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UNIT:I ELECTROSTATICS

POTENTIAL AND CAPACITANCE

02

CONTACT US:

+91-9939586130

+91-9955930311

www.aepstudycircle.com

aepstudycircle@gmail.com

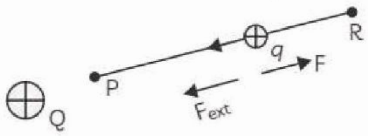
2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

ELECTROSTATIC POTENTIAL 1

TOPIC 1

WHAT IS ELECTROSTATIC POTENTIAL?

Have you ever seen a brilliant flash of lightning; ever wondered why birds sit on the high tension wires but don't get electrocuted; ever saw the white cylinder attached to the fan in your home and wondered what it does? If the answer to all these is yes then the answer to your questions lie in this chapter. It is all about how work is done in electrostatics and how the work done is stored in the form of potential energy.



Potential Due to a Charge

Consider a positive test charge q which is being brought from a point R to P inside an electric field created by the charge Q. Since both charges are positive hence a repulsive force $F = qE$ is being applied on the test charge by the main charge Q. To overcome this repulsion an external force F_{ext} is required which is governed by the equation,

$$F = -F_{ext}$$

Hence a certain amount of work is being done in moving the test charge as given by,

$$W_{RP} = \int_R^P F.dr = -\int_R^P F_{ext}.dr$$

From this relation we describe the potential energy as;

$$\Delta U = U_P - U_R = W_{RP}$$

Now if we consider that the test charge has been brought from infinity to the point P then we get electric potential as;

$$U = U_P - U_\infty$$

Hence, electrical potential energy is the work done by the external force to bring a unit positive charge from infinity to a point inside the electric field.

From the above discussion we may define electrostatic potential as;

$$V_P - V_R = \frac{U_P - U_R}{q}$$

where, V_P and V_R are the Electric Potential at P and R and defined as;

The work done per unit charge in bringing the charge from infinity to a point inside the electric field.

TOPIC 2

POTENTIAL DUE TO A POINT CHARGE

Electrostatic potential can be defined as work done to bring a unit positive charge from infinity to a point inside the electric field.

Hence we assume, $q = +1$ C. This transforms our equation to,

$$W_{RP} = \int_R^P F.dr$$

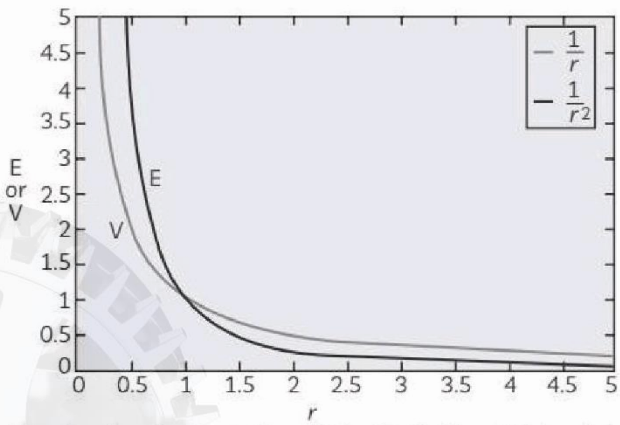
$$= -\int_R^P F_{ext}.dr$$

$$= -\int_\infty^r \frac{Qq}{4\pi\varepsilon_0 R^2}dr = \frac{Qq}{4\pi\varepsilon_0 r}$$

which transforms as;

$$V_r = \frac{Q}{4\pi\varepsilon_0 r}$$

which gives us the mathematical description of Electrostatic Potential of a charge Q at a distance r from it.



Graphs showing variation of Electric field and Potential due to a point charge as a function of distance.

Caution

➥ *Students should confirm that electrostatic field varies as $\frac{1}{r^2}$ while electrostatic potential varies as $\frac{1}{r}$. While attempting problems, both conceptual and numerical, one must stay alert regarding this condition.*

Example 1.1: Calculate the potential at a point P due to a charge of 2 × 10^{-7} C located 90 cm away. Also obtain the work done in bringing a charge of 2 × 10^{-9} C from infinity to the point P.

Ans. Given: Q = 2 × 10^{-7}C and r = 0.9 m.

Hence, $$V_p = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$= 9 \times 10^9 \times \frac{2\times10^{-7}}{0.9}$$

$$= 2 \times 10^3 \text{ V}$$

The work done is given by, $W_p = qV_p$.

Since, $q = 2 \times 10^{-7}$ C

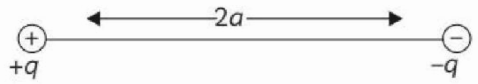
Hence, $W_p = 2 \times 10^3 \times 2 \times 10^{-7}$

$$= 4 \times 10^{-4} \text{ J}$$

TOPIC 3

POTENTIAL DUE TO AN ELECTRIC DIPOLE

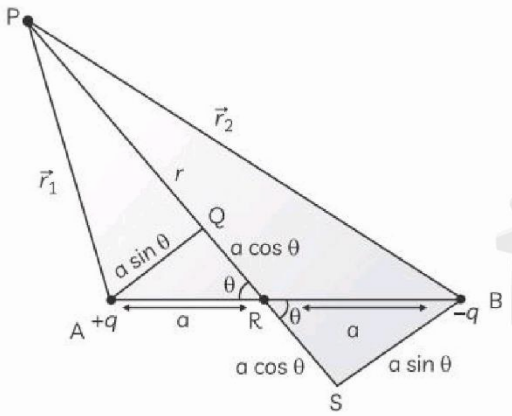
Two charges of equal magnitude but opposite signs separated by a distance "2a" forms an electric dipole. The dipole moment of this dipole is given by



$$\vec{p} = 2qa$$

The direction of the dipole moment is from negative to the positive charge and its unit is Coulomb-metre(C-m).

Now let us see how the electrostatic potential of a dipole looks like.



Potential due to a dipole

Let us assume an arbitrary point P at a distance r_1 from $+q$ and r_2 from $-q$ of the dipole. We assume a distance of r from the point P to the center of the dipole. Then we extend the line joining P with the center and extend the lines from $+q$ and $-q$ as shown. Let us calculate potential for $+q$ at P and then we can generalise for the charge $-q$.

Potential at P due to the dipole is given as

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1}-\frac{q}{r_2}\right)$$

$$r_1^2 = (r - a\cos\theta)^2 + (a\sin\theta)^2$$

$$\Rightarrow \quad r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

$$\Rightarrow \quad r_1 = r\left(1+\frac{a^2}{r^2}-\frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{1}{r_1} = \frac{1}{r}\left(1+\frac{a^2}{r^2}-\frac{2a\cos\theta}{r}\right)^{-\frac{1}{2}}$$

Applying binomial expansion and assuming $a << r$, we get,

$$\frac{1}{r_1} = \frac{1}{r}\left(1+\frac{a\cos\theta}{r}\right)$$

Applying similar steps to calculate for r_2, we get,

$$\frac{1}{r_2} = \frac{1}{r}\left(1-\frac{a\cos\theta}{r}\right)$$

Substituting the two equations we get,

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1}-\frac{q}{r_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(1+\frac{a\cos\theta}{r}-1+\frac{a\cos\theta}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{2a\cos\theta}{r}\right)$$

$$V_r = \frac{2qa\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad [\because \vec{p} = 2qa]$$

$$V_r = \frac{\vec{p}.\hat{r}}{4\pi\varepsilon_0 r^2}$$

Hence, the electric potential due to a dipole falls off as $\frac{1}{r^2}$. We can now generalise the equation into a few cases.

On the axis of a dipole, θ = 0°

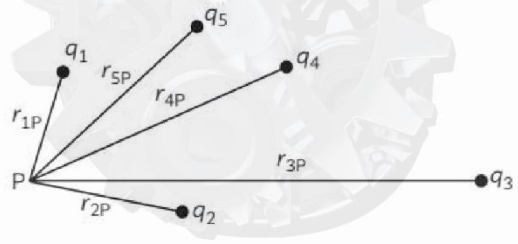
$$V_r = \pm\frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \pm\frac{p}{4\pi\varepsilon_0 r^2}$$

On the equatorial plane, the V_r = 0, since the plane makes an angle 90° with the dipole which leads to cos θ = cos 90° = 0 .

TOPIC 4

POTENTIAL DUE TO A SYSTEM OF CHARGES

As can be seen, potential is a scalar quantity. Hence, if we have to find out the potential at a point due to multiple charges then all we need to do is to do a numerical addition of all individual potentials due to the individual charges.



Potential due to Multiple Charges

Potential at any point P is given as,

$$V_P = V_1 + V_2 + V_3 ... + V_n$$

$$= \sum_{i=1}^{n} V_i$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} .... + \frac{q_n}{r_{nP}}\right)$$

Important

➥ *For a uniformly charged spherical shell of radius r the potential outside the shell at a distance R from the centre is given by; $V_R = \frac{q}{4\pi\varepsilon_0 R}$, whereas for a point inside the sphere is given by; $V_r = \frac{q}{4\pi\varepsilon_0 r}$, where R >> r. The potential inside is constant because the field inside is zero which means no work is done to move a charge inside the shell.*

Example 1.2: N small conducting liquid droplets, each of radius *r*, are charged to a potential V each. These droplets coalesce to form a single large drop. Find the potential of the large drop.

Ans. Let us assume q as the charge on the smaller drop. Then,

$$V_0 = k\frac{q}{r} \text{ where, } k = \frac{1}{4\pi\varepsilon_0}.$$

The volume of each sphere is $v = \frac{4}{3}\pi r^3$.

When N drops coalesce the total volume becomes N times the original. When compared with the radius R of the bigger sphere, we get,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}N\pi r^3$$

$$R = N^{\frac{1}{3}}r$$

The total charge on the bigger sphere also becomes, Q = Nq

This gives us the potential of the bigger sphere as;

$$V_1 = \frac{kQ}{R} = k\frac{Nq}{N^{\frac{1}{3}}r} = k\frac{N^{\frac{2}{3}}q}{r} = N^{\frac{2}{3}}V_0$$

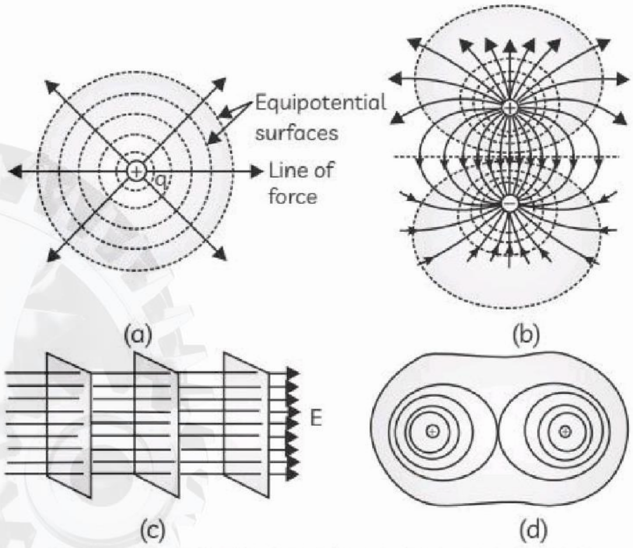
TOPIC 5

EQUIPOTENTIAL SURFACES

In case of a dipole at the equatorial region the potential is zero. This implies that if a charged particle is taken from one point to another along the equatorial line no work will be done since the potential at every point on the line is zero which further implies that the potential difference on the plane between any two points will also be zero. Hence, you may say it is an equipotential surface.

Equipotential surfaces are those surfaces where the potential is equal throughout or the potential difference is zero.

For example, if you consider a point charge then at any distance r from the charge a spherical surface can be assumed. That spherical surface will be equipotential in nature.



Various Equipotential Surfaces: (a) Due to a Point Charge, (b) Due to a Dipole, (c) Due to a Plane Sheet of Charge, (d) Due to Two Similar Charges

Relation Between Field and Potential

$$W_{RP} = \int_R^P F.dr = -\int_R^P F_{ext}.dr$$

which means work done is negative when an external force is acting on the charge. This can be re-written as;

$$W = \int_R^P qE.dr$$

This can be then written as;

$$\frac{W}{q} = V = -\int E.dr$$

which becomes, $dV = -E.dr$

Hence, $E = -\frac{dV}{dr}$.

The above equation can be writter as,

$$E = -\nabla V$$

where, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

This means that the electric field at a point is the negative gradient of the potential at that point.

⚠ Caution

➥ *Student must know that electric field is in the direction in which the potential decreases steepest and that its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.*

Example 1.3: The electric potential at any point is given by the equation V = $4x^2$. What is the electric field at the point (1, 2)?

Ans. We know that $\vec{E} = -\frac{dV}{dr}$ which can be further written as, $\vec{E} = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$.

If we replace the value of V here we will get,

$$\vec{E} = -\frac{\partial(4x^2)}{\partial x}\hat{i}$$

Since the y and z axis values will become zero.

Hence, $\vec{E} = -8x\hat{i}$ in which we will replace the value of $x = 1$ and we will get, $\vec{E} = -8\hat{i}$ N/C.

Hence, electric field at the point (1, 2) is $-8\hat{i}$ N/C.

Example 1.4: "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify.

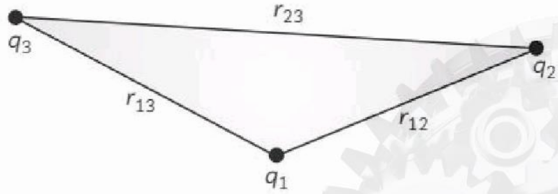
Ans. For any given surface, the surface should be normal to the field because if that were not true then there would be a non-zero component of potential along the surface, which will lead to the movement of charges on the surface, thereby generating a current on the surface with no external source, leading to the violation of law of conservation of energy.

TOPIC 6

POTENTIAL ENERGY OF A SYSTEM OF CHARGES

Let us see what happens if we bring a few charges from infinity

Initially, when q_1 is brought from infinity to its original place there was no external charge present there and hence no work was done and no potential energy was gained. Hence, $U_1 = 0$



Multiple Charge Configuration

Now when q_2 is brought in and kept at a distance of r_{12} from q_1 then the potential energy gained is given by $U_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

Now, if we bring in q_3 then some work will be done:

Hence, $U_3 = q_3\left(\frac{q_1}{4\pi\varepsilon_0 r_{13}} + \frac{q_2}{4\pi\varepsilon_0 r_{23}}\right)$

$$= \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$$

Hence, if we need to find the total work done, the potential energy of the system then,

$$U = U_1 + U_2 + U_3$$

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Hence, the net work done in assembling a system of charged particle completely depends on the charge configuration and not on the path taken or order of the charged particles accumulated.

Example 1.5: Calculate the amount of work done to dissociate a system of three charges 1 μC, 1 μC, -4 μC placed on the vertices of an equilateral triangle of side length 10 cm.

Ans. The amount of work needed to dissociate the three charges is the same as required to put the three charges together in the first place.

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= 9 \times \frac{10^9}{0.1} \times [(1\ \mu C \times 1\ \mu C)$$

$$- (1\ \mu C \times 4\ \mu C) - (1\ \mu C \times 4\ \mu C)]$$

$$= 9 \times 10^{10} \times 10^{-12} \times (1 - 4 - 4)$$

$$= -63 \times 10^{-2}\ J$$

Hence, the work done is -63×10^{-2} Joules.

TOPIC 7

POTENTIAL ENERGY IN AN EXTERNAL FIELD

Single Charge

If a charged particle q is brought inside an electric field E from infinity some amount of work will have to be done. This work done is stored in the charge in the form of potential energy.

$$W_{RP} = \int_R^P F.dr = -\int_R^P F_{ext}.dr$$

which leads to,

$$\Delta U = U_P - U_R = W_{RP}$$

So, if $R = \infty$ then,

$$\Delta U = U_P = W$$

Hence, in an external field E a charge q will have a potential energy;

$$U = W = -\int_R^P F_{ext}.dr$$

$$U = -q\int E.dr = qV(r)$$

Hence, the potential energy of a single charge q in an external field E is given by $qV(r)$.

System of Charges

We will again calculate the potential energy for bringing together the three charges but this time we will do so in the presence of an external field E.

Initially when q_1 is brought from infinity to its original place in the presence of no external charge but in the presence of an external field. Hence, work was done and potential energy was gained and hence $U_1 = q_1V(r_1)$.

Now when q_2 is brought in and kept at a distance of r_{12} from q_1 then the potential energy gained is given by, $U_{21} = \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$ due to the first charge q_1 and

$$U_{22} = q_2\,V(r_2).$$

Hence, the total work done to bring in q_2 is,

$$U_2 = q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}.$$

Now, if we bring in q_3 then some work will have to be done due to the combined presence of the previous two charges and due to the external field.

Hence,
$$U_{31} = q_3\left(\frac{q_1}{4\pi\varepsilon_0 r_{13}} + \frac{q_2}{4\pi\varepsilon_0 r_{23}}\right)$$

$$U_{31} = \frac{q_1q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2q_3}{4\pi\varepsilon_0 r_{23}}$$

due to the presence of the first two charges and $q_3V(r_3)$ due to the external field. Hence, the total potential energy for the particle q_3 is given by

$$U_3 = \frac{q_1q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2q_3}{4\pi\varepsilon_0 r_{23}} + q_3V(r_3).$$

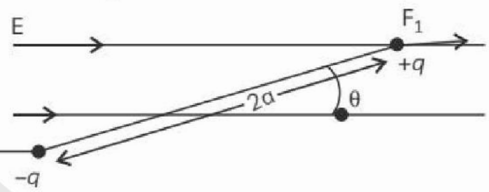
Hence, if we now need to find the total work done and hence the potential energy of the system then

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) + \Sigma_i q_iV(r_i)$$

So, if multiple charges are to be brought together to form a system, the total energy will be due to the external field as well as due to the charge configuration.

Dipole

Till now we have seen how work is done for a single charge and a charge configuration in an external field. Now let us assume there is a uniform electric field in which a dipole has been kept making a certain angle as shown in the figure.



Dipole in an Electric Field.

As can be seen in the figure, the dipole makes an angle θ with the electric field. The work done will then be given by,

$$W = \int F.dr$$

But as can be seen in a uniform field the net force on the charges will be zero as the electric field is uniform and the charges are equal and opposite. Hence, net linear force will be zero and there will be no net translational motion.

Having said that it also needs to be noted that they make an angle which leads to a torque on the system

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = 2a \times q\vec{E} = \vec{p} \times \vec{E}$$

$$\tau = pE \sin\theta. \qquad [\because \vec{p} = 2a.q]$$

Now we can redefine work as;

$$W = \int \tau d\theta$$

$$W = \int_{\theta_0}^{\theta_1} pE\sin\theta d\theta$$

$$W = pE\cos(\theta_0 - \theta_1)$$

Hence, the work done to turn a dipole from an angle θ_0 to θ_1 in a uniform external field is given above. This work is stored in the form of potential energy of the dipole. If we assume that a dipole is taken from 90° to 0° inside a electric field then,

$$U(\theta) = pE\left(\cos\frac{\pi}{2} - \cos\theta\right)$$

$$= -pE\cos\theta = -\vec{p}.\vec{E}$$

Important

➥ ***Stable and Unstable configurations:***

In case of dipoles in an external field the dipoles can have two extreme equilibrium condition-Stable and Unstable. When the dipole is placed such that the angle between the dipole moment and the field is 0° dipole is in stable equilibrium. In case of the angle being 180° then dipole is said to be in the an unstable equilibrium and the slightest of perturbation can immediately make the dipole rotate on its axis to achieve the stable equilibrium condition.

Example 1.6:

Assertion (A): A dipole in a non-uniform electric field will move in the direction of the field.

Reason (R): The net force on the dipole will be zero.

(a) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If both Assertion and Reason are false.

Ans. *(c) If Assertion is true but Reason is false.*

Explanation: When kept in a non-uniform electric field the forces acting on the two charges are not equal and opposite and hence will force the dipole to move in the direction of the electric field. Hence the assertion is true but the reason is false.

Example 1.7: What is the net work done to turn a dipole of dipole moment 10^{-6} Cm to turn from an angle of 30° to stable configuration when kept in an electric field of 10 N/m?

Ans. Given: $p = 10^{-6}$ Cm, E = 10 N/m.

$$\theta_0 = 30^\circ$$

$$\theta_1 = 0^\circ$$

$$\therefore \quad W = pE(\cos\theta_0 - \cos\theta_1)$$

$$W = 10^{-6} \times 10[\cos 30^\circ - \cos 0^\circ]$$

$$W = 10^{-5}\left[\frac{\sqrt{3}}{2} - 1\right]$$

$$W = -0.14 \times 10^{-5}\text{ J}$$

Caution

➥ *Student must know about dipole in non-uniform field. When a dipole is kept in a non-uniform electric field it experiences a non-zero linear force apart from a non-zero torque, unlike a dipole in a uniform field where the torque is non-zero but the linear force is zero. This is because the force on each charge is different.*

Example 1.8:

(A) Assertion (A): Electric lines of force never cross each other.

Reason (R): Electric field at a point superimpose to give one resultant electric field.

(a) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If both Assertion and Reason are false.

(B) What do the positive and negative potential imply?

(C) How does the energy of dipole change when it is rotated from unstable equilibrium to stable equilibrium in a uniform electric field?

[Delhi Gov. QB 2022]

Ans.

(A) *(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.*

Explanation: Electric lines of force never cross each other. Electric field at a point add up to give one resultant electric field. So, they do not have independent existence at the point of superposition. So, electric lines of force do not cross each other (crossing of electric lines of force at a point means at a point two fields are having independent existence).

(B) Positive potential implies that work can be obtained by releasing the charges while negative potential energy indicates that an external agency will have to do work to separate the charges in finite distance apart.

(C) Work done $= pE(\cos 180^\circ - \cos 0^\circ)$

$= -2pE$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. An electric dipole of moment p is placed parallel to the uniform electric field. The amount of work done in rotating the dipole by 90° is:

(a) $2pE$ (b) pE

(c) $\frac{pE}{2}$ (d) zero

[CBSE Term-1 SQP 2021]

Ans. *(b) pE*

$$W = pE(\cos\theta_1 - \cos\theta_2)$$
$$\theta_1 = 0°$$
$$\theta_2 = 90°$$
$$W = pE(\cos 0° - \cos 90°)$$
$$= pE(1 - 0) = pE$$

[CBSE Marking Scheme Term-1 SQP 2021]

2. Equipotential surfaces:

(a) are closer in regions of large electric fields compared to regions of lower electric fields.

(b) will be more crowded near sharp edges of a conductor.

(c) will never be equally spaced.

(d) both (a) and (b) are correct.

[Delhi Gov. SQP 2022]

Ans. *(d) both (a) and (b) are correct.*

Explanation: Electric field, $E = -\frac{dV}{dr}$

dr is inversely proportional to electric field. Equipotential surfaces are closer in regions of large electric fields compared to regions of lower electric field. At sharp edges of a conductor, charge density is more. Therefore, electric field is stronger. Hence, equipotential surfaces are more crowded.

3. Which of the following is not the property of an equipotential surface?

(a) They do not cross each other.

(b) The work done in carrying a charge from one point to another on an equipotential surface is zero.

(c) For a uniform electric field, they are concentric spheres.

(d) They can be imaginary spheres.

[CBSE SQP 2023]

Ans. *(c) For a uniform electric field, they are concentric spheres.*

[CBSE Marking Scheme SQP 2023]

Explanation: No gradient may exist in a direction that is parallel to the surface. Equipotential surfaces are any surfaces over which the electric potential is constant throughout. Lines of force are everywhere at an angle to the equipotential surface because of the electric field because there is no electric field parallel to the surface and no potential gradient along any path parallel to the surface. As a result, the electric field and the lines of force that flow from it are always perpendicular to the equipotential surface. As a result, they are not spheres with an even electric field that are concentric. They are spheres with a central point of isolation.

4. Two thin wire rings each having a radius R are placed at a distance d apart with their axis coinciding. The charge on the two rings are $+q$ and $-q$. The potential difference between the centre of two rings is:

(a) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

(b) $\frac{qR}{4\pi\varepsilon_0 d^2}$

(c) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

(d) zero

5. The potential energy between two atoms in a molecule, is given as $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ where a, b are constants and x is distance between atoms then equilibrium is established when:

(a) $x = 0$ (b) $x = \left(\frac{a}{2b}\right)^{\frac{1}{6}}$

(c) $x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$ (d) $x = \left(\frac{11a}{5b}\right)^{\frac{1}{6}}$

Ans. *(c)* $x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$

Explanation:

Given $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

$\therefore$ $F = -\frac{dU}{dx}$

$$= \left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right]$$

For equilibrium $\quad F = 0$

$$\frac{12a}{x^{13}} = \frac{6b}{x^7}$$

$$\therefore \quad \frac{2a}{b} = \frac{x^{13}}{x^7}$$

$$\frac{2a}{b} = x^6$$

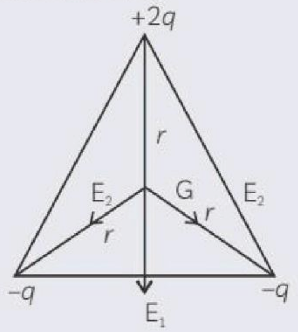
$$\Rightarrow \quad x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

6. Three charges $2q$, $-q$ and $-q$ lie at vertices of a triangle. The value of E and V at centroid of triangle will be:

(a) $E \neq 0$ and $V \neq 0$ (b) $E = 0$ and $V = 0$
(c) $E \neq 0$ and $V = 0$ (d) $E = 0$ and $V \neq 0$

[CBSE Term-1 SQP 2021]

Ans. *(c) $E \neq 0$ and $V = 0$*



Net EFI (Electric field intensity) at $G \neq O$

Net Potential at G,

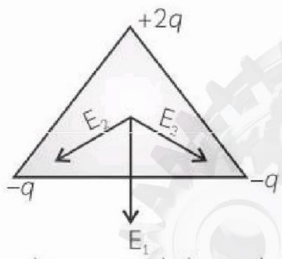
$$V = \frac{k2Q}{r} - \frac{KQ}{r} - \frac{KQ}{r}$$

$$V = 0$$

Correct option is (c).

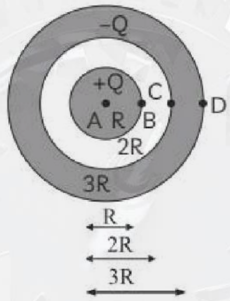
[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: As can be seen from the diagram the net electric field is non-zero.



Potential at the centroid depends on the net charge which is zero and hence, potential is zero.

7. A solid spherical conductor has charge +Q and radius R. It is surrounded by a solid spherical shell with charge -Q, inner radius 2R, and outer radius 3R.



Which of the following statements is true?

(a) The electric potential has a maximum magnitude at C and the electric field has a maximum magnitude at A.

(b) The electric potential has a maximum magnitude at D and the electric field has a maximum magnitude at B.

(c) The electric potential at A is zero and the electric field has a maximum magnitude at D.

(d) Both the electric potential and electric field achieve a maximum magnitude at B.

[CBSE Term-1 SQP 2021]

8. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately:

(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

[NCERT Exemplar]

Ans. *(a) spheres*

Explanation: The collection of charges, whose total sum is not zero, with regard to great distance can be considered as a single point charge electric potential due to point charge is given by,

$$V = \frac{kq}{r} \quad ...(i)$$

According to equation (i), the electric potential due to a point charge is same for all equidistant points and the equipotential surface due to a point charge is spherical.

9. In a given region, electric potential varies with position as $V(x) = 3 + 2x^2$.

Identify which of the following statements is correct.

(a) Potential difference between the two point x = 2 and x = –2 is 2 V.

(b) A charge of 1 C placed at x = 2 experiences a force of 6 N.

(c) The force experienced by the above charge is along +x –axis.

(d) The electric field in the given region is non-uniform along x-axis.

[CBSE Question Bank 2023]

Ans. *(d) The electric field in the given region is non-uniform along x-axis.*

Explanation:

$$E = -\frac{dV}{dx}$$

$$= -\frac{d(3+2x^2)}{dx} = -4x$$

The value varies in x direction hence the electric field in the given region is non-uniform along x-axis.

Related Theory

➥ *The electric field intensity E is inversely proportional to the separation between equipotential surfaces. So, equipotential surfaces are closer in regions of large electric fields.*

10. The work done to move a charge along an equipotential from A to B:

(a) cannot be defined as $-\int_A^B E.dl$

(b) must be defined as $-\int_A^B E.dl$

(c) is zero.

(d) can have a non-zero value.

[NCERT Exemplar]

Ans. *(c) is zero.*

Explanation: We know that, work done,

$$W = q(V_A - V_B) = -q\int_A^B \vec{E}.\vec{dl}$$

For equipotential surface,

$$V_A = V_B.$$

Hence, $W = 0.$

Also, electric field is perpendicular to equipotential surface,

hence, $\vec{E}.\vec{dl} = 0$

$\Rightarrow$ $W = 0$

11. In a region of constant potential:

(a) the electric field is uniform

(b) the electric field is zero

(c) there can be no charge inside the region

(d) the electric field shall necessarily change if a charge is placed outside the region

[NCERT Exemplar]

Ans. *(b) the electric field is zero*

(c) there can be no charge inside the region.

Explanation: As we know that electric field intensity, $E = -\frac{dV}{dr}$

$V = 0$

So by the above relation,

$$E = -\frac{dV}{dr};$$

$$E = 0$$

Hence, the electric field is not uniform.

Hence, option (b) is correct.

If some charge is present inside the region then electric field cannot be zero at that region, for this V = constant is not valid.

Hence, option (c) is correct.

12. The electric potential on the axis of an electric dipole at a distance 'r' from it's centre is V. Then the potential at a point at the same distance on its equatorial line will be

(a) 2V **(b) -V**

(c) $\frac{V}{2}$ **(d) Zero**

[CBSE SQP 2022]

Ans. *(d) Zero*

[CBSE marking Scheme SQP 2022]

Explanation: For equatorial line, $\theta = 90°$

For axial line $\theta = 0°$

$$V = \frac{kp}{r^2} \times \cos\theta$$

$$V = \frac{kp}{r^2}$$

For equatorial line V = 0 as cos 90° = 0.

13. If a unit positive charge is taken from one point to another over an equipotential surface:

(a) work is done on the charge

(b) work is done by the charge

(c) work done is constant

(d) no work is done **[Delhi Gov. SQP 2022]**

Ans. *(d) no work is done*

Explanation: Since the potential at each point of an equipotential surface is the same, the potential does not change while we move a unit positive charge from one point to another. Therefore, work done in the process is zero.

14. The shape of the equipotential surfaces for a uniform electric field is:

(a) parallel planes perpendicular to the direction of electric field.

(b) parallel planes parallel to the direction of electric field.

(c) parallel planes which are inclined at 45° with the electric field.

(d) none of these. **[Delhi Gov. QB 2022]**

15. In bringing an electron towards another electron, the electrostatic potential energy of the system:

(a) increases

(b) decreases

(c) remains unchanged

(d) remains zero **[Delhi Gov. QB 2022]**

Ans. *(a) increases*

Explanation: In bringing an electron towards another electron, work has to be done (since same charges repel each other). The work done stored as electrostatic potential energy, and hence, electrostatic potential energy of the system increases.

16. **Two charges 14 μC and –4 μC are placed at (– 12 cm, 0, 0) and (12 cm, 0, 0) in an external electric field $E = \left(\frac{B}{r^2}\right)$, where $B = 1.2 \times 10^6$ N/cm^2 and *r* is in metres.**

The electrostatic potential energy of the configuration is:

(a) 97.9 J **(b) 102.1 J**
(c) 2.1 J **(d) – 97.9 J**

[CBSE Term-1 2021]

Ans. *(b) 102.1 J*

Explanation:

$$U = k\frac{q_1q_2}{r} + q_1V + q_2V$$

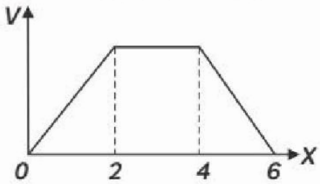
$$V = -\int E.dr$$

$$V = \frac{B}{r}$$

By putting the values, we get

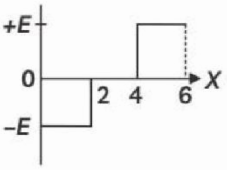
$$U = 102.1 \text{ J}$$

17. **The electric potential V as a function of distance X is shown in the figure.**

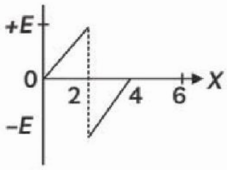


The graph of the magnitude of electric field intensity E as a function of X is:

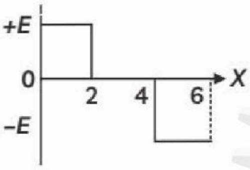
(a)



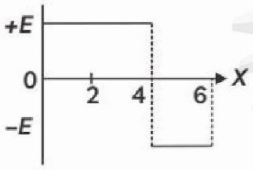
(b)



(c)

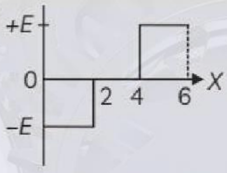


(d)



[CBSE SQP 2022]

Ans. *(a)*



[CBSE Marking Scheme SQP 2022]

Explanation: $E = -\frac{dV}{dx}$

For 0 – 2 seconds, $\frac{dV}{dx}$ = +*ve* and constant

So, E = –*ve* and constant

For 2 – 4 seconds, $\frac{dV}{dx} = 0$

So, $E = 0$

For 4 – 6 seconds, $\frac{dV}{dx}$ = – *ve* and constant

So, E = +*ve*

Assertion-Reason Questions

For Questions 18 to 22, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If both Assertion and Reason are false.

18. **Assertion (A):** An electron has a higher potential energy when it is at a location associated with a negative value of potential and has a lower potential energy when at a location associated with a positive potential.

Reason (R): Electrons move from a region of higher potential to a region of lower potential.

[CBSE SQP 2023]

Ans. *(c) If Assertion is true but Reason is false.*

[CBSE Marking Scheme SQP 2023]

Explanation: We know that,

$$U = qV$$

where, V is potential and U is potential energy. As electron has a negative charge, the higher negative potential and negative charged electron will give maximum potential energy.

$$U = (-e)(-V)$$
$$= eV = \text{maximum}$$

Current flows from higher potential to lower potential and electron flows in opposite to the current. Hence, electrons flow from lower to higher potential.

19. Assertion (A): Surface of asymmetrical conductor can be treated as equipotential surface.

Reason (R): Charges can easily flow in a conductor.

20. Assertion (A): Conductors having equal positive charge and volume, must also have same potential.

Reason (R): Potential depends only on charge and volume of conductor.

Ans. *(d) If both Assertion and Reason are false.*

Explanation: Electric potential of a charged conductor depends not only on the amount of charge and volume but also on the shape of the conductor. Hence if their shapes are different, they may have different electric potential.

21. Assertion (A): The total charge stored in a capacitor is zero.

Reason (R): The field just outside the capacitor is $\frac{\varepsilon_0}{\sigma}$. ($\sigma$ is the charge density)

Ans. *(c) If Assertion is true but Reason is false.*

Explanation: The net charge on either of the capacitor's plates is equal, and in the opposite direction of the net charge that is kept there, which is 0. The surface won't have any charge on it, and the Gauss theorem predicts that the flux will be zero if we picture a surface surrounding the capacitor's plates. Therefore, the field outside of the capacitor is zero.

22. Assertion (A): Work done in moving a charge around a closed path, in an electric field is always zero.

Reason (R): Electrostatic force is a conservative force.

[CBSE 2023]

Ans. *(a) If both Assertion and Reason are true and Reason is correct explanation of Assertion.*

Explanation: The given statement, work done in moving a charge around a closed path, in an electric field, is always zero is true. This is because the electric field is a conservative field, which means that the work done in moving a charge between two points in the field is independent of the path taken by the charge.

The electrostatic force is a conservative force. This means that the work done by the electrostatic force on a charge moving between two points in an electric field is independent of the path taken by the charge.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

23. *An electric potential is the amount of work needed to move a unit of electric charge from a reference point to a specific point in an electric field without producing an acceleration. Typically, the reference point is the earth or a point at infinity, although any point can be used. This value can be calculated in either a static (time - invariant) or a dynamic (varying with time) electric field at a specific time in units of joules per coulomb (JC^{-1}), or volts (V).*

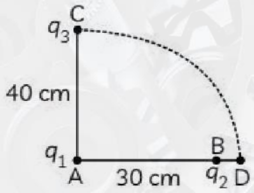
The electric potential at infinity is assumed to be zero.

(A) In bringing an electron towards another electron, the electrostatic potential energy of the system:
- (a) increases
- (b) decreases
- (c) remains unchanged
- (d) becomes zero

(B) Some charge is being given to conductor. Then, its potential:
- (a) is maximum at surface
- (b) is maximum at centre
- (c) remains the same throughout the conductor
- (d) is maximum somewhere between surface and centre.

(C) A point charge $+q$ is placed at the origin O. What is the work done in taking another point charge $-Q$ from the point A $(0, a)$ to another point B $(a, 0)$ along the straight path AB?
- (a) $\left(\frac{1}{4\pi\varepsilon_0}, \frac{qQ}{a^2}\right)\sqrt{2a}$
- (b) $\left(-\frac{1}{4\pi\varepsilon_0}, \frac{qQ}{a^2}\right)\sqrt{2a}$
- (c) $\left(-\frac{1}{4\pi\varepsilon_0}, \frac{qQ}{a^2}\right)\frac{a}{\sqrt{2}}$
- (d) Zero

(D) Two charge q_1 and q_2 are placed 30 cm apart as shown in the figure. A third charge q_3 is moved along the circular path of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$. What is the value of k?



(a) $8q_1$ (b) $6q_1$

(c) $8q_2$ (d) $6q_2$

(E) The electric potential at a point (x, y) in $(x - y)$ plane is given by $V = -xy$. The field intensity at a distance r from origin varies as:

(a) 2 (b) r

(c) $\frac{1}{r}$ (d) $\frac{1}{r^2}$

Ans. (A) *(a) increases*

Explanation: The electron has negative charge. When an electron is approaching towards another electron, then due to same negative charge repulsive force is produced between them. So, to bring them closer a work is done against the repulsive force. This work is stored in the form of electrostatic potential energy. Thus, electrostatic potential energy of system increases.

(C) *(d) Zero*

Explanation: If U_A and U_B are the electrostatic potential energies of the charges at the points A and B respectively, then work done.

$$W = U_B - U_A$$

Since, $Q_A = Q_B = -Q,$

$$U_B = U_A = \frac{-KQ}{a^2}$$

$$W = U_B - U_A = 0$$

(D) *(c) $8q_2$*

Explanation:

$$dU = U_D - U_C$$

Now, $U_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{BC}$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{\sqrt{(0.3)^2+(0.4)^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{0.5}$$

and $U_D = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{BD} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{0.1}$

$$\therefore \quad dU = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{0.1} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{0.5}$$

$$dU = \frac{q_3}{4\pi\varepsilon_0}(10q_2 - 2q_2) = \frac{q_3}{4\pi\varepsilon_0}(8q_2)$$

Hence, from question,

$$\frac{q_3}{4\pi\varepsilon_0}k = \frac{q_3}{4\pi\varepsilon_0}8q_2$$

$$k = 8q_2$$

24. *Electrostatic potential energy of system of point charges is defined as the total amount of work done in bringing the different charges to their respective position from infinitely large mutual separations.*

By definition, work done in carrying charge from E to any point is W = Potential × charge

This work is stored in the system of the point charge in the form of electrostatic potential energy U of the system.

(A) What is the difference between potential energy of a charge at a certain point and the potential at that point?

(B) Discuss the statement that "Potential energy of a system of charge is a unique value".

(C) What is the amount of work done in displacing a charge of 1 C through a distance of 1 m along an equipotential surface of potential 'V'?

Ans. (A) Potential energy at any point, in an electrostatic field is defined as the amount of work done in moving the charge from infinity to that point while the potential is the work done in moving a unit charge from infinity to that point.

(B) The potential energy of a system of charge doesn't depend upon the order of path along which the charges are brought to assemble that constitution. Hence it has a unique value which is characteristic of that constitution.

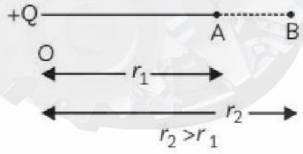
VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

25. A point charge +Q is placed at point O as shown in the figure. Is the potential difference ($V_A - V_B$) positive, negative or zero?

[CBSE 2016, 11]

Ans. According to question,



Potential at point A due to charge +Q,

$$V_A = \frac{kQ}{r_1}$$

Potential at point B due to charge +Q,

$$V_B = \frac{kQ}{r_2}$$

As, $V_A \propto \frac{1}{r_1}$

and $V_B \propto \frac{1}{r_2}$

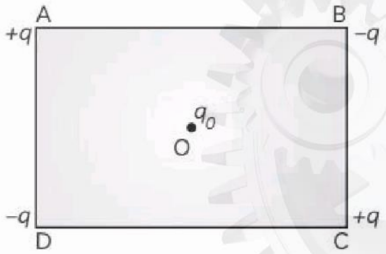
and $r_2 > r_1$

So, $V_A > V_B$

Thus, ($V_A - V_B$) is positive.

26. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one. [NCERT Exemplar]

27. Four charges $+q$, $-q$, $+q$, $-q$ are placed as shown in the figure. What is the work done in bringing a test charge from ∞ to point O.



Here, OA = OB = OC = OD and q_0 = Test charge [Delhi Gov. QB 2022]

Ans. $V_0 = \frac{kq}{AO} + \frac{kq}{OC} - \frac{kq}{OB} - \frac{kq}{OD} = 0$

$W = q_0 \times V_0 = 0$

28. Can a metal sphere of radius 1 cm hold a charge of 1 Coulomb?

Ans. The potential 'V' on the surface of sphere will be,

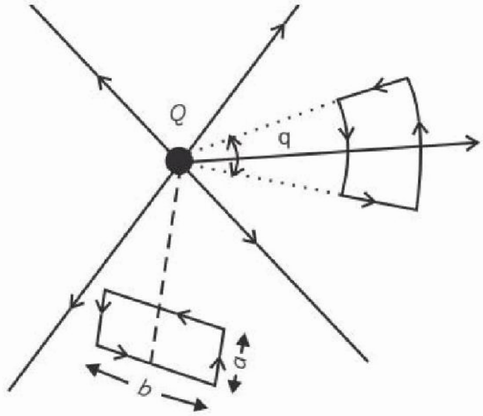
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$V = 9\times10^9 \frac{1}{(0.01)}$$

$$= 9 \times 10^{11} \text{ volt}$$

This value of potential is much higher than the ionisation potential of air. Therefore, the surplus charge will leak to the atmosphere.

29. A test charge q is made to move in the electric field of a point charge Q along two different closed paths. First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



[NCERT Exemplar]

30. In an uniform electric field of strength E, a charged particle Q moves from point A to point B in the direction of the field and back from B to A. Calculate the ratio of the work done by the electric field in taking the charge particle from A to B and from B to A. [Delhi Gov. QB 2022]

Ans. $\frac{W_{AB}}{W_{BA}} = -1$

$\therefore \quad W_{AB} + W_{BA} = 0$

$|W_{AB}| = |-W_{BA}|$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

31. Two hollow spherical conductors are charged positively. The smaller is at 20 V while the bigger one is at 50 V. How would you arrange them so that on connecting them together the charge flows from smaller on to the bigger one?

Ans. Charge will flow from conductor with bigger potential to conductor at smaller potential. This can be achieved by making them concentric.

Potential of smaller sphere

V_1 = (Potential due to its own charge q) + (potential due to charge Q on outer sphere)

$$\therefore \quad V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}+\frac{Q}{R}\right) \quad ...(i)$$

Potential of bigger sphere,

V_2 = (Potential due to charge on smaller sphere) + (potential due to charge on itself)

$$\therefore \quad V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}+\frac{Q}{R}\right) \quad ...(ii)$$

Comparing equation (i) and (ii) we see that,

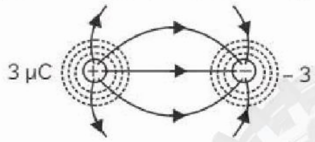
$$V_1 > V_2$$

Thus, the inner sphere is always at a higher potential. Hence, the flow of charge will take place from smaller sphere to the bigger one.

32. Calculate potential on the axis of a disc of radius *R* due to a charge *Q* uniformly distributed on its surface. [NCERT Exemplar]

33. Two point charges +3 μC and -3 μC are placed at point A and B, 5 cm apart.
(A) Draw the equipotential surfaces of the system.
(B) Why do equipotential surfaces get close to each other near the point charge?
[CBSE 2011]

Ans. (A) Equipotential surfaces of the system;



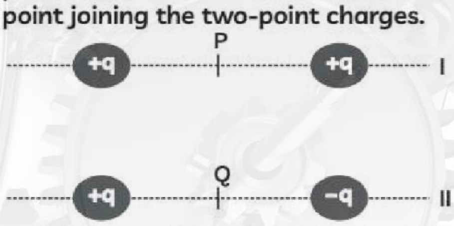
(B) Equipotential surface get closer to each other near the point charges as strong electric field is produced there.

$$E = -\frac{\Delta V}{\Delta r}$$

i.e., $$E \propto -\frac{1}{\Delta r}$$

For a given equipotential surface, Δr represent strong electric field and vice-versa.

34. (A) If electric field strength at a point is zero at a given point, then what can you say about the electric potential at that point? Explain.
(B) In the two instances below, state whether electric field intensity and electric potential are zero or non-zero at the mid-point joining the two-point charges.



[CBSE Question Bank 2023]

Ans. (A) As, $E = -\frac{\Delta V}{\Delta r}$

If $E = 0$, at a given point,

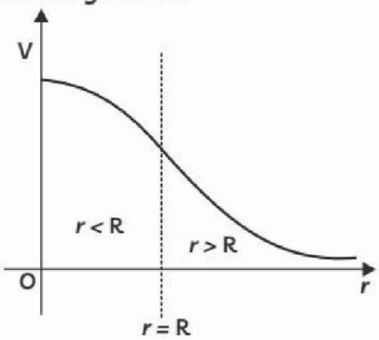
then $\frac{\Delta V}{\Delta r} = 0$

i.e., V = 0 or constant at that point.

(B) At mid-point P in Fig I, E is zero, but V is non-zero.

At mid-point Q in Fig II, E is non-zero, but V adds up to zero.

35. A given solid sphere of radius R made of an insulating material carries a charge *q* distributed uniformly throughout its volume. The potential due to this charge distribution as a function of distance *r* from the center of the sphere is given as:



(A) At which location with respect to the sphere, is the potential V maximum in this case?
(B) In case the above sphere is made up of a conducting material instead of an insulating material, what would be your answer for part (A)? How is the charge *q* distributed across a charged conducting sphere? [CBSE Question Bank 2023]

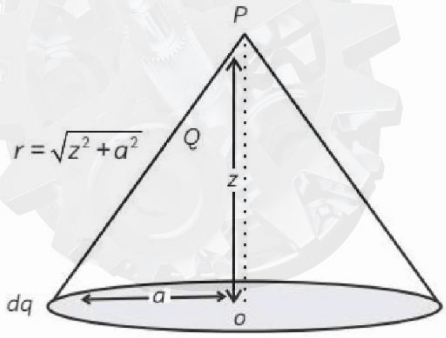
Ans. (A) At the center of the sphere

(B) Potential is constant and maximum across the volume of the sphere of conducting material.

Charges are distributed only on the surface of the conducting sphere. The charge inside the surface of the conducting sphere is always zero.

36. Calculate potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R. [NCERT Exemplar]

Ans.



Consider a point P to be at a distance z from the centre of the ring, as shown in figure. The charge element dq is at a distance z from the point P. Therefore,

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+a^2}}$$

Since each element dq is at the same distance from point P, so net potential

$$V=\frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+a^2}}\int dq$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+a^2}}(Q)$$

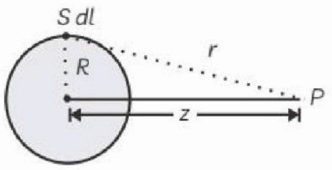
$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{z^2+a^2}}$$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

37. Calculate potential energy of a point charge $-q$ placed along the axis due to a charge $+Q$ uniformly distributed along a ring of radius R. Sketch P.E. as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis)? [NCERT Exemplar]

Ans. Suppose a negatively charged particle is placed at the axis of the ring of radius R.



The potential at an axial distance z from the centre of the ring is:

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{z^2+R^2}}$$

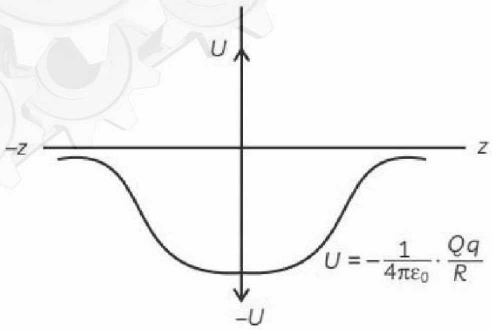
Hence potential energy of a point charge $-q$ is:

$$U=qV=(-q)\left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{z^2+R^2}}\right]$$

$$U=-\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\sqrt{z^2+R^2}}$$

At $z=0$, $\quad U=-\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}$.

At $z\to\infty$, $\quad U\to 0$



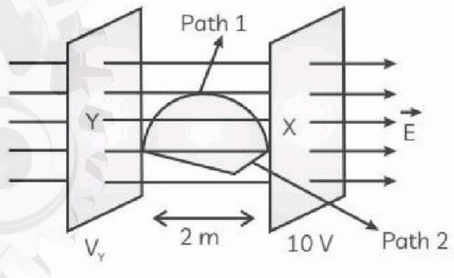
The variation of potential energy with z is shown in the above figure.

The charge $-q$ displaced would perform oscillations.

38. X and Y are two equipotential surfaces separated by a distance of 2 m in a uniform electric field of 10 V/m. Surface X has a potential of 10 V

(A) Calculate the potential of surface Y.

(B) What is the work done in moving a + 2 C charge from surface Y to surface X along path 1? How will this work change when the charge is moved along Path 2? Give a reason for your answer.



[CBSE Practice Set-1 2023]

Ans. (A) Given $E = 10$ V/m

$V_x = 10$ V

$\Delta r = 2$ m

$$|\Delta V| = \vec{E}.\Delta\vec{r}$$

$$= 10 \times 2 = 20 \text{ V}$$

Since, the potential decreases in the direction of the electric field, the potential at surface Y will be more than the potential at surface X.

$$V = 20 + 10 = 30 \text{ V}$$

(B) Given: $q = 2$ C

Work done in moving charge from Y to X along Path 1.

$$W = (V_x - V_y)_q$$

$$= (10 - 30) \times 2$$

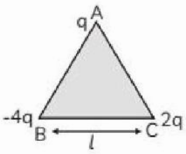
$$= -20 \times 2$$

$$= -40 \text{ J}$$

Work done in moving charge along Path 2 will be the same as work done along Path 1.

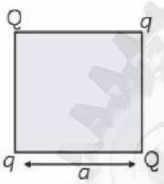
This is because the work done between two surfaces is independent of the path since the force acting on the charge is conservative in nature.

39. **(A)** **Three point charge q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side l as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.**



(B) **Find out the amount of the work done to separate the charges at infinite distance.** **[CBSE 2018]**

40. **Four point charge Q, q, Q and q are placed at the corners of a square of side a as shown in figure.**



Find the:

(A) resultant electric force on a charge Q and

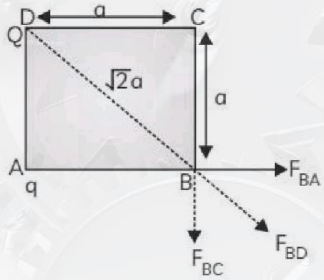
(B) potential energy of this system.

[CBSE 2018]

Ans. (A) Force acting on charge Q placed at point B, is due to charges placed at point A, C and D.

Here, magnitude of force on charge at point B due to charge at point A is,

$$F_{BA} = \frac{KQq}{a^2}$$

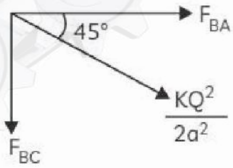


Similarly, magnitude of force on charge at point B due to charge at point C is,

$$F_{BC} = \frac{KQq}{a^2}$$

Also, the magnitude of force on charge at point B due to charge at point D is,

$$F_{BD} = \frac{KQ^2}{(\sqrt{2}a)^2} = \frac{KQ^2}{2a^2}$$



Let F is resultant of F_{BA} and F_{BC},

$$\therefore \quad F = \sqrt{2}\frac{KQq}{a^2}$$

As, $$F_{BA} = F_{BC} = \frac{KQq}{a^2}$$

$\therefore$ The resultant electric force,

$$F_{net} = F + \frac{KQ^2}{2a^2} = \sqrt{2}\frac{KQq}{a^2} + \frac{KQ^2}{2a^2}$$

$$= \frac{KQ}{a^2}\left(\sqrt{2}q + \frac{Q}{2}\right)\text{N}$$

(B) The potential energy of the system is given by

$$U = U_{AB} + U_{BC} + U_{CD} + U_{DA} + U_{AC} + U_{BD}$$

$$= \frac{KQq}{a} + \frac{KQq}{a} + \frac{KQq}{a} + \frac{KQq}{a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{\sqrt{2}a}$$

$$= \left[4\left(\frac{KQ_q}{a}\right) + \frac{\sqrt{2}Kq^2}{a}\right]$$

41. **Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point P (P lies somewhere in between the charges) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.** **[Delhi Gov. SQP 2022]**

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

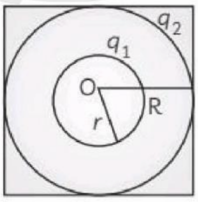
42. A charge Q is distributed over two concentric hollow sphere of radius r and R (R > r) such that the surface densities are equal. What will be the potential at their common centre?

Ans. Let q_1 and q_2 be charges acquired by two sphere.

$$\therefore \quad Q = q_1 + q_2 \quad \text{...(i)}$$

Since their surface charge densities are equal

$$\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$



$$\therefore \quad \frac{q_2}{q_1} = \frac{R^2}{r^2}$$

$$\therefore \quad q_2 = \frac{R^2}{r^2}q_1 \quad \text{...(ii)}$$

Substituting in equation (i)

$$Q = q_1 + \frac{R^2}{r^2}q_1 = q_1\left(\frac{R^2+r^2}{r^2}\right)$$

$$\therefore \quad q_1 = \left(\frac{Qr^2}{R^2+r^2}\right) \quad \text{...(iii)}$$

Potential at O due to charge q_1 on sphere of radius,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

Potential at O due to charge q_2 on sphere of radius R,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1R^2}{r^2R} = \frac{1}{4\pi\varepsilon_0}\frac{q_1R}{r^2}$$

Net potential 'V' at O,

$$V = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_1R}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\left(1+\frac{R}{r}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\left(\frac{R+r}{r}\right) = \frac{1}{4\pi\varepsilon_0}q_1\frac{(R+r)}{r^2}$$

Substituting for q_1 from equation (iii),

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{(R^2+r^2)} \times \frac{R+r}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)}$$

43. Two point charge q and $-q$ are located at points (0, 0, $-a$) and (0, 0, a) respectively.

(A) Find the electrostatic potential at (0, 0, z) and (x, y, 0).

(B) How much work is done in moving a small test charge from the point (5, 0, 0) to (–7, 0, 0) along the x-axis?

(C) How would your answer change if the path of the test charge between the same points is not along the x-axis but along any other random path?

Justify your answer in each case.

44. A capacitor is made of two circular plates of radius R each, separated by a distance d<<R. The capacitor is connected to a constant voltage. A thin conducting disc of radius r<<R and thickness t << r is placed at a centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m. [NCERT Exemplar]

Ans. When a disc is in touch with the botton plate the entire plate is a equipotential. A charge q' is transferred to the disc. The electric field on the disc is $= \frac{V}{d}$

$$q' = -\varepsilon_0\frac{V}{d}\pi r^2$$

The force acting on the disc is $-\frac{V}{d} \times q'$

$$= \varepsilon_0\frac{V^2}{d^2}\pi r^2$$

If the disc is to be lifted,

Then

$$\varepsilon_0\frac{V^2}{d^2}\pi r^2 = mg$$

$$V^2 = \frac{mgd^2}{\pi r^2\varepsilon_0}$$

$$\Rightarrow \quad V = \sqrt{\frac{mgd^2}{\pi r^2\varepsilon_0}}$$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

42. A charge Q is distributed over two concentric hollow sphere of radius r and R (R > r) such that the surface densities are equal. What will be the potential at their common centre?

Ans. Let q_1 and q_2 be charges acquired by two sphere.

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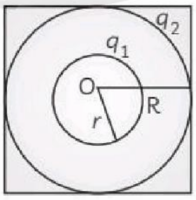

$$\therefore \quad \frac{q_2}{q_1} = \frac{R^2}{r^2}$$

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Potential at O due to charge q_1 on sphere of radius,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

Potential at O due to charge q_2 on sphere of radius R,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 R^2}{r^2 R} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 R}{r^2}$$

Net potential 'V' at O,

$$V = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 R}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\left(1+\frac{R}{r}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\left(\frac{R+r}{r}\right) = \frac{1}{4\pi\varepsilon_0}q_1\frac{(R+r)}{r^2}$$

Substituting for q_1 from equation (iii),

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Justify your answer in each case.

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$$q' = -\varepsilon_0\frac{V}{d}\pi r^2$$

The force acting on the disc is $-\frac{V}{d} \times q'$

$$= \varepsilon_0\frac{V^2}{d^2}\pi r^2$$

If the disc is to be lifted,

Then

$$\varepsilon_0\frac{V^2}{d^2}\pi r^2 = mg$$

$$V^2 = \frac{mgd^2}{\pi r^2\varepsilon_0}$$

$$\Rightarrow \quad V = \sqrt{\frac{mgd^2}{\pi r^2\varepsilon_0}}$$

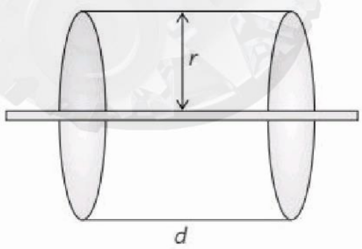
NUMERICAL Type Questions

45. Find the equation of the equipotentials for an infinite cylinder of radius r_0, carrying charge of linear density. [NCERT Exemplar](3m)

Ans. Let a cylindrical Gaussian surface of radius r from the line charge.

Let the field lines be radially outward,

$$V_{(r)} - V_{(r_0)} = V_r - V_{(r_0)} = -\int_{r_0}^{r} \vec{E}.\vec{dr} \quad ...(i)$$



$$\oint \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0}(\lambda l)$$

$$E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l$$

$$\Rightarrow \quad E_r = \frac{\lambda}{2\pi\varepsilon_0 r} \quad ...(ii)$$

If r_0 is the radius of cylindrical wire, then from equation (i) and (ii),

$$V_{(r)} - V_{(r_0)} = -\int_{r_0}^{r} \frac{\lambda}{2\pi\varepsilon_0 r}.dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}(\log r)_{r_0}^{r}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{r}{r_0}\right)$$

For a given V,

$$\log\frac{r}{r_0} = -\frac{2\pi\varepsilon_0}{\lambda}[V_{(r)} - V_{(r_0)}]$$

The equipotential surfaces are cylinders of radius $r_0 e^{\frac{-2\pi\varepsilon_0}{\lambda}[V(r)-V(r_0)]}$.

Caution

➥ *Students are often get confused about the integral relation of electric field. The integral relation between electric field gives potential difference between two points. The electric field due to line charge need to be obtained in order to find the potential at distance r from the line charge.*

46. A region is specified by potential function:

$$V = 2x^2 + 3y^3 - 5z^2$$

Calculate the electric field strength at a point (2, 4, 5) in this region. (3m)

Ans. Potential function is given as

$$V = 2x^2 + 3y^3 - 5z^2$$

Let E_x, E_y, & E_z be the rectangular component of the electric field at any point (x, y, z),

$$\therefore \quad E_x = -\frac{dV}{dx}, E_y = -\frac{dV}{dy}, E_z = -\frac{dV}{dz}$$

$$E_x = -\frac{d}{dx}(2x^2 + 3y^3 - 5z^2) = -4x$$

$$E_y = -\frac{d}{dy}(2x^2 + 3y^3 - 5z^2) = -9y^2$$

$$E_z = -\frac{d}{dz}(2x^2 + 3y^3 - 5z^2) = +10z$$

The electric field at any point is,

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$= -4x\hat{i} - 9y^2\hat{j} + 10z\hat{k}$$

$\therefore$ Electric field at point (2, 4, 5)

$$E_x = \frac{-dV}{dx}$$

$$= -4x = -4 \times 2 = -8$$

$$E_y = \frac{-dV}{dy}$$

$$= -9y^2 = -9 \times (4)^2 = -144$$

$$E_z = \frac{-dV}{dz}$$

$$= +10z = +10 \times 5 = +50$$

$$\vec{E} = -8\hat{i} - 144\hat{j} + 50\hat{k}$$

At point (2, 4, 5)

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$= \sqrt{(8)^2 + (144)^2 + (50)^2}$$

$$= 152.64 \text{ units}$$

47. A proton is placed in a uniform electric field of strength 6000 N/C and is allowed to go from one point to another in the direction of field. If the distance between the two point is 0.1 m. Calculate:

(A) The potential difference between the two point.

(B) Work done in moving the proton.

(C) Velocity of proton at final point. (3m)

48. Two charges +2 μC and –2 μC are placed at point A and B, 5 cm apart. At what distance is the potential zero? [CBSE 2013](3m)

ELECTROSTATICS OF CONDUCTORS, DIELECTRICS AND CAPACITANCE 2

TOPIC 1

ELECTROSTATICS OF CONDUCTORS

Till now we have seen how charges behave inside an electric field. Now we will see how conductors and dielectrics behave inside an electric field. You all know what conductors are materials which can let electrons and charges to freely flow through them. You also know about insulators. These are materials that simply don't allow the passage of electrons or charges through them. But dielectrics are a tad special in the sense that they also belong to the category of insulators but are slightly different in the sense that they can be polarised and they do allow the limited movement of its electrons. But before that let us see the major points of difference between conductors, insulators and dielectrics.

Conductors	Insulators	Dielectrics
Have free electrons which can move throughout the materials.	The electrons are strongly bound and cannot move around.	In comparison to insulators dielectrics have electrons which can move around in a restricted space.
Allow the flow of electrons/ charges freely.	Do not allow the flow of electrons or charges at all.	Electrons or charges can move a bit such that under electric field the system can be polarised.
Can be polarised.	Cannot polarize in electric field.	Can polarize in electric field.
Charges can flow	Charges are obstructed	Charge can be stored.

Inside Conductors

Imagine you have a solid conductor, say, solid iron sphere. By some method you place a charge at the centre of the sphere. What happens next? The sphere being a conductor allows the charges to move freely. Being similar in nature the charges will tend to repel each other and try to reach the maximum distance away from each other which incidentally happens to be the surface. Hence if charges are placed inside a conductor they will always move to the surface leaving the inside of the conductor charge free. So, no charge inside means no electric field inside the conductor.

At Surface

On the surface the field should be perpendicular to the surface. That is because if the field is not perpendicular it will have a component along the surface which will create a force on the electrons of the surface leading to an electric current without an input. Hence, Electric field at the surface of a conductor is perpendicular to the surface of the conductor.

No Excess Charge at Interiors of a Conductor

Since we know that electric field is zero inside a conductor it also means that by Gauss' law since no electric field will lead to zero electric flux from within the surface which leads us to conclude that there cannot be any excess charge inside a conductor.

Constant Potential Inside

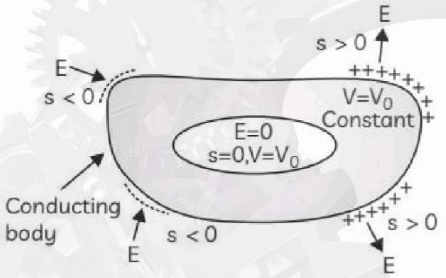
We know that,

$$E = -\frac{dV}{dR}$$

which means if the electric field inside a conductor is zero it implies that the potential inside a conductor is constant.

Electrostatic Shielding

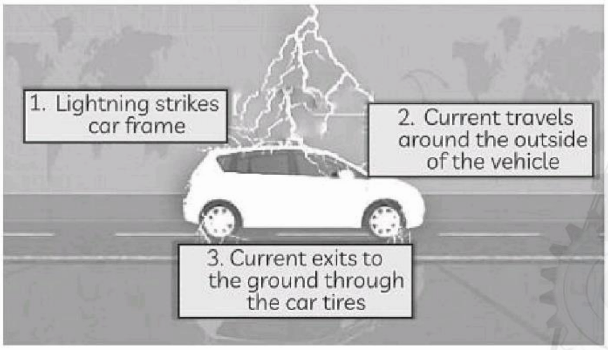
You may have heard or read that if faced with a thunderstorm it is a good idea to stay inside a car with windows rolled up. The reason is something that we have discussed in the previous points. The situation of what happens inside of a conductor is equally applicable for any cavity inside a conductor. As can be seen here if any charge is placed inside the cavity it will immediately spread out to the surface thereby keeping the cavity charge free. Same happens if an external electric field is applied on the cavity. Due to induction the charges will spread out throughout the surface keeping the cavity free of electric field. Hence, anything which is surrounded by a conductor remains unaffected by external fields. This is known as Electrostatic Shielding.



Electrostatic Shielding

Example 2.1: Case Based :

You may have studied in your earlier classes that during thunderstorms the best place to hide from lightning strikes is inside a car. The reason being that a car is generally made of metal sheet and in case of a lightning strike the charges so deposited on the car remains on the outer surface of the car and moves through the surface to the ground thereby saving anyone inside. This is a practical application of electrostatic shielding.



(A) Which of the following can be used to make an electrostatic shield?

(a) Plastic **(b) Wood**

(c) Copper **(d) All of these**

(B) Why is E = 0, V = constant inside a conductor?

(C) Which among (i) and (ii) will be a better car to be in when thunderstorm strikes?

(i) An open car

(ii) A closed car

(D) How are sensitive devices protected from electric disturbance?

(E) Potential at the point of a pointed conductor is greater than the other points on surface. Is the given statement true or false?

Ans. (A) *(c) Copper*

Explanation: For electrostatic shielding to take place we need a metallic sheet.

(B) Electric field inside a conductor is zero due to the absence of charges. Hence, the potential remains constant.

(C) (ii) A closed car

For electrostatic shielding to take place we need to have a car that is completely covered in metal.

(D) Sensitive components of electronic devices protected or shielded from external electric disturbances by placing metal sheets around them.

(E) False

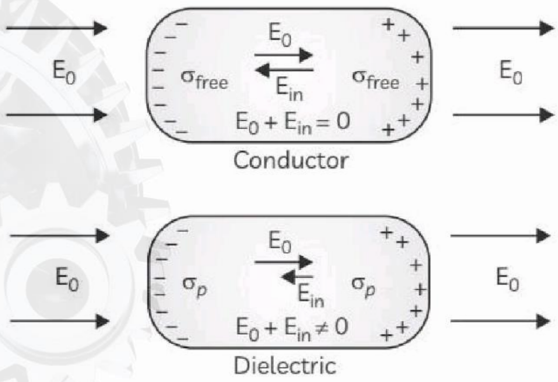
Explanation: A conductor is an equipotential body. Therefore, the potential at a point of a pointed conductor is same as the potential of any other point on the conductor.

TOPIC 2

DIELECTRICS AND POLARISATION

As stated earlier dielectrics are those special breed of insulators which can be easily polarised under the influence of an electric field.

In case of conductors, the external field gives rise to an internal field which cancels out completely. But in dielectrics it is not so. The internal field doesn't cancel out and a net internal field is left out. This net internal field is because of dipoles that have got created inside. Let us understand the mechanism of dielectrics and how they get polarised.



Dielectrics in an External Field

When a dielectric is placed in an external field it gets polarised due to the induction of dipole moments inside the dielectrics. This net dipole moment per unit volume is known as Polarisation (P) and is given by,

$$P = \varepsilon_0 \chi_e E$$

where, χ_e is known as electric susceptibility. The higher the susceptibility the more the ability of a dielectric to form dipoles under the influence of an external electric field.

TOPIC 3

CAPACITORS AND CAPACITANCE

A capacitor can be considered as a system containing two conducting plates separated by a dielectric such that when connected to the opposite ends of a battery the system has the ability to store charge in it. The charge stored in the capacitor q is related to the applied potential difference as

$$q \propto V$$

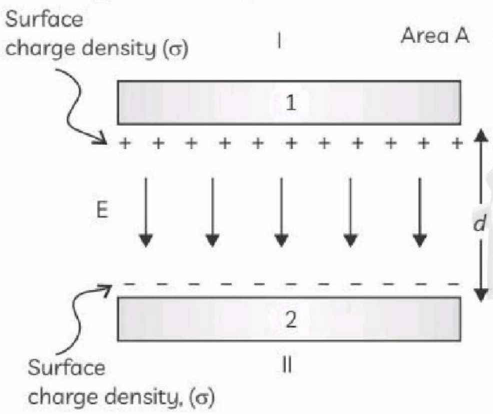
$$\Rightarrow \quad q = CV$$

Here, C is known as the capacity (or capacitance) of the capacitor. As can be seen the charge in the capacitor depends on the applied voltage. Similarly if the charge is taken out of a capacitor its voltage will also drop.

The unit of capacitance is Coulomb/Volt or Farad.

Parallel Plate Capacitors

As shown in the figure two parallel plates have opposite charge densities given as + σ and – σ.



On the two opposite plates. The electric field at the region between the two plates is given by,

$$E_{in} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

The electric field in regions I and II being zero.

Hence the net electric field can now be written as,

$$E_{in} = \frac{Q}{\varepsilon_0 A}$$

Now the potential difference between the two plates can be related to the field equation as,

$$V = \frac{Qd}{\varepsilon_0 A}$$

Now, since we know that,

$$C = \frac{Q}{V}$$

Hence, we can write

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

Equation gives us the capacitance of a parallel plate capacitor with air as the dielectric.

Effects of Dielectrics on Capacitance

Imagine a dielectric is placed inside a capacitor such that it covers the entire space. In such a situation the external field will generate dipole moment in the dielectric which will lead to reduction of electric field inside.

Hence potential difference inside will be given by,

$$V = \frac{(\sigma - \sigma_p)d}{\varepsilon_0}$$

where, σ is the charge density of the plates while σ_p is the charge density due to dipole moment. For linear dielectrics this should be proportional to the applied field E_0 which means $V = E_0 d$.

This gives rise to the equation

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

Where K is the characteristic property of the material. Hence, we get

$$C = \frac{K\varepsilon_0 A}{d}$$

This leads to the fact that,

$$K = \frac{\varepsilon}{\varepsilon_0}$$

Here, K is known as dielectric constant and as can be seen it increases the capacitance of the capacitor while decreasing the field strength as well as the force between charged plates.

Important

➡ *Every insulator, including dielectrics, has a maximum limit till which it can withstand an external applied electric field. As the electric field is applied the centers of positive and negative charges tend to shift leading to the rise of dipoles. At extremely high values of field strength the molecular* *bonds, which are under strain due to the field, may break leading to ionisation of the material. This maximum strength is known as dielectric strength. For example the dielectric strength of air is approximately 6×10^5 V/m. Beyond this the air molecules tend to break down and we can see lightning and hear thunder.*

Example 2.2: A dielectric slab of constant k, thickness $t < d$ is placed inside a capacitor of Capacitance C. What will be its new capacitance?

Ans. Potential difference inside a capacitor is given by,

$$V = V_1 + V_2$$

where,

$$V_1 = \frac{\sigma}{\varepsilon_0}(d-t) = \frac{q}{A\varepsilon_0}(d-t)$$

and $$V_2 = \frac{\sigma}{k\varepsilon_0}t = \frac{q}{Ak\varepsilon_0}t.$$

The corresponding potential difference will be then,

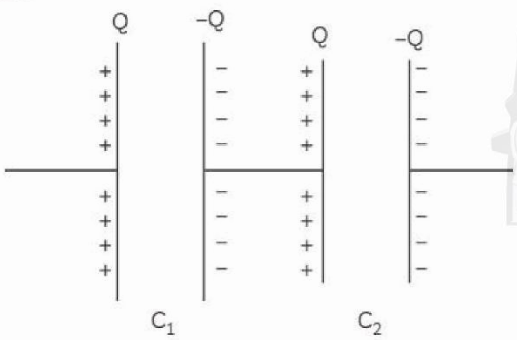
$$V = \frac{q}{A\varepsilon_0}\left(\frac{t}{k}+d-t\right)$$

Since capacitance is calculated as,

$$C = \frac{q}{V} = \frac{k\varepsilon_0 A}{k(d-t)+t}$$

Combination of Capacitors

Series



In series combination, the potential difference across individual capacitors drop while the net charge remains the same.

Hence,

$$V = V_1 + V_2$$

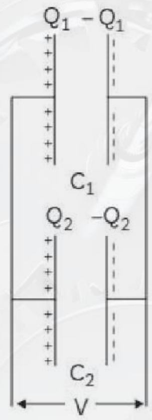
which can be further written as

$$V = \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Hence, the net capacitance of a series connection of capacitors is given as,

$$C_s = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}$$

Parallel



In case of parallel combination, the potential difference across the capacitors remain the same while the charge is distributed.

Hence,

$$Q = Q_1 + Q_2$$

This can be further written as

$$Q = CV = C_1V + C_2V$$

$$C = C_1 + C_2$$

Hence, in parallel the capacitances added as

$$C_p = C_1 + C_2$$

Example 2.3:

Assertion (A): Increasing the charge on the plates of a capacitor means increasing the capacitance.

Reason (R): For a capacitor $Q \neq CV$.

(a) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If both Assertion and Reason are false.

Ans. *(d) If both Assertion and Reason are false.*

Explanation: Capacitance is a function of geometrical configuration and has nothing to do with the voltage or charge. Hence assertion is not correct. And reason is also incorrect because charge does depend on both voltage and capacitance.

Example 2.4: A parallel-plate capacitor contains one mica sheet of thickness d_1 = 1 mm and one fibre sheet of thickness d_2 = 0.5 mm. The dielectric constants of mica and fibre are 8 and 2.5 respectively. Fibre breaks down in an electric field of 6.4×10^6 V/m. What maximum voltage can be applied in the capacitor?

Ans. Let σ be the surface charge density on the capacitor plates. Electric fields in mica (k_1) of thickness d_1 and fibre (k_2) of thickness d_2 will be:

$$E_1 = \frac{\sigma}{\varepsilon_0 k_1} \text{ and } E_2 = \frac{\sigma}{\varepsilon_0 k_2}$$

which leads to $E_1 = \frac{k_2}{k_1} E_2$

and the maximum permissible value for E_2 is 6.4×10^6 V/m.

Hence, the Maximum permissible value for

$$E_1 = \frac{2.5}{8} \times 6.4 \times 10^6 = 2 \times 10^6 \text{ V/m}$$

Maximum voltage that can be applied to capacitor is,

$$V = E_1 d_1 + E_2 d_2$$

$$V = 2 \times 10^6 \times 10^{-3} + 6.4 \times 10^6 \times 0.5 \times 10^{-3}$$

$$V = 5200 \text{ V}$$

Example 2.5: Three identical capacitors of capacitances 6 μF each are connected in parallel to a battery of 12 V. What is the total capacitance, the charge on each of the capacitors and the total energy stored in the system?

Ans. The total capacitance is given by,

$$C = C_1 + C_2 + C_3$$

$$= 3 \times 6 \text{ μF} = 18 \text{ μF}.$$

Charge on each is given by,

$$q = CV$$

$$= 6 \text{ μF} \times 12 \text{ V} = 72 \text{ μC}.$$

Total energy stored is given by,

$$E = \frac{1}{2} CV^2$$

$$= 0.5 \times 18 \text{ μF} \times 144$$

$$= 1296 \text{ μJ}$$

$$= 1.296 \text{ mJ}.$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Three capacitors 2 μF, 3 μF and 6 μF are joined in series with each other. The equivalent capacitance is:

(a) $\frac{1}{2}$ **μF** **(b) 1 μF**

(c) 2 μF **(d) 11 μF**

Ans. *(b) 1 μF*

Explanation:

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{series}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

$$= \frac{3+2+1}{6} = \frac{6}{6}$$

$$C_{series} = 1 \text{ μF}$$

2. Two metal spheres of capacitance C_1 and C_2 carry some charges. They are put in contact and then separated. The final charges Q_1 and Q_2 on them will satisfy:

(a) $\frac{Q_1}{Q_2} < \frac{C_1}{C_2}$ **(b)** $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$

(c) $\frac{Q_1}{Q_2} > \frac{C_1}{C_2}$ **(d)** $\frac{Q_1}{Q_2} < \frac{C_1}{C_2}$

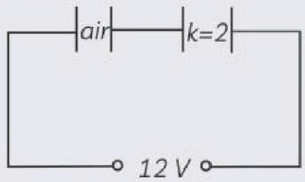
3. Two parallel plate capacitors X and Y, have the same area of plates and same separation between plates. X has air and Y with dielectric of constant 2, between its plates. They are connected in series to a battery of 12 V. The ratio of electrostatic energy stored in X and Y is:

(a) 4 : 1 **(b) 1 : 4**

(c) 2 : 1 **(d) 1 : 2**

[CBSE Term-1 SQP 2021]

Ans. *(c) 2 : 1*



A = Same
d = Same
Q = Same in series

$$C_X = \frac{\varepsilon_0 A}{d}, C_Y = \frac{2\varepsilon_0 A}{d}$$

$$U_X = \frac{Q^2}{2C_X}, U_Y = \frac{Q^2}{2C_Y}$$

$$\therefore \quad \frac{U_X}{U_Y} = \frac{C_Y}{C_X} = \frac{2C_X}{C_X} = \frac{2}{1}$$

[CBSE Marking Scheme Term-1 SQP 2021]

4. A capacitor plates are charged by a battery with 'V' volts. After charging battery is disconnected and a dielectric slab with

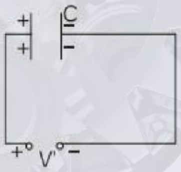
dielectric constant 'K' is inserted between its plates, the potential across the plates of a capacitor will become:

(a) zero (b) $\frac{V}{2}$

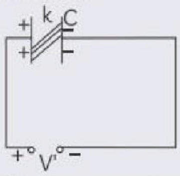
(c) $\frac{V}{K}$ (d) KV

[CBSE Term-1 SQP 2021]

Ans. (c) $\frac{V}{K}$



Battery is disconnected



Q = Charge remains constant

$C' = K C$

$Q' = C' V'$

$Q = C' V'$

$Q = K C V'$

$V' = \frac{Q}{KC} = \frac{V}{K}$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: Given:

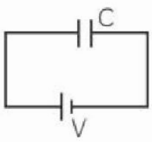
Capacitor of capacitance = C

Charging voltage = V

Dielectric constant = K

Case 1: When a capacitor is initially charged it acquires charge.

$q = CV$



Case 2: When battery is disconnected its charge remains "q". But when a dielectric slab is introduced within the plates the only parameter that remains constant is the charge "q".

Hence, $'q' = C_0V_0 = CV$

Since, $C_0 = \varepsilon_0\frac{A}{d}$ in case 1 and $C = K\varepsilon_0\frac{A}{d}$ in case 2. which means $C = KC_0$.

$q = C_0V_0 = KC_0V$

$\Rightarrow \quad V = \frac{C_0V_0}{KC_0} = \frac{V_0}{K}$

5. The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of the plates d with a potential difference V between the plates, is

(a) $\frac{CV^2}{2d}$ (b) $\frac{C^2}{V^2}$

(c) $\frac{C^2V^2}{d^2}$ (d) $\frac{V^2d}{C}$

Ans. (a) $\frac{CV^2}{2d}$

Explanation: Total electric field between the plates of the capacitor:

$E_T = \frac{V}{d}$

Then, electric field due to only one plate:

$E = \frac{V}{2d}$

∴ Force of one plate on another:

$F = E \times Q$

$= E \times CV$

$= \frac{V}{2d} \times CV$

$= \frac{CV^2}{2d}$

6. To increase the charge on the plate of a capacitor means to:

(a) decrease the potential difference between the plates

(b) decrease the capacitance of the capacitor

(c) increase the capacitance of the capacitor

(d) increase the potential difference between the plates.

7. The capacitance of a capacitor is C_o. It is connected to a battery of voltage V which charges the capacitor. With the capacitor still connected to the battery, a slab of dielectric material is introduced between the plates of the capacitor.

Which of the following explains the effect of the dielectric slab in the above situation?

(a) The electric field between the plates of the capacitor rises.

(b) The potential difference between the plates falls.

(c) The total charge on the capacitor increases.

(d) The ability of the capacitor to store charge decreases.

[CBSE Question Bank 2023]

Ans. *(c) The total charge on the capacitor increases.*

Explanation: The strength of the electric field is reduced due to the presence of dielectric. If the total charge on the plates is kept constant, then the potential difference is reduced across the capacitor plates which is not possible when battery is still connected. Hence, charge on the capacitor increases. In this way, dielectric increases the capacitance of the capacitor.

8. **The capacity of a condenser in which a dielectric of dielectric constant 5 has been used is C. If the dielectric is replaced by another with dielectric constant 20, the capacity will becomes:**

(a) $\frac{C}{4}$ (b) 4 C

(c) $\frac{C}{2}$ (d) 2 C

9. **A capacitor of capacitance 6 μF is charged upto 100 volt. The energy stored in the capacitor is:**

(a) 0.6 joule (b) 0.06 joule

(c) 0.03 joule (d) 0.3 joule

Ans. *(c) 0.03 joule*

Explanation:

$$U = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 6 \times 10^{-6} \times (100)^2$$

$$= 0.03 \text{ J.}$$

10. **An air filled parallel plate capacitor has capacity C. If distance between plates is doubled and it is immersed in a liquid then capacity becomes twice. Dielectric constant of the liquid is:**

(a) 1 (b) 2

(c) 3 (d) 4

11. **A parallel plate capacitor of capacity C_o is charged to a potential V_0. The energy stored in the capacitor when the battery is disconnected and the plate separation is doubled is E_1 and the energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled is E_2. Then value of $\frac{E_1}{E_2}$ is:**

(a) 4 (b) $\frac{3}{2}$

(c) 2 (d) $\frac{1}{2}$

Ans. *(a) 4*

Explanation:

Let $E = \frac{1}{2}C_0V_0^2 = \frac{Q_0^2}{2C_0}$

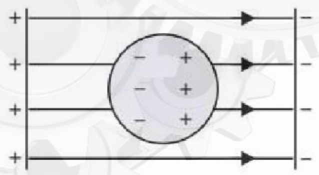
then $E_1 = \frac{Q_0^2}{2\left(\frac{C_0}{2}\right)} = 2E$

and $E_2 = \frac{1}{2}\left(\frac{C_0}{2}\right)V_0^2 = \frac{E}{2}$

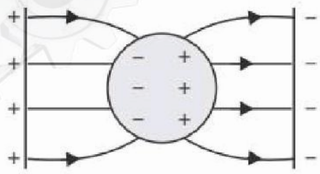
So, $\frac{E_1}{E_2} = \frac{4}{1}$.

12. **An uncharged metal ball is placed in the uniform electric field of a plane capacitor. Which one of them correctly represents the field lines on the ball?**

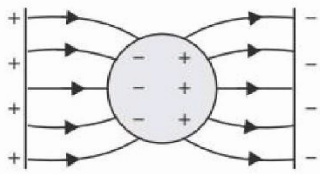
(a)



(b)



(c)



(d) None [Diksha]

13. **Three capacitors each of capacitance C and of breakdown voltage V are joined in series. The capacitance and breakdown voltage of the combination will be:**

(a) $\frac{C}{3}, \frac{V}{3}$ (b) $3C, \frac{V}{3}$

(c) $\frac{C}{3}$, 3V (d) 3C, 3V

Ans. *(c) $\frac{C}{3}$, 3V*

Explanation:

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C' = \frac{C}{3}$$

$$V' = V_1 + V_2 + V_3$$

$$= V + V + V = 3V.$$

14. **A capacitor of capacitance 50 μF is charged to 100 volts. Its energy is equal to:**

(a) 25×10^{-2} J **(b)** 225×10^{-3} J

(c) 25×10^{2} J **(d)** 25×10^{3} J

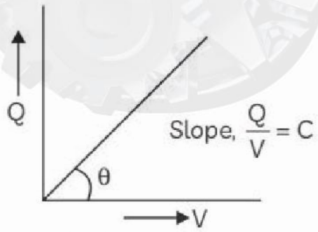
[Delhi Gov. 2022]

Ans. *(a) 25×10^{-2} J*

Explanation: Energy stored in the capacitor,

$$U = \frac{1}{2}CV^2$$
$$= 0.5 \times (50 \times 10^{-6}) \times (100)^2$$
$$= 25 \times 10^{-2} \text{J}$$

15.



Above graph shows:

(a) $Q \propto \frac{1}{V}$ **(b)** $Q \propto \frac{1}{V^2}$

(c) $Q \propto V$ **(d)** $Q \propto V^2$

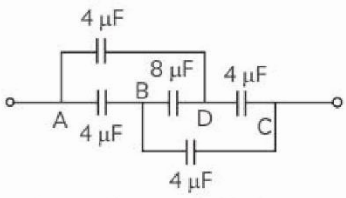
[Delhi Gov. 2022]

Ans. *(c) $Q \propto V$*

Explanation: Given, slope

$$\frac{Q}{V} = C$$
$$Q = VC$$
$$Q \propto V$$

16.



In the given figure equivalent capacitances of the given combination of five capacitors is:

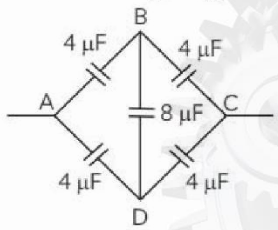
(a) 4 μF **(b)** 10 μF

(c) 8 μF **(d)** 120 μF

[Delhi Gov. 2022]

Ans. *(a) 4 μF*

Explanation: The given circuit can also be made in the following way.



This is just like the Wheatstone bridge. As the bridge is balanced $\left(\frac{4}{4} = \frac{4}{4}\right)$, there will be no effect of 8 μF capacitor. Equivalent capacitance in upper arm.

$$C' = C + C = 2\ \mu F + 2\ \mu F$$
$$= 4\ \mu F$$

17. **When a dielectric material is introduced between the plates of a charged condenser, then electric field between the plates:**

(a) decreases

(b) remains constant

(c) increases

(d) first increases then decreases

[Delhi Gov. 2022]

18. **A car battery is charged by a 12 V supply, and energy stored in it is 7.20×10^5 J. The charge passed through the battery is:**

(a) 6.0×10^4 C **(b)** 5.8×10^3 J

(c) 8.64×10^6 J **(d)** 1.6×10^5 C

[CBSE Term-1 2021]

Ans. *(a) 6.0×10^4 C*

Explanation: Given,

$$V = 12 \text{ V}$$
$$W = 7.20 \times 10^5 \text{ J}$$
$$V = \frac{W}{Q}$$
$$Q = \frac{W}{V} = \frac{7.20 \times 10^5 \text{ J}}{12 \text{ V}}$$
$$= 6.0 \times 10^4 \text{ C}$$

Assertion-Reason Questions

For Questions 19 to 22, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If both Assertion and Reason are false.

19. **Assertion (A):** If three capacitors of capacitance $C_1 < C_2 < C_3$ are connected in parallel then their equivalent capacitance $C_p > C_3$

Reason (R): $\frac{1}{C_p} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Ans. *(c) If Assertion is true but Reason is false.*

Explanation: Equivalent capacitance of parallel combination

$$C_p = C_1 + C_2 + C_3$$

20. **Assertion (A):** If the distance between parallel plates of a capacitor is halved and dielectric constant is made three times, then the capacitance becomes 6 times.

Reason (R): Capacity of the capacitor does not depend upon the nature of the material.

21. **Assertion (A):** A parallel plate capacitor is connected across battery through a key. A dielectric slab of constant K is introduced between the plates. The energy which is stored becomes *k* times.

Reason (R): The surface density of charge on the plate remains constant or unchanged.

Ans. (c) *If Assertion is true but Reason is false.*

Explanation: Given, $V = V_0$ (Constant)

Energy stored in the capacitor $= \frac{1}{2}CV^2$

$$C \rightarrow KC,$$

So energy stored will become K times

$$Q = CV$$

So Q will become K times

∴ Surface charge density,

$$\sigma = \frac{Kq}{A} = K\sigma_0.$$

22. **Assertion (A):** Energy stored in a capacitor is not equal to the work done in charging it.

Reason (R): A charged capacitor is disconnected from a battery. Now, if its plate are separated further, the potential energy will fall.

Ans. (d) *If both Assertion and Reason are false.*

Explanation: The battery is disconnected from the capacitor, so the charge Q is constant. The capacitance of a parallel plate capacitor is given by,

$$C = \frac{\varepsilon_0 A}{d}$$

The energy stored in the capacitor is equal to the net work done in charging it and it is given by,

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

$$U \propto d;$$

Hence, if its plates are separated further the potential energy will increase.

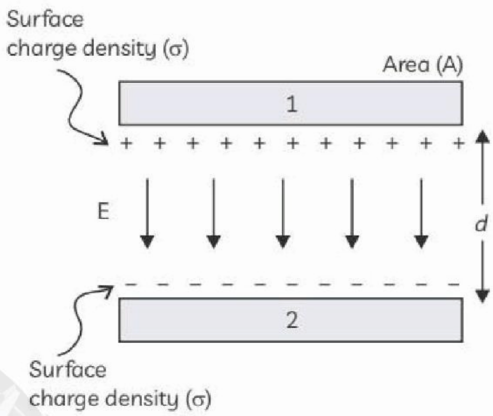
CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

23. *Today capacitor are widely used in electronic circuit for blocking dc current while allowing ac current to pass. In electric power transmission system, they stabilized voltage and power flow. The property of energy storage in capacitor was exploited as dynamic memory in early digital computers, and still in modern DRAM.*

The simplest model of capacitor consist of two thin parallel conductive plates each with an area filled with a dielectric with permittivity ε. It is assumed that the gap d is much smaller than the dimensions of the plates. Since, the separations between the plates is uniform over the plates area, the electric field between the plates E is constant and is perpendicular to the plate surface, except for an area near the edges of the plate where field decreases because the electric field lines bulge out of the sides of capacitor.



(A) If a parallel plate capacitor has *n* number of interleaved plates, area of plates is A and separation between them is *d*, then the total capacitance would be:

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{\varepsilon_0 nA}{d}$

(c) $\frac{\varepsilon_0 (n-1)A}{d}$ (d) $\frac{\varepsilon_0 n^2 A}{d}$

(B) A capacitor's dielectric material has dielectric strength U_d which sets the capacitor's breakdown voltage at $V = U_d d$. The maximum energy stored in the capacitor is:

(a) $\frac{1}{2} 2\varepsilon A d U_d$ (b) $\varepsilon A d U_d$

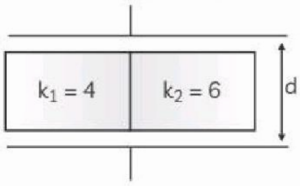
(c) $\frac{1}{2} \varepsilon A d U_d^2$ (d) $\varepsilon A d U_d^2$

(C) A capacitor is constructed from two conductive metal plates having 30 cm × 50 cm dimension which are spaced 6 mm apart from each other and use dry air as its only dielectric material, then the capacitance of the capacitor is:

(a) 0.22 nF (b) 0.221 nF

(c) 2.21 nF (d) 2.21 nF

(D) A capacitor of capacitance 1 μF is filled with two dielectric of dielectric constant 4 and 6. The new capacitance would be:



(a) 10 μF (b) 7 nF

(c) 5 μF (d) 4 nF

(E) A parallel plate capacitor is charged. If the plates are pulled apart:

(a) the charge and potential difference remain the same.

(b) the total charge increases

(c) the potential difference increases

(d) the capacitance increases

Ans. (A) *(c)* $\frac{\varepsilon_0 (n-1) A}{d}$

Explanation: For n number of plates in an interleaved capacitor, the total capacitance would be

$$C = \frac{A\varepsilon_0}{d}(n-1) = (n-1)\,C_0.$$

where, $C_0 = \frac{\varepsilon_0 A}{d}$ is the capacitance for a single plate and n is the number of interleaved plates.

(D) *(c) 5 μF*

Explanation: The arrangement is equivalent to a parallel combination of two capacitor, each with plate area $\frac{A}{2}$ and separation d,

$$C = C_1 + C_2$$

$$C = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

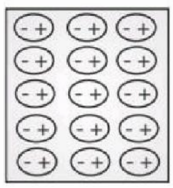
$$\left[\because C_0 = \frac{A\varepsilon_0}{d} = 1\ \mu F\right]$$

$$C = \frac{1}{2}(k_1 + k_2)$$

$$\therefore \quad C = \frac{1}{2}(4 + 6) = 5\ \mu F.$$

24. *Dielectric are non-conducting substances. In contrast to conductor, they have no charge carriers. When a conductor is placed in an external electric field, the free charge carriers move and charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. In a dielectric, this free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching or re-orienting molecules of the dielectric. The collective effect of all the moelcular dipole moments is net charges on the surface of the dielectric which produce a field that opposes the external electric field. The extent of the effect depend on the nature of dielectric.*

In an external electric field, the positive and negative charges of a non-polar molecule are displaced in opposite direction. The displacement stops when the external force on the constituent charges of the molecule is balanced by restoring force (due to internal fields in the molecule). The non-polar molecule develops an induced dipole moment, the dielectric is said to be polarised by the external field. When an external field is applied, the individual dipole moments tend to align with the field. Thus in either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and its magnitude is usually referred to as the polarisation density.



(A) Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

(B) Write the relation between relative permittivity (ε_r) and electric susceptibility (χ) of the medium.

(C) Define polar and non-polar molecules with example.

Ans. (A) On account of polarisation of dielectric an electric field is developed inside the dielectric in a direction opposite to the external electric field. As a result. The net field decreases.

Reduced field inside dielectric $E = E_0 - \frac{P}{\varepsilon_0}$

(B) $\varepsilon_r = 1 + \chi$.

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

25. The plates of a charged capacitor are connected to voltmeter. If the plates of the capacitor are separated further, what will be the effect on the reading of the voltmeter?

Ans. $V = \frac{q}{C}$ and $C = \frac{\varepsilon_0 A}{d}$

As the capacitor plates are separated, C decrease. Since, charge on the plates remains the same, value of V increases. Hence, the reading of the voltmeter will increase.

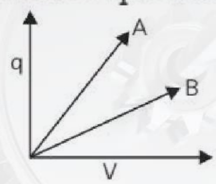
26. If a solid dielectric is placed between the plates of a capacitor, its capacitance increases. Is there any other advantage of solid dielectric?

27. Define the dielectric constant of a medium. What is its unit?

Ans. When a dielectric slab is introduced between the plates of charges, capacitor or in the region of electric field, an electric field E_p induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence net electric field inside the dielectric get reduced to $E_0 - E_p$, where E_0 is external electric field. The ratio of applied external electric field and reduced electric field is known as dielectric constant K of dielectric medium *i.e.*, $K = \frac{E_0}{E_0 - E_p}$

and it is a dimensionless quantity.

28. The given graph shows the variation of charge *q* versus potential difference V for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that C_1. Which line (A or B) corresponds to C_1 and why?



Ans. Line B corresponds to C_1 because slope $\left(\frac{q}{V}\right)$ of B is less than slope of A.

29. For an isolated parallel plate capacitor of capacitance C and potential difference V, what will be the change in (A) charge on the plates (B) potential difference across the plates (C) electric field between the plates (D) energy stored in the capacitor, when the distance between the plates is increased? [Delhi Gov. QB 2022]

Ans. (A) No change (B) increase (C) No change (D) increases.

30. In a parallel plate capacitor of capacitance C, a metal sheet is inserted between the plates, parallel to them. If the thickness of the sheet is half the separation between the plates. What will be its capacitance?

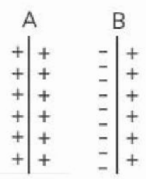
31. Capacitors are basic units that store charge and used in different fields of electronics. The arrangement alters the ability of storing the charge in capacitors. They are used in motherboards and help in reading different values. When the capacitor is charged, one value is read and when discharged, another value is read. What is the basic use of a capacitor?

Ans. Basic use of the capacitor is to store charge and energy.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

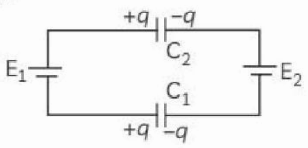
32. Show that capacitance of a metal plate A can be increased by bringing another metal plate B near A.



Ans. The charge holding property of a conductor is called its capacitance. Figure shows an insulated metal plate A. Let positive charge be given to it till its potential becomes maximum. No further charge can be given to the plates as it would leak out. Now bring another insulated metal plate B near the plate A shown in figure. The plate A will induce negative charge on the inner face of B and equal positive charge tends to increase the potential of plate A while induced negative charge tends to decrease the potential of plate A. Since, the induced negative charge is nearer to plate A than the induced positive charge, the net effect is that the potential of plate A decreases. Therefore, more charge can be given to plate A to raise its potential to maximum value. Thus, the capacitance of conductor A is increased by bringing another uncharged conductor B near it.

33. **A capacitor has some dielectric between its plates, and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.** **[NCERT Exemplar]**

34. **Determine the potential difference across the plates of the capacitor C_1 and C_2 of the network shown in the figure [Assume $E_2 > E_1$].**



[CBSE 2012]

Ans. $$\frac{-q}{C_1} - E_1 - \frac{q}{C_2} + E_2 = 0$$

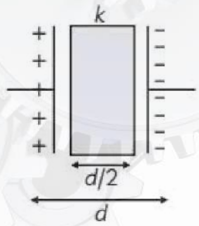
$$\therefore \quad \frac{q}{C_1} + \frac{q}{C_2} = E_2 - E_1$$

Now, $$V_1 = \frac{q}{C_1}$$

and $$V_2 = \frac{q}{C_2}.$$

35. **A slab of material of dielectric constant k has the same area as that of the plates of a parallel plate capacitor but has the thickness $\frac{d}{2}$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.** **[CBSE 2013]**

Ans.



Capacitance of a capacitor partially filled with dielectric.

$$C = \frac{\varepsilon_0 A}{d - t + t} = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}}$$

$$\therefore \quad C = \frac{2\varepsilon_0 Ak}{d(k+1)}$$

36. **Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel?**
Find the ratio of energy stored in the two configurations if they are connected to the same source. **[CBSE 2011]**

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

37. **A parallel plate capacitor of capacitance C is charged to a potential V by a battery without disconnecting battery, a dielectric medium of K = 10 is introduced between the plates of the capacitor. Explain giving reasons, how will the following be affected.**

(A) capacitance of the capacitor

(B) charge on the capacitor

(C) energy density of the capacitor?

[CBSE 2017]

Ans. On introducing the dielectric slab to fill the gap between plates of capacitor completely when capacitor is connected with battery.

(A) The capacitance of capacitor becomes K times of original capacitor.

$$C' = KC = 10\ C$$

(B) The potential difference V between capacitor is same due to connectivity with battery and hence, charge q' becomes K times of original charge as:

$$q' = C'V' = (KC)(V) = K(CV)$$

$$q' = Kq = 10\ CV$$

(C) When a dielectric slab is inserted between the plates of a charged capacitor, and battery is disconnected. Then, the charge on the plates remains unchanged and energy stored in the capacitor becomes $\frac{U_0}{K}$, *i.e.*, energy decreases.

38. (A) **How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening regions?**

(B) **A slab of material of dielectric constant *k* has the same area as the plates of a parallel plate capacitor but has thickness $\frac{1}{2}d$, where *d* is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates.**

Ans. (A) In a dielectric under the effect of an external field, a net dipole moment is induced in the dielectric. Due to molecular dipole moments, a net charge appears on the surface of the dielectric.

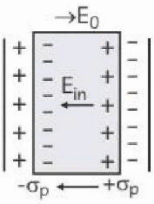
These induced charges (of densities $-\sigma_p$ and $+\sigma_p$) produce a field opposing the external field. Induced field is lesser in magnitude than the external field.

So, field inside the dielectric gets reduced.

$$E = |E_0| - |E_{in}|$$

where, E = resultant electric field in the dielectric,

E_0 = external electric field between two plates and E_{in} = electric field inside the dielectric.



(B) The thickness of dielectric slab is $\frac{d}{2}$, *i.e.*,

$$t = \frac{d}{2}$$

The capacitance of a capacitor due to dielectric slab is

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

$$C = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}}$$

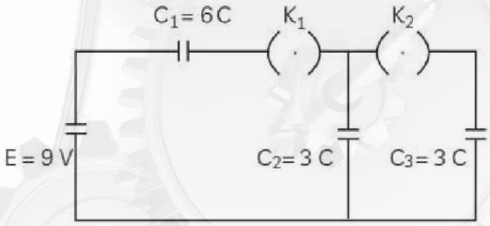
$$C = \frac{2\varepsilon_0 A}{d\left(1 + \frac{1}{K}\right)}$$

39. **Three capacitors of capacitance 2 pF, 3 pF and 4 pF are connected in parallel.**

(A) What is the total capacitance of the combination?

(B) Determine the charge on each capacitor if the combination is connected to a 100 V supply. [Delhi Gov. SQP 2022]

40. In the circuit shown in figure initially K_1 is closed and K_2 is open. What are the charges on each capacitors. Then K_1 was opened and K_2 was closed (order is important), What will be the charge on each capacitor now? [C = 1 μF]



[NCERT Exemplar]

Ans. When initially K_1 is closed and K_2 is open then capacitor C_1 and C_2 are connected in series with battery and have equal charge on capacitors C_1 and C_2 are,

$$Q_1 = Q_2 = q = \left(\frac{C_1 C_2}{C_1 + C_2}\right)E$$

$$= \left(\frac{6C \times 3C}{6C + 3C}\right) \times 9 = 18\ \mu C$$

$$Q_1 = Q_2 = 18\ \mu C \text{ and}$$

$$Q_3 = 0$$

Now, when K_1 is opened and K_2 is closed, the battery and capacitor C_1, are disconnected from the circuit. The charge in capacitor C, will remain constant equal to $Q_1 = Q_2 = 18\ \mu C$.

The charged capacitor C_2 now connects in parallel with uncharged capacitor C_3, consider common potential of parallel combination at V'.

Then, $C_2V' + C_3V' = Q_2$

$$\Rightarrow \quad V' = \frac{Q_2}{C_2 + C_3} = \frac{18}{3C + 3C} = 3\ V$$

Hence, $Q_2' = 3CV' = 9\ \mu C$

$$Q_3' = 3CV' = 9\ \mu C$$

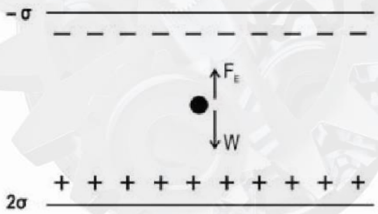
$$Q_1' = 18\ \mu C$$

41. **Two charged sheets having charge density 2σ and -σ are placed parallel and close to each other in a vertical plane as shown in the figure. A particle having positive charge *q* and mass *m* is placed between these sheets and released from rest under gravity. What is the acceleration of this particle?**

[CBSE Competency QB 2023]

Ans. Acceleration of the particle (a) = $\frac{F_{net}}{m}$

F_{net} = Electric force (F_E) + Gravitational force (W)



Here,

$$F_E = qE$$

Where, E = Electric field

Magnitude of electric field due to a charged sheet is given by $\frac{\sigma}{2\varepsilon_0}$,

where σ is its surface charge density.

Thus,

E = Electric field between the sheets

$$= \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{3\sigma}{2\varepsilon_0}$$

(As fields due to both the sheets are in the same direction so they add up)

Thus, $F_E = qE = \frac{3\sigma q}{2\varepsilon_0}$

and W = mg

Now, as both the forces are in opposite direction, thus

$$F_{net} = \frac{3\sigma q}{2\varepsilon_0} - mg$$

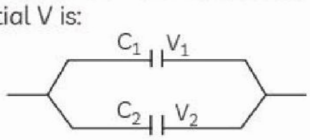
and acceleration $a = \frac{F_{net}}{m} = \frac{3q\sigma}{2m\varepsilon_0} - g$.

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

42. Obtain an expression for the common potential and loss in energy in the combining of the capacitors.

Ans. Common potential: When two capacitor charged to the different potential are connected, the redistribution of charge takes place and both the capacitors acquire same potential difference. Let two capacitors C_1 and C_2 charged to the potentials V_1 and V_2 respectively are connected, then the common potential V is:



$$V = \frac{\text{Total charge}}{\text{Total capacitance}}$$

$$V = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

Loss in energy in combining the capacitors: Let two capacitors of capacitance C_1 and C_2 charged to the potential, V_1 and V_2 respectively are connected. Due to the redistribution of charge the both capacitors acquire common potential,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

∴ The initial energy of the system,

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

And final energy of the system,

$$U_f = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$$

$$= \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

Thus, loss in energy is:

$$\Delta U = U_i - U_f$$

$$= \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2 - \frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}\right]$$

$$= \frac{1}{2}\left[[C_1V_1^2 + C_2V_2^2 - \frac{(C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2)}{(C_1 + C_2)}\right]$$

$$= \frac{1}{2}\left[\frac{C_1C_2V_1^2 + C_1C_2V_2^2 - 2C_1C_2V_1V_2}{(C_1 + C_2)}\right]$$

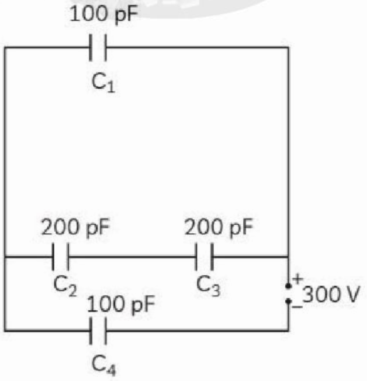
$$= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}\left(V_1^2+V_2^2-2V_1V_2\right)$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

$\therefore$ $(V_1 - V_2)^2$ is always positive, it means there is always loss in energy ($U_i > U_f$) and this loss in energy dissipates in the form of heat in connecting wires.

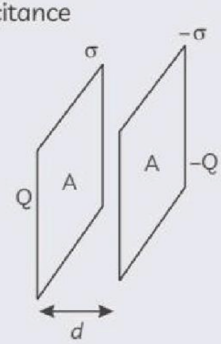
43. **(A) Derive an expression for the capacitance of a parallel plate capacitor with air present between the two plates.**

(B) Obtain the equivalent capacitance of the network shown in figure. For a 300 V supply, determine the charge on each capacitor.



[CBSE SQP 2023]

Ans. (A) Derivation of the expression for the capacitance



Let the two plates be kept parallel to each other separated by a distance d and cross-sectional area of each plate is A. Electric field by a single thin plate

$$E = \frac{\sigma}{2\varepsilon_0}$$

Total electric field between the plates

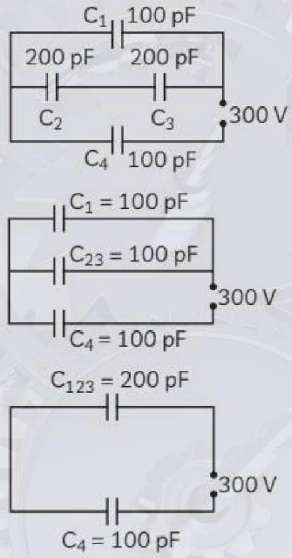
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Potential difference between the plates

$$V = Ed = \left[\frac{Q}{A\varepsilon_0}\right]d$$

$$\text{Capacitance, } C = \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

(B)



The equivalent capacitance $= \frac{200}{3}$ pF

$$\text{charge on } C_4 = \frac{200}{3} \times 10^{-12} \times 300$$

$$= 2 \times 10^{-8}\ C,$$

Potential difference across.

$$C_4 = \frac{200 \times 10^{-12} \times 300}{3 \times 100 \times 10^{-12}}$$

$$= 200\ V$$

Potential difference across C_1

$$= 300 - 200$$

$$= 100\ V$$

$$\text{charge on } C_1 = 100 \times 10^{-12} \times 100$$

$$= 1 \times 10^{-8}\ C$$

Potential difference across C_2 and C_3 series combination = 100 V

potential difference across C_2 and C_3 each

$$= 50\ V$$

charge on C_2 and C_3 each

$$= 200 \times 10^{-12} \times 50$$

$$= 1 \times 10^{-8}\ C$$

[CBSE Marking Scheme SQP 2023]

44. **(A) A camera usually operates at 1.5 V and this potential difference is not sufficient to emit light energy using flash. For this purpose, the flash circuit of the camera has a capacitor that is charged to 300 V-330 V using various electrical components. If the voltage generated across the plates of the capacitor is 300 V and the capacitance of the parallel plate capacitor used is 100 μF, then find the energy released when the trigger button on the camera is pressed.**

(B) How much charge does the 100 μF capacitor charged to 300 V hold?

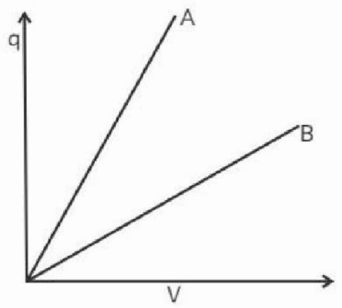
(C) If the distance between the parallel plate capacitor of capacitance 100 μF is increased two times, then calculate the capacitance of the capacitor.

(D) The given graph shows the variation of charge 'q' with potential difference 'V' for a parallel plate capacitor 'C' for scenarios P and Q.

Scenario P - the space between the capacitor 'C' is filled with air.

Scenario Q - the space between the capacitor 'C' is filled with a substance of dielectric constant K.

Which of the two lines A or B corresponds to scenario Q? Give a reason for your answer.



[CBSE Practice Set-1 2023]

Ans. (A) $V = 300\ V$

$C = 100\ \mu F$

$$\text{Energy} = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 100 \times 10^{-6} (300)^2$$

$$= 4.5\ J$$

(B) $q = CV$

$$q = 100 \times 10^{-6} \times 300$$

$$= 0.03\ C$$

(C) Capacitance of a parallel plate capacitor C

$$= \frac{\varepsilon_0 A}{d}$$

$C = 100\ \mu F$

$d' = 2d$

$$C' = \frac{\varepsilon_0 A}{d'}$$

$$C' = \frac{\varepsilon_0 A}{2a} = \frac{100}{2} = 50\ \mu F$$

Hence, if the distance between the plates of the capacitor is increased two times the capacitance of the capacitor decreases by $\frac{1}{2}$ *i.e.*, becomes 50 μF.

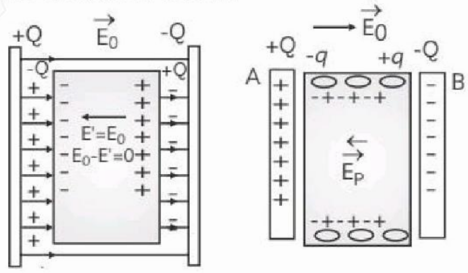
(D) The slope of the q vs V graph gives the capacitance of a parallel plate capacitor.

When the space between the plates of a capacitor is filled with a substance of dielectric constant K, its capacitance increases K times. Greater the slope of the q vs V graph, the higher the capacitance. As line A has a greater slope it represents greater capacitance and corresponds to scenario Q.

[CBSE Marking Scheme Practice Set-1 2023]

45. Explain, using suitable diagrams, the difference in the behaviour of a (A) conductor and (B) dielectric in the presence of external electric field. Define the terms polarisation of a dielectric and write its relation with susceptibility.

Ans. (A) When a conductor is placed in a uniform electric field $\vec{E}_0$, electrons present inside conductor drift in a direction opposite to the external electrical field, till the electric field $\vec{E}'$, produced by drift of free electron with in the conductor just becomes equal and opposite to the external field $\vec{E}_0$. Thus, the net electric field $\vec{E}_0 - \vec{E}'$ at any point inside the conductor is zero.



(B) When a dielectric slab is placed in the electric field $\vec{E}_0$, the electrons in atoms or molecules of dielectric get pulled in a direction opposite to the applied electric field. The separation between the charges is such that the force due to the external electric field is just balanced by the restoring force due to internal electric fields in the atom or molecule. The molecules, thus, develop an induced dipole moment and the dielectric is said to be polarised. Let E, be the internal electric field in the dielectric due to polarisation of charge. Thus, net electric field in the presence of dielectric becomes,

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

It is observed that,

$$\vec{E} = \vec{E} - \vec{E}_p = \frac{\vec{E}_0}{K},$$

where, K is the dielectric constant of the given dielectric.

The polarisation vector $\vec{p}$ of a dielectric is defined as the dipole moment developed in the dielectric per unit volume when placed in an external electric field.

Thus, polarisation vector,

$$\vec{p} = \frac{\text{Net dipole moment}}{\text{Volume of dielectric}}$$

For linear isotropic dielectrics, the polarisation vector $\vec{p}$ is found to be directly proportional to external electric field $\vec{E_0}$ *i.e.*,

$$\vec{p} \propto \vec{E_0} \text{ or } \vec{p} = \chi_e \vec{E_0}$$

Hence, χ_e is a constant which is characteristic of a dielectric and is known as the 'electric susceptibility' of given dielectric.

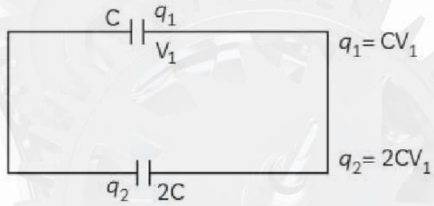
46. A parallel plate capacitor (A) of capacitance C is charged by a battery to voltage V. The battery is disconnected and an uncharged capacitor (B) of capacitance 2C is connected across A. Find the ratio of:

(A) final charges on A and B.

(B) total electrostatic energy stored in A and B finally and that stored in A initially.

[CBSE 2023]

Ans.



$$q_1 + q_2 = Q$$

$$q_1 + q_2 = CV$$

$$CV_1 + 2CV_1 = CV$$

$$V_1 = \frac{V}{3}$$

$$q_1 = CV_1 = C\frac{V}{3} = \frac{Q}{3}$$

$$q_2 = 2CV_1$$

$$= 2C\frac{V}{3} = \frac{2Q}{3}$$

$$\frac{q_1}{q_2} = \frac{1}{2}$$

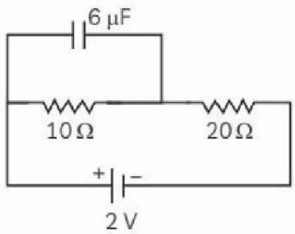
$$U_1 = \frac{1}{2}CV_1^2 = \frac{1}{18}CV^2$$

$$U_2 = \frac{1}{9}CV^2$$

$$\frac{U_1}{U_2} = \frac{1}{2}$$

NUMERICAL Type Questions

47. Find the charge on the capacitor as shown in the circuit.



(2m)

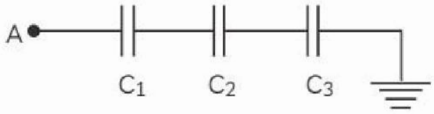
Ans. Total resistance, R = 10 Ω + 20 Ω = 30 Ω

The current, $I = \frac{V}{R} = \frac{2\text{ V}}{30\ \Omega} = \frac{1}{15}$ A

Potential difference, $V = IR = \frac{1}{15} \times 10 = \frac{2}{3}$ V

Charge, $q = CV = 6 \times \frac{2}{3} = 4$ μC

48. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given, potential at A is 90 V, C_1 = 20 μF, C_2 = 30 μF, C_3 = 15 μF.



(3m)

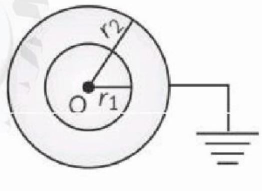
49. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(A) Determine the capacitance of the capacitor.

(B) What is the potential of the inner sphere?

(C) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain, why the later is much smaller.

Ans. From question



Radius of inner sphere, $r_1 = 12$ cm

Radius of outer sphere, $r_2 = 13$ cm

and charge on inner sphere, $q = 2.5\ \mu C$

The dielectric constant, $k = 32$

(A) Capacitance of a spherical capacitor,

$$C = \frac{4\pi\varepsilon_0 k r_1 r_2}{r_1 - r_2}$$

$$= \frac{1}{9\times10^9}\ \frac{32\times12\times13\times10^{-4}}{(13-12)\times10^{-2}}$$

$$= 5.5 \times 10^{-9}\ F$$

(B) Electric potential of inner sphere,

$$V = 4.5 \times 10^2\ V$$

(C) Capacitance of an isolated sphere of radius

$$r = 12\ cm$$

$$C = 4\pi\varepsilon_0 r = \frac{1}{9\times10^9}\times12\times10^{-2}$$

$$= 1.33 \times 10^{-11}\ F$$

The capacitance of an isolated sphere is much smaller as compared to the inner spherical capacitor because the outer sphere is earthed. The potential difference decreases and hence the capacitance increases.

50. **Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, then determine the value of C_1 and C_2. Also, calculate the charge on each capacitor in parallel combination. (3m)**

51. **(A) Draw equipotential surfaces for (i) an electric dipole and (ii) two identical positive charges places near each other.**

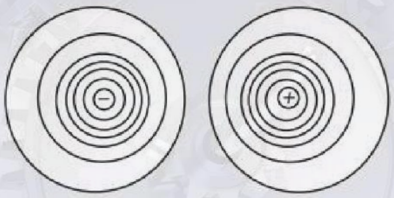
(B) In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

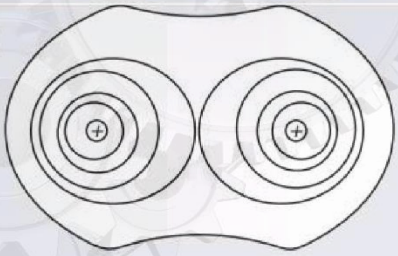
(ii) If the capacitor is connected to 100 V supply, what would be the charge on each plate?

(iii) How would charge on the plate be affected if a 3 mm thick mica sheet of K = 6 is inserted between the plates while the voltage supply remains connected? [CBSE SQP 2022](5m)

Ans. (A) (i)



(ii)



(B) Here, $A = 6\times10^{-3}\ m^2$, $d = 3$ mm $= 3\times10^{-3}$ m

(i) Capacitance, $C = \varepsilon_0 \dfrac{A}{d}$

$$= \left(\frac{8.85\times10^{-12}\times 6\times10^{-3}}{3\times10^{-3}}\right)$$

$$= 17.7\times10^{-12} F$$

(ii) Charge, $Q = CV = 17.7\times10^{-12}\times100$

$$= 17.7\times10^{-10} C$$

(iii) New charge, $Q' = KQ = 6\times17.7\times10^{-10}$

$$= 1.062\times10^{-8} C$$

[CBSE Marking Scheme SQP 2022]

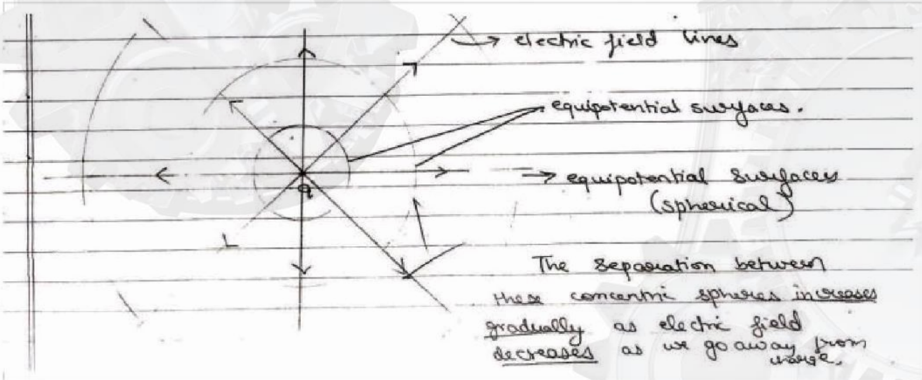
TOPPER'S CORNER

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

1. Draw the equipotential surfaces due to an isolated point charge.

Ans.



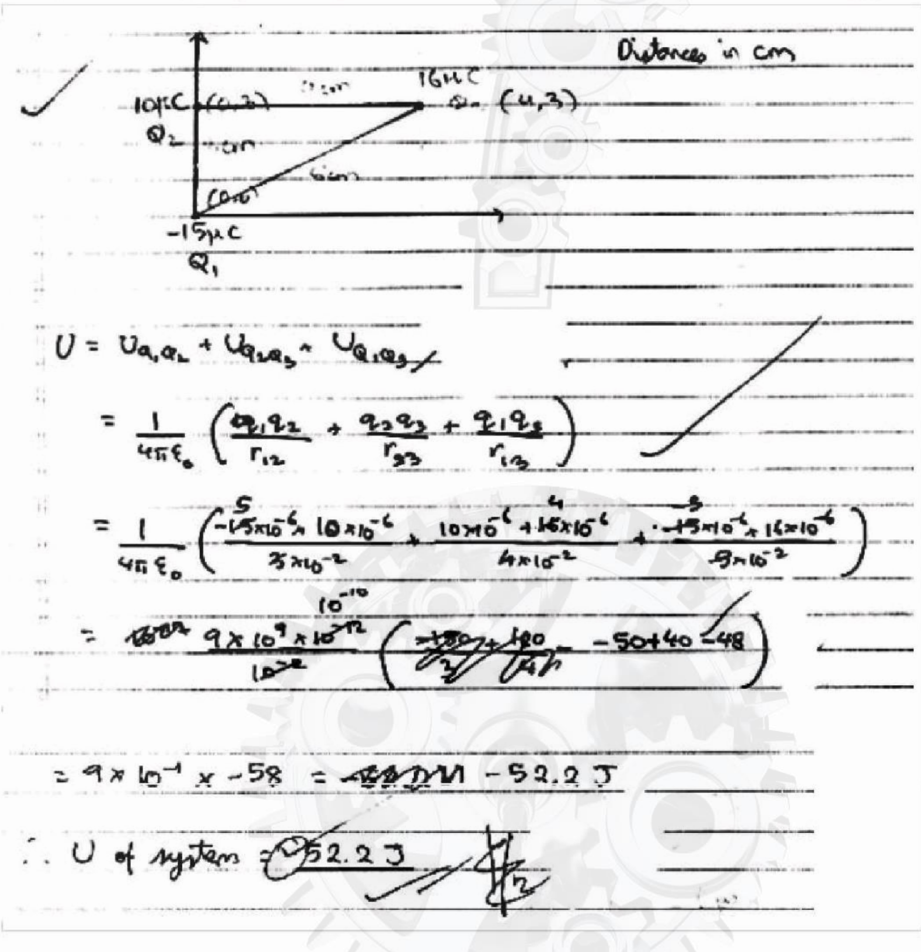
[CBSE Topper 2019]

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

2. Three point charges $Q_1(-15\ \mu C)$, Q_2 (10 μC) and Q_3(16 μC) are locate at (0 cm, 0 cm), (0 cm, 3 cm) and (4 cm, 3 cm) respectively. Calculate the electrostatic potential energy of this system of charges.

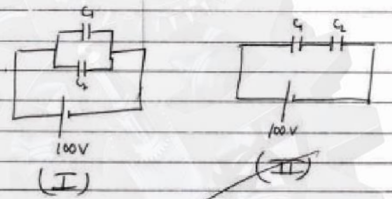
Ans.



[CBSE Topper 2023]

3. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination.

Ans.



In (I) $C_{eq} = C_1 + C_2$

$E = \frac{1}{2} C_{eq} V^2$

$E = \frac{1}{2}(C_1 + C_2) V^2$

$0.25 = \frac{1}{2}(C_1 + C_2) \times 100 \times 100$

$C_1 + C_2 = 0.5 \times 10^{-4}$ —— (1)

In (II), $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

$E = \frac{1}{2} C_{eq} V^2$

$0.045 = \frac{1}{2} \times \frac{C_1 C_2}{C_1 + C_2} \times 100 \times 100$

$0.09 \times 10^{-4} = \frac{C_1 C_2}{0.5 \times 10^{-4}}$

$C_1 C_2 = 0.045 \times 10^{-8}$

Solving we get $C_1 = \frac{2.5 \times 10^{-5}}{2} F$, $C_2 = 3.6 \times 10^{-5} F$ ($1.25 \times 10^{-5} F$, $3.6 \times 10^{-5} F$)

in parallel $q_{net} = (C_1 + C_2) V = 5 \times 10^{-5} \times 100 = 5 \times 10^{-3} C$

$q_1 = \frac{C_1}{C_1 + C_2} q_{net}$

$q_2 = \frac{C_2}{C_1 + C_2} q_{net}$

$q_2 = \frac{C_1}{5 \times 10^{-5}} \times 5 \times 10^{-3} = C_1 \times 100 = \frac{2.5}{2} \times 10^{-5} \times 100 = \frac{2.5 \times 10^{-3}}{2} C$

$q_1 = C_2 \times 100 = 3.6 \times 10^{-5} \times 100 = 3.6 \times 10^{-3} = 3.6 \text{ mC}$

$q_2 = 1.25 \text{ mC}$ → Charge on C_2

$q_1 = 3.6 \text{ mC}$ → Charge on C_1

[CBSE Topper 2015]