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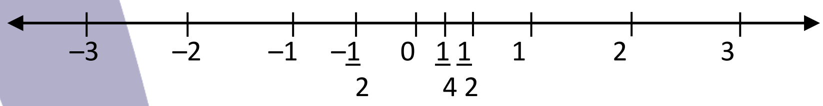
NUMBER 01 SYSTEM

IMPORTANT POINTS

1. An Important Sequence of Various Types of Numbers

Natural Numbers (N)
↓
Whole Numbers (W)
↓
Integers (Z)
↓
Rational Numbers (Q)
↓
Irrational Number
↓
Real Numbers (R)

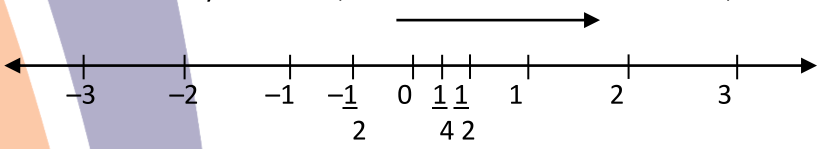
2. Number Line



3. Various Types of Numbers on the Number Line

■ (I) Search of Natural Numbers

Suppose that we start from zero and go on walking along the number line in the positive direction. Then as far as our eyes can see, we find that there are numbers, numbers and numbers.



Start collecting some of the numbers and get ready to store them. We might begin with picking up only **natural numbers like 1, 2, 3, and so on**. So, we have infinitely many natural numbers{N}.

■(II) Search of whole Numbers

Now let us turn and walk the entire way back, pick up zero and get it. We now have the collection of **whole numbers** [**W]**.

■(III) Search of Integers

Now, stretching in front of us are many, many negative integers. Let us collect all the negative integers. We now have a new collection which is the collection of all integers [**Z]**.

■(IV) Search of rational Numbers

We find that there are still numbers left of the number line. There are numbers like ½, ¾, or even $-\frac{101}{105}$.

If we collect all such numbers also, it will now be the collection of **rational numbers**. **Q.** 'Rational' comes from the word 'ratio'.

4. Facts About rational Numbers (Rationals)

■ (I) Definition of Rational Numbers

A number 'r' is called a rational number, if it can be written in the form p/q, where p and q are integers and q ≠ 0. We insist that q ≠ 0 because division by zero is not defined.

■ (II) Rational Numbers include the Natural Numbers, Whole Numbers and Integers

All the numbers can be written in the form p/q, where p and q are integers and q ≠ 0. For example,

(*i*) 10 can be written as $\frac{20}{1}$. Here p = 20 and q = 1.

(ii) 0 can be written as $\frac{0}{1}$. Here p = 0 and q = 1.

(iii) –25 can be written as $\frac{-25}{1}$. Here p = –25 and q = 1.

∴ the rational numbers also include the natural numbers, whole numbers and integers.

(III) Equivalent Rational Numbers

Rational numbers do not have a unique representation in the form p/q, where p and q are integers and q ≠ 0. For example,

$\frac{1}{2} = \frac{2}{4} = \frac{10}{20} = \frac{25}{50} = \frac{47}{94}$, and so on ; These are equivalent rational numbers (or fractions).

(IV) Standard Form of Rational Number

p/q is a rational number, or when we represent p/q on the number line, we assume that q ≠ 0 and that p and q have no common factors other than 1 (that is, p and q are co-prime). So, on the number line, among the infinitely many fractions equivalent to ½, we will choose ½ to represent all of them.

(V) Rational Numbers between any Two Given Rational Numbers

There are infinitely many rational numbers between any two given rational numbers (Say a & b).

► To find a rational number between a and b, we add a and b and divide by 2 that is $\frac{a+b}{2}$ lies between a and b.

NCERT Exercise

1. Is zero a rational number? Can you write it in the form p/q, where p and q are integers and q ≠ 0?

Sol. Yes, zero is a rational number. We can write zero in the form p/q, where p and q are integers and q ≠ 0 as follows:

$0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc., denominator q can also be taken as negative integer.

2. Find six rational numbers between 3 and 4.

Sol. $\frac{3+4}{2} = \frac{7}{2}$ ------------------- [i]

=> $\frac{3+7/2}{2} = \frac{13}{4}$ ------------------- [ii]

=> $\frac{3+13/4}{2} = \frac{25}{8}$ ------------------- [iii]

=> $\frac{3+25/8}{2} = \frac{49}{16}$ ------------------- [iv]

=> $\frac{3+49/16}{2} = \frac{97}{32}$ ------------------- [v]

=> $\frac{3+97/32}{2} = \frac{193}{64}$ ------------------- [vi]

Thus, six rational numbers between 3 and 4 are $\frac{7}{2}, \frac{13}{4}, \frac{25}{8}, \frac{49}{16}, \frac{97}{32}$ and $\frac{193}{64}$.

Alternate method: We write 3 and 4 as rational numbers with denominator 6 + 1 (=7), i.e.,

$3 = \frac{3}{1} = \frac{3\times 7}{1\times 7} = \frac{21}{7}$

and $\frac{4}{1} = \frac{4\times 7}{1\times 7} = \frac{28}{7}$

Thus, six rational numbers between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. $\frac{3}{5} = \frac{30}{50}$

$\frac{4}{5} = \frac{40}{50}$

∴ five rational between 3 and 4 are $\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$.

4. State whether the following statements are true or false? Give reasons for your answer.
(i) Every natural number is a whole number.
(ii) Every integer is a whole number.
(iii) Every rational number is a whole number.

Sol. (i) True, since the collection of whole numbers contains all natural numbers.
(ii) False, for example –2 is not a whole number.
(iii) False, for example ½ is a rational number but not a whole number.

AEP'S Questions :

1. Prove that between two distinct rational numbers a and b, there exists another rational number.

Sol. Since $a \neq b$, therefore without any loss of generality, let us suppose that $a < b$.

Now, $a < b$
=> $a + a < b + a$ [Adding a to both sides]
=> $2a < b + a$
=> $2a < a + b$
=> $a < \frac{a+b}{2}$ ... (1)

Again, $a < b$ [Adding b to both sides]
=> $a + b < b + b$
=> $a + b < 2b$
=> $\frac{a+b}{2} < b$ ... (2)

Combining (1) and (2), we get
$a < \frac{a+b}{2} < b$

∵ a, b and $2(\neq 0)$ are rational numbers.
∴ $\frac{a+b}{2}$ is also a rational number.
Thus, there exists another rational number between two distinct rational numbers a and b.

2. Give a method of finding n rational numbers between two distinct rational numbers.

Sol. For inserting n rational numbers between a and b where a < b, divide (b – a) by (n + 1). Then the required n rational numbers between a and b will be

$a + \frac{(b-a)}{(n+1)}, a + \frac{2(b-a)}{(n+1)}$

$a + \frac{3(b-a)}{(n+1)} \ldots\ldots a + \frac{n(b-a)}{(n+1)}$

3. Prove that between two distinct rational numbers, there lie an infinite number of rational numbers.

Sol. Let a and b two distinct rational numbers. We know that between two distinct rational numbers, there lies another rational number.

Let the rational number c lie between a and b such that
$a < c < b$

Now, between two rational numbers a and c, there lies a rational number d such that
$a < d < c$ ... (1)

Similarly, between two rational number c and b, there lies a rational number d such that
$c < e < b$ ... (2)

Combining (1) and (2), we get
$a < d < c < e < b$

Now, between each pair of rational numbers (a, d), (d, c), (c, e) and (e, b), there lies a rational number.
If we continue this process indefinitely, we find that there lies an infinite number of rational numbers between two distinct rational numbers.

4. Give three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

Sol. Here $a = \frac{1}{3}, b = \frac{1}{2}, n = 3$

$\therefore$ $$\frac{b-a}{n+1} = \frac{\frac{1}{2} - \frac{1}{3}}{3+1} = \frac{\frac{3-2}{6}}{4} = \frac{\frac{1}{6}}{4} = \frac{1}{24}$$

$\therefore$ Three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$ are

$$\frac{1}{3} + \frac{1}{24}, \quad \frac{1}{3} + 2\left(\frac{1}{24}\right), \quad \frac{1}{3} + 3\left(\frac{1}{24}\right)$$

i.e., $$\frac{1}{3} + \frac{1}{24}, \quad \frac{1}{3} + \frac{1}{12}, \quad \frac{1}{3} + \frac{1}{8}$$

i.e., $$\frac{3}{8}, \quad \frac{5}{12}, \quad \frac{11}{24}$$

5. Insert three rational numbers between $-\frac{2}{5}$ and $-\frac{1}{5}$.

Sol. Here $a = -\frac{2}{5}, b = -\frac{1}{5}, n = 3$

$\therefore$ $$\frac{b-a}{n+1} = \frac{-\frac{1}{5} - \left(-\frac{2}{5}\right)}{3+1} = \frac{-\frac{1}{5} + \frac{2}{5}}{4} = \frac{\frac{1}{5}}{4} = \frac{1}{20}$$

$\therefore$ There rational numbers between $-\frac{2}{5}$ and $-\frac{1}{5}$ are

$$-\frac{2}{5} + \frac{1}{20}, \quad -\frac{2}{5} + 2\left(\frac{1}{20}\right), \quad -\frac{2}{5} + 3\left(\frac{1}{20}\right)$$

$$-\frac{7}{20}, -\frac{3}{10}, -\frac{1}{4}$$

6. Find a rational number lying between $\frac{5}{6}$ and $\frac{6}{7}$.

Sol. A rational number lying between $\frac{5}{6}$ and $\frac{6}{7}$ is

$$\frac{\frac{5}{6} + \frac{6}{7}}{2} = \frac{\frac{35+36}{42}}{2} = \frac{\frac{71}{42}}{2} = \frac{71}{84}$$

7. Find four rational numbers lying between $\frac{1}{4}$ and $\frac{1}{3}$.

Sol. Here $a = \frac{1}{4}, b = \frac{1}{3}, n = 4$

$\therefore$ $$\frac{b-a}{n+1} = \frac{\frac{1}{3} - \frac{1}{4}}{4+1} = \frac{\frac{4-3}{12}}{5} = \frac{\frac{1}{12}}{5} = \frac{1}{60}$$

$\therefore$ Four rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$ are

$$\frac{1}{4} + \frac{1}{60}, \quad \frac{1}{4} + 2\left(\frac{1}{60}\right), \quad \frac{1}{4} + 3\left(\frac{1}{60}\right), \quad \frac{1}{4} + 4\left(\frac{1}{60}\right)$$

i.e., $$\frac{16}{60}, \quad \frac{17}{60}, \quad \frac{6}{20}, \quad \frac{19}{60}$$

i.e., $$\frac{4}{15}, \quad \frac{17}{60}, \quad \frac{3}{10}, \quad \frac{19}{60}$$

8. Give a method to determine any number of rational numbers between 0 and 0.1. Find three rational numbers lying between 0 and 0.1. Find twenty rational numbers lying between 0 and 0.1.

Sol. To determine any number of rational numbers between 0 and 0.1, Write 0 immediately after the decimal and any digit (s) in the next place (s).

Three rational numbers between 0 and 0.1 are 0.01, 0.02 and 0.03.

Twenty rational numbers between 0 and 0.1 are

0.01,	0.02,	0.03,	0.04,	0.05,
0.06,	0.07,	0.08,	0.09,	0.011,
0.012,	0.013,	0.014,	0.015,	0.016,
0.017,	0.018,	0.019,	0.021,	0.022

◘ *Mathematical tools:*

1.More Numbers on the Number Line: If we look at the number line again, we observe that there are infinitely many more numbers left on the number line. There are gaps in between the places of the numbers, we picked up. Moreover, these gaps are not one or two but infinitely many. Also, there are infinitely many numbers lying between any two of these gaps too. It is obvious that these numbers are not rationales. These numbers are called irrational numbers (irrationals), because they cannot be written in the form p/q, where p and q are integers and $q \neq 0$.

2. Definition of Irrational Numbers: A number 's' is called irrational, if it cannot be written in the form p/q where p and q are integers and $q \neq 0$.

We already know that there are infinitely many rationales. It turns out that there are infinitely many irrational numbers too. Some examples are:

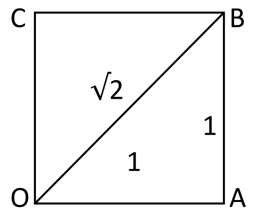
$\sqrt{2}, \sqrt{3}, \sqrt{15}, \frac{1}{1+\sqrt{2}}, \pi$, 0.10110111011110........

➤: When we use the symbol $\sqrt{\ }$, we assume that it is the positive square root of a number. So, $\sqrt{4} = 2$, though both 2 and -2 are square roots of 4.

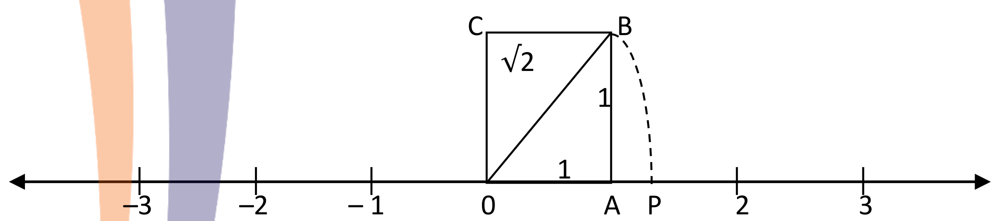
3.Real Numbers: If we put all irrational numbers into the bag of rational numbers, then no number will be left on the number lines. It turns out that the collection of all rational numbers and irrational numbers together make up what we call the collection of real numbers which is denoted by R. therefore, a real number is either rational or irrational. So, we can say that every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number; This is why we called the number line, the real number line.

4.Location of √2 on the Number Line: Consider a unit square (a square with each side 1 unit in length) OABC. Then by the Pythagoras theorem, we see that

$$OB = \sqrt{1^2 + 1^2} = \sqrt{2}$$

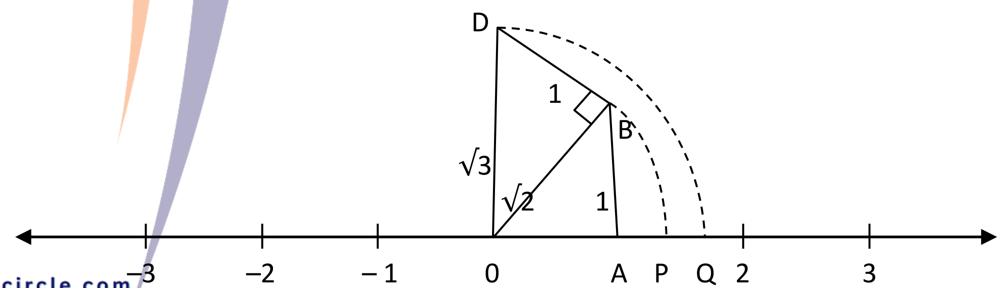


To represent √2 on the number line, we transfer the above figure onto the number line making sure that the vertex O coincides with zero.



Using a compass with centre O and radius OB, draw an arc which intersects the number line in the point P. Then P corresponds to √2 on the number line.

7. Location of √3 on the Number Line:



In the proceeding figure, construct BD of unit length perpendicular to OB. Then using the Pythagoras theorem, we see that OD = $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$. Using a compass, with centre O and radius OD, draw an arc which intersects the number line in the point Q. Then Q corresponds to $\sqrt{3}$.

Note: In the same way, we can locate $\sqrt{n}$ for any positive integer n, after $\sqrt{n-1}$ has been located.

NCERT Exercise

1. State whether following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.
(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
(iii) Every real number is an irrational number.

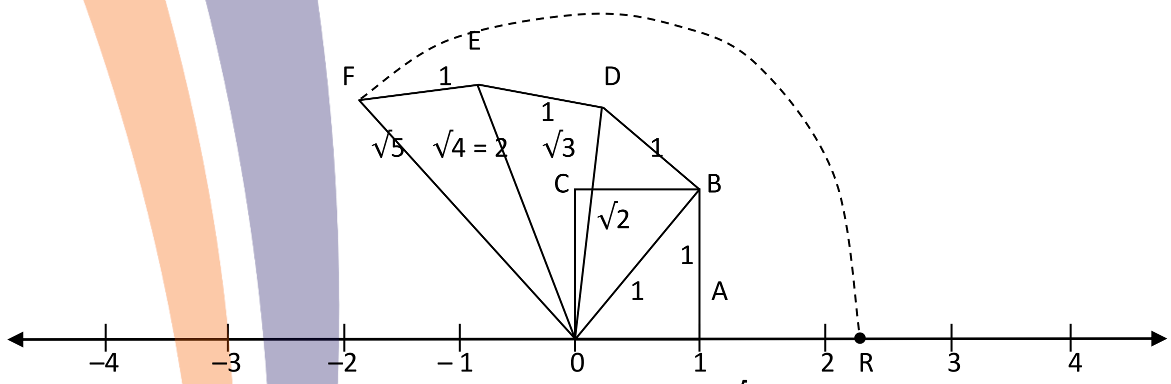
Sol. (i) True, since collection of real numbers is made up of rational and irrational numbers.
(ii) False, because no negative number can be the square root of any natural number.
(iii) False, for example 2 is real but not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol. No, For example, $\sqrt{4} = 2$ is a rational number

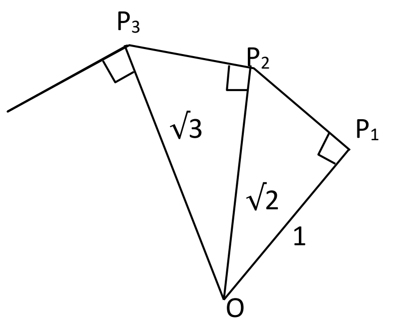
3. Show how $\sqrt{5}$ can be represented on the number line.

Sol. (i) Representation of $\sqrt{5}$ on the number line
Consider a unit square OABC and transfer it onto the number line making sure that the vertex O coincides with zero.
Then OB = $\sqrt{1^2 + 1^2} = \sqrt{2}$; Construct BD of unit length perpendicular to OB.
Then OD = $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$
Construct DE of unit length perpendicular OD.
Then OE = $\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$
Construct EF of unit length perpendicular to OE.
Then OF = $\sqrt{2^2 + 1^2} = \sqrt{5}$
Using a compass, with centre O and radius OF, draw an arc which intersects the number line in the point R. Then R corresponds to $\sqrt{5}$.



(i) Representation of $\sqrt{5}$

4. Class room activity (Constructing the 'square root spiral'): Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length [see figure]. Now draw a line segment P_2P_3 perpendicular to OP_3. Continuing in this manner, we can get the line segment $P_{n-1} P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, we will have created the points: P1, P2, P3, ..., P_n, ..., and joined them to create a beautiful spiral depicting $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, ...



Constructing square root spiral

Mathematical Tools :

1. **Decimal Expansions of Real Numbers**: The decimal expansions of real numbers can be used to distinguish between rationales and irrationals. Let us consider the decimal expansions of $\frac{10}{3}$, $\frac{7}{8}$ and $\frac{1}{7}$.

(i) Decimal expansion of $\frac{10}{3}$

```
      | 3.33333.......
 -----+--------------
   3  | 10
      |  9
      |------
      |  10
      |   9
      |------
      |   10
      |    9
      |------
      |    10
      |     9
      |------
      |     1
```

Remainders: 1, 1, 1, 1, 1...
Divisor: 3

(ii) Decimal expansion of $\frac{7}{8}$

```
   | 0.875
   |------
 8 | 7.0
   | 64
   |------
   |  60
   |  56
   |------
   |   40
   |   40
   |------
   |    0
```

Remainders: 6, 4, 0 ; Divisor: 8
(iii) Decimal Expansion of $\frac{1}{7}$

```
   | 0.142857...
   |------------
 7 | 1.0
   |  7
   |------------
   |   30
   |   28
   |------------
   |    20
   |    14
   |------------
   |     60
   |     56
   |------------
   |      40
   |      35
   |------------
   |       50
           49
```

Remainders: 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, ...
Divisor: 7

Conclusion:

- (*i*) In all these cases, the remainder is smaller than the divisor, which is true for any divisor.
- (*ii*) The remainders either become 0 after a certain stage, or start repeating themselves.
- (*iii*) The number of entries in the repeating string of remainders is less than the divisor (in 1/3 one number repeats itself and the divisor is 3, in 1/7 there is six entries 326451 in the repeating string of remainders and 7 is the divisor).

(*iv*) If the remainders repeat, then we get a repeating block of digits in the quotients (for 1/3, 3 repeats in the quotients and for 1/7, we get the repeating block 142857 in the quotient).

►► On division of p by q, ***two main things happen***-either the remainder becomes zero or never becomes zero and we get a repeating string of remainders.

2. Terminating Decimal Expansions: Decimal expansion terminals or ends after a finite number of steps. We call such a decimal expansion as terminating. For example:

$$\frac{7}{8} = 0.875$$

$$\frac{1}{2} = 0.5$$

$$\frac{639}{250} = 2.556$$

3. Non-terminating Recurring Expansions: The remainder repeat after a certain stage forcing the decimal expansion to go on for ever. In other words, we have a repeating block of digits in the quotient. We say that this expansion is non-terminating recurring. For example:

$$\frac{1}{3} = 0.3333........$$

$$\frac{1}{7} = 0.1428571428571428757........$$

►**4. How to Write Non-terminating Recurring Expansions in Short**: The usual way of showing that 3 repeats in the quotient of 1/3 is to write it as $0.\overline{3}$. Similarly, since the block of digits 142857 repeats in the quotient of 1/7, we write 1/7 as $0.\overline{142857}$, where the bar above the digits indicates the block of digits that repeats. Also 3.57272...... can be written as $3.5\overline{72}$. So, all these examples give us non-terminating recurring (repeating) decimal expansions.

5. Decimal Expansions of Rational Numbers: In the decimal expansion of rational numbers have only two choices: either they are terminating or non-terminating recurring. Thus, a number like 3. 142678 whose decimal expansion is terminating or a number like 1.272727.........., that is $1.\overline{27}$, whose decimal expansion is non-terminating recurring are both rational numbers.

➢ Identification of the nature of real numbers from their decimal expansions:

Case I: When the decimal expansion is terminating.

Consider the real number 3.142678, whose decimal expansion is terminating.

We have $3.142678 = \frac{3142678}{1000000}$

Here, $p = 3142678$

$q = 1000000 \ (\neq 0)$

Hence 3.142678 is a rational number.

So, every number with a terminating decimal expansion can be expressed in the form p/q (q ≠ 0), where p and q are integer and hence such a number is a rational number.

Case II. When the decimal expansion is non-terminating.

Example 1: Consider the real number 0.3333....... (Or $0.\overline{3}$).

Let $x = 0.3333......\ (-0.\overline{3})$

Since one-digit repeats, we multiply x by 10 to get

$10x = 10 \times (0.3333......) = 3.333......$

=> $10x = 3 + 0.333......$ => $10x = 3 + x$

=> $9x = 3$ => $x = 3/9$ [Solving for x]

=> $x = \frac{1}{3}$

Hence 0.3333........ (or $0.\overline{3}$) is a rational number.

[We may check the reverse that = $0.\overline{3}$].

Example 2: Consider the number 1.272727......... (or $1.\overline{27}$).

Let $x = 1.272727.........\ (= 1.\overline{27})$

Since two digits are repeating, we multiply x by 100 to get

$100x = 127.2727.........$

=> $100x = 126 + 1.272727.......$ => $100x = 126 + x$

=> $99x = 126$ => $x = \frac{126}{99} = \frac{14}{11}$ [Solving for x]

Here p = 14

Q = 11 ($\neq 0$)

Hence 1.272727.......... (or $1.\overline{27}$) is a rational number.

[We may check the reverse that $\frac{14}{11} = 1.\overline{27}$]

Example 3: Consider the real number 0.2353535......... (or $0.2\overline{35}$).

Let x = 0.2353535.... (= $0.2\overline{35}$)

[Here 2 does not repeat, but the block 35 repeats]

Since two digits are repeating, we multiply x by 100 to get

$100x = 23.53535.......$

=> $100x = 23.3 + 0.23535........$ => $100x = 23.3 + x$

=> $99x = 23.3$ => $9x = \frac{233}{10}$

=> $x = \frac{233}{990}$ [Solving for x]

Here p = 233

q = 990 ($\neq 0$)

Hence 0.2353535........ (or $0.2\overline{35}$) is a rational number.

[We may check the reverse that $\frac{233}{990} = 0.2\overline{35}$]

So, every number with a non-terminating recurring decimal expansion can be expressed in the form p/q ($q \neq 0$), where p and q are integers and hence such a number is a rational number.

6. **An Important Conclusion**: The decimal expansion of a rational number is either terminating or non-terminating recurring. Moreover, a number whose decimal expansion is terminating or non-terminating recurring is rational.

7. **Decimal Expansions of Irrational Numbers**: Since the decimal expansion of a rational number is either terminating or non-terminating recurring. Therefore, we conclude that the decimal expansions of irrational numbers are non-terminating non-recurring.

8. **An Important Conclusion**: The decimal expansion of an irrational number is non-terminating non-recurring. Moreover, a number whose decimal expansion is non-terminating non-recurring is irrational.

For example: the numbers

0.10110111011110........, $\sqrt{2}$ = 1.4142125623730950 488016887242096.......

And π = 3.141592653 58979323846264338327950........

are irrational numbers as each of these has a non-terminating and non-recurring decimal expansion.

$\left(\text{Note: We often take } \frac{22}{7} \text{ as an approximate value for } \pi, \text{ but } \pi \neq \frac{22}{7}\right)$

NCERT Exercise

1. **Write the following in decimal form and say what kind of decimal expansion each has:**

(i) $\frac{36}{100}$ **(ii) $\frac{1}{11}$** **(iii) $4\frac{1}{8}$**

(iv) $\frac{3}{13}$ **(v) $\frac{2}{11}$** **(vi) $\frac{329}{400}$**

Sol. **(i)** $\frac{36}{100} = 0.36$

The decimal expansion is terminating.

(ii) $\frac{1}{11}$

11) 1. 000000 (0.090909.....
99
100
99
100
99
1

∴ $\frac{1}{11} = 0.090909........ = 0.\overline{09}$

The decimal expansion is non-terminating repeating.

(iii) $4\frac{1}{8}$

$4\frac{1}{8} = \frac{4 \times 8 + 1}{8} = \frac{32 + 1}{8} = \frac{33}{8}$

8) 33.000 (4.125
32
10
8
20
16
40
40
×

∴ $4\frac{1}{8} = 4.125$

The decimal representation is terminating.

(iv) $\frac{3}{13}$

13) 3.0000000000 (0.230769230769........
26
40
39
100
91
90
78
120
119
100
91
90
78
120
117
30
26
40
39
100
91
90
78
120
117
3

∴ $\frac{3}{13} = 0.230769230769........ = 0.\overline{230769}$.

The decimal expansion is non-terminating repeating.

(v) $\frac{2}{11}$

11) 2.0000 (0.1818.........
11
90
88
20
11
90
88
2

$\therefore$ $\frac{2}{11} = 0.1818........ = 0.\overline{18}$.

The decimal expansion is non-terminating repeating.

(vi) $\frac{329}{400}$

400) 329.0000 (0.8225
3200
900
800
1000
800
2000
2000
×

$\therefore$ $\frac{329}{400} = 0.8225$

The decimal expansion is terminating.

2.You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how? [◘Study the remainders while finding the value 1/7 carefully.]

Sol. Yes! We can predict the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$, without actually doing the long divisions as follows:

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ **(ii) $0.4\overline{7}$** **(iii) $0.\overline{001}$**

Sol. (i) $0.\overline{6}$

Let $x = 0.\overline{6} = 0.666...........$

=> $10x = 6 + 0.6666......$ => $10x = 6 + x$

=> $100x - x = 6$ => $9x = 6$

=> $x = \frac{6}{9} = \frac{2}{3}$

Thus, $0.6 = \frac{2}{3}$

Here $p = 2$ $q = 3\ (\neq 0)$

(ii) $0.4\overline{7}$

Let $x = 0.4\overline{7} = 0.47777.........$

Multiplying both sides by 10 (since one digit is repeating), we get

$10x = 0.47777.........$

=> $10x = 4.3 + 0.47777.........$ => $10x = 4.3 + x$

=> $10x - x = 4.3$ => $9x = 4.7$

=> $x = \frac{4.3}{9} = \frac{43}{90}$

Thus, $4.\overline{7} = \frac{43}{90}$

Here, $p = 43$

$q = 90 \ (\neq 0)$.

(iii) $0.\overline{001}$

Let $x = 0.\overline{001} = 0.001001001..........$

Multiplying both sides by 1000 (since three digits are repeating), we get

$1000x = 1.001001..........$

=> $1000x = 1 + 0.001001001...........$ => $1000x = 1 + x$

=> $1000x - x = 1$ => $999x = 1$

=> $x = \frac{1}{999}$

Thus, $0.001 = \frac{1}{999}$

Here $p = 1$

$q = 999 \ (\neq 0)$.

4. Express 0.99999.......... in the form p/q. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Sol. Let $x = 0.99999........$

Multiplying both sides by 10 (since one digit is repeating), we get

$10x = 9.9999.....$

=> $10x = 9 + 0.99999.......$ => $10x = 9 + x$

=> $10x - x = 9$ => $9x = 9$

=> $x = \frac{9}{9} = 1$

Thus, $0.99999...... = 1 = \frac{1}{1}$

Here $p = 1$

$q = 1$

Since 0.99999....... goes on for ever, so there is no gap between 1 and 0.99999....... end hence they are equal.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol. The maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ can be 16.

17) 1.0000000000000000000000000 (0.0588235294117647058829647........
85
150
136
140
136
40
34
60
51
90
85

50
34
160
153
70
68
20
17
30

17
130
119
110
102
80
68
120
119
100
85
150
136
140
136
40
34
160
153
110
102
80
68
120
119
1

Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$

By Long Division, the number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ = 16.

∴ The answer is verified.

6. Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1, have terminating decimal representation (expansions). Can you guess what property q must satisfy?

Sol. the property that q must satisfy in order that the rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1, have terminating decimal representation (expansions) is that the prime factorisation of q has only powers of 2 or powers of 5 or both, i.e., q must be of the form 2m × 5n; m = 0, 1, 2, 3, ..., n = 0, 1, 2, 3, ...

7. Write three numbers whose decimal expansions and non-terminating non-recurring.

Sol. 0.01001 0001 00001........,
0.20 2002 20003 200002.........,
0.003000300003..........,

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. 7) 5.000000 (0.714285..........
49
10
7
30
28
20
14
60
56
40
35
5

Thus, $\frac{5}{7}$ = 0.714285...... = $0.\overline{714285}$

11) 9.0000 (0.8181........
88
20
11
90
88
20
11
9

Thus, $\frac{9}{11}$ = 0.8181........ = $0.\overline{81}$

Three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ can be taken as

0.75 075007000750000 75......, 0.767076000767..........., 0.808008000800008.........

9. Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478..... (v) 1.101001000100001.........

Sol. (i) $\sqrt{23}$

	4.795831523
4	23.00 00 00 00 00 00 00 00 00
	16
87	700
	609
949	9100
	8541
9585	55900
	47925
95908	797500
	767264
959163	3023600
	2877489
9591661	14611100
	9591661
95916625	501943900
	479583125
959166302	2236077500
	1918332604
9591663043	31774489600
	28774989129
	2999500471

Thus, $\sqrt{23} = 4.795831523.........$

$\because$ The decimal expansion is non-terminating non-recurring.

$\therefore$ $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

```
        15
   1 | 225
     | 1
  25 | 125
     | 125
     |   ×
```

$\because$ $\sqrt{225} = 15 = \frac{15}{1}$ $\therefore$ $\sqrt{225}$ is a rational number.

Here $p = 15$

$q = 1\ (\neq 0)$.

(iii) 0.3796

$\therefore$ The decimal expansion is terminating.

$\therefore$ 0.3796 is a rational number.

(iv) 7.478478........

$7.478478......... = 7.\overline{478}$

$\because$ The decimal expansion is non-terminating recurring.

$\therefore$ 7.478478........ is a rational number.

(v) 1.101001000100001..........

$\because$ The decimal expansion is non-terminating non-recurring.

$\therefore$ 1.101001000100001..... is an irrational number.

Additional Examples

5. Express each of the following decimals in the form p/q where p and q are integers with no common factors other than 1. (i) 2.15 (ii) 9.90 (iii) 0.750

Sol. (i) 2.15

$2.15 = \frac{215}{100} = \frac{215 \div 5}{100 \div 5}$ [$\because$ H.C.F. (215, 100) = 5]

$= \frac{43}{20}$ Here $p = 43$ $q = 20$

(ii) 9.90

$9.90 = \frac{990}{100} = \frac{990 \div 10}{100 \div 10}$ [$\because$ H.C.F. (990, 100) = 10]

$= \frac{99}{10}$ Here $p = 99$

$q = 10$

(iii) 0.750

$0.750 = \frac{750}{1000} = \frac{750 \div 250}{1000 \div 250}$ [$\because$ H.C.F. (750, 1000) = 250]

$= \frac{3}{4}$ Here $p = 3$ $q = 4$

6. Find two irrational numbers between 0.12 and 0.13.

Sol. The two irrational numbers between 0.12 and 0.13 can be taken as

0.1210100100010001.........

And 0.1220100100010001.......

7. Find two irrational numbers between 0.1 and 0.12.

Sol. The two irrational numbers between 0.1 and 0.12 can be taken as

0.1010010001......... And 0.11010010001..........

8. Find two irrational numbers between 2 and 2.5.

Sol. First Method

The two irrational numbers between 2 and 2.5 can be taken as

2.101001000100001………

And 2.201001000100001………

Second Method

☞**:** If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is a irrational number lying between a and b.

∴ An irrational number between 2 and 2.5 = $\sqrt{2 \times (2.5)} = \sqrt{5}$

Similarly, an irrational number between 2 and $\sqrt{5}$

$= \sqrt{2\sqrt{5}}$

∴ Two irrational numbers between 2 and 2.5 are $\sqrt{5}$ and $\sqrt{2\sqrt{5}}$

9. Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol.☞: If a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.

∴ An irrational number lying between $\sqrt{2}$ and $\sqrt{3} = \sqrt{\sqrt{2}\,\sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$.

Similarly, an irrational number lying between $\sqrt{2}$ and $6^{1/4}$

$= \sqrt{\sqrt{2}.6^{1/4}} = 2^{1/4}\,6^{1/8}$

∴ Two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$ are $6^{1/4}$ and $2^{1/4}\,6^{1/8}$.

11. Find one irrational number between the numbers a and b given below:

a = 0.1111 …… = $0.\overline{1}$

b = 0.1101.

Sol. An irrational number between 0.1111…….. = $0.\overline{1}$ and 0.1101 can be taken as 0.111010010001.

12. Express the following in the form $\frac{p}{q}$.

(i) 0.6666…….. **(ii) 0.272727………..**

(iii) 3.7777……… **(iv) 18.484848…….**

Sol. (i) Let x = 0.6666………

Multiplying both sides by 10 (since one digit is repeating), we get

10x = 6.666……

=> 10x = 6 + 0.6666

=> 10x = 6 + 0.6666 => 10x = 6 + x

=> 10x – x = 6 => 9x = 6

=> 10x – x = 6 => 9x = 6

=> $x = \frac{6}{9}$ => $x = \frac{2}{3}$

∴ 0.6666…….. $= \frac{2}{3}$

(ii) 0.272727……..

Let x = 0.272727………..

Multiplying both sides by 100 (since two digits are repeating), we get

100x = 27.2727………

=> 100x = 27 + 0.272727………=> 100x = 27 + x

=> 100x – x = 27 => 99x = 27

=> $x = \frac{27}{99}$ => $x = \frac{3}{11}$

∴ 0.272727…….. $= \frac{3}{11}$

(iii) 3.7777………

Let x = 3.7777………..

Multiplying both sides by 10 (since one digit is repeating), we get

10x = 37.777………

=> 10x = 34 + 3.7777………. => 10x = 34 + x

=> 10x – x = 34 => 9x = 34

=> $x = \frac{34}{9}$

∴ 3.7777………. $= \frac{34}{9}$.

(iv) 18.484848………

Let x = 118.484848…….

Multiplying both sides by 100 (since two digits are repeating), we get

$100x = 18.4848........$

=> $100x = 1830 + 18.484848............$ => $100x = 1830 + x$

=> $100x - x = 1830$ => $99x = 1830$

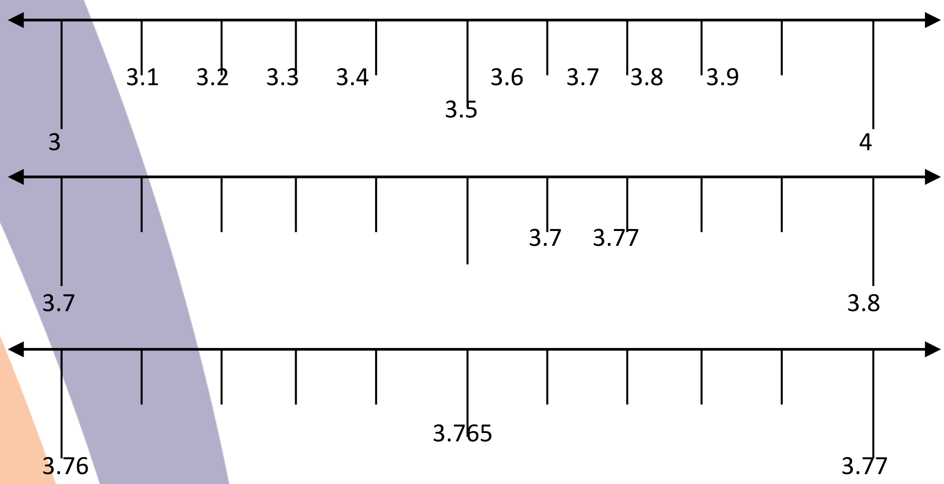
=> $x = \frac{1830}{99}$ => $x = \frac{610}{33}$

∴ $18.484848........ = \frac{610}{33}$

NCERT Exercise

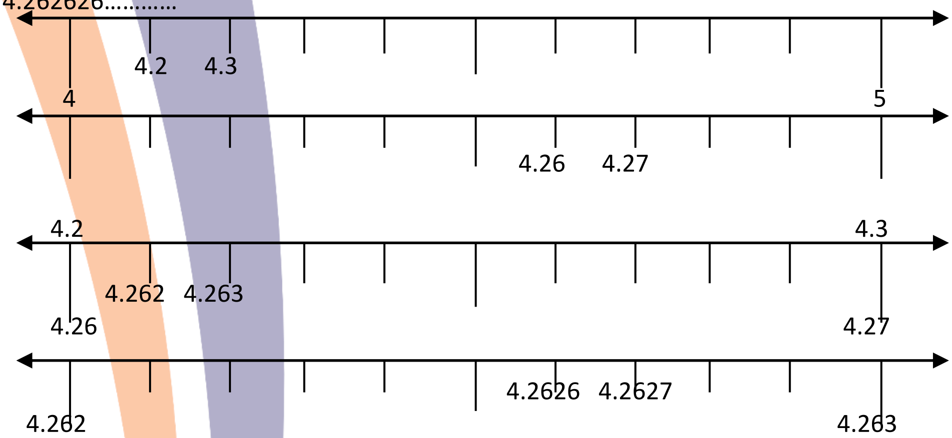
1. Visualise 3.765 on the number line, using successive magnification.

Sol.



2. Visualize $4.2\overline{6}$ on the number line, up to 4 decimal places.

Sol. $4.2\overline{6} = 4.262626...........$



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IMPORTANT POINTS

1. **Operations on Rational Numbers**: We know that rational number satisfy the commutative, associative and distributive laws for addition and multiplication. Moreover, if we add, subtract, multiply or divide (except by zero) two rational numbers, we still get a rational number (that is, rational numbers are 'closed' with respect to addition, subtraction, multiplication and division).

2. **Operation on Irrational Numbers**: Irrational numbers also satisfy the commutative, associative and distributive laws for addition and multiplication. However, the sum, difference, quotients and products of irrational numbers are not always irrational. For example, $\sqrt{3} + (-\sqrt{3})$, $(\sqrt{2}) - (\sqrt{2})$, $(\sqrt{3}).(\sqrt{3})$ and $\frac{\sqrt{17}}{\sqrt{17}}$ are rationales.

3. **Operations on Real Numbers**:

(i) **The sum or difference of a rational number and an irrational number is irrational**.

Example: If we add the rational number 2 and the irrational number $\sqrt{3}$, we get $2 + \sqrt{3}$. Since $\sqrt{3}$ has a non-terminating non-recurring decimal expansion, the same is true for $2 + \sqrt{3}$. Therefore, $2 + \sqrt{3}$ is an irrational number. Similarly, $2 - \sqrt{3}$ is an irrational number.

(ii) ***The product or quotient of a non-zero rational number with a irrational number is irrational.***

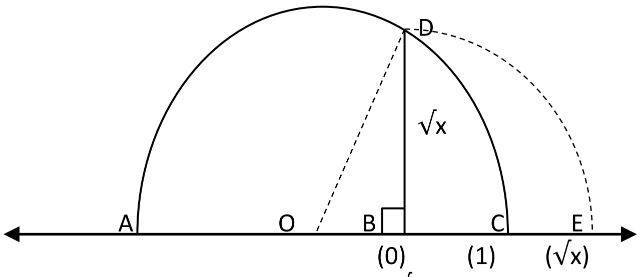
Example: If we multiply the rational number 2 and the irrational number $\sqrt{3}$, we get $2\sqrt{3}$. Since $\sqrt{3}$ has a non-terminating non-recurring decimal expansion, the same is true for $2\sqrt{3}$. Therefore, $2\sqrt{3}$ is an irrational number. Similarly, $\underline{2}$ is an irrational number.

[$\frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$. See that $\frac{2}{3}$ is rational and $\frac{\sqrt{3}}{3}$ is irrational]

4. **Operation of Taking Square Root of Real Numbers**: We know that, if a is a natural number, then $\sqrt{a} = b$ means $b^2 = a$ and $b \geq 0$. The same definition can be extended for positive real numbers.

Let $a > 0$ be a real number. Then $\sqrt{a} = b$ means $b^2 = a$ and $b \geq 0$.

5. **Representation of $\sqrt{x}$ for any Given Positive Real Number x Geometrically Method**: Let A be a fixed point on a given line. Let x be any positive real number. Mark a point B on the given line such that AB = x. Again mark a new point C on the given line such that BC = 1. Find the midpoint of AC and mark that point as O. Draw a semi-circle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Then BD = $\sqrt{x}$.



Proof: We shall use Pythagoras theorem to prove that BD = $\sqrt{x}$.

Join OD.

∵ BD ⊥ AC

∴ ΔOBD is a right angled triangle.

Also, radius of the circle = OC = OA = OD = ½ AC [∵ O is the mid-point of AC]

= ½ (AB + BC) = ½ (x + 1) units

∴ $OB = AB - OA = x - \frac{1}{2}(x + 1) = \frac{x-1}{2}$

In right angled triangle OBD.

$BD^2 = OD^2 - OB^2$ [By Pythagoras Theorem]

$= \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$

$= \frac{(x^2 + 2x + 1) - (x^2 + 2x + 1)}{4} = \frac{4x}{4} = x$

=> BD = $\sqrt{x}$

This construction gives us a visual and geometric way of showing that $\sqrt{x}$ exists for all real numbers x > 0. If we want to know the position of $\sqrt{x}$ on the number line, then let us treat the line BC as the number line, with B as zero, C as I, and so on. Draw an arc with centre B and radius BD, which intersect the number line in E. Then E represents $\sqrt{x}$.

6. **n^{th} Root of a Real Number**: Let > 0 be a real number and n be a positive integer. Then $\sqrt[n]{a} = b$, if $b^n = a$ 25 and b > 0.

For example: $\sqrt[3]{8} = 2$ as $2^3 = 8$

$5\sqrt{243} = 3$ as $3^5 = 243$

7. **Some Identities Related to Square Roots**: Let a and b be positive real numbers. Then

(i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ (iv) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - bP$

(v) $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

(vi) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

8. **Rationalisation**: When the denominator of an expression contains a term with a square root, the procedure of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator. For example:

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\frac{1}{\sqrt{5}+1} = \frac{1}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{\sqrt{5}-1}{5-1} = \frac{\sqrt{5}-1}{4}$

$\frac{1}{8+3\sqrt{5}} = \frac{1}{8+3\sqrt{5}} \times \frac{8-3\sqrt{5}}{8-3\sqrt{5}} = \frac{8-3\sqrt{5}}{64-45} = \frac{8-3\sqrt{5}}{19}$

☞: Rationalisation helps in locating the number on the number line. For example: when we rationalise the denominator of $\frac{1}{\sqrt{2}}$, we get $\frac{\sqrt{2}}{2}$. Obviously it is now easy to locate $\frac{1}{\sqrt{2}}$ on the number line. It is half way between 0 and $\sqrt{2}$.

NCERT Exercise

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$ (ii) $(3 + \sqrt{23}) - \sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Sol. (i) $2 - \sqrt{5}$

$\because$ 2 is a rational number and $\sqrt{5}$ is an irrational number.

$\therefore$ $2 - \sqrt{5}$ is an irrational number.[$\because$ The difference of a rational number and an irrational number is irrational.]

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23} = 3$ This is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

$\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ This is rational number.

(iv) $\frac{1}{\sqrt{2}}$

$\because$ 1 ($\neq$ 0) is a rational number and $\sqrt{2}$ ($\neq$ 0) is an irrational number.

$\therefore$ $\frac{1}{\sqrt{2}}$ is an irrational number. [$\because$ The quotient of a non-zero rational number with an irraitional number is irrational number is irrational.]

(v) 2π

$\because$ 2 is a rational number and π is an irrational number.

$\therefore$ 2π is an irrational number. [$\because$ The product of a non-zero rational number with an irrational number is irrational.]

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$ (ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

(iii) $(\sqrt{5} + \sqrt{2})^2$ (iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Sol. (i) $(3 + \sqrt{3})(2 + \sqrt{2})$

$(3 + \sqrt{3})(2 + \sqrt{2}) = 3(2 + \sqrt{2})\sqrt{3}(2 + \sqrt{2})$ [Left Distributive law of multiplication over addition]

$= (3)(2) + 3\sqrt{2} + (\sqrt{3})(2) + (\sqrt{3})(\sqrt{2})$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{(3)(2)}$ [$\because \sqrt{a}\sqrt{b} = \sqrt{ab}$]

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

$(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6$

(iii) $(\sqrt{5} + \sqrt{2})^2$

$(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2 = 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

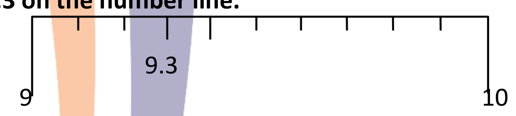
$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = c/d$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

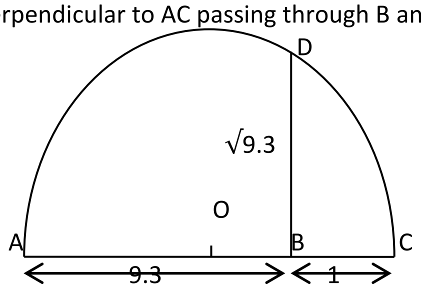
Sol. Actually $\frac{c}{d}$ = 22 which is an approximate value of π.

4. Represent $\sqrt{9.3}$ on the number line.

Sol.



Mark the distance 9.3 from a fixed-point A on a given line to obtain a point B such that AB = 9.3 units. From B mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semi-circle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Then BD = $\sqrt{9.3}$.



5. **Rationalise the denominators of the following:**

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Sol. (i) $\frac{1}{\sqrt{7}}$

$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ [Multiplying and dividing by √7]

$= \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ [Multiplying and dividing by √7 + √6]

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}+\sqrt{6}}$ [Multiplying and dividing by √5 – √2]

$= \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2}$

$\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$ [Multiplying and dividing by √7 + 2]

$= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$

Additional Examples

1. **Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers.**

(i) $\sqrt{4}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$ (v) $-\sqrt{0.64}$ (vi) $\sqrt{100}$

Sol. (i) $\sqrt{4}$

$\sqrt{4} = 2$ which is a rational number.

Its decimal representation is 2.0

(ii) $3\sqrt{18}$

$3\sqrt{18} = 3\sqrt{3 \times 3 \times 2} = 3 \times 3\sqrt{2} = 9\sqrt{2}$

∵ 9 (≠ 0) is a rational number and √2 is an irrational number.

∴ 9 √2 is an irrational number ∵ The product of a non-zero rational number with an irrational number is irrational.]

∴ 3√18 is an irrational number.

(iii) $\sqrt{1.44}$

$\sqrt{1.44} = \sqrt{1.2 \times 1.2} = 1.2 = \frac{12}{10} = \frac{6}{5}$ This is a rational number.

Here p = 6

q = 5 (≠ 0)

Its decimal representation is 1.2.

```
5) 6.0 ( 1.2
   5
   --
   10
   10
   --
    ×
```

(iv) $\sqrt{\frac{9}{27}}$

$\sqrt{\frac{9}{27}} = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

∵ 1 (≠ 0) is a rational number and √3 is an irrational number.

∴ 1/√3 is an irrational number. [∵ The quotient of a non-zero rational number with an irrational number is irrational.]

(v) $-\sqrt{0.64}$

$-\sqrt{0.64} = -\sqrt{0.8 \times 0.8}$

$= -0.8 = -\frac{8}{10} = -\frac{4}{5}$

This is a rational number.

Here $p = -4$, $q = 5\ (\neq 0)$

Its decimal representation is -0.8.

5) 4.0 (0.8
4.0
×

(vi) $\sqrt{100}$ $\sqrt{100} = \sqrt{10 \times 10} = 10$ This is a rational number. Its decimal representation is 10.0.

2. In the following equations, find which variables x, y, z etc. represent rational numbers and which represent irrational numbers:

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = .04$ (iv) $u^2 = \frac{17}{4}$

(v) $v^2 = 3$ (vi) $w^3 = 27$ (vii) $t^2 = 0.4$

Sol. (i) $x^2 = 5$

$x^2 = 5$ => $x = \sqrt{5}$ This is an irrational number.

(ii) $y^2 = 9$

$y^2 = 9$ => $y = \sqrt{9} = \sqrt{3 \times 3} = 3$ This is a rational number.

(iii) $z^2 = .04$

$z^2 = .04$ => $z = \sqrt{0.4} = \sqrt{0.2 \times 0.2} = 0.2 = \frac{2}{10} = \frac{1}{5}$ This is an rational number.

Here $p = 1$
$q = 5\ (\neq 0)$

(iv) $u^2 = \frac{17}{4}$

=> $u = \sqrt{\frac{17}{4}}$ => $u = \frac{\sqrt{17}}{\sqrt{4}}$

=> $u = \frac{\sqrt{17}}{2}$ => $u = \frac{1}{2} \times \sqrt{17}$

∵ $\frac{1}{2}\ (\neq 0)$ is a rational number and $\sqrt{17}$ is an irrational number.

∴ $\frac{1}{2} \times \sqrt{17}$ is an irrational number. [∵ The quotient of a non-zero rational number with an irrational number is irrational.]

∴ u is an irrational number.

(v) $v^2 = 3$

$v^2 = 3$ => $v = \sqrt{3}$ This is an irrational number.

(vi) $w^3 = 27$

$w^3 = 27$ => $w = (27)^{1/3}$

=> $w = (3 \times 3 \times 3)^{1/3}$ => $w = 3$ This is a rational number.

(vii) $t^2 = 0.4$

$t^2 = 0.4$

=> $t = \sqrt{0.4} = 0.63245553203$..........

This is an irrational number since the decimal representation is non-terminating non-recurring.

	0.6324555 3203........
6	0.4000000000000000000000
	36
123	400
	369
1262	3100
	2524
12244	57600
	50576
126485	702400
	632425
1264905	6997500
	6324525
12649105	67297500
	63245525
126491103	405197500

	379473309
1264911062	2572419100
	2529822124
126491106403	425969760000
	379473319209
	46496440791

3. Prove that $\sqrt{3} + 2$ is irrational.

Sol. ∵ $\sqrt{3}$ is irrational and 2 is rational.
∴ $\sqrt{3} + 2$ is irrational. [∵ The sum of a rational number and an irrational number is irrational.]

4. Prove that $\sqrt{5} - 2$ is irrational.

Sol. ∵ $\sqrt{5}$ is a irrational and 2 is rational.
∴ $\sqrt{5} - 2$ is irrational. [∵ The difference of a rational number and an irrational number is irrational.]

5. Examine whether the following numbers are rational or irrational.
(i) $(\sqrt{2} + 2)^2$ (ii) $(\sqrt{2} + \sqrt{3})^2$ (iii) $(2 - \sqrt{2})(2 + \sqrt{2})$ (iv) $6/3\sqrt{2}$

Sol. (i) $(\sqrt{2} + 2)^2$
$$(\sqrt{2} + 2)^2 = (\sqrt{2})^2 + 2(\sqrt{2})(2) + (2)^2$$
$$= 2 + 4\sqrt{2} + 4 = 6 + 4\sqrt{2}$$
∵ 4 ($\neq 0$) is rational number and $\sqrt{2}$ is irrational.
∴ $4\sqrt{2}$ is irrational. [∵ The product of a non-zero rational number with an irrational number is irrational.]
Again 6 is rational.
∴ $6 + 4\sqrt{2}$ is irrational. [∵ The sum of a rational number and an irrational number is irrational.]
∴ $(\sqrt{2} + 2)^2$ is irrational.
(ii) $(\sqrt{2} + \sqrt{3})^2$
$$(\sqrt{2} + \sqrt{3})^2 = (\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + \sqrt{3}^2$$
$$= 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$$
∵ 2 ($\neq 0$) is rational and $\sqrt{6}$ is irrational.
∴ $2\sqrt{6}$ is irrational. [∵ The product of a non-zero rational number with an irrational number is irrational.]
Again 5 is rational.
∴ $5 + 5\sqrt{6}$ is irrational. [∵ The sum of a rational number and an irrational number is irrational.]
∴ $(\sqrt{2} + \sqrt{3})^2$ is irrational.
(iii) $(2 - \sqrt{2})(2 + \sqrt{2})$
$$(2 - \sqrt{2})(2 + \sqrt{2}) = (2)^2 - (\sqrt{2})^2$$
$$= 4 - 2 = 2.$$ This is rational.

(iv) $6/3\sqrt{2}$
$$6/3\sqrt{2} = \sqrt{2}.$$ This is an irrational number.

6. Give an example to show that the product of a rational number and an irrational number may be a rational number.

Sol. Rational number '0' multiplied by any irrational number 'a' gives a rational number '0'.

7. Give an example each of two irrational numbers, whose
(i) Difference is a rational number. (ii) Difference is an irrational number.
(iii) Sum is a rational number. (iv) Sum is an irrational number.
(v) Product is a rational number. (vi) Product is an irrational number.
(vii) Quotient is a rational number. (viii) Quotient is an irrational number.

Sol. (i***) Let us consider two irrational numbers $2 + \sqrt{3}$ and $-2 + \sqrt{3}$.***
Then their difference = $(2 + \sqrt{3}) - (-2 + \sqrt{3}) = 4$. This is a rational number.
(ii) **Let us consider two irrational numbers $2\sqrt{3}$ and $\sqrt{3}$.**
Then their difference = $2\sqrt{3} - \sqrt{3} = \sqrt{3}$. This is an irrational number.
(iii) **Let us consider two irrational numbers $5 + \sqrt{5}$ and $5 - \sqrt{5}$.**
Then their sum = $(5 + \sqrt{5}) + (5 - \sqrt{5}) = 10$. This is a rational number.
(iv) **Let us consider two irrational numbers $5\sqrt{7}$ and $2\sqrt{7}$.**
Then their difference = $5\sqrt{7} - 2\sqrt{7} = 3\sqrt{7}$
∵ 3 ($\neq 0$) is a rational number and $\sqrt{7}$ is an irrational number.
∴ $3\sqrt{7}$ is an irrational number. [∵ The product of a non-zero rational number with an irrational number is an irrational.]
(v) Let us consider two irrational numbers $2 - \sqrt{3}$ and $2 + \sqrt{3}$.
Then their product = $(2 - \sqrt{3})(2 + \sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$. This is a rational number.
(vi) **Let us consider two irrational numbers $\sqrt{2}$ and $\sqrt{3}$.**
Then their product = $\sqrt{2}.\sqrt{3} = \sqrt{6}$. This is an irrational number.
(vii) **Let us consider two irrational numbers $2\sqrt{5}$ and $\sqrt{5}$.**
Then their quotient = $\frac{2\sqrt{5}}{\sqrt{5}} = 2$. This is a rational number.

(viii) **Let us consider two irrational numbers $2\sqrt{15}$ and $2\sqrt{5}$.**

Then their quotient = $\dfrac{2\sqrt{15}}{2\sqrt{5}} = \sqrt{\dfrac{15}{5}} = \sqrt{3}$ .This is an irrational number.

8. **Rationalise the denominators of the following:**

(i) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ **(ii)** $\dfrac{3+\sqrt{2}}{3-\sqrt{2}}$ **(iii)** $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}}$

(iv) $\dfrac{5+\sqrt{6}}{5-\sqrt{6}}$ **(v)** $\dfrac{3+\sqrt{7}}{3-4\sqrt{7}}$

Sol. (i) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ [Multiplying the numerator and denominator by $\sqrt{3}-1$].

$= \dfrac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2} = \dfrac{(\sqrt{3})^2-2(\sqrt{3})(1)+(1)^2}{3-1} = \dfrac{3-2\sqrt{3}+1}{2} = \dfrac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$

(ii) $\dfrac{3+\sqrt{2}}{3-\sqrt{2}}$

$\dfrac{3+\sqrt{2}}{3-\sqrt{2}} = \dfrac{3+\sqrt{2}}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$ [Multiplying the numerator and denominator by $3+\sqrt{2}$]

$= \dfrac{(3+\sqrt{2})^2}{(3)^2-(\sqrt{2})^2} = \dfrac{(3)^2+2(3)(\sqrt{2})+(\sqrt{2})^2}{9-2}$

$= \dfrac{9+6\sqrt{2}+2}{7} = \dfrac{11+6\sqrt{2}}{7}$

(iii) $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}}$

$\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = \dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$ [Multiplying the numerator and denominator by $7-4\sqrt{3}$]

$= \dfrac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2} = \dfrac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{49-48}$

$= \dfrac{35-20\sqrt{3}+14\sqrt{3}-24}{1} = \dfrac{11-6\sqrt{3}}{1}$

(iv) $\dfrac{5+\sqrt{6}}{5-\sqrt{6}}$

$\dfrac{5+\sqrt{6}}{5-\sqrt{6}} = \dfrac{5+\sqrt{6}}{5-\sqrt{6}} \times \dfrac{5+\sqrt{6}}{5+\sqrt{6}}$ [Multiplying the numerator and denominator by $5+\sqrt{6}$]

$= \dfrac{(5+\sqrt{6})^2}{(5)^2-(\sqrt{6})^2} = \dfrac{(5)^2+2(5)(\sqrt{6})+(\sqrt{6})^2}{25-6}$

$= \dfrac{25+10\sqrt{6}+6}{19} = \dfrac{31+10\sqrt{6}}{19}$

(v) $\dfrac{3+\sqrt{7}}{3-4\sqrt{7}}$

$\dfrac{3+\sqrt{7}}{3-4\sqrt{7}} = \dfrac{3+\sqrt{7}}{3-4\sqrt{7}} \times \dfrac{3+4\sqrt{7}}{3+4\sqrt{7}}$ [Multiplying the numerator and denominator by $3+4\sqrt{7}$]

$= \dfrac{(3+\sqrt{7})(3+4\sqrt{7})}{(3)^2-(4\sqrt{7})^2} = \dfrac{3(3+4\sqrt{7})+\sqrt{7}(3+4\sqrt{7})}{9-112}$

$= \dfrac{9+12\sqrt{7}+3\sqrt{7}+28}{-103} = \dfrac{37+15\sqrt{7}}{-103} = -\dfrac{37+15\sqrt{7}}{103}$

9. **Simplify each of the following by rationalising the denominator:**

(i) $\dfrac{1}{\sqrt{6}-\sqrt{5}}$ (ii) $\dfrac{4}{\sqrt{7}+\sqrt{3}}$ (iii) $\dfrac{30}{5\sqrt{3}-3\sqrt{5}}$ (iv) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (v) $\dfrac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}}$

(vi) $\dfrac{3}{5-\sqrt{2}} + \dfrac{2}{5+\sqrt{2}}$ (vii) $\dfrac{4+\sqrt{5}}{4-\sqrt{5}} + \dfrac{4-\sqrt{5}}{4+\sqrt{5}}$ (viii) $\dfrac{\sqrt{5}-2}{\sqrt{5}+2} - \dfrac{\sqrt{5}+2}{\sqrt{5}-2}$

(ix) $\dfrac{4}{2+\sqrt{3}+\sqrt{7}}$ (x) $\dfrac{1}{\sqrt{3}+\sqrt{2}-\sqrt{5}}$

Sol. (i) $\frac{1}{\sqrt{6}-\sqrt{5}}$

$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ [Multiplying the numerator and denominator by $\sqrt{6}+\sqrt{5}$]

$= \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \frac{\sqrt{6}+\sqrt{5}}{1}$

(i1) $\frac{4}{\sqrt{7}+\sqrt{3}}$

$= \frac{4}{\sqrt{7}+\sqrt{3}} = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ [Multiplying the numerator and denominator by $\sqrt{7}-\sqrt{3}$]

$= \frac{4(\sqrt{7}-\sqrt{3})}{4} = \frac{\sqrt{7}-\sqrt{3}}{1}$

NCERT Examples

1. Find:

(i) $64^{1/2}$ (ii) $32^{1/5}$ (iii) $125^{1/3}$

Sol. (i) $64^{1/2}$

$64^{1/2} = (8^2)^{1/2}$

$= 8^{2 \times 1/2} = 8^1 = 8$

(ii) $32^{1/5}$

$32^{1/5} = (2^5)^{1/5} = 2^{5 \times 1/5} = 2^1 = 2$

(iii) $125^{1/3}$

$125^{1/3} = (5^3)^{1/3}$

$= 5^{3 \times 1/3} = 5^1 = 5$

2. Find:

(i) $9^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$ (iv) $125^{-1/3}$.

Sol. (i) $9^{3/2}$

$9^{3/2} = (9^{1/2})^2 = 3^3 = 27$

(ii) $32^{2/5}$

$32^{2/5} = (2^5)^{2/5} = 2^{5 \times 2/5} = 2^2 = 4$

(iii) $16^{3/4}$

$16^{3/4} = (2^4)^{3/4} = 2^{4 \times 3/4} = 2^3 = 4$

(iv) $125^{-1/3} = (5^3)^{-1/3}$

$= 5^{3 \times (-1/3)} = 5^{-1} = \frac{1}{5}$

3. Simplify:

(i) $2^{2/3} . 2^{1/5}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2} . 8^{1/2}$

Sol. (i) $2^{2/3} . 2^{1/5}$

$2^{2/3} . 2^{1/5} = 2^{2/3 + 1/5}$

$= 2^{\frac{10+3}{15}} = 2^{13/15}$

(ii) $\left(\frac{1}{3^3}\right)^7$

$\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} = 3^{-21}$

(iii) $\frac{11^{1/2}}{11^{1/4}}$

$\frac{11^{1/2}}{11^{1/4}} = 11^{1/2 - 1/4} = 11^{1/4}$

(iv) $7^{1/2} . 8^{1/2}$

$7^{1/2} . 8^{1/2} = (7 . 8)^{1/2} = 56^{1/2}$