \frac{39V}{40}$$

New mass of the metal

$$= V'\times\rho' = \frac{39V}{40}\rho' = V\times11$$

or $$\rho' = \frac{40\times11}{39} = \frac{440}{39} \text{ gcm}^{-3}$$

20. Longitudinal strain,

$$\Rightarrow \quad \alpha = \frac{l_2 - l_1}{l_1} = 10^{-3}$$

$$\frac{l_2}{l_1} = 1.001$$

$$\text{Poisson's ratio, } \sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\beta}{\alpha}$$

or $$\beta = \sigma\alpha = 0.1\times10^{-3} = 10^{-4} = \frac{r_1 - r_2}{r_1}$$

or $$\frac{r_2}{r_1} = 1 - 10^{-4} = 0.9999$$

$$\%\text{ increase in volume} = \left(\frac{V_2 - V_1}{V_1}\right)\times100$$

$$= \left(\frac{\pi r_2^2 l_2 - \pi r_1^2 l_1}{\pi r_1^2 l_1}\right)\times100$$

$$= \left(\frac{r_2^2 l_2}{r_1^2 l_1} - 1\right)\times100$$

$$= [(0.9999)^2\times1.001 - 1]\times100$$

$$= 0.08\%$$

21. Decrease in volume, $\Delta V = \frac{\Delta p\times V}{K}$

Final volume, $V' = V - \Delta V$

$$= V - \frac{V\Delta p}{K} = V(1 - \Delta p / K)$$

or $$\frac{m}{\rho'} = \frac{m}{\rho}\left(1 - \frac{\Delta p}{K}\right)$$

or $$\rho' = \frac{\rho}{\left(1 - \frac{\Delta p}{K}\right)}$$

or $$\rho = \frac{10.5 \times 10^3}{(1 - 10^7 / 17 \times 10^{10})} = 0.61 \text{ kg m}^{-3}$$

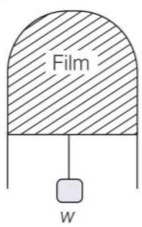
22. As, $Y = \frac{w}{A} \times \frac{L}{l}$ or $l = \frac{wL}{YA}$

When wire goes over a pulley and weight w is attached each free end of wire, then the tension in the wire is doubled, but the original length of wire is reduced to half, so extension in the wire is

$$l' = \frac{2w \times (L/2)}{YA} = \frac{wL}{YA} = l$$

23. Frequency, $f = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{Ad}}$

where, T is tension in the wire and μ is the mass per unit length of wire.



Also, Young's modulus, $Y = \frac{Tl}{A\Delta l}$

$$\Rightarrow \quad \frac{T}{A} = \frac{Y\Delta l}{l}$$

Putting this value in expression of frequency, we have

$$f = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{ld}}$$

Given, $l = 1.5$ m, $\frac{\Delta l}{l} = 0.01$

$$d = 7.7 \times 10^3 \text{ kg / m}^3, Y = 2.2 \times 10^{11} \text{N / m}^2$$

Putting these values, we have

$$f = \frac{1}{2l}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}}$$

$$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3}$$

$$\Rightarrow \quad f \approx 178.2 \text{ Hz}$$

24. We know that, time period,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{L + \Delta L}{g}}$$

where, ΔL is increase in length.

We know that,

$$Y = \frac{Mg / A}{\Delta L / L} = \frac{MgL}{A\Delta L} \Rightarrow \Delta L = \frac{MgL}{AY}$$

$$\Rightarrow \therefore \quad T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY} \text{ or } \frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$$

or $$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

25. Twisting couple per unit twist for solid cylinder,

$$C_1 = \frac{\pi\eta r^4}{2l}$$

$\therefore$ Hollow cylinder,

$$C_2 = C_1\left(\frac{r_2^4 - r_1^4}{r^4}\right) = \frac{0.1 \times (5^4 - 4^4)}{3^4} = \frac{36.9}{81}$$

$$= 0.455 \text{ N-m}$$

26. Young's modulus of copper $(Y_1) = 110 \times 10^9$ N/m^2

Young's modulus of steel $(Y_2) = 190 \times 10^9$ N/m^2

Let d_1 and d_2 be the diameters of steel and copper wires, respectively.

Since tension in each wire is same, therefore each wire has same extension. As each wire is of same length, hence each wire has same strain.

Young's modulus $(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{F / A}{\text{Strain}}$

or $$Y = \frac{F}{\left(\frac{\pi d^2}{4}\right)\text{Strain}} = \frac{4F}{\pi d^2 \times \text{Strain}}$$

$$\therefore \quad Y \propto \frac{1}{d^2} \text{ or } d^2 \propto \frac{1}{Y}$$

$$\therefore \quad \frac{d_1^2}{d_2^2} = \frac{Y_2}{Y_1}$$

or $$\frac{d_1}{d_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = \sqrt{1.73}$$

$$= 1.31$$

$$\therefore \quad d_1 : d_2 = 1.31 : 1$$

27. Here, tension in B, $T_B = F = mg/3$

Tension in A, $T_A = T_B + mg = \frac{mg}{3} + mg = \frac{4mg}{3}$

$$\therefore \quad T_A = 4T_B$$

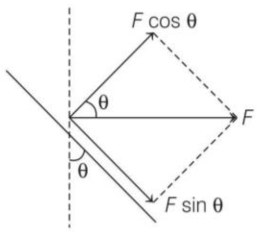
A wire will break when the stress is breaking stress (s).

$$S = \frac{\text{Tension}}{\text{Area of cross-section}} = \frac{T}{\pi r^2}$$

For $r_A = 2r_B, S_A = 4S_B$

So, A will break before B.

28. Consider A' be the area of cross-section of plane inclined at an angle θ shown in figure



$\therefore$ $$A' = \frac{A}{\cos\theta}$$

Now, the tangential force on plane is $F \sin\theta$.

Shear stress $= \dfrac{F\sin\theta}{\dfrac{A}{\cos\theta}} = \dfrac{F}{A}\sin\theta\cos\theta$

$$= \frac{F}{2A}2\sin\theta\cos\theta = \frac{F}{2A}(\sin 2\theta)$$

29. $Y = \dfrac{F/A}{\Delta L/L}$

$\Rightarrow$ $$Y = \frac{\Delta L}{L} = \frac{F}{AY} \quad \text{...(i)}$$

Now, by definition of Poisson's ratio,

$$\sigma = -\frac{\Delta r/r}{\Delta L/L}$$

$\Rightarrow$ $$\frac{\Delta r}{r} = -\frac{\sigma \Delta L}{L} = -\frac{\sigma F}{AY} \quad \text{[by using Eq. (i)]}$$

$\Rightarrow$ $$\frac{\Delta r}{r} = -\frac{\sigma F}{AY} \quad \text{...(ii)}$$

Now, $$V = \pi r^2 L$$

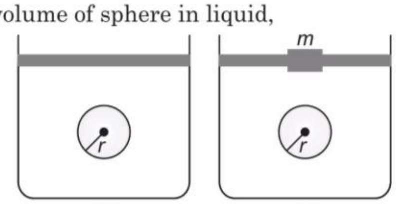
$\Rightarrow$ $$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta L}{L} \quad \text{...(iii)}$$

By using Eqs. (i), (ii) and (iii), we get

$$\frac{\Delta V}{V} = 2\left(\frac{-\sigma F}{AY}\right) + \frac{F}{AY}$$

$\Rightarrow$ $$\frac{\Delta V}{V} = \frac{F}{AY}[1 - 2\sigma]$$

30. The volume of sphere in liquid,



(a) (b)

$$V = \frac{4}{3}\pi r^3 \quad \text{...(i)}$$

When mass m is placed on the piston, the increased pressure $p = \dfrac{mg}{a}$.

Since, this increased pressure is equally applicable to all directions on the sphere, so there will be decrease in volume of sphere, due to decrease in its radius.

From Eq. (i), change in volume is

$$\Delta V = \frac{4}{3}\pi \times 3r^2\Delta r = 4\pi r^2 \Delta r$$

$$\frac{\Delta V}{V} = \frac{4\pi r^2 \Delta r}{(4/3)\pi r^3} = \frac{3\Delta r}{r}$$

$\therefore$ Bulk modulus ,

$$K = \frac{p}{dV/V} = \frac{mg}{a} \times \frac{r}{3\Delta r}$$

$\therefore$ $$\frac{\Delta r}{r} = \frac{mg}{3Ka}$$

31. Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l is length of pendulum and g is acceleration due to gravity.

Change in time period of a pendulum is

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

When clock gains 12 s, we get

$$\frac{12}{T} = \frac{1}{2}\alpha(40 - \theta) \quad \text{...(i)}$$

When clock loses 4 s, we get

$$\frac{4}{T} = \frac{1}{2}\alpha(\theta - 20) \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$3 = \frac{40 - \theta}{\theta - 20}$$

$\Rightarrow$ $$3\theta - 60 = 40 - \theta$$

$\Rightarrow$ $$4\theta = 100$$

$\Rightarrow$ $$\theta = 25°C$$

Substituting the value of θ in Eq. (i), we have

$$\frac{12}{T} = \frac{1}{2}\alpha(40 - 25)$$

$\Rightarrow$ $$\frac{12}{24 \times 3600} = \frac{1}{2}\alpha(15)$$

$\Rightarrow$ $$\alpha = \frac{24}{24 \times 3600 \times 15}$$

$\Rightarrow$ $$\alpha = 1.85 \times 10^{-5}/°C$$

Thus, the coefficient of linear expansion in a pendulum clock $= 1.85 \times 10^{-5}/°C$

32. Given, **for steel wire**

Length $(l_1) = 4.7$ m

Area of cross-section $(A_1) = 3.0 \times 10^{-5}$ m^2

For copper wire

Length $(l_2) = 3.5$ m

Area of cross-section $(A_2) = 4.0 \times 10^{-5}$ m^2

Let F be the given load under which steel and copper wires be stretched by the same amount Δl.

Young's modulus $(Y) = \dfrac{F/A}{\Delta l/l} = \dfrac{F \times l}{A \times \Delta l}$

For steel, $$Y_s = \frac{F \times l_1}{A_1 \times \Delta l} \quad \text{...(i)}$$

For copper, $Y_c = \dfrac{F \times l_2}{A_2 \times \Delta l}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{Y_s}{Y_c} = \frac{F \times l_1}{A_1 \times \Delta l} \times \frac{A_2 \times \Delta l}{F \times l_2}$$

$$= \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{4.7}{3.5} \times \frac{4.0 \times 10^{-5}}{3.0 \times 10^{-5}}$$

$$= \frac{18.8}{10.5} = 1.79 = 1.8$$

Hence, $Y_s : Y_c = 1.8 : 1$

33. Poisson's ratio, $\sigma = 0.4 = \dfrac{\Delta d / d}{\Delta l / l}$

Area, $A = \pi r^2 = \dfrac{\pi d^2}{4}$ or $d^2 = \dfrac{4A}{\pi}$

Differentiating, we get

$$2d\,\Delta d = \frac{4}{\pi} \cdot \Delta A$$

As, $A = \dfrac{\pi d^2}{4}$

So, $\Delta A = \dfrac{2\pi d \Delta d}{4}$

$$\frac{\Delta A}{A} = \frac{\pi \frac{d}{2} \Delta d}{\pi d^2/4} = 2\frac{\Delta d}{d}$$

Given $\dfrac{\Delta A}{A} \times 100 = 2\%$

$\Rightarrow$ $\dfrac{2\Delta d}{d} \times 100 = 2\%$

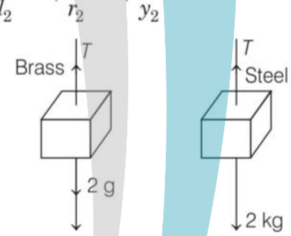
or $\dfrac{\Delta d}{d} \times 100 = 1\%$

Given $\sigma = \dfrac{\Delta d / d}{\Delta l / l} = 0.4$

or $\dfrac{\Delta d}{d} = 0.4 \dfrac{\Delta l}{l} = \dfrac{\Delta l}{l} = \dfrac{1}{0.4}\left(\dfrac{\Delta d}{d}\right)$

$\Rightarrow$ $\dfrac{\Delta l}{l} \times 100 = 2.5 \times 1\% = 2.5\%$

34. Given, $\dfrac{l_1}{l_2} = a, \dfrac{r_1}{r_2} = b, \dfrac{y_1}{y_2} = c$



Let Young modulus of steel be Y_1 and that of brass be Y_2

$$Y_1 = \frac{F_1 l_1}{A_1 \Delta l_1} \quad \text{...(i)}$$

and $Y_2 = \dfrac{F_2 l_2}{A_2 \Delta l_2}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{Y_1}{Y_2} = \frac{F_1 A_2 \cdot l_1 \Delta l_2}{F_2 A_1 l_2 \Delta l_1} \quad \text{...(iii)}$$

Force on steel wire,

$$F_1 = T = 2g$$

Force on brass wire,

$$F_2 = T_1' = T + 2g = 4g$$

Now putting the value of $F_1 \cdot F_2$ in Eq. (iii), we get

$$\frac{Y_2}{Y_1} = \left(\frac{2g}{4g}\right)\left(\frac{\pi r_2^2}{\pi r_1^2}\right)\left(\frac{l_1}{l_2}\right)\left(\frac{\Delta l_2}{\Delta l_1}\right) \quad \text{...(iv)}$$

Now given that, $\dfrac{l_1}{l_2} = a, \dfrac{r_1}{r_2} = b$

and $\dfrac{Y_1}{Y_2} = c$

From Eq. (iv), we get

$$\frac{\Delta l_1}{\Delta l_2} = \frac{c}{2b^2 a}$$

35. From figure, the increase in length,

$$\Delta l = (PR + RQ) - PQ = 2PR - PQ$$

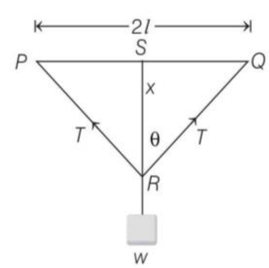
$$= 2(l^2 + x^2)^{1/2} - 2l$$

$$= 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$$

$$= 2l\left[1 + \frac{1}{2}\frac{x^2}{l^2}\right] - 2l$$

$$= x^2/l \quad \text{(By Binomial theorem)}$$

$\therefore$ Strain $= \Delta l / 2l = x^2/2l^2$



36. Here $m = 0.02$ kg; $v = 20\ \text{ms}^{-1}$;

$$l = 42\ \text{cm} = 0.42\ \text{m}$$

$$\Delta l = 20\ \text{cm} = 0.20\ \text{cm}$$

$$r = 3\ \text{mm} = 3 \times 10^{-3}\ \text{m}$$

Due to extension, energy is stored in the cord. This is converted into kinetic energy when the stone flies off.

$\therefore$ Work done $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}F\Delta l$

or $F = \dfrac{mv^2}{\Delta l} = \dfrac{0.02 \times (20)^2}{0.20} = 40$ N

$$\text{Stress} = \frac{F}{\pi r^2} = \frac{40}{(22/7)(3 \times 10^{-3})^2}$$

$$= 1.4 \times 10^6\ \text{Nm}^{-2}$$

37. According to Boyle's law, $p_2V_2 = p_1V_1$

or $$p_2 = p_1\left(\frac{V_1}{V_2}\right)$$

or $$p_2 = 72 \times 1000/900 = 80 \text{ cm of Hg}$$

Stress = Increase in pressure

$$= p_2 - p_1 = 80 - 72 = 8$$
$$= (8 \times 10^{-2}) \times 13.6 \times 10^3 \times 9.8$$
$$= 10662.4 \text{ Nm}^{-2}$$

Volumetric strain $= \dfrac{V_1 - V_2}{V_1} = \dfrac{1000 - 900}{1000} = 0.1$

38. Elastic energy per unit volume,

$$U = \frac{1}{2}F\Delta l = \frac{F^2 l}{2AY}$$

$$U \propto \frac{l}{r^2} \text{ (} F \text{ and } Y \text{ are constants)}$$

$\therefore$ $$\frac{U_A}{U_B} = \left(\frac{l_A}{l_B}\right)\left(\frac{r_B}{r_A}\right)^2 = (3) \times \left(\frac{1}{2}\right)^2 = \frac{3}{4}$$

39. For A $\quad \dfrac{F}{\pi r^2} = Y\dfrac{2\text{ mm}}{a}$ ...(i)

For B $\quad \dfrac{F}{\pi \cdot 16r^2} = Y\dfrac{4\text{ mm}}{b}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$\therefore$ $$16 = \frac{2b}{4a}$$

$$\frac{a}{b} = \frac{1}{32}$$

$\therefore$ $$x = 32$$

40. Density of wire, $d = 9 \times 10^{-3}\ \dfrac{\text{kg}}{\text{cm}^3} = 9 \times 10^3 \text{ kg/m}^3$

Strain in the wire, $\varepsilon = 4.9 \times 10^{-4}$

Young's modulus of wire is

$$Y = 9 \times 10^{10} \text{ N/m}^2$$

Lowest frequency of vibration in wire will be

$$f = \frac{1}{2L}\sqrt{\frac{T}{(M/L)}} \quad \text{...(i)}$$

Now, $$\frac{T}{M/L} = \frac{T}{V\rho/L} = \frac{T}{\left(\frac{LA \cdot \rho}{L}\right)} = \frac{T}{A\rho}$$

But $\dfrac{T}{A}$ = stress = Y × strain = $Y \times \varepsilon$

$\Rightarrow$ $$\frac{T}{M/L} = \frac{Y \times \varepsilon}{\rho} \quad \left(\because A = \frac{M}{\rho L}\right)$$

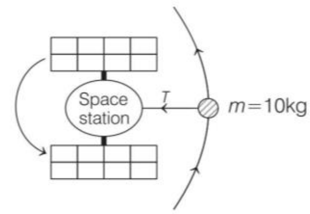
So, from Eq (i), frequency will be

$$f = \frac{1}{2L}\sqrt{\frac{T}{(M/L)}} = \frac{1}{2L}\sqrt{\frac{Y \times \varepsilon}{\rho}}$$

$$= \frac{1}{2 \times 1} \times \sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9 \times 10^3}}$$

$$= 35 \text{ Hz}$$

41. Centripetal force is provided by the tension in wire.



So, $$T = m\omega^2 l$$

Stress in wire, $$\sigma = \frac{T}{A} = \frac{m\omega^2 l}{A} \quad \text{...(i)}$$

Here, $\sigma_{max} = 4.8 \times 10^7 \text{ Nm}^{-2}$,

$A = 10^{-2} \text{ cm}^2 = 10^{-2} \times 10^{-4} \text{ m}^2 = 10^{-6} \text{ m}^2$,

$m = 10$ kg and $l = 0.3$ m

If maximum angular speed of rotation is ω_{max}, then from Eq. (i), we have

$$\omega_{max}^2 = \frac{\sigma_{max} A}{ml} = \frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3} = 16$$

or $$\omega_{max} = 4 \text{ rad s}^{-1}$$

42. Here, $2r = 3 \times 10^{-2}$ m

or $r = (3/2) \times 10^{-2}$ m

$\Delta p = 10 \text{ kgfm}^{-2} = 10^3 \times 9.8 \text{ Nm}^{-2}$

$K = 10^7 \text{ dyne cm}^2 = 10^6 \text{ Nm}^{-2}$

Volume of the ball,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2}\right)^3 \text{ m}^3$$

Now, $$K = \frac{\Delta p \times V}{\Delta V} \text{ or } \Delta V = \frac{V\Delta p}{K}$$

$$= \frac{\frac{4}{3} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2}\right)^3 \times 10^3 \times 9.8}{10^6}$$

$$= 0.1386 \times 10^{-6} \text{ m}^3 = 0.1386 \text{ cm}^3$$

43. Given $l = 7 \text{ cm} = 7 \times 10^{-2}$ m

$$F = 300 \text{ kgf} = 300 \times 10 \text{ N}$$
$$\eta = 2 \times 10^7 \text{ dyne cm}^{-2}$$
$$= 2 \times 10^6 \text{ Nm}^{-2}$$

As, $$\eta = \frac{F/A}{\theta}$$

or $$\theta = \frac{F}{A\eta} = \frac{F}{l^2\eta}$$

$$= \frac{300 \times 10}{(7 \times 10^{-2})^2 \times 2 \times 10^6} = 0.3 \text{ rad}$$

$$\theta = \frac{\Delta l}{l}$$

or $$\Delta l = l\theta = 7 \times 0.3 = 2.1 \text{ cm}$$