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OSCILLATIONS

When an object moves back and forth repeatedly over the same path, it is said to be oscillating or vibrating. An important type of oscillatory motion is **simple harmonic motion (S.H.M.)** for which displacement-time graph is a sine curve or cosine curve. The motion of a mass vibrating up and down on a spring, the motion of the prongs of a vibrating tuning fork and the motion of a swinging pendulum bob are the familiar examples of motion that is simple harmonic motion or very nearly so. The study of simple harmonic motion involves no new laws or principles of physics. Instead, we use Newton's second law, kinematics and energy principles to analyse this simple type of motion.

PERIODIC AND OSCILLATORY MOTION

1. Periodic motion. *The motion which repeats itself after a regular interval of time is called* **periodic motion.** *The regular time interval is called the time period of the periodic motion.*

Examples.

(*i*) The revolution of earth around the sun is a periodic motion. Its period of revolution is 1 year.

(*ii*) The revolution of moon around the earth is a periodic motion. Its period of revolution is 27.3 days.

(*iii*) The motion of the hands of a clock is a periodic motion.

2. Oscillatory motion. *If a body in periodic motion moves along the same path to and fro about a definite point (equilibrium position), then the motion of the body is* **vibratory motion** *or* **oscillatory motion.**

All objects that vibrate or oscillate have one thing in common : each object is subjected to a *restoring force* that increases with the increase in displacement from the equilibrium position. A restoring force is one that tries to pull or push a displaced object back to its equilibrium position.

☐ All oscillatory motions are periodic motions but all periodic motions are not oscillatory.
For example, the motion of moon around the earth is periodic but not oscillatory.

Examples.

(*i*) The motion of the bob of swinging simple pendulum is oscillatory motion.

(*ii*) When a loaded spring is pulled and then released, the load attached to the spring executes oscillatory motion.

SIMPLE HARMONIC MOTION (S.H.M)

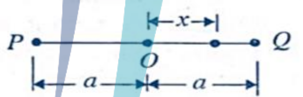
A particle is said to execute SHM, if it moves to and fro about the mean position(equilibrium) Position in a straight line and the graph between displacement of the particle from equilibrium Position and time are sine or cosine curve.

In simple harmonic motion, the oscillatory motion of the particle is in a straight line such that its displacement from the *equilibrium position (or mean position) varies sinusoidally (or cosinusoidally) with time. Since this is the simplest type of periodic motion (or harmonic motion), it is called simple harmonic motion.

* Equilibrium position is the position at which the particle would come to rest if it were to lose all of its energy.

ANALYSIS OF S H M

Consider a particle [See Fig. 25.1 (*i*)] executing simple harmonic motion about point O (equilibrium position) with an amplitude a (*i.e.*, maxim[illegible] displacement from O)



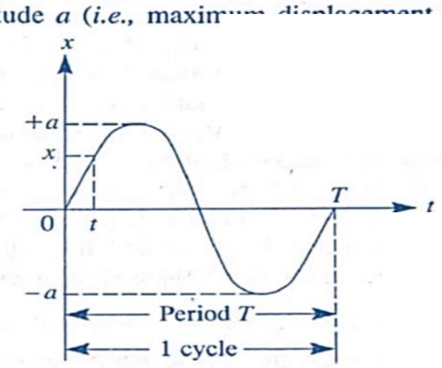


Fig. 25.1

The particle is said to have completed **1 vibration** (or oscillation) if starting from *O*, it has moved through positions *O* – *Q* – *P* – *O* *i.e.*, when it has returned to its starting position and is moving in the *same* direction. The time taken to complete 1 vibration is called **time period** *T* of the motion. The number of complete oscillations in one second is called the **frequency** *f* of the motion. Clearly, $f = 1/T$. If the motion is taken to **start from the equilibrium position (*i.e.*, $x = 0$ at $t = 0$), then by definition, displacement *x* of the particle from the equilibrium position varies with time (*t*) according to the relation :

$$x = a \sin \omega t = a \sin \frac{2\pi t}{T}$$

where *a* and ω(or *T*) are constants of the motion. In order to give physical significance to these constants, it is convenient to plot *x* – *t* (displacement – time) graph as in Fig. 25.1 (*ii*).

(*i*) Since $\sin\omega t$ varies between +1 and –1, the displacement *x* varies between + *a* and – *a*. The constant *a* is called the *amplitude* of the simple harmonic motion; it is simply the *maximum displacement* of the particle from the equilibrium position in either the positive or negative *x* – direction.

(*ii*) The constant ω is called the *angular frequency* of simple harmonic motion and as we shall see, its value depends on the rate of the oscillations. Let us interpret constant ω. The displacement *x* must return to its initial value after one period *T* of the motion. Thus in one cycle, the quantity ωt increases by 2π rad while the time *t* increases by *T* *i.e.*,

$$\omega(t+T) = \omega t + 2\pi$$

or $$\omega T = 2\pi$$

$$\therefore \quad \boxed{\omega = \frac{2\pi}{T} = 2\pi f} \qquad \left(\because f = \frac{1}{T}\right)$$

- □ Since, time period T is inversely proportional to ω .
  This means that larger the angular frequency, the smaller the time period and the more quickly the Particle completes a cycle.
- □ Since, In SHM., the body periodically passes through the equilibrium position and therefore, there, must act a restoring force on the body to bring it back to the equilibrium position time and again.

□□ PERIODIC functions:

PERIODIC FUNCTIONS are those which represent periodic motion.

. The sines and cosines functions of time are simple periodic functions. A function is said to be periodic if it repeats itself after time period *T* *i.e.* the same function is obtained when the variable *t* is changed to *t* + *T*. Consider the following periodic functions :

$$f(t) = \sin\frac{2\pi}{T}t$$

and $$g(t) = \cos\frac{2\pi}{T}t$$

Here *T* is the time period of the periodic motion and is equal to 2π radians. We shall see that if the variable *t* is changed to *t* + *T*, the same function results.

$$f(t+T) = \sin\left[\frac{2\pi}{T}(t+T)\right] = \sin\left[\frac{2\pi t}{T} + 2\pi\right] = \frac{{}^*\sin 2\pi}{T}t$$

$\therefore$ $$f(t+T) = f(t)$$

Similarly, $$g(t+T) = g(t)$$

It can be easily verified that :

$$f(t+nT) = f(t)$$

and $$g(t+nT) = g(t) \qquad \text{where } n = 1, 2, 3, \ldots\ldots$$

Therefore, infinite sets of periodic function of period *T* may be represented as :

$$f_n(t) = \sin\frac{2\pi nt}{T}$$

and $$g_n(t) = \cos\frac{2\pi nt}{T}$$

Further, a linear combination of sine and cosine functions like :

$$f(t) = a \sin \omega t + b \cos \omega t \qquad ...(i)$$

is also a periodic function with a period T.

Let $a = A \cos\phi$ and $b = A \sin\phi$

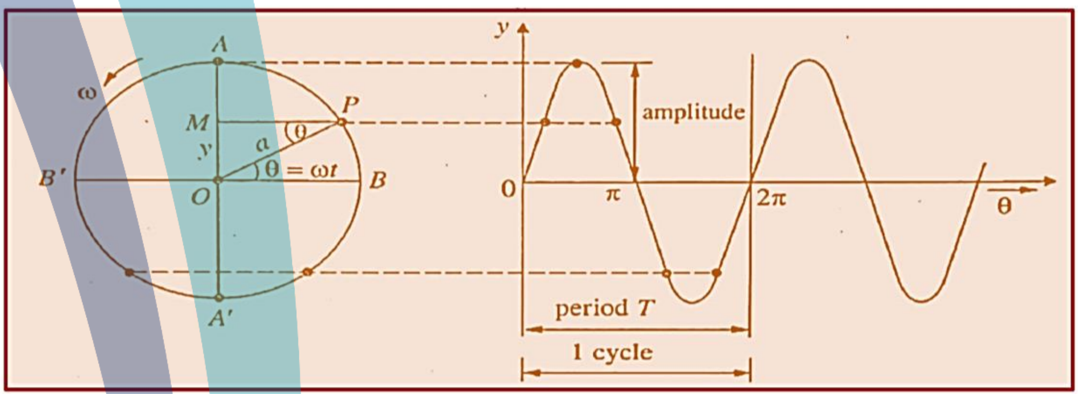
$\therefore$ Eq. (i) can be written as : $f(t) = A \sin(\omega t + \phi)$

where $A = \sqrt{a^2 + b^2}$; $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

$\because \sin(\theta + 2\pi) = \sin\theta$

S H M IN TERMS OF UNIFORM CIRCULAR MOTION:

:. Suppose a particle is moving (anticlockwise) with a uniform angular speed ω along the circumference of a circle of radius a and centre O. When the particle is at point P, then the foot of the perpendicular drawn from the particle on the diameter AA' of the circle is at the point M [See Fig. 25.2 (i)]. Let us study the motion of M (*i.e.*, foot of the perpendicular drawn from the particle on the diameter AA') as the particle moves along the circle. When the particle was at point B, the foot of the perpendicular was at the point O. As the particle moves along the arc BA, the foot of the perpendicular (M) moves along the diameter OA. When the particle reaches point A, the foot of the perpendicular is also at A. When the particle moving along the arc AB' reaches point B', the foot of the perpendicular moving along diameter AO reaches point O. As the particle moves along the arc $B'A'B$, the foot of the perpendicular moves from O to A' and then from A' to O. Note that as the particle completes one circle, the foot of the perpendicular (M) completes one vibration. Note that M moves to and fro along the straight line AA'; O being the equilibrium position.



(i) (ii)

Fig. 25.2

The motion of M is the simple harmonic motion. To test it, let us find the equation of motion of M. Suppose the particle starts from point B and rotates through an angle θ rad in time t. Then $\theta = \omega t$. At this instant, the displacement of M is OM (= y).

Now $$\frac{OM}{OP} = \sin\theta = \sin\omega t$$

or $$OM = OP \sin\omega t$$

or $$y = a \sin\omega t \qquad (\because OP = a = \text{radius of circle})$$

This is the displacement equation of simple harmonic motion. The circle is called the *reference circle* of simple harmonic motion. Note that the particle is executing circular motion but the foot of the perpendicular drawn from the particle on the diameter AA' is executing SHM. We arrive at an important conclusion :

A particle travelling around a reference circle of radius a at constant angular speed $\omega(= 2\pi f)$ can be used to represent simple harmonic motion of amplitude a and frequency f.

Because of this relationship between uniform circular motion and simple harmonic motion (S.H.M.), we often use the terminology of circular motion in discussing S.H.M. For example, ω is often called the angular frequency of S.H.M.

Now $y = a \sin \omega t = a \sin 2\pi f t = a \sin \frac{2\pi}{T} t$

where $\omega = 2\pi f = \frac{2\pi}{T}$

GENERAL DISPLACMENT EQUATION OF S H M

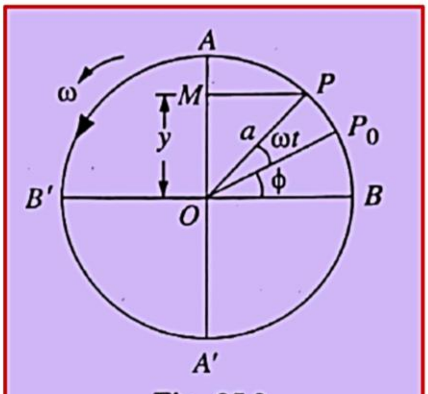


Fig. 25.3

The displacement equation of S.H.M., $y = a \sin \omega t$ represents the condition when the motion is taken to start from the equilibrium position (*i.e.*, at $t = 0$; $y = 0$). This corresponds to point B on the reference circle of S.H.M. [See Fig. 25.3]. The general case requires that timing may commence when the particle is at any point of its oscillation. Suppose the time is measured from the instant when the particle moving on the reference circle was at a point different from B, say at P_0 (See Fig. 25.3) where $\angle P_0OB = \phi$. In time t, the particle has reached point P such that $\angle POP_0 = \omega t$. Therefore, displacement of M from the equilibrium position is OM (= y).

$\therefore \quad \frac{OM}{OP} = \sin MPO = \sin POB$

or $\quad OM = OP \sin(\omega t + \phi) \qquad (\because \angle MPO = \angle POB = \omega t + \phi)$

or $\quad y = a \sin(\omega t + \phi) \qquad (\because OP = a) \qquad ...(i)$

Eq. (i) is the *general displacement equation of S.H.M. Note that if time is measured from the instant the particle is at the equilibrium position, then $\phi = 0$ so that $y = a \sin \omega t$. Note that ϕ is called the initial phase (or phase constant) or epoch of M.

Finally, it should be noted that cosines instead of sines could have been used throughout our discussion because :

$$\cos \theta = \sin\left(\theta + \frac{\pi}{2}\right)$$

This difference in representation is equivalent to adding a $\pi/2$ phase constant. This causes no difficulty because the choice of phase is arbitrary when only one oscillator is being described. However, once a choice has been made, that choice must be adhered to throughout the problem at hand.

□ $y = a \sin(\omega t + \phi)$

(*i*) If $\phi = 0°$, then $y = a \sin \omega t$. This means that at $t = 0$, the particle is at the mean position.

(*ii*) If $\phi = 90°$, $y = a \sin(\omega t + 90°) = a \cos \omega t$. This means that at $t = 0$, the particle is at the extreme position. Therefore both $y = a \sin \omega t$ and $y = a \cos \omega t$ are valid representation of sinusoidal motion. Motion described by a sine or cosine function of time is known as *sinusoidal motion*. Sinusoidal motion and S.H.M. are the same.

CHARECTERISTIC OF S H M

Consider a particle moving (anticlockwise) with uniform angular speed ω along the circumference of a reference circle of radius a and centre O. Suppose the particle starts from point B and rotates through an angle θ rad in time t and assumes the position P. Then $\theta = \omega t$ [See Fig. 25.4 (i)]. When the particle is at the point P, then the foot of the perpendicular drawn from the particle on the diameter AA' of the reference circle is at the point M. As the particle moves along the circumference of the reference circle, M executes simple harmonic motion along the diameter AA'. Note that O is the equilibrium position for this S.H.M.

DISPLAMENT: The displacement of a particle executing SHM at any instant is the distance from the equilibrium position at that instant.

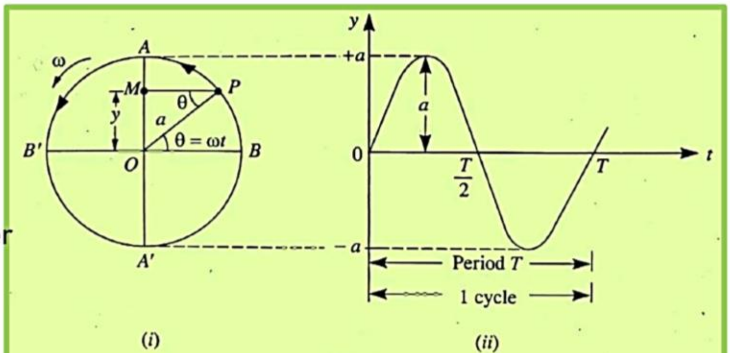


Fig. 25.4

here, M is executing SHM along the diameter AA' .At the considered instant , the displacement of M is OM = y.

DISPLAMENT EQUATION When the particle is at point P, M has a displacement $OM = y$.

Now $$\frac{OM}{OP} = \sin \omega t$$

or $$OM = OP \sin \omega t$$

or $$y = a \sin \omega t \quad (\because OP = a) \quad ...(i)$$

Equation (*i*) is the displacement equation of simple harmonic motion. It is clear that the displacement-time graph for the motion of M [See Fig. 25.4 (*ii*)] is a sine curve — a test for S.H.M. In general, if the time is measured from the instant when the particle is at point P' between B and A such that $\angle P'OB = \phi$, then displacement equation becomes :

$$y = a \sin(\omega t + \phi)$$

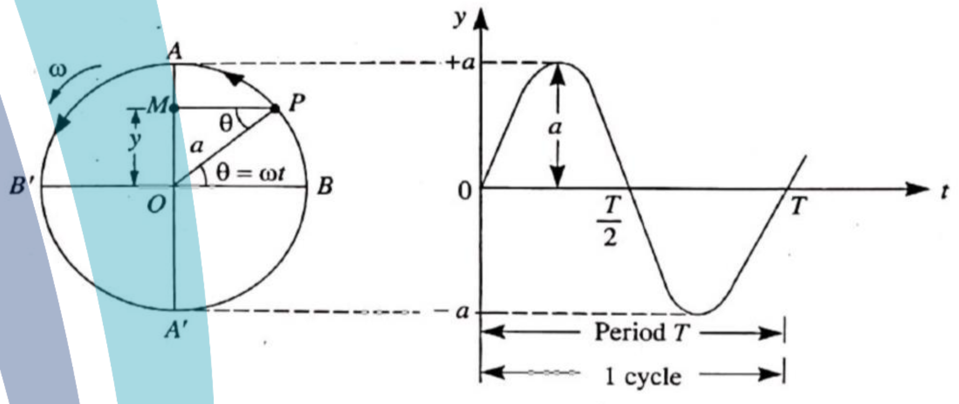
AMPLITUDE *The maximum displacement of the particle executing S.H.M. on either side of the equilibrium position is called amplitude of the motion.* Thus referring to Fig. 25.4 (*i*), the displacement of M at any time t is given by ;

$$y = a \sin \omega t$$

The value of y will be maximum when $\sin \omega t$ is maximum. Now the maximum value of $\sin \omega t$ (or $\cos \omega t$) is 1.

$\therefore$ $$\text{Amplitude} = a \times 1 = a$$

Note that when the particle reaches point A, the displacement of M is maximum (= $OA = a$ = radius of reference circle). Thus amplitude is equal to the radius of the reference circle.



(*i*) (*ii*)

Fig. 25.4

PHASE *The phase angle of a particle executing S.H.M. is its position at $t = 0$.* The phase constant (or phase angle) tells us what the displacement was at $t = 0$. If at $t = 0$, the particle is at point B (starting point), $\phi = 0$. If at $t = 0$, the particle is at point A, $\phi = \pi/2$ rad.

$\therefore$ $$y = a \sin(\omega t + \phi)$$

The quantity $(\omega t + \phi)$ is called the *phase* of motion *i.e.*, displacement of particle in time t and is useful in comparing the motion of two systems of particles. Note that function y is *periodic* and repeats itself when ωt increases by 2π radians.

VELOCITY: *The velocity of a particle executing S.H.M. at any instant is the time rate of change of its displacement at that instant.* Thus referring to Fig. 25.4 (*i*), M is executing S.H.M. along the diameter AA'. At the considered instant, its displacement is given by ;

$$y = a \sin \omega t$$

$\therefore$ $$\text{Velocity, } V = \frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t) = a\omega \cos \omega t$$

$$= a\omega\sqrt{1 - \sin^2 \omega t} = a\omega\sqrt{1 - y^2/a^2}$$

$\therefore$ $$V = \omega\sqrt{a^2 - y^2} \quad ...(ii)$$

Eq. (*ii*) gives the instantaneous velocity of the particle executing S.H.M. Thus the velocity of the particle executing S.H.M. changes with the displacement y of the particle.

Eq. (*ii*) gives the instantaneous velocity of the particle executing S.H.M. Thus the velocity of the particle executing S.H.M. changes with the displacement y of the particle.

At equilibrium position, $y = 0$ so that $V = \omega a$

At extreme positions, $y = a$ so that $V = 0$

Thus the velocity of a particle executing S.H.M. is maximum when $y = 0$ *i.e.,* when the particle passes through its equilibrium position. However, when the displacement is maximum ($y = a$), the velocity of the particle is zero. Note that maximum value of velocity is called *velocity amplitude* in S.H.M.

ACCELERATION: *The acceleration of a particle executing S.H.M. at any instant is the time rate of change of velocity at that instant.* Thus referring to Fig. 25.4 (*i*), M is executing S.H.M. along the diameter AA'. At the considered instant,

$$y = a\sin\omega t$$

$$\therefore \quad \text{Velocity, } V = \frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t) = a\omega\cos\omega t$$

$$\therefore \quad \text{Acceleration, } A = \frac{dV}{dt} = \frac{d}{dt}(a\omega\cos\omega t) = -\omega^2 a\sin\omega t$$

$$\therefore \quad A = -\omega^2 y \qquad (\because a\sin\omega t = y) \qquad ...(iii)$$

Eq. (*iii*) gives the instantaneous acceleration of the particle executing S.H.M. Note that acceleration of the particle executing S.H.M. changes with the displacement y of the particle.

At equilibrium position, $y = 0$ so that $A = 0$

At extreme positions, $y = a$ so that $A = -\omega^2 a$

Thus the acceleration of a particle executing S.H.M. is zero when $y = 0$ *i.e.,* when the particle passes through its equilibrium position. However, when the displacement of the particle is maximum, its acceleration is also maximum. Note that maximum value of acceleration is called *acceleration amplitude* in S.H.M.

$$A = -\omega^2 y$$

or $$A \propto -y \qquad (\because \omega \text{ is constant})$$

Thus we find that the acceleration of a particle executing S.H.M. is proportional to its displacement y and its direction (negative sign) is opposite to the direction of displacement. This fact provides another definition of S.H.M.

> *A particle is said to execute S.H.M. if its acceleration at any instant is directly proportional to its displacement from the equilibrium position and is always directed towards the equilibrium position.*

TIME PERIOD: *The time taken by a particle executing S.H.M. to complete *one vibration is called time period (T) of the motion. This means that the value of y at any time t is equal to the value of y at time t + T. We can show that time period of motion is given by* $T = 2\pi/\omega$ by using the fact that phase increases by 2π radians in a time T :

$$\omega t + 2\pi = \omega(t + T)$$

or $$T = \frac{2\pi}{\omega} \qquad ...(iv)$$

Neglecting the negative sign from eq. (*iii*), we have,

$$A = \omega^2 y$$

or $$\omega = \sqrt{A/y}$$

$$\therefore \quad \text{Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{y}{A}} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

time period (T) and frequency (f) of S.H.M. do not depend on the amplitude

GRAPHICAL REPRESENTATION OF S H M:

DISPLACEMNT- TIME VELOCITY – TIME ACCELERTION – TIME GRAPH:

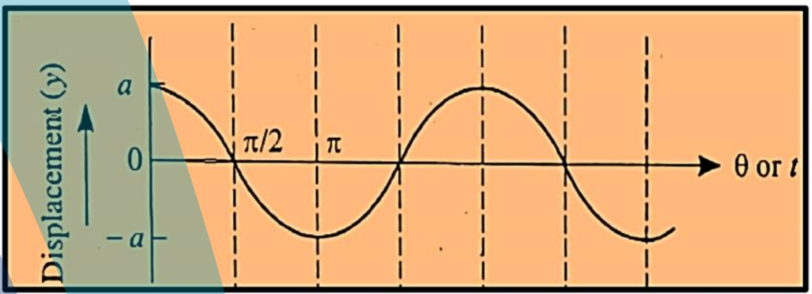
Let us suppose,

$$y = a\cos\omega t = a\cos\theta \quad \text{...}(i)$$

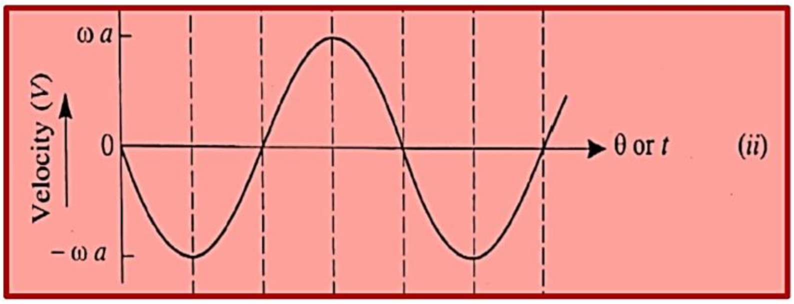
Note that we choose a phase constant (ϕ) equal to zero for the displacement.

(i) Velocity, $V = \dfrac{dy}{dt} = \dfrac{d}{dt}(a\cos\omega t) = -a\omega\sin\omega t$

$$\therefore \quad V = a\omega\cos\left(\omega t + \frac{\pi}{2}\right) \quad \text{... }(ii)$$



--(i)



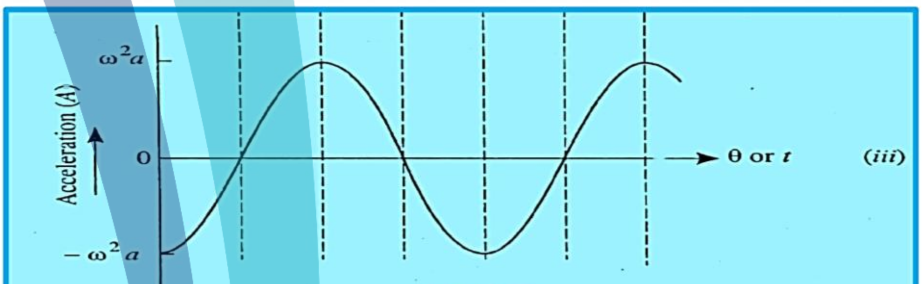


Fig. 25.5

Comparing eqs. (i) and (ii), we note that phase of the velocity differs from the phase of the displacement by $\pi/2$ rad or 90°. This means that when y is a maximum or minimum, the velocity is zero. Likewise, when $y = 0$, the velocity is maximum. Note that velocity of the particle at a particular time can be determined by taking the slope of displacement – time graph at that time.

(ii) Acceleration, $A = \dfrac{dV}{dt} = -a\omega^2\cos\omega t = a\omega^2\cos(\omega t + \pi)$

$$\therefore \quad A = a\omega^2\cos(\omega t + \pi) \quad \text{...}(iii)$$

Comparing eqs. (i) and (iii), we note that acceleration (A) is 180° out of phase with the displacement. Note that acceleration-time graph looks like the displacement-time graph but with opposite sign. This is expected because in S.H.M., acceleration is proportional to the negative of the displacement. In other words, when y is positive, acceleration is directed towards $-y$.

Important points. The following are the important properties of a particle executing S.H.M. :

(i) *The particle moves to and fro about the mean (equilibrium) position in a straight line.*

(ii) *The displacement, velocity and acceleration all vary *sinusoidally with time but are not in phase.*

(iii) *The acceleration of the particle is proportional to the displacement but in the opposite direction.*

(iv) *The frequency and period of motion are independent of the amplitude of the motion.*

TOTAL ENERGY IN S H M

When a body oscillates in simple harmonic motion, it is acted upon by a restoring force which tends to bring it to the equilibrium position. Due to this force, there is a potential energy in the body. Moreover, as the body is in motion, it has kinetic energy. During oscillation of the body, its energy continuously interchanges between kinetic energy and potential energy but their sum remains constant (taking friction negligible).

Potential energy. Consider a particle of mass m executing S.H.M. with amplitude a and constant angular frequency ω. Suppose at any time t, the displacement of the particle from the equilibrium position is y. If A is the acceleration of the particle at that instant, then by definition of S.H.M., we have,

$$A = -\omega^2 y$$

Magnitude of the restoring force acting on the particle at that instant is

$$F = m\,A = -m\omega^2 y = -k\,y$$

where $m\omega^2 = k$ = force constant of S.H.M.

If the particle undergoes a further very small displacement dy, the small work done against the restoring force is

$$dW = -F\,dy = -(-k\,y)\,dy = k\,y\,dy$$

∴ Total work done for displacement y is

$$W = \int_0^y k\,y\,dy = \frac{1}{2}k\,y^2$$

This work done is equal to the potential energy E_P of the particle at displacement y i.e,

$$E_P = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2 y^2 \qquad (\because k = m\omega^2) \qquad ...(i)$$

Kinetic energy. Suppose V is the velocity of the particle when its displacement is y. Then kinetic energy E_K of the particle is

$$E_K = \frac{1}{2}mV^2$$

Now $$V = \omega\sqrt{a^2 - y^2}$$ where a = amplitude of S.H.M.

∴ $$E_K = \frac{1}{2}m\omega^2(a^2 - y^2) \qquad ...(ii)$$

Total energy. The total energy E of the particle for displacement y is given by ;

$$E = E_P + E_K = \frac{1}{2}m\omega^2 y^2 + \frac{1}{2}m\omega^2(a^2 - y^2)$$

or $$E = \frac{1}{2}ma^2\omega^2 \qquad ...(iii)$$

We know that $\omega = 2\pi f$ where f is the frequency of S.H.M.

∴ $$E = \frac{1}{2}ma^2(2\pi f)^2$$

or $$E = 2m\pi^2 f^2 a^2 \qquad ...(iv)$$

Discussion. The following points may be noted :

(*i*) For a given particle in S.H.M., m, ω and a are constants [See eq. (*iii*)]. Thus the total energy (E_P + E_K) of a particle executing S.H.M. remains constant at all times.

(*ii*) The total energy of a particle executing S.H.M. is directly proportional to the square of the amplitude (a^2) and to the square of the frequency (f^2). It is also directly proportional to the mass (m) of the particle.

(*iii*) The energy of the particle continuously interchanges between kinetic energy and potential energy but their sum at all times remains constant. Thus in the position of maximum displacement (when $y = a$), the total energy is in the form of potential energy while in the equilibrium position (when $y = 0$), the total energy is in the form of kinetic energy.

Graphical representation. Fig. 25.6 shows the graph of kinetic energy and potential energy versus displacement. Each curve is a parabola centred at $y = 0$.

$$E_P = \frac{1}{2} m\omega^2 y^2 \; ; \; E_K = \frac{1}{2} m\omega^2 (a^2 - y^2)$$

When the particle is at the mean position, $y = 0$ so that $E_P = 0$ and $E_K = \frac{1}{2} m\omega^2 a^2$. In this position, the total energy of the particle is in the form of kinetic energy since $E_P = 0$. This is quite clear from Fig. 25.6. Since the kinetic energy of the particle passing through mean position is maximum, it is clear that at this position, the speed of the particle will be maximum.

When the particle is at the extreme positions, $y = a$ so that $E_P = \frac{1}{2} m\omega^2 a^2$ and $E_K = 0$. In this position, the total energy of the particle is in the form of potential energy since $E_K = 0$. This is also clear from Fig. 25.6. Since the kinetic energy of the particle at the extreme positions is zero, the speed of the particle at these positions will be zero.

It is clear from Fig. 25.6 that energy is being continuously transferred between potential energy and kinetic energy of the particle. Note that both E_K and E_P are always positive and their sum at all times is a constant equal to $\frac{1}{2} m\omega^2 a^2$. This is represented by a straight line parallel to the displacement axis.

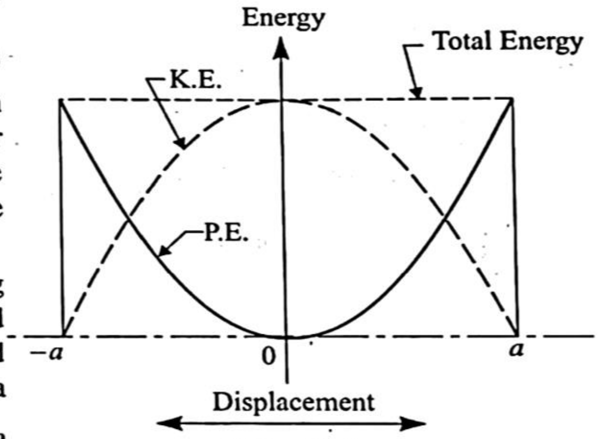


Fig. 25.6

DYNAMICS OF SHM(cause): Consider a particle of mass m executing S.H.M. with amplitude a and constant angular frequency ω. Then acceleration A of the particle is given by ;

$$A = -\omega^2 y$$

Here y is the displacement of the particle from the mean position. The negative sign shows that acceleration is always directed towards the mean position. Therefore, the particle is subjected to a force F which tries to bring the particle back to the mean position. For this reason, it is called *restoring force.*

$\therefore$ Restoring force, F = Mass × Acceleration $= m \times (-\omega^2 y)$

or $$F = -m\omega^2 y$$

or $$F = -ky \qquad ...(i)$$

where k $(= m\omega^2)$ is a constant of proportionality called **force constant* for the system. **Eq. (*i*) is the general expression to produce S.H.M.** In other words, the motion of any system governed by a force of the form of eq. (*i*) is called simple harmonic motion.

TIME PERIOD AND FREQUENCY OF S H M:

(*i*) As shown above, the force constant (k) of the particle executing S.H.M. is

$$k = \omega^2 m \quad \text{or} \quad \omega = \sqrt{k/m}$$

$$\therefore \quad \text{Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\text{Mass}}{\text{Force constant}}}$$

$$\therefore \quad \text{Frequency, } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\text{Force constant}}{\text{Mass}}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} \; ; \; f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus time period (T) and frequency (f) of S.H.M. depend on the *mass* (m) of the body and its force constant (k).

(*ii*) The time period and frequency of S.H.M. can also be expressed in the following alternate form :

$$\text{Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\text{Mass}}{\text{Force constant}}}$$

Now,
$$k = \frac{F}{y} = \frac{\text{Force}}{\text{Displacement}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{\text{Force/Displacement}}} = 2\pi\sqrt{\frac{m\times\text{Displacement}}{m\times\text{Acceleration}}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$\therefore \quad \text{Frequency, } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

EXAMPLES OF S H M : *The test for S.H.M. is that force (F) acting on the body must be in the form : F = – k y where y is the displacement of the body from the mean position and k is the force constant.*

(*i*) Oscillations of a loaded spring
(*ii*) Oscillations of a simple pendulum
(*iii*) Oscillations of a liquid in a U-tube
(*iv*) Oscillations of a floating cylinder
(*v*) Body dropped in a tunnel along earth diameter.

[A] OSCILLATIONS OF LOADED SPRING:

If load attached to a spring is pulled a little from its mean position and then released, the load will execute S.H.M. We shall consider the following two cases :

1. Vibrations (oscillations) of a horizontal spring
2. Vibrations of a vertical spring.

1. Vibrations of a horizontal spring. Fig. 25.7 (*i*) shows a block of mass m attached to one end of a horizontal spring while the other end of the spring is fixed to a rigid support. The block is at rest but is free to move along a frictionless horizontal surface. In Fig. 25.7 (*i*), the block is at rest at the mean position $x = 0$. If the block is displaced through a small distance x to the right [See Fig. 25.7 (*ii*)], the spring gets stretched. According to Hooke's law, the spring exerts a restoring force F to the left given by ;

$$F = -kx \qquad ...(i)$$

Here k is the force constant (generally called *spring constant) and x is the displacement of mass m from the mean position. Clearly, eq. (*i*) satisfies the condition to produce S.H.M. Therefore, if the block is released from the displaced position and left to itself, the block will execute S.H.M. The time period (T) and frequency (f) of the vibrations is given by ;

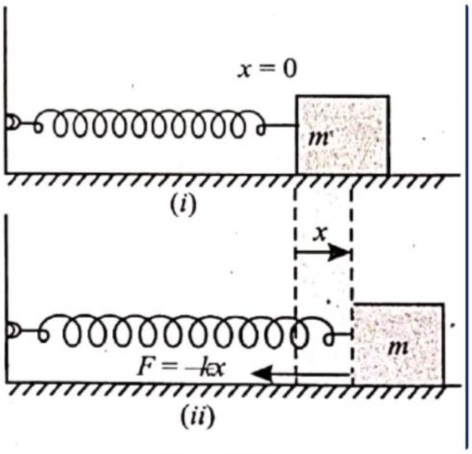


Fig. 25.7

$$T = 2\pi\sqrt{\frac{m}{k}}\ ; \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

We see that increasing the m or using a less stiff spring (smaller k) will increase the time period of oscillations.

2. Vibrations of a vertical spring. Suppose that a light spring with a spring constant k is suspended vertically from a rigid support as shown in Fig. 25.8 (*i*). Initially, the spring is neither compressed nor stretched. Suppose a mass m is suspended from the lower end. Due to weight mg, the spring stretches and comes to equilibrium as shown in Fig. 25.8 (*ii*). In this position, the spring is stretched by an amount l and exerts a restoring force $F_1 = -kl$* on the mass m. Since the mass m is in equilibrium, the sum of the forces on it is zero *i.e.*,

$$-kl + mg = 0 \qquad ...(i)$$

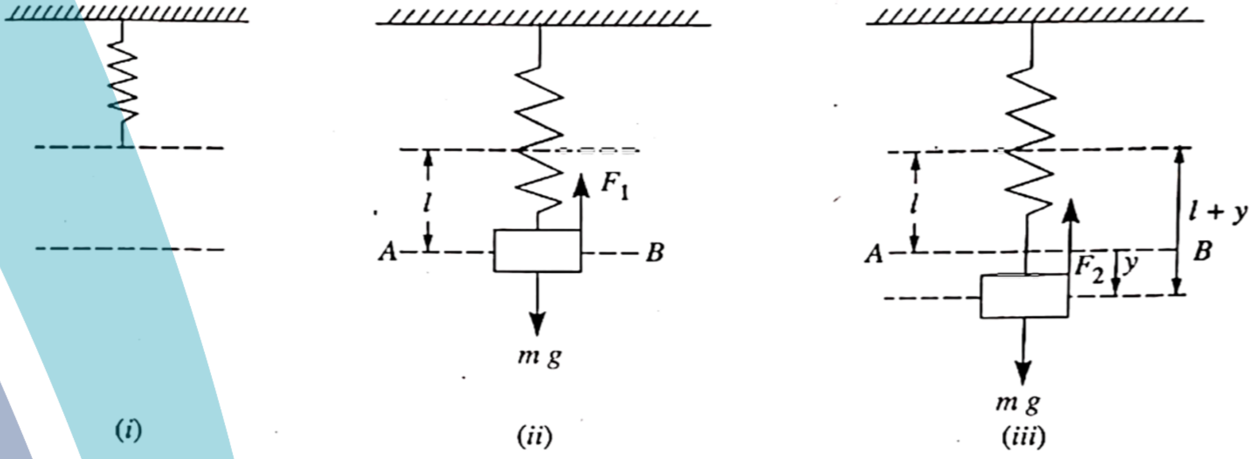


Fig. 25.8

If the mass is displaced downward an *additional* distance y, then the restoring force exerted by the spring is $F_2 = -k(l + y)$ as shown in Fig. 25.8 (*iii*). Now the net force F on the mass is

$$\text{Net force, } F = -k(l + y) + mg = -kl - ky + mg$$

But
$$-kl + mg = 0 \text{ so that :}$$

$$F = -ky$$

We note that restoring force is directly proportional to displacement and is directed towards the equilibrium position. Therefore, if the pull from the mass is released, it will execute S.H.M. about the horizontal dashed line AB. According to Newton's second law, $F = ma$ so that

$$ma = -ky$$

or
$$a = -\frac{k}{m}y$$

Now
$$\text{Time period, } T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{y}{a}}$$

$$= 2\pi\sqrt{\frac{y}{^{**}ky/m}} = 2\pi\sqrt{\frac{m}{k}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} \qquad ...(ii)$$

$$\therefore \quad \text{Frequency, } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad ...(iii)$$

Note that the expressions for T and f are exactly the same as we obtained for a horizontally vibrating mass. The only difference is that here the mass oscillates about an equilibrium point displaced a distance l from the end of the unstretched spring. Therefore, we conclude that *gravity shifts the equilibrium point but does not otherwise effect the vibration.*

*	Downward forces are being considered positive.
**	Magnitude of acceleration.

Alternate formula. From eq. (*i*), $k = mg/l$. Putting this value of k in eq. (*ii*), we have,

$$T = 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}$$

SPRING IN SERIES AND PARALLEL :

(*i*) **Springs in series.** Consider two springs S_1 and S_2 of force constants k_1 and k_2 connected in series as shown in Fig. 25.9. When weight mg is attached to this series combination, the restoring force F in each spring will be the same. Since force constants of the two springs are different, the extensions in their lengths will be different. Let the extension in the length of spring S_1 be y_1 and that of S_2 be y_2. Then,

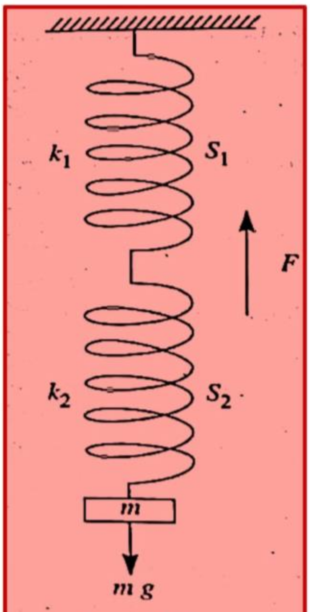


Fig. 25.9

$$F = -k_1 y_1 \quad \text{and} \quad F = -k_2 y_2$$

$$\therefore \quad y_1 = -\frac{F}{k_1} \quad \text{and} \quad y_2 = -\frac{F}{k_2}$$

Total extension y in the series combination is

$$y = y_1 + y_2 = \left(-\frac{F}{k_1}\right) + \left(-\frac{F}{k_2}\right) = -\left(\frac{1}{k_1} + \frac{1}{k_2}\right)F$$

$$= -\left(\frac{k_1 + k_2}{k_1 k_2}\right)F$$

$$\therefore \quad F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right)y$$

$$\text{or} \quad F = -ky$$

where $k = \dfrac{k_1 k_2}{k_1 + k_2}$ = Force constant of series combination

If the two springs are identical, $k_1 = k_2$. Therefore, the force constant of the series combination is $k = k_1/2$ and time period is

$$T = 2\pi\sqrt{\frac{2m}{k_1}}$$

when the springs are connected in series, the force constant of the combination is decreased. Therefore, the mass m will oscillate with a longer period as compared with the case for a single spring.

(*ii*) **Springs in parallel.** Consider two springs S_1 and S_2 of force constants k_1 and k_2 connected in parallel as shown in Fig. 25.10. When weight mg is attached at the middle of a short horizontal connecting bar, each spring will undergo the same amount of extension y. Since the force constants of the two springs are different, the restoring force exerted by each spring will be different. Let F_1 and F_2 be the restoring forces exerted by springs S_1 and S_2 respectively. Then,

$$F_1 = -k_1 y \quad \text{and} \quad F_2 = -k_2 y$$

Both the restoring forces will be in the same direction (opposite to displacement). Therefore, the resultant restoring force F acting on mass m is

$$F = F_1 + F_2 = (-k_1 y) + (-k_2 y) = -(k_1 + k_2)y$$

$$\text{or} \quad F = -ky$$

where $k = k_1 + k_2$ = Force constant of parallel combination

The period of oscillation of mass m is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} \qquad \ldots(ii)$$

If the two springs are identical, $k_1 = k_2$. Therefore, the force constant of the parallel combination is $k = k_1 + k_1 = 2\,k_1$ and time period is

$$T = 2\pi\sqrt{\frac{m}{2k_1}}$$

when the springs are connected in parallel, the force constant of the combination is increased. Therefore, the period of a parallel system is less than that for a single spring.

Another case. Consider two springs S_1 and S_2 of force constants k_1 and k_2 attached to a mass m and two fixed supports as shown in Fig. 25.11. When the mass is pulled downward, then length of the spring S_1 will be extended by y while that of spring S_2 will be compressed by y. Since the force constants of the two springs are different, the restoring force exerted by each spring will be different. Let F_1 and F_2 be the restoring forces exerted by springs S_1 and S_2 respectively. Then,

$$F_1 = -k_1 y \quad \text{and} \quad F_2 = -k_2 y$$

Both the restoring forces will be directed upward (opposite to displacement). Therefore, the resultant restoring force F is

$$F = F_1 + F_2 = (-k_1 y) + (-k_2 y) = -(k_1 + k_2)\,y$$

or $$F = -k\,y$$

where $k = k_1 + k_2$ = Effective force constant of the system

The period of oscillation of mass m is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} \qquad ...(iii)$$

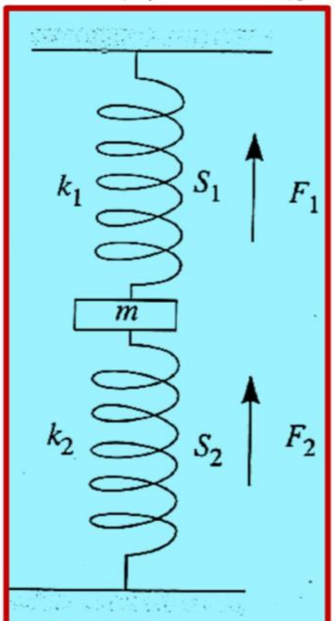


Fig. 25.11

[B] OSCILLATIONS OF SIMPLE PENDULUM:

A simple pendulum consists of a small mass (the pendulum bob) suspended from the end of a light thread. The distance from the point of suspension to the centre of gravity of the bob is called the 'effective length' of the pendulum. The equilibrium or mean position [point O in Fig. 25.15 (*i*)] is where the pendulum hangs vertically. When the bob is displaced slightly to one side from its mean position and released, then it oscillates about the mean position in an arc of a circle. The linear displacement of the bob is the arc length [See Fig. 25.15 (*ii*)] from the mean position O. If we can show that the restoring force is proportional to the linear displacement from mean position and directed opposite to the displacement, we can conclude that the bob moves with S.H.M. Indeed, it is so.

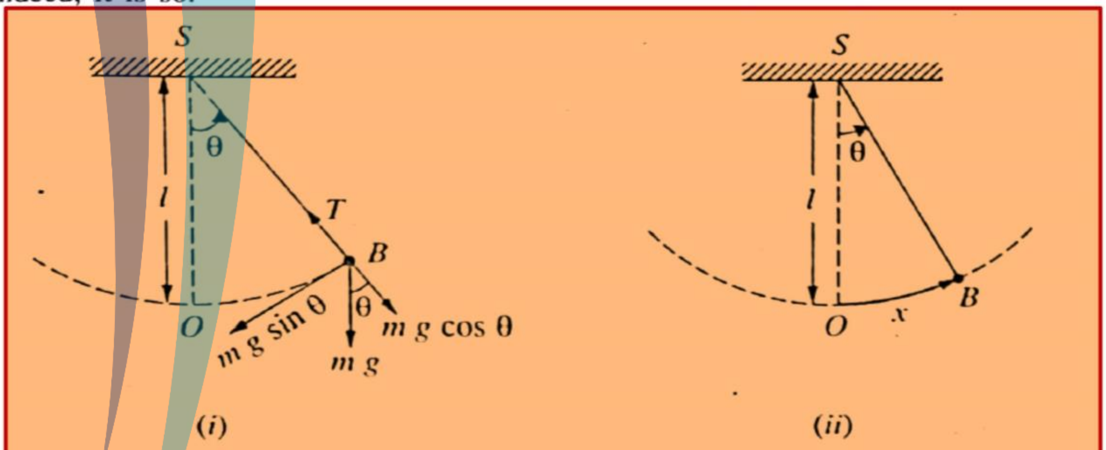


Fig. 25.15

Time period of simple pendulum. Suppose l is the effective length of a simple pendulum and m is the mass of its bob. The pendulum is suspended from point S and its mean position is O. Suppose at any instant, the vibrating bob is in the position B where its linear displacement is arc $OB = x$ and the thread makes an angle θ with the vertical. As shown in Fig. 25.15 (*i*), there are two forces acting on the mass m : the tension T and the weight mg. The tension T is always directed along the thread toward the point of suspension and mg is always vertically downward. The weight mg can be resolved into two rectangular components *viz.*

(*i*) the component $mg \cos\theta$ in line with the thread and directed opposite to T. The net radial force ($= T - mg\cos\theta$) along the thread causes the mass m to move in a circular arc whose radius is the length of the pendulum l.

(*ii*) the component $mg \sin\theta$ acts tangent to the arc along which the bob moves. This component restores the bob toward its equilibrium position and serves as a restoring force. Thus we can write :

$$\text{Restoring force, } F = -m\,g\,\sin\theta$$

The negative sign shows that the restoring force points opposite to the direction of the displacement.

If θ is small, $\sin\theta = \theta$ so that restoring force can be written as :

$$\text{Restoring force, } F = -m\,g\,\theta$$

The linear displacement x of the bob from its mean position is related to its angular displacement θ as :

$$x = l\,\theta \qquad \text{where } \theta \text{ must be in the units of radians.}$$

$$\therefore \quad \text{Restoring force, } F = -\frac{m\,g\,x}{l} \qquad \left(\because \theta = \frac{x}{l}\right)$$

$$\therefore \quad \text{Acceleration of bob, } a = \frac{F}{m} = -\frac{mgx}{lm} = -\left(\frac{g}{l}\right)x \qquad \text{...(i)}$$

Now g/l is constant for a given pendulum at a place.

$$\therefore \quad {}^{*}a \propto -x$$

Thus the acceleration a of the bob is directly proportional to its displacement x and its direction is opposite to the displacement. *Therefore, the motion of the bob is simple harmonic.* The time period of a body executing S.H.M. is given by ;

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$\text{From eq. (i) above, } \frac{\text{displacement } (x)}{\text{acceleration } (a)} = \frac{l}{g} \quad \text{(magnitude)}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{...(ii)}$$

period of simple pendulum does not depend on the mass of the pendulum bob. It depends only on length l and on g.

Frequency of vibrations of simple pendulum is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \qquad \text{...(iii)}$$

Discussion.

(*i*) The frequency (and hence time period) of a simple pendulum depends *only* on the length l and on g. This fact provides a simple method to measure g. A pendulum of known length (l) is made to oscillate. The frequency f (no. of vibrations per second) is recorded. Then g can be easily calculated from eq. (*iii*).

(*ii*) On the moon, g is about one-sixth that on the earth. From eq. (*ii*), we see that a pendulum of given length on the moon would have a period over twice as long as on the earth.

Second's pendulum. *A simple pendulum whose time period is two seconds is called second's pendulum.* It is called second's pendulum because it makes a swing from one side to the other in one second. Let us find the length of such a pendulum.

Now,
$$T = 2\pi\sqrt{\frac{l}{g}}$$

or
$$l = \frac{T^2 g}{4\pi^2}$$

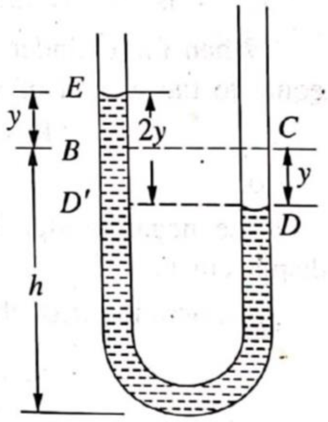
Here $T = 2$ s ; $g = 9.8$ m s^{-2}

$$\therefore \quad l = \frac{(2)^2 \times 9.8}{4\pi^2} = 0.993 \text{ m} = 99.3 \text{ cm}$$

length of second's pendulum depends upon the value of g; the value of length changes from place to place due to change in g.

C] OSCILLATIONS OF LIQUID IN U - TUBE:

Consider a liquid filled in a vertical U-tube of uniform area of cross-section upto a height h as shown in Fig. 25.16. In other words, initially (equilibrium position), the liquid level is at the same height in both the limbs (at B and C). On pressing down level C upto D through a distance y, the level B rises up the same distance upto E. As a result, in the left limb of the U-tube, there is an additional liquid column $D'E$ of length $2y$ as shown in Fig. 25.16. When the pressed level of the liquid is left free, the levels of the liquid in the two limbs will oscillate about their respective initial (undisturbed) positions B, C. Note that the weight of the liquid column of height $2y$ serves as the restoring force and tends to restore the liquid column to its equilibrium position. If the mass of the liquid per unit length of the tube is m, then restoring force F acting on the liquid is



$$\text{Restoring force, } F = -(m \times 2\, y)\, g$$

The negative sign indicates that the restoring force points opposite to the direction of the displacement of the liquid.

The mass of the whole liquid in the tube is $m \times 2\,h$ where $2\,h$ is the length of the whole liquid column.

$\therefore$ Acceleration a of the liquid column is

$$a = \frac{\text{Force}}{\text{Mass}} = \frac{-(m \times 2y)g}{m \times 2h} = -\left(\frac{g}{h}\right)y \qquad \text{...}(i)$$

Since g/h is constant, $a \propto -y$.

Thus the acceleration a of the liquid column is directly proportional to its displacement y and its direction is opposite to the displacement. *Therefore, motion of the liquid is simple harmonic.* The time period for S.H.M. is

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

From eq. (i) above, $\dfrac{\text{displacement } (y)}{\text{acceleration } (a)} = \dfrac{h}{g}$ (magnitude only)

$$\therefore \quad T = 2\pi\sqrt{\frac{h}{g}} \quad \text{...}(ii) \; ; \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{h}} \qquad \text{...}(iii)$$

The period of oscillation does not depend on the density of the liquid and the cross-sectional area of the tube. It depends upon the length of liquid column and on g. In practice, the oscillations are heavily damped due to friction, which we have ignored.

D] OSCILLATIONS OF A FLOATING CYLINDER:

Consider a wooden cylinder of mass m and cross-sectional area A floating in a liquid of density ρ. At equilibrium, the cylinder is floating with a depth h submerged [See Fig. 25.17]. If the cylinder is pushed downward a small distance y and then released, it will move up and down with S.H.M. It is desired to find the time period and the frequency of oscillations.

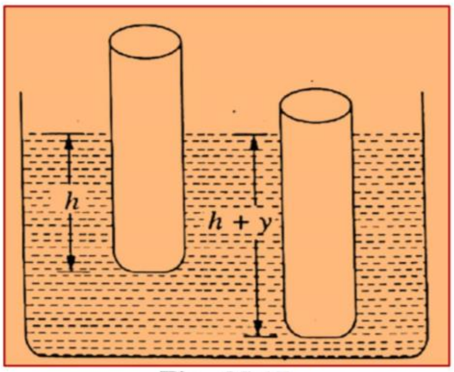


Fig. 25.17

According to the principle of floatation, the weight of the liquid displaced by the immersed part of the body is equal to the weight of the body. Therefore, at equilibrium,

Weight of cylinder = Weight of liquid displaced by the immersed part of cylinder

or $$m\,g = (\rho A\,h)\,g$$

$\therefore$ Mass of cylinder, $m = \rho A\,h$

When the cylinder is pushed down an additional distance y, the restoring force F (upward) equal to the weight of additional liquid displaced acts on the cylinder.

$\therefore$ Restoring force, $F = -$ (weight of additional liquid displaced)

or $$F = -(\rho A\,y)\,g$$

The negative sign indicates that the restoring force acts opposite to the direction of the displacement.

Acceleration a of the cylinder is given by ;

$$a = \frac{F}{m} = -\frac{\rho A\,y\,g}{\rho\,Ah} = -\left(\frac{g}{h}\right)y \quad ...(i)$$

Since g/h is constant, $a \propto -y$.

Thus the acceleration a of the body (wooden cylinder) is directly proportional to the displacement y and its direction is opposite to the displacement. *Therefore, motion of the cylinder is simple harmonic.* The time period for S.H.M. is

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

From eq. (*i*) above, $\dfrac{\text{displacement }(y)}{\text{acceleration }(a)} = \dfrac{h}{g}$ (magnitude only)

$\therefore$ $$\text{Time period, } T = 2\pi\sqrt{\frac{h}{g}} \quad ...(ii)$$

$\therefore$ $$\text{Frequency, } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{h}} \quad ...(iii)$$

These very interesting results show that time period and frequency have the same form as that of the simple pendulum. The submerged depth at equilibrium takes the place of the length of the pendulum.

E] BODY DROPPED IN A TUNNEL ALONG EARTH'S DIAMETER:

Suppose earth to be a sphere of radius R with centre O. Let a tunnel be dug along the diameter of the earth as shown in Fig. 25.18. If a body of mass m is dropped at one end of the tunnel, the body will execute S.H.M. about the centre O of the earth. Suppose at any instant, the body in the tunnel is at a distance y from the centre O of the earth. Because the body is *inside* the earth, only the inner sphere of radius y will exert gravitational force F on the body. The force F serves as the restoring force that tends to bring the body to the equilibrium position O.

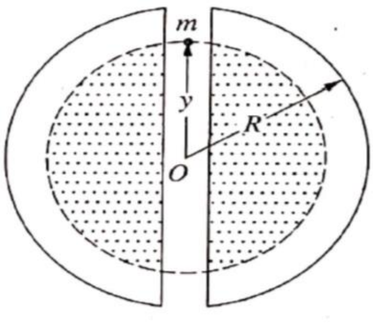


Fig. 25.18

$\therefore$ $$\text{Restoring force, } F = -G \times \frac{\left(\frac{4}{3}\pi y^3 \rho\right)m}{y^2}$$

where ρ is the density of the earth. The negative sign is assigned because the force is of attraction.

Acceleration of the body, $a = \frac{F}{m} = -\left(\frac{4}{3}\pi G\rho\right)y$ ...(*i*)

Now the quantity $(4/3)\,\pi G\rho$ is constant so that :

$$a \propto -y$$

Thus the acceleration of the body is directly proportional to the displacement y and its direction is opposite to the displacement. *Therefore, the motion of the body is simple harmonic.* The time period for S.H.M. is

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

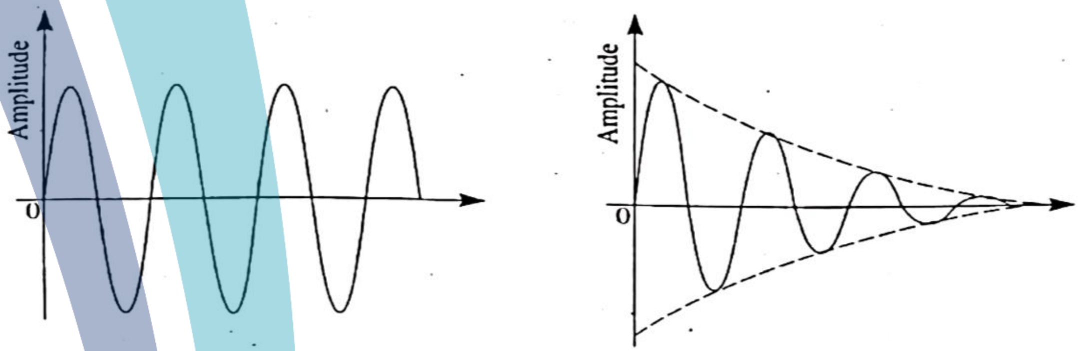
From eq. (*i*) above, $\frac{\text{displacement}\,(y)}{\text{acceleration}\,(a)} = \frac{3}{4\pi G\rho}$ (magnitude only)

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$ or $T = \sqrt{\frac{3\pi}{G\rho}}$ ...(*ii*)

D AMPED ABD U NDAMPED OSCILLATION:

A system executing simple harmonic motion is called a *harmonic oscillator.* A harmonic oscillator produces sinusoidal oscillations. The sinusoidal oscillations can be of two types *viz.*, *undamped oscillations* and *damped oscillations.*

(*i*) Undamped oscillations. *The oscillations whose amplitude remains constant with time are called* **undamped oscillations.** Fig. 25.19 (*i*) shows waveform of undamped oscillations. Such oscillations can occur if frictional forces are absent. For example, if the bob of a pendulum is displaced in vacuum and then released, the bob will continue to execute S.H.M. of constant amplitude.



(*i*) (*ii*)

Fig. 25.19

(*ii*) **Damped oscillations.** *The oscillations whose amplitude goes on decreasing with time are called* ***damped oscillations** [See Fig. 25.19 (*ii*)]. In real oscillating systems, forces like friction are always present that dissipate the oscillating energy. Unless energy is somehow added, dissipation eventually brings the system to rest at equilibrium.

We can define damped motion only if we have a mathematical expression for the damping force. In many systems—especially those involving friction associated with slow motion through a viscous fluid—the damping force is approximately proportional to the velocity and acts opposite to motion.

$$\text{Damping force, } F_d = -bV = -b\frac{dx}{dt}$$

where b is a constant giving the strength of damping. We can write Newton's law, now including damping force along with the restoring force. For a spring-mass system, we have,

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

or $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

We will not solve this equation but its solution will be stated. Provided the damping force is not too large, the solution of this equation is

$$x = ae^{-bt/2m}\cos(\omega t + \phi) \quad ...(i)$$

Eq. (*i*) describes sinusoidal motion whose amplitude (a) decreases exponentially with time. How fast the amplitude drops depends on the damping constants b and m.

The frequency of this damped motion is given by ;

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

If the frictional forces are absent, $b = 0$ so that :

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{...undamped oscillations}$$

FREE, FORCED AND RESONANT OSCILLATIONS

(*i*) Free oscillations. When a body capable of oscillating is displaced from its equilibrium position and then left free, it begins to oscillate with a definite frequency. This frequency depends upon the intrinsic properties (shape, elasticity etc.) of the body and is called *natural frequency* of the body. Such oscillations are called free oscillations.

When a body capable of oscillating is made to vibrate with its own natural frequency, it is said to execute **free oscillations.**

For example, when a simple pendulum is displaced from its mean position and then left free, it executes free oscillations. The natural frequency of the simple pendulum depends upon its length and is given by ;

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Examples. (*a*) When a tuning fork is struck with a rubber pad, then its prongs produce free oscillations. The frequency of free oscillations (*i.e.*, natural frequency) depends on tuning fork's length, its thickness and the material of which it is made.

(*b*) If we pluck the string of a sitar and then left free, it executes free oscillations. The natural frequency depends upon the length, density and tension of the string.

(*c*) When the bob of a simple pendulum is displaced from its mean position and then left free, it executes free oscillations. The natural frequency depends upon the length of the pendulum.

In the absence of frictional forces, the amplitude of free oscillations would remain constant (undamped oscillations). However, in actual practice, the amplitude of free oscillations decreases gradually with time due to the frictional forces. When whole of the originally imparted energy to the body is used up in overcoming frictional forces, the body stops executing oscillations and comes to rest at the mean position.

(*ii*) Forced oscillations. *When a body is maintained in a state of oscillations by an external periodic force of frequency other than the natural frequency of the body, the oscillations are called* **forced oscillations.**

The frequency of forced oscillations is equal to the frequency of the periodic force. The external applied force on the body is called the *driver* and the body set into oscillations is called *driven oscillator.*

Examples. (*a*) When the stem of a vibrating tuning fork is held in hand, only a feeble sound is heard. However, if the stem is pressed against a table top, the sound becomes louder. It is because the tuning fork *forces* the table to vibrate with fork's frequency. Since the table has a large vibrating area than the tuning fork, these *forced oscillations* produce a more intense sound.

(*b*) A vibrating violin string stretched tightly between two clamps does not produce intense sound. When the string is stretched across the wooden bridge of a violin, the wooden bridge is *forced to vibrate at the same frequency. Since the wooden bridge has a much larger area than the string, these forced oscillations produce a more intense sound.

Mathematical analysis. Most of the oscillations that occur in systems (*e.g.*, machinery) are forced oscillations; oscillations that are produced and sustained by an external force. The simplest driving force is one that itself oscillates as a sine or a cosine. Suppose such an external force F_{ext} is applied to an oscillator that moves along x axis such as a block connected to a spring. We can represent the external force as :

$$F_{ext} = F_0 \cos \omega t$$

where F_0 is the maximum magnitude of the force and ω ($= 2\pi f$) is the angular frequency of the force.

Examples. (*a*) A tuning fork is often mounted on a sound box. The box is constructed in such a way that the natural frequency of air column inside it has the same frequency as that of the fork. When the tuning fork is struck, it sets the air column into resonant vibrations. This reinforces the sound of the fork.

(*b*) Soldiers are asked to break step while crossing a bridge. If the soldiers march in step, there is a possibility that the frequency of the footsteps may become equal to natural frequency of the bridge. Due to resonance, the bridge may start vibrating violently, thereby damaging itself

(*c*) During earthquake, resonance can cause disaster. If the natural frequency of a building matches the frequency of earthquake, the building will begin to vibrate with a large amplitude. Consequently, the building may collapse.

COUPLED O SCILLATIONS

A system of two bodies connected by a spring so that both can oscillate in a straight line along the length of the string is called a **coupled oscillator.**

If the spring is stretched and then released, the two bodies will oscillate along the length of the spring. Such oscillations are called **coupled oscillations**. It can be proved that oscillations produced by such a system are S.H.M. in nature.

Fig. 25.22 (*i*) shows two masses m_1 and m_2 connected at the ends of a massless spring of length l and the system is placed on a frictionless horizontal surface. When the spring is stretched and then released, the system oscillates in S.H.M. We can describe the motion of the system in terms of the separate motions of the two masses. Suppose at any time :

x_1 = distance of mass m_1 from the origin O

x_2 = distance of mass m_2 from the origin O

The relative separation $x_1 - x_2$ gives the length of the spring at any time. If the spring is stretched by an amount x, then,

$$x = (x_1 - x_2) - l \quad ...(i)$$

The magnitude of force exerted by spring on each mass is $F = kx$ where k is spring constant. If the spring exerts a force $-F$ on m_1, it exerts a force $+F$ on m_2. Let us apply Newton's second law of motion separately to the two masses.

$$m_1 \frac{d^2x_1}{dt^2} = -kx \quad ...(ii) \qquad m_2 \frac{d^2x_2}{dt^2} = +kx \quad ...(iii)$$

Multiplying eq. (*ii*) by m_2 and eq. (*iii*) by m_1 and then subtracting eq. (*iii*) from eq. (*ii*), we have,

$$m_1 m_2 \frac{d^2x_1}{dt^2} - m_1 m_2 \frac{d^2x_2}{dt^2} = -m_2 kx - m_1 kx$$

or
$$m_1 m_2 \frac{d^2}{dt^2}(x_1 - x_2) = -k(m_1 + m_2)x$$

or
$$\frac{m_1 m_2}{m_1 + m_2} \frac{d^2}{dt^2}(x_1 - x_2) = -kx \quad ...(iv)$$

Taking second derivative of both sides of eq. (*i*), we have,

$$\frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(x_1 - x_2)$$

Therefore, eq. (*iv*) becomes :

$$\frac{m_1 m_2}{m_1 + m_2} \frac{d^2x}{dt^2} = -kx \quad ...(v)$$

The quantity $\frac{m_1 m_2}{m_1 + m_2}$ has the dimensions of mass and is called *reduced mass* μ.

$$\therefore \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Note that reduced mass is always smaller than either mass. Therefore, eq. (*v*) becomes :

$$\mu \frac{d^2x}{dt^2} = -kx$$

or $$\frac{d^2x}{dt^2} = -\frac{k}{\mu}x$$

or Acceleration, $$a = -\frac{k}{\mu}x \quad ...(vi)$$

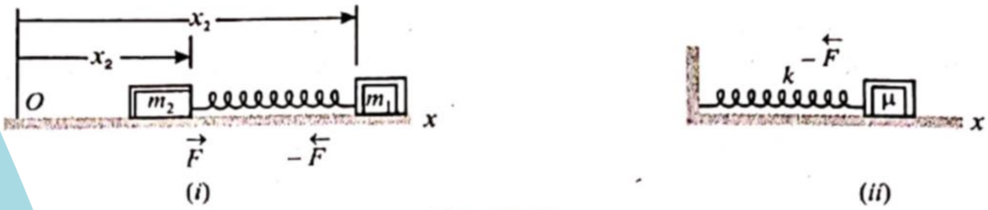


Fig. 25.22

Eq. (*vi*) is identical in form as for the motion of single mass connected to a spring. Therefore, the system shown in Fig. 25.22. (*i*) can be replaced by a single mass equal to the reduced mass $\mu(= m_1m_2/ m_1+ m_2)$ as shown in Fig. 25.22 (*ii*). If one of the masses is very much smaller than the other, then μ is roughly equal to the smaller mass. If the masses are equal, then μ is half as large as either mass.

Since k/μ is constant, acceleration, $a \propto - x$. Therefore, the acceleration of the motion of the system is directly proportional to the relative displacement of the masses from the mean position. Further, negative sign shows that acceleration is always directed towards the mean position. *Therefore, coupled oscillations are simple harmonic in nature.* The time period of S.H.M. is given by ;

$$\text{Time period, } T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{k}{\mu}x}} = 2\pi\sqrt{\frac{\mu}{k}}$$

Note that time period of the system is identical in form as for a single oscillating mass except that now mass is equal to the reduced mass of the system.

CONCEPTUAL...

Q.1. What is the origin of the name simple harmonic motion?

Ans. Of all the periodic or harmonic motions (*e.g.*, circular motion, elliptical motion etc.), it is the simplest periodic motion. For this reason, it is called simple harmonic motion.

Q.2. What are the characteristics of simple harmonic motion?

Ans. The characteristics of simple harmonic motion are : (*i*) It is a periodic motion (*ii*) The particle moves to and fro about the mean position in a straight line (*iii*) The acceleration of the particle is directly proportional to the displacement from mean position and its direction is opposite to the displacement.

Q.3. What type of force causes a particle to execute S.H.M.?

Ans. When the force on a particle is directly proportional to its displacement from the mean position and acts opposite to the displacement, the particle will execute S.H.M.

Q.4. Is the motion of a simple pendulum strictly simple harmonic?

Ans. It is not strictly simple harmonic because we make the assumption that $\sin\theta = \theta$ which is nearly valid only if θ is very small.

Q.5. Why do we say that velocity and acceleration of a body executing S.H.M. are out of phase?

Ans. When a body is executing S.H.M., its acceleration is zero at the mean position and maximum at the extreme positions. On the other hand, the velocity is maximum at the mean position and zero at the extreme positions. Further, acceleration is always directed towards the mean position. For this reason, the velocity and acceleration of a body executing S.H.M. are out of phase.

Q.6. The amplitude of a simple harmonic oscillator is doubled. How does this effect (*i*) the period (*ii*) the total energy (*iii*) the maximum velocity of the oscillator?

Ans. (*i*) The time period of a simple harmonic oscillator is independent of the amplitude of the oscillations. Therefore, time period is not affected.

(*ii*) Total energy of oscillator is $E = (1/2)\, m\, \omega^2 a^2$ where a is the amplitude. When a is doubled, E becomes four times.

(*iii*) $V_{max} = \omega a$. When a is doubled, V_{max} is also doubled.

Q.7. What is the relation between displacement and velocity of a body executing S.H.M. ?

Ans. The velocity of a body executing S.H.M. is maximum at the point where displacement is zero (mean position); it decreases with displacement and becomes zero at the extreme position.

Q.8. What is the relation between uniform circular motion and S.H.M.?

Ans. Uniform circular motion can be thought of as two simple harmonic motions operating at right angles.

Q.9. What is the most important characteristic of simple harmonic motion?

Ans. The most important characteristic of S.H.M. is that acceleration is directly proportional to the displacement and is directed opposite to the displacement. Mathematically, $A = -\omega^2 y$.

Q.10. Can simple pendulum experiment be done inside a satellite?

Ans. The time period of a simple pendulum is given by ;

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Inside a satellite, a body is in a state of weightlessness so that the effective value of g for it is zero. When $g = 0$, $T \to \infty$. Therefore, inside the satellite, the pendulum does not oscillate at all. Consequently, the simple pendulum experiment cannot be performed inside a satellite.

Q.11. Which of the following is not a sufficient condition for S.H.M. and why? (*i*) acceleration $\propto$ displacement (*ii*) restoring force $\propto$ displacement.

Ans. Both the conditions are insufficient because the minus sign is missing. The missing minus sign implies that the acceleration increases as displacement increases and the object would then never return to its mean position.

Q.12. Show that time period of S.H.M. is given by $2\pi/\omega$ where ω is the angular frequency of S.H.M.

Ans. The displacement-time equation of S.H.M. is $y = a \sin(\omega t + \phi)$

Note that function y is periodic *i.e.* value of y at time t equals the value at time $t + T$ where T is the time period of the motion. Since the phase increases by 2π radians in time T,

$$\therefore \quad \omega t + \phi + 2\pi = \omega(t + T) + \phi \quad \therefore \quad T = \frac{2\pi}{\omega}$$

Q.13. The acceleration of a spring-mass system is given by ;

$$a = -\frac{k}{m}x$$

What conclusions you draw from this equation?

Ans. If the mass is displaced a maximum distance a (amplitude) from the mean position and then released, its *initial* acceleration will be $-ka/m$ (*i.e.* it has a maximum negative value). When it passes through the mean position ($x = 0$), the acceleration is zero. At this instant, the velocity is maximum. It will then travel to the other side of mean position and finally reaches $x = -a$ at which time its acceleration is ka/m (maximum positive) and its velocity is zero. In one full cycle of motion, the mass travels a distance of $4a$.

Q.14. Show that the total energy of a body executing S.H.M. is independent of time.

Ans. Let us discuss the total energy of mass-spring system. Assuming initial phase to be zero,

$$y = a \sin \omega t, \quad V = \frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t) = a\omega \cos \omega t$$

$$\therefore \quad K.E. = \frac{1}{2}mV^2 = \frac{1}{2}m\omega^2 a^2 \cos^2 \omega t = \frac{1}{2}ka^2 \cos^2 \omega t \quad (\because m\omega^2 = k)$$

$$P.E. = \frac{1}{2}ky^2 = \frac{1}{2}ka^2 \sin^2 \omega t \quad (\because y = a \sin \omega t)$$

Note that K.E. and P.E. are always positive quantities.

$$\text{Total energy} = K.E. + P.E. = \frac{1}{2}ka^2(\cos^2 \omega t + \sin^2 \omega t) = \frac{1}{2}ka^2$$

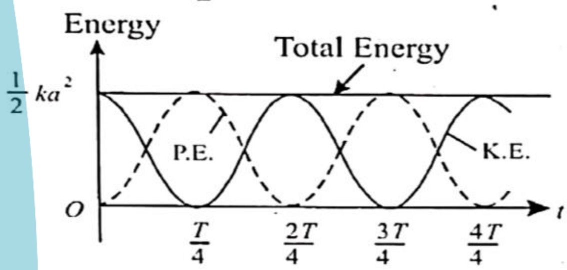


Fig. 25.23

Thus total mechanical energy is always constant and is equal to $k a^2/2$. Note that total energy is independent of time. Fig. 25.23 shows kinetic energy and potential energy versus time for the mass-spring system. The potential energy oscillates with time and has a maximum value of $ka^2/2$. Similarly, K.E. oscillates with time and has a maximum value of $k a^2/2$. At any instant, K.E. + P.E. = constant = $k a^2/2$. Note that K.E. or P.E. oscillates at double the frequency of S.H.M.

Q.15. You have a light spring, a metre scale and a known mass. How will you find the time period of oscillations of mass without the use of a clock?

Ans. Suspend the spring from a rigid support and attach the mass at its lowest end. Measure the extension (l) in the spring with a metre scale. If k is the force constant of the spring, then restoring force F is

$$F = -kl = -mg \quad \therefore \quad \frac{m}{k} = \frac{l}{g}$$

$$\text{Time period, } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$$

Q.16. Given some practical examples of S.H.M.

Ans. Some practical examples of S.H.M. are :

(*i*) Motion of piston in a gas-filled cylinder.

(*ii*) Atoms vibrating in a crystal lattice.

Q.1. What is a periodic motion?

Ans. The motion which repeats itself after a regular interval of time is called a periodic motion *e.g.*, motion of earth around the sun, motion of moon around the earth etc.

Q.2. What is an oscillatory motion?

Ans. It is a periodic motion (*i.e.*, having definite time period) in which the body moves to and fro along the same path about a fixed point (called mean position) *e.g.*, a pendulum swinging back and forth.

Q.3. What are the characteristics of oscillatory motion?

Ans. (*i*) It is a periodic motion (*ii*) It is to and fro motion along the same path about a fixed point called equilibrium position or mean position (*iii*) The body is confined within well defined limits called extreme positions.

Q.4. What is simple harmonic motion (S.H.M.)?

Ans. When the oscillatory motion of a body is along straight line and the graph between displacement of the body from the mean position and time is a sine curve (or cosine curve), the body is said to execute S.H.M.

Q.5. Define (*i*) displacement (*ii*) amplitude of S.H.M.

Ans. (*i*) The distance of the body at any time from the mean position is called displacement of the body at that time.

(*ii*) The maximum displacement of the body from its mean position is called amplitude of the body.

Q.6. What is the test for S.H.M.?

Ans. The displacement-time graph for a body executing S.H.M. is a sine curve or cosine curve.

Q.7. Why is S.H.M. called so?

Ans. It is the simplest type of oscillatory motion.

Q.8. Is the motion of earth around the sun periodic or oscillatory?

Ans. Periodic. Oscillatory motion is to and fro motion about a mean position.

Q.9. A particle is executing S.H.M. What are its velocity and acceleration at (*i*) mean position (*ii*) extreme positions?

Ans. (*i*) Velocity = maximum; Acceleration = zero (*ii*) Velocity = 0; Acceleration = maximum.

Q.10. How is time period of simple harmonic motion related to its acceleration?

Ans. Time period, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$.

Q.11. How are time period and frequency of S.H.M. affected by the amplitude of motion?

Ans. The time period (T) and frequency (f) of S.H.M. are independent of amplitude of motion.

Q.12. A particle is executing S.H.M. What are its K.E. and P.E. at (*i*) mean position (*ii*) extreme positions?

Ans. (*i*) K.E. = maximum ; P.E. = 0 (*ii*) P.E. = maximum ; K.E. = 0.

Q.13. A particle executing S.H.M. has amplitude 0.01 m and frequency 60 Hz. What is the maximum acceleration of the particle?

Ans. Amplitude, a = 0.01 m ; $\omega = 2\pi f = 2\pi \times 60$ rad s^{-1}

$\therefore$ Max. acceleration of particle $= \omega^2 a = (2\pi \times 60)^2 \times 0.01 = 144\,\pi^2$ m/s^2.

Q.14. The acceleration of a particle performing S.H.M. is 12 cm/s^2 at a distance of 3 cm from the mean position. What is its time period?

Ans. Acceleration, $A = \omega^2 y$ or $\omega^2 = A/y = 12/3 = 4$ $\quad\therefore\quad \omega = 2$ rad/s

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.14$ s.

Q.15. The length of a simple pendulum is doubled. How will its time period be affected?

Ans. Time period, $T \propto \sqrt{l}$. Therefore, the time period will become $\sqrt{2}$ times the original value.

Q.16. What is a seconds pendulum?

Ans. A simple pendulum that has a time period of 2 seconds is called a seconds pendulum.

Q.17. Why does a swinging simple pendulum eventually stop?

Ans. Due to friction between air and bob, the amplitude of the pendulum goes on decreasing and eventually it comes to rest.

Q.18. A swinging simple pendulum is placed in a lift which is falling freely. What is the frequency of the pendulum?

Ans. The time period of a simple pendulum falling with acceleration a is

$T = 2\pi\sqrt{\frac{l}{g-a}}$. For a freely falling lift, $a = g$ so that $T = \infty$. Hence $f = 0$.

Q.19. A swinging simple pendulum is placed in a lift which is accelerating downwards. How is its time period affected?

Ans. The time period of a simple pendulum falling downwards with acceleration a is

$T = 2\pi\sqrt{\frac{l}{g-a}}$. Therefore, the time period will increase because the effective acceleration due to gravity decreases from g to $(g - a)$.

Q.20. A swinging simple pendulum is placed in a lift which is accelerating upwards. How is its time period affected?

Ans. The time period of a simple pendulum moving upwards with acceleration a is

$T = 2\pi\sqrt{\frac{l}{g+a}}$. Therefore, the time period will decrease because the effective acceleration due to gravity increases from g to $(g + a)$.

Q.21. Is the tension in the string of a simple pendulum constant throughout the oscillations?

Ans. No. The tension in the string is $T = mg \cos\theta$ where θ is the angle which the string makes with the vertical. As θ varies, the tension T in the string also varies.

Q.22. When is tension (*i*) maximum (*ii*) minimum in the string of a simple pendulum?

Ans. (*i*) Tension in the string is $T = mg \cos\theta$. Therefore, tension in the string is maximum when $\cos\theta = 1$ or $\theta° = 0°$ *i.e.*, when the bob is at the mean position.

(*ii*) Tension in the string is minimum when $\cos\theta$ is minimum or θ is maximum *i.e.*, when the bob is at either extreme position.

Q.23. Will a pendulum clock gain or lose time when taken to the top of a mountain?

Ans. Time period, $T = 2\pi\sqrt{l/g}$. As g is less at the top of mountain, value of T will increase. Therefore, the pendulum will take more time to complete one vibration. As a result, pendulum clock will lose time.

Q.24. A wrist watch is taken to the top of a mountain. Will it give correct time?

Ans. Yes. It is because the time period of wrist watch (spring controlled) does not depend upon the value of g but depends upon the potential energy stored in the spring.

Q.25. Will a simple pendulum vibrate at the centre of earth?

Ans. No. It is because $g = 0$ at the centre of earth.

Q.26. If the metal bob of a simple pendulum is replaced by a wooden bob, what will be the change in the time period of the pendulum?

Ans. No change. It is because time period $T(= 2\pi\sqrt{l/g})$ of a simple pendulum is independent of the mass of the bob.

Q.27. The length of a seconds pendulum is *l*. If the length is halved, what will be its time period?

Ans. The time period T of seconds pendulum is 2s. Now $T \propto \sqrt{l}$.

$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} \text{ or } \frac{T_2}{2} = \sqrt{\frac{l_1/2}{l_1}} = \frac{2}{\sqrt{2}}\text{s}$$

Q.28. A particle is executing S.H.M. What is the phase difference between (*i*) velocity and displacement (*ii*) acceleration and velocity?

Ans. (*i*) The velocity leads the displacement by $\pi/2$ radians.

(*ii*) The acceleration leads the velocity by $\pi/2$ radians.

Q.29. A body is executing S.H.M. with amplitude *a*. In time equal to its period, what is the (*i*) distance moved (*ii*) displacement?

Ans. (*i*) The body reaches its initial position after a time equal to its period. Therefore, the distance covered in one time period $= 4 \times a = 4a$.

(*ii*) Since body comes to its initial position after one time period, displacement is zero.

Q.30. What is spring constant?

Ans. In a spring, the magnitude of restoring force is $F = kx$ where k $(= F/x)$ is the spring constant. It is defined as the force required to produce unit extension or compression in the spring.

Q.31. What is the SI unit and dimensional formula of spring constant?

Ans. The SI unit of spring constant k $(= F/x)$ is Nm^{-1} and its dimensional formula is $[ML^0T^{-2}]$.

$$[k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{[L]} = [M\ L^0\ T^{-2}]$$

Note that spring constant is *dimensional constant.*

Q.32. A body of mass 5 kg hangs from a spring and oscillates with a time period of 2π seconds. What is the force constant of the spring?

Ans. Time period, $T = 2\pi\sqrt{\frac{m}{k}}$ or $2\pi = 2\pi\sqrt{\frac{5}{k}}$ $\therefore\ k = 5\ \text{N m}^{-1}$.

Q.33. A mass *m* is suspended from a spring of negligible mass and the system oscillates with a frequency *f*. What will be the frequency of oscillations if a mass of 4 m is suspended from the same spring?

Ans. Frequency, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Therefore, $f \propto \sqrt{\frac{1}{m}}$.

Now $\frac{f'}{f} = \sqrt{\frac{m}{m'}}$ or $\frac{f'}{f} = \sqrt{\frac{m}{4\,m}} = \frac{1}{2}$ $\therefore\ f' = \frac{f}{2}$

Q.34. A spring of spring constant *K* is cut into three equal pieces. What will be the spring constant of each part?

Ans. Let *k* be the spring constant of each part. We can consider the original spring of spring constant *K* to be made of three springs in series, each of spring constant *k*.

$$\therefore \quad \frac{1}{K} = \frac{1}{k} + \frac{1}{k} + \frac{1}{k} = \frac{3}{k} \quad \text{or} \quad k = 3K$$

This means that spring constant of each part will become three times that of the full length of the spring.

Q.35. Two springs of force constants 100 N/m and 400 N/m are connected in series. What will be the effective spring constant of the spring system?

Ans. The spring constant *k* of the system is given by ;

$$\frac{1}{k} = \frac{1}{100} + \frac{1}{400} \quad \text{or} \quad k = \frac{100 \times 400}{100 + 400} = 80\ \text{N/m}$$

Q.36. Two unequal springs of the same material are loaded with same load. Which one will have a larger value of time period?

Ans. Time period, $T = 2\pi\sqrt{\frac{m}{k}}$. The longer spring will have smaller value of *k* and its time period will be larger.

Q.37. There are two springs, one delicate and other stout one. For which spring, the time period will be less?

Ans. Time period, $T = 2\pi\sqrt{\frac{m}{k}}$. For stout (stiffer) spring, the spring constant will be larger than the delicate spring. Therefore, time period for stout spring will be less.

Q.38. What will be the change in the time period of a loaded spring when taken to moon?

Ans. No change. It is because the time period of a loaded spring is independent of acceleration due to gravity.

Q.39. The spring-mass system oscillating vertically has a time period *T*. What will be the effect on the time period if the spring is made to oscillate horizontally?

Ans. The time period will remain the same ($\because\ T = 2\pi\sqrt{k/m}$).

Q.40. Is the oscillation of a mass suspended by a spring S.H.M.?

Ans. Yes, because it satisfies the conditions of S.H.M.

SHORT TYPE QUESTIONS...

Q.1. A restoring force is a must for a body to execute S.H.M. Explain.

Ans. In S.H.M., the body oscillates back and forth about the mean position. For this to happen, a restoring force must act on the body to permit it to perform back and forth journey about the mean position. When the body is at the extreme position, this restoring force brings it to the mean position. When the body reaches mean position, it has momentum and overshoots. The restoring force slows it down till the velocity of the body becomes zero at the other extreme position.

Q.2. What should be the characteristics of the restoring force in S.H.M.?

Ans. (*i*) The direction of the restoring force should be in a direction opposite to the direction of displacement of the body.

(*ii*) The magnitude of the restoring force should be directly proportional to the displacement of the body.

Q.3. Give mathematical equation of restoring force in S.H.M. How is it provided?

Ans. In S.H.M., the restoring force (F) can be expressed mathematically as :

$$F = -ky$$

Here y is the displacement of the body and k is a constant of proportionality. The negative sign shows that restoring force acts opposite to displacement. The restoring force may be provided due to gravity, elasticity etc.

Q.4. What are the characteristics of S.H.M.?

Ans. The important characteristics of S.H.M. are :

(*i*) The particle moves to and fro about the mean (equilibrium) position in a straight line.

(*ii*) The displacement, velocity and acceleration all vary sinusoidally with time but are not in phase.

(*iii*) The acceleration is always directed towards the mean position.

(*iv*) The restoring force is directly proportional to displacement but acts in a direction opposite to displacement.

Q.5. Show that in S.H.M., the acceleration of the particle is directly proportional to the displacement at the given instant.

Ans. In S.H.M., the displacement of the particle at any instant is

$$y = a \sin \omega t$$

$$\therefore \quad \text{Velocity, } V = \frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t) = a\omega \cos \omega t$$

$$\therefore \quad \text{Acceleration, } A = \frac{dV}{dt} = \frac{d}{dt}(a\omega \cos \omega t) = -\omega^2 a \sin \omega t$$

or $$A = -\omega^2 y \quad (\because a \sin \omega t = y)$$

Thus in S.H.M., acceleration (A) is directly proportional to the displacement *i.e.*, $A \propto y$.

Q.6. Show that velocity and displacement of a body executing S.H.M. are out of phase by π/2 radians.

Ans. In S.H.M., displacement equation is $y = a \sin \omega t$ ...(*i*)

$$\text{Velocity, } V = \frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t) = a\omega \cos \omega t$$

$$\therefore \quad V = a\omega \sin(\omega t + \pi/2) \quad ...(ii)$$

From eqs. (*i*) and (*ii*), it is clear that velocity (V) and displacement (y) are out of phase by $\pi/2$ radians.

Q.7. The displacement of a particle in S.H.M. may be given by ;

$$y = a \sin(\omega t + \phi)$$

Show that if the time t is increased by $2\pi/\omega$, the value of y remains the same.

Ans. $y = a \sin(\omega t + \phi)$. Therefore, displacement at any time $(t + 2\pi/\omega)$ is

$$y = a \sin[\omega(t + 2\pi/\omega) + \phi]$$
$$= a \sin[(\omega t + \phi) + 2\pi]$$
$$= a \sin(\omega t + \phi) \quad [\because \sin(2\pi + \theta) = \sin\theta]$$

Therefore, the displacements at time t and $(t + 2\pi/\omega)$ are the same.

Q.8. A simple pendulum hangs from the ceiling of a car. If the car accelerates with uniform acceleration, will the frequency of the pendulum increase or decrease?

Ans. Frequency of simple pendulum, $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \therefore f \propto \sqrt{g}$.

As the car accelerates with uniform acceleration a, the resultant acceleration $= \sqrt{a^2 + g^2}$. Since $\sqrt{a^2 + g^2}$ is greater than $\sqrt{g}$, frequency of the simple pendulum will increase.

Q.9. A S.H.M. of amplitude a has a time period T. What will be the acceleration of the oscillator when (*i*) at mean position (*ii*) extreme position (*iii*) its displacement is half of the amplitude ?

Ans. Acceleration, $A = -\omega^2 y$; $\omega = 2\pi/T$.

(*i*) At mean position, $y = 0$ so that $A = 0$.

(*ii*) At extreme position, $y = a$ so that $A = -\omega^2 a = -\frac{4\pi^2}{T^2}a$.

(*iii*) At $y = a/2$, $A = -(2\pi/T)^2 \times \frac{a}{2} = -\frac{2\pi^2 a}{T^2}$.

Q.10. A S.H.M. of amplitude a has time period T. What will be the velocity of the oscillator at (*i*) equilibrium position (*ii*) half amplitude?

Ans. Velocity, $V = \omega\sqrt{a^2 - y^2}$; $\omega = 2\pi/T$.

(*i*) At equilibrium position, $y = 0$ so that $V = a\omega = a\,2\pi/T$.

(*ii*) At $y = a/2$, $V = \frac{2\pi}{T}\sqrt{a^2 - a^2/4} = \frac{\sqrt{3}\,a\pi}{T}$.

Q.11. A particle executing S.H.M. has maximum speed when passing through the mean position. Why?

Ans. P.E., $E_P = \frac{1}{2}m\omega^2 y^2$; K.E., $E_K = \frac{1}{2}m\omega^2(a^2 - y^2)$

At mean position, $y = 0$ so that $E_P = 0$ and $E_K = \frac{1}{2}m\omega^2 a^2$.

Since K.E. of the particle at mean position is maximum, its speed at this position will also be maximum.

Q.12. A particle executes S.H.M. with amplitude a. At what distance from the mean position its K.E. is equal to its P.E.?

Ans. Suppose K.E. of the particle is equal to its P.E. at a distance y from the mean position.

$\therefore$ K.E. $= \frac{1}{2}m\omega^2(a^2 - y^2)$; P.E. $= \frac{1}{2}m\omega^2 y^2$

Now $\frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2 y^2$ or $a^2 - y^2 = y^2$ $\therefore$ $y = a/\sqrt{2}$

Q.13. The bob of a simple pendulum is a hollow sphere full of water. If a fine hole is made at the bottom of the sphere, how will time period of the pendulum be affected?

Ans. Time period of simple pendulum, $T = 2\pi\sqrt{l/g}$. Here l is the distance between point of suspension and centre of gravity (C.G) of the sphere. Initially, when the sphere is full of water, C.G is at the centre of sphere. As the water starts flowing out of sphere, the C.G of sphere will first move down and then comes up. Finally, when all water flows out, the C.G. shifts to the centre of the sphere. Due to this, length l and hence time period T of the pendulum changes. Therefore, time period T first increases, then decreases and finally becomes equal to the initial value.

Q.14. A simple pendulum has metallic bob. What will be the effect on its time period if (*i*) metallic bob is replaced by ice bob (*ii*) metallic bob pendulum is taken to moon?

Ans. Time period of simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

(*i*) $T \propto \sqrt{l}$. Here l is the distance between the point of suspension and centre of gravity (C.G.) of the ice bob. If on melting of ice, l increases, T will increase and vice-versa.

(*ii*) $T \propto \sqrt{1/g}$. At moon, g is less (1/6th that on earth) so that T increases.

Q.15. The amplitude of an oscillating simple pendulum is doubled. How will it affect (*i*) time period (*ii*) total energy (*iii*) maximum velocity?

Ans. Suppose a is the amplitude of the pendulum.

(*i*) $T = 2\pi\sqrt{l/g}$. There will be no affect on the time period because it is independent of amplitude (a) of oscillation provided it is small.

(*ii*) Total energy, $E = \frac{1}{2}m\omega^2 a^2$. If amplitude ($a$) is doubled, E will become four times.

(*iii*) Max. velocity, $V_{max} = \omega a$. If amplitude (a) is doubled, V_{max} will become 2 times.

Q.16. In S.H.M., if displacement is half of the amplitude, then which part of total energy will be the kinetic energy?

Ans. In S.H.M., K.E., $E_K = \frac{1}{2}m\omega^2(a^2 - y^2)$; Total energy, $E = \frac{1}{2}m\omega^2 a^2$

For $y = \frac{a}{2}$; $E_K = \frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right) = \frac{3}{8}m\omega^2 a^2$ $\therefore$ $\frac{E_K}{E} = \frac{3}{4}$