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UNITS AND MEASUREMENTS
IIT-NEET-PHYSICS

1. PHYSICAL QUANTITY

Any quantity which can be measured is called a physical quantity.

Examples: length, weight, time etc

Fig. 1.1

1.1 Types of Physical Quantities

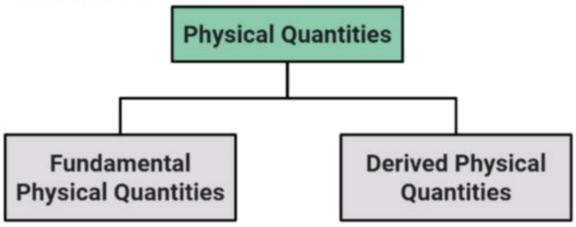


Fig. 1.2

1.2 Fundamental Physical Quantities:

Physical quantities which are independent of other physical quantities are called fundamental physical quantity. These are the quantities we take as fundamental quantities.

Quantity
Length
Mass
Time
Electric Current
Temperature
Amount of Substance
Luminous Intensity

1.3 Derived Physical Quantities

Physical quantities which are dependent on other physical quantities are called derived physical quantities.

For Example:

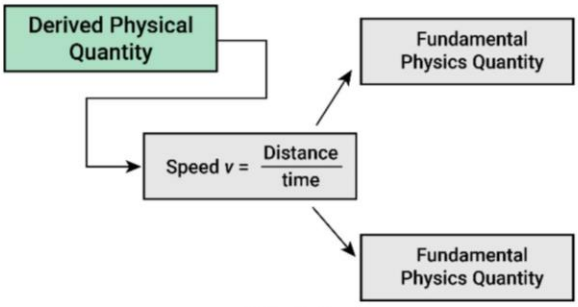


Fig. 1.3

1.4 Derived Physical Quantities

Examples:

- Acceleration $= \frac{\text{length}}{\text{time}^2}$
- Density $= \frac{\text{mass}}{\text{length}^3}$
- Volume $= \text{length}^3$
- Force $= \text{mass}.\frac{(\text{length})}{\text{time}^2}$
- Momentum $= \text{mass}.\frac{\text{Length}}{\text{time}}$

Pressure $= \frac{\text{mass}}{\text{length.time}^2}$

1.5 How to Measure a Physical Quantity

For measuring a physical quantity we have to compare it with some reference, we call it unit.

A unit is a standard amount of a physical quantity.

Example: In old times people used to measure length by hand span or foot span.

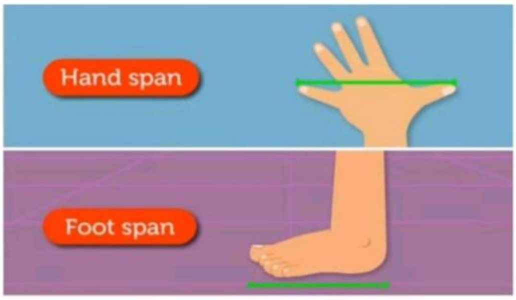


Fig. 1.4

Let's measure the length of a book using our hand span.

In this case unit for measurement is hand span.

But the length of hand span varies from person to person. So everyone will get a different result for measuring the same object.

So, there was a need of precise standardization of units.

1.6 Standard Units

Some of the standard units:

For measuring length: meter, centimeter, foot etc.

For measuring weight: kilogram, gram, pound etc

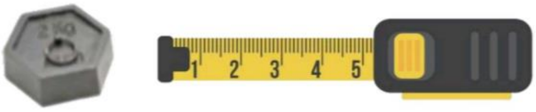


Fig. 1.5

1.7 Expressing Measurement of physical quantity

Suppose we measure length of a rod and write

length = 28

By this expression we didn't get any idea about the size of rod it can be anything like

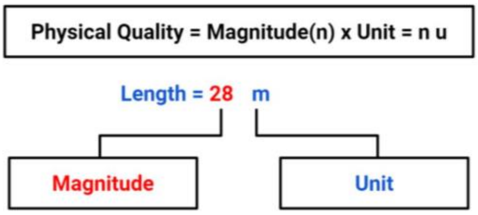
28 m

28 mm

28 km

28 foot or 28 steps

So we should always express a measurement with the unit of measurement.



NOTE:

We always write a measurement of physical quantity as its magnitude multiplied by its unit.

If we measure a physical quantity in more than one unit then the multiplication of magnitude and unit is a constant.

If Magnitude of a Physical Quantity is

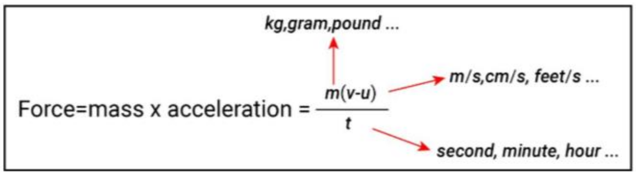
$= n_1$ in u_1 unit and n_2 in u_2 unit.

We can say that $= n_1u_1 = n_2u_2$

1.8 Need of System of Units

What if everyone uses a unit of their choice for every measurement.

For Example:



LET'S SEE A SHORTCUT HERE:

Imagine the no. of units force could have.

If everyone decides to have his own way of measurement, then it will not be possible to come to correct conclusion. Thus a well-defined, universally accepted system must be developed.

1.9 System of Units

A system of units is a complete set of units which is used to measure all kinds of fundamental and derived quantities.

Let's see example of some of the major system of units

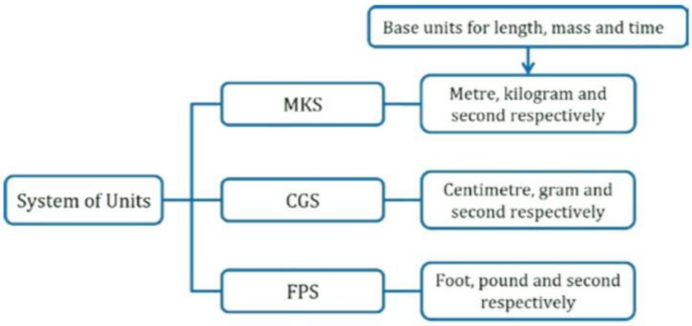


Fig. 1.6

1.10 The SI System of Units

Earlier different system of units are used in different countries.

So, there was need of an internationally accepted system of units as a complete set of units.

Here comes the "The International SI System of Units".

Currently it is the most popular system of units worldwide.

In SI system there are 7 base units and 2 supplementary units.

1.11 Fundamental Units:

Quantity	Name of units	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	Cd

1.12 Supplementary Units:

Quantity	Name of units	Symbol
Plane angle	Radian	rad
Solid angle	Steradian	sr

1.13 Plane Angle

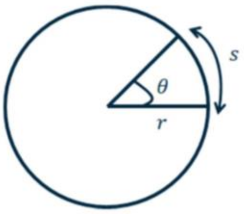


Fig. 1.7

$$\theta = \frac{s}{r} rad$$

1.14 Solid Angle

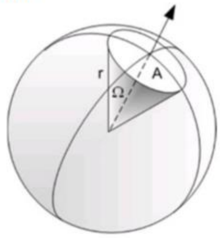


Fig. 1.8

$$\Omega = \frac{A}{r^2} sr$$

1.15 Dimensions

Dimensions of a physical quantity are the powers to which the fundamental units must be raised in order to get the unit of physical quantity.

Fundamental quantity	Dimension
Length	Meter
Mass	Kilogram
Time	Second
Electric Current	Ampere
Luminous Intensity	Candela
Temperature	Kelvin
Amount of substance	Mole

1.16 Writing Dimensions of Physical Quantities

Dimensions of a physical quantity are the powers to which the fundamental units must be raised in order to get the unit of that physical quantity.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \frac{\text{length}}{\text{time}}$$

$$\Rightarrow \text{Dimension of velocity} = \left[L^1T^{-1}\right]$$

$$\text{Acceleration}(a) = \frac{\text{change in velocity}}{\text{time}} = \frac{\text{length}}{(\text{time})^2}$$

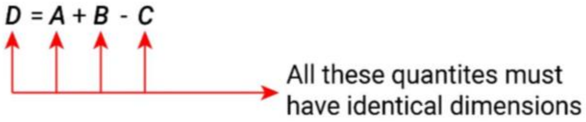
$\Rightarrow$ Dimension of acceleration $= [L^1T^{-2}]$

Force = Mass × Acceleration = Mass × $\frac{\text{length}}{(\text{time})^2}$

$\Rightarrow$ Dimension of force $= [M^1L^1T^{-2}]$

1.17 How do Dimensions behave in Mathematical Formulae?

Rule 1: All terms that are added or subtracted must have same dimensions.



Rule 2: Dimensions obey rules of multiplication and division.

$D = \frac{AB}{C}$

Given $A = [ML^0T^{-2}], B = [M^0L^{-1}T^2], C = [ML^{-2}T^0]$

$[D] = \frac{[ML^0T^{-2}] \times [M^0L^{-1}T^2]}{[ML^{-2}T^0]}$

$\Rightarrow [D] = [M^{1-1}L^{0-1+2}T^{-2+2}]$

2. DIMENSIONAL ANALYSIS

Dimensional analysis is a tool to find or check relations among physical quantities by using their dimensions.

By using dimensional analysis, we can

1. Convert a physical quantity from one system of units to another.
2. Check the dimensional consistency of equation.
3. Deduce relation among physical quantities.

2.1 Converting a Physical quantity from one System of Units to another

If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are their corresponding magnitude then $Q = n_1u_1 = n_2u_2$

Let M_1, L_1 and T_1 be the fundamental units of mass, length and time in one system; and M_2, L_2, T_2 be corresponding units in another system. If the dimensional formula of quantity be $[M^aL^bT^c]$

then $u_1 = [M_1^aL_1^bT_1^c]$ and $u_2 = [M_2^aL_2^bT_2^c]$

$Q = n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$

$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

This equation can be used to find the numerical value in the second or new system of units. Space between lines

Let us convert one joule into erg.

Joule is SI unit of energy and erg is the CGS unit of energy. Dimensional formula of energy is $[ML^2T^{-2}]$

- a = 1, b = 2, c = - 2.

SI	CGS
$M_1 = 1kg = 1000g$	$M_2 = 1g$
$L_1 = 1m = 100cm$	$L_2 = 1cm$
$T_1 = 1s$	$T_2 = 1s$
$n_1 = 1(\text{Joule})$	$n_2 = ?(\text{erg})$

$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

This equation can be used to find the numerical value in the second or new system of units.

$= 1\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^2\left[\frac{1}{1}\right]^{-2}$

$= 1 \times 10^3 \times 10^4 = 10^7$

$\therefore 1 \text{ joule} = 10^7 \text{ erg}.$

2.2 Checking the Dimensional Consistency of Equations

Principle of Homogeneity of Dimensions: For an equation to be valid, the dimensions on the left side must match the dimensions on the right side, It is then dimensionally correct. Checking this is the basic way of performing dimensional analysis.

Let's check that the second equation of motion is correct or not.

$s = ut + \frac{1}{2}at^2$

$s = \text{distance} = \text{length} = [L]$

$ut = \frac{\text{length}}{\text{time}} \times \text{time} = \text{length} = [L]$

$at^2 = \frac{\text{length}}{(\text{time})^2} \times (\text{time})^2 = \text{length} = [L]$

$\boxed{[L] = [L] + [L]}$

NOTE:

If an equation is dimensionally incorrect, it must be wrong. On the other hand, dimensionally correct equation may or may not be correct.

Let's take an example to make it simple for you.

If I say the area of a circle = 2 x radius^2;

- this is dimensionally correct (both sides have dimensions [L^2])

- but it is wrong, as, it should be '**π**' instead of '2'.

2.3 Deducing Relation among the Physical Quantities

The method of dimensions can sometimes be used to deduce relation among the physical quantities.

For this, we should know the dependence of the physical quantity on other quantities and consider it as a product type of the dependency.

Let's find the time period of a simple pendulum by using dimensional analysis. The period of oscillation of the simple pendulum depends on its length (***L***), mass of the bob (***m***) and acceleration due to gravity (***g***).

Time period $T \propto m^a g^b L^c$

$\Rightarrow T = km^a g^b L^c$

Where k is dimensionless constant.

By considering dimensions on both sides,

$$\left[M^0L^0T^1\right] = \left[M^1\right]^a \cdot \left[LT^{-2}\right]^b \left[L^1\right]^c$$

$$\Rightarrow \left[M^0L^0T^1\right] = \left[M^aL^{b+c}T^{-2b}\right]$$

Comparing both sides $a = 0, b = -\frac{1}{2}, c = \frac{1}{2}$

$$T = km^0 g^{-\frac{1}{2}} L^{\frac{1}{2}} = k\sqrt{\frac{L}{g}}$$

2.4 Limitations of Dimensional Analysis

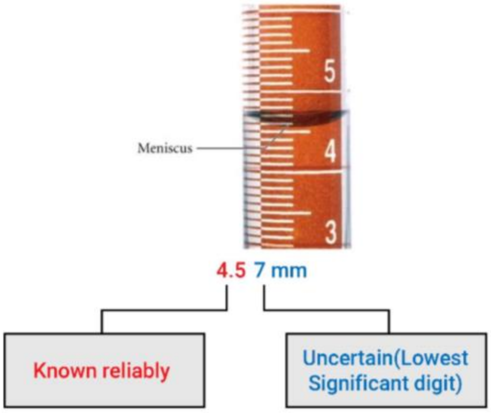
- Dimensionless quantities cannot be determined by this method.
- Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
- This method is not applicable to trigonometric, logarithmic and exponential functions.
- In the case of physical quantities which are dependent upon more than three physical quantities, this method will be difficult.
- In some cases, the constant of proportionality also possesses dimensions. In such cases, we cannot use this system.
- If one side of the equation contains addition or subtraction of physical quantities, we cannot use this method to derive the expression.

3. SIGNIFICANT FIGURES

The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.

let see an example shown in figure case a student takes reading 4.57 *mm*.

Here the digit 4 and 5 are certain and the digit 7 is an estimate.



3.1 Rules for Determining Significant Figures

Rule 1: Every non zero digit in a reported measurement is assumed to be significant.

Example:

24.7 meters, no. of significant figures = 3

0.743 meter, no. of significant figures = 3

714 meters, no. of significant figures = 3

Rule 2: Zeros appearing between nonzero digits are significant.

Example:

70003 meters, no. of significant figures = 5

40.79 meters, no. of significant figures = 4

1.503 meters, no. of significant figures = 4

Rule 3: Leftmost zeros appearing in front of nonzero digits are not significant

Example:

0.0073 meter, no. of significant figures = 2

0.423 meter, no. of significant figures = 3

0.000099 meter, no. of significant figures = 2

NOTE:

Leftmost zeros act as placeholders. By writing the measurements in scientific notation, we can eliminate such place holding zeros.

Leftmost zeros appearing in front of nonzero digits are not significant

0.0073 meter = 7.3×10^{-3} meter

0.423 meter = 4.23×10^{-1} meter

0.000099 meter = 9.9×10^{-5} meter

NOTE:

As power of ten does not contribute in significant figures thus even by changing units the number of significant digits will remain same.

Rule 4: Zeros at the end of a number will be counted as significant, only if they are at the right side of a decimal point.

Example:

300 meter, no. of significant figures = 1

3.00 meter, no. of significant figures = 3

27210 meter, no. of significant figures = 4

10.010 meter, no. of significant figures = 5

3.2 Significant Figures in Calculations

In general, a calculated answer cannot be more precise than the least precise measurement from which it was calculated.

The calculated value must be rounded to make it consistent with the measurements from which it was calculated.

So let's read about rounding off.

3.3 Rounding Off

Rule 1: If last significant digit(d) < 5 then drop it.

Example:

Round off 12.3 to 2 significant figures.

Last significant digit is 3 < 5

So, the answer is 12.

Rule 2: If last significant digit(d) > 5, then increase the preceding digit by 1 and drop 'd'.

Example:

Round off 14.56 to 3 significant figures.

Last significant digit is 6 > 5

So, the answer is 14.6.

Rule 3: If last significant digit(d) = 5, then look at the preceding digit.

(i) If preceding digit is even, drop 'd'.

(ii) If preceding digit is odd then increase preceding digit by 1 and drop 'd'.

Example:

Round off 1.45 to 2 significant figures.

Last significant digit is 5 and preceding digit is 4 which is even. So, the answer is 1.4

Example:

Round off 147.5 to 3 significant figures.

Last significant digit is 5 and preceding digit is 7 which is odd. So, the answer is 148

4. ERRORS

An error is a mistake of some kind causing an error in your results so the result is not accurate.

4.1 Types of Errors

Errors can be divided into two main classes

- Random errors
- Systematic errors

4.2 Random Errors

Random error has no pattern. One instant your readings might be too small. The next instant they might be too large. You can't predict random error and these errors are usually unavoidable.

- Random errors cannot be rectified but can be minimized.
- Random errors can be reduced by taking a lot of readings, and then calculating the average (mean).

4.3 Causes of Random errors

1. Human Error

Example:

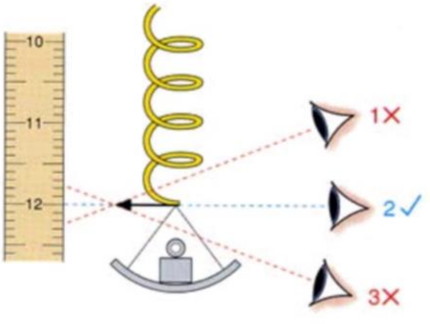


Fig. 1.9

Way of taking reading 2 is best, 1 and 3 give the wrong readings. This is called a parallax error.

2. Faulty Technique

Using the instrument wrongly.

Fig. 1.10

4.4 Systematic Errors

Systematic error is consistent, repeatable error associated with faulty equipment or a flawed experiment design. These errors are usually caused by measuring instruments that are incorrectly calibrated.

- These errors cause readings to be shifted one way (or the other) from the true reading.

4.5 Causes of Systematic Errors

1. Faulty Instruments

Example:

- There is no any weight and the can't be nothing weighed but weighing machines are not showing zero.

Fig. 1.11

Example:

- If a ruler is wrongly calibrated, or if it expands, then all the readings will be too low (or all too high).

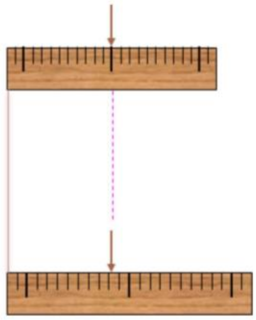


Fig. 1.12

2. Personal Error

Example:

- If someone has habit of taking measurements always from above the reading, then due to parallax you will get a systematic error and all the readings will be different from the actual reading

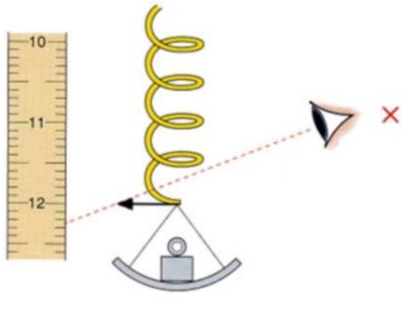


Fig. 1.13

Now, Let's learn about some common terms used during, measurements and error analysis

4.6 Accuracy and Precision

Accuracy is an indication of how close a measurement is to the accepted value.

- An accurate experiment has a low systematic error.
  Precision is an indication of the agreement among a number of measurements.
- A precise experiment has a low random error

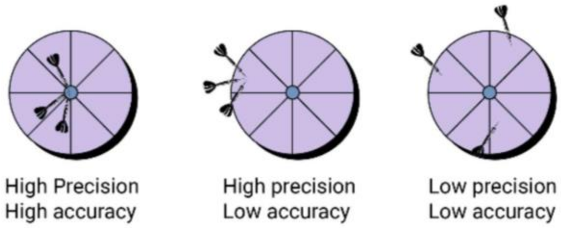


Fig. 1.14

4.7 Calculation of Errors

For calculation purpose we divide the errors in three types

Absolute. error

Relative. error

Percentage. error

4.8 Absolute Errors

The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement.

Absolute error is denoted by Δa, and it is always taken positive.

For Example:

Let say the: Values obtained in several measurements are $a_1, a_2, a_3, \ldots,$

If true value is not available, we can consider arithmetic mean as true value.

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

Absolute Errors in measurements =

$$\Delta a_1 = a_1 - a_{mean},$$

$$\Delta a_2 = a_2 - a_{mean},$$

... ...

... ...

$$\Delta a_n = a_n - a_{mean}$$

Mean Absolute Error

$$|\Delta a_{mean}| = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|}{n}$$

So, we show the measurement by $a \pm \Delta a_{mean}$ so on.

4.9 Relative Errors

The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

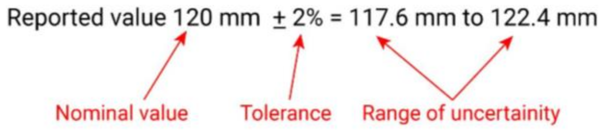
$$\text{Relative error } = \frac{\Delta a_{mean}}{a_{mean}}$$

When the relative error is expressed in percent, it is called the percentage error (δ).

$$\text{Percentage error } \delta = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

4.10 Range of Uncertainty

Range of uncertainty is reported as a nominal value plus or minus an amount called the tolerance or percent tolerance.



as

2% of 120 = 2.4,

120 - 2.4 = 117.6,

120 +2.4 = 122.4

4.11 Limit of Reading or Least Count

The limit of reading of a measurement is equal to the smallest graduation of the scale of an instrument.

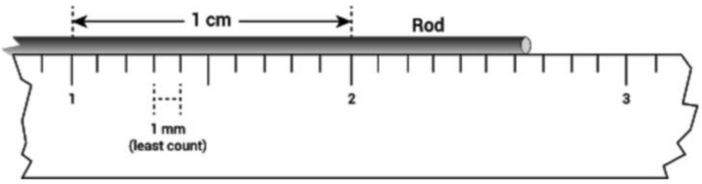


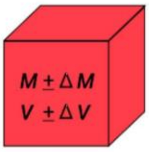
Fig. 1.15

Least count of this scale is 1 mm

4.12 Least Count Error

When a measurement falls between two divisions, then errors due to approximate measurement made by the observer is called least count error.

4.13 Propagation of Errors



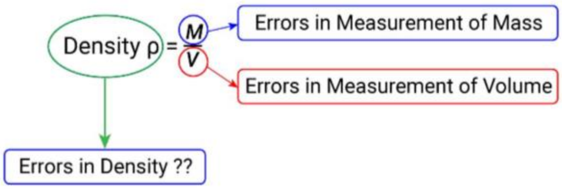


Fig. 1.16

4.14 Errors of a Sum or a Difference

When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Measured values of physical quantity A and B are respectively $A \pm \Delta A$ and $B \pm \Delta B$

If a Physical Quantity $Z = A + B$ or

$Z = A - B$

Then Maximum possible Error in Z is given by $\Delta Z = \Delta A + \Delta B$

4.15 Errors of a Multiplication or Division

Measured value of physical quantity A and B are respectively $A \pm \Delta A$ and $B \pm \Delta B$

If a Physical Quantity $Z = A \times B$ or

$Z = A/B$

Then maximum relative error in Z,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

4.16 Error of a Measured Quantity Raised to a Power

The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

Measured values of physical quantity A and B are respectively $A \pm \Delta A$ and $B \pm \Delta B$

If a Physical Quantity $Z = A^2$

Then maximum relative error in Z,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A} = 2\frac{\Delta A}{A}$$

In general, if $Z = A^p B^q C^r$

Then maximum relative error in Z,

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Dimensional Formulae of Physical Quantities

S. No.	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
1.	Area	Length × breadth	$[L^2]$	$[M^0L^2T^0]$
2.	Volume	Length × breadth × height	$[L^3]$	$[M^0L^3T^0]$
3.	Mass density	Mass/volume	$[M]/[L^3]$ or $[ML^{-2}]$	$[ML^{-3}T^0]$
4.	Frequency	1/time period	$1/[T]$	$[M^0L^0T^{-1}]$
5.	Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0LT^{-1}]$
6.	Acceleration	Velocity/time	$[LT^{-1}]/[T]$	$[M^0LT^{-2}]$
7.	Force	Mass × acceleration	$[M][LT^{-2}]$	$[MLT^{-2}]$
8.	Impulse	Force × time	$[MLT^{-2}][T]$	$[MLT^{-1}]$
9.	Work, Energy	Force × distance	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
10.	Power	Work/time	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
11.	Momentum	Mass × velocity	$[M][LT^{-1}]$	$[MLT^{-1}]$
12.	Pressure, stress	Force/area	$[MLT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
13.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[L]/[L]$	$[M^0L^0T^0]$
14.	Modulus of elasticity	Stress/strain	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$	$[ML^{-1}T^{-2}]$
15.	Surface tension	Force/length	$[MLT^{-2}]/[L]$	$[ML^0T^{-2}]$
16.	Surface energy	Energy/area	$[ML^2T^2]/[L^2]$	$[ML^0T^{-2}]$
17.	Velocity gradient	Velocity/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
18.	Pressure gradient	Pressure/distance	$[ML^{-1}T^{-2}]/[L]$	$[M^1L^{-2}T^{-2}]$
19.	Pressure energy	Pressure × volume	$[ML^{-1}T^{-2}][L^3]$	$[ML^2T^{-2}]$
20.	Coefficient of viscosity	Force/area × velocity gradient	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1}T^{-1}]$
21.	Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0L^0T^0]$
22.	Trigonometric ratio $(\sin\theta, \cos\theta, \tan\theta, \text{etc.})$	Length/length	$[L]/[L]$	$[M^0L^0T^0]$
23.	Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0L^0T^{-1}]$

BASIC MATHEMATICS

1. QUADRATIC EQUATION

A quadratic equation is an equation of second degree, meaning it contains at least one term that is squared.

The standard form of quadratic equation is

$ax^2 + bx + c = 0$ where $a \neq 0$

The solution of the above quadratic equation is the values of variable 'x' which will satisfy it. It basically have 2 solutions $(x_1$ and $x_2)$

If we try to calculate time when football is at height H then we will observe that we will get 2 answers

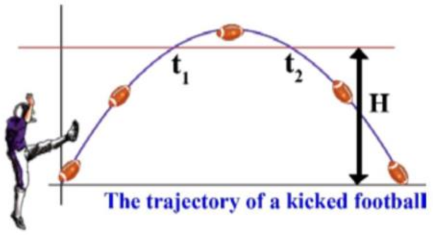


Fig. 1.17

t_1 - While going up

t_2 - While Coming down

What if we take a height which is greater than maximum height covered by ball and we are trying to find the time?

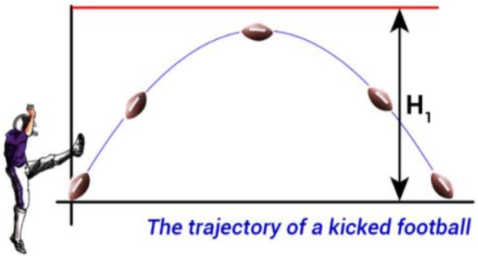


Fig. 1.18

By this diagram we can easily say that at no real value of time, the ball is at height H_1. We will not have a diagram everytime though.

For finding out if a quadratic equation has a real solution or not, we shall use the **'DISCRIMINANT'**.

1.1 Discriminant of a Quadratic equation

Discriminant of a quadratic equation $ax^2 + bx + c = 0$ is represented by D.

$D = b^2 - 4ac$

Nature of the roots of a quadratic equation

The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- If $D < 0$, No real roots for given equation.

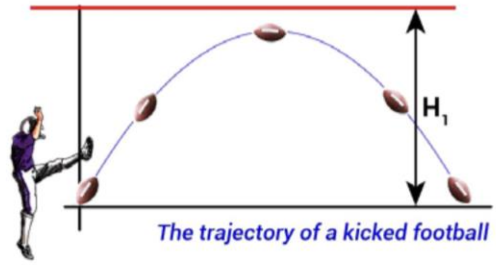


Fig. 1.19

- If $D > 0$, Two distinct real roots

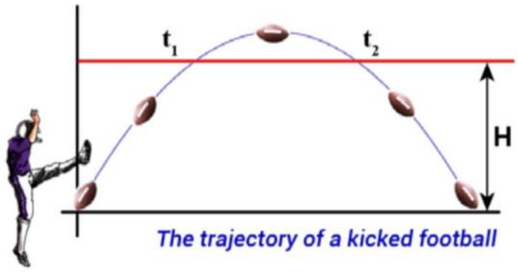


Fig. 1.20

- The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $D = 0$, Equal and real roots. Then we will get only one root

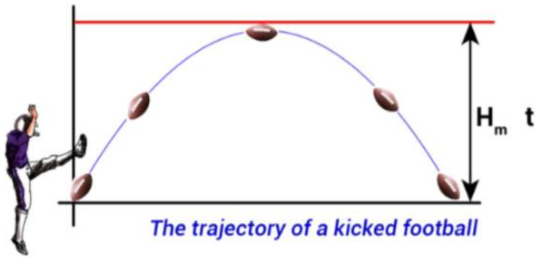


Fig. 1.21

- The roots are given by $-\dfrac{b}{2a}$

i. Sum of roots $= x_1 + x_2 = -\dfrac{b}{a}$

ii. Product of roots $= x_1 x_2 = \dfrac{c}{a}$

iii. Difference of the roots $= x_1 - x_2 = \dfrac{\sqrt{D}}{a}$

If the value of discriminant = 0 i.e. $b^2-4ac=0$	The quadratic equation will have equal roots i.e. $\alpha=\beta=-\dfrac{b}{2a}$
If the value of discriminant $<0 i.e. b^2-4ac<0$	The quadratic equation will have imaginary Roots i.e. $\alpha=(p+iq)$ and $\beta=(p-iq)$. Where 'iq' is the imaginary part of complex number
If the value of discriminant $(D)>0 i.e. b^2-4ac>0$	The quadratic equation will have real roots
If the value of discriminant > 0 and D is perfect square	The quadratic equation will have rational roots
If the value of discriminant (D) > 0 and D is not a perfect square	The quadratic equation will have irrational roots i.e. $\alpha=(p+\sqrt{q})$ and $\beta=(p-\sqrt{q})$
If the value of discriminant > 0, D is perfect square, a = 1 and b and c are integers	The quadratic equation will have integral roots

2. BASIC GRAPHS

(i) Straight line graph

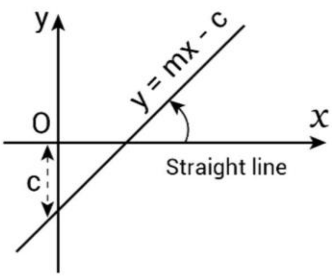


Fig. 1.22

Equation of graph: $y=mx-c$

(ii) Straight line graph

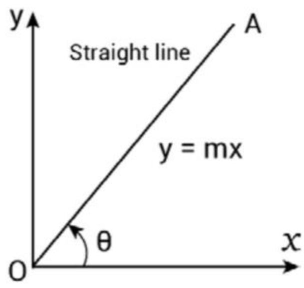


Fig. 1.23

Equation of graph: $y=mx$

(iii) Straight line graph

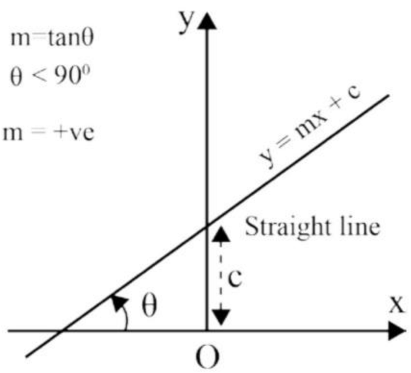


Fig. 1.24

Equation of graph: $y=mx+c$

$m=\tan\theta$

$\theta<90°$

$m=+ve$

(iv) Straight line graph

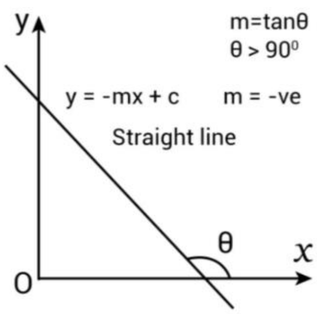


Fig. 1.25

Equation of graph: $y=mx+c$

$m=\tan\theta$

$\theta>90°$

(v) Parabola graph

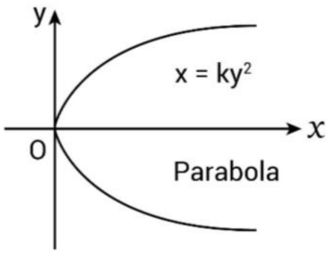


Fig. 1.26

Equation of graph: $x = ky^2$

(vi) Parabola graph

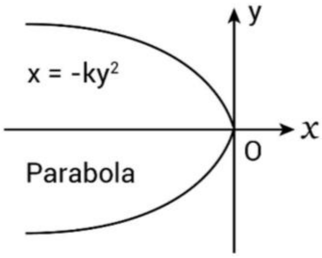


Fig. 1.27

Equation of graph: $x = -ky^2$

(vii) Parabola graph

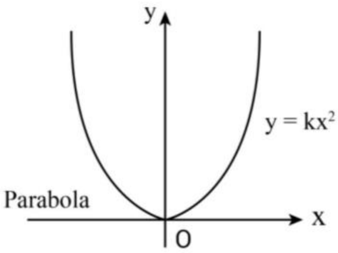


Fig. 1.28

Equation of graph: $y = kx^2$

(viii) Parabola graph

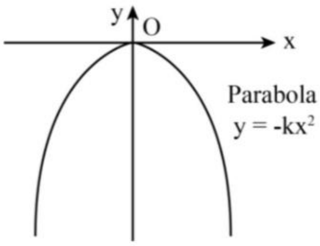


Fig. 1.29

Equation of graph: $y = -kx^2$

(ix) Rectangular Hyperbola graph

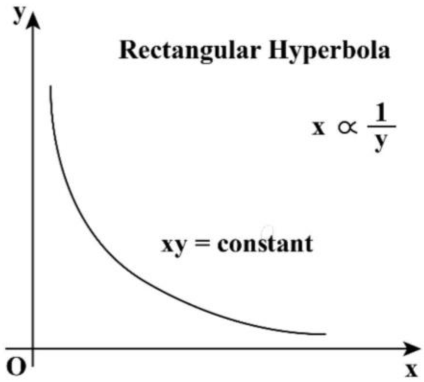


Fig. 1.30

Equation of graph: $xy = \text{constant}$

$x \propto \frac{1}{y}$

(x) Circle graph

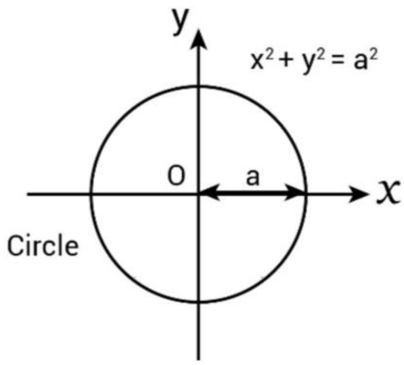


Fig. 1.31

Equation of graph: $x^2 + y^2 = a^2$

(xi) Ellipse graph

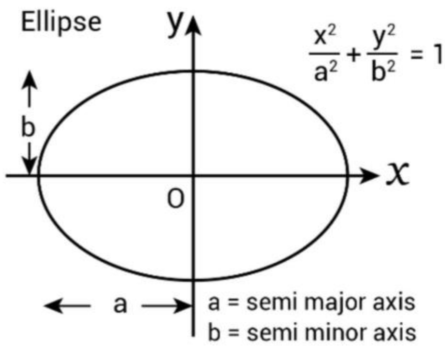


Fig. 1.32

Equation of graph: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(xii) Exponential Decay graph

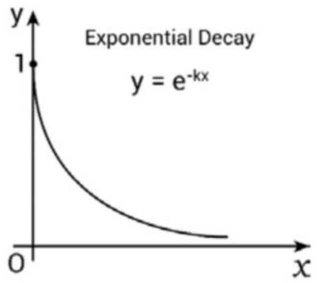


Fig. 1.33

Equation of graph: $y = e^{-kx}$

(xiv) sin graph:

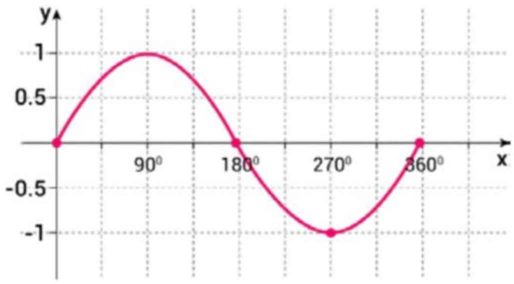


Fig. 1.34

Max value of Graph	**Min value of graph**
1 at $90°, 450°, 810°$...etc.	-1 at $270°, 630°, 990°$...etc.

- y = sin x
- The roots or zeros of y = sin x is at the multiples of π
- The sin graph passes the x-axis as sin x = 0.
- Period of the sine function is 2 π
- The height of the curve at each point is equal to the line value of sine

(xv) cos graph:

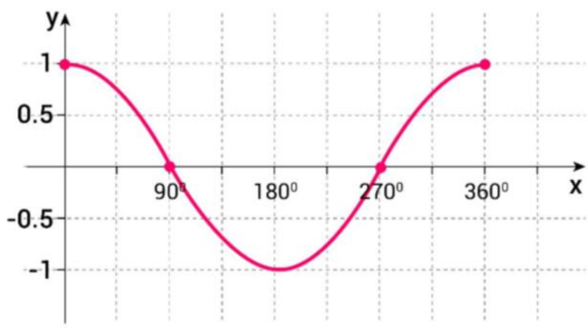


Fig. 1.35

Max value of Graph	**Min value of graph**
1 at $0, 360°, 720°$...etc.	-1 at $180°, 540°, 900°$...etc.

- y = cos x
- sin (x + π/2) = cos x
- The y = cos x graph is obtained by shifting the y = sin x, π/2 units to the left
- Period of the cosine function is 2π

There are a few similarities between the sine and cosine graphs they are:

- Both have the same curve which is shifted along the x-axis.
- Both have an amplitude of 1
- Have a period of 360° or 2π radians

The combined graph of sine and cosine function can be represented as follows:

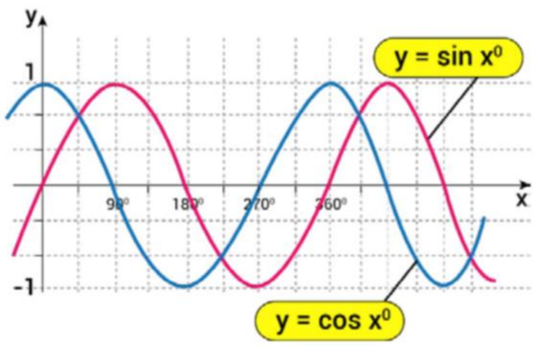


Fig. 1.36

(xvi) tan graph:

The tan function is completely different from sin and cos function. The function here goes between negative and positive infinity, crossing through y = 0 over a period of π radian.

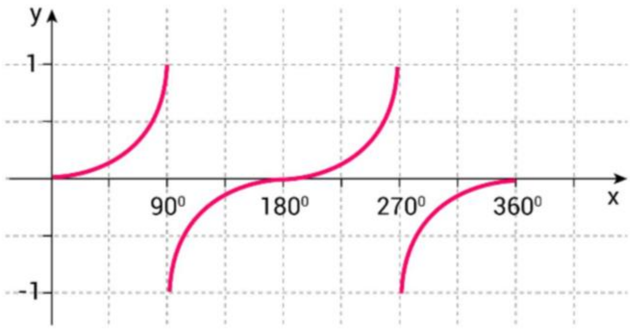


Fig. 1.37

- y = tan x
- The tangent graph has an undefined amplitude as the curve tends to infinity
- It also has a period of 180°. i.e. π

3. BINOMIAL EXPANSION

A binomial is a polynomial with two terms.

For example, $3x^2 + 2$ is a binomial.

what happens when we multiply the binomial with itself?

Example: a + b

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2)$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = (a+b)(a^3 + 3a^2b + 3ab^2 + b^3)$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Calculations get longer and longer as we increase the power, but we can say that there is a pattern,

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

Power of a	2	1	0
Power of b	0	1	2

$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2)$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

Power of a	3	2	1	0
Power of b	0	1	2	3

$$(a+b)^4 = (a+b)(a^3 + 3a^2b + 3ab^2 + b^3)$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Power of a	4	3	2	1	0
Power of b	0	1	2	3	4

This pattern is summed up by binomial theorem

$$(a+b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2\times 1}a^{n-2}b^2$$

$$+\frac{n(n-1)(n-2)}{3\times 2\times 1}a^{n-3}b^3 ......+b^n$$

$$(1+x)^n = 1 + nx^1 + \frac{n(n-1)}{2\times 1}x^2 + \frac{n(n-1)(n-2)}{3\times 2\times 1}x^3 ...$$

If value of x is very small, we can neglect higher powers of x

So, $(1+x)^n = 1 + nx$

4. VECTORS

Scalars and Vectors

4.1 What is a scalar?

A scalar is a quantity that is fully described by a magnitude only. It is described by just a number.

Examples:

Speed, volume, mass, temperature, power, energy, time, etc.

4.2 What is a vector?

Vector is a physical quantity which has magnitude as well as direction and follows the rule of vector addition.

Vector quantities are important in the study of physics.

Examples:

Force, velocity, acceleration, displacement, momentum, etc.

4.3 Representation of Vectors

- A vector is drawn as an arrow with a head and a tail.
- The magnitude of the vector is often described by the length of the arrow.
- The arrow points in the direction of the vector.

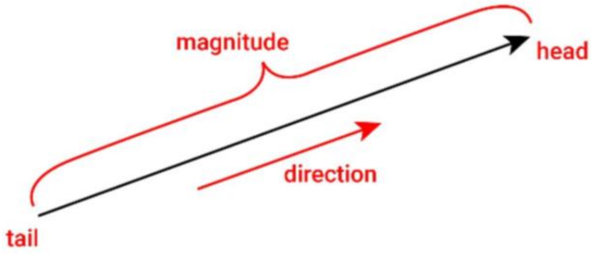


Fig. 1.38

- Vectors can be defined in two dimensional or three dimensional space

How to write a vector?

Vectors are generally written with an arrow over the top of the letter. (Ex: $\vec{a}$)

They can also be written as boldface letters. (Ex: **a**)

$$\boxed{\overrightarrow{AB} = \vec{a}}$$

Magnitude:

$$\boxed{\left|\overrightarrow{AB}\right| = a}$$

4.4 Properties of Vectors

Vectors are mathematical objects and we will now study some of their mathematical properties.

(1). Equality of vectors

Two vectors are equal if they have the same magnitude and the same direction.

(2). Negative Vector

A negative vector is a vector that has the opposite direction to the reference positive direction.

4.5 Types of Vectors

1. Zero Vector
2. Unit Vector
3. Position Vector
4. Co-initial Vector
5. Like and Unlike Vectors
6. Coplanar Vector
7. Collinear Vector
8. Displacement Vector

4.6 Zero Vector

- A zero vector is a vector when the magnitude of the vector is zero and the starting point of the vector coincides with the terminal point.
- In other words, a vector $\overrightarrow{AB}'s$ coordinates of the point A are the same as that of the point B then the vector is said to be a zero vector and is denoted by 0.

4.7 Unit Vector

A vector which has a magnitude of unit length is called a unit vector.

Suppose if $\vec{x}$ is a vector having a magnitude $|\vec{x}|$ then the unit vector is denoted by $\hat{x}$ in the direction of the vector $\vec{x}$ and has the magnitude equal to 1.

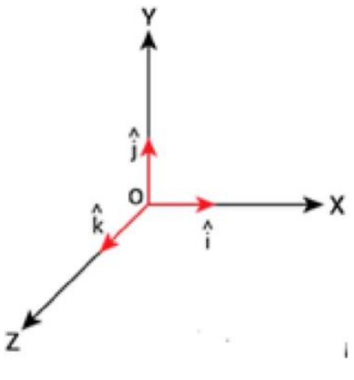


Fig. 1.39

Unit vector

$$\therefore \hat{x} = \frac{\vec{x}}{|\vec{x}|}$$

It must be carefully noted that any two unit vectors must not be considered as equal, because they might have the same magnitude, but the direction in which the vectors are taken might be different.

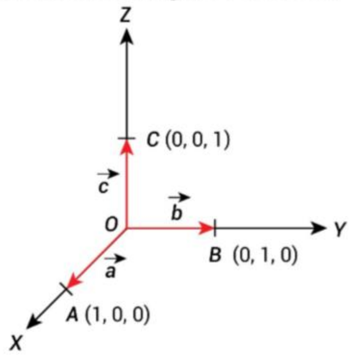
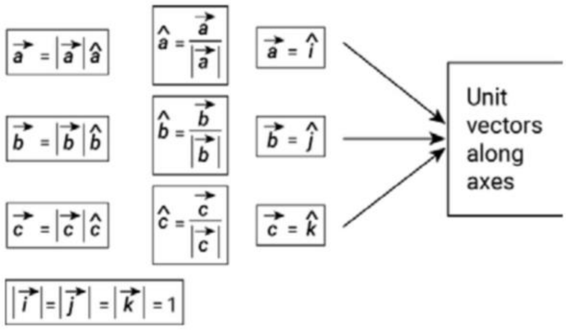


Fig. 1.40

$\vec{a}$ $\quad |\vec{a}| = 1 \quad \times \quad \boxed{\hat{a}} \rightarrow \boxed{\hat{i}}$

$\vec{b}$ $\quad |\vec{b}| = 1 \quad \times \quad \boxed{\hat{b}} \rightarrow \boxed{\hat{j}}$

$\vec{c}$ $\quad |\vec{c}| = 1 \quad \times \quad \boxed{\hat{c}} \rightarrow \boxed{\hat{k}}$



$\vec{a} = |\vec{a}|\hat{a}$, $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$, $\hat{a} = \hat{i}$

$\vec{b} = |\vec{b}|\hat{b}$, $\hat{b} = \frac{\vec{b}}{|\vec{b}|}$, $\hat{b} = \hat{j}$

$\vec{c} = |\vec{c}|\hat{c}$, $\hat{c} = \frac{\vec{c}}{|\vec{c}|}$, $\hat{c} = \hat{k}$

$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$

4.8 Position Vector:

If O is taken as reference origin and P is an arbitrary point in space then the vector OP is called as the position vector of the point. Position vector simply denotes the position or location of a point in the three-dimensional Cartesian system with respect to a reference origin.

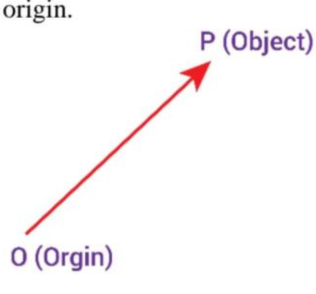


Fig. 1.41

4.9 Co-initial Vector:

The vectors which have the same starting point are called co-initial vectors.

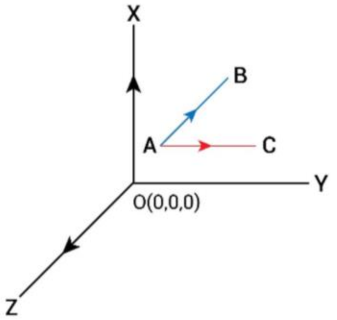


Fig. 1.42

The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called co-initial vectors as they have same starting point.

4.10 Like and Unlike Vectors:

The vectors having the same direction are known as like vectors. On the contrary, the vectors having the opposite direction with respect to each other are termed to be unlike vectors.

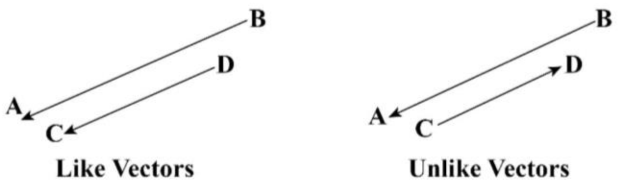


Fig. 1.43

4.11 Coplanar Vectors:

Three or more vectors lying in the same plane or parallel to the same plane are known as coplanar vectors.

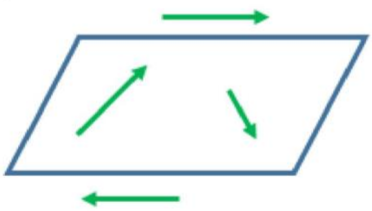


Fig. 1.44

4.12 Collinear Vectors:

Vectors which lie along the same line are known to be collinear vectors.

Fig. 1.45

4.13 Displacement Vector:

If a point is displaced from position A to B then the displacement AB represents a vector $\overrightarrow{AB}$ which is known as the displacement vector.

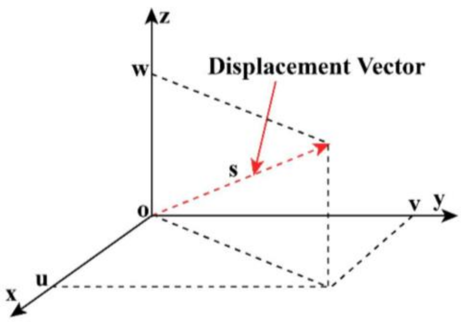


Fig. 1.46

4.14 Multiplication of a vector with a scalar

- When a vector is multiplied by a scalar quantity, then the magnitude of the vector changes in accordance with the magnitude of the scalar but the direction of the vector remains unchanged.
- Suppose we have a vector $\vec{a}$, then if this vector is multiplied by a scalar quantity k then we get a new vector with magnitude as $|\overrightarrow{ka}|$ and the direction remains same as the vector $\vec{a}$.

4.15 Multiplication of vectors with scalar

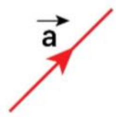


Fig. 1.47

Now let us understand visually the scalar multiplication of the vector.

Let us take the values of 'k' to be = 2, 3, -3, -1/2 and so on.

Fig. 1.48

4.16 Position Vector

A vector representing the straight-line distance and the direction of any point or object with respect to the origin, is called position vector.

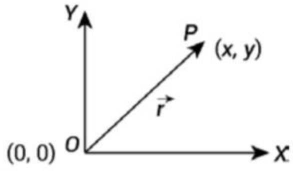


Fig. 1.49

$$\overrightarrow{OP} = x\hat{i} + y\hat{j}$$

$$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2} = \left|\vec{r}\right| = r$$

4.17 Displacement Vector

A vector representing the straight-line distance and the direction of any point or object with respect to another point is called displacement vector.

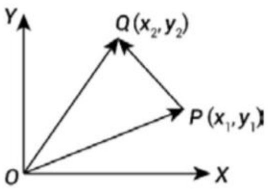


Fig. 1.50

$$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j}$$

$$\overrightarrow{OQ} = x_2\hat{i} + y_2\hat{j}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= \left(x_2 - x_1\right)\hat{i} + \left(y_2 - y_1\right)\hat{j}$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

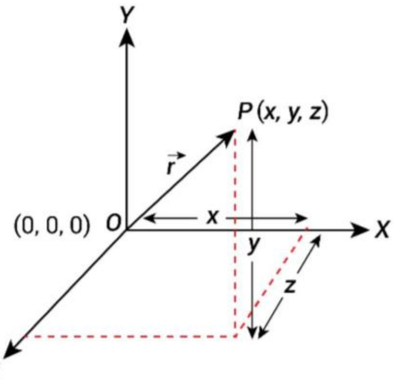


Fig. 1.51

$$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2} = \left|\vec{r}\right| = r$$

4.18 Components of a Vector

In physics, when you break a vector into its parts, those parts are called its components.

Typically, a physics problem gives you an angle and a magnitude to define a vector

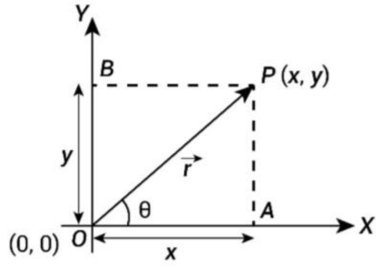


Fig. 1.52

$$\overrightarrow{OP} = x\hat{i} + y\hat{j}$$

$$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2} = \left|\vec{r}\right| = r$$

$$\overrightarrow{OA} = x\hat{i} \Rightarrow r\cos\theta = \left|\overrightarrow{OA}\right|$$

$$\left|\overrightarrow{OB}\right| = y\hat{i} \Rightarrow r\sin\theta = \left|\overrightarrow{OB}\right|$$

$$\tan\theta = \frac{\left|\overrightarrow{OB}\right|}{\left|\overrightarrow{OA}\right|}$$

$$\overrightarrow{OA} = x\hat{i}$$

$$\overrightarrow{OB} = y\hat{i} = \overrightarrow{AD}$$

$$\overrightarrow{OC} = z\hat{k} = \overrightarrow{DP}$$

In ΔODP

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{DP} = x\hat{i} + y\hat{j} + z\hat{k}$$

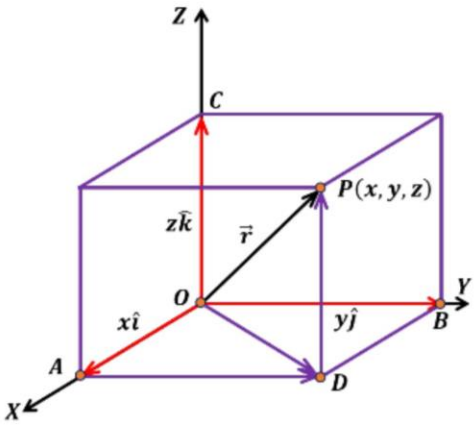


Fig. 1.53

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$ → unit vector along $\vec{r}$

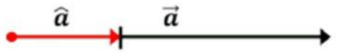
$\overrightarrow{OA} = x\hat{i}$, is the component of vector $\vec{r}$ in X-axis

$\overrightarrow{OB} = y\hat{j}$, is the component of vector $\vec{r}$ in Y-axis

$\overrightarrow{OC} = z\hat{k}$, is the component of vector $\vec{r}$ in Z-axis

4.19 Finding a Unit Vector (2D/3D)

- We have already studied about it in previous classes. Just to recall:
- Unit vector in the direction of $\vec{a}$ is $\hat{a}$



$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$ $$|\hat{a}| = 1$$

Fig. 1.54

- It will be more clear by solving some problems pertaining 2D/3D cases.

5. ADDITION OF VECTORS

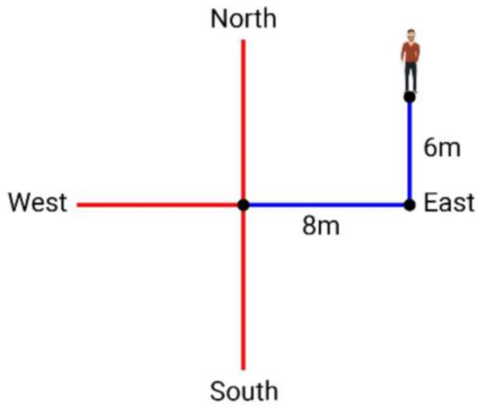


Fig. 1.55

Can we add these vectors directly as (8 m + 6 m) = 14 m ?

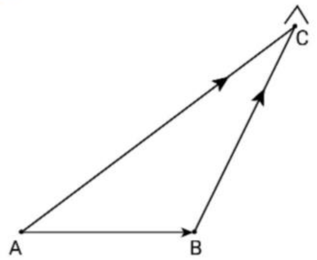
(a) Yes

(b) No

Sol: We add vectors considering their directions.

So, now we will learn about the addition of vectors.

5.1 Triangle Law of Vector Addition



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ Triangle law of Addition

$0 = \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC}$ $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

Fig. 1.56

5.2 Both Addition and Subtraction can be shown as:

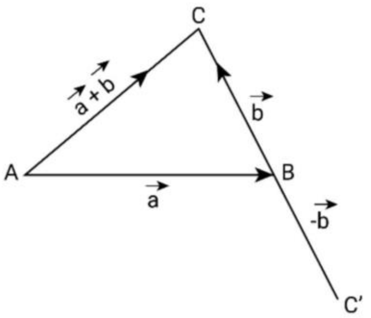


Fig. 1.57

5.3 Polygon Law of Vector Addition

It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector R is represented in magnitude and direction by the closing side of polygon taken in opposite order. In fact, polygon law of vectors is the outcome of triangle law of vectors.

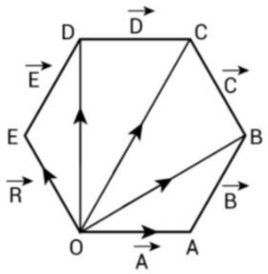


Fig. 1.58

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$$

NOTE:

- Resultant of two unequal vectors cannot be zero.
- Resultant of three coplanar vectors may or may not be zero.
- Resultant of three non-coplanar vectors cannot be zero, minimum number of non-coplanar vectors whose sum can be zero is four.
- Polygon law should be used only for diagram purpose for calculation of resultant vector (For addition of more than 2 vectors), we use components of vector.
- Minimum no. of coplanar vector for zero resultant is 2 (for equal magnitude) & 3 (for unequal magnitude).

5.4 Addition of Vectors

Adding Vectors Analytically

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$

$$\vec{r} = \vec{r}_1 + \vec{r}_2$$

$$= \left(x_1\hat{i} + y_1\hat{j}\right) + \left(x_2\hat{i} + y_2\hat{j}\right)$$

$$= x_1\hat{i} + y_1\hat{j} + x_2\hat{i} + y_2\hat{j} = x_1\hat{i} + x_2\hat{i} + y_1\hat{j} + y_2\hat{j}$$

$$= \left(x_1 + x_2\right)\hat{i} + \left(y_1 + y_2\right)\hat{j}$$

5.5 Addition of Vectors: Components

Step 1: Identify the x-and y-axes that will be used in the problem.

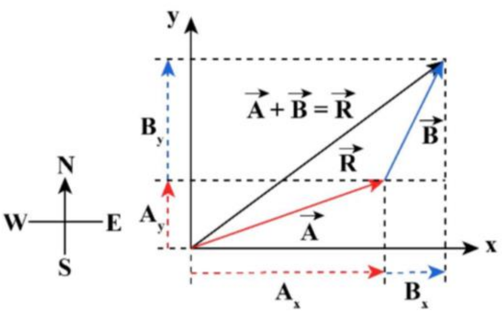


Fig. 1.59

Then, find the components of each vector to be added along the chosen perpendicular axes. Use the equations $\vec{A}_x = \vec{A}\cos\theta$, $\vec{A}_y = \vec{A}\sin\theta$ to find the components. In figure, these components are $\vec{A}_x$, $\vec{A}_y$, $\vec{B}_x$ and $\vec{B}_y$.

The angles that vectors $\vec{A}$ and $\vec{B}$ make with the x-axis are θ_A and θ_B. respectively.

Step 2: Find the components of the resultant along each axis by adding the components of the individual vectors along that axis. That is, as shown in figure,

$$R_x = A_x + B_x$$

and $$R_y = A_y + B_y$$

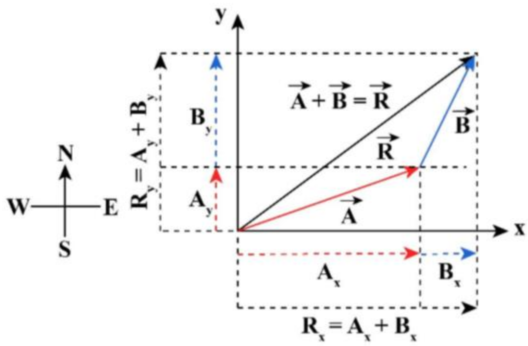


Fig. 1.60

components along the same axis, say the x-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the y-axis. So resolving vectors into components along common axes makes it easier to add them. Now that the components of R are known, its magnitude and direction can be found.

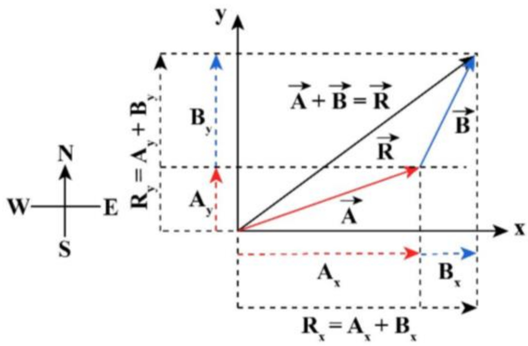


Fig. 1.61

Step 3: To get the magnitude R of the resultant, use the Pythagorean theorem;

$$R = \sqrt{R_x^2 + R_y^2}$$

Step 4: To get the direction of the resultant;

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

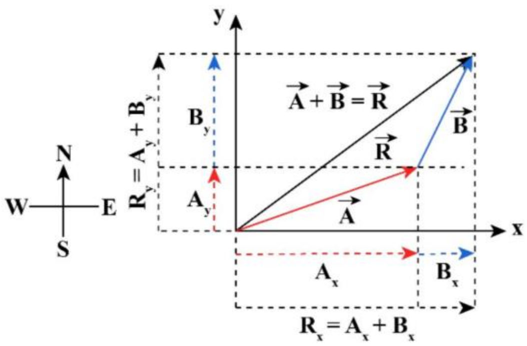


Fig. 1.62

5.6 Parallelogram Law of Vector Addition

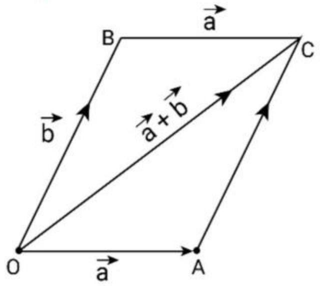


Fig. 1.63

Suppose the magnitude of $\vec{a} = a$ and that of $\vec{b} = b$.

What is the magnitude of $\vec{a}+\vec{b}$ and what its direction?

Suppose the angle between $\vec{a}$ and $\vec{b}$ is θ.

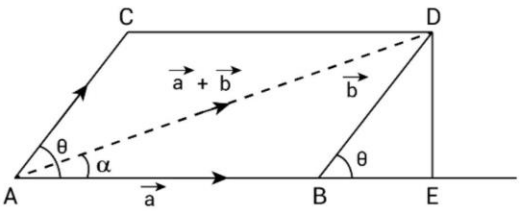


Fig. 1.64

It is easy to say from fig. that

$AD^2 = (AB+BE)^2 + (DE)^2$

$= (a+b\cos\theta)^2 + (b\sin\theta)^2$

$= a^2 + 2ab\cos\theta + b^2$

Thus, the magnitude of is

$\sqrt{a^2+b^2+2ab\cos\theta}$

Its angle with $\vec{a}$ is α where

$$\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{a+b\cos\theta}$$

5.7 Some Properties of Vector Addition

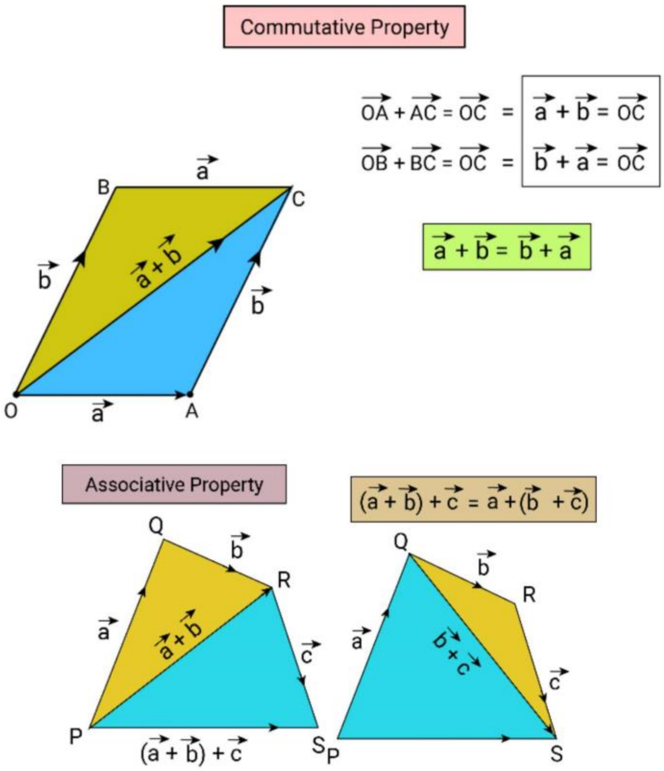


Fig. 1.65

5.8 Subtraction of Vectors

- **Subtracting Vectors Algebraically**

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{b} = b_x\hat{i} + b_y\hat{j}$

$\vec{a}-\vec{b} = \vec{a}+(-\vec{b}) = a_x\hat{i}+a_y\hat{j}+(-b_x\hat{i}-b_y\hat{j})$

$= (a_x-b_x)\hat{i}+(a_y-b_y)\hat{j}$

- **Subtracting Vectors Geometrically**

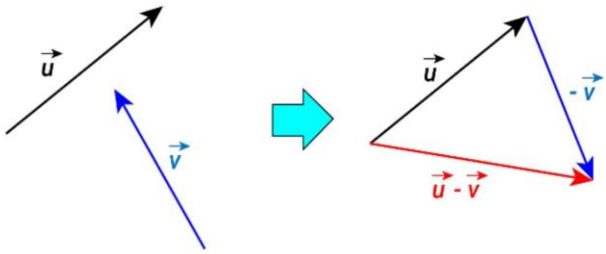


Fig. 1.66

5.9 Change in Vectors

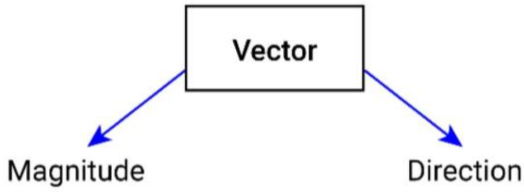


Fig. 1.67

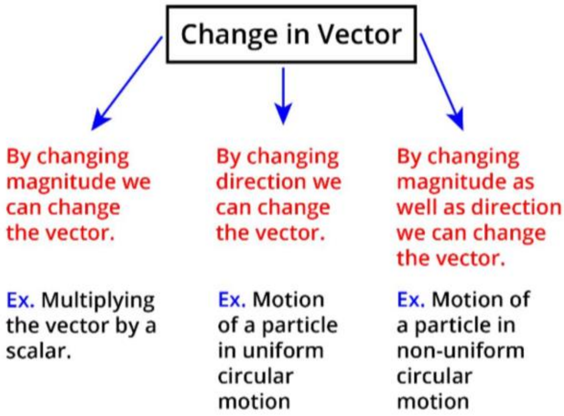


Fig. 1.68

ALWAYS REMEMBER:

A Vector can be changed either its magnitude or direction or by changing both of them.

6. PRODUCT OF TWO VECTORS

- A vector can be multiplied by another but may not be divided by another vector.
- There are two kinds of products of vectors used broadly in physics and engineering.
- One kind of multiplication is a scalar multiplication of two vectors. Taking a scalar product of two vectors results in a number (a scalar), as its name indicates.
- Scalar products are used to define work and energy relations.
- For example, the work that a force (a vector) performs on an object while causing its displacement (a vector) is defined as a scalar product of the force vector with the displacement vector.
- A quite different kind of multiplication is a vector multiplication of vectors. Taking a vector product of two vectors returns a vector, as its name suggests.
- Vector products are used to define other derived vector quantities.
- For example, in describing rotations, a vector quantity called torque is defined as a vector product of an applied force (a vector) and its distance from pivot to force (a vector).
- It is important to distinguish between these two kinds of vector multiplication because the scalar product is a scalar quantity and a vector product is a vector quantity.

6.1 Scalar Product or Dot Product

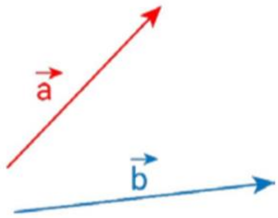


Fig. 1.69

Dot product of vector $\vec{a}$ and $\vec{b}$ is donated by $\vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$0\le\theta\le\pi$

- Dot product give us a scalar quantity.
- Angle between vectors, $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

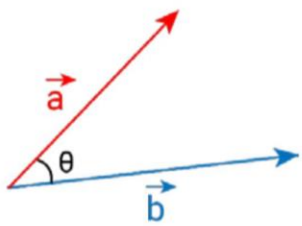


Fig. 1.70

- When $\theta=0°$,

$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=|\vec{a}||\vec{b}|$

$\vec{a}\cdot\vec{b}$ is maximum

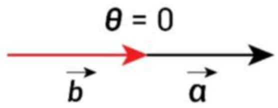


Fig. 1.71

- When $\theta = \pi$,

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\pi = -|\vec{a}||\vec{b}|$

$\vec{a}\cdot\vec{b}$ is minimum.

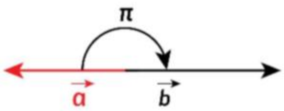


Fig. 1.72

Condition for vector a. vector b = 0

6.2 Properties of Dot Product

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{b}\cdot\vec{a} = |\vec{b}||\vec{a}|\cos\theta$

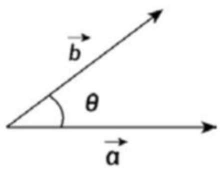


Fig. 1.73

- Dot product is commutative.

$\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}$

- Dot product is distributive over addition or subtraction.

$\vec{a}\cdot(\vec{b}\pm\vec{c}) = \vec{a}\cdot\vec{b}\pm\vec{a}\cdot\vec{c}$

- When vectors are given in component form,

$\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j})\cdot(B_x\hat{i} + B_y\hat{j})$

- We know that, $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = 0$

$\Rightarrow \vec{A}\cdot\vec{B} = A_xB_x + A_y + B_y$

- Thus for 3D, when

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

6.3 Application of Dot product in physics

Work done (W): It is defined as the scalar product of the force $(\vec{F})$, acting on the body and the Displacement $(\vec{s})$ produced.

Thus $W = \vec{F}\cdot\vec{s}$

Instantaneous power (P): It is defined as the scalar product of force $(\vec{F})$ and the instantaneous velocity $(\vec{v})$ of the body.

Thus $P = \vec{F}\cdot\vec{v}$

Magnetic flux (ϕ): The magnetic flux linked with a surface is defined as the scalar product of magnetic intensity $(\vec{B})$ and the area $(\vec{A})$ vector. Thus

$\phi = \vec{B}\cdot\vec{A}$

NOTE:

As the scalar product of two vectors is a scalar quantity, so work, power and magnetic flux are all scalar quantities

6.4 Cross Product of Two Vectors

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n} \quad 0^\circ \le \theta \le 180^\circ$

$\hat{n}$ is the unit vector in direction normal to the a and b

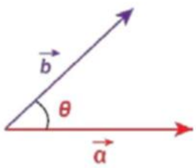


Fig. 1.74

It is also called Vector Product.

6.5 Direction of Cross Product

Right Hand Thumb Rule: Curl the fingers of the right hand in such a way that they point in the direction of rotation from vector $\vec{a}$ to $\vec{b}$ through the smaller angle, then the stretched thumb points in the direction of $\vec{a}\times\vec{b}$

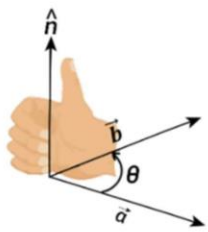


Fig. 1.75

Direction of $\vec{a} \times \vec{b}$

Direction of $\vec{b} \times \vec{a}$

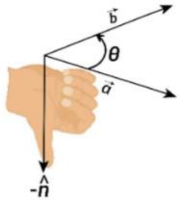


Fig. 1.76

$$\vec{b} \times \vec{a} = |\vec{b}||\vec{a}| \sin\theta\left(-\hat{n}\right)$$

6.6 Properties of Vector Product

- Vector product is anti - commutative i.e., $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- Vector product is distributive over addition i.e., $\vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
- Vector of two parallel or antiparallel vectors is a null vector. Thus

$$\vec{A} \times \vec{B} = AB \sin\left(0° \text{ or } 180°\right)\hat{n} = \vec{0}$$

- Vector product of a vector with itself is a null vector.

$$\vec{A} \times \vec{A} = AA \sin 0° \hat{n} = \vec{0}$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

- The magnitude of the vector product of two mutually perpendicular vectors is equal to the product of their magnitudes.

$$|\vec{A} \times \vec{B}| = AB \sin 90° = AB$$

- Sine of the angle between two vectors. If θ is the angle between two vectors $\vec{A}$ and $\vec{B}$, then $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

$$\sin\theta = \frac{\left|\vec{A} \times \vec{B}\right|}{\left|\vec{A}\right|\left|\vec{B}\right|}$$

- If $\hat{n}$ is a unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$, then $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$
- Vector product of orthogonal unit vectors

$$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$$

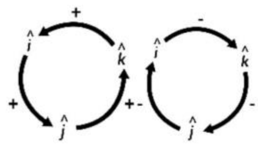


Fig. 1.77

A Force of 15 N at angle 60° from horizontal is used to push a box along the floor a distance of 3 meter. How much work was done?

6.7 Application of Scalar and Vector Products

1. We use Dot product in finding value of a vector.
2. Finding component of one vector along another vector.
3. Finding angle between two vectors.
4. Finding unit vector perpendicular to plane consisting both the vectors.
5. Finding work done.
6. Finding area of triangle and parallelogram.
7. Finding Torque.
8. Finding power.

SUMMARY

UNITS AND MEASUREMENTS

FUNDAMENTAL AND DERIVED UNITS

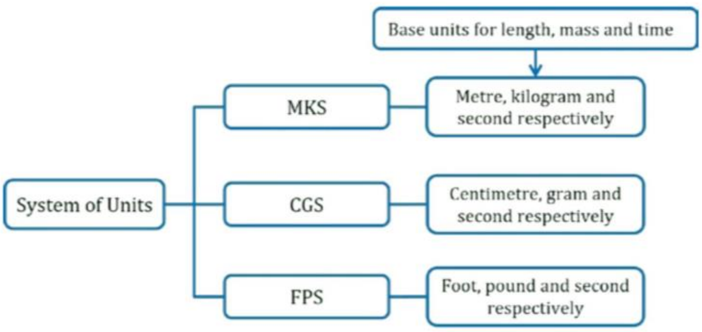
Fundamental Unit:

Quantity	Name of units	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	Cd

Supplementary Units:

Quantity	Name of units	Symbol
Plane angle	Radian	rad
Solid angle	Steradian	sr

System of Units:



DIMENSIONS AND SIGNIFICIENT FIGURES

Dimensional and Dimensional Analysis:

Dimensions of a physical quantity are the powers to which the fundamental units must be raised in order to get the unit of derived quantity.

- Dimensional analysis is a tool to find or check relations among physical quantities by using their dimensions.

By using dimensional analysis, we can

1. Convert a physical quantity from one system of unit to another.
2. Check the dimensional consistency of equations
3. Deduce relation among physical quantities.

Limitations of Dimensional Analysis

- In some cases, the constant of proportionality also possesses dimensions. In such cases, we cannot use this system.
- If one side of the equation contains addition or subtraction of physical quantities, we cannot use this method to derive the expression.

ERROR ANALYSIS

Systematic Errors

Systematic error is consistent, repeatable error associated with faulty equipment or a flawed experiment design. These errors are usually caused by measuring instruments that are incorrectly calibrated.

- These errors cause readings to be shifted one way (or the other) from the true reading.

Now, Let's learn about some common terms used during, measurements and error analysis.

Accuracy and Precision

- Accuracy is an indication of how close a measurement is to the accepted value.
- An accurate experiment has a low systematic error.
- Precision is an indication of the agreement among a number of measurements.

- A precise experiment has a low random error
- If N division of vernier scale are equal in length to (N -1) MS divisions, then:

$$L.C. = 1MS - 1VS \Rightarrow L.C. = \frac{1MS}{N}$$

- Least count of a screw gauge

$$L.C. = \frac{Pitch}{N}$$

BASIC MATHEMATICS

BASIC ALGEBRA

Quadratic Equation

A quadratic equation is an equation of second degree, meaning it contains at least one term that is squared.

The standard form of quadratic equation is

$ax^2 + bx + c = 0$ where $a \neq 0$

Discriminant of a Quadratic equation:

Discriminant of a quadratic $ax^2 + bx + c = 0$ equation is represented by D.

$$D = b^2 - 4ac$$

The roots are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial Expansion

A binomial is a polynomial with two terms.

There are a few similarities between the sine and cosine graphs they are:

- Both have the same curve which is shifted along the x-axis.
- Both have an amplitude of 1

Have a period of 360^o or 2π radians

SCALARS & VECTORS AND VECTOR OPERATIONS

Vectors

- Scalar and Vector
- Representation and Properties of Vectors
- Types of Vectors

Negative Vector:

A negative vector is a vector that has the opposite direction to the reference positive direction.

Types of Vectors

(i) Zero Vector

(ii) Unit Vector

(iii) Position Vector

(iv) Co-initial Vector

(v) Like and Unlike Vectors

(vi) Coplanar Vector

(vii) Collinear Vector

(viii) Displacement Vector

- When a vector is multiplied by a scalar quantity, then the magnitude of the vector changes in accordance with the magnitude of the scalar but the direction of the vector remains unchanged.
- A unit vector is a vector that has a magnitude of 1.
- Any vector can become a unit vector on dividing it by the vector's magnitude.
- A vector representing the straight line distance and the direction of any point or object with respect to the origin, is called position vector.

Polygon Law:

- It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector R is represented in magnitude and direction by the closing side of polygon taken in opposite order.

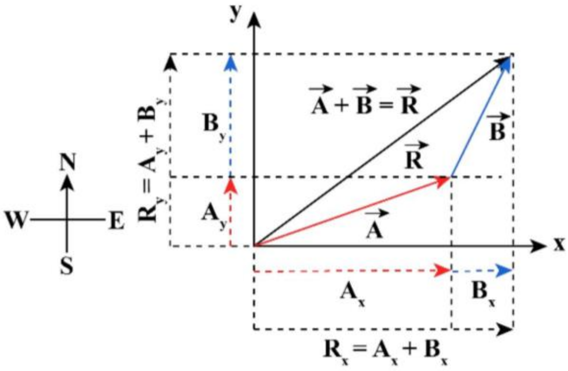
Addition of vectors:

Components to get the magnitude R of the resultant, use the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

To get the direction of the resultant;

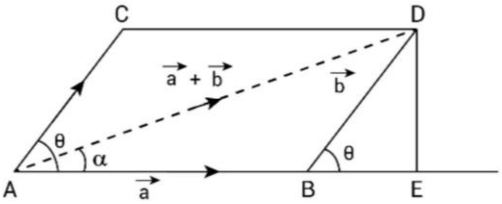
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Addition of vectors:

Law of Parallelogram of vector addition. Thus, the magnitude of

$\vec{a}+\vec{b}$ is $\sqrt{a^2+b^2+2ab\cos\theta}$



Its angle with $\vec{a}$ is α where $\tan\alpha = \dfrac{DE}{AE} = \dfrac{b\sin\theta}{a+b\cos\theta}$

Vector Subtraction:

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{b} = b_x\hat{i} + b_y\hat{j}$

$\vec{a}-\vec{b} = \vec{a}+\left(-\vec{b}\right) = a_x\hat{i}+a_y\hat{j}+\left(-b_x\hat{i}-b_y\hat{j}\right)$

$= \left(a_x-b_x\right)\hat{i}+\left(a_y-b_y\right)\hat{j}$

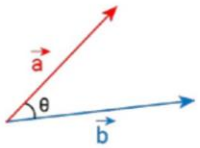
Scalar Product or Dot Product

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$0 \le \theta \le \pi$

- Dot product give us a scalar quantity.
- Angle between vectors,

  $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$



- Dot product is commutative.

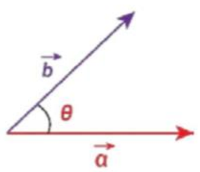
  $\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}$
- Dot product is distributive over addition or subtraction.

  $\vec{a}\cdot\left(\vec{b}\pm\vec{c}\right) = \vec{a}\cdot\vec{b}\pm\vec{a}\cdot\vec{c}$

Cross Product:

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n} \qquad 0^\circ \le \theta \le 180^\circ$

$\hat{n}$ is the unit vector in direction normal to the a and b



Properties of Cross Product:

- Vector product is anti - commutative i.e., $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- Vector product is distributive over addition i.e., $\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$
- Vector of two parallel or antiparallel vectors is a null vector. Thus

  $\vec{A}\times\vec{B} = AB\sin\left(0^\circ \text{ or } 180^\circ\right)\hat{n} = \vec{0}$
- Vector product of a vector with itself is a null vector.

  $\vec{A}\times\vec{A} = AA\sin 0^\circ\hat{n} = \vec{0}$

  $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
- $\left(\vec{a}\times\vec{b}\right) = -\left(\vec{b}\times\vec{a}\right)$
- $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
- $\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{i} = -\hat{k}$
- $\hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{j} = -\hat{i}$
- $\hat{k}\times\hat{i} = \hat{j} \qquad \hat{i}\times\hat{k} = -\hat{j}$
- If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $= b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then

  $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Applications of Vector Product

- **Torque** $\vec{\tau}$: The torque acting on a particle is equal to the vector product of its position vector $\left(\vec{r}\right)$ and force vector $\left(\vec{F}\right)$. Thus $\vec{r} = \vec{r}\times\vec{F}$
- **Angular momentum** $\vec{L}$ **:** The angular momentum of a particle is equal to the cross product of its position vector $\left(\vec{r}\right)$ and linear momentum $\left(\vec{p}\right)$. Thus $\vec{L} = \vec{r}\times\vec{p}$
- **Instantaneous velocity** $\vec{v}$ **:** The instantaneous velocity of a particle is equal to the cross product of its angular velocity $\left(\vec{\omega}\right)$ and the position vector $\left(\vec{r}\right)$. Thus $\vec{v} = \vec{\omega}\times\vec{r}$

Solved Examples

Example - 1

The unit of surface tension in SI system is

(a) Dyne / cm^2 (b) Newton/m

(c) Dyne/cm (d) Newton/m^2

Ans. (b)

Sol. From the formula of surface tension $T = \frac{F}{l}$

By substituting the S.I. units of force and length, we will get the unit of surface tension = Newton/m.

Example – 2

The unit of absolute permittivity is

(a) Farad – meter (b) Farad / meter

(c) Farad/meter^2 (d) Farad

Ans. (b)

Sol. From the formula $C = 4\pi\varepsilon_0 R \therefore \varepsilon_0 = \frac{C}{4\pi R}$

By substituting the unit of capacitance and radius: unit of ε_0 = Farad / meter.

Example – 3

If $x = at + bt^2$, where x is the dis

tance travelled by the body in kilometre while t the time in seconds, then the units of b are

(a) km/s (b) km-s

(c) km/s^2 (d) km-s^2

Ans. (c)

Sol. From the principle of dimensional homogeneity $[x] = [bt^2] \Rightarrow [b] = \left[\frac{x}{t^2}\right]$

$\therefore$ Unit of b = km/s^2

Example – 4

Which relation is wrong?

(a) 1 Calorie = 4.18 Joules

(b) 1Å $=10^{-10}$ m

(c) 1 MeV = 1.6×10^{-13} Joules

(d) 1 Newton $=10^{-5}$ Dynes

Ans. (d)

Sol. Because 1 Newton = 10^5 Dyne.

Example – 5

The equation of a wave is given by $Y = A\sin\omega\left(\frac{x}{v} - k\right)$ where ω is the angular velocity and v is the linear velocity. The dimension of k is

(a) LT (b) T

(c) T^{-1} (d) T^2

Ans. (b)

Sol. According to principle of dimensional homogeneity $[k] = \left[\frac{x}{v}\right] = \left[\frac{L}{LT^{-1}}\right] = [T]$

Example – 6

E, m, L and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\frac{El^2}{m^5G^2}$ is

(a) Angle (b) Length

(c) Mass (d) Time

Ans. (a)

Sol. [E]= energy = $[ML^2T^{-2}]$, [m] = mass

= [M], [l] = Angular momentum

= $[ML^2T^{-1}]$

[G] = Gravitational constant

= $[M^{-1}L^3T^{-2}]$

Now substituting dimensions of given expression

$$\left[\frac{El^2}{m^5G^2}\right] = \frac{[ML^2T^{-2}] \times [ML^2T^{-1}]^2}{[M^5] \times [M^{-1}L^3T^{-2}]^2}$$

$= [M^0L^0T^0]$

The given expression is dimensionless. As angle is also dimensionless the answer is (a).

Example – 7

Each edge of a cube is measured to be 7.203 m. The volume of the cube up to appropriate significant figures is

(a) 373.714 (b) 373.71

(c) 373.7 (d) 373

Ans. (c)

Sol. Volume $= a^3 = (7.023)^3 = 373.715 m^3$

In significant figures volume of cube will be $373.7 m^3$ because its edge has four significant figures.

Example – 8

Each edge of a cube is measured to be 5.402 cm. The total surface area and the volume of the cube in appropriate significant figures are:

(a) $175.1 cm^2, 157 cm^2$

(b) $175.1 cm^2, 157.6 cm^3$

(c) $175 cm^2, 157 cm^2$

(d) $175.08 cm^2, 157.639 cm^3$

Ans. (b)

Sol. Total surface area

$= 6\times(5.402)^2 = 175.09 cm^2 = 175.1 cm^2$

(Upto correct number of significant figure)

Total volume

$= (5.402)^3 = 175.64 cm^3 = 175.6 cm^3$

(Upto correct number of significant figure).

Example – 9

The resistance $R = \frac{V}{i}$ where V= 100 ± 5 volts and i = 10 ± 0.2 amperes. What is the total error in R

(a) 5% (b) 7%

(c) 5.2% (d) $\frac{5}{2}$%

Ans. (b)

Sol. $R = \frac{V}{I} \therefore \left(\frac{\Delta R}{R}\times 100\right)_{max} = \frac{\Delta V}{V}\times 100 + \frac{\Delta I}{I}\times 100$

$= \frac{5}{100}\times 100 + \frac{0.2}{10}\times 100 = (5+2)\% = 7\%$

Example – 10

The SI unit of universal gas constant (R) is

(a) Watt K^{-1} mol^{-1} (b) Newton K^{-1} mol^{-1}

(c) Joule K^{-1} mol^{-1} (d) Erg K^{-1} mol^{-1}

Ans. (c)

Sol. Ideal gas equation $PV = nRT$

$$\therefore [R] = \frac{[P][V]}{[nT]} = \frac{[ML^{-1}T^{-2}][L^3]}{[mole][K]} = \frac{[ML^2T^{-2}]}{[mole]\times[K]}$$

So the unit will be Joule K^{-1} mol^{-1}.

Example – 11

The equation $\left(P + \frac{a}{V^2}\right)(V - b) = \text{constant}$. The units of a is

(a) Dyne × cm^5 (b) Dyne × cm^4

(c) Dyne × cm^3 (d) Dyne × cm^2

Ans. (b)

Sol. According to the principle of dimensional homogeneity

$[P] = \left[\frac{a}{V^2}\right]$

$\Rightarrow [a] = [P][V^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$

or unit of $a = gm \times cm^5 \times \sec^{-2} = Dyne \times cm^4$

Example – 12

Suppose we employ a system in which the unit of mass equals 100 kg, the unit of length equals 1 km and the unit of time 100 s and call the unit of energy eluoj (joule written in reverse order), then

(a) 1 eluoj = 10^4 joule (b) 1 eluoj = 10^{-3} joule

(c) 1 eluoj = 10^{-4} joul (d) 1 joule = 10^3 eluoj

Ans. (a)

Sol. $[E] = [ML^2T^{-2}]$

$1 eluoj = [100kg]\times[1km]^2\times[100\sec]^{-2}$

$= 100kg \times 10^6 m^2 \times 10^{-4} \sec^{-2}$

$= 10^4 kgm^2 \times \sec^{-2} = 10^4 Joule$

Example – 13

$X = 3YZ^2$ find dimension of Y in (MKSA) system, if X and Z are the dimension of capacitance and magnetic field respectively

(a) $M^{-3}L^{-2}T^{-4}A^{-1}$ (b) ML^{-2}

(c) $M^{-3}L^{-2}T^{4}A^{4}$ (d) $M^{-3}L^{-2}T^{8}A^{4}$

Ans. (d)

Sol. $X = 3YZ^2$

$$\therefore [Y] = \frac{[X]}{[Z^2]} = \frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$$

$$= [M^{-3}L^{-2}T^8A^4]$$

Example – 14

Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are

(a) $[LT^{-1}]$ (b) $[L^{-1}T]$

(c) $[L^{-2}T^2]$ (d) $[L^2T^{-2}]$

Ans. (d)

Sol. We know that velocity of light

$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \therefore \frac{1}{\mu_0\varepsilon_0} = C^2$$

$$\therefore \text{So}\left[\frac{1}{\mu_0\varepsilon_0}\right] = [LT^{-1}]^2 = [L^2T^{-2}]$$

Example – 15

If the value of resistance is 10.845 ohms and the value of current is 3.23 amperes, the potential difference is 35.02935 volts. Its value in significant number would be

(a) 35 V (b) 35.0 V

(c) 35.03 V (d) 35.025 V

Ans. (b)

Sol. Value of current (3.23 A) has minimum significant figure (3) so the value of potential difference

V(= IR) has only 3 significant figure. Hence its value be 35.0 V.

Example – 16

A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = b^\alpha c^\beta / d^\gamma e^\delta$. If the maximum errors in the measurement of b, c, d and e are $b_1\%, c_1\%, d_1\%$ and $e_1\%$, then the maximum error in the value of a determined by the experiment is

(a) $(b_1 + c_1 + d_1 + e_1)\%$

(b) $(b_1 + c_1 - d_1 - e_1)\%$

(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)\%$

(d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$

Ans. (d)

Sol. $a = b^\alpha c^\beta / d^\gamma e^\delta$

So maximum error in a is given by $\left(\frac{\Delta a}{a} \times 100\right)_{\max}$

$$= \alpha \cdot \frac{\Delta b}{b} \times 100 + \beta \cdot \frac{\Delta c}{c} \times 100 + \gamma . \frac{\Delta d}{d} \times 100 + \delta \cdot \frac{\Delta e}{e} \times 100$$

$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

Example – 17

Unit of Stefan's constant is

(a) Js^{-1} (b) $Jm^{-2}s^{-1}K^{-4}$

(c) Jm^{-2} (d) Js

Ans. (b)

Sol. Stefan's formula $\frac{Q}{At} = \sigma T^4 \therefore \sigma = \frac{Q}{AtT^4}$

$\therefore$ unit of $\sigma = \frac{Joule}{m^2 \times \sec \times K^4} = Jm^{-2}s^{-1}K^{-4}$

Example – 18

In C.G.S. system the magnitude of the force is 100 dynes. In another system where the fundamental physical quantities are kilogram, metre and minute, the magnitude of the force is

(a) 0.036 (b) 0.36

(c) 3.6 (d) 36

Ans. (c)

Sol. $n_1 = 100$, M_1 = g, L_1 = cm, T_1 = sec and M_2 = kg,

L_2 = meter, T_2 = minute, x = 1, y = 1, z = - 2

By substituting these values in the following conversion formula

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

$$n_2 = 100\left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^1\left[\frac{\sec}{\text{minute}}\right]^{-2}$$

$$n_2 = 100\left[\frac{gm}{10^3\,gm}\right]^1\left[\frac{cm}{10^2\,cm}\right]^1\left[\frac{\sec}{60\sec}\right]^{-2}$$

$$= 3.6kg - metre/s^2$$

Example – 19

Conversion of 1 MW power on a new system having basic units of mass, length and time as 10kg, 1dm and 1 minute respectively is

(a) $2.16\times10^{12}\,unit$ (b) $1.26\times10^{12}\,unit$

(c) $2.16\times10^{10}\,unit$ (d) $2\times10^{14}\,unit$

Ans. (a)

Sol. $[P] = [ML^2T^{-3}]$

Using the relation

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

$$= 1\times10^6\left[\frac{1kg}{10kg}\right]^1\left[\frac{1m}{1dm}\right]^2\left[\frac{1s}{1\min}\right]^{-3}$$

[As 1 MW $=10^6$ W]

$$= 10^6\left[\frac{1kg}{10kg}\right]\left[\frac{10dm}{1dm}\right]^2\left[\frac{1\sec}{60\sec}\right]^{-3}$$

$$= 2.16\times10^{12}\text{ unit}$$

Example – 20

The equation of the stationary wave is $y = 2a\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$, which of the following statements is wrong

(a) The unit of ct is same as that of λ

(b) The unit of x is same as that of λ

(c) The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$

(d) The unit of c/λ is same as that of x/λ

Ans. (d)

Sol. Here, $\frac{2\pi ct}{\lambda}$ as well as $\frac{2\pi x}{\lambda}$ are dimensionless (angle)

i.e., $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$

So (i) unit of c t is same as that of λ

(ii) unit of x is same as that of λ

(iii) $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right]$ and (iv) $\frac{x}{\lambda}$ is unit less. It is not the case with $\frac{c}{\lambda}$.

Example – 21

The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x^2+B}$, where A and B are dimensional constants then dimensional formula for AB is

(a) $ML^{7/2}T^{-2}$ (b) $ML^{11/2}T^{-2}$

(c) $M^2L^{9/2}T^{-2}$ (d) $ML^{13/2}T^{-3}$

Ans. (b)

Sol. From the dimensional homogeneity

$[x^2] = [B] \therefore [B] = [L^2]$

As well as

$$[U] = \frac{[A][x^{1/2}]}{[x^2]+[B]} \Rightarrow [ML^2T^{-2}] = \frac{[A][L^{1/2}]}{[L^2]}$$

$$\therefore [A] = [ML^{7/2}T^{-2}]$$

Now $[AB] = [ML^{7/2}T^{-2}]\times[L^2] = [ML^{11/2}T^{-2}]$

Example – 22

If L, C and R denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is

(a) $[ML^{-2}T^{-1}I^0]$ (b) $[M^0L^0T^3I^0]$

(c) $[M^{-1}L^{-2}T^6I^2]$ (d) $[M^0L^0T^2I^0]$

Ans. (b)

Sol. $[C^2LR] = \left[C^2L^2\frac{R}{L}\right] = \left[(LC)^2\left(\frac{R}{L}\right)\right]$

and we know that frequency of LC circuits is given by $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

i.e., the dimension of LC is equal to $\left[T^2\right]$ and $\left[\frac{L}{R}\right]$ gives the time constant of L□R circuit so the dimension of $\frac{L}{R}$ is equal to [T].

By substituting the above dimensions in the given formula $\left[(LC)^2\left(\frac{R}{L}\right)\right]=\left[T^2\right]^2\left[T^{-1}\right]=\left[T^3\right]$

Example – 23

If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be

(a) $V^{-2}F^0E$ (b) V^0FE^2

(c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

Ans. (d)

Sol. Let $M=V^aF^bE^c$

Putting dimensions of each quantities in both side

$[M]=\left[LT^{-1}\right]^a\left[MLT^{-2}\right]^b\left[ML^2T^{-2}\right]^c$

Equating powers of dimensions. We have $b+c=1, a+b+2c=0$ and

$-a-2b-2c=0$

Solving these equations, $a=-2, b=0$ and c = 1

So $M=\left[V^{-2}F^0E\right]$

Example – 24

If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is

(a) 25% (b) 50%

(c) 100% (d) 125%

Ans. (c)

Sol. Kinetic energy $E=\frac{1}{2}mv^2$

$\therefore \frac{\Delta E}{E}\times 100=\left(\frac{\Delta m}{m}+\frac{2\Delta v}{v}\right)\times 100$

Here, $\Delta m=0$ and $\frac{\Delta v}{v}\times 100=50\%$

$\therefore \frac{\Delta E}{E}\times 100=2\times 50=100\%$

Example – 25

The length of a cylinder is measured with a meter rod having least count 0.1 cm. Its diameter is measured with vernier callipers having least count 0.01 cm. Given that length is 5.0 cm. and radius is 2.0 cm. The percentage error in the calculated value of the volume will be

(a) 1% (b) 2%

(c) 3% (d) 4%

Ans. (c)

Sol. Volume of cylinder $V=\pi r^2 l$

Percentage error in volume

$\frac{\Delta V}{V}\times 100=\frac{2\Delta r}{r}\times 100+\frac{\Delta l}{l}\times 100$

$=\left(2\times\frac{0.01}{2.0}\times 100+\frac{0.1}{5.0}\times 100\right)$

$=(1+2)\%=3\%$

Example – 26

If the Vectors $\vec{P}=a\hat{i}+a\hat{j}+3\hat{k}$ and $\vec{Q}=a\hat{i}-2\hat{j}-\hat{k}$ are perpendicular to each other. Find the value of a?

Sol. If vectors $\vec{P}$ and $\vec{Q}$ are perpendicular

$\Rightarrow \vec{P}.\vec{Q}=0$

$\Rightarrow a^2-2a-3=0$

$\Rightarrow a(a-3)+1(a-3)=0$

$\Rightarrow \left(a\hat{i}+a\hat{j}+3\hat{k}\right).\left(a\hat{i}-2\hat{j}-\hat{k}\right)=0$

$\Rightarrow a^2-3a+a-3=0$

$\Rightarrow a=-1,3$

Example – 27

Find the component of $3\hat{i}+4\hat{j}$ along $\hat{i}+\hat{j}$?

Sol. Component of $\vec{A}$ along $\vec{B}$ is given by $\frac{\vec{A}.\vec{B}}{B}$ hence required component

$=\frac{\left(3\hat{i}+4\hat{j}\right).\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}=\frac{7}{\sqrt{2}}$

Example – 28

Find angle between $\vec{A}=3\hat{i}+4\hat{j}$ and $\vec{B}=12\hat{i}+5\hat{j}$?

Sol. We have $\cos\theta=\frac{\vec{A}.\vec{B}}{AB}=\frac{(3\hat{i}+4\hat{j}).(12\hat{i}+5\hat{j})}{\sqrt{3^2+4^2}\sqrt{12^2+5^2}}$

$\cos\theta=\frac{36+20}{5\times13}=\frac{56}{65}$ $\qquad\theta=\cos^{-1}\frac{56}{65}$

Example – 29

Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ. Find a unit vector which is perpendicular to both $\vec{A}$ and $\vec{B}$

Sol. $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}\quad\Rightarrow\hat{n}=\frac{\vec{A}\times\vec{B}}{AB\sin\theta}$

Here $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$

Example – 30

A particle under constant force $\hat{i}+\hat{j}-2\hat{k}$ gets displaced from point A(2, -1, 3) to B(4, 3, 2). Find the work done by the force

Sol. Force $=\hat{i}+\hat{j}-2\hat{k}$

Displacement

$=d=\overrightarrow{AB}=(4\hat{i}+3\hat{j}+2\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})=(2\hat{i}+4\hat{j}-\hat{k})$

Work done $=F.d=(\hat{i}+\hat{j}-2\hat{k}).(2\hat{i}+4\hat{j}-\hat{k})$

$=1\times2+1\times4+(-2)\times(-1)=2+4+2$

= 8 units

Example – 31

The vector from origin to the points A and B are $\vec{a}=3\hat{i}-6\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$ respectively. Find the area of

(i) the triangle OAB

(ii) The parallelogram formed by OA and OB as adjacent sides.

Sol. Given $OA=\vec{a}=3\hat{i}-6\hat{j}+2\hat{k}$ and

$OB=\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$

$$\therefore\ (\vec{a}\times\vec{b})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-6&2\\2&1&-2\end{vmatrix}$$

$=(12-2)\hat{i}-(-6-4)\hat{j}+(3+12)\hat{k}$

$=10\hat{i}+10\hat{j}+15\hat{k}$

$\Rightarrow|\vec{a}\times\vec{b}|=\sqrt{10^2+10^2+15^2}=\sqrt{425}=5\sqrt{17}$

(i) Area of $\Delta OAB=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{5\sqrt{17}}{2}$ sq.units

(ii) Area of parallelogram formed by OA and OB as adjacent sides $=|\vec{a}\times\vec{b}|=5\sqrt{17}$ sq.units

Example – 32

The torque of a force $\vec{F}=-3\hat{i}+\hat{j}+5\hat{k}$ acting at the point $\vec{r}=7\hat{i}+3\hat{j}+\hat{k}$ is

(a) $14\hat{i}-38\hat{j}+16\hat{k}$ (b) $4\hat{i}+4\hat{j}+6\hat{k}$

(c) $-21\hat{i}+4\hat{j}+4\hat{k}$ (d) $-14\hat{i}+34\hat{j}-16\hat{k}$

Ans. (a)

Sol. The torque is defined as $\vec{\tau}=\vec{r}\times\vec{F}$

$$\vec{r}\times\vec{F}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\7&3&1\\-3&1&5\end{vmatrix}$$

$=\hat{i}\begin{vmatrix}3&1\\1&5\end{vmatrix}+\hat{j}\begin{vmatrix}1&7\\5&-3\end{vmatrix}+\hat{k}\begin{vmatrix}7&3\\-3&1\end{vmatrix}$

$=\hat{i}(15-1)+\hat{j}(-3-35)+\hat{k}(7-(-9))$

$=14\hat{i}-38\hat{j}+16\hat{k}$

Thus the answer is (a)

Example – 33

A force $\vec{F}=(3\hat{i}+2\hat{j}+\hat{k})N$ acts on a particle. As a result the particle move with a constant velocity $\vec{v}=(2\hat{i}+\hat{j})$m/s. The power applied by the force is

(a) 4W (b) 6W

(c) 8W (d) 16 W

Ans. (c)

Sol. Power $P=\vec{F}.\vec{V}$

$=(3\hat{i}+2\hat{j}+\hat{k}).(2\hat{i}+\hat{j})$

$=(3\times2+2\times1+1\times0)$

= 8 N-m/s = 8 W

Hence correct answer is (c).

Example – 34

Show that the vectors

$a = 3\hat{i} - 2\hat{j} + \hat{k},\ b = \hat{i} - 3\hat{j} + 5\hat{k},\ c = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right triangle.

Sol. We have

$\vec{b} + \vec{c} = \left(\hat{i} - 3\hat{j} + 5\hat{k}\right) + \left(2\hat{i} + \hat{j} - 4\hat{k}\right) = 3\hat{i} - 2\hat{j} + \hat{k} = a$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar

Hence no two of these vectors are parallel, therefore, 'the given vectors form a triangle.

$\vec{a}.\vec{c} = \left(3\hat{i} - 2\hat{j} + \hat{k}\right).\left(2\hat{i} + \hat{j} - 4\hat{k}\right)$

$= 3 \times 2 - 2 \times 1 - 4 \times 1 = 0$

Hence the given vectors form a right angled triangle.

Example – 35

If $A = 3\hat{i} + 4\hat{j}$ and $B = 7\hat{i} + 24\hat{j}$, the vector having the same magnitude as B and parallel to A is

(a) $5\hat{i} + 20\hat{j}$
(b) $15\hat{i} + 10\hat{j}$
(c) $20\hat{i} + 15\hat{j}$
(d) $15\hat{i} + 20\hat{j}$

Ans. (d)

Sol. $|B| = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$

Unit vector in the direction of A will be $\hat{A} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

So required vector $= 25\left(\dfrac{3\hat{i} + 4\hat{j}}{5}\right) = 15\hat{i} + 20\hat{j}$